

# Chapter - Laws of Motion



## Topic-1: 1st, 2nd & 3rd Laws of Motion



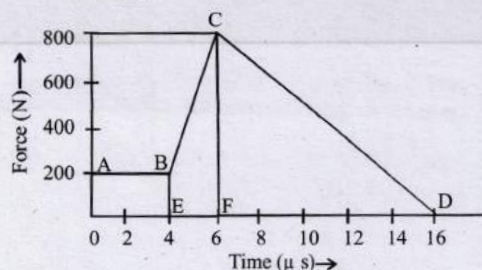
### 1 MCQs with One Correct Answer

1. A block of mass 5 kg moves along the  $x$ -direction subject to the force  $F = (-20x + 10)$  N, with the value of  $x$  in metre. At time  $t = 0$  s, it is at rest at position  $x = 1$  m. The position and momentum of the block at  $t = \left(\frac{\pi}{4}\right)$  s are [Adv. 2024]
  - (a)  $-0.5$  m, 5 kg m/s
  - (b)  $0.5$  m, 0 kg m/s
  - (c)  $0.5$  m,  $-5$  kg m/s
  - (d)  $-1$  m, 5 kg m/s
2. A particle of mass  $m$  is moving in the  $xy$ -plane such that its velocity at a point  $(x, y)$  is given as  $\vec{v} = \alpha(y\hat{x} + 2x\hat{y})$ , where  $\alpha$  is a non-zero constant. What is the force  $\vec{F}$  acting on the particle? [Adv. 2023]
  - (a)  $\vec{F} = 2m\alpha^2(x\hat{x} + y\hat{y})$
  - (b)  $\vec{F} = m\alpha^2(y\hat{x} + 2x\hat{y})$
  - (c)  $\vec{F} = 2m\alpha^2(y\hat{x} + x\hat{y})$
  - (d)  $\vec{F} = m\alpha^2(x\hat{x} + 2y\hat{y})$
3. A particle moves in the  $X$ - $Y$  plane under the influence of a force such that its linear momentum is  $\vec{p}(t) = A[\hat{i} \cos(kt) - \hat{j} \sin(kt)]$ , where  $A$  and  $k$  are constants. The angle between the force and the momentum is [2007]
  - (a)  $0^\circ$
  - (b)  $30^\circ$
  - (c)  $45^\circ$
  - (d)  $90^\circ$



### 4 Fill in the Blanks

4. The magnitude of the force (in newtons) acting on a body varies with time  $t$  (in micro seconds) as shown in the fig  $AB, BC$  and  $CD$  are straight line segments. The magnitude of the total impulse of the force on the body from  $t = 4 \mu\text{s}$  to  $t = 16 \mu\text{s}$  is .....Ns. [1994 - 2 Marks]



### 5 True / False

5. A rocket moves forward by pushing the surrounding air backwards. [1980]



### 6 MCQs with One or More than One Correct Answer

6. A reference frame attached to the earth [1986 - 2 Marks]
  - (a) is an inertial frame by definition.
  - (b) cannot be an inertial frame because the earth is revolving round the sun.
  - (c) is an inertial frame because Newton's laws are applicable in this frame.
  - (d) cannot be an inertial frame because the earth is rotating about its own axis.



### 9 Assertion and Reason Type Questions

7. **Statement-1** : It is easier to pull a heavy object than to push it on a level ground and  
**Statement-2** : The magnitude of frictional force depends on the nature of the two surfaces in contact. [2008]
  - (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
  - (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
  - (c) Statement-1 is True, Statement-2 is False
  - (d) Statement-1 is False, Statement-2 is True
8. **Statement-1** : A cloth covers a table. Some dishes are kept on it. The cloth can be pulled out without dislodging the dishes from the table.  
**Statement-2** : For every action there is an equal and opposite reaction. [2007]
  - (a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
  - (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
  - (c) Statement-1 is True, Statement-2 is False
  - (d) Statement-1 is False, Statement-2 is True.





## Topic-2: Motion of Connected Bodies, Pulley & Equilibrium of Forces

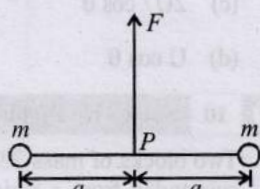


### 1 MCQs with One Correct Answer

1. Two particles of mass  $m$  each are tied at the ends of a light string of length  $2a$ . The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance ' $a$ ' from the centre  $P$  (as shown in the figure).

Now, the mid-point of the string is pulled vertically upwards with a small but constant force  $F$ . As a result, the particles move towards each other on the surface. The magnitude of acceleration, when the separation between them becomes  $2x$ , is [2007]

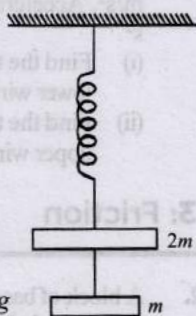
- (a)  $\frac{F}{2m} \frac{a}{\sqrt{a^2 - x^2}}$   
 (b)  $\frac{F}{2m} \frac{x}{\sqrt{a^2 - x^2}}$   
 (c)  $\frac{F}{2m} \frac{x}{a}$   
 (d)  $\frac{F}{2m} \frac{\sqrt{a^2 - x^2}}{x}$



2. The string between blocks of mass  $m$  and  $2m$  is massless and inextensible. The system is suspended by a massless spring as shown. If the string is cut find the magnitudes of accelerations of mass  $2m$  and  $m$  (immediately after cutting)

[2006 - 3M, -1]

- (a)  $g, g$  (b)  $\frac{g}{2}, \frac{g}{2}$   
 (c)  $\frac{g}{2}, g$  (d)  $\frac{g}{2}, \frac{g}{2}$



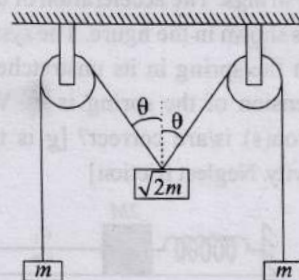
3. A string of negligible mass going over a clamped pulley of mass  $m$  supports a block of mass  $M$  as shown in the figure. The force on the pulley by the clamp is given by [2001S]

- (a)  $\sqrt{2} Mg$   
 (b)  $\sqrt{2} mg$   
 (c)  $\sqrt{(M+m)^2 + m^2} g$   
 (d)  $\sqrt{(M+m)^2 + M^2} g$



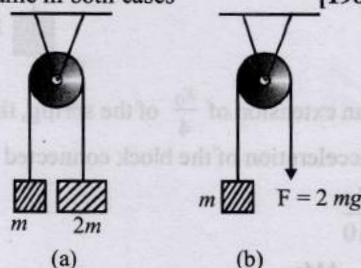
4. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle  $\theta$  should be [2001S]

- (a)  $0^\circ$   
 (b)  $30^\circ$   
 (c)  $45^\circ$   
 (d)  $60^\circ$



### 5 True / False

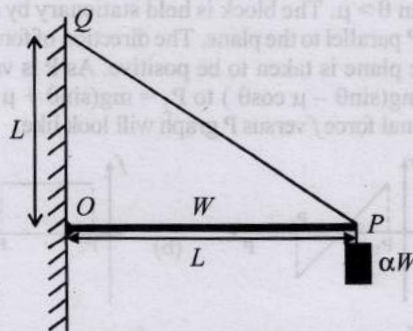
5. The pulley arrangements of Figs. (a) and (b) are identical. The mass of the rope is negligible. In (a) the mass  $m$  is lifted up by attaching a mass  $2m$  to the other end of the rope. In (b),  $m$  is lifted up by pulling the other end of the rope with a constant downward force  $F = 2mg$ . The acceleration of  $m$  is the same in both cases [1984 - 2 Marks]



### 6 MCQs with One or More than One Correct Answer

6. One end of a horizontal uniform beam of weight  $W$  and length  $L$  is hinged on a vertical wall at point  $O$  and its other end is supported by a light inextensible rope. The other end of the rope is fixed at point  $Q$ , at a height  $L$  above the hinge at point  $O$ . A block of weight  $\alpha W$  is attached at the point  $P$  of the beam, as shown in the figure (not to scale). The rope can sustain a maximum tension of  $(2\sqrt{2}) W$ . Which of the following statement(s) is(are) correct?

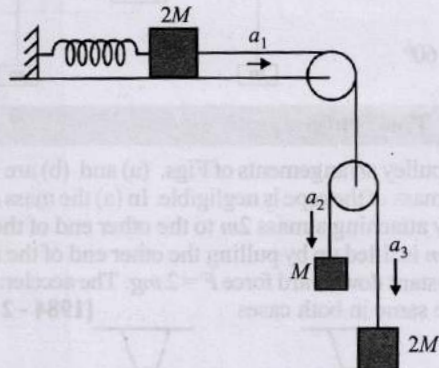
[Adv. 2021]



- (a) The vertical component of reaction force at  $O$  does not depend on  $\alpha$   
 (b) The horizontal component of reaction force at  $O$  is equal to  $W$  for  $\alpha = 0.5$   
 (c) The tension in the rope is  $2W$  for  $\alpha = 0.5$   
 (d) The rope breaks if  $\alpha > 1.5$



7. A block of mass  $2M$  is attached to a massless spring with spring-constant  $k$ . This block is connected to two other blocks of masses  $M$  and  $2M$  using two massless pulleys and strings. The acceleration of the blocks are  $a_1$ ,  $a_2$  and  $a_3$  as shown in the figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is  $x_0$ . Which of the following option(s) is/are correct? [ $g$  is the acceleration due to gravity. Neglect friction] [Adv. 2019]



- (a) At an extension of  $\frac{x_0}{4}$  of the spring, the magnitude of acceleration of the block connected to the spring is  $\frac{3g}{10}$
- (b)  $x_0 = \frac{4Mg}{k}$

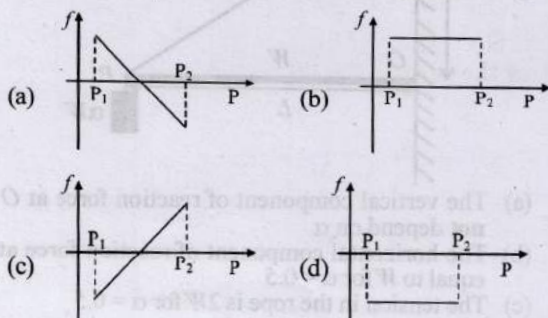


### Topic-3: Friction



#### 1 MCQs with One Correct Answer

1. A block of mass  $m$  is on an inclined plane of angle  $\theta$ . The coefficient of friction between the block and the plane is  $\mu$  and  $\tan \theta > \mu$ . The block is held stationary by applying a force  $P$  parallel to the plane. The direction of force pointing up the plane is taken to be positive. As  $P$  is varied from  $P_1 = mg(\sin \theta - \mu \cos \theta)$  to  $P_2 = mg(\sin \theta + \mu \cos \theta)$ , the frictional force  $f$  versus  $P$  graph will look like [2010]



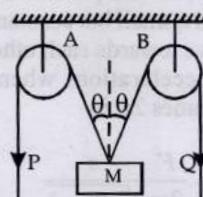
- (c) When spring achieves an extension of  $\frac{x_0}{2}$  for the first time, the speed of the block connected to the

spring is  $3g \sqrt{\frac{M}{5k}}$

- (d)  $a_2 - a_1 = a_1 - a_3$

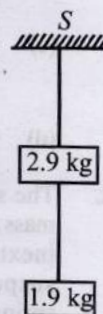
8. In the arrangement shown in the Fig, the ends  $P$  and  $Q$  of an unstretchable string move downwards with uniform speed  $U$ . Pulleys  $A$  and  $B$  are fixed. [1982-3 Marks] Mass  $M$  moves upwards with a speed

- (a)  $2U \cos \theta$   
(b)  $U / \cos \theta$   
(c)  $2U / \cos \theta$   
(d)  $U \cos \theta$



#### 10 Subjective Problems

9. Two blocks of mass  $2.9 \text{ kg}$  and  $1.9 \text{ kg}$  are suspended from a rigid support  $S$  by two inextensible wires each of length  $1 \text{ meter}$ , see fig. The upper wire has negligible mass and the lower wire has a uniform mass of  $0.2 \text{ kg/m}$ . The whole system of blocks wires and support have an upward acceleration of  $0.2 \text{ m/s}^2$ . Acceleration due to gravity is  $9.8 \text{ m/s}^2$ . [1989 - 6 Marks]

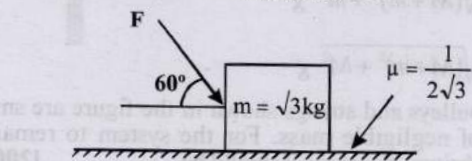


- (i) Find the tension at the mid-point of the lower wire.  
(ii) Find the tension at the mid-point of the upper wire.

2. A block of base  $10 \text{ cm} \times 10 \text{ cm}$  and height  $15 \text{ cm}$  is kept on an inclined plane. The coefficient of friction between them is  $\sqrt{3}$ . The inclination  $\theta$  of this inclined plane from the horizontal plane is gradually increased from  $0^\circ$ . Then

[2009]

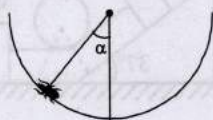
- (a) at  $\theta = 30^\circ$ , the block will start sliding down the plane  
(b) the block will remain at rest on the plane up to certain  $\theta$  and then it will topple  
(c) at  $\theta = 60^\circ$ , the block will start sliding down the plane and continue to do so at higher angles  
(d) at  $\theta = 60^\circ$ , the block will start sliding down the plane and on further increasing  $\theta$ , it will topple at certain  $\theta$ .
3. What is the maximum value of the force  $F$  such that the block shown in the arrangement, does not move? [2003S]



- (a)  $20 \text{ N}$  (b)  $10 \text{ N}$  (c)  $12 \text{ N}$  (d)  $15 \text{ N}$



4. An insect crawls up a hemispherical surface very slowly (see fig.). The coefficient of friction between the insect and the surface is  $1/3$ . If the line joining the center of the hemispherical surface to the insect makes an angle  $\alpha$  with the vertical, the maximum possible value of  $\alpha$  is given by [2001S]



- (a)  $\cot \alpha = 3$  (b)  $\tan \alpha = 3$   
(c)  $\sec \alpha = 3$  (d)  $\operatorname{cosec} \alpha = 3$
5. A block of mass  $0.1 \text{ kg}$  is held against a wall applying a horizontal force of  $5 \text{ N}$  on the block. If the coefficient of friction between the block and the wall is  $0.5$ , the magnitude of the frictional force acting on the block is :

[1994 - 1 Mark]

- (a)  $2.5 \text{ N}$  (b)  $0.98 \text{ N}$  (c)  $4.9 \text{ N}$  (d)  $0.49 \text{ N}$
6. A block of mass  $2 \text{ kg}$  rests on a rough inclined plane making an angle of  $30^\circ$  with the horizontal. The coefficient of static friction between the block and the plane is  $0.7$ . The frictional force on the block is [1980]
- (a)  $9.8 \text{ N}$  (b)  $0.7 \times 9.8 \times \sqrt{3} \text{ N}$   
(c)  $9.8 \times \sqrt{3} \text{ N}$  (d)  $0.7 \times 9.8 \text{ N}$



2

## Integer Value Answer

7. A block is moving on an inclined plane making an angle  $45^\circ$  with the horizontal and the coefficient of friction is  $\mu$ . The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define  $N = 10 \mu$ , then  $N$  is [2011]
8. A block of mass  $1 \text{ kg}$  lies on a horizontal surface in a truck. The coefficient of static friction between the block and the surface is  $0.6$ . If the acceleration of the truck is  $5 \text{ m/s}^2$ , the frictional force acting on the block is ..... newtons.

[1984 - 2 Marks]



5

## True / False

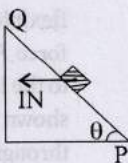
9. When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of his motion. [1981 - 2 Marks]



6

## MCQs with One or More than One Correct Answer

10. A small block of mass of  $0.1 \text{ kg}$  lies on a fixed inclined plane  $PQ$  which makes an angle  $\theta$  with the horizontal. A horizontal force of  $1 \text{ N}$  acts on the block through its centre of mass as shown in the figure.



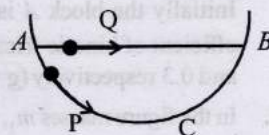
[2012]

The block remains stationary if (take  $g = 10 \text{ m/s}^2$ )

- (a)  $\theta = 45^\circ$   
(b)  $\theta > 45^\circ$  and a frictional force acts on the block towards  $P$ .  
(c)  $\theta > 45^\circ$  and a frictional force acts on the block towards  $Q$ .  
(d)  $\theta < 45^\circ$  and a frictional force acts on the block towards  $Q$ .

11. A particle  $P$  is sliding down a frictionless hemispherical bowl. It passes the point  $A$  at  $t = 0$ . At this instant of time, the horizontal component of its velocity is  $v$ . A bead  $Q$  of the same mass as  $P$  is ejected from  $A$  at  $t = 0$  along the horizontal string  $AB$ , with the speed  $v$ . Friction between the bead and the string may be neglected. Let  $t_P$  and  $t_Q$  be the respective times taken by  $P$  and  $Q$  to reach the point  $B$ . Then : [1993-2 Marks]

- (a)  $t_P < t_Q$   
(b)  $t_P = t_Q$   
(c)  $t_P > t_Q$   
(d)  $\frac{t_P}{t_Q} = \frac{\text{length of arc } ACB}{\text{length of arc } AB}$



7

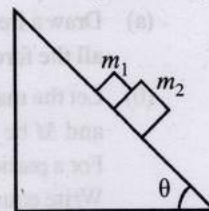
## Match the Following

12. A block of mass  $m_1 = 1 \text{ kg}$  another mass  $m_2 = 2 \text{ kg}$ , are placed together (see figure) on an inclined plane with angle of inclination  $\theta$ . Various values of  $\theta$  are given in List-I. The coefficient of friction between the block  $m_1$  and plane is always zero. The coefficient of static and dynamic friction between the block  $m_2$  and the plane are equal to  $\mu = 0.3$ . In List-II expressions for the friction on block  $m_2$  are given. Match the correct expression of the friction in List-II with the angles given in List-I, and choose the correct option. The acceleration due to gravity is denoted by  $g$ .  
[Useful information:  $\tan(5.5^\circ) \approx 0.1$ ;  $\tan(11.5^\circ) \approx 0.2$ ;  $\tan(16.5^\circ) \approx 0.3$ ]  
[Adv. 2014]

List-I	List-II
P. $\theta = 5^\circ$	1. $m_2 g \sin \theta$
Q. $\theta = 10^\circ$	2. $(m_1 + m_2) g \sin \theta$
R. $\theta = 15^\circ$	3. $\mu m_2 g \cos \theta$
S. $\theta = 20^\circ$	4. $\mu(m_1 + m_2) g \cos \theta$

Code:

- (a) P-1, Q-1, R-1, S-3 (b) P-2, Q-2, R-2, S-3  
(c) P-2, Q-2, R-2, S-4 (d) P-2, Q-2, R-3, S-3



## 10 Subjective Problems

13. A circular disc with a groove along its diameter is placed horizontally on a rough surface.

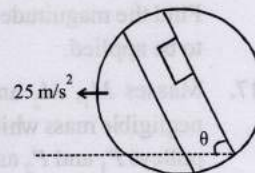
A block of mass  $1 \text{ kg}$  is placed as shown. The co-efficient of friction between the block and all surfaces of groove and horizontal surface in contact is

$$\mu = \frac{2}{5}$$

The disc has an acceleration of  $25 \text{ m/s}^2$  towards left. Find the acceleration of the block with respect to disc. Given

$$\cos \theta = \frac{4}{5}, \sin \theta = \frac{3}{5}$$

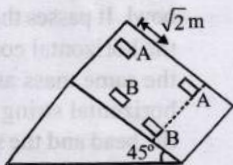
[2006 - 6M]





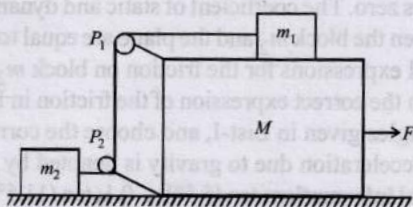
14. Two block  $A$  and  $B$  of equal masses are placed on rough inclined plane as shown in figure.

When and where will the two blocks come on the same line on the inclined plane if they are released simultaneously?



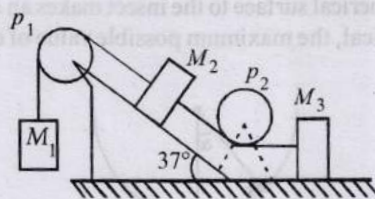
Initially the block  $A$  is  $\sqrt{2} m$  behind the block  $B$ . Coefficient of kinetic friction for the blocks  $A$  and  $B$  are 0.2 and 0.3 respectively ( $g=10 \text{ m/s}^2$ ). [2004 - Marks]

15. In the figure masses  $m_1$ ,  $m_2$  and  $M$  are 20 kg, 5 kg and 50 kg respectively. The coefficient of friction between  $M$  and ground is zero. The coefficient of friction between  $m_1$  and  $M$  and that between  $m_2$  and ground is 0.3. The pulleys and the strings are massless. The string is perfectly horizontal between  $P_1$  and  $m_1$  and also between  $P_2$  and  $m_2$ . The string is perfectly vertical between  $P_1$  and  $P_2$ . An external horizontal force  $F$  is applied to the mass  $M$ . Take  $g = 10 \text{ m/s}^2$ . [2000 - 10 Marks]



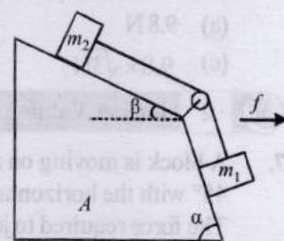
- Draw a free body diagram for mass  $M$ , clearly showing all the forces.
  - Let the magnitude of the force of friction between  $m_1$  and  $M$  be  $f_1$  and that between  $m_2$  and ground be  $f_2$ . For a particular  $F$  it is found that  $f_1 = 2f_2$ . Find  $f_1$  and  $f_2$ . Write equations of motion of all the masses. Find  $F$ , tension in the string and acceleration of the masses.
16. A particle of mass  $m$  rests on a horizontal floor with which it has a coefficient of static friction  $\mu$ . It is desired to make the body move by applying the minimum possible force  $F$ . Find the magnitude of  $F$  and the direction in which it has to be applied. [1987 - 7 Marks]
17. Masses  $M_1$ ,  $M_2$  and  $M_3$  are connected by strings of negligible mass which pass over massless and friction less pulleys  $P_1$  and  $P_2$  as shown in fig. The masses move such that the portion of the string between  $P_1$  and  $P_2$  is parallel to the inclined plane and the portion of the string between  $P_2$  and  $M_3$  is horizontal. The masses  $M_2$  and  $M_3$  are 4.0 kg each and the coefficient of kinetic friction between the

masses and the surfaces is 0.25. The inclined plane makes an angle of  $37^\circ$  with the horizontal. [1981 - 6 Marks]

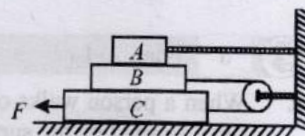


If the mass  $M_1$  moves downwards with a uniform velocity, find

- the mass of  $M_1$
  - The tension in the horizontal portion of the string ( $g = 9.8 \text{ m/sec}^2$ ,  $\sin 37^\circ \approx 3/5$ )
18. A horizontal uniform rope of length  $L$ , resting on a frictionless horizontal surface, is pulled at one end by force  $F$ . What is the tension in the rope at a distance  $l$  from the end where the force is applied? [1978]
19. Two cubes of masses  $m_1$  and  $m_2$  be on two frictionless slopes of block  $A$  which rests on a horizontal table. The cubes are connected by a string which passes over a pulley as shown in the figure. To what horizontal acceleration  $f$  should the whole system (that is blocks and cubes) be subjected so that the cubes do not slide down the planes. What is the tension of the string in this situation? [1978]
20. In the diagram shown,



the blocks  $A$ ,  $B$  and  $C$  weighs, 3 kg, 4 kg and 5 kg respectively. The coefficient of sliding friction between any



two surface is 0.25.  $A$  is held at rest by a massless rigid rod fixed to the wall while  $B$  and  $C$  are connected by a light flexible cord passing around a frictionless pulley. Find the force  $F$  necessary to drag  $C$  along the horizontal surface to the left at constant speed. Assume that the arrangement shown in the diagram,  $B$  on  $C$  and  $A$  on  $B$ , is maintained all through. ( $g = 9.8 \text{ m/s}^2$ ) [1978]



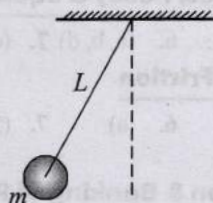


### Topic-4: Circular Motion & Banking of Road



#### 1 MCQs with One Correct Answer

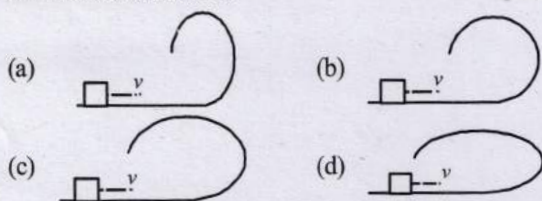
1. A ball of mass ( $m$ ) 0.5 kg is attached to the end of a string having length ( $L$ ) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is



[2011]

- (a) 9 (b) 18 (c) 27 (d) 36
2. A small block is shot into each of the four tracks as shown below. Each of the tracks rises to the same height. The speed with which the block enters the track is the same in all cases. At the highest point of the track, the normal reaction is maximum in

[2001S]

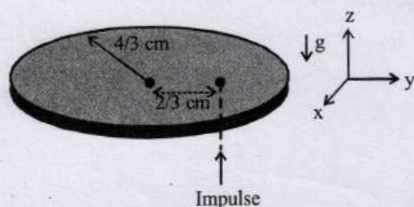


#### 2 Integer Value Answer

3. A thin circular coin of mass 5 gm and radius  $4/3$  cm is initially in a horizontal  $xy$ -plane. The coin is tossed vertically up ( $+z$  direction) by applying an impulse of  $\sqrt{\frac{\pi}{2}} \times 10^{-2}$  N-s at a distance  $2/3$  cm from its center. The coin spins about its diameter and moves along the  $+z$  direction. By the time the coin reaches back to its initial position, it completes  $n$  rotations. The value of  $n$  is \_\_\_\_\_.

[Given: The acceleration due to gravity  $g = 10 \text{ m s}^{-2}$ ]

[Adv. 2023]



#### 5 True / False

4. A simple pendulum with a bob of mass  $m$  swings with an angular amplitude of  $40^\circ$ . When its angular displacement

is  $20^\circ$ , the tension in the string is greater than  $mg \cos 20^\circ$ .  
[1984 - 2 Marks]

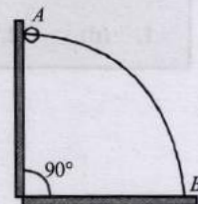


#### 6 MCQs with One or More than One Correct Answer

5. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from  $A$  to  $B$ , the force it applies on the wire is

[Adv. 2014]

- (a) always radially outwards  
(b) always radially inwards  
(c) radially outwards initially and radially inwards later  
(d) radially inwards initially and radially outwards later



6. A simple pendulum of length  $L$  and mass (bob)  $M$  is oscillating in a plane about a vertical line between angular limit  $-\phi$  and  $+\phi$ . For an angular displacement  $\theta$  ( $|\theta| < \phi$ ), the tension in the string and the velocity of the bob are  $T$  and  $V$  respectively. The following relations hold good under the above conditions :

[1986 - 2 Marks]

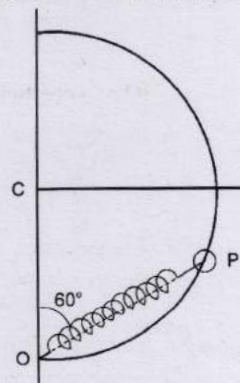
- (a)  $T \cos \theta = Mg$ .  
(b)  $T - Mg \cos \theta = \frac{MV^2}{L}$   
(c) The magnitude of the tangential acceleration of the bob  $|a_T| = g \sin \theta$   
(d)  $T = Mg \cos \theta$



#### 10 Subject Problems

7. A smooth semicircular wire-track of radius  $R$  is fixed in a vertical plane. One end of a massless spring of natural length  $3R/4$  is attached to the lowest point  $O$  of the wire-track. A small ring of mass  $m$ , which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point  $P$  such that the spring makes an angle of  $60^\circ$  with the vertical. The spring constant  $K = mg/R$ . Consider the instant when the ring is released, and (i) draw the free body diagram of the ring, (ii) determine the tangential acceleration of the ring and the normal reaction.

[1996 - 5 Marks]







## Answer Key

### Topic-1 : 1st, 2nd & 3rd Laws of Motion

1. (c) 2. (a) 3. (d) 4. (0.005) 5. False 6. (b, d) 7. (b) 8. (b)

### Topic-2 : Motion of Connected Bodies, Pulley & Equilibrium of Forces

1. (b) 2. (c) 3. (d) 4. (c) 5. False 6. (a, b, d) 7. (d) 8. (b)

### Topic-3 : Friction

1. (a) 2. (b) 3. (a) 4. (a) 5. (b) 6. (a) 7. (5) 8. (5) 9. False 10. (a, c)  
11. (a) 12. (d)

### Topic-4 : Circular Motion & Banking of Road

1. (d) 2. (a) 3. (30) 4. False 5. (d) 6. (b, c)



# Hints & Solutions



## Topic-1: 1st, 2nd & 3rd Laws of Motion

1. (c) Given mass of block = 5 kg moving along the x-direction subject to the force  $F = (-20x + 10)$  N with the value of x in metre.

$$\text{Acceleration } a = \frac{F}{m} \leftarrow \frac{F = (-20x + 10) \text{ N}}{m = 5 \text{ kg}} \quad t = 0;$$

$$v = 0; x = 1 \text{ m}$$

$$= \frac{-20x + 10}{5} = -4x + 2$$

$$\text{Also, } a = \frac{v dx}{dx} = -4x + 2$$

$$\therefore \int v dv = \int_1^x (-4x + 2) dx \rightarrow \frac{v^2}{2} = (-2x^2 + 2x)_1^x$$

$$\text{or, } v = -2\sqrt{x - x^2} \quad [\text{since particle starts moving in -ve x-direction}]$$

$$\therefore \frac{dx}{dt} = -2\sqrt{x - x^2} \Rightarrow \int_{x=1}^{x=x} \frac{dx}{\sqrt{x - x^2}} = -2 \int_0^{\frac{\pi}{4}} dt$$

$$\text{or, } \sin^{-1} [2x - 1]_1^x = -\frac{\pi}{2}$$

$$\therefore \text{Position } x = 0.5 \text{ m}$$

$$\text{And since } v = -2\sqrt{x - x^2} = -2\sqrt{0.5 - (0.5)^2} = -1 \text{ m/s}$$

$$\therefore \text{Momentum } P = mv = 5(-1) = -5 \text{ kg ms}^{-1}$$

2. (a)  $\therefore \vec{v} = \alpha(y\hat{x} + 2x\hat{y})$   
 $\therefore a = \frac{d\vec{v}}{dt} = \alpha \left( \frac{dy}{dt} \hat{x} + 2 \frac{dx}{dt} \hat{y} \right)$   
 $= \alpha(v_y \hat{x} + 2v_x \hat{y}) = \alpha(2x\alpha \hat{x} + 2\alpha y \hat{y}) = 2\alpha^2 [x\hat{x} + y\hat{y}]$

3. (d) Given : momentum  $\vec{p}(t) = A[\hat{i} \cos(kt) - \hat{j} \sin(kt)]$

$$\text{And, force, } \vec{F} = \frac{d\vec{p}}{dt} = Ak[-\hat{i} \sin(kt) - \hat{j} \cos(kt)]$$

$$\text{Here, } \vec{F} \cdot \vec{p} = 0 \quad \text{But } \vec{F} \cdot \vec{p} = Fp \cos \theta$$

$$\therefore \cos \theta = 0 \Rightarrow \theta = 90^\circ$$

$$\text{Hence, angle between the force momentum, } \theta = 90^\circ$$

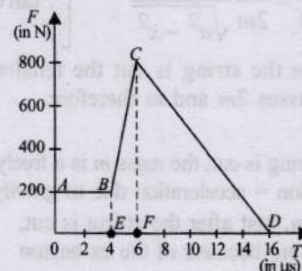
4. (0.005) Area under the  $F - t$  graph gives the impulse imparted to the body.

$$\text{The magnitude of total impulse of force on the body from}$$

$$t = 4 \mu\text{s} \text{ to } t = 16 \mu\text{s}$$

$$= \text{area (BCDFEB)}$$

$$= \text{area of BCFEB} + \text{area CDFC}$$



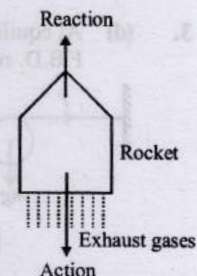
$$= \frac{1}{2} (200 + 800) \times 2 \times 10^{-6} + \frac{1}{2} \times 10 \times 800 \times 10^{-6}$$

$$= 0.001 + 0.004 = 0.005 \text{ N s}$$

5. **False** : The forward motion of rocket is due to the exhaust gases, thrown backward not due to surrounding air pushing backwards.

Here exhaust gases thrown backwards is action and rocket moving forward is reaction.

This phenomenon takes place in the absence of air as well.



6. (b, d) Earth is an accelerated frame and hence, cannot be an inertial frame.

Earth is revolving round the sun and is rotating about its own axis.

7. (b) It is easier to pull a heavy object than to push it on a level ground. This is because the normal reaction in the case of pulling is less as compared by pushing. ( $f = \mu N$ ). Therefore the frictional force is small in case of pulling.

The magnitude of frictional force depends on the nature of the two surfaces in contact. But is not the correct explanation of statement-1.

8. (b) Cloth can be pulled out without dislodging the dishes from the table because of inertia.

Law of inertia is the Newton's first law of motion.

For every action there is an equal and opposite reaction. This is Newton's third law of motion.

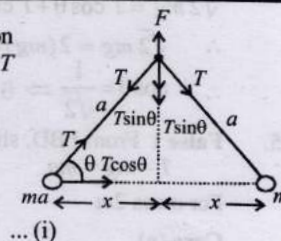


## Topic-2: Motion of Connected Bodies, Pulley & Equilibrium of Forces

1. (b) From figure, acceleration of mass  $m$  is due to the force  $T \cos \theta$

$$\therefore T \cos \theta = ma$$

$$\Rightarrow a = \frac{T \cos \theta}{m}$$





$$\text{also, } F = 2T \sin \theta \Rightarrow T = \frac{F}{2 \sin \theta}$$

Putting this value of  $T$  in eqn. (i)

$$a = \left( \frac{F}{2 \sin \theta} \right) \frac{\cos \theta}{m} = \frac{F}{2m \tan \theta} = \frac{F}{2m} \frac{x}{\sqrt{a^2 - x^2}} \left[ \because \tan \theta = \frac{\sqrt{a^2 - x^2}}{x} \right]$$

2. (c) Before the string is cut the tension  $T$  has to hold both the masses  $2m$  and  $m$  therefore,

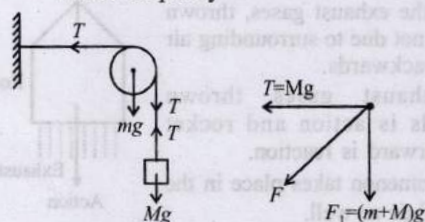
$$T = 3mg$$

When the string is cut, the mass  $m$  is a freely falling body and its acceleration = acceleration due to gravity =  $g$ .

For mass  $2m$ , just after the string is cut,  $T$  remains  $3mg$  because of the extension of string.

$$\therefore 3mg - 2mg = 2m \times a \Rightarrow \frac{g}{2} = a$$

3. (d) At equilibrium  $T = Mg$   
F.B.D. of pulley



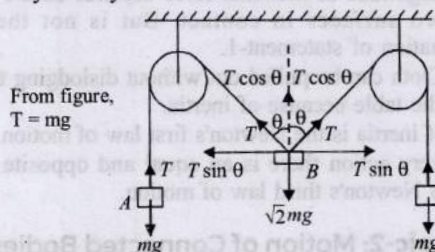
$$F_1 = (m + M)g$$

The resultant force on pulley

$$F = \sqrt{F_1^2 + T^2} = [\sqrt{(m + M)^2 + M^2}]g$$

As pulley is on rest. So force applied by clamp should be equal to ' $F$ ' and opposite to it.

4. (c) The tension in both strings will be same due to symmetry.



For equilibrium

$$\sqrt{2}mg = T \cos \theta + T \cos \theta = 2T \cos \theta$$

$$\therefore \sqrt{2}mg = 2(mg) \cos \theta$$

$$\therefore \cos \theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^\circ$$

5. **False** : From FBD, shown in case (a) for mass  $m$

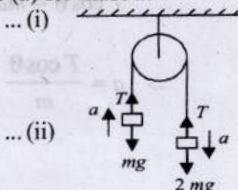
$$T - mg = ma \quad \dots (i)$$

For mass  $2m$

Case (a)

$$2mg - T = 2ma$$

From (i) and (ii)



$$a = g/3$$

$$\text{Case (b) } T - mg = ma'$$

$$\Rightarrow 2mg - mg = ma'$$

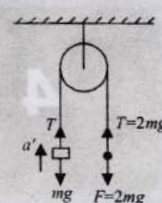
$$[\because T = 2mg]$$

$$\therefore a' = g$$

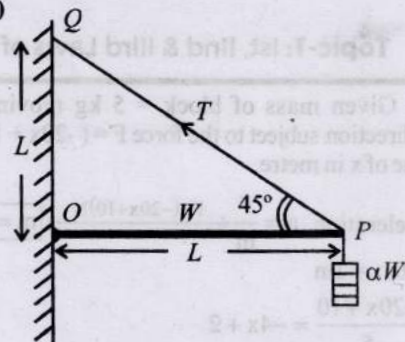
Hence, from eq (iii) & (iv)  $a < a'$

... (iii)

... (iv)



6. (a, b, d)



Since  $OQ = OP$

$$\therefore \angle P = \angle Q = 45^\circ$$

At equilibrium, about point 'O'

$$R_y + \frac{T}{\sqrt{2}} = W + \alpha W \quad \dots (i)$$

$$\text{and } R_x = \frac{T}{\sqrt{2}} \quad \dots (ii)$$

Torque about point 'O' is zero

$$\text{So, } W \frac{L}{2} + \alpha WL = \frac{T}{\sqrt{2}} L \therefore T = \sqrt{2} \left( \frac{W}{2} + \alpha W \right) \quad \dots (iii)$$

$$\therefore R_x = \frac{T}{\sqrt{2}} = \left( \frac{W}{2} + \alpha W \right)$$

Therefore for  $\alpha = 0.5$

$$R_x = \frac{W}{2} + \alpha W = \frac{W}{2} + 0.5W$$

$$\text{or } R_x = W$$

i.e., the horizontal component of reaction force at, O,  $R_x = W$  for  $\alpha = 0.5$

Now torque about point P

$$T_y L = W \frac{L}{2}$$

$$\Rightarrow R_y = \frac{W}{2}$$

The vertical component of reaction force at O does not depend on  $\alpha$

As per question, rope can sustain a maximum tension of  $2\sqrt{2}W$

$$\therefore 2\sqrt{2}W = \sqrt{2} \left( \frac{W}{2} + \alpha W \right)$$



$$\Rightarrow 2 = \frac{1}{2} + \alpha$$

$$\therefore \alpha = \frac{3}{2}$$

7. (d) According to constraint relation from figure,

$$a_1 = \frac{a_2 + a_3}{2}$$

$$\Rightarrow a_2 + a_3 = 2a_1$$

$$\Rightarrow a_2 + a_3 = a_1 + a_1$$

$$\Rightarrow a_1 - a_3 = a_2 - a_1$$

$$\Rightarrow \text{Option (d) is correct}$$

Let 'x' be the extension of the spring at a certain instant

$$2T - Kx = 2Ma_1$$

$$2Mg - T = 2Ma_3$$

$$Mg - T = Ma_2$$

On solving we get,

$$a_1 = \frac{4g}{7} - \frac{3kx}{14M} = \frac{-3K}{14M} \left( x - \frac{8mg}{3K} \right) \quad \dots(i)$$

Comparing it with  $a = -\omega^2(x - x_0)$

$$\therefore \omega^2 = \frac{3k}{14M} \quad \therefore \omega = \sqrt{\frac{3k}{14M}}$$

$$\text{and } T = \frac{4Mg}{7} + \frac{2kx}{7} \quad \dots(ii)$$

For  $a_1 = 0$  (Maximum extension of spring) we have from (i)

$$\frac{4g}{7} - \frac{3kx}{14M} = 0$$

$$\therefore 4g = \frac{3kx}{2M} \quad \therefore x = \frac{8Mg}{3k}$$

$$\therefore x_0 = 2x = \frac{16Mg}{3k}$$

$$\text{For } x = \frac{x_0}{4} = \frac{1}{4} \left( \frac{16Mg}{3k} \right) = \frac{4Mg}{3k}$$

$$\text{From eqn. (i) } a_1 = \frac{4g}{7} - \frac{3k}{14M} \times \frac{4Mg}{3k} = \frac{2g}{7}$$

At  $x = \frac{x_0}{2}$  particle is at mean position and its velocity =  $A\omega$

$$= \frac{x_0}{2} \sqrt{\frac{3k}{14M}} = \frac{8Mg}{3k} \sqrt{\frac{3k}{14M}}$$

8. (b) Here from figure,  $AN = x$  (= constant as pulley A and

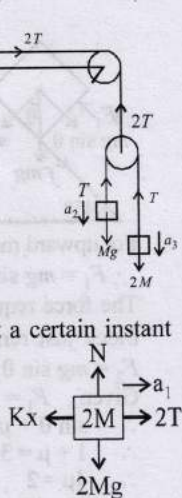
B are fixed),  $NO = z$ . Then velocity of mass  $m = \frac{dz}{dt}$ . Also,

$$\text{let } OA = \ell \text{ then } \frac{d\ell}{dt} = U$$

From  $\triangle ANO$

$$x^2 + z^2 = \ell^2 \quad \dots(i)$$

Differentiating

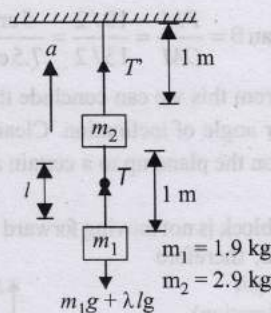


equation (i) w.r.t to t

$$0 + 2z \frac{dz}{dt} = 2\ell \frac{d\ell}{dt} \Rightarrow zv_M = \ell U$$

$$\Rightarrow v_M = \frac{\ell}{z} U = \frac{U}{z/\ell} = \frac{U}{\cos \theta} \quad \left( \because \cos \theta = \frac{z}{\ell} \right)$$

9.  $\ell$  = Mass of unit length of wire = 0.2 kg/m.



- (i) Tension T at midpoint of lower wire :

$$\ell = \text{Half-length} = 0.5 \text{ m}$$

$$\therefore T - (m_1 + \lambda \ell)g = (m_1 + \lambda \ell)a$$

$$T = (m_1 + \lambda \ell)(a + g)$$

$$= [1.9 + (0.2 \times 0.5)](0.2 + 9.8) = 2 \times 10 = 20 \text{ N.}$$

- (ii) Tension T' at mid-point of upper wire :

$$\therefore T' = [m_1 + (\lambda \times 2\ell) + m_2]a + [m_2g + \lambda \times 2\ell g + m_1g]$$

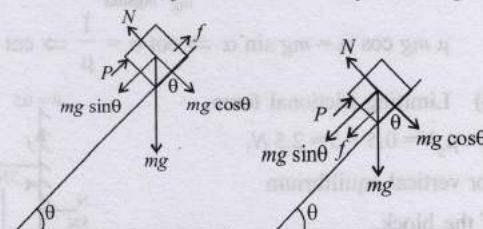
$$\text{or } T' = [m_1 + (\lambda \times 2\ell) + m_2](a + g)$$

$$= [1.9 + (0.2 \times 1) + 2.9][0.2 + 9.8] = 5 \times 10 = 50 \text{ N.}$$



### Topic-3: Friction

1. (a) According to question,  $\tan \theta > \mu$ , so block has a tendency to move down the incline. Force P is applied upwards along the incline to keep the block stationary. Here, at equilibrium  $P + f = mg \sin \theta \Rightarrow f = mg \sin \theta - P$ . Now as P increases, f decreases linearly with respect to P.



When  $P = mg \sin \theta$ ,  $f = 0$ .

When force P is increased further, the block has a tendency to move upwards along the incline and hence frictional force acts downwards along the incline.

Here, at equilibrium  $P = f + mg \sin \theta$

$$\Rightarrow f = P - mg \sin \theta$$

Now as P increases, f increases linearly w.r.t P.

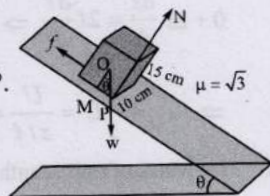
Hence graph (a) correctly depicts the situation.



2. (b) Maximum angle not to slide the block, angle of inclination = angle of repose,

$$\text{i.e., } \tan^{-1} \mu = \tan^{-1} \sqrt{3} = 60^\circ.$$

For the block to topple, the condition of the block has been shown in the figure.



$$\text{In } \triangle POM, \tan \theta = \frac{PM}{OM} = \frac{10/2}{15/2} = \frac{5 \text{ cm}}{7.5 \text{ cm}} = \frac{2}{3}$$

So,  $\theta < 60^\circ$ . From this we can conclude that the block will topple at lesser angle of inclination. Clearly the block will remain at rest on the plane up to a certain angle  $\theta$  and then it will topple.

3. (a) Since the block is not moving forward for the maximum force  $F$  applied, therefore

$$F \cos 60^\circ = f = \mu N$$

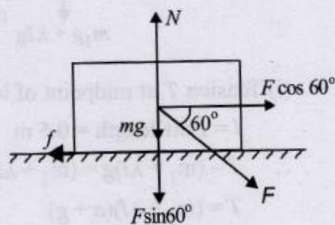
(Horizontal direction)

For vertical equilibrium of the block,

$$N = mg + F \sin 60^\circ$$

$$\therefore F \cos 60^\circ = \mu N = \mu$$

$$[F \sin 60^\circ + mg]$$



$$\Rightarrow F = \frac{\mu mg}{\cos 60^\circ - \mu \sin 60^\circ} = \frac{\frac{1}{2\sqrt{3}} \times \sqrt{3} \times 10}{\frac{1}{2} - \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{2}} = \frac{5}{\frac{1}{4}} = 20 \text{ N}$$

4. (a) The two forces acting on the insect are  $mg$  and  $N$ . Two components of  $mg$  are

$mg \cos \alpha$  balances  $N$ .

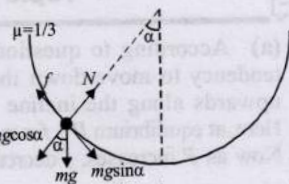
$mg \sin \alpha$  is balanced by the frictional force.

$$\therefore N = mg \cos \alpha$$

$$f = mg \sin \alpha$$

$$\text{But } f = \mu N = \mu mg \cos \alpha$$

$$\therefore \mu mg \cos \alpha = mg \sin \alpha \Rightarrow \cot \alpha = \frac{1}{\mu} \Rightarrow \cot \alpha = 3$$

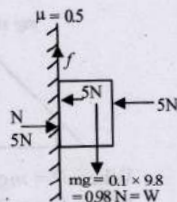


5. (b) Limiting frictional force,

$$f_l = \mu_s N = 0.5 \times 5 = 2.5 \text{ N.}$$

For vertical equilibrium of the block,

$$\text{Frictional force, } f = mg = 0.98 \text{ N.}$$

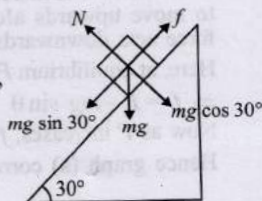


6. (a) The block is at rest.

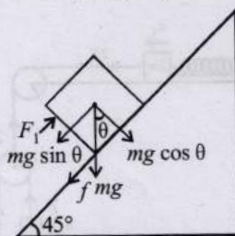
For equilibrium, frictional force,

$$f = mg \sin \theta = mg \sin 30^\circ$$

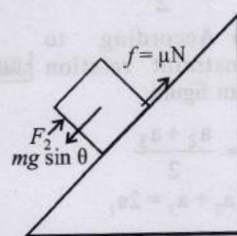
$$= 2 \times 9.8 \times \frac{1}{2} = 9.8 \text{ N}$$



7. (5) Block moving upward



Block just remains stationary



For upward moving of block, pushing force  $F_1 = mg \sin \theta + f$

$$\therefore F_1 = mg \sin \theta + \mu mg \cos \theta = mg (\sin \theta + \mu \cos \theta)$$

The force required to just prevent it from sliding down or block just remains stationary.

$$F_2 = mg \sin \theta - \mu N = mg (\sin \theta - \mu \cos \theta)$$

$$\text{Given, } F_1 = 3F_2$$

$$\therefore \sin \theta + \mu \cos \theta = 3(\sin \theta - \mu \cos \theta)$$

$$\therefore 1 + \mu = 3(1 - \mu) \quad [\because \sin \theta = \cos \theta]$$

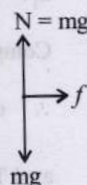
$$\therefore 4\mu = 2 \Rightarrow \mu = 0.5$$

$$\therefore N = 10\mu = 10 \times 0.5 = 5 \text{ N}$$

8. (5) The frictional force is responsible to move the block of mass 1 kg with an acceleration of  $5 \text{ m/s}^2$ .

Therefore, frictional force,

$$f = m \times a = 1 \times 5 = 5 \text{ N.}$$



9. **False** : Friction force opposes the relative motion of the surface of contact.

As the feet pushes the surface in backward direction, so frictional force exerted by the surface on the person is in the direction of his motion.

10. (a, c) The various forces acting on the block are as shown in the figure.

When  $\theta = 45^\circ$ ,  $\sin \theta = \cos \theta$

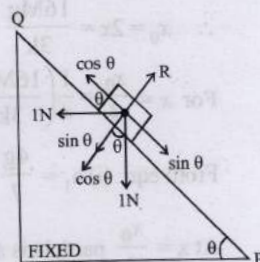
The block will remain stationary and the frictional force is zero.

When  $\theta > 45^\circ$ ,  $\sin \theta > \cos \theta$

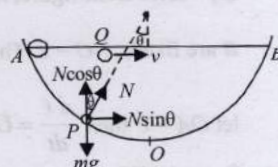
Therefore a frictional force acts towards Q.

When  $\theta < 45^\circ$ ,  $\cos \theta > \sin \theta$

Therefore a frictional force acts towards P.



11. (a) According to question, at A the horizontal speeds of both the masses is the same. As no force is acting in the horizontal direction the velocity of Q remains the same in horizontal.



In case of P as shown in figure at any intermediate position, the horizontal velocity first increases due to  $N \sin \theta$ , reaches a max value at O and then decreases.

But, it always remains greater than v. So,  $t_P < t_Q$ .



12. (d) Block will not slip or will be at rest if

$$(m_1 + m_2)g \sin \theta \leq \mu m_2 g \cos \theta$$

$$\tan \theta \leq \frac{\mu m_2 g}{(m_1 + m_2)g}$$

$$\Rightarrow \tan \theta \leq \frac{\mu m_2}{m_1 + m_2}$$

$$\Rightarrow \tan \theta \leq \frac{0.3 \times 2}{1 + 2} \leq \frac{1}{5}$$

$$\Rightarrow \tan \theta \leq 0.2 \text{ i.e., } \theta \leq 11.5^\circ$$

i.e., If the angle  $\theta < 11.5^\circ$  the frictional force is less than

$$\mu N_2 = \mu m_2 g = 0.3 \times 2 \times g = 0.6 g$$

and is equal to  $(m_1 + m_2)g \sin \theta$

Blocks will not slip on the inclined plane and friction is static.

At  $\theta > 11.5^\circ$  the bodies start moving on the inclined plane and friction is kinetic and equal to  $\mu m_2 g \cos \theta$

13. Normal reaction,
- $N_1 = ma \sin \theta$
- and
- $N_2 = mg$
- 
- Applying pseudo force
- $ma$
- and resolving it.

$$F_{\text{net}} = ma_r$$

$$ma \cos \theta - (f_1 + f_2) = ma_r$$

$$ma \cos \theta - \mu N_1 - \mu N_2 = ma_r$$

$$ma \cos \theta - \mu ma \sin \theta - \mu mg = ma_r$$

$$\Rightarrow a_r = a \cos \theta - \mu a \sin \theta - \mu g$$

$$= 25 \times \frac{4}{5} - \frac{2}{5} \times 25 \times \frac{3}{5} - \frac{2}{5} \times 10 = 10 \text{ m/s}^2$$

14. Acceleration of block down the plane

$$a = \frac{mg \sin \theta - \mu_k mg \cos \theta}{m}$$

$$\therefore a_A = g \sin \theta - \mu_{kA} g \cos \theta$$

$$= g \sin 45^\circ - \mu \cos 45^\circ$$

$$= 10 \left( \frac{1}{\sqrt{2}} \right) - (0.2)(10) \left( \frac{1}{\sqrt{2}} \right) = 4\sqrt{2}$$

$$\text{And } a_B = g \sin \theta - \mu_{kB} g \cos \theta$$

$$= g \sin 45^\circ - \mu_{kB} g \cos 45^\circ$$

$$= 10 \left( \frac{1}{\sqrt{2}} \right) - (0.3)(10) \left( \frac{1}{\sqrt{2}} \right) = 3.5\sqrt{2} \text{ m/s}^2$$

Let  $a_{AB}$  is relative acceleration of A w.r.t. B. Then

$$a_{AB} = a_A - a_B$$

The relative distance between A and B,  $L$ .

$$L = \frac{1}{2} a_{AB} t^2$$

$$\text{or } t^2 = \frac{2L}{a_{AB}} = \frac{2L}{a_A - a_B} = \frac{2(\sqrt{2})}{(4\sqrt{2}) - (3.5\sqrt{2})}$$

$$\Rightarrow t^2 = 4 \text{ or } t = 2 \text{ s.}$$

Distance moved by A during that time is given by

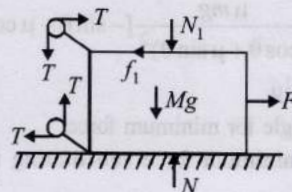
$$S_A = \frac{1}{2} a_A t^2 = \frac{1}{2} \times 4.5\sqrt{2} \times 4 = 8\sqrt{2} \text{ m}$$

Similarly for B =  $7\sqrt{2} \text{ m}$ .

Hence both the blocks will come in line after A has travelled

a distance  $8\sqrt{2} \text{ m}$  down the plane

15. (a) Free body diagram of mass M



- (b) The maximum value of force of friction between
- $m_1$
- and M

$$(f_1)_{\text{max}} = (0.3)(20)(10) = 60 \text{ N} \quad \dots(i)$$

The maximum value of force of friction between  $m_2$  and M

$$(f_2)_{\text{max}} = (0.3)(5)(10) = 15 \text{ N} \quad \dots(ii)$$

Forces on  $m_1$  and  $m_2$  in horizontal direction are as follows:



There are only two possibilities.

**Case I** Either both  $m_1$  and  $m_2$  will remain stationary (w.r.t. ground)

**Case II** both  $m_1$  and  $m_2$  will move (w.r.t. ground).

First case is possible when

$$\text{or } T \leq (f_1)_{\text{max}} \text{ or } T \leq 60 \text{ N and } T \leq (f_2)_{\text{max}} \text{ or } T \leq 15 \text{ N}$$

These conditions will be satisfied when  $T \leq 15 \text{ N}$  say  $T = 14$  then  $f_1 = f_2 = 14 \text{ N}$ .

Therefore the condition  $f_1 = 2f_2$  will not be satisfied.

Thus  $m_1$  and  $m_2$  both can't remain stationary.

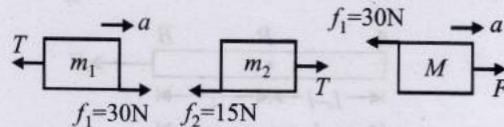
In the second case, when  $m_1$  and  $m_2$  both move

$$f_2 = (f_2)_{\text{max}} = 15 \text{ N}$$

$$\therefore f_1 = 2f_2 = 30 \text{ N}$$

Since  $f_1 < (f_1)_{\text{max}}$ , there is no relative motion between  $m_1$  and M, i.e., all the masses move with same acceleration, say 'a'.

Free body diagrams and equations of motion are as follows:



$$\text{For mass, } m_1 : 30 - T = 20a \quad \dots(iii)$$

$$\text{For mass, } m_2 : T - 15 = 5a \quad \dots(iv)$$

$$\text{For mass, } M : F - 30 = 50a \quad \dots(v)$$

Adding eq. (iii) & (iv), we get.

$$\text{acceleration } a = \frac{3}{5} \text{ m/s}^2.$$

$$\text{From eq. (iv) } T - 15 = \frac{5 \times 3}{5} \Rightarrow T = 18 \text{ N}$$

$$\text{From eq. (v) } F - 30 = 50 \times \frac{3}{5} \Rightarrow F = 60 \text{ N}$$

16. Let force F be applied to move the body at an angle
- $\theta$
- to the horizontal.

The body will move when

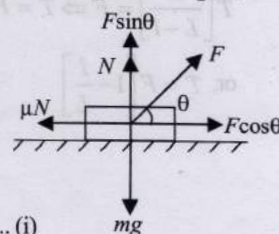
$$F \cos \theta = \mu N$$

from figure, normal reaction

$$N = mg - F \sin \theta$$

$$F \cos \theta = \mu(mg - F \sin \theta)$$

$$\Rightarrow F = \frac{\mu mg}{\cos \theta + \mu \sin \theta} \quad \dots(i)$$





Differentiating the above equation w.r.t.  $\theta$ , we get

$$\frac{dF}{d\theta} = \frac{\mu mg}{(\cos\theta + \mu \sin\theta)^2} [-\sin\theta + \mu \cos\theta] = 0$$

$$\therefore \theta = \tan^{-1}\mu$$

This is the angle for minimum force.

To find the minimum force substituting these values in equation (i)

$$\begin{aligned} \sin\theta &= \frac{\mu}{\sqrt{\mu^2+1}}, \quad \cos\theta = \frac{1}{\sqrt{\mu^2+1}} \\ F &= \frac{\mu mg}{\frac{1}{\sqrt{\mu^2+1}} + \frac{\mu}{\sqrt{\mu^2+1}} \times \mu} \\ \Rightarrow F &= \frac{\mu mg (\sqrt{\mu^2+1})}{\mu^2+1} = \frac{\mu mg}{\sqrt{\mu^2+1}} \end{aligned}$$

$$\Rightarrow F = mg \sin\theta$$

17. According to question, mass  $M_1$  moves downwards with a uniform velocity i.e., net acceleration of the system is zero. Or net pulling force on the system is zero.

For equilibrium,

$$(a) \quad M_1 g = M_2 g \sin 37^\circ + \mu M_2 g \cos 37^\circ + \mu M_3 g$$

$$\text{or } M_1 = M_2 \sin 37^\circ + \mu M_2 \cos 37^\circ + \mu M_3$$

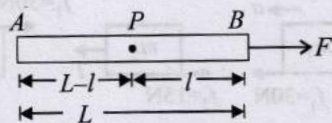
$$= (4) \left( \frac{3}{5} \right) + (0.25)(4) \left( \frac{4}{5} \right) + (0.25)(4) = 4.2 \text{ kg}$$

$$(b) \quad \text{Since, } M_3 \text{ is moving with uniform velocity}$$

$$T = \mu_1 m_2 g = (0.25) \times 4 \times 9.8 = 9.8 \text{ N}$$

18. Let  $T$  be the tension in the rope at point  $P$  and  $a$  be the acceleration produced in the rope.

Mass per unit length of the rope is  $\mu = \frac{F}{L}$



For the part AP,

$$T = \mu(L-l)a \quad \dots(i)$$

For the part PB,

$$F - T = \mu a \quad \dots(ii)$$

$$F - T = \mu l \left[ \frac{T}{\mu(L-l)} \right] \quad [\text{Using eq. (i)}]$$

$$F - T = \frac{Tl}{L-l} \Rightarrow T \left[ \frac{l}{L-l} + 1 \right] = F;$$

$$T \left[ \frac{L}{L-l} \right] = F \Rightarrow T = F \left( \frac{L-l}{L} \right)$$

$$\text{or, } T = F \left[ 1 - \frac{l}{L} \right]$$

19. As cubes do not slide down the planes hence they have same acceleration.

Consider the FBD of the cubes along incline

$$T + m_1 f \cos \alpha = m_1 g \sin \alpha \rightarrow (i)$$

$$T + m_2 g \sin \beta = m_2 f \cos \beta \rightarrow (ii)$$

$$\text{Eq (i) - Eq (ii)}$$

$$(m_1 \cos \alpha + m_2 \cos \beta) f = (m_1 \sin \alpha + m_2 \sin \beta) g$$

$$\Rightarrow f = \frac{(m_1 \sin \alpha + m_2 \sin \beta)}{(m_1 \cos \alpha + m_2 \cos \beta)} g$$

$$\text{from eq (i) } T = (m_1 g \sin \alpha) - (m_1 \cos \alpha) \left[ \frac{m_1 \sin \alpha + m_2 \sin \beta}{m_1 \cos \alpha + m_2 \cos \beta} \right] g$$

$$\text{or, } T = g \left[ \frac{m_1^2 \cos \alpha \sin \alpha + m_1 m_2 \cos \beta \sin \alpha - m_1^2}{(m_1 \cos \alpha + m_2 \cos \beta)} \right]$$

$$\text{or, } T = \frac{m_1 m_2 [\cos \beta \sin \alpha - \sin \beta \cos \alpha] g}{(m_1 \cos \alpha + m_2 \cos \beta)}$$

20. When force  $F$  is applied on block  $C$  will move towards left and the block 'B' will move towards right due to reaction of  $C$  on  $B$ , while block  $A$  always remains at rest.

The F.B.D. for mass  $C$  is

$$F \leftarrow \boxed{C} \xrightarrow{T} \quad \begin{aligned} f_2 &= \mu(m_A + m_B)g \\ f_1 &= \mu(m_A + m_B + m_C)g \end{aligned}$$

$$\text{As } C \text{ is moving with constant speed } F = f_1 + f_2 + T \quad \dots(i)$$

F.B.D. for mass  $B$  is

$$\begin{aligned} \mu m_A g &= f_3 \leftarrow \boxed{B} \xrightarrow{T} \\ \mu(m_A + m_B)g &= f_2 \leftarrow \end{aligned}$$

$$\text{As } B \text{ is moving with constant speed } f_2 + f_3 = T \quad \dots(ii)$$

Subtracting eq. (ii) from (i)

$$\begin{aligned} F - (f_2 + f_3) &= f_1 + f_2 + T - T = f_1 + f_2 \\ \Rightarrow F &= f_1 + 2f_2 + f_3 = \mu(m_A + m_B + m_C)g + 2\mu(m_A + m_B)g + \mu m_A g \end{aligned}$$

$$\begin{aligned} F &= \mu(4m_A + 3m_B + m_C)g \\ (\text{Given: } m_A &= 3\text{ kg, } m_B = 4\text{ kg, } m_C = 5\text{ kg and } \mu = 0.25) \\ &= 0.25[4 \times 3 + 3 \times 4 + 5] \times 9.8 = 71.05 \text{ N} \end{aligned}$$

Hence, force necessary to drag,  $F = 71.05 \text{ N}$



#### Topic-4: Circular Motion & Banking of Road

$$1. \quad (d) \quad T \sin \theta = mR\omega^2 \quad \dots(i)$$

$$T \cos \theta = mg \quad \dots(ii)$$

Dividing (ii) by (i), we get

$$\tan \theta = \frac{\omega^2}{Rg} \Rightarrow \omega = \sqrt{Rg \tan \theta}$$





Clearly,  $\omega$  is maximum, when  $\tan \theta$  is maximum

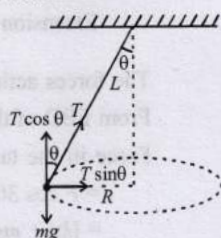
i.e.  $\theta = 90^\circ$

$$\text{So, } T \sin 90^\circ = mR\omega^2$$

$$T = mL\omega^2 \text{ [Here, } R = L]$$

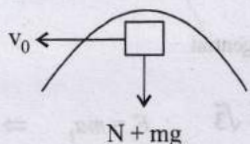
$$\Rightarrow \omega = \sqrt{\frac{T}{mL}} = \sqrt{\frac{324}{0.5 \times 0.5}}$$

$$= \frac{18}{0.5} = 36 \text{ rad/s}$$



2. (a) According to question, the speed with which the block enters the track is the same in all the tracks and the block rises to the same height so from law of conservation of energy, speed of the block at highest point will be same in all four cases.

Let the velocity at the highest point be  $v$



$(N + mg)$  provides the centripetal force  $\frac{mv^2}{R}$  to the body

$$N + mg = \frac{mv^2}{R}$$

$$\text{or } N = \frac{mv^2}{R} - mg$$

$R$  (the radius of curvature) in first case is minimum. Hence, normal reaction  $N$  will be maximum in first case.

3. (30) From impulse-momentum theorem,

$$J = MV_{CM} \Rightarrow V = \frac{J}{M} = \frac{\sqrt{\pi/2}}{100 \times \frac{5}{1000}} = \sqrt{2\pi} \text{ m/s}$$

$$\text{Total time taken, } t = \frac{2v}{g}$$

$$= \frac{2 \times \sqrt{2\pi}}{g} = \frac{2 \times \sqrt{2\pi}}{10} = \frac{\sqrt{2\pi}}{5} \text{ s}$$

By angular impulse-momentum theorem,

$$J \times \frac{R}{2} = I_c \omega = \left[ \frac{1}{4} MR^2 \right] \omega \quad \therefore \omega = \frac{J \times \frac{R}{2}}{\frac{MR^2}{4}} = \frac{J \times 2}{MR}$$

$$= \frac{\frac{\sqrt{\pi/2}}{100} \times 2}{\frac{5}{1000} \times \frac{4}{3} \times \frac{1}{100}} = 2 \times 75 \sqrt{2\pi} \text{ rad/s}$$

$$\therefore \theta = 2\pi n = \omega t \quad \therefore n = \frac{\omega t}{2\pi}$$

$$\therefore n = \frac{2 \times 75 \sqrt{2\pi} \times \frac{\sqrt{2\pi}}{5}}{2\pi} = 30$$

4. False : The angular amplitude of the pendulum is  $40^\circ$  given, when its angular displacement is  $20^\circ$  then

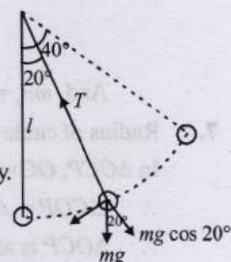
$$\text{For equilibrium of the bob, } T - mg \cos 20^\circ = \frac{mv^2}{l}, \text{ where}$$

$l$  is the length of the pendulum and  $v$  is the velocity of the bob.

$$\therefore T = mg \cos 20^\circ + \frac{mv^2}{l}$$

$$\frac{mv^2}{l} \text{ is always a positive quantity.}$$

Hence, clearly  $T > mg \cos 20^\circ$ .



5. (d) Suppose 'N' is acting radially outward

$$\text{Then, } mg \cos \theta - N = \frac{mv^2}{R}$$

$$\Rightarrow N = mg \cos \theta - \frac{mv^2}{R} \dots (i)$$

And by energy conservation,

$$\frac{1}{2} mv^2 = mg[R - R \cos \theta]$$

$$\therefore \frac{v^2}{R} = 2g(1 - \cos \theta)$$

Putting this value of  $\frac{v^2}{R}$  in eqn. (i)

$$N = mg \cos \theta - m[2g - 2g \cos \theta]$$

$$\Rightarrow N = mg \cos \theta - 2mg + 2mg \cos \theta$$

$$\Rightarrow N = 3mg \cos \theta - 2mg \Rightarrow N = mg(3 \cos \theta - 2)$$

Clearly when  $\cos \theta > \frac{2}{3}$ ,  $N$  is positive acts radially outwards

So, force on wire is inward and if  $\cos \theta < \frac{2}{3}$ ,  $N$  acts radially inwards.

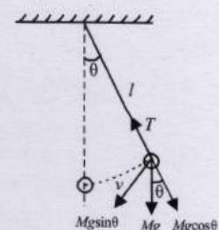
So, force on wire is outward.

6. (b, c) A long radius net force = centripetal force

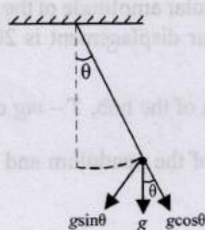
$$\left( \frac{Mv^2}{\ell} \right).$$

And along tangent net force =  $ma_t$  as the motion of a pendulum is the part of circular motion.

$$\therefore T - Mg \cos \theta = \frac{Mv^2}{\ell}$$







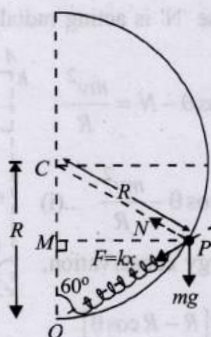
$$\text{And, } ma_t = mg \sin \theta \Rightarrow a_t = g \sin \theta$$

7. Radius of circle = R

In  $\triangle OCP$ ,  $OC = CP = R$

$$\therefore \angle COP = \angle CPO = 60^\circ \Rightarrow \angle OCP = 60^\circ$$

$$\therefore \triangle OCP \text{ is an equilateral triangle} \Rightarrow OP = R$$



$$\therefore \text{Extension of string} = R - \frac{3R}{4} = \frac{R}{4} = x$$

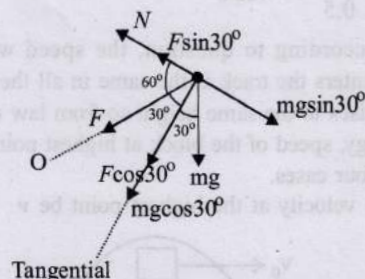
The forces acting are shown in the figure (i)

From FBD of the ring

Force in the tangential direction

$$= F \cos 30^\circ + mg \cos 30^\circ$$

$$= [kx + mg] \cos 30^\circ$$



$$F_t = \frac{5mg}{8} \sqrt{3} \quad \therefore F_t = ma_t \Rightarrow a_t = \frac{5\sqrt{3}}{8} g$$

Also, when the ring is just released

$$N + F \sin 30^\circ = mg \sin 30^\circ$$

$$\Rightarrow N = (mg - F) \sin 30^\circ = \left( mg - \frac{mg}{4} \right) \times \frac{1}{2} = \frac{3mg}{8}$$