

★ Counter:

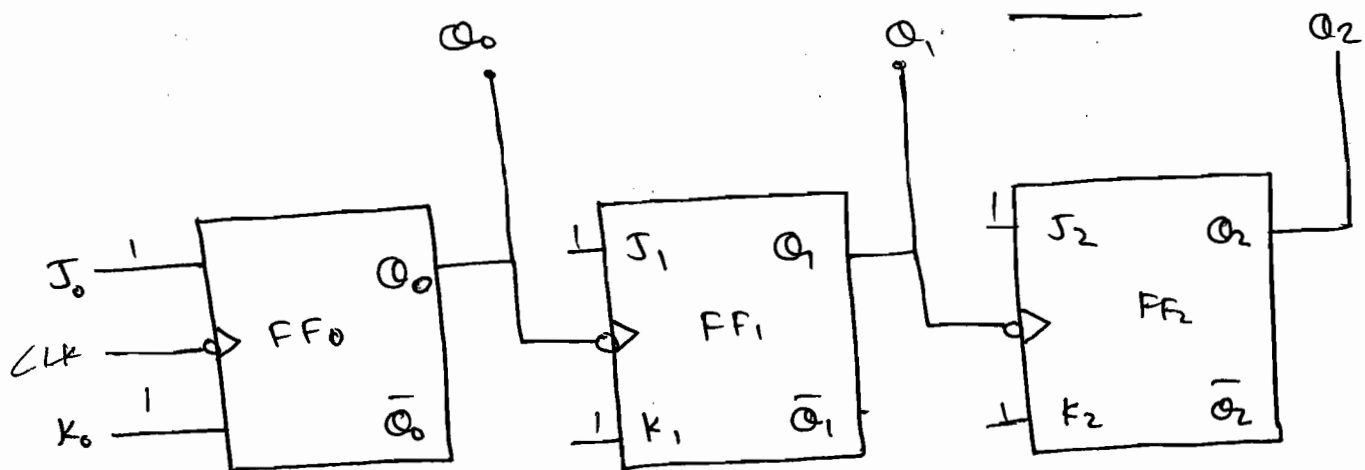
① Asynchronous (Ripple) Counter:

→ Any FF, but in toggle mode:

② Synchronous (parallel) Counters:

→ Any FF

① 3-Bit Asynchronous Counter: (UP Counter) (Ripple counter)



→ As $J=K=1 \Rightarrow$ All FFs are in toggle mode.

$$\overline{Q(t+1)} = Q(t).$$

→ Q_0 is toggle for every clock pulse.

→ Q_1 is toggle when Q_0 changes from '1 to 0'

→ Q_2 is toggle when Q_1 changes from '1 to 0'.

CLK	Q ₀ (LSB)	Q ₁	Q ₂ (MSB)	
0	0	0	0	0
1	1	0	0	1
2	0	1	0	2
3	1	1	0	3
4	0	0	1	4
5	1	0	1	5
6	0	1	1	6
7	1	1	1	7
8	0	0	0	8

} Up Counter

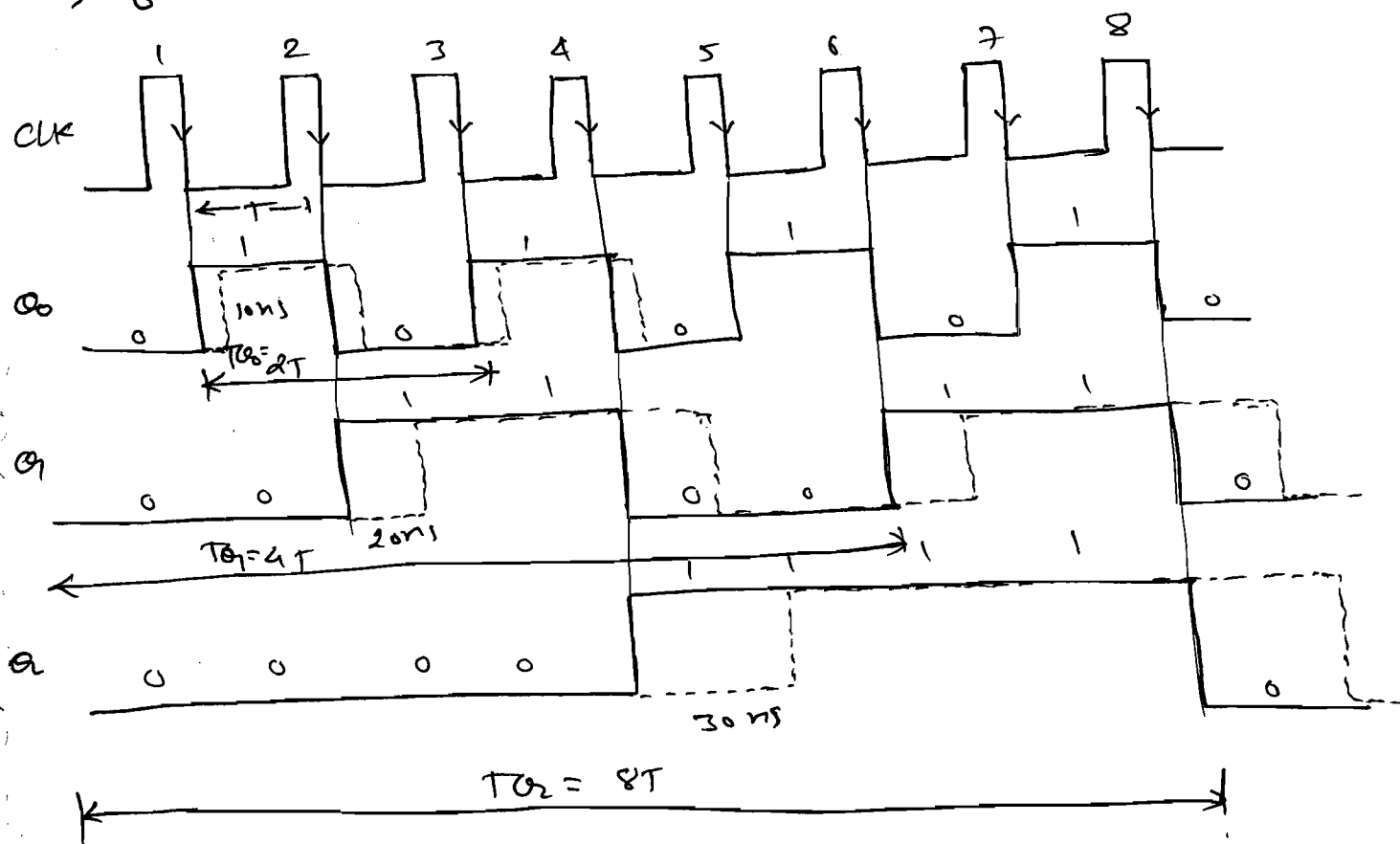
$t_{\text{prop}} = 10\text{ns}$

Q₀ is delay by 10ns

Q₁ is delay by 20ns

Q₂ is delay by 30ns

→ freq. is divided by 2 after each flip flop.



* Max. Conv. time = $3 \times 10\text{ns} = 30\text{ns}$.

Clock period $T \geq 30\text{ns}$.

$$\therefore f = \frac{1}{T} \leq \frac{1}{30 \times 10^{-9}}$$

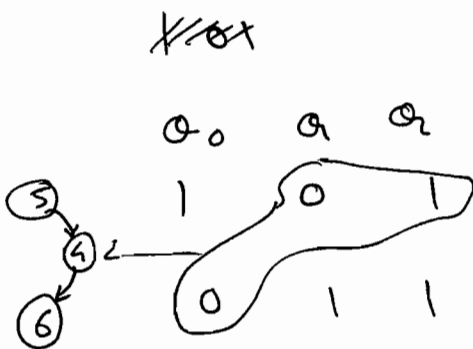
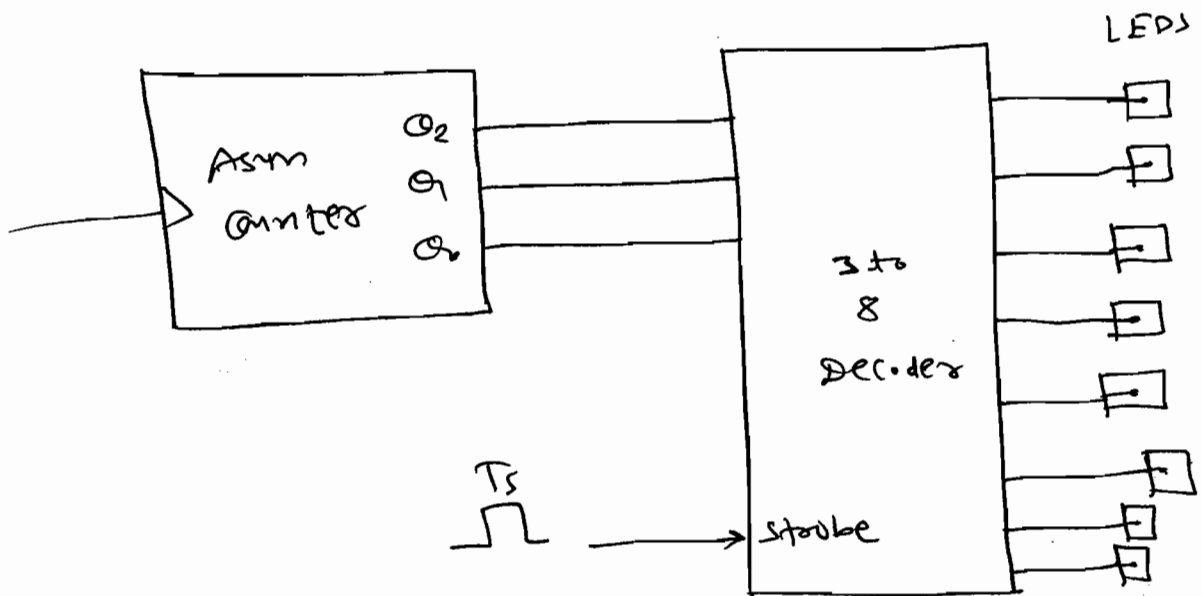
i.e. $f \leq \frac{1}{30 \times 10^{-9}}$

In general,

N-Bit Asynchronous Counter,

$$f \leq \frac{1}{N \cdot t_{pd,PF}}$$

→ Freq. of Async. Counter using strobe pulse.



$$f \leq \frac{1}{N \cdot t_{pd,PF} + T_S}$$

→ Asynchronous Counter uses Strobe
Where as Strobe are not required in Synchronous.

* 3-bit

asynchronous

Counter.

(Down

Counter).

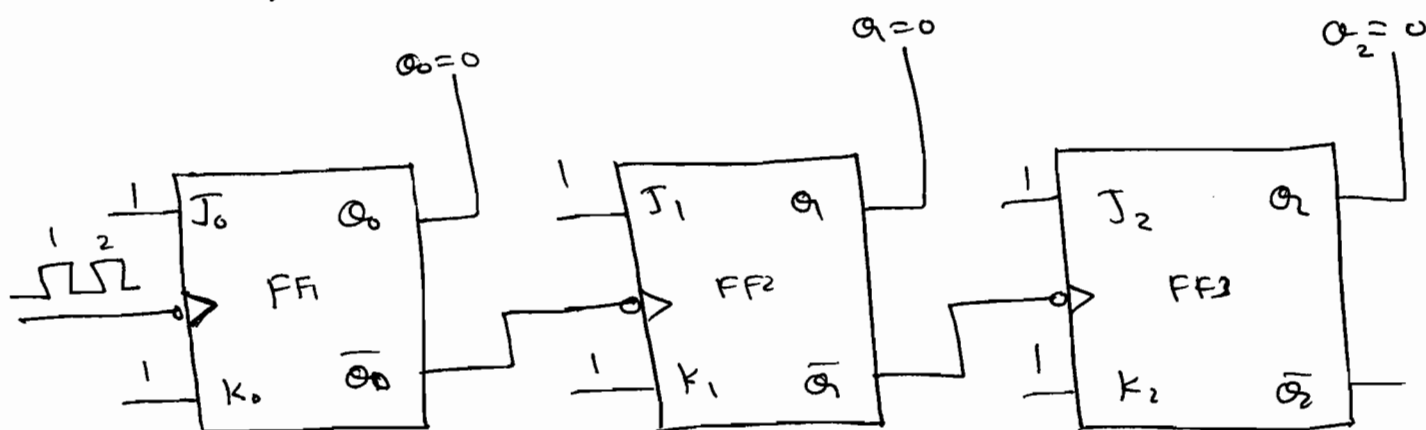
140

→ In the following circuits,

(1) Q_0 (toggles) Changes for each clock pulse.

(2) Q_1 toggles when Q_0 changes from 0 to 1.

(3) Q_2 toggles when Q_1 changes from 0 to 1.



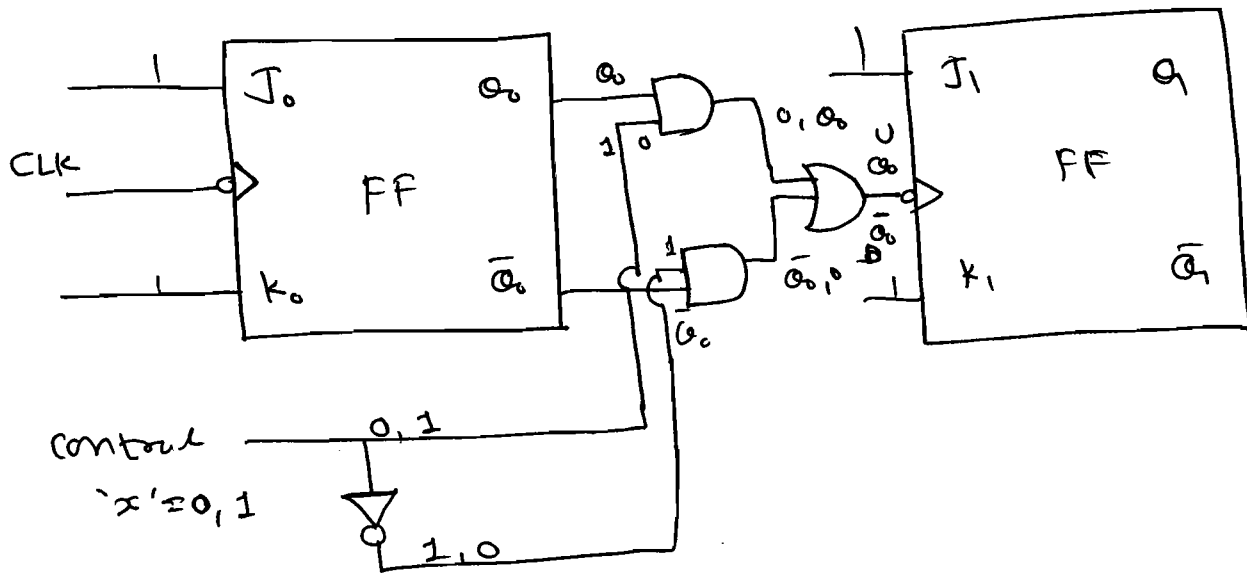
Note: All FFs are in toggle mode.

CLK	(LSB) Q_0	Q_1	(MSB) Q_2	Decimal
0	0	0	0	0
1	↓	↓	↓	7
2	0 ^x	1	1	6
3	↓	0 ^x	1	5
4	0 ^x	0	1	4
5	↓	↓	0	3
6	0 ^x	1	0	2
7	↓	0	0	1
8	0 ^x	0	0	0

→ Down Counter.

* 2-Bit "Asynchronous Up-Down" Counter.

147



→ If $x=0 \Rightarrow \underline{\bar{Q}_0}$ is connected to the clock of next FF.

→ If $x=1 \Rightarrow \underline{Q_0}$ is connected to the clock of next FF.
i.e. 00, 11, 10, 01, 00, ...

→ If $x=1 \Rightarrow \underline{Q_0}$ is connected to the clock of next FF.

$\Rightarrow$ Up Counter
i.e. 00, 01, 10, 11, 00, ...

Imp
NOTE: In the above Up-Down Counter if the edge triggered FF circuit then

- ① $x=0 \rightarrow$ It act as Up counter.
- ② $x=1 \rightarrow$ It act as Down counter.

Hint *

Up Counter

triggering

Q

neg. $\rightarrow$ Positive (or)

pos. $\rightarrow$ negative.

*

Down Counter

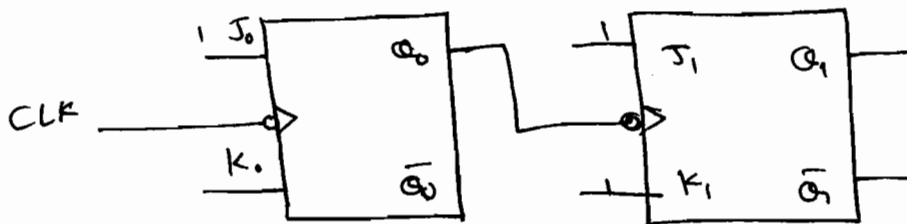
triggering

Q

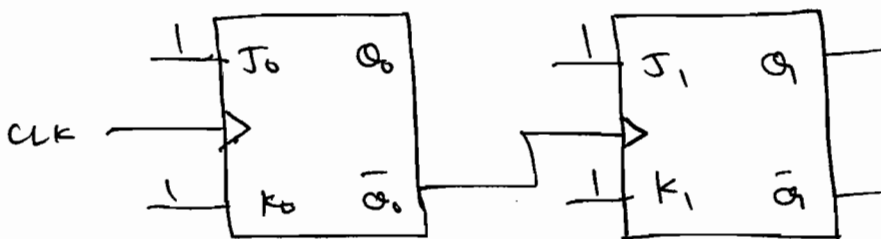
neg. $\rightarrow$ negative

pos. $\rightarrow$ Positive

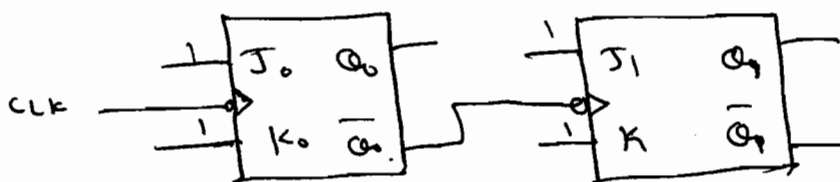
① Up Counter:



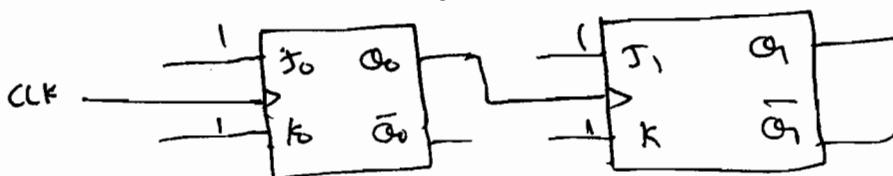
(or)



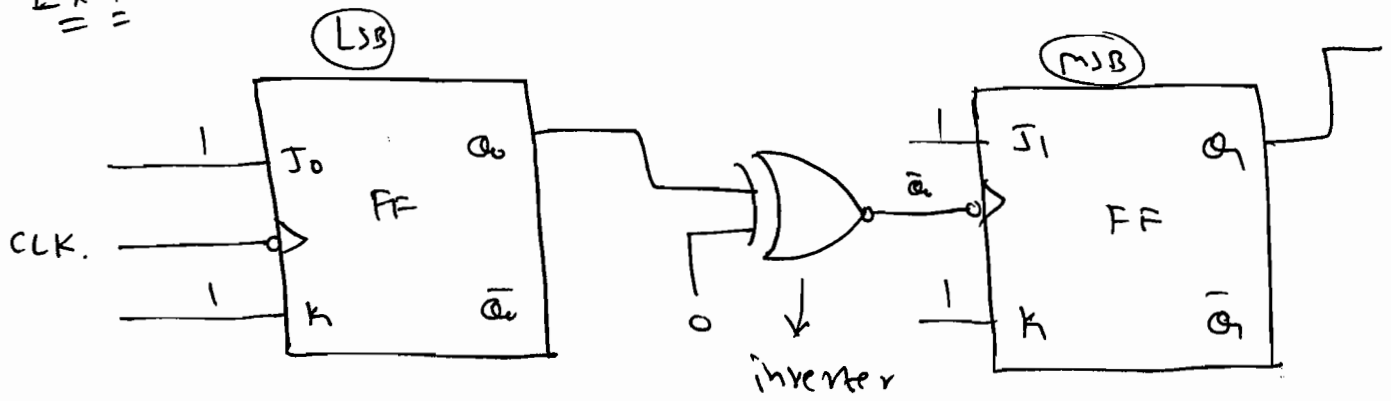
② Down Counter:



(or)



Ex-1

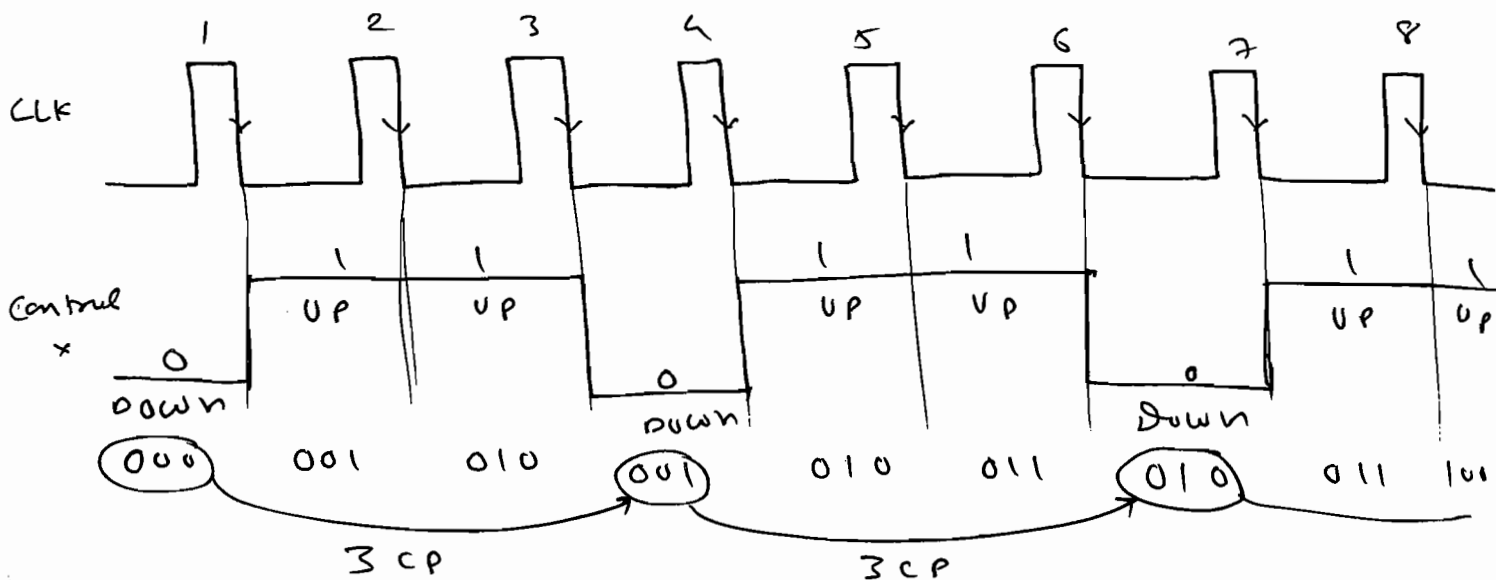


Ans

→ FF is -ve edge triggered, $\bar{Q}_0 \rightarrow \text{CLK}$
 hence it is **Down Counter.**

Imp

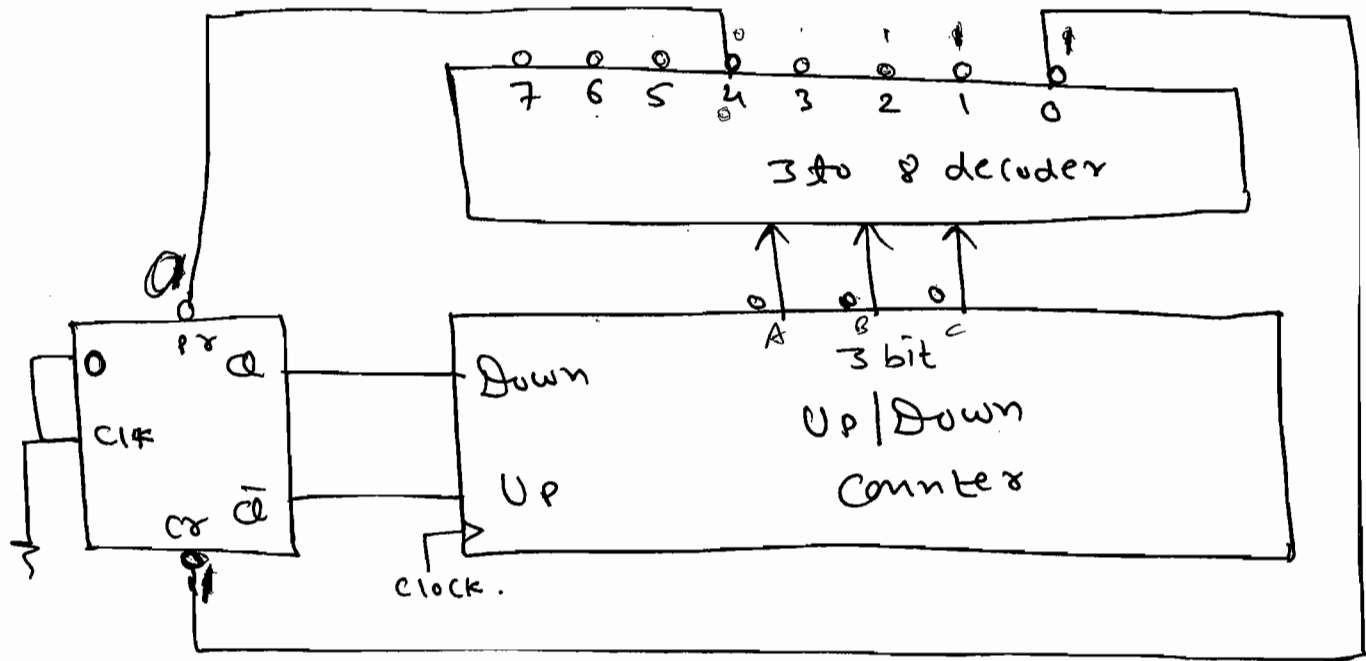
Ex-1 In a 3-bit asynchronous up-Down Counter the clock and control inputs are as given below: Determine the no. of clock pulses required to bring the counter to the initial counting seq. starting with 000.



→ In this Counter each increment requires three clock pulses.

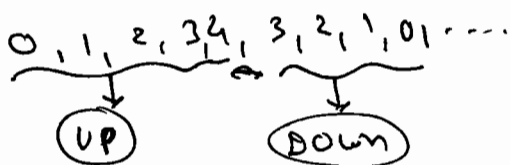
→ To increment 8 times No. of clock pulses are $8 \times 3 = 24$.

Ex-2 In the following Up down Counter determine the counting sequences of the Counter.



Ans:

CLK	A	B	C	
0	0	0	0	
1	0	0	1	UP
2	0	1	0	UP
3	0	1	1	UP
4	1	0	0	UP
5	0	1	1	Down
6	0	1	0	Down
7	0	0	1	Down
8	0	0	0	Down



$$\left[\because P_2 = 1, C_2 = 0 \right] \left[\therefore P_2 = 0, C_2 = 1 \right]$$

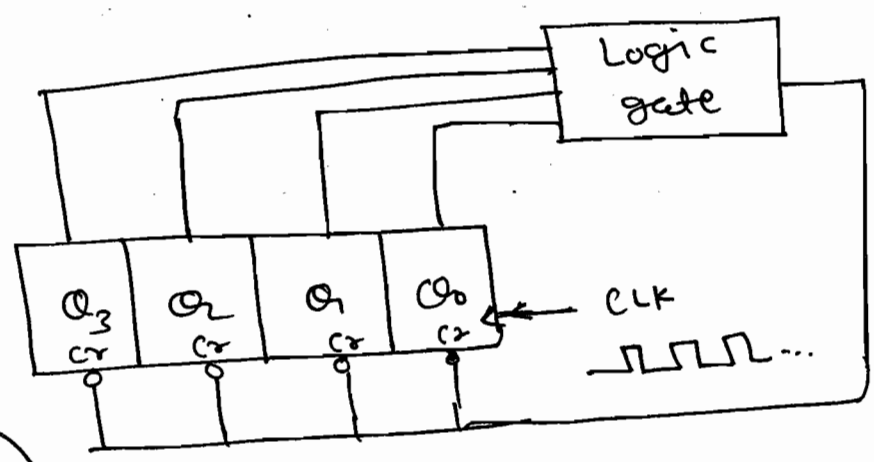
$$\text{i.e. } Q = 0, \bar{Q} = 1 \quad \left[\text{i.e. } Q = 1, \bar{Q} = 0 \right]$$

★ Modulus of a Counter.

→ It is the number of CLK pulses required to bring the counter in the initial state.

→ A Mod-N Counter ^{Counts from 0 to N-1} and output freq. is f/N .

E.g.



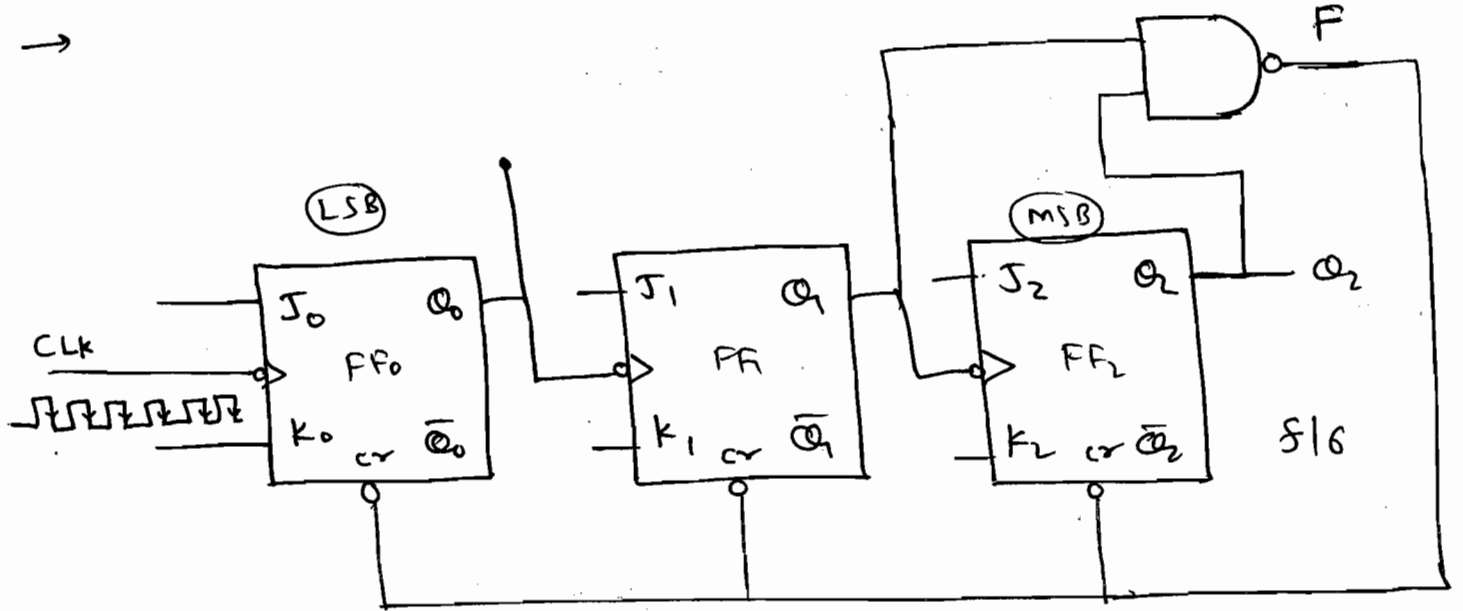
Q_3	Q_2	Q_1	Q_0
0	0	0	0
0	0	0	1
0	0	1	0

	1	0	0	1
10 →	1	0	1	0
	1	1	1	1

After the required value, F should be zero
 $F=0$.
i.e for e.g.
12

* Construct a Mod-6 Asynchronous Counter.

→ As soon as it reaches 6 CLK pulse it should be reset.



→ $0 \text{ to } 5 \Rightarrow 2^N \geq \text{No. of mode of counter.}$

$$\therefore 2^N \geq 6$$

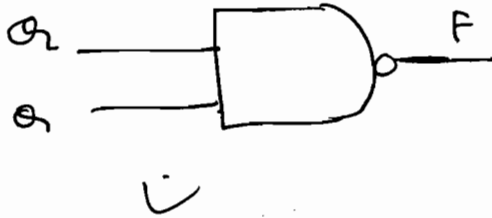
$$N = 3 \text{ FFs}$$

CLK	Serial inputs	Q_2	Q_1	Q_0	Decimal
0	—	0	0	0	0
1	1	0	0	1	1
2	1	0	1	0	2
3	1	0	1	1	3
4	0	1	0	0	4
5	0	1	0	1	5
6	0	1	1	0	6
		0	0	0	

(i) At 5^{th} CLK pulse $Q_2 = 1, Q_1 = 1$ 199

(ii) Hence when $Q_2 = 1, Q_1 = 1 \Rightarrow$ o/p of gate $F = 0$. All FFs are cleared.

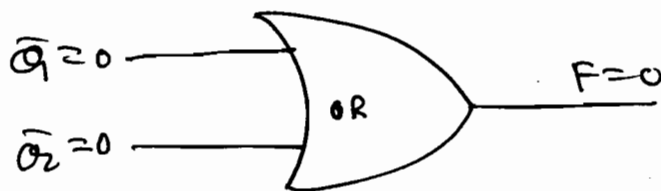
$\Rightarrow$ NAND gate



$\rightarrow$ In the above mod-6 asynchronous counter determine the feedback logic gate if its inputs are $\bar{Q}_2$ & $\bar{Q}_1$

$\rightarrow$ i.e. $\bar{Q}_2 = 0$ and $\bar{Q}_1 = 0 \Rightarrow$ o/p of gate $F = 0$.

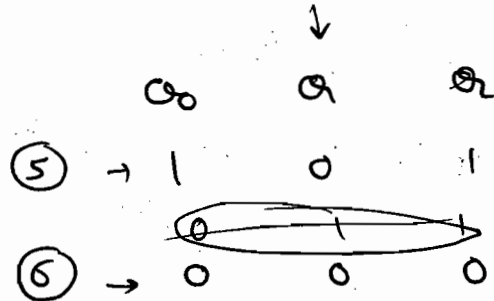
$\therefore \Rightarrow$ OR gate



* Disadvantages:

$\rightarrow$ In Asynchronous counters whose modulus is not equal to 2^N the output produces unwanted spikes called as glitches.

Eg.

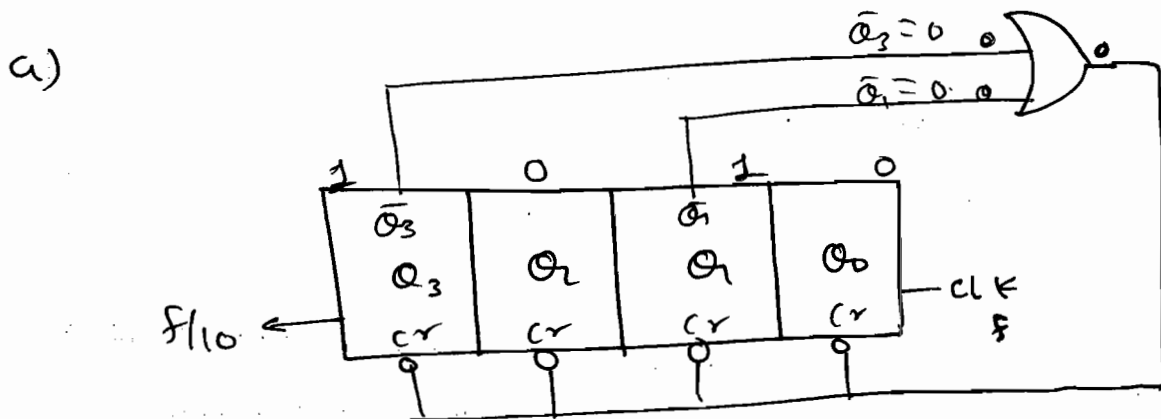


→ Latch output is latched on input; output doesn't change even if input changes



- ② If the FF are having unequal clearing times then all the FF cannot be cleared at the required clock pulse. To overcome this a latch is used in the feedback path such that its output remains zero until all the FF are cleared.

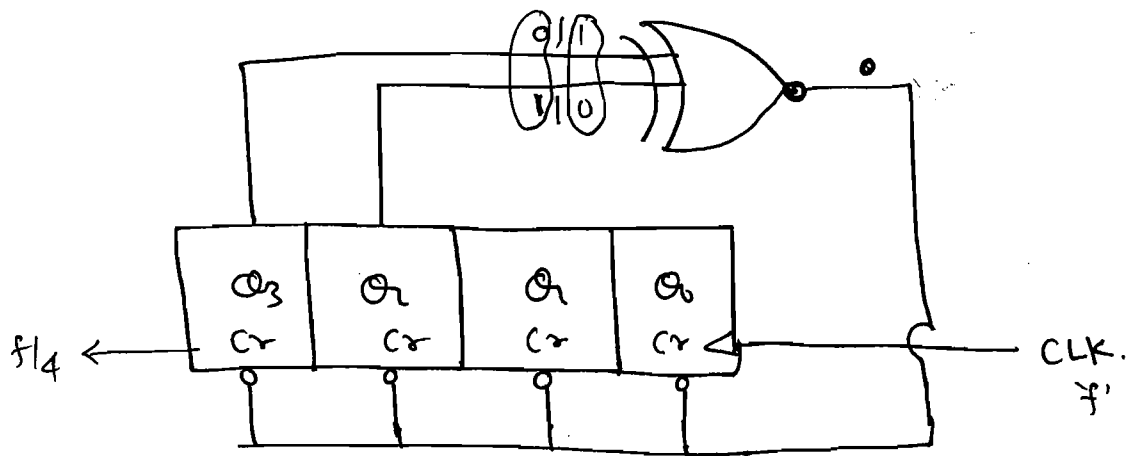
Ex-1 Determine the modulus of the following Asynchronous Counter:



→ For $Q_3 Q_2 Q_1 Q_0 = 1000$ → All the FF are cleared.

→ It is Mod-10 Counter (decade counter)

(b)



Ans:

Q_3	Q_2	Q_1	Q_0
0	0	0	0
0	0	0	1
	!		

$$f \propto \frac{1}{N}$$

0	1	0	0
:	:	:	:
1	0	0	0
1	1	1	1

4 will appear first
i.e. 01 will appear first then 10.

0 1 2 3 *

For $Q_3 Q_2 Q_1 Q_0 = 0100 \rightarrow$ All the FFs are cleared
 $\rightarrow$ It is Mod - 4 Counter.

$\rightarrow$ The disadvantages of Asynchronous Counter is the freq. of operation is inversely proportional to the no. of FFs. To overcome this we use Synchronous or Parallel Counters.

* Flip Flop Excitation table:

① S-R FF:

$Q(t)$	$Q(t+1)$	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

② D- Flip Flop:

$Q(t)$	$Q(t+1)$	D
0	0	0
0	1	1
1	0	0
1	1	1

③ T- Flip Flop:

$Q(t)$	$Q(t+1)$	T
0	0	0
0	1	1
1	0	1
1	1	0

④ J-k- Flip Flop:

$Q(t)$	$Q(t+1)$	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

$Q(t)$	$Q(t+1)$	FF Inputs					
		J	K	S	R	D	T
0	0	0	X	0	X	0	0
0	1	1	X	1	0	1	1
1	0	X	1	0	1	0	1
1	1	X	0	X	0	1	0

Ex-1 Determine the excitation table of X-Y FF whose T.T. is as given below.

X	Y	$Q(t+1)$
0	0	1
0	1	$Q(t)$
1	0	$\overline{Q}(t)$
1	1	0

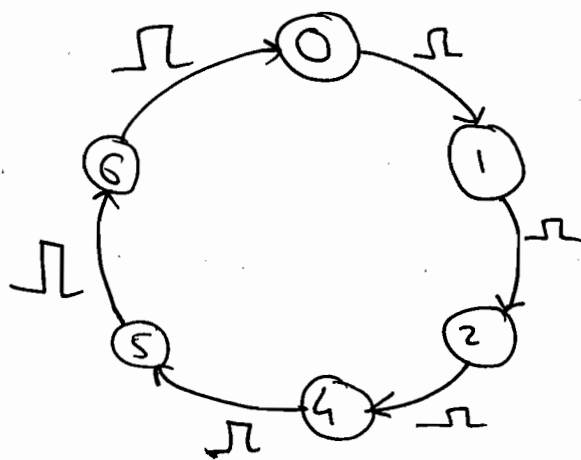
$Q(t)$	$Q(t+1)$	X	Y
0	0	X	0
0	1	0	X
1	0	1	X
1	1	0	0

* Design a Synchronous Counter using J-K Flip Flops. which comes through the states 0, 1, 2, 4, 5, 6, 0, ...

(b) is it a self starting Counter?

Ans:

(a) State Diagram.



(b) State Assignment:

① → 000

② → 001

③ → 010

④ → 100

⑤ → 101

⑥ → 110.

(c) Excitation Table:

	FF Inputs			$J_2 K_2$	$J_1 K_1$	$J_0 K_0$
	Q_2	Q_1	Q_0			
②	0	0	0	0 X	0 X	1 X
①	0	0	1	0 X	1 X	X 1
②	0	1	0	1 X	X 1	0 X
④	1	0	0	X 0	0 X	1 X
⑤	1	0	1	X 0	0 X	1 X
⑥	1	1	0	X 1	X 1	0 X
⑦	0	0	0			

* ③, ⑦ are unused states. Takes them as don't cares.

$$J_0 = \bar{Q}_1 \quad K_0 = 1$$

$$J_1 = Q_1 + Q_2 \quad K_1 = 1$$

$$J_2 = Q_1 \quad K_2 = Q_1$$

$Q_1 Q_2$	00	01	11	10
Q_0				
0	0	0	X	1
1	X	X	X	X

$$J_2 = Q_1$$

$Q_1 Q_2$	00	01	11	10
Q_0				
0	X	X	X	X
1			X	1

$$K_2 = Q_1$$

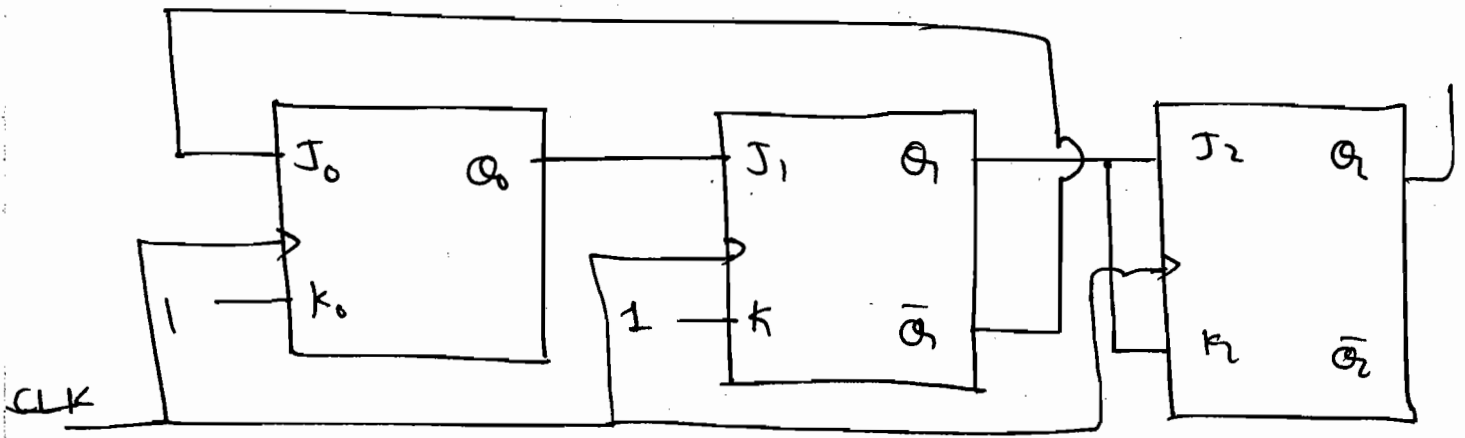
$Q_1 Q_2$	00	01	11	10
Q_0				
0	X	X	X	
1	1	X	X	

$$J_0 = \bar{Q}_1$$

$Q_1 Q_2$	00	01	11	10
Q_0				
0		X	X	X
1		X	X	X

$$J_1 = Q_2 + Q_1$$

Ex/2 Sum

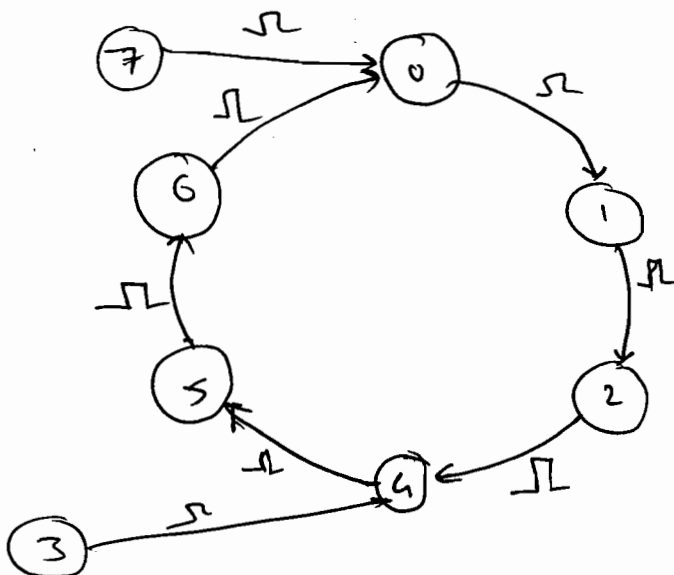


→ In Sync Counter, Freq of operation

$$f \leq \frac{1}{t_{pd} \cdot FF}$$

(b)

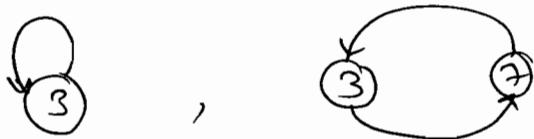
	Present State			FF Input			Next State		
	Q_2	Q_1	Q_0	$J_2 K_2$	$J_1 K_1$	$J_0 K_0$	Q_2	Q_1	Q_0
③ →	0	1	1	<u>1</u> <u>1</u>	<u>1</u> <u>1</u>	<u>0</u> <u>1</u>	1	0	0
⑦ →	1	1	1	<u>1</u> <u>1</u>	<u>1</u> <u>1</u>	<u>0</u> <u>1</u>	0	0	0



It is a self starting counter as we get ③ after ① and ② after ⑦

→ The above Counter is a self starting Counter because it is able to enter the used states from all unused state.

e.g. of non self starting Counter



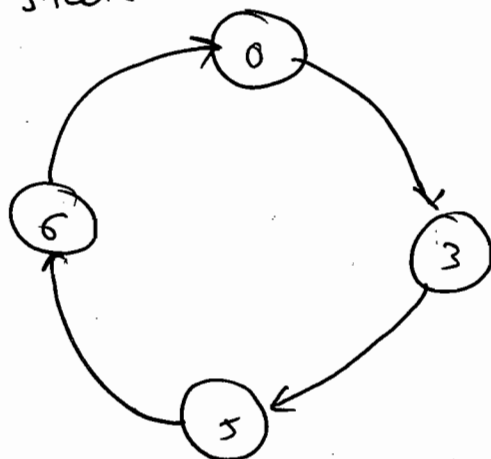
Ex-2 =

A Synchronous Counter comes through the states 0, 3, 5, 6, 0, ... and FF inputs are $T_2 = Q_1$, $T_1 = 1$, $T_0 = \bar{Q}_1$. Is it a self starting Counter.

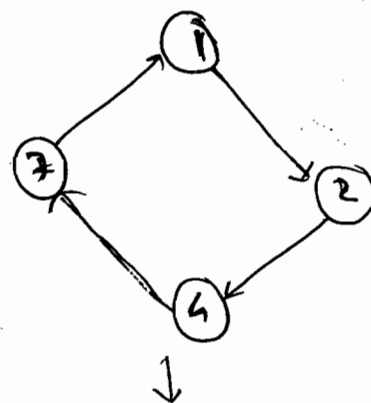
⇒ Table for analysis:

	Pr. State			FF Inputs			Next State		
	Q_2	Q_1	Q_0	$T_2 = Q_1$	$T_1 = 1$	$T_0 = \bar{Q}_1$	Q_2	Q_1	Q_0
①	0	0	1	0	1	1	0	1	0
②	0	1	0	1	1	0	1	0	0
④	1	0	0	0	1	1	1	1	1
⑦	1	1	1	1	1	0	0	0	1

Used state



Unused state

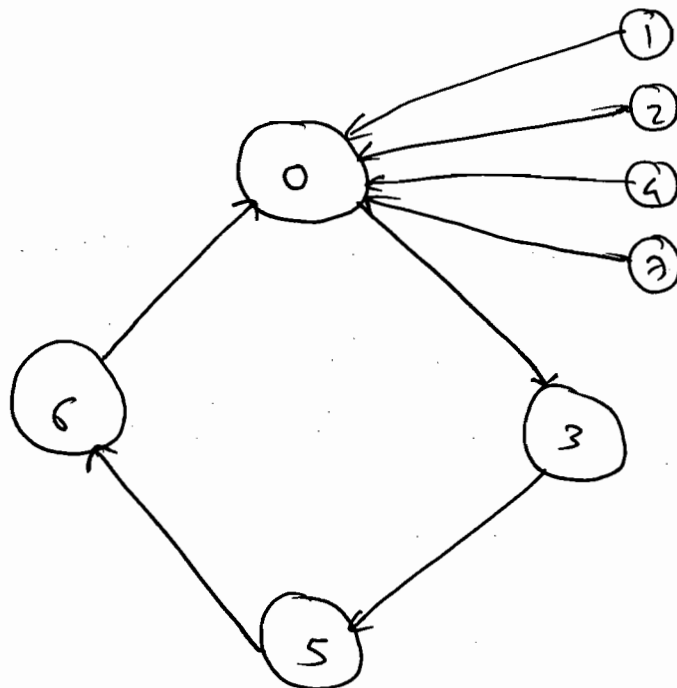


→ It is an ~~in efficient~~ counter.

'Lock-out' of Counter
Not a self starting Counter

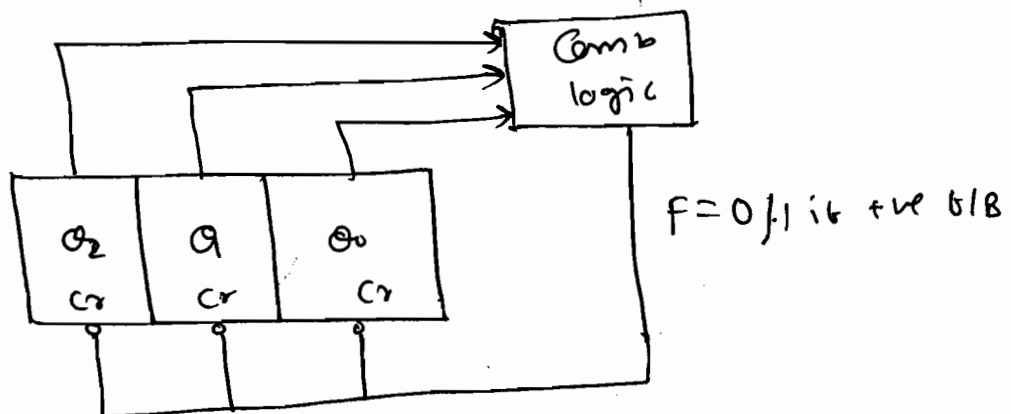
→ To avoid the Lock-out condition

① Redesign the Counter according to the following state diagram.



→ No. of FF state is increasing $\Rightarrow$ no. of FF is increased in it will increase the Complexity. of ckt

② Clearing the Counter as soon it enters into Unused state. as shown below:



$$F(Q_2, Q_1, Q_0) = \sum m(1, 2, 4, 7).$$

	00	01	11	10
00				
01	1			1
10				
11				

$$\therefore F = Q_2 \oplus Q_1 \oplus Q_0$$

(OR)

$$F = Q_2 \odot Q_1 \odot Q_0$$

NOTE:

⇒ In the above synchronous counter the Combinational logic is determine as follows:

→ When Counter enters into the states ①, ②, ④, ⑦ the F should become 1. So, then the Counter can be clear.

→ The sum of minterm expression for F is

$$F(Q_2, Q_1, Q_0) = \Sigma (1, 2, 4, 7).$$

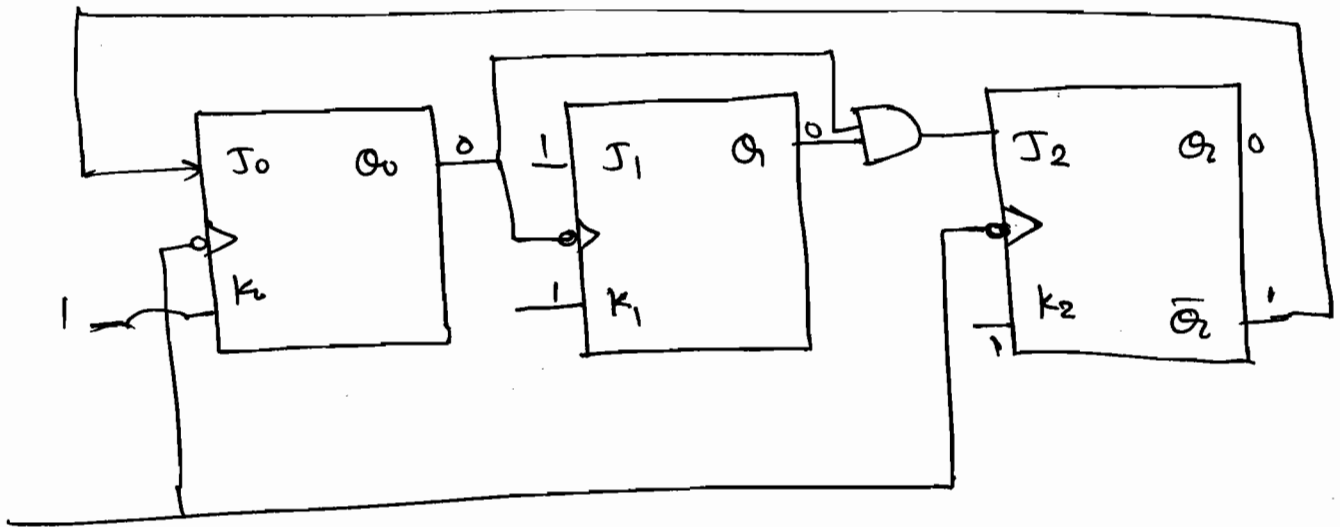
	00	01	11	10
00				
01		1		1
10	1		1	
11				

$$\therefore F = Q_2 \oplus Q_1 \oplus Q_0$$

(OR)

$$F = Q_2 \odot Q_1 \odot Q_0$$

Ex-2 Determine the modulus of the following counter

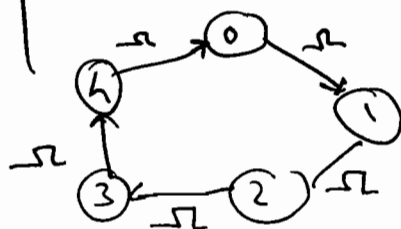


Ans: 'Q' is in Asynchronous mode. It toggles when Q_0 changes from 1 to 0.

Now $J_0 = \bar{Q}_2$, $J_2 = Q_0 Q_1$
 $K_0 = 1$, $K_2 = 1$.

⇒ Table for analysis:

	P.S.			FF Inputs		N.S.		
	Q_2	Q_1	Q_0	$J_2 K_2$	$J_0 K_0$	Q_2	Q_1^+	Q_0
⑤	0	0	0	0 1	1 1	0	0	0*
①	0	0	1	0 1	1 1	0	1	0
②	0	1	0	0 1	1 1	0	1	1
③	0	1	1	1 1	1 1	1	0	0
④	1	0	0	0 1	0 1	0	0	0



"Modulus = 5".

→ If $t_{pd,FF} = 10\text{ns}$ → $f_{max} = ?$ 158

Max. Conv. time = $10 + 10 = 20\text{ns}$.

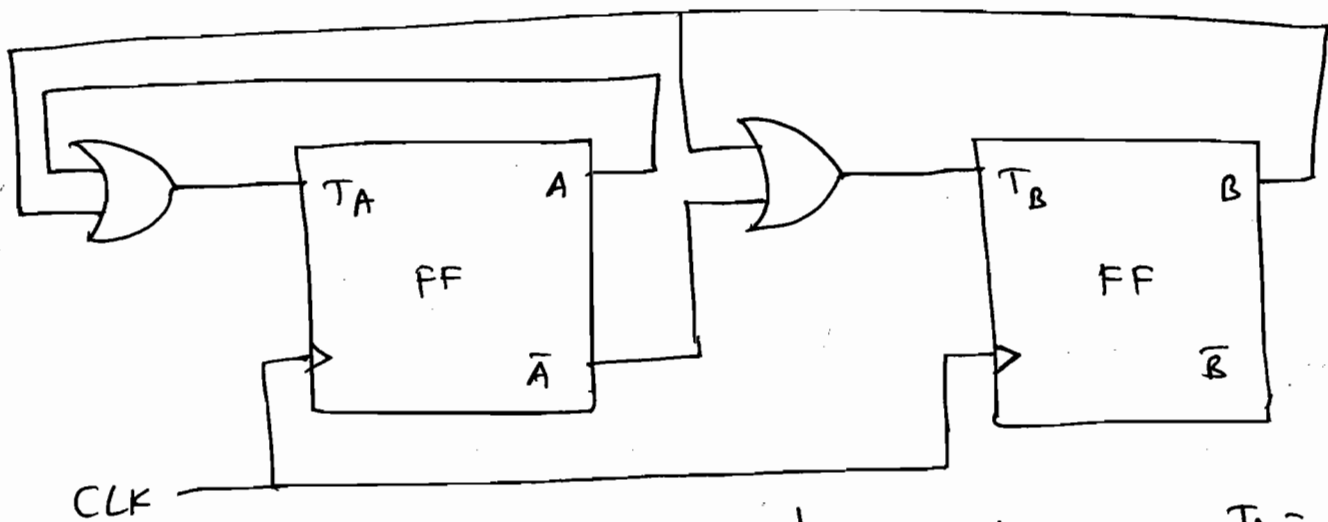
C ∴ FF₀, FF₂ are in synch. mode,
FF₁ is in Async mode].

Clock period $T \geq 20\text{ns}$.

$$\therefore f \leq \frac{1}{20 \times 10^{-9}}$$

$$\therefore f_{max} = \frac{1}{20 \times 10^{-9}} \text{ Hz}$$

Ex-3 Determine the f_n of the following
Seq. Circuits by obtaining its state
diagram.

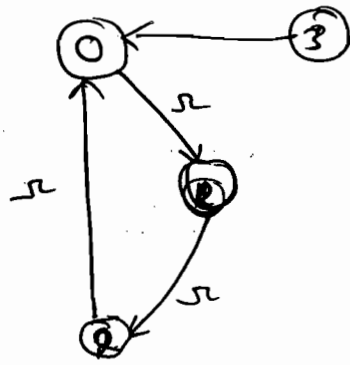


	P.S.		FF inputs		N.S.	
	A	B	TA	TB	A	B
①	0	0	0	1	0	1
②	0	1	1	1	1	0
③	1	0	1	0	0	0
④	1	1	1	1	0	0

$$T_A = A + B$$

$$T_B = \bar{A} + B$$

State diagram



→ it is a Mod-3, self starting synchronous counter.

* State Diagram:

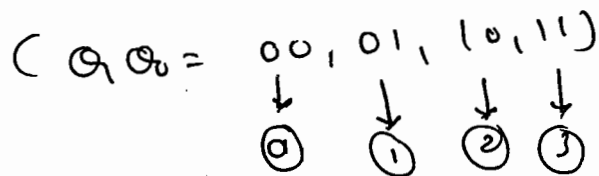
Hint:

(i) No of states = 2^N ; N = No of FFs

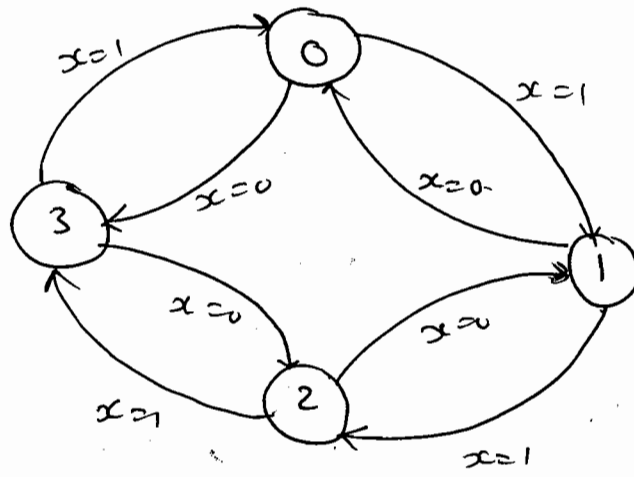
(ii) No of branches from each state = 2^x ;
 x = no of direct inputs.

① 2-Bit Up/Down Sync Counter:

→ 2 FFs $\Rightarrow$ No. of states = $2^2 = 4$.



→ 1 direct i/p (i.e. x) $\Rightarrow$ No. of branches from each state = $2^1 = 2$.

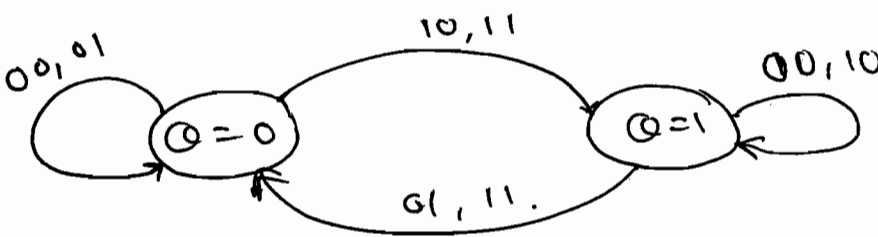


[State Diagram]

② J-K FF state diagram:

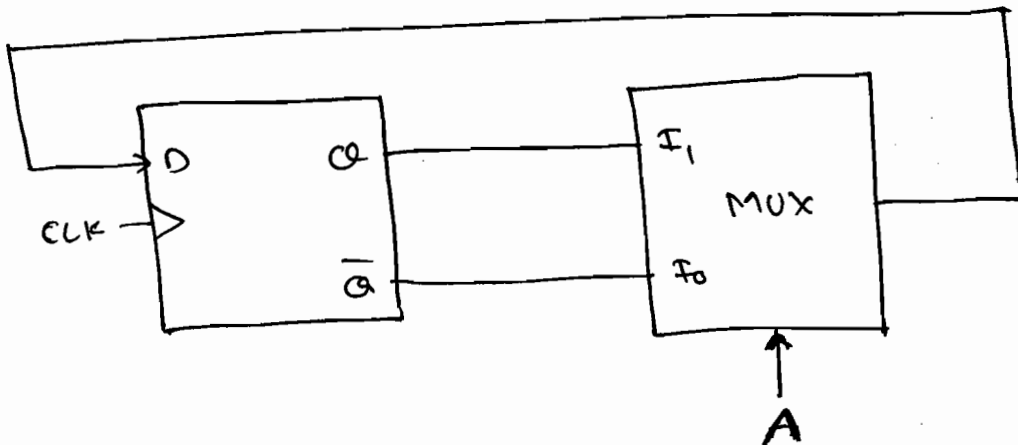
→ 1 FF $\Rightarrow$ 2 states ($Q=0, Q=1$).

→ 2 Direct input $\Rightarrow$ 4 branches from each state.



Q(k)	J	K	Q(k+1)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

③ Gate - 2013



→ 1 FF $\Rightarrow$ 2 States ($Q=0, Q=1$).

→ Input (i.e. A) $\Rightarrow$ 2 branches from each state.

For D-FF $\rightarrow Q(t+1) = D$ — (1)

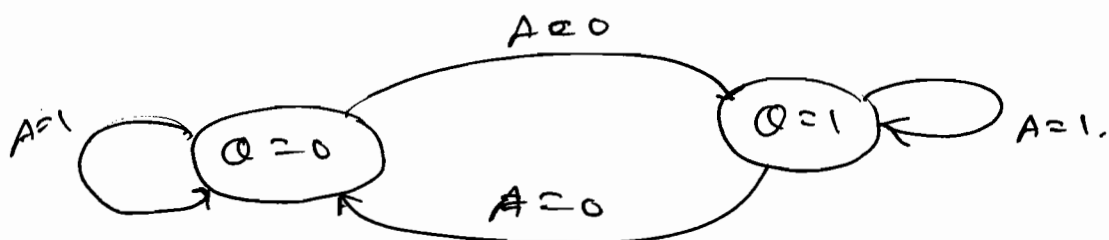
For MUX $\rightarrow D = \bar{A} \cdot \bar{Q} + A \cdot Q$ — (2)

from (1) & (2)

$$\therefore Q(t+1) = \bar{A} \cdot \bar{Q}(t) + A \cdot Q(t).$$

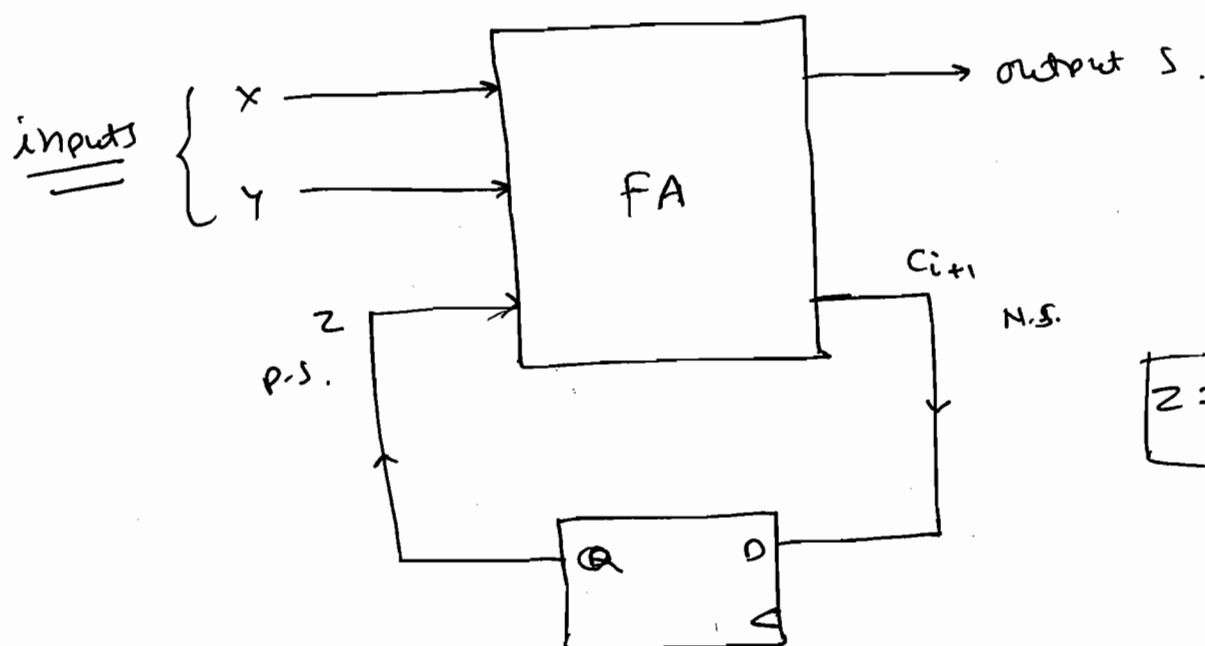
When $A=0$, $Q(t+1) = \bar{Q}(t)$

$A=1$, $Q(t+1) = Q(t)$



④ Serial Adder State diagram:

183
160

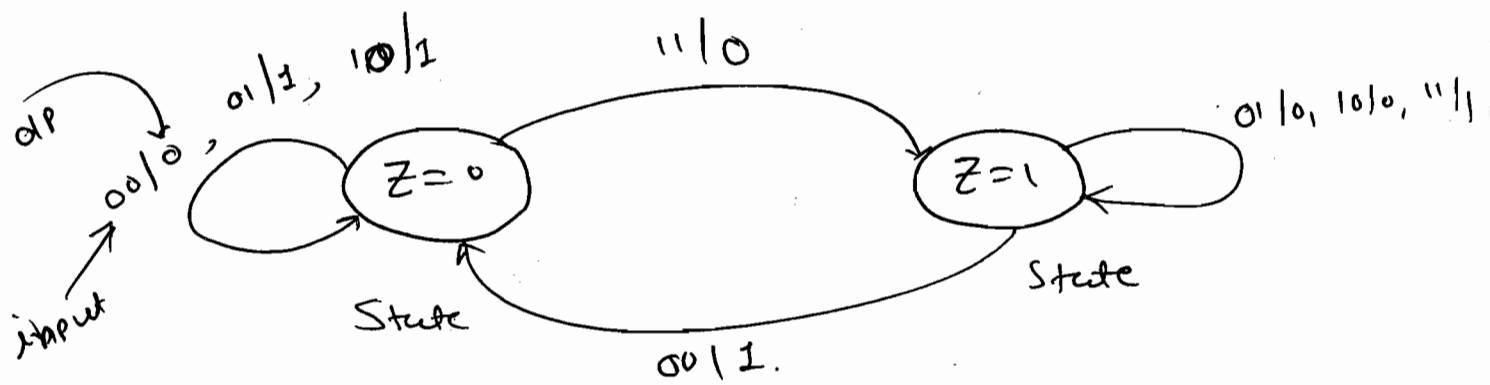


$$Z = 0 = C_i + 1 \\ = Q(t+1)$$

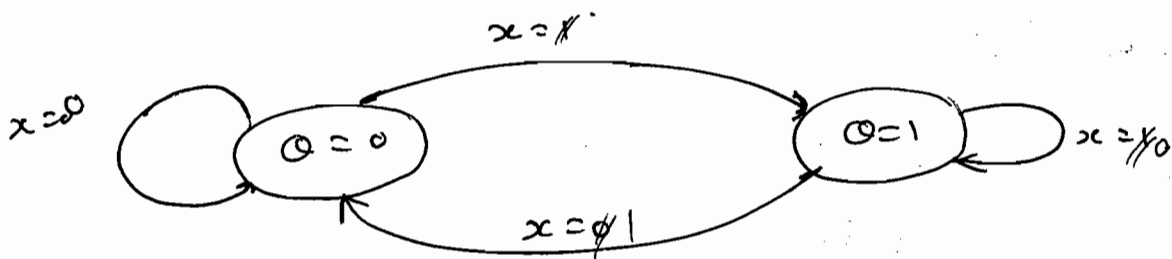
→ 2 inputs 1 FF → 2 states ($z=0, z=1$).

→ 2 input → $2^2 = 4$ branches from each state.

P.S. (z)	input x y		N.S. C_{i+1}	output Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	1	0
1	1	1	1	1

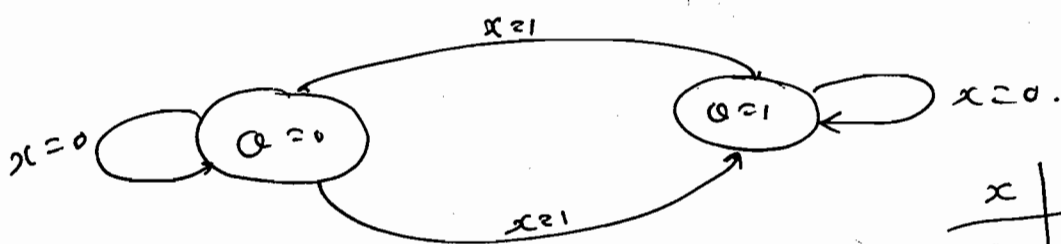


Ex-1 Identify the following FFs.



x	$Q(t+1)$
0	0
1	1

→ T-FF State diagram

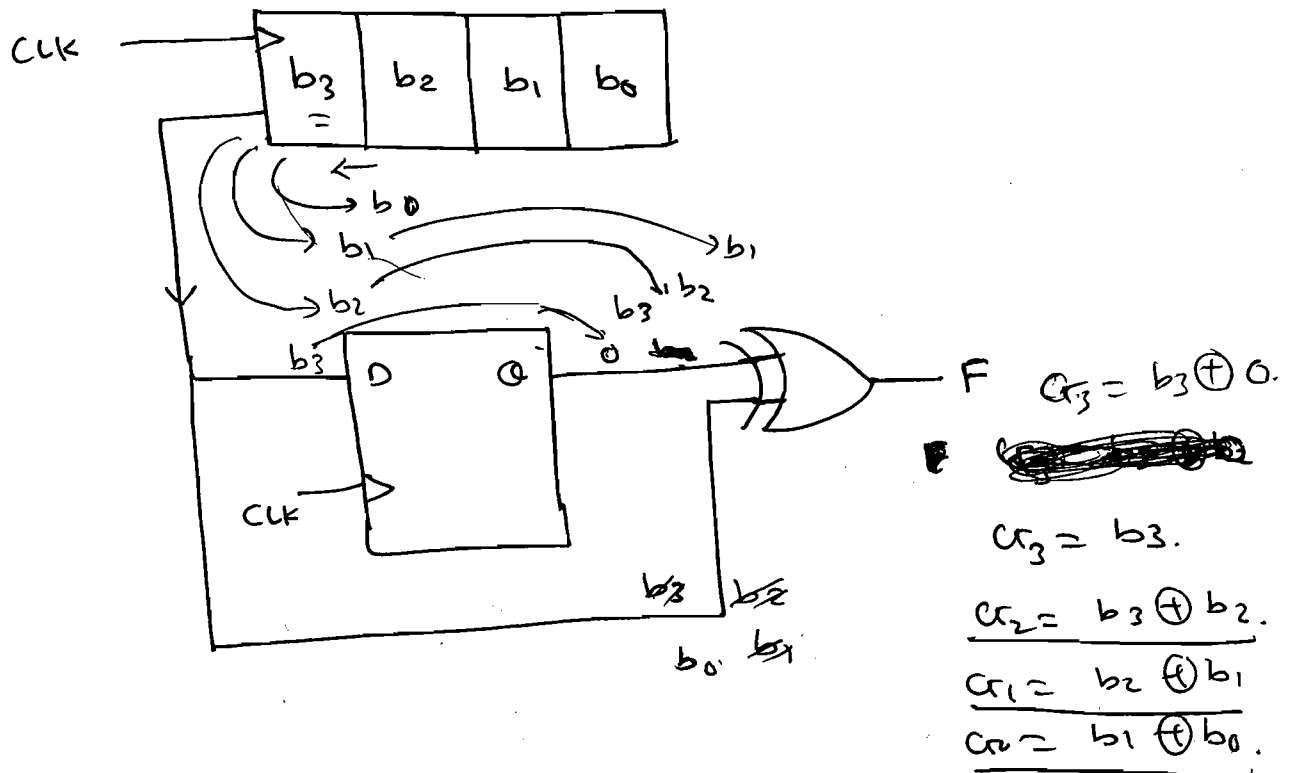


x	$Q(t+1)$
0	$Q(t)$
1	$\bar{Q}(t)$

Ex-1 Determine the b^n of the following circuits.

①

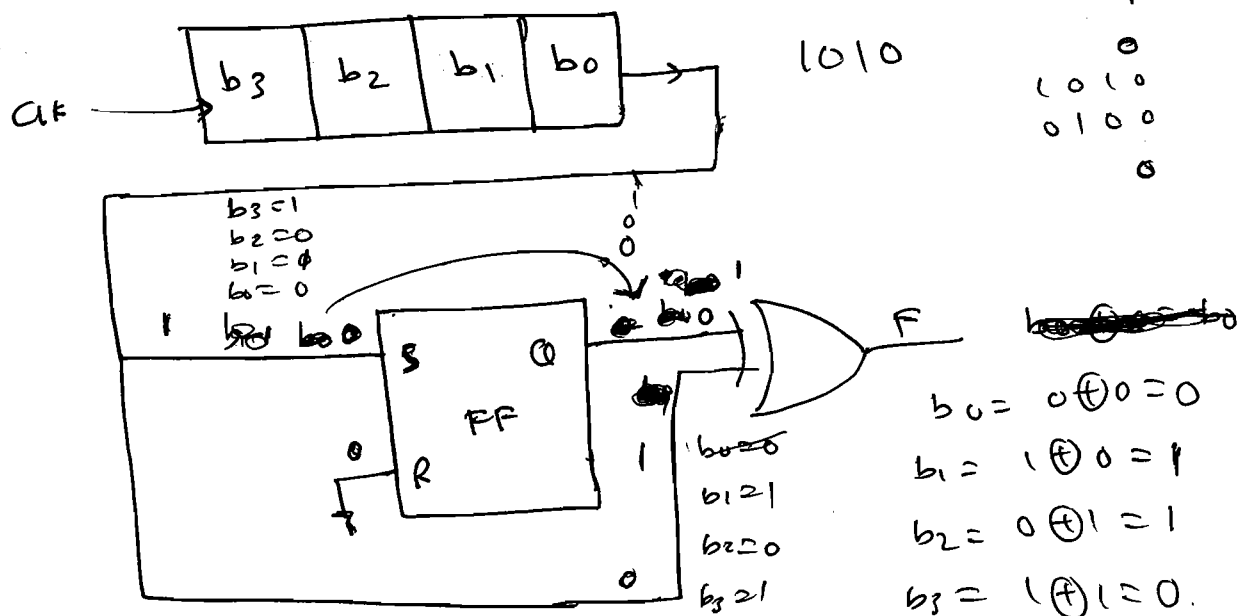
Shift left shift Register.



→ Binary to Gray Code Conversion.

②

Shift Right Shift Register



Input = $\boxed{1 \ 0 \ 1 \ 0}$

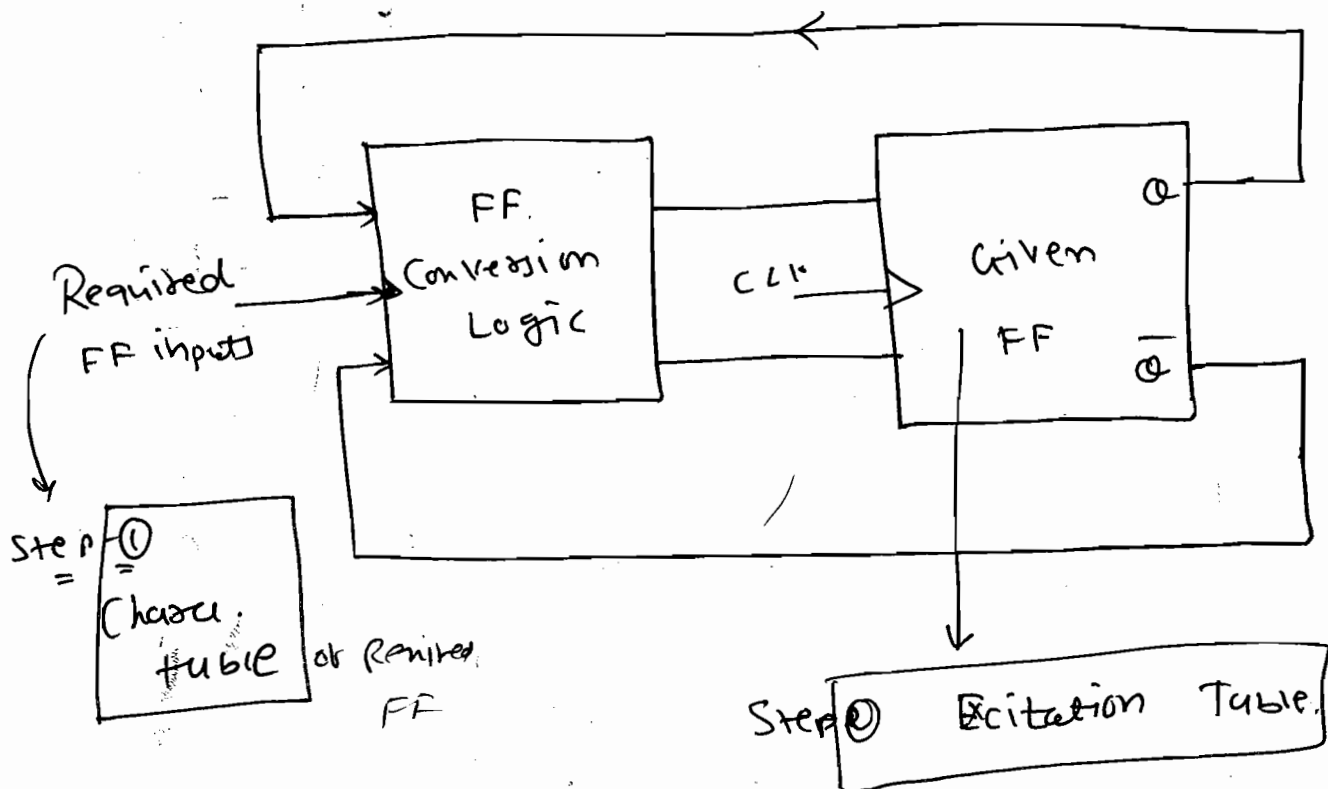
output = $\boxed{0 \ 1 \ 1 \ 0}$

→ It is 2's Comp. of binary no.

→ so, $\boxed{\text{it is 2's Complement circuit}}$

* Conversion of Flipflops:

⇒ Universal Principle:



Ex-1 Convert S-R FF to T flip flop.

167
162

Ans: Step-① → Characteristics table of T-Flip flop.
Step-② → Excitation table of SR flip flop.

T	Q(t)	Q(t+1)	S	R
0	0	0	0	x
	1	1	x	0
1	0	1	1	0
	1	0	0	1

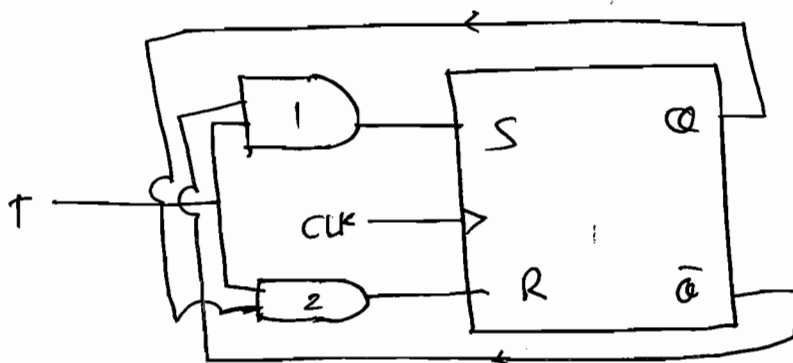
→ ⑤

	Q(t)	0	1
T	0	0	x
	1	①	0

$$S = T\bar{Q}(t).$$

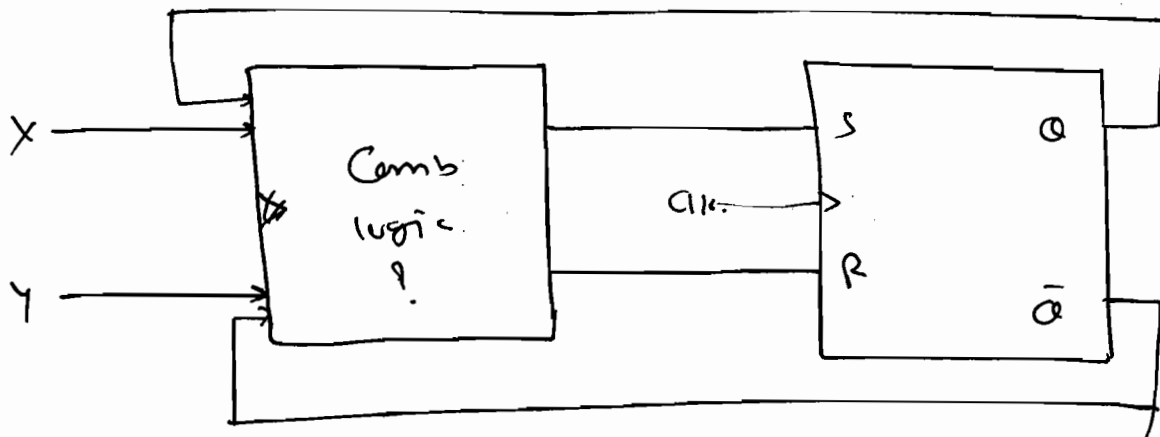
	Q(t)	0	1
T	0	x	0
	1	0	①

$$R = TQ(t).$$



[T-Flip Flop]

Ex-2 In the following diagram determine the
Combinational logic to be used.



Given as

X Y	Q(t+1)	S R
0 0	0	
0 1	Q(t)	
1 0	$\bar{Q}(t)$	
1 1	1	

X	Y	Q(t)	Q(t+1)	S	R
0	0	0	0	0	X
0	0	1	0	X 0	0 1
0	1	0	0	0	X
0	1	1	1	X	0
1	0	0	1	1	0
1	0	1	0	0	1
1	1	0	1	1	0
1	1	1	1	X	0

① S

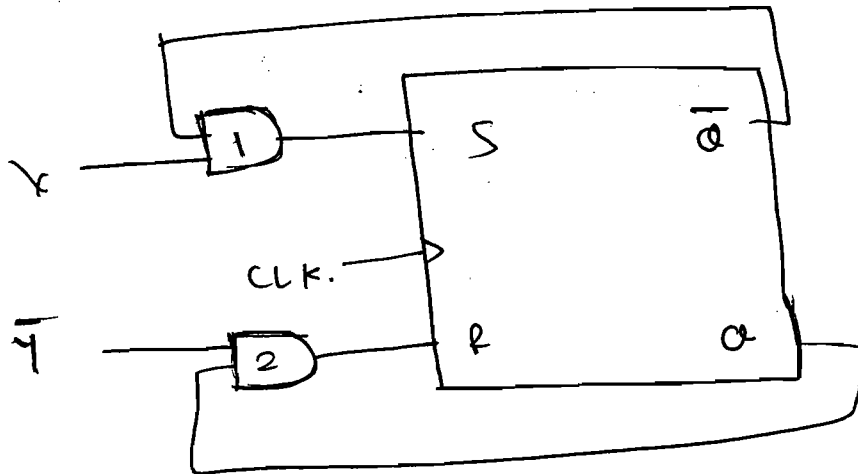
	$ya(t)$			
	00	01	11	10
X	0	0	1	0
0	0	0	1	0
1	1	0	1	1

$\therefore S = \cancel{X} \cdot \cancel{y} \cdot a(t)$

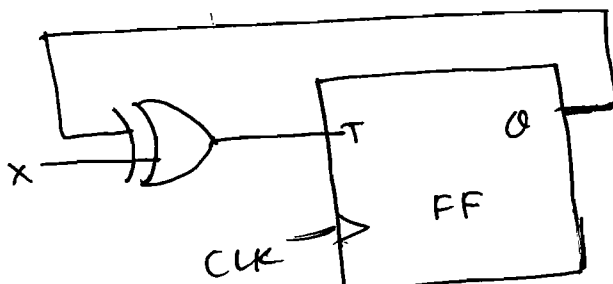
② R

	$ya(t)$			
	00	01	11	10
X	0	1	0	0
0	1	0	0	0
1	0	1	0	0

$\therefore R = \cancel{X} \cdot \bar{y} \cdot a(t)$
 $R = \bar{y} \cdot a(t)$



Ex-3 (a) Identify the following flip flop.



$T = x \oplus a(t) \quad \text{--- (1)}$

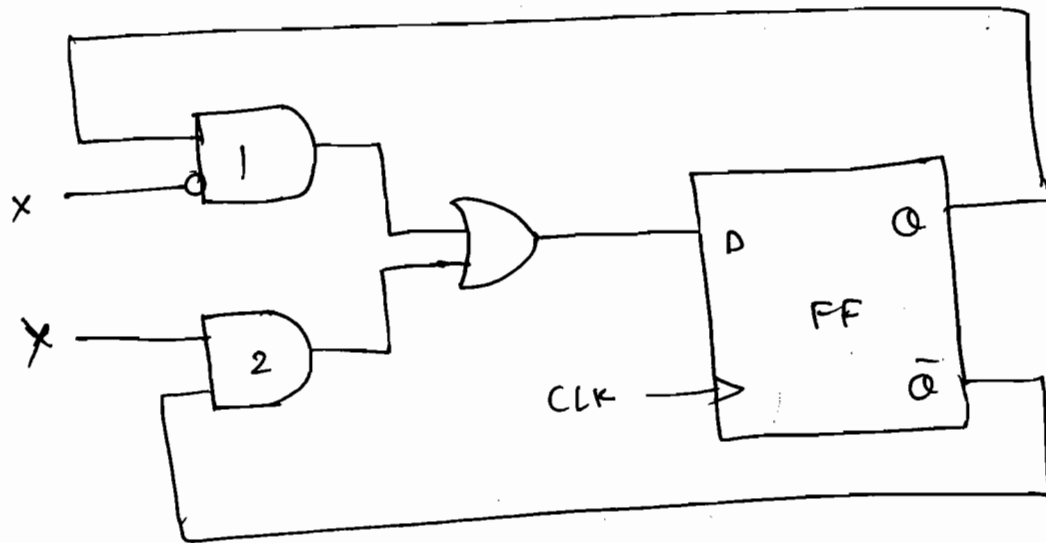
For T FF
 $T \Rightarrow a(t+1) = T \oplus a(t)$

$a(t+1) = x \oplus a(t) \oplus a(t)$

$\therefore a(t+1) = x \oplus 0$
 $\boxed{a(t+1) = x}$

We know $\Rightarrow$ ③ is similar to D-FF Character
 $\Rightarrow$ It is D-Flip Flop.

b)



$$D = Q\bar{X} + \bar{Q}Y.$$

$$\therefore Q(t+1) = Q(t)(\bar{X}) + \bar{Q}(t)(Y). \quad \text{--- ③}$$

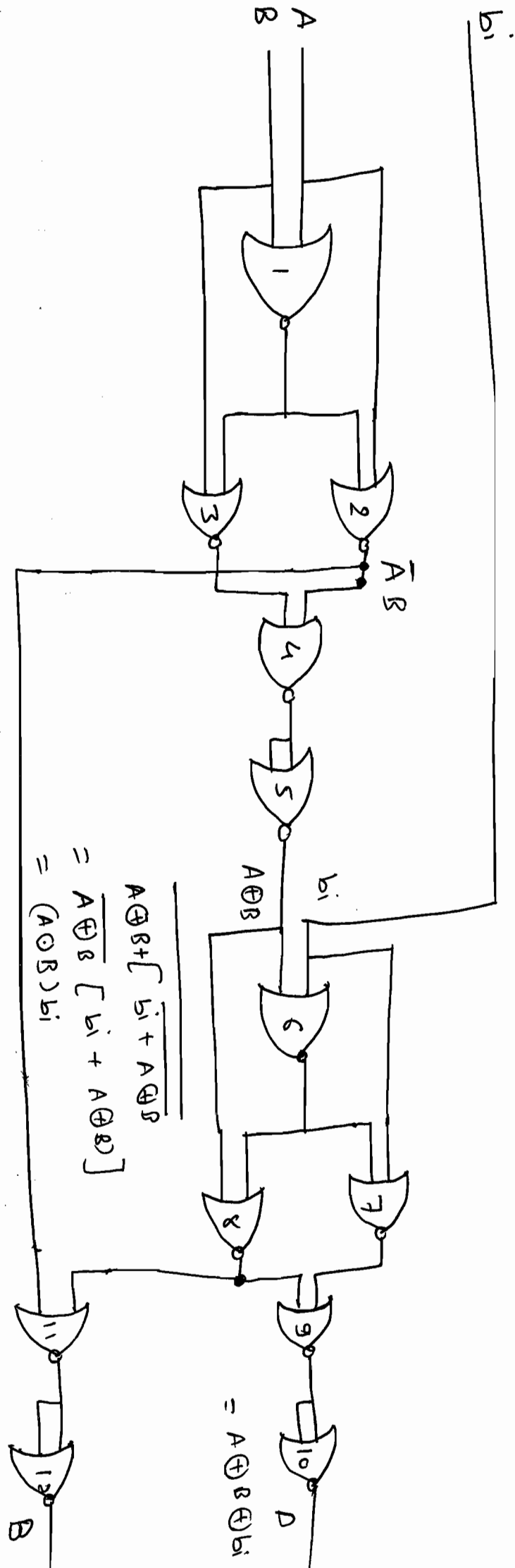
$$\begin{matrix} J\bar{Q} & \bar{J}Q \\ K\bar{Q} & KQ \end{matrix}$$

$$\therefore Q(t+1) = (\bar{J})\bar{Q}(t) + (K)Q(t). \quad \text{--- ④}$$

So, $\bar{X} = \bar{J}$ $X = K$ $Y = J$

So, it is J-K flip flop.

Full Subtractor using NOR gate only.



$$A \oplus B + [b_i + A \odot B]$$

$$= A \oplus B + (A \odot B) b_i$$

$$D = A \oplus B \oplus b_i$$

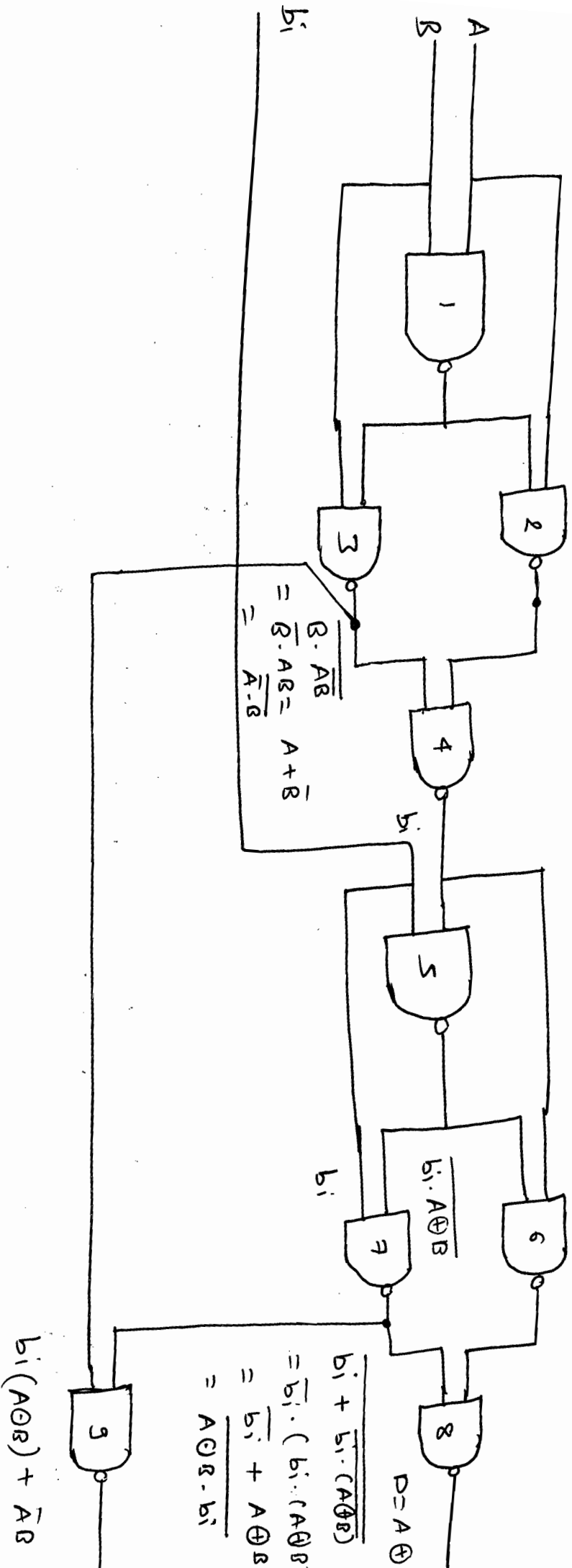
$$B = b_i (A \odot B) + \bar{A} B$$

$$D = A \oplus B \oplus b_i$$

$$B = b_i (A \odot B) + \bar{A} B$$

* Full Subtractor using NAND gate only:

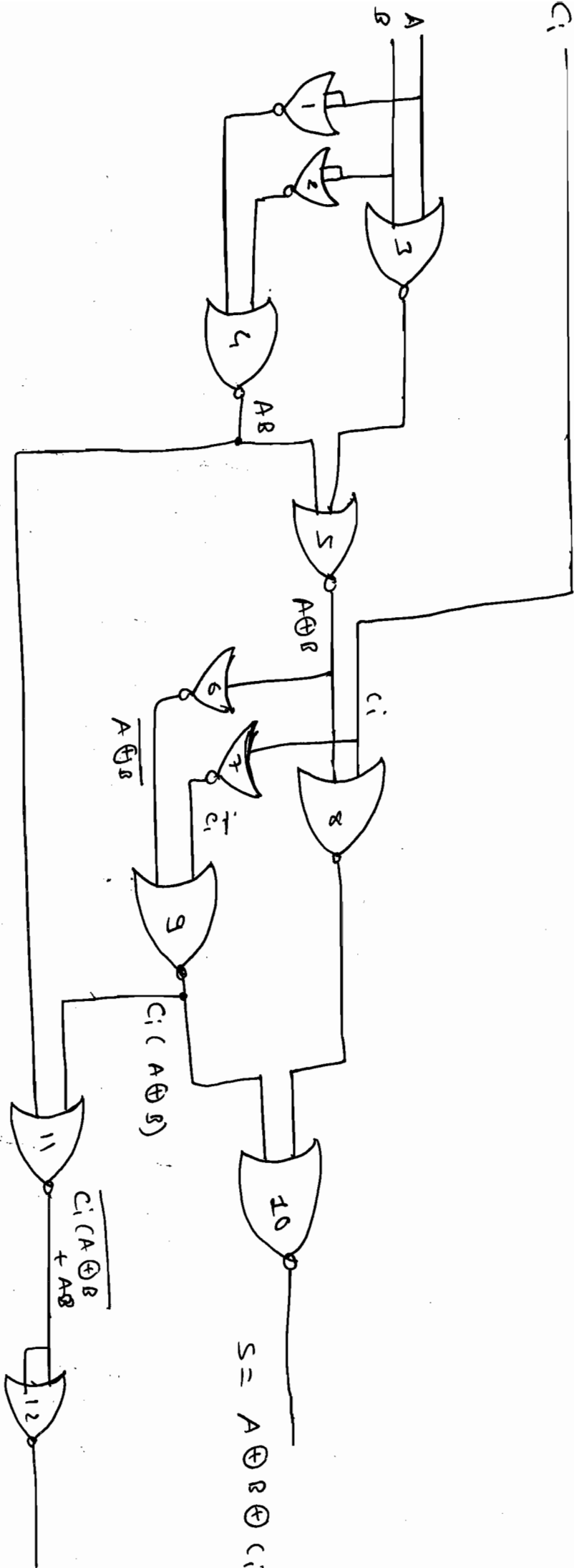
$$\overline{A \oplus AB} = \overline{A} + AB = \overline{A} + B$$



$$D = A \oplus B \oplus b_i$$

$$B = b_i(A \oplus B) + \overline{A}B$$

* Full Adder using Nor gate only:

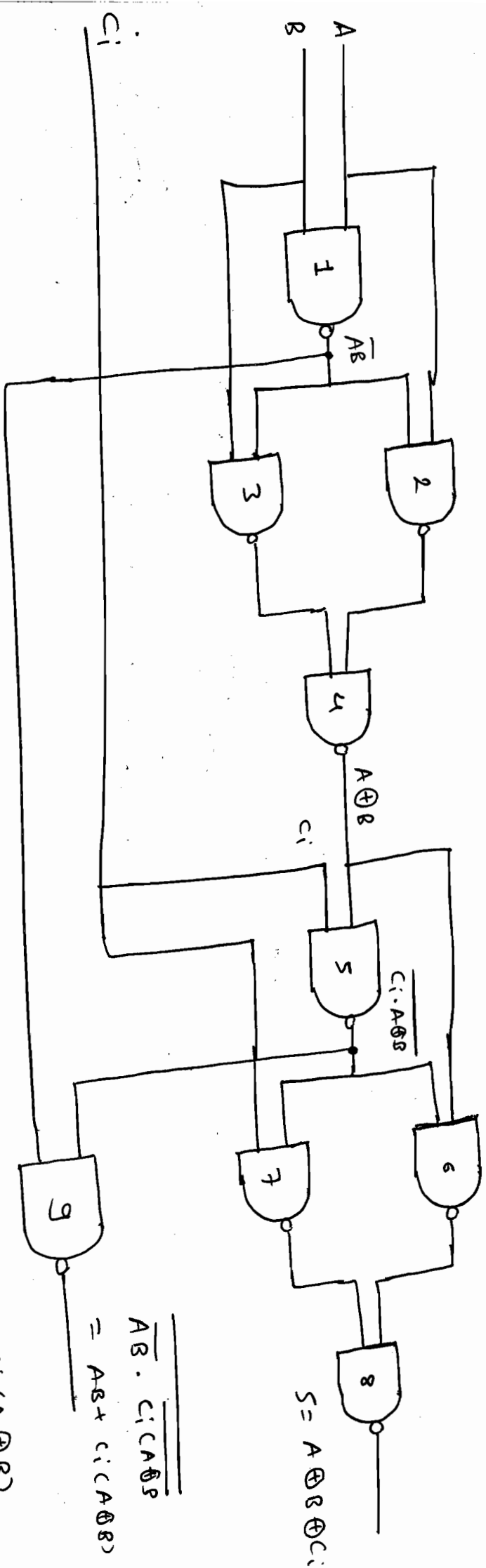


$$S = A \oplus B \oplus C_i$$

$$C_{i+1} = C_i(A \oplus B) + AB$$

$$C_{i+1} = C_i(A \oplus B) + AB$$

* Full Adder using NAND gate only



$$C_{i+1} = \overline{\overline{AB \cdot C_i(A \oplus B)}} = AB + C_i(A \oplus B)$$

$$S = A \oplus B \oplus C_i$$

$$C = C_i \cdot (A \oplus B) + AB$$