Integers

- All positive and negative numbers including zero are called **integers**. It is usually denoted by **I** or **Z**.

  I or **Z** = { ... –3, –2, –1, 0, 1, 2, 3 …}

  Here, –1, –2, –3 … are called negative integers whereas 1, 2, 3 … are called positive integers and 0 is taken as neutral.

- The **absolute value** of an integer is its numerical value regardless of its sign. The absolute value of an integer n is denoted as $|n|$.

  Therefore, $|-10| = 10$, $|-2| = 2$, $|0| = 0$, $|7| = 7$ etc.

- The **opposite of an integer** is the integer with its sign reversed. The opposite of integer a is $-a$ and the opposite of integer $-b$ is $+b$ or b.

  Thus, opposite of 5 is −5, opposite of −8 is 8.

- **Multiplication of integers**

  Rules for the product of integers:

  (i)The product of two positive integers is always positive.

  (ii) The product of one positive integer and one negative integer is always negative.

  For example, $5 \times (-9) = -(5 \times 9) = -45$

  (iii) The product of two negative integers is always positive.
  (iv) If the number of negative integers in a product is even, then the product is a positive integer. If the number of negative integers in a product is odd, then the product is a negative integer.

  For example, $(-1) \times (-2) \times (-3) = -6$, $(-7) \times (-2) = 14$ etc.

- Integers are commutative under multiplication.

  For example, $(-2) \times (5) = 5 \times (-2) = -10$

- The product of an integer and zero is zero.

  $(-2) \times 0 = 0$
  $7 \times 0 = 0$

- When an integer, say a, is multiplied by 1, it gives the same integer.

  $1 \times a = a \times 1 = a$
  Therefore, 1 is the multiplicative identity for integers.

- Integers are associative under multiplication. For integers a, b and c,

  $a \times (b \times c) = (a \times b) \times c$
  For instance, $(-25) \times [4 \times 39] = [(-25) \times 4] \times 39 = (-100) \times 39 = -3900$

- Multiplication is distributive over addition and subtraction for integers.

  For integers, a, b, and c,

  $a \times (b + c) = a \times b + a \times c$
  $a \times (b - c) = a \times b - a \times c$

- **Division of integers**
  - To divide a positive integer by a negative integer or a negative integer by a positive integer, the division is carried out as in whole numbers and then a negative sign (–) is put before the quotient.

For example, $(12) \div (-4) = (-12) \div 4 = -3$

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  - When a negative integer is divided by another negative integer or a positive integer is divided by another positive integer, a positive quotient is obtained.

For example, $(-6) \div (-3) = 6 \div 3 = 2$

Properties of Division

Property 1: If a and b are two integers, then $a \div b$ might not be an integer.

Property 2: If a is an integer and $a \neq 0$, then $a \div a = 1$.

Property 3: If a is an integer and $a \neq 0$, then $a \div 1 = a$

Property 4: If a is an integer and $a \neq 0$, then $0 \div a = 0$

Property 5: If a is a non-zero integer, then $a \div 0$ is not defined.

Property 6: If a, b and c are non-zero integers, then
$(a \div b) \div c \neq a \div (b \div c)$ except when $c = 1$
Note: when $c = 1$, $(a \div b) \div c = a \div (b \div c)$

Property 7: If a, b and c are integers, such that
(i) $a > b$ and c is positive, then $(a \div c) > (b \div c)$

- **Order of performing operations:**

  The correct order of performing the operations in a given expression is: **Division, Multiplication, Addition, Subtraction (DMAS).** If any of the operations is not present in an expression, then we skip it and move to the next operation.

  - In case of expressions having one operation more than once**, we move from left to right to perform the repeated operations.**

- **Use and order of brackets:**

  There are mainly three types of brackets namely **square bracket [ ], curly bracket { }** and **simple bracket ( ).** These are used to separate the terms as well as to define that which operation should be performed first in an expression.

  - In expressions having more than one bracket, simple bracket is solved first, then curly bracket is solved and finally the square bracket is solved.

For example, $[25 + \{(100 \div 4 + 12) - 7\} \times (2 + 3)] - 10$ can be solved as follows:

$[25 + \{(100 \div 4 + 12) - 7\} \times (2 + 3)] - 10$

$= [25 + \{(25 + 12) - 7\} \times (2 + 3)] - 10$ (By solving division in the innermost simple bracket)

$= [25 + \{37 - 7\} \times 5] - 10$ (By addition in both the simple brackets)

$= [25 + 30 \times 5] - 10$ (By solving curly bracket)

$= [25 + 150] - 10$ (By solving multiplication in square bracket)

$= 175 - 10$ (By solving square bracket)

$= 165$ (By solving subtraction)