WAVES AND ACOUSTICS [JEE ADVANCED PREVIOUS YEAR SOLVED PAPERS]

JEE Advanced

Single Correct Answer Type

1. A cylindrical tube, open at both ends, has a fundamental frequency 'f' in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column

   a. $f/2$ b. $3f/4$
   c. f d. $2f$ **(IIT-JEE 1981)**

2. A transverse wave is described by the equation

$$y = y_0 \sin 2\pi \left(ft - \frac{x}{\lambda} \right)$$

   The maximum particle velocity is equal to four times the wave velocity if

   a. $\lambda = \pi \frac{y_0}{4}$ b. $\lambda = \pi \frac{y_0}{2}$
   c. $\lambda = \pi y_0$ d. $\lambda = 2\pi y_0$
   (IIT-JEE 1984)

3. A tube, closed at one end and containing air, produces, when excited, the fundamental mode of frequency 512 Hz. If the tube is open at both ends the fundamental frequency that can be excited is (in Hz).

   a. 1024 b. 512
   c. 256 d. 128 **(IIT-JEE 1986)**

4. A wave represented by the equation $y = a\cos(kx - \omega t)$ is superposed with another wave to form a stationary wave such that point $x = 0$ is a node. The equation for the other wave is

   a. $a\sin(kx + \omega t)$ b. $a\sin(kx - \omega t)$
   c. $-a\cos(kx + \omega t)$ d. $-a\sin(kx - \omega t)$
   (IIT-JEE 1988)

5. An organ pipe P_1 closed at one end vibrating in its first harmonic and another pipe P_2 open at both the ends vibrating in its third harmonic are in resonance with a given tuning fork. The ratio of the length of P_1 to that of P_2 is

   a. 8/3 b. 3/8
   c. 1/6 d. 1/3 **(IIT-JEE 1988)**

6. An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water so that one half of its volume is submerged. The new fundamental frequency in Hz is

   a. $300\left(\frac{2\rho - 1}{2\rho}\right)^{1/2}$ b. $300\left(\frac{2\rho}{2\rho - 1}\right)^{1/2}$
   c. $300\left(\frac{2\rho}{2\rho - 1}\right)$ d. $300\left(\frac{2\rho - 1}{2\rho}\right)$
   (IIT-JEE 1995)

7. The extension in a string, obeying Hooke's law, is x. The speed of sound in the stretched string is v. If the extension in the string is increased to $1.5x$, the speed of sound will be

   a. $1.22\ v$ b. $0.61\ v$
   c. $1.50\ v$ d. $0.75\ v$ **(IIT-JEE 1996)**

8. An open pipe is suddenly closed at one end and with the result frequency of third harmonic of the closed pipe is found to be higher by 100 Hz than the fundamental frequency of the open pipe. The fundamental frequency of the open pipe is

   a. 200 Hz b. 300 Hz
   c. 240 Hz d. 480 Hz **(IIT-JEE 1996)**

9. A whistle giving out 450 Hz approaches a stationary observer at a speed of 33 m/s. The frequency heard by the observer in Hz is (speed sound = 330 m/s)

   a. 409 b. 429
   c. 517 d. 500 **(IIT-JEE 1997)**

10. A travelling wave in a stretched string is described by the equation $y = A\sin(kx - \omega t)$. The maximum particle velocity is

    a. $A\omega$ b. ω/k
    c. $d\omega/dk$ d. x/t **(IIT-JEE 1997)**

11. A string of length 0.4 m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulse is produced at one end at equal intervals of time, Δt. The minimum value of Δt which allows constructive interface between successive pulses is

    a. 0.05 s b. 0.10 s
    c. 0.20 s d. 0.40 s **(IIT-JEE 1998)**

12. The ratio of the speed of sound in nitrogen gas to that in helium gas at 300 K is

    a. $\sqrt{(2/7)}$ b. $\sqrt{(1/7)}$
    c. $(\sqrt{3})/5$ d. $(\sqrt{6})/5$
    (IIT-JEE 1999)

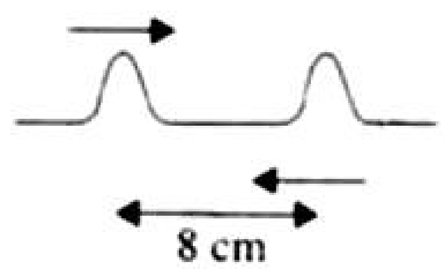
13. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the train's speed is reduced to 17 m/s, the frequency registered is f_2. If the speed of the sound is 340 m/s, then the ratio f_1/f_2 is

    a. 18/19 b. 1/2
    c. 2 d. 19/18 **(IIT-JEE 2000)**

14. Two vibrating strings of the same material but lengths L and $2L$ have radii $2r$ and r, respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes, the one of length L with frequency n_1 and the other with frequency n_2. The ratio n_1/n_2 is given by

    a. 2 b. 4
    c. 8 d. 1 **(IIT-JEE 2000)**

15. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 s, the total energy of the pulse will be



a. zero
b. purely kinetic
c. purely potential
d. partly kinetic and partly potential

(IIT-JEE 2000)

16. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A\sin(2\pi x/L)\sin 2\omega t$ and energy is E_2. Then

a. $E_2 = E_1$
b. $E_2 = 2E_1$
c. $E_2 = 4E_1$
d. $E_2 = 16E_1$

(IIT-JEE 2001)

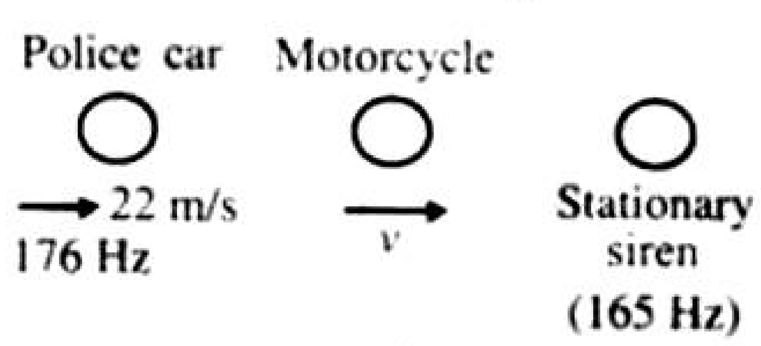
17. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train *A* records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train *B* he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train *B* to that of train *A* is

a. 242/252
b. 2
c. 4
d. 6 **(IIT-JEE 2002)**

18. A sonometer wire resonates with a tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by a mass M, the wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is

a. 25 kg
b. 5 kg
c. 12.5 kg
d. 1/25 kg **(IIT-JEE 2002)**

19. A police car, moving at 22 m/s, chases a motorcyclist. The police man sounds his horn at 176 Hz, while both of them move towards a stationary siren of frequency 165 Hz. Calculate the speed of the motorcycle, if it is given that he does not observe any beats.



a. 33 m/s
b. 22 m/s
c. zero
d. 11 m/s

(IIT-JEE 2003)

20. In the experiment for the determination of the speed of sound in air using the resonance column method, the length of the air column that resonates in the fundamental mode, with a tuning fork is 0.1 m. When this length is changed to 0.35 m, the same tuning fork resonates with the first overtone. Calculate the end correction.

a. 0.012 m
b. 0.025 m
c. 0.05 m
d. 0.024 m

(IIT-JEE 2003)

21. A source of sound of frequency 600 Hz is placed inside water. The speed in water is 1500 m/s and in air it is 300 m/s. The frequency of sound recorded by an observer standing in air is

a. 200 Hz
b. 3000 Hz
c. 120 Hz
d. 600 Hz

(IIT-JEE 2004)

22. A pipe of length l_1, closed at one end, is kept in a chamber of gas of density ρ_1. A second pipe open at both ends is placed in a second chamber of gas of density ρ_2. The compressibility of both the gases is equal. Calculate the length of the second pipe if frequency of first overtone in both the cases is equal

a. $\frac{4}{3}l_1\sqrt{\frac{\rho_2}{\rho_1}}$
b. $\frac{4}{3}l_1\sqrt{\frac{\rho_1}{\rho_2}}$
c. $l_1\sqrt{\frac{\rho_2}{\rho_1}}$
d. $l_1\sqrt{\frac{\rho_1}{\rho_2}}$ **(IIT-JEE 2004)**

23. In a resonate tube with tuning fork of frequency 512 Hz, first resonance occurs at water level equal to 30.3 cm and second resonance occurs at 63.7 cm. The maximum possible error in the speed of sound is

a. 5.12 cm/s
b. 102.4 cm/s
c. 204.8 cm/s
d. 153.6 cm/s

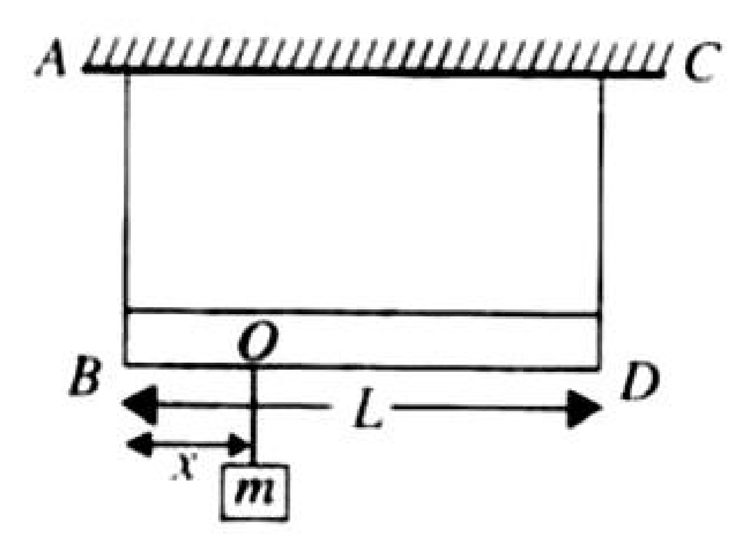
(IIT-JEE 2005)

24. An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option.

a. $n = 3, f_2 = \frac{3}{4}f_1$
b. $n = 3, f_2 = \frac{5}{4}f_1$
c. $n = 5, f_2 = \frac{3}{4}f_1$
d. $n = 5, f_2 = \frac{5}{4}f_1$

(IIT-JEE 2005)

25. A massless rod of length L is suspended by two identical strings AB and CD of equal length. A block of mass m is suspended from point O such that BO is equal to 'x'. Further it is observed that the frequency of 1st harmonic in AB is equal to 2nd harmonic frequency in CD. 'x' is

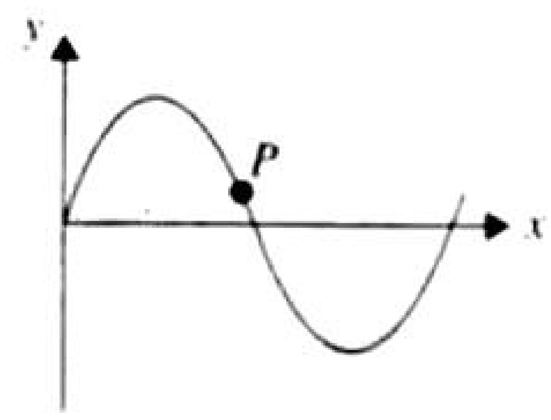


a. $\frac{L}{5}$
b. $\frac{4L}{5}$
c. $\frac{3L}{4}$
d. $\frac{L}{4}$ **(IIT-JEE 2006)**

26. In the experiment to determine the speed of sound using a resonance column

a. prongs of the tuning fork are kept in a vertical plane
b. prongs of the tuning fork are kept in a horizontal plane
c. in one of the two resonances observed, the length of the resonating air column is close to the wavelength of sound in air
d. in one of the two resonances observed, the length of the resonating air column is close to half of the wavelength of sound in air **(IIT-JEE 2007)**

27. A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap-shot of the wave is shown in the figure. The velocity of point P when its displacement is 5 cm is



a. $\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s b. $-\frac{\sqrt{3}\pi}{50}\hat{j}$ m/s
c. $\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s d. $-\frac{\sqrt{3}\pi}{50}\hat{i}$ m/s

(IIT-JEE 2008)

28. A vibrating string of certain length l under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased, the number of beats reduces to 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is
a. 344 b. 336
c. 117.3 d. 109.3 **(IIT-JEE 2008)**

29. A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is
a. 5 g b. 10 g
c. 20 g d. 40 g **(IIT-JEE 2010)**

30. Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2, respectively, are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by
a. $\sqrt{\frac{m_1}{m_2}}$ b. $\sqrt{\frac{m_2}{m_1}}$
c. $\frac{m_1}{m_2}$ d. $\frac{m_2}{m_1}$ **(IIT-JEE 2010)**

31. A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/h towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is
a. 8.50 kHz b. 8.25 kHz
c. 7.75 kHz d. 7.50 kHz
(IIT-JEE 2011)

32. A student is performing the experiment of resonance column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the resonance column tube. When the first resonance occurs, the reading of the water level in the column is
a. 14.0 cm b. 15.2 cm
c. 16.4 cm d. 17.6 cm
(IIT-JEE 2012)

33. A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s^{-1}.
He is told that the air in the tube has been replaced by another gas (assume that the column remains filled with the gas). If the minimum height at which resonance occurs is $(0.350 \pm 0.005)m$, the gas in the tube is
(**Useful information:** $\sqrt{167RT} = 640$ J$^{1/2}$ mole$^{-1/2}$; $\sqrt{140RT} = 590$ J$^{1/2}$ mole$^{-1/2}$. The molar masses M in grams are given in the options. Take the values of $\sqrt{\frac{10}{M}}$ for each gas as given there.)
a. Neon $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$
b. Nitrogen $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$
c. Oxygen $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$
d. Argon $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$ **(JEE Advanced 2014)**

Multiple Correct Answer Type

1. A wave equation which gives the displacement along the y-direction is given by $y = 10^{-4} \sin(60t + 2x)$ where x and y are in metres and t is time in seconds. This represents a wave
a. travelling with a velocity of 30 m/s in the negative x-direction
b. of wavelength π m
c. of frequency $30/\pi$ Hz
d. of amplitude 10^{-4} m travelling along the negative x-direction. **(IIT-JEE 1981)**

2. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the column in cm is (Speed of sound = 330 m/s)
a. 31.25 b. 62.50
c. 93.75 d. 125 **(IIT-JEE 1985)**

3. The displacement of particle in a string stretched in the x-direction is represented by y. Among the following expression for y, those describing wave motion are
   a. $\cos kx \sin \omega t$ b. $k^2x^2 - \omega^2t^2$
   c. $\cos^2(kx + \omega t)$ d. $\cos(k^2x^2 - \omega^2t^2)$

(IIT-JEE 1987)

4. Velocity of sound in air is 320 m/s. A pipe closed at one end has a length of 1 m. Neglecting end corrections, the air column in the pipe can resonate for sound of frequency
   a. 80 Hz b. 240 Hz
   c. 320 Hz d. 400 HZ **(IIT-JEE 1989)**

5. A wave is represented by the equation

$$y = A\sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$$

where x is in metres and t is in seconds. The expression represents:
   a. a wave travelling in the positive x-direction with a velocity 1.5 m/s.
   b. a wave travelling in the negative x-direction with a velocity 1.5 m/s.
   c. a wave travelling in the negative x-direction having a wavelength 0.2 m.
   d. a wave travelling in the positive x-direction having a wavelength 0.2 m. **(IIT-JEE 1990)**

6. Two identical straight wires are stretched so as to produce 6 beats per second when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Let T_1 and T_2 represent the higher and the lower initial tensions in the strings. While making the above change in tension:
   a. T_2 was decreased b. T_2 was increased
   c. T_1 was increased d. T_1 was decreased

(IIT-JEE 1991)

7. A sound wave of frequency f travels horizontally to the right. It is reflected from a large vertical plane surface moving to left with a speed v. The speed of sound in medium is c.
   a. The number of waves striking the surface per second is $f\frac{(c+v)}{c}$
   b. The wavelength of reflected wave is $\frac{c(c-v)}{f(c+v)}$
   c. The frequency of the reflected wave is $f\frac{(c+v)}{(c-v)}$
   d. The number of beats heard by a stationary listener to the left of the reflecting surface is $\frac{vf}{c-v}$

(IIT-JEE 1995)

8. A wave disturbance in a medium is described by $y(x, t) = 0.02\cos\left(50\pi t + \frac{\pi}{2}\right)\cos(10\pi x)$, where x and y are in metre and t is in second.
   a. A node occurs at $x = 0.15$ m
   b. An antinode occurs at $x = 0.3$ m
   c. The speed of wave is 5 m/s
   d. The wave length is 0.2 m **(IIT-JEE 1995)**

9. The (x, y) coordinates of the corners of a square plate are (0, 0), (L, L) and $(0, L)$. The edges of the plate are clamped and transverse standing waves are set up in it. If $u(x, y)$ denotes the displacement of the plate at point (x, y) at some instant of time, the possible expression(s) for u is (are) (a = positive constant).
   a. $a\cos(\pi x/2L)\cos(\pi y/2L)$
   b. $a\sin(\pi x/L)\sin(\pi y/L)$
   c. $a\sin(\pi x/L)\sin(2\pi y/L)$
   d. $a\cos(2\pi x/L)\sin(\pi y/L)$ **(IIT-JEE 1998)**

10. A transverse sinusoidal wave of amplitude a, wavelength λ and frequency f is travelling on a stretched string. The maximum speed of any point on the string is $v/10$, where v is the speed of propagation of the wave. If $a = 10^{-3}$ m and $v = 10$ m/s, then λ and f are given by
    a. $\lambda = 2\pi \times 10^{-2}$ m b. $\lambda = 10^{-3}$ m
    c. $f = \frac{10^3}{2\pi}$ Hz d. $f = 10^4$ Hz

(IIT-JEE 1998)

11. $y(x, t) = 0.8/[(4x + 5t)^2 + 5]$ represents a moving pulse, where x and y are in metre and t in second. Then
    a. pulse is moving in $+x$-direction
    b. in 2 s it will travel a distance of 2.5 m
    c. its maximum displacement is 0.16 m
    d. it is a symmetric pulse **(IIT-JEE 1999)**

12. In a wave motion $y = a\sin(kx - \omega t)$, y can represent
    a. electric field b. magnetic field
    c. displacement d. pressure

(IIT-JEE 1999)

13. Standing waves can be produced
    a. on a string clamped at both the ends.
    b. on a string clamped at one end and free at the other.
    c. when incident wave gets reflected from a wall.
    d. when two identical waves with a phase difference of π are moving in the same direction.

(IIT-JEE 1999)

14. The function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents SHM for which of the option(s)
    a. For all value of A, B and C (except $C = 0$)
    b. $A = -B$, $C = 2B$, amplitude $= |B\sqrt{2}|$
    c. $A = B$, $C = 0$
    d. $A = B$, $C = 2B$, amplitude $= |B|$ **(IIT-JEE 2006)**

15. A student performed an experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air-column is the second resonance. Then,
    a. the intensity of the sound heard at the first resonance was more than that at the second resonance.

b. the prongs of the tuning fork were kept in a horizontal plane above the resonance tube.

c. the amplitude of vibration of the ends of the prongs is typically around 1 cm.

d. the length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air. **(IIT-JEE 2009)**

16. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe.

a. a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open

b. a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is open

c. a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

d a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed **(IIT-JEE 2012)**

17. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are)

a. If the wind blows from the source to the observer, $f_2 > f_1$

b. If the wind blows from the observer to the source, $f_2 > f_1$

c. If the wind blows from observer to the source, $f_2 < f_1$

d. If the wind blows from the source to the observer $f_2 < f_1$

(JEE Advanced 2013)

18. One end of a taut string of length 3 m along the x axis is fixed at $x = 0$. The speed of the waves in the string is $100\ \text{ms}^{-1}$. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are)

a. $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

b. $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

c. $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

d. $y(t) = A\sin\frac{5\pi x}{2}\cos 250\pi t$ **(JEE Advanced 2014)**

Linked Comprehension Type

For Problems 1–3

Waves $y_1 = A\cos(0.5\pi x - 100\pi t)$ and $y_2 = A\cos(0.46\pi x - 92\pi t)$ are travelling along the x-axis. (Here x is in metre and t is in second) **(IIT-JEE 2006)**

1. Find the number of times intensity is maximum in time interval of 1 s.

a. 4 b. 6

c. 8 d. 10

2. The wave velocity of louder sound is

a. 100 m/s b. 192 m/s

c. 200 m/s d. 96 m/s

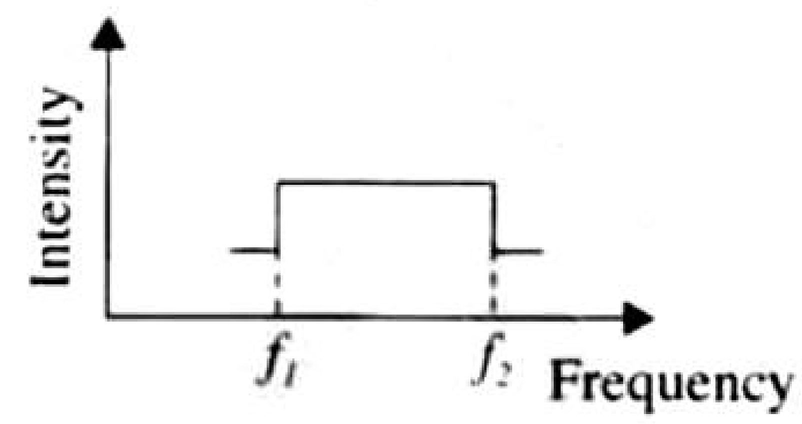
3. The number of times $y_1 + y_2 = 0$ at $x = 0$ in 1 s is

a. 100 b. 46

c. 192 d. 96

For Problems 4–6

Two trains A and B are moving with speeds 20 m/s and 30 m/s, respectively in the same direction on the same straight track with B ahead of A. The engines are at the front ends. The engine of train A blows a long whistle.



Assume that the sound of the whistle is composed of components varying in frequency from $f_1 = 800$ Hz to $f_2 = 1120$ Hz, as shown in the figure. The spread in the frequency (highest frequency – lowest frequency) is thus 320 Hz. The speed of sound in still air is 340 m/s.

(IIT-JEE 2007)

4. The speed of sound of the whistle is

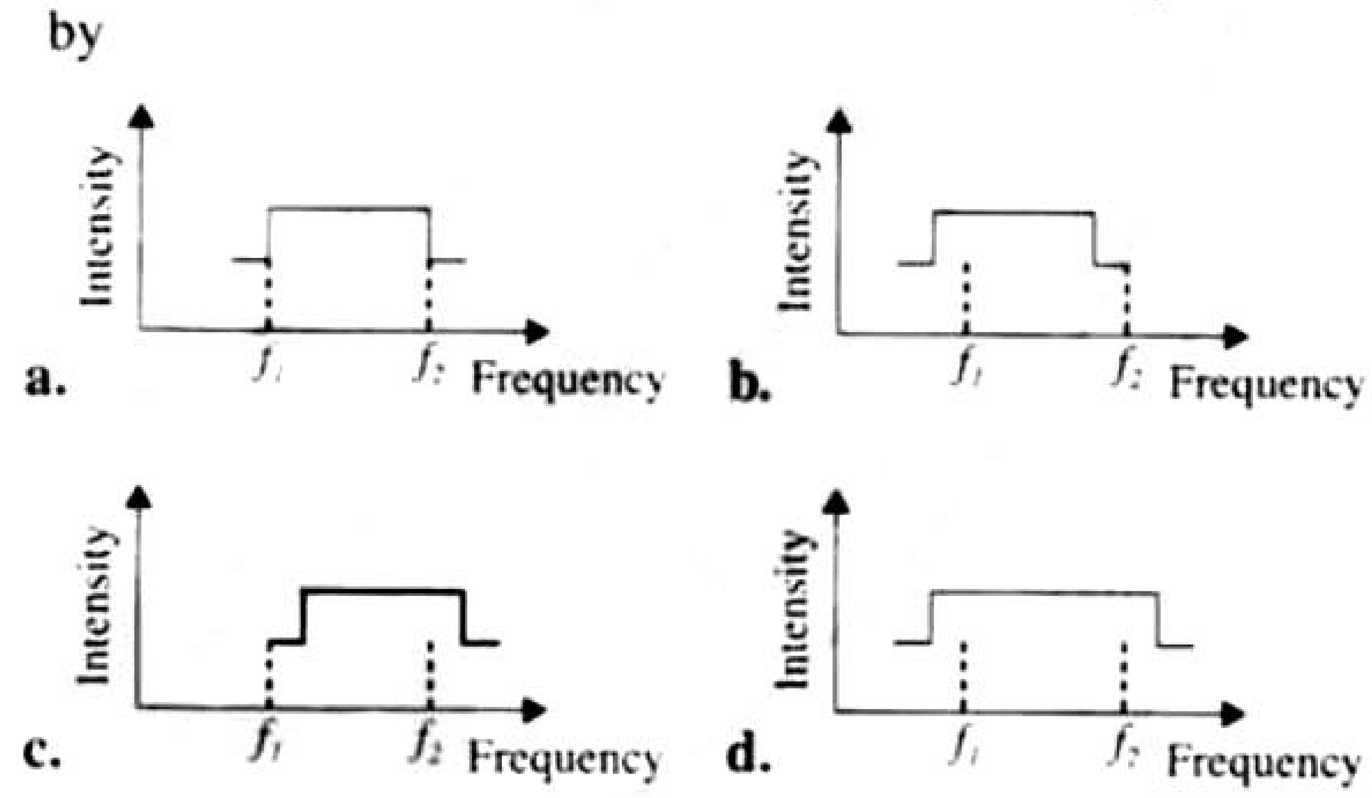
a. 340 m/s for passengers in A and 310 m/s for passengers in B.

b. 360 m/s for passengers in A and 310 m/s for passengers in B.

c. 310 m/s for passengers in A and 360 m/s for passengers in B.

d. 340 m/s for passengers in both the trains.

5. The distribution of the sound intensity of the whistle as observed by the passengers in train A is best represented by



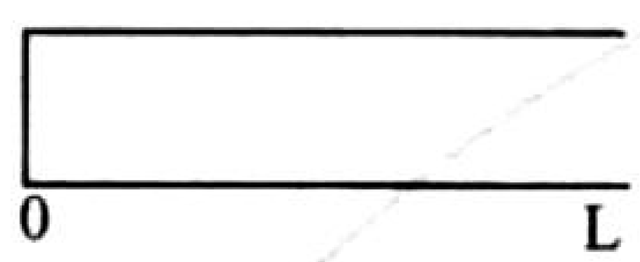
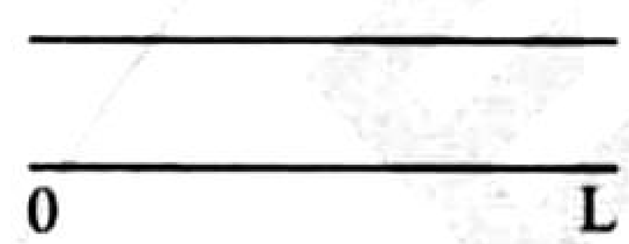
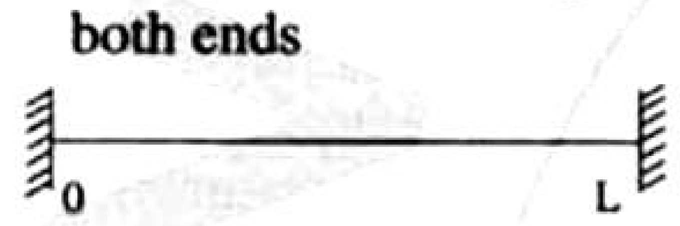
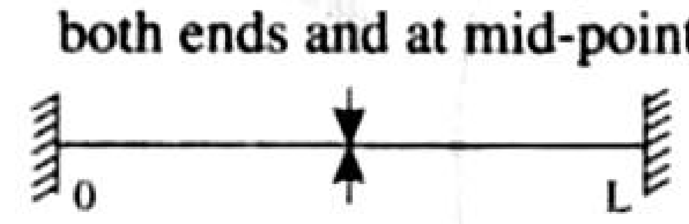
6. The spread of frequency as observed by the passengers in train B is

a. 310 Hz b. 330 Hz

c. 350 Hz d. 290 Hz

Matching Column Type

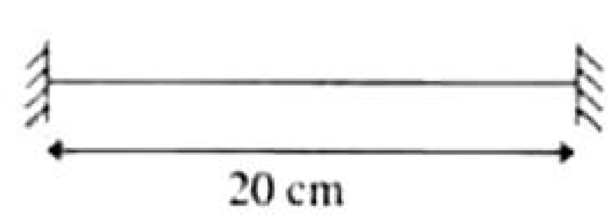
1. Column I shows four systems, each of the same length L, for producing standing waves. 'The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in Column II describing the nature and wavelength of the standing waves.

Column I	Column II
i. Pipe closed at one end 	**a.** Longitudinal waves
ii. Pipe open at both ends 	**b.** Transverse waves
iii. Stretched wire clamped at both ends 	**c.** $\lambda_f = L$
iv. Stretched wire clamped at both ends and at mid-point 	**d.** $\lambda_f = 2L$
	e. $\lambda_f = 4L$

(IIT-JEE 2011)

Integer Answer Type

1. A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. Find the separation (in cm) between the successive nodes on the string.



(IIT-JEE 2009)

2. A stationary source is emitting sound at a fixed frequency f_0, which is reflected by two cars approaching the source. The difference between the frequencies of sound reflected from the cars is 1.2% of f_0. What is the difference in the speeds of the cars (in km per hour) to the nearest integer? The cars are moving at constant speeds much smaller than the speed of sound which is 330 ms^{-1}.

(IIT-JEE 2010)

3. When two progressive waves $y_1 = 4\sin(2x - 6t)$ and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ superimposed, the amplitude of the resultant wave is **(IIT-JEE 2010)**

4. Four harmonic waves of equal frequencies and equal intensities I_0 have phase angles 0, $\frac{\pi}{3}$, $\frac{2\pi}{3}$ and π. When they are superposed, the intensity of the resulting wave is nI_0. The value of n is: **(JEE Advanced 2015)**

Fill in the Blanks Type

1. A travelling wave has the frequency v and the particle displacement amplitude A. For the wave the particle velocity amplitude is ____ and the particle acceleration amplitude is ____. **(IIT-JEE 1983)**
2. Sound waves of frequency 660 Hz fall normally on perfectly reflecting wall. The shortest distance from the wall at which the air particles have maximum amplitude of vibration is ____ metres. **(IIT-JEE 1984)**
3. In a sonometer wire, the tension is maintained by suspending a 50.7 kg mass from the free end of the wire. The suspended mass has a volume of 0.0075 m^3. The fundamental frequency of vibration of the wire is 260 Hz. If the suspended mass is completely submerged in water, the fundamental frequency will become ____ Hz. **(IIT-JEE 1987)**
4. The amplitude of a wave disturbance propagating in the positive x-direction is given by $y = 1/(1 + x^2)$ at time $t = 0$ and by $y = 1/[1 + (1 - x)^2]$ at $t = 2$ s, where x and y are in metres. The shape of the wave disturbance does not change during the propagation. The velocity of the wave is ____ m/s. **(IIT-JEE 1990)**
5. A cylindrical resonance tube open at both ends has fundamental frequency f in air. Half of the length of the tube is dipped vertically in water. The fundamental frequency to the air column now is ——. **(IIT-JEE 1992)**
6. A bus is moving towards a huge wall with a velocity of 5 m/s. The driver sounds a horn of frequency 200 Hz. The frequency of the beats heard by a passenger of the bus will be ____ Hz. (Speed of sound in air = 342 m/s.) **(IIT-JEE 1992)**
7. A plane progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m and initial phase zero propagates along the negative x-direction with a velocity of 300 m/s. At any instant, the phase difference between the oscillations at two points 6 m apart along the line of propagation is ____, and the corresponding amplitude difference is ____ m. **(IIT-JEE 1997)**

True/False Type

1. The ratio of the velocity of sound in hydrogen gas ($\gamma = 7/5$) to that in helium gas ($\gamma = 5/3$) at the same temperature is $\sqrt{21/5}$. **(IIT-JEE 1980)**

2. A man stands on the ground at a fixed distance from a siren which emits sound of fixed amplitude. The man hears the sound to be louder on a clear night than on a clear day. **(IIT-JEE 1980)**

3. A plane wave of sound travelling in air is incident upon a plane water surface. The angle of incidence is 60°. Assuming Snell's law to be valid for sound waves, it follows that the sound wave will be refracted into water away from the normal. **(IIT-JEE 1984)**

4. A source of sound with frequency 256 Hz is moving with a velocity V towards a wall and an observer is stationary between the source and the wall. When the observer is between the source and the wall, he will hear beats. **(IIT-JEE 1985)**

Subjective Type

1. A tube of a certain diameter and length 48 cm is open at both ends. Its fundamental frequency is found to be 320 Hz. The velocity of sound in air is 320 m/s. Estimate the diameter of the tube. One end of the tube is now closed. Calculate the lowest frequency of resonance for the tube. **(IIT-JEE 1980)**

2. A source of sound of frequency 256 Hz is moving rapidly towards wall with a velocity of 5 m/sec. How many beats per second will be heard by the observer on source itself if sound travels at a speed of 330 m/sec? **(IIT-JEE 1981)**

3. A string 25 cm long and having a mass 2.5 gm is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases beat frequency. If the speed of sound in air is 320 m/s, find the tension in the string. **(IIT-JEE 1982)**

4. A sonometer wire under tension of 64 Newtons vibrating in its fundamental mode is in resonance with a vibrating tuning fork. The vibrating portion of the sonometer wire has a length of 10 cm and a mass of 1 gm. The vibrating tuning fork is now moved away from the vibrating wire with a constant speed and an observer standing near the sonometer hears one beat per second. Calculate the speed with which the tuning fork is moved if the speed of sound in air is 300 m/s. **(IIT-JEE 1983)**

5. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope? **(IIT-JEE 1984)**

6. A steel wire of length 1 m, mass 0.1 kg and uniform cross sectional area 10^{-6} m^2 is rigidly fixed at both ends. The temperature of wire is lowered by 20°C. If transverse waves are set up by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration. Young's module of steel $= 2 \times 10^{11}$ N/m^2, coefficient of linear expansion of steel $= 1.21 \times 10^{-5}$ (deg C)$^{-1}$. **(IIT-JEE 1984)**

7. The vibrations of a string of length 60 cm fixed at both ends are represented by the equation

$$y = 4 \sin\left(\frac{\pi x}{15}\right) \cos(96\pi t)$$

where x and y are in cm and t in second.

i. What is the maximum displacement at $x = 5$ cm?
ii. Where are the nodes located along the string?
iii. What is the velocity of the particle at $x = 7.5$ cm at $t = 0.25$ s?
iv. Write down the equations of component waves whose superposition gives the above waves.

(IIT-JEE 1985)

8. Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork. Speed of sound = 340 m/s. **(IIT-JEE 1986)**

9. The following equations represent transverse waves:
$z_1 = A\cos(kx - \omega t)$; $z_2 = A\cos(kx + \omega t)$;
$z_3 = A\cos(ky - \omega t)$
Identify the combination(s) of the waves which will produce (i) standing wave (c), (ii) a wave travelling in the direction making an angle of 45° degrees with the positive x and positive y axes. In each case, find the positions at which the resultant intensity is always zero. **(IIT-JEE 1987)**

10. A train approaching a hill at a speed of 40 km/hr sounds a whistle of frequency 580 Hz when it is at a distance of 1 km from a hill. A wind with a speed of 40 km/hr is blowing in the direction of motion of the train. Find **(IIT-JEE 1988)**

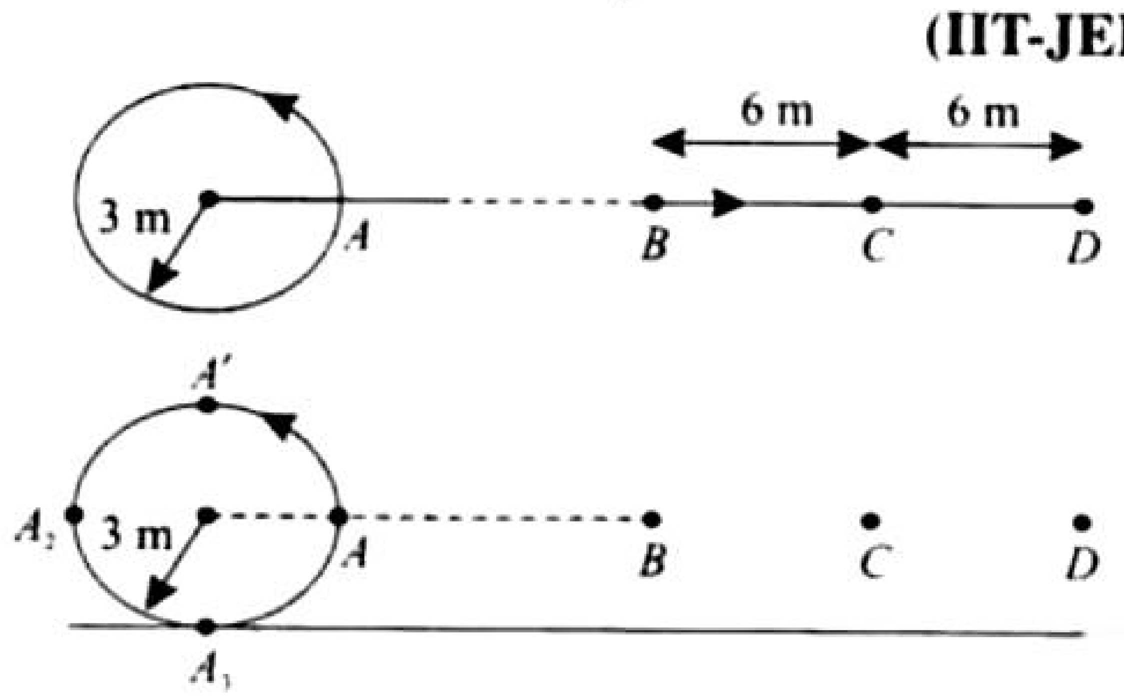
i. the frequency of the whistle as heard by an observer on the hill,
ii. the distance from the hill at which the echo from the hill is heard by the driver and its frequency.

(Velocity of sound in air = 1200 km/hr)

11. A source of sound is moving along a circular orbit of radius 3 metres with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD with an amplitude $BC = CD = 6$ metres. The frequency of oscillation of the detector is $5/\pi$ per second. The source is at the point A when the detector is at the point B. If the source emits a continuous sound wave of frequency 340

Hz, find the maximum and the minimum frequencies recorded by the detector. (Speed of sound = 340 m/s)

(IIT-JEE 1990)



12. The displacement of the medium in a sound wave is given by the equation $y_1 = A\cos(ax + bt)$ where A, a and b are positive constants. The wave is reflected by an obstacle situated at $x = 0$. The intensity of the reflected wave is 0.64 times that of the incident wave.

a. What are the wavelength and frequency of incident wave?

b. Write the equation for the reflected wave.

c. In the resultant wave formed after reflection, find the maximum and minimum values of the particle speeds in the medium.

d. Express the resultant wave as a superposition of a standing wave and a travelling wave. What are the positions of the antinodes of the standing wave? What is the direction of propagation of travelling wave?

(IIT-JEE 1991)

13. Two radio stations broadcast their programmes at the same amplitude A, and at slightly different frequencies ω_1 and ω_2 respectively, where $\omega_2 - \omega_1 = 10^3$ Hz. A detector receives the signals from the two stations simultaneously. It can emit signals only of intensity $\geq 2A^2$.

i. Find the time intervals between successive maxima of the intensity of the signal received by the detector.

ii. Find the time for which the detector remains idle in each cycle of the intensity of the signal.

(IIT-JEE 1993)

14. A metallic rod of length 1 m is rigidly clamped at its midpoint. Longitudinal stationary waves are set up in the rod in such a way that there are two nodes on either side of the midpoint. The amplitude of an antinode is 2×10^{-6} m. Write the equation of motion at a point 2 cm from the midpoint and those of constituent waves in the rod. ($Y = 2 \times 10^{11}$ N/m^2 and $\rho = 8 \times 10^3$ kg/m^3)

(IIT-JEE 1994)

15. A whistle emitting a sound of frequency 440 Hz is tied to a string of 1.5 m length and rotated with an angular velocity of 20 rad s^{-1} in the horizontal plane. Calculate the range of frequencies heard by an observer stationed at a large distance from the whistle. **(IIT-JEE 1996)**

16. The first overtone of an open organ pipe beats with the first overtone of a closed organ pipe with a beat frequency 2.2 Hz. The fundamental frequency of the closed organ pipe is 110 Hz, find the lengths of the pipes. Take velocity of sound = 330 m/s. **(IIT-JEE 1997)**

17. A band plying music at a frequency f is moving towards a wall at a speed v_b. A motorist is following the band with a speed v_m. If v is the speed of sound, obtain an expression for the beat frequency heard by the motorist.

(IIT-JEE 1997)

18. The air in a pipe closed at one end is made to vibrate in its second overtone by a tuning fork of frequency 440 Hz. The speed of sound in air 330 m/s. End correction may be neglected. Let P_0 denotes the mean pressure of any point in the pipe and ΔP_0 the maximum amplitudes of pressure variation.

a. Find the length L of the air column.

b. What is the amplitude of pressure variation at the middle of the column

c. What are maximum and minimum pressures at the open end of the pipe? **(IIT-JEE 1998)**

19. A long wire PQR is made by joining two wires PQ and QR of equal radii. PQ has length 4.8 m and mass 0.06 kg. QR has length 2.56 m and mass 0.2 kg. The wire PQR is under a tension of 80 N. A sinusoidal wave pulse of amplitude 3.5 cm is sent along the wire PQ from the end? No power is dissipated during the propagation of the wave-pulse. Calculate

a. The time taken by the wave pulse to reach the other end R of the wire, and

b. The amplitude of the reflected and transmitted wave pulses after the incident wave pulse crosses the joint Q. **(IIT-JEE 1999)**

20. A 3.6 m long vertical pipe resonates with a source of frequency 212.5 Hz when water level is at certain height in the pipe. Find the height of water level (from the bottom of the pipe) at which resonance occurs. Neglect end correction. Now, the pipe is filled to a height H ($\approx$ 3.6 m). A small hole is drilled very close to its bottom and water is allowed to leak. Obtain an expression for the rate of fall of water level in the pipe as a function of H. If the radii of the pipe and the hole are 2×10^{-2} m and 1×10^{-3} m respectively, calculate the time interval between the occurrence of first two resonances. (Speed of sound in air is 340 m/s and g = 10 m/s^2). **(IIT-JEE 2000)**

21. A boat is travelling in a river with a speed 10 m/s along the stream flowing with a speed 2 m/s. From this boat, a sound transmitter is lowered into the river through a rigid support. The wavelength of the sound emitted from the transmitter inside the water is 14.45 mm. Assume that attenuation of sound in water and air is negligible.

a. What will be the frequency detected by a receiver kept inside the river downstream?

b. The transmitter and the receiver are now pulled up into air. The air is blowing with a speed 5 m/s in the direction opposite the river stream. Determine the frequency of the sound detected by the receiver.

(Temperature of the air and water = 20°C; Density of river water = 10^3 kg/m^3; Bulk modulus of the water = 2.088×10^9 Pa; Gas constant $R = 8.31$ J/mol-K; Mean molecular mass of air = 28.8×10^{-3} kg/mol; C_P/C_V for air = 1.4) **(IIT-JEE 2001)**

22. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature.

a. If the frequency to the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B; determine the value of M_A/M_B.

b. Now the open end of pipe B is also closed (so that pipe B is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B.

(IIT-JEE 2002)

23. In a resonance tube experiment to determine the speed of sound in air, a pipe of diameter 5 cm is used. The air column in pipe resonates with a tuning fork of frequency 480 Hz, when minimum length of air column in 16 cm. Find the speed of sound in air at room temperature. **(IIT-JEE 2003)**

24. A string tied between $x = 0$ and $x = l$ vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. **(IIT-JEE 2003)**

25. A whistling train approaches a junction. An observer standing at junction observes the frequency to be 2.2 KHz and 1.8 KHz of the approaching and the receding train respectively. Find the speed of the train (speed of sound = 300 m/s). **(IIT-JEE 2005)**

26. A transverse harmonic disturbance is produced in a string. The maximum transverse velocity is 3 m/s and the maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s, then find the waveform. **(IIT-JEE 2005)**

ANSWER KEY

JEE Advanced

Single Correct Answer Type

1. c.	**2.** b.	**3.** a.	**4.** c.	**5.** c.
6. a.	**7.** a.	**8.** a.	**9.** d.	**10.** a.
11. b.	**12.** b.	**13.** d.	**14.** d.	**15.** b.
16. c.	**17.** b.	**18.** a.	**19.** b.	**20.** b.
21. d.	**22.** b.	**23.** c.	**24.** d.	**25.** a.
26. a.	**27.** a.	**28.** a	**29.** b.	**30.** b.
31. a.	**32.** b.	**33.** d.		

Multiple Correct Answers Type

1. a, b, c, d.	**2.** a, c.	**3.** a, c.
4. a, b, d.	**5.** b, c.	**6.** b, d.
7. a, b, c.	**8.** a, b, c, d.	**9.** b, c.
10. a, c.	**11.** b, c, d.	**12.** a, b, c, d.
13. a, b, c.	**14.** b, d.	**15.** a, d.
16. b, d.	**17.** a, b.	**18.** a, c, d.

Linked Comprehension Type

1. a. **2.** c. **3.** a. **4.** b. **5.** a. **6.** a.

Matching Column Type

1. i. – a., e.; ii. – a., d.; iii. – b., d.; iv. – b., c.

Integer Answer Type

1. (5) **2.** (7) **3.** (5) **4.** (3)

Fill in the Blanks Type

1. $A(2\pi\nu)^2$ **2.** 0.125 m **3.** 240 Hz
4. –0.5 m/s **5.** f **6.** 6 Hz
7. 0

True/False Type

1. False **2.** False **3.** True **4.** False

Subjective Type

1. 63.27 Hz **2.** 8 **3.** 27.04 N
4. 0.75 m/s **5.** 0.12 m **6.** 11 Hz

7. (i) $2\sqrt{3}$ cm (ii) $x = 0$, 15 cm, 30 cm, etc., (iii) zero

(iv) $y_1 = 2\sin\left(\frac{\pi x}{15} - 96\pi t\right)$ and $y_2 = 2\sin\left(\frac{\pi x}{15} + 96\pi t\right)$

8. 1.5 m/s

9. (i) z_1 and z_2; $x = (2n+1)\frac{\pi}{2k}$ where $n = 0, \pm1, \pm2....$ etc.

(ii) z_1 and z_2; $x - y = (2n+1)\frac{\pi}{k}$ where $n = 0, \pm1, \pm2....$ etc.

10. (i) 599 Hz (ii) 0.902 km, 621 Hz

11. 485.7 Hz, 257.3 Hz

12. (a) $\frac{2\pi}{a}$ and $\frac{b}{2\pi}$

(b) $y_2 = -0.8\, A \cos(ax + bt)$

(c) 1.8 Ab, zero

(d) $y = -1.6\, A \sin ax \sin bt + 0.2\, A \cos(ax + bt)$

Antinodes are at $x = \left[n + \frac{(-1)^n}{2}\right]\frac{\pi}{a}$

Travelling wave is propagating in negative x-direction.

13. (i) 6.28×10^{-3} s (ii) 1.57×10^{-3} s

14. $y = 2 \times 10^{-6} \sin(0.1\pi) \sin(25000\pi t)$,

$y_1 = 10^{-6} \sin(25000\pi t - 5\pi x)$,

$y_2 = 10^{-6} \sin(25000\pi t + 5\pi x)$

15. 403.3 Hz to 484 Hz.

16. 99.3 cm, 100.6 cm

17. $\frac{2v_b(v + v_m)}{(v^2 - v_b^2)} f$

18. (a) 93.75 cm (b) $\frac{\Delta P_0}{\sqrt{2}}$ (c) $P_{max} = P_{min} = P_0$

19. (a) 0.14 s (b) $A_r = 1.5$ cm, $A_t = 2$ cm

20. 3.2 m, 2.4 m, 1.6 m, 0.8 m, $\frac{-dH}{dt} = (1.11 \times 10^{-2})\sqrt{H}$, 43 s

21. (a) 1.007×10^5 Hz (b) 1.03×10^5 Hz

22. (a) $\frac{400}{189}$ (b) $\frac{3}{4}$

23. 336 m/s

24. $\frac{\pi^2 T a^2}{4l}$

25. $v_T = 30$ m/s

26. $y = (0.1\text{ m}) \sin[(30\text{ rad/s})t \pm (1.5\text{ m}^{-1})x + \phi]$

HINTS AND SOLUTIONS

JEE Advanced
Single Correct Answer Type

1. c. **Case I** Here $\lambda/2 = l$

$\therefore \quad \lambda = 2l$

Now, $v = f \times \lambda$

$$\therefore \quad f = \frac{v}{\lambda} = \frac{v}{2l} \qquad \text{(i)}$$

Case I Here $\lambda'/4 = l/2$

$\therefore \quad \lambda' = 2l$

Now, $v = f' \times \lambda'$

$$\therefore \quad f' = \frac{v}{\lambda'} = \frac{v}{2l} = f$$

2. b. $y = y_0 \sin 2\pi \left[ft - \frac{x}{\lambda}\right]$

$$\therefore \quad \frac{dy}{dt} = \left[y_0 \cos 2\pi \left(ft - \frac{x}{\lambda}\right)\right] \times 2\pi f$$

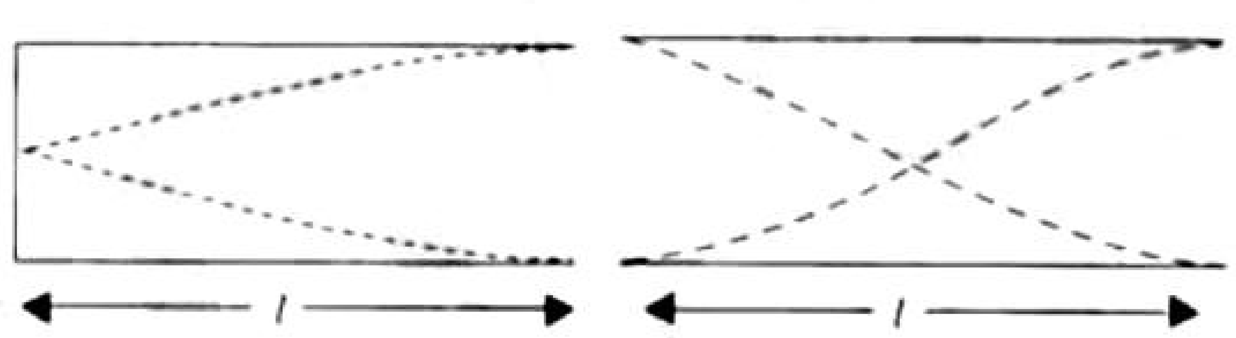
$$\Rightarrow \left[\frac{dy}{dt}\right]_{max} = y_0 \times 2\pi f$$

Given that the maximum particle velocity is equal to four times the wave velocity ($c = f\lambda$).

$\therefore \quad y_0 \times 2\pi f = 4f \times \lambda$

$$\lambda = \frac{\pi y_0}{2}$$

3. a. $\lambda/4 = l$ (Fundamental mode), $\lambda = 4l$, $c = v\lambda$



$$\therefore \quad v = \frac{c}{\lambda} = \frac{c}{4l} = 512 \text{ Hz}$$

Given, $\lambda'/2 = l$

Fundamental mode.

$\therefore \quad \lambda' = 2l$ but $c = v'\lambda'$

$$\therefore \quad v' = \frac{c}{\lambda'} = \frac{c}{2l} = 2\left(\frac{c}{4l}\right)$$

$$= 2 \times 512 = 1024 \text{ Hz}$$

4. c. Stationary wave is produced when two waves travel in opposite directions. Now,

$$y = a\cos(kx - \omega t) - a\cos(kx + \omega t)$$

$y = 2a \sin kx \sin \omega t$ is equation of stationary wave which gives a node at $x = 0$.

5. c. Given $\frac{v}{4l_1} = \frac{3v}{2l_2} \quad \Rightarrow \frac{l_1}{l_2} = \frac{1}{6}$

6. a. We know that

$$f = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

In air, $T = mg = \rho V g$

$$\therefore \quad f = \frac{1}{2l}\sqrt{\frac{\rho V g}{m}} \qquad \text{(i)}$$

In water, $T = mg$ − upthrust

$$= V\rho g - \frac{V}{2}\rho_\omega g = \frac{Vg}{2}(2\rho - \rho_\omega)$$

Therefore,

$$\therefore f' = \frac{1}{2l}\sqrt{\frac{\frac{Vg}{2}(2\rho - \rho_\omega)}{m}}$$

$$= \frac{1}{2l}\sqrt{\frac{Vg\rho}{m}}\sqrt{\frac{(2\rho - \rho_\omega)}{2\rho}} = 300\left[\frac{2\rho - 1}{2\rho}\right]^{\frac{1}{2}}$$

$\because \rho_\omega = 1$ g/cc and from Eq. (i)

7. a. According to Hooke's law, $F_g \propto x$ [Restoring force $F_g = T$, tension of spring]

Velocity of sound by a stretched string

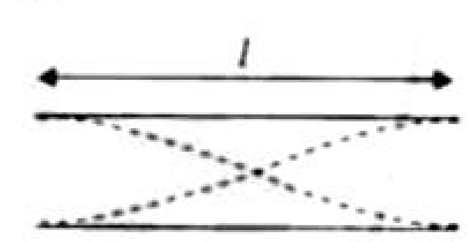
$$v = \sqrt{\frac{T}{m}}$$

where m is the mass per unit length

$$\therefore \quad \frac{v}{v'} = \sqrt{\frac{T}{T'}} \quad \Rightarrow v' = v\sqrt{\frac{T'}{T}} = v\sqrt{\frac{1.5x}{x}} = 1.22v$$

8. a. For both ends open, fundamental frequency

$$\frac{2\lambda_1}{4} = l \quad \Rightarrow \lambda_1 = 2l$$



$$\therefore \quad v_1 = \frac{c}{\lambda_1} = \frac{c}{2l} \qquad \text{(i)}$$

For one end closed the third harmonic

$$\frac{3\lambda_2}{4} = l \quad \Rightarrow \lambda_2 = \frac{4l}{3}$$

$$v_2 = \frac{c}{\lambda_2} = \frac{3c}{4l} \qquad \text{(ii)}$$

Given $v_2 - v_1 = 100$

From Eqs. (i) and (ii)

$$\frac{v_2}{v_1} = \frac{3/4}{1/2} = \frac{3}{2}$$

On solving, we get $v_1 = 200$ Hz

9. d. $f' = f\left[\frac{v}{v - v_s}\right] = 450\left[\frac{330}{330 - 33}\right] = 500 \text{ Hz}$

10. a. $V = \frac{dy}{dt} = -A\omega\cos(kx - \omega t)$

$\therefore \quad V_{max} = A\omega$

11. b. Velocity of wave: $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{1.6}{10^{-2}/0.4}} = 8$ m/s

The wave will be in same phase after travelling a distancne of $2l = 2 \times 0.4 = 0.8$ m.

And constructive interference will take place. So time Δt

$$\Delta t = \frac{0.8}{v} = \frac{0.8}{8} = 0.10 \text{ s}$$

12. b. $\frac{(C)N_2}{(C)He} = \sqrt{\frac{M_{He}}{M_{N_2}}} = \sqrt{\frac{4}{28}} = \sqrt{\frac{1}{7}}$

13. d. $n_1 = n_0 \frac{340}{340 - 34} = \frac{10}{9} n_0$

$$n_2 = n_0 \frac{340}{340 - 17} = \frac{20}{19} n_0$$

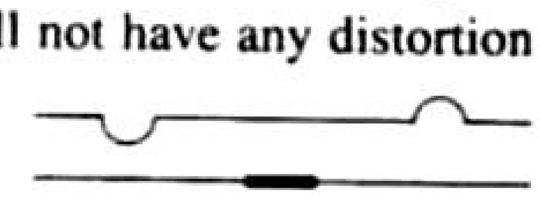
$$\frac{n_1}{n_2} = \frac{10}{9} \times \frac{19}{20} = \frac{19}{18}$$

14. d. $n_1 = \frac{1}{2l}\sqrt{\left[\frac{T}{4\pi r^2 \rho}\right]}$

and $\quad n_2 = \frac{1}{4l}\sqrt{\left[\frac{T}{\pi r^2 \rho}\right]}$

$\therefore \quad \frac{n_1}{n_2} = 2 \times \frac{1}{2} = 1$

15. b. After 2 s, tubes will overlap each other. According to superposition principle, the string will not have any distortion and will be straight. Hence, there will be no PE. The total energy will be kinetic.



16. c. We know that $E \propto A^2 v^2$, where A = amplitude and v = frequency.

Also, $\omega = 2\pi v = \omega \propto v$

In case 1: Amplitude = A and $v_1 = v$

In case 2: Amplitude = A and $v_2 = 2v$

$\therefore \quad \frac{E_2}{E_1} = \frac{A^2 v_2^2}{A^2 v_1^2} = 4 \quad \Rightarrow E_2 = 4E_1$

17. b. $\frac{V_A + V}{V} = \frac{5.5}{5}$

$$\frac{V_B + V}{V} = \frac{6}{5}$$

Solve to get $\frac{V_B}{V_A} = 2$

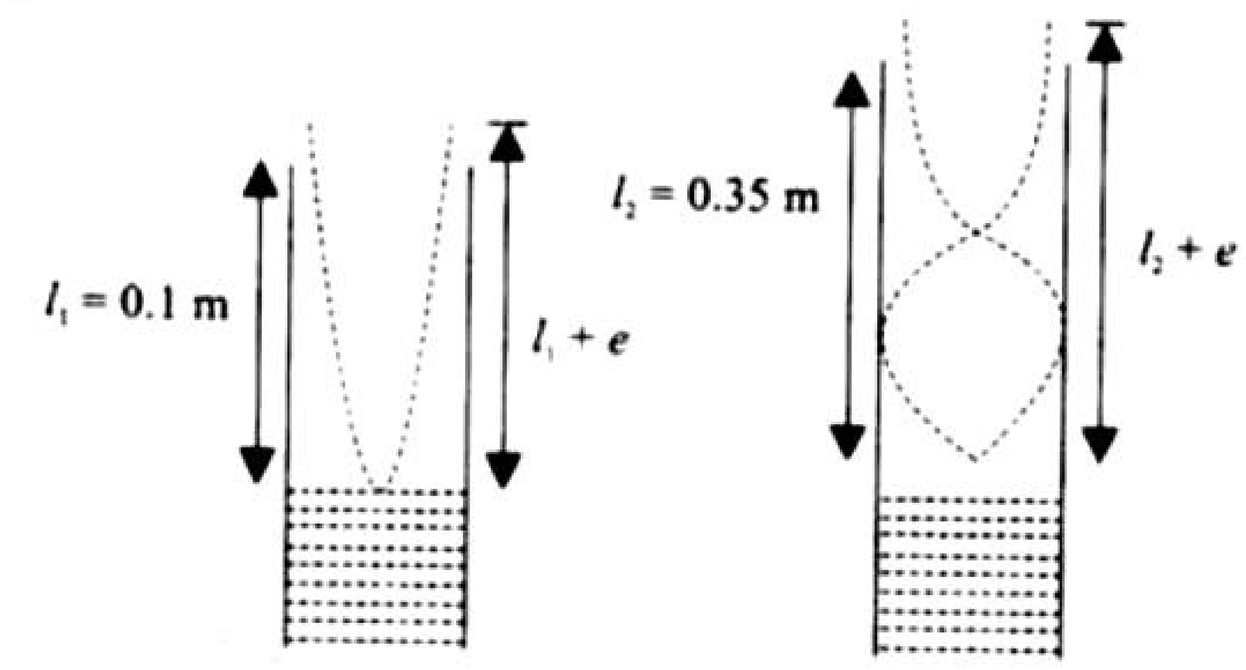
18. a. $f_0 = \frac{5}{2l}\sqrt{\frac{9g}{\mu}} = \frac{3}{2l}\sqrt{\frac{Mg}{\mu}} \Rightarrow M = 25$ kg

19. b. f_1 = frequency of the police car heard by motorcyclist, f_2 = frequency of the siren heard by motorcyclist.

$$f_1 = \frac{330 - v}{330 - 22} \times 176, \quad f_2 = \frac{330 + v}{330} \times 165$$

$\therefore \quad f_1 - f_2 = 0 \quad \Rightarrow \quad v = 22$ m/s

20. b.



$$l_1 + x = \frac{\lambda}{4} \quad \Rightarrow \lambda = 4(l_1 + x)$$

$$(l_2 + x) = \frac{3\lambda}{4} \quad \Rightarrow \lambda = \frac{4}{3}(l_2 + x)$$

$\therefore \quad v_1 = \frac{v}{\lambda_1} = \frac{v}{4(l_1 + x)}$

$\therefore \quad v_2 = \frac{v}{\lambda_2} = \frac{3v}{4(l_2 + x)}$

Given $v_1 = v_2$

$$\Rightarrow \frac{v}{4(l_1 + x)} = \frac{3v}{4(l_2 + x)} \quad \Rightarrow x = 0.025 \text{ m}$$

21. d. The frequency is a characteristic of source. It is independent of the medium. Hence, the correct option is (d).

22. b. Frequency of first overtone in closed pipe,

$$v = \frac{3}{4l_1}\sqrt{\frac{B}{\rho_1}} \tag{i}$$

Frequency of the first overtone in open pipe,

$$v = \frac{1}{l_2}\sqrt{\frac{B}{\rho_2}} \Rightarrow l_2 = \frac{4}{3} l_1 \sqrt{\frac{\rho_1}{\rho_2}} \tag{ii}$$

23. c. For first the resonance $l_1 + e = \frac{\lambda}{4}$

But $\quad v = f\lambda$

$\therefore \quad v = f4(l_1 + e)$

$$\Rightarrow \quad l_1 + e = \frac{v}{4f} \tag{i}$$

For the second resonance

$$l_2 + e = \frac{3\lambda}{4}$$

$\therefore \quad v = f\frac{4}{3}(l_2 + e)$

$$\Rightarrow \quad l_2 + e = \frac{3v}{4f} \tag{ii}$$

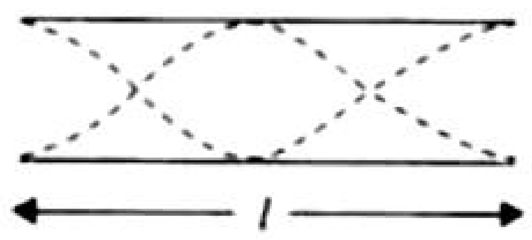
From Eqs. (i) and (ii), we get

$v = 2f(l_2 - l_1)$

$\therefore \quad \Delta v = 2f(\Delta l_2 + \Delta l_1)$

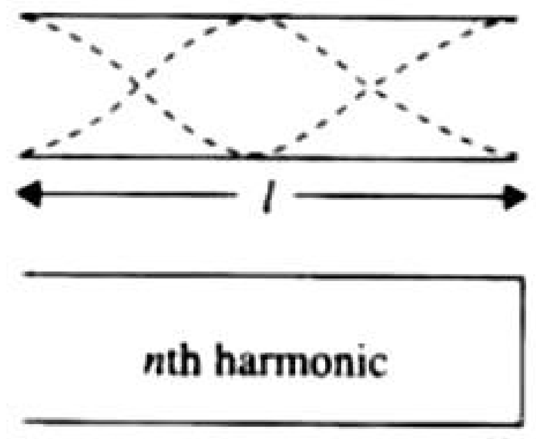
$= 2 \times 512 \times (0.1 + 0.1)$ cm/s

$= 204.8$ cm/s

24. d. 

$\lambda = l$

$\therefore \quad f_1 = \frac{v}{\lambda} = \frac{v}{l}$

(i) 

(ii)

$\lambda = \frac{4l}{n}$

$\therefore \quad f_2 = \frac{v}{\lambda} = \frac{nv}{4l}$

Here n is an odd number

$f_2 = \frac{n}{4} f_1$

From Eqs. (i) and (ii), we get

For the first resonance

$n = 5,$

$f_2 = \frac{5}{4} f_1$

Given $f_2 > f_1$

25. a. Frequency of the first harmonic of

$$AB = \frac{1}{2l}\sqrt{\frac{T_{AB}}{m}}$$

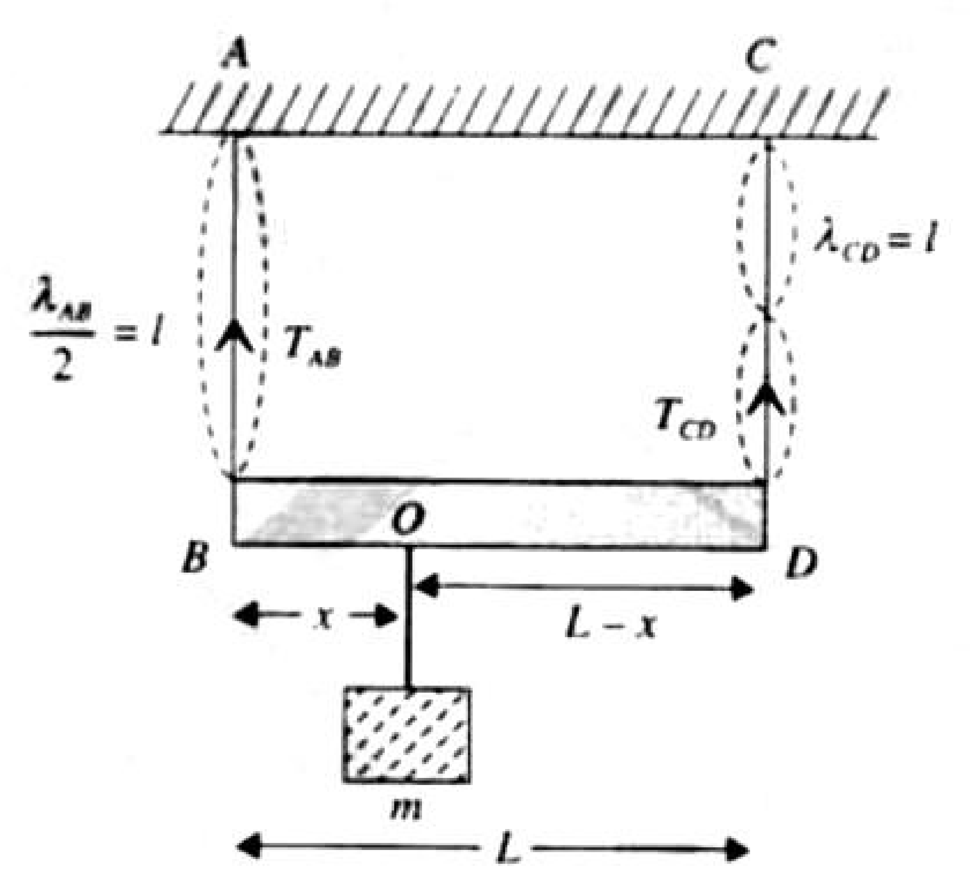
Frequency of the 2nd harmonic of

$$CD = \frac{1}{l}\sqrt{\frac{T_{CD}}{m}}$$

Given that the two frequencies are equal.

$$\therefore \quad \frac{1}{2l}\sqrt{\frac{T_{AB}}{m}} = \frac{1}{l}\sqrt{\frac{T_{CD}}{m}} \Rightarrow \frac{T_{AB}}{4} = T_{CD}$$

$\Rightarrow \quad T_{AB} = 4T_{CD}$ (i)

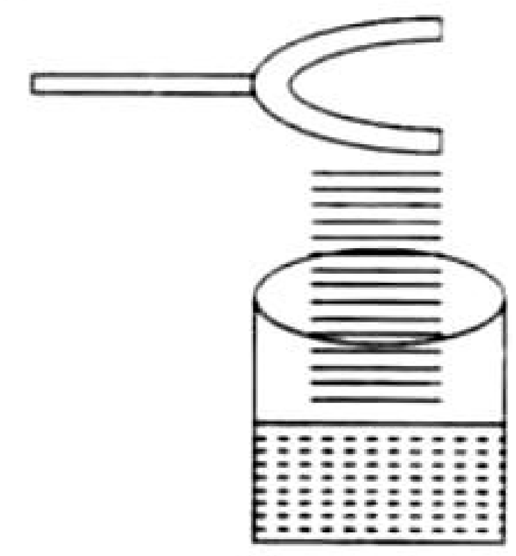


For rotational equilibrium of massless rod taking torque about point O,

$T_{AB} \times x = T_{CD}(L - x)$ (ii)

Solve to get $x = \frac{L}{5}$

26. a. As shown in the figure, the prongs of the tuning fork are kept in a vertical plane.



27. a. Particle velocity, $v_P = -v$ (slope of y-x graph)

Here, v = +ve, as the wave is travelling in positive direction.

Slope at P is negative.

Therefore, velocity of particle is in negative y (or $\hat{j}$) direction.

28. a. With increase in tension, frequency of vibrating string will increase. Since the number of beats are decreasing, therefore frequency of vibrating string or the third harmonic frequency of closed pipe should be less than the frequency of tuning fork by 4.

Therefore, frequency of tuning fork = third harmonic frequency of closed pipe + 4

$$= 3\left(\frac{v}{4l}\right) + 4 = 3\left(\frac{340}{4 \times 0.75}\right) + 4 = 344 \text{ Hz}$$

29. b. $\frac{v_s}{4L_P} = \frac{2}{2l_S}\sqrt{\frac{T}{\mu}} \Rightarrow \frac{320}{4 \times 0.8} = \frac{1}{0.5}\sqrt{\frac{50}{\mu}}$

$\Rightarrow \quad \mu = 1/50$

Mass of string,

$$m = \mu l_S = \frac{1}{50} \times 0.5 = 0.01 \text{ kg} = 10 \text{ g}$$

30. b. We know that

$$C = \sqrt{\left(\frac{\gamma RT}{M}\right)}$$

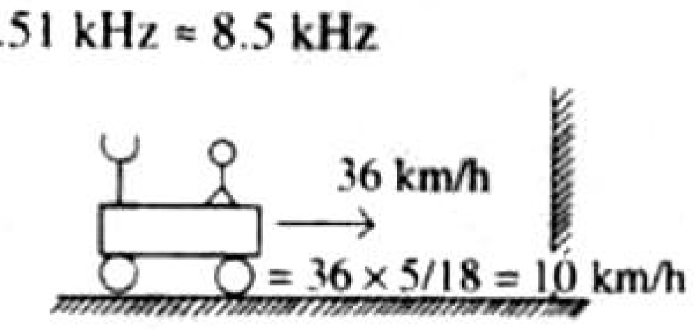
Hence $C \propto \sqrt{\frac{1}{m}}$

$$\therefore \quad \frac{C_1}{C_2} = \sqrt{\left(\frac{m_2}{m_1}\right)}$$

31. a. $f_{incident} = f_{reflected} = \frac{320}{320 - 10} \times 8 \text{ kHz}$

$$f_{observed} = \frac{320 + 10}{320} f_{reflected} = 8 \times \frac{320}{310}$$

$= 8.51 \text{ kHz} \approx 8.5 \text{ kHz}$



32. b. $\frac{V}{4(l + e)} = f$

$\Rightarrow l + e = \frac{V}{4f}$

$$\Rightarrow l = \frac{V}{4f} - e$$

Here $e = (0.6)\, r = (0.6)\,(2) = 1.2$ cm

So $\quad l = \dfrac{336 \times 10^2}{4 \times 512} - 1.2 = 15.2$ cm

33. d. For minimum height, $\dfrac{\lambda}{4} = e$

$$\frac{\lambda}{4} = 0.356 \pm 0.005 \text{ or } \lambda = 1.400 \pm 0.020$$

$$v = f\lambda = 244(1.4000 \pm 0.020)$$

$$\Rightarrow \quad v = (341.6 \pm 4.88)\frac{\text{m}}{\text{s}}$$

$$v = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}} = \sqrt{\frac{100\gamma RT \times 10}{M}}$$

For monoatomic $\gamma = 1.67$

$$v = \sqrt{167RT} \cdot \sqrt{\frac{10}{M}}$$

For neon (monoatomic): $640 \times \dfrac{7}{10} = 448 \text{ ms}^{-1}$

For argon (monoatomic): $640 \times \dfrac{17}{32} = 340 \text{ ms}^{-1}$

For diatomic $\gamma = 1.4$

$$v = \sqrt{140RT} \cdot \sqrt{\frac{10}{M}}$$

For nitrogen (diatomic): $590 \times \dfrac{3}{5} = 384 \text{ ms}^{-1}$

For oxygen (diatomic): $590 \times \dfrac{9}{16} = 351.875 \text{ ms}^{-1}$

Hence possible answer is (d).

Multiple Correct Answer Type

1. a., b., c., d. $y = 10^{-4} \sin(60t + 2x)$. Comparing the given equation with the standard wave equation travelling in negative x-direction,

$$y = a \sin(\omega t + kx)$$

We get amplitude $a = 10^{-4}$ m

Also, $\omega = 60$

$$\therefore \quad 2\pi f = 60 \quad \Rightarrow f = \frac{30}{\pi} \text{ Hz}$$

Also, $k = 2$

$$\Rightarrow \frac{2\pi}{\lambda} = 2 \quad \Rightarrow \lambda = \pi \text{ m}$$

We know that

$$v = f\lambda = \frac{30}{\pi} \times \pi = 30 \text{ m/s}$$

2. a., c. The wavelengths possible in an air column in a pipe which has one closed end is

$$\lambda = \frac{4l}{(2n+1)}$$

so, $\quad c = v\lambda \Rightarrow 330 = 264 \times \dfrac{4l}{2n+1}$

as it is in resonance with a vibrating tuning fork of frequency 264 Hz

$$l = \frac{330 \times (2n+1)}{264 \times 4}$$

For $\quad n = 0, l = 0.315$ m $= 31.25$ cm

For $\quad n = 1, l = 0.9375$ m $= 93.75$ cm

3. a., c. For wave motion, the differential equation is

$$\frac{\partial^2 y}{\partial t^2} = \left(\text{constant} \frac{\omega^2}{k^2}\right) \frac{\partial^2 y}{\partial x^2}$$

or $\quad \dfrac{\partial^2 y}{\partial t^2} = v^2 \dfrac{\partial^2 y}{\partial x^2} \quad$ (i)

The wave motion is characterized by the two conditions viz.

$f(x, t) = f(x, t + T) \quad$ (ii)

$f(x, t) = f(x, +\lambda, t) \quad$ (iii)

Hence option (a) and (c) represent correct answer.

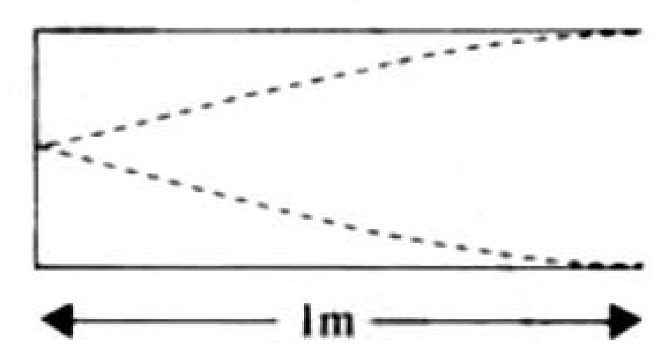
4. a., b., d. In general we can write for a closed end pipe

$$v = \frac{(2n-1)c}{4l}$$

where $n = 1, 2, 3, \ldots$

$$\therefore \quad v = \frac{c}{4l}, \frac{3c}{4l}, \frac{5c}{4}, \ldots$$

$$= 80, 240, 400, \ldots$$



5. b., c. $y = A \sin(10\pi x = 15\pi t + \pi/3)$

The standard equation of a wave travelling in X-direction is

$$y = A \sin\left[\frac{2\pi}{\lambda}(vt + x) + (\phi)\right]$$

$$\Rightarrow y = A \sin\left[\frac{2\pi v}{\lambda} t + \frac{2\pi}{\lambda} x + \phi\right]$$

Comparing it with the given equation we find

$$\frac{2\pi v}{\lambda} = 15\pi$$

and $\quad \dfrac{2\pi}{\lambda} = 10\pi$

$$\lambda = \frac{1}{5} = 0.2 \text{ m}$$

$$\therefore \quad v = \frac{15\pi}{2\pi} \times \frac{1}{5} = 1.5 \text{ m/s}$$

6. b., d. T_1 and T_2 are the higher and lowest tensions initially. Now, frequency $\propto \sqrt{\text{tension}}$.

Therefore, frequency produced in wire with tension T_1 is higher and that with tension T_2 is lower. If we lower the tension T_2 then beat frequency will increase. Therefore, the tension T_1 is decreased. If tension has to be increased, then tension T_2 should be increased.

7. a., b., c. Moving plane is like a moving observer. Therefore, number of waves encountered by moving plane

$$f_1 = f\left(\frac{v + v_0}{v}\right) = f\left(\frac{c + v}{c}\right)$$

Frequency of reflected wave,

$$f_2 = f_1\left(\frac{v}{v-v_s}\right) = f\left(\frac{c+v}{c-v}\right)$$

Wavelength of reflected wave,

$$\lambda_2 = \frac{v}{f_2} = \frac{c}{f_2} = \frac{c}{f}\left(\frac{c-v}{c+v}\right)$$

Number of beats heard $= f_2 - f = \dfrac{2vf}{c-v}$

8. a., b., c., d. It is given that $y(x, t) = 0.02\cos(50\pi t + \pi/2)\cos(10\pi x)$

$$\cong A\cos\left(\omega t + \frac{\pi}{2}\right)\cos kx$$

Node occurs when $kx = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$, etc.

$$\Rightarrow 10\pi x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$\Rightarrow \quad x = 0.05$ m, 0.15 m option (a)

Anitnode occurs when $kx = \pi, 2\pi, 3\pi$ etc.

$\Rightarrow \quad 10\,\pi x = \pi, 2\pi, 3\pi$ etc.

$\Rightarrow \quad x = 0.1$ m, 0.02 m, 0.3 m option (b)

Speed of the wave is given by

$$v = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5 \text{ m/s} \qquad \text{option (c)}$$

Wavelength is given by

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = \left(\frac{1}{5}\right)\text{m} = 0.2 \text{ m} \qquad \text{option (d)}$$

9. b., c. Due to the clamping of the square plate at the edges, its displacements along the x- and y-axes will individually be zero at the edges. Only the choices (b) and (c) predict these displacements correctly. This is because $\sin 0 = 0$.

10. a., c. For a transverse sinusoidal wave travelling on a string, the maximum velocity is $a\omega$. Also, the maximum velocity is

$$\frac{v}{10} = \frac{10}{10} = 1 \text{ m/s}$$

$\therefore \quad a\omega = 1 \quad \Rightarrow 10^{-3} \times 2\pi f = 1$

$$\Rightarrow f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi} \text{ Hz}$$

The velocity $v = f\lambda$

$$\therefore \quad \lambda = \frac{v}{f} = \frac{10}{10^3/2\pi} = 2\pi \times 10^{-2} \text{ m}$$

11. b., c., d. Given,

$$y = \frac{0.8}{(4x+5t)^2+5} = \frac{0.8}{16\left[x+\frac{5}{4}t\right]^2+5} \qquad \text{(i)}$$

We know that equation of moving pulse is

$$y = f(x + vt) \qquad \text{(ii)}$$

On comparing Eqs. (i) and (ii), we get

$$v = \frac{5}{4} \text{ m/s} = \frac{2.5}{2} \text{ m/s}$$

Wave will travel a distance of 2.5 m in 2 s.

12. a., b., c., d.

The wavemotion $y = a\sin(kx - \omega t)$ represents

(a) electric field in electromagnetic wave

(b) magnetic field in electromagnetic wave

(c) displacement in sound wave

(d) pressure in sound wave.

Hence all the four options are correct.

13. a., b., c. Standing waves are produced by the superposition of two of two identical waves travelling in opposite direction.

Option (d) is accordingly not correct.

Option (a), (b) and (c) are correct

14. b., d. For $A = -B$ and $C = 2B$

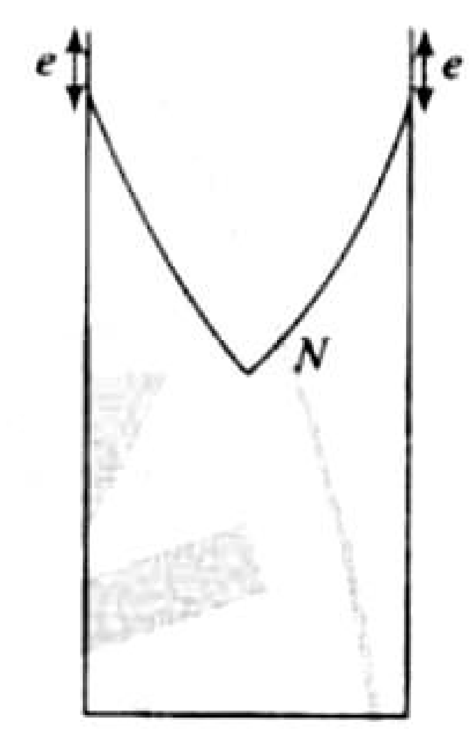
$$X = B\cos 2\omega t + B\sin 2\omega t = \sqrt{2}B\sin\left[2\omega t + \frac{\pi}{4}\right]$$

This is equation of SHM of amplitude $\sqrt{2}B$.

If $A = B$ and $C = B$, then $X = B + B\sin 2\omega t$

This is also equation of SHM about the point $X = B$. Function oscillates between $X = 0$ and $X = 2B$ with amplitude B.

15. a., d.



(a) Intensity of the fundamental is more than that of the overtones. Therefore the 1[st] resonance was having more intensity.

(b) The prongs should not be in the horizontal position but vertical over the resonance tube.

(c) The amplitude of vibrations are very small to be observed.

(d) The antinodes are formed always a little above the open end of the tube.

This is called end correction.

This effect will be there for overtones also.

$\therefore$ Length of the air column is less than $\lambda/4$.

16. b., d. At open end phase of pressure wave charge by π so compression returns as rarefraction. While at closed end phase of pressure wave does not change so compression return as compression.

17. a., b. If wind blows from source to observer

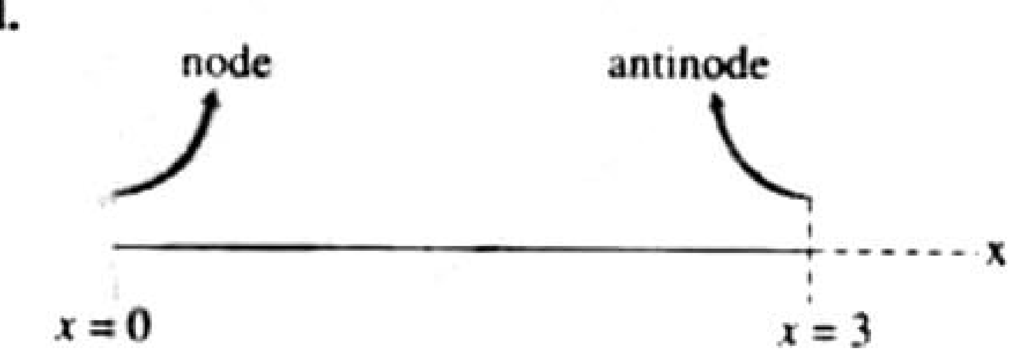
$$f_2 = f_1\left(\frac{V+w+u}{V+w-u}\right)$$

When wind blows from observer towards source

$$f_2 = f_1\left(\frac{V-w+u}{V-w-u}\right)$$

In both cases, $f_2 > f_1$.

18. a, c, d.



The fixed end is a node while the free end is an antinode. Therefore, at $x = 0$ is a node and at $x = 3$ m is an antinode. Possible modes of variation are

$$L = (2n+1)\frac{\lambda}{4} \quad \text{where } n = 0, 1, 2, 3, \ldots$$

or $\lambda = \frac{4L}{2n+1} = \frac{12}{2n+1}$ ($\because L = 3$ m (given)

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{12/(2n+1)} = \frac{(2n+1)\pi}{6}$$

$$\omega = vk = 100(2n+1)\frac{\pi}{6} = \frac{(2n+1)50\pi}{3}$$

We have stationary wave equation $y = A \sin kx \cdot \cos \omega t$

For $n = 0$

$$y(t) = A \sin\frac{\pi x}{6}\cos\frac{50xt}{3}$$

For $n = 2$

$$y(t) = A \sin\frac{5\pi x}{6}\cos\frac{250xt}{3}$$

For $n = 7$

$$y(t) = A \sin\frac{5\pi x}{2}\cos 250xt$$

Linked Comprehension Type

For Problems 1–3

1. a., 2. c., 3. a.

1. a. Number of maxima in 1 s is called the beat frequency. Hence,

$$f_b = f_1 - f_2$$

$$= \frac{100\pi}{2\pi} - \frac{92\pi}{2\pi} = 4 \text{ Hz}$$

2. c. Speed of wave,

$$v = \frac{\omega}{k}$$

$$v = \frac{100\pi}{0.5\pi} \text{ or } \frac{92\pi}{0.46\pi} = 200 \text{ m/s}$$

3. a. At $x = 0, y = y_1 + y_2$

$$= 2A \cos 96\pi t \cos 4\pi t$$

Frequency of cos $(96\pi t)$ function is 48 Hz and that of cos $(4\pi t)$ function is 2 Hz. In 1 s, cos function becomes zero at $2f$ times, where f is the frequency. Therefore, first function will become zero at 96 times and the second at 4 times. But second will not overlap with the first. Hence, net y will become zero 100 times in 1 s.

For Problems 4–6

4. b., 5. a., 6. a.

4. b. $v_{SA} = 340 + 20 = 360$ m/s

$$v_{SB} = 340 - 30 = 310 \text{ m/s}$$

5. a. For the passengers in train A, there is no relative motion between source and observer, as both are moving with velocity 20 m/s. Therefore, there is no change in observed frequencies and correspondingly there is no change in their intensities. Therefore, the correct option is (a).

6. a. For the passengers in train B, observer is receding with velocity 30 m/s and source is approaching with velocity 20 m/s.

$$f_1 = 800\left(\frac{340-30}{340-20}\right) = 775 \text{ Hz}$$

$$f_2 = 1120\left(\frac{340-30}{340-20}\right) = 1085 \text{ Hz}$$

Therefore, spread of frequency, $f'_2 - f'_1 = 310$ Hz

Matching Column Type

1. i. → a., e.; ii. → a., d.; iii. → b., d.; iv. → b., c.

i. Sound waves are longitudinal waves

$$\frac{\lambda_f}{4} = L \Rightarrow \lambda_f = 4L$$

ii. Sound waves are longitudinal waves

$$\frac{\lambda_f}{2} = L \Rightarrow \lambda_f = 2L$$

iii. String waves are transverse waves

$$\frac{\lambda_f}{2} = L \Rightarrow \lambda_f = 2L$$

iv. String waves are transverse waves

$$\frac{2\lambda_f}{2} = L \Rightarrow \lambda_f = L$$

Integer Answer Type

1. (5)

$$v = \sqrt{\frac{T}{\mu}} = 10 \text{ m/s}$$

$$\lambda = \frac{v}{f} = \frac{10}{100} = 10 \text{ cm}$$

Distance between the successive nodes = $\lambda/2 = 5$ cm

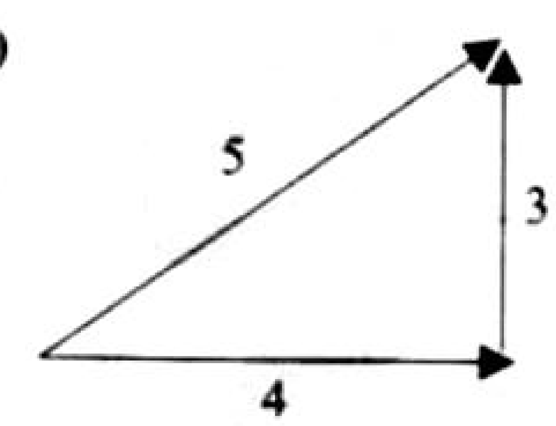
2. (7) $f_{app} = f_0\frac{c+v}{c-v}$, where c is speed of sound

$$\Rightarrow df = \frac{2f_0c}{(c-v)^2}dv \approx \frac{2f_0c}{(c-v)^2}dv = \frac{2f_0}{c}dv \qquad \text{(i)}$$

Given $df = \frac{1.2}{100}f_0$

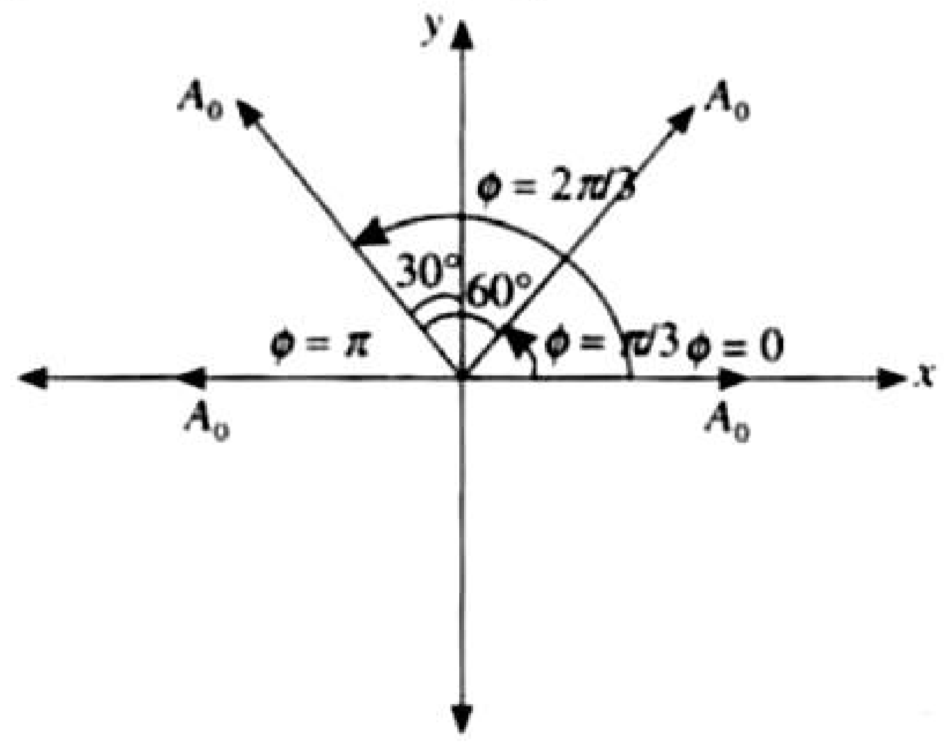
From Eqs. (i) and (ii) $dv \approx 2$ m/s = 7 km/h.

3. (5)



Two waves have phase difference $\pi/2$.

4. (3) Using phasor method of superposition,



Resultant amplitude $A = 2A_0 \cos 30° = A_0\sqrt{3}$

As $I \propto A^2$

$\therefore$ Resultant intensity $= 3I_0$

Fill in the Blanks Type

1. If $y = A\sin(\omega t - kx)$, displacement amplitude $= A$
(Maximum displacement)
Particle velocity,

$$v = \frac{dy}{dt} = A\omega\cos(\omega t - kx)$$

Therefore, velocity amplitude $= A\omega = A(2\pi v)$
(Maximum velocity)
Particle acceleration,

$$\text{Acc} = \frac{dv}{dt} = -A\omega^2 \sin(\omega\tau - \kappa x)$$

Therefore, acceleration (maximum acceleration) amplitude $= A\omega^2 = A(2\pi v)^2$

2. $c = v\lambda$

$$\therefore \quad \lambda = \frac{c}{v} = \frac{330}{660} = 0.5\,\text{m}$$

The antinode will be at a distance of

$$\frac{\lambda}{4} = \frac{0.5}{4} = 0.125\text{ m}$$

3. $c = v\lambda$ and $c = \sqrt{T/m}$

$$\therefore \quad \sqrt{\frac{T}{m}} = v\lambda$$

where T = tension in the string and m = mass per unit length of wire

When 50.7 kg mass is suspended for fundamental mode $\lambda = 2l$

$$v_1 \times 2l = \sqrt{\frac{50.7 \times g}{m}} \quad \text{(i)}$$

when mass is submerged in water, new tension

T_2 = weight – upthrust
$= 50.7\text{ g} - 0.0075 \times 1000 \times g = 43.2\text{ g}$

$$\therefore v_2 \times 2l = \sqrt{\frac{43.2g}{m}} \quad \text{(ii)}$$

On dividing Eqs. (i) and (ii), we get

$$\frac{v_2}{v_1} = \sqrt{\frac{43.2}{50.7}} \Rightarrow v_2 = v_1\sqrt{\frac{43.2}{50.7}}$$

$= 260 = 240$ Hz

4. Let the wave velocity is v towards right.

(Displacement at $t = 0$, $x = x$)
$=$ [Displacement at $t = 2$ s, $x = x + v(2)$]

$$\Rightarrow \quad \frac{1}{1+x^2} = \frac{1}{1+(1+x+2v)^2}$$

$$\Rightarrow \quad v = -0.5\text{ m/s}$$

The negative sign indicates that wave is travelling towards left.

5. In Fig. (i)

$$\frac{\lambda}{2} = l \Rightarrow \lambda = 2l$$

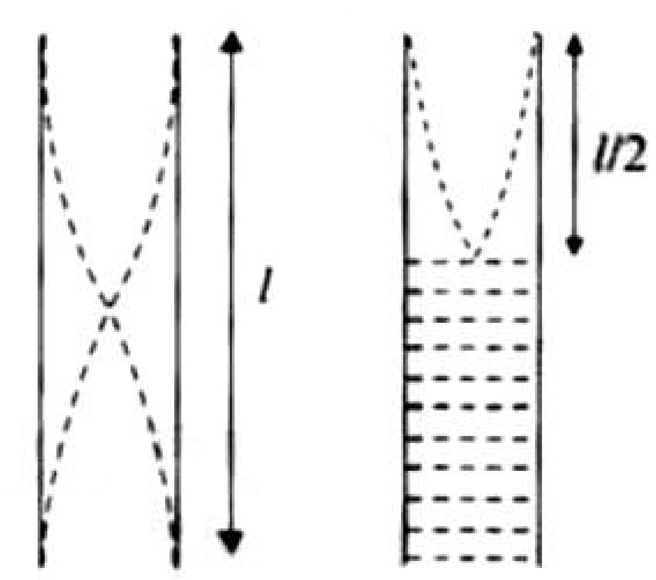
$$f = \frac{c}{\lambda} = \frac{c}{2l}$$

In Fig. (ii)

$$\frac{\lambda'}{4} = \frac{l}{2} \Rightarrow \lambda' = 2l$$

From Figs. (ii) and (i),

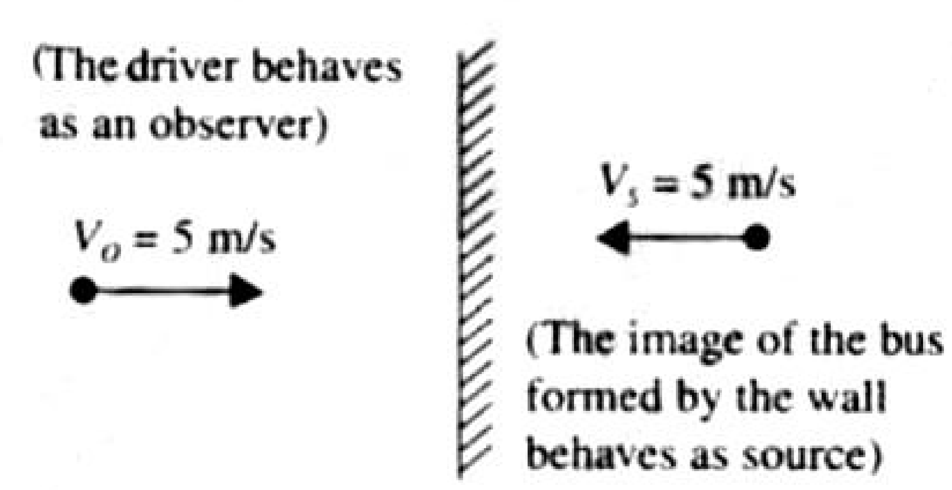
$$f' = \frac{c}{\lambda'} = \frac{c}{2l} = f$$



(i) (ii)

6. The first frequency the driver of bus hears is the original frequency 200 Hz. The second frequency the driver hears is the frequency of sound reflected from the wall. The two frequencies of sound heard by the driver are

i. Original frequency (200 Hz)
ii. Frequency of sound reflected from the wall (v')



The frequency of sound reflected from the wall

$$v' = v\left[\frac{u + u_0}{u - u_s}\right]$$

$$v' = 200\left[\frac{342+5}{342-5}\right] = 205.93\text{ Hz}$$

∴ Frequency of beats

$= v' - v = 205.93 - 200 = 5.93 = 6$ Hz

7. $u = 300$ m/s and $n = 25$

∴ $\lambda = \frac{u}{n} = \frac{300}{25} = 12$ m

For a path difference of λ, the phase difference is 2π.
For a path difference of 6 m, the phase difference is

$$\frac{2\pi \times 6}{\lambda} = \pi \text{ rad}$$

The amplitude is the same at both points, hence amplitude difference is zero.

True/False Type

1. False.

$$v_{sound} = \sqrt{\frac{\gamma RT}{M}}$$

$$\frac{v_{H_2}}{v_{He}} = \frac{\sqrt{\gamma_{H_2}/M_{H_2}}}{\sqrt{\gamma_{He}/M_{He}}} = \frac{\sqrt{(7/5)/2}}{\sqrt{(5/3)/4}} = \sqrt{\frac{42}{25}}$$

2. False. The intensity of sound at a given point is the energy per second received by a unit area perpendicular to the direction of propagation.

$$I = \frac{1}{2}\rho V\omega^2 A^2$$

Also intensity varies as distance from the point source as $I \propto \frac{1}{r^2}$.

None of the parameters are changing in case of a clear night or a clear day. Therefore the intensity will remain the same.

3. True. Speed of sound waves in water is greater than that in air. So water is rare medium for sound waves.

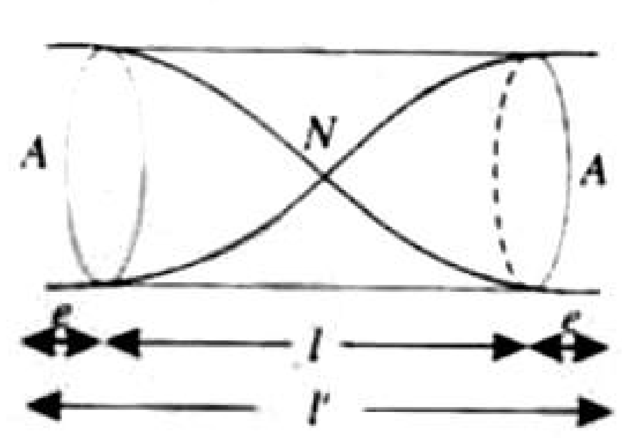
4. False. If the sound reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source will become the source of reflected sound. Thus, in both the cases, one sound coming directly from the source and the other coming after reflection will have the same frequency (since velocity of source w.r.t. observer is same in both the cases). Therefore, no beats will be heard.

Subjective Type

1. In case of vibration in air column, the antinodes at the open end(s) are located at a small distance outside the open end. This small distance is known as end correction.

The value of end correction $e = 0.3d$, where d is the diameter of the tube.

In case of a tube open at both ends, the effective length of the air column.



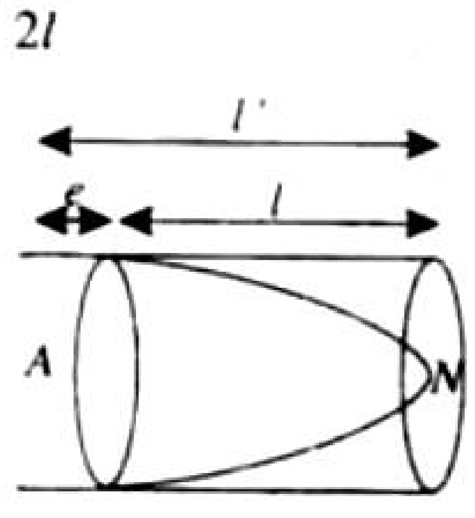
⇒ $l' = l + 2e$ where $e = 0.3d$

Fundamental frequency in this case $f_0 = \frac{v}{2l}$

⇒ $320 = \frac{320}{2(l + 2e)}$

$l + 2e = \frac{1}{2}$

$0.48 + 2(0.3d) = \frac{1}{2}$



⇒ $d = 1/30$ m $= 3.33$ m

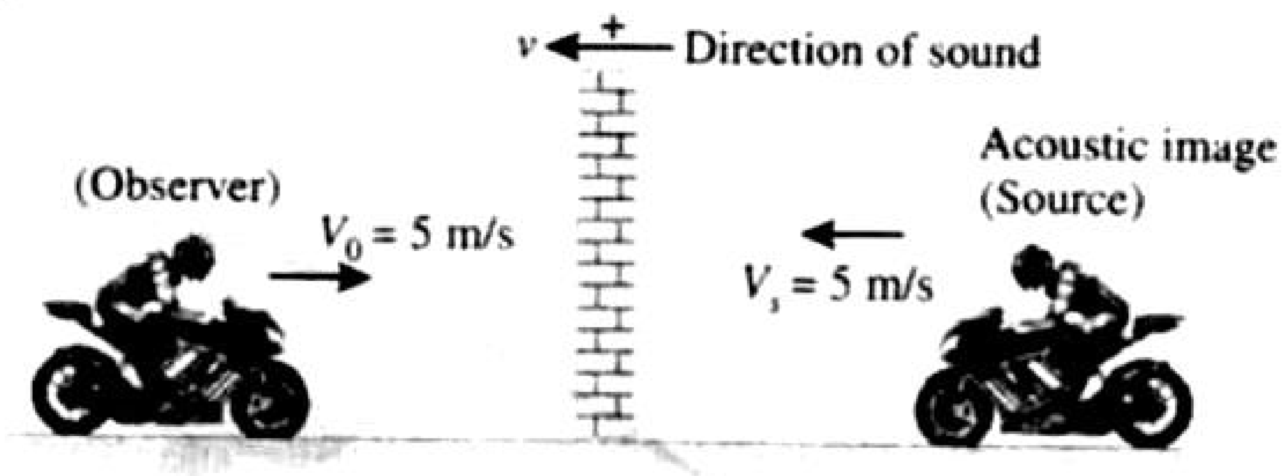
Now one end is closed, the effective length of the tube

$l' = l + e$

$$f_0 = \frac{v}{4l'} = \frac{v}{4(l+e)}$$

⇒ $$f_0 = \frac{320 \times 100}{4(48 + 0.3 \times 3.33)} = 163.27 \text{ Hz}$$

2.



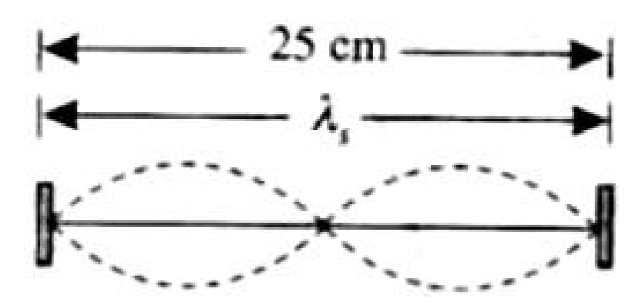
If the second reaches the observer after being reflected from a stationary surface and the medium is also stationary, the image of the source in the reflecting surface will become the source of the reflected sound.

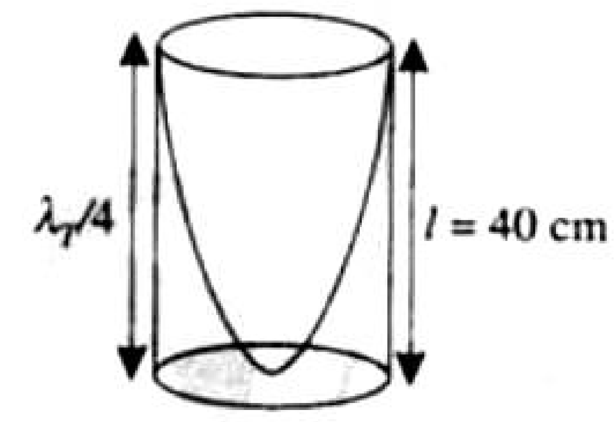
$$f' = f_0\left[\frac{v - v_0}{v - v_s}\right]$$

$$f' = 256\left[\frac{330 - (-5)}{330 - 5}\right] = 264$$

∴ Number of beats/sec = Beat frequency = 264 − 256 = 8.

3.





For first overtone of string

Linear mass density of the string = Mass of string per unit length $= \frac{2.5 \times 10^{-3}}{0.25} = 0.01$ kg/m

$$f_s = \frac{1}{\lambda}\sqrt{\frac{T}{m}} = \frac{1}{0.25}\sqrt{\frac{T}{0.01}} \quad \text{(i)}$$

Fundamental frequency of closed organ

∴ $\frac{\lambda_T}{4} = 0.4 \Rightarrow \lambda_T = 1.6$ m

$v = f_T\lambda_T$

$$f_T = \frac{c}{\lambda_T} = \frac{320}{1.6} = 200 \text{ Hz} \quad \text{(ii)}$$

In this case 8 beats/second is heard. The beat frequency decreases with the decreasing tension. This means that beat frequency decreases with decreasing f_s. This means that beat frequency is given by the expression. $f_{beat} = f_s - f_T$

∴ $$8 = \frac{1}{0.25}\sqrt{\frac{T}{0.01}} - 200$$

$$\Rightarrow \quad \sqrt{\frac{T}{0.01}} = 208 \times 0.25 = 52$$

$$\Rightarrow \quad T = 0.01 \times 52^2 = 27.04 \text{ N}$$

4. Linear mass density of sonometer wire

$$\mu = \frac{m}{l} = \frac{0.001}{0.1} = 0.01 \text{ kg/m}$$

Velocity of wave

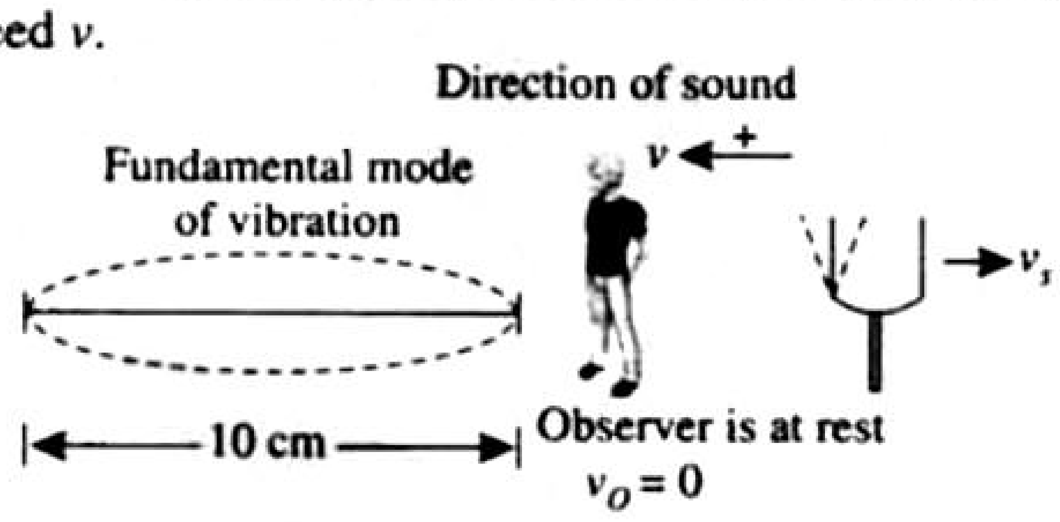
$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{64}{0.01}} = 80 \text{ m/s}$$

For fundamental frequency

Also, $\frac{\lambda}{2} = 0.1 \Rightarrow \lambda = 0.2$

$$\therefore \quad f = \frac{v}{\lambda} = \frac{80}{0.2} = 400 \text{ Hz}$$

Since tuning forks is in resonance, therefore frequency of tuning fork is 400 Hz. The observer is hearing one beat per second when the tuning fork is moved away with a constant speed v.



The frequency of tuning fork as heard by the observer standing stationary near sonometer wire can be found with the help of Doppler effect.

$$\text{Using } f' = f_0\left[\frac{v - v_0}{v - (-v_s)}\right] = \frac{v f_0}{v + v_s}$$

$$\therefore \quad f' = \frac{400 \times 300}{300 + v_s}$$

Since the beat frequency is 1 and as the tuning fork is going away from the observer, its apparent frequency should be 399 Hz.

$$\therefore \quad 399 = \frac{400 \times 300}{300 + v_s}$$

$$\Rightarrow \quad v_s = \frac{300}{399} = 0.752 \text{ m/s}$$

5. The velocity of wave in string is given by $v = \sqrt{\frac{T}{\mu}}$ where T is the tension and μ is the mass per unit length.

Since the tension in the string will increase as we move up the string (as the string has mass), therefore the velocity will also increase μ is the same as the rope is uniform

$$\therefore \quad \frac{v_{top}}{v_{bottom}} = \sqrt{\frac{T_{top}}{T_{bottom}}} = \sqrt{\frac{(6+2)g}{2g}} = 2$$

$$\therefore \quad v_{top} = 2v_{bottom}$$

Since frequency remains the same

$$\therefore \quad \lambda_{top} = 2\lambda_{bottom} = 2 \times 0.06 = 0.12 \text{ m}$$

6. The frequency of the wire in fundamental mode

$$f_0 = \frac{1}{2\ell}\sqrt{\left(\frac{T}{\mu}\right)}$$

When temperature of wire is changed, the tension developed

$T = YA\alpha(\Delta\theta)$ and

mass per unit length $\mu = A\rho$

$$\therefore \quad f_0 = \frac{1}{2\ell}\sqrt{\left(\frac{YA\alpha\,\Delta\theta}{M}\right)}$$

Here we are given $\ell = 1$ m, $Y = 2 \times 10^{11}$ N/m^2

$\alpha = 1.21 \times 10^{-5}$ per deg

$$\Delta\theta = 20°\text{C}, \mu = \frac{M}{\ell} = \frac{0.1 \text{ kg}}{1 \text{ m}} = 0.1 \text{ kg/m}$$

Area $A = 10^{-6}\text{m}^2$.

$$\therefore \quad f_0 = \frac{1}{2 \times 1}\sqrt{\left(\frac{2 \times 10^{11} \times 10^{-6} \times 1.21 \times 10^{-5} \times 20}{0.1}\right)}$$

$$= 11 \text{ vib/sec}$$

7. The given equation for standing waves in the string is

$$y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t) \qquad \text{(i)}$$

i. The amplitude of the waves is given by

$$A = 4\sin\frac{\pi x}{15} \qquad \text{(ii)}$$

Therefore, the maximum displacement or amplitude at $x = 5$ cm is

$$A = 4\sin\frac{\pi \times 5}{15} = 4\sin\frac{\pi}{3}$$

$$= 4\sin 60° = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3} = 2 \times 1.732 = 3.464 \text{ cm}$$

ii. The position of zero displacement or nodes are given by

$$\sin\frac{\pi x}{15} = 0 \quad \text{or} \quad \frac{\pi x}{15} = r\pi \qquad (\text{where } r = 0, 1, 2, 3, \ldots)$$

$$\Rightarrow x = 15r \quad \Rightarrow \quad x = 0, 0.15 \text{ cm}, 0.30 \text{ cm}, \ldots$$

iii. Differentiating Eq. (i) with respect to t, we get velocity of particle

$$u = \frac{dy}{dt} = -4 \times 96\pi \sin\left(\frac{\pi x}{15}\right)\sin(96\pi t)$$

Substituting $x = 7.5$ cm and $t = 0.25$ s.

$$u = -384\pi\sin\left(\frac{\pi \times 7.5}{15}\right)\sin(96\pi \times 0.25)$$

$$= -384\sin(\pi/2)\sin(24\pi) = 0$$

iv. Using the relation

$2\sin A\cos B = \sin(A + B) + \sin(A - B)$

Equation (i) may be expressed as

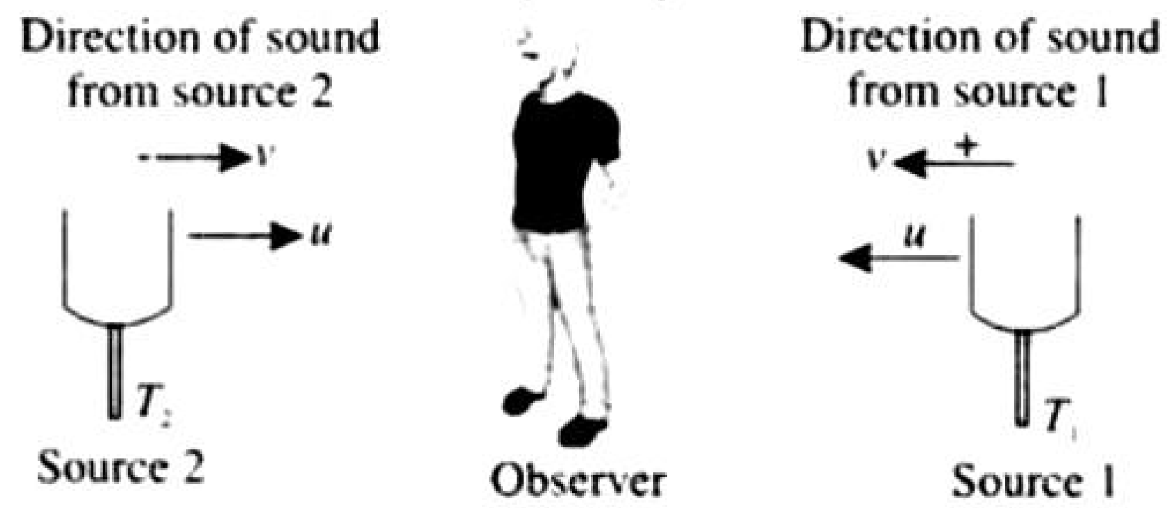
$$y = 2\left[\sin\left\{\frac{\pi x}{15} + (96\pi t)\right\} + 2\sin\left\{\frac{\pi x}{15} - (96\pi t)\right\}\right]$$

$$= 2\sin\left\{\frac{\pi x}{15} + 96\pi t\right\} + 2\sin\left\{\frac{\pi x}{15} - 96\pi t\right\} = y_1 + y_2$$

Therefore, the component waves are given by

$$\left.\begin{aligned} y_1 &= 2\sin\left(\frac{\pi x}{15} - 96\pi t\right) \\ \text{and} \quad y_2 &= -2\sin\left(96\pi t + \frac{\pi x}{15}\right) \end{aligned}\right\}$$

8. The apparent frequency from tuning fork T_1 as heard by the observer will be $f_1 = f_0\left(\frac{v}{v-u}\right)$ (i)



The apparent frequency from tuning fork T_2 as heard by the observer will be $f_2 = f_0\left(\frac{v}{v+u}\right)$ (ii)

Given that beat frequency is 3, $f_1 - f_2 = 3$

$$f_0\left(\frac{v}{v-u}\right) - f_0\left(\frac{v}{v+u}\right) = 3$$

$$f_0 v\left[\frac{v+u-v+u}{v^2-u^2}\right] = 3 \Rightarrow f_0\frac{v\cdot 2u}{v^2-u^2} = 3$$

As $v >> u$ hence $f_0\frac{2uv}{v^2} = 3 \Rightarrow u = \frac{3v}{2f_0} = 1.5$ m/s

9. i. If two progressive waves having the same amplitude and time period, but traveling in opposite direction with same velocity superimpose, the produce standing waves.

The following two equations qualify the above criteria and hence produce standing wave

$z_1 = A\cos(kx - \omega t)$

$z_2 = A\cos(kx - \omega t)$

The resultant wave is $z = z_1 + z_2$

$\Rightarrow \quad z = A\cos(kx - \omega t) + A\cos(kx + \omega t)$

Hence, $z = 2A\cos kx \cos \omega t$

The resultant intensity will be zero when $2A\cos kx = 0$

$$\Rightarrow \quad \cos kx = \cos\frac{(2n+1)}{2}\pi$$

$$\Rightarrow \quad kx = \frac{2n+1}{2}\pi$$

$$\Rightarrow \quad x = \frac{(2n+1)\pi}{2k} \text{ where } n = 0, 1, 2, \dots \qquad \text{(iii)}$$

ii. The equations of transverse waves

$z_1 = A\cos(kx - \omega t)$

$z_3 = A\cos(ky - \omega t)$

Combine to produce a wave traveling in the direction making an angle of 45° with the positive x and positive y axes.

The resultant wave; $z = z_1 + z_3$

$z = A\cos(kx - wt) + A\cos(ky - wt)$

$$z = 2A\cos\frac{k(x-y)}{2}\cos\left[\frac{k(x+y)-2wt}{2}\right]$$

The resultant intensity will be zero when

$$2A\cos\frac{k(x-y)}{2} = 0 \Rightarrow \cos\frac{k(x-y)}{2} = 0$$

$$\Rightarrow \quad \frac{k(x-y)}{2} = \frac{2n+1}{2}\pi$$

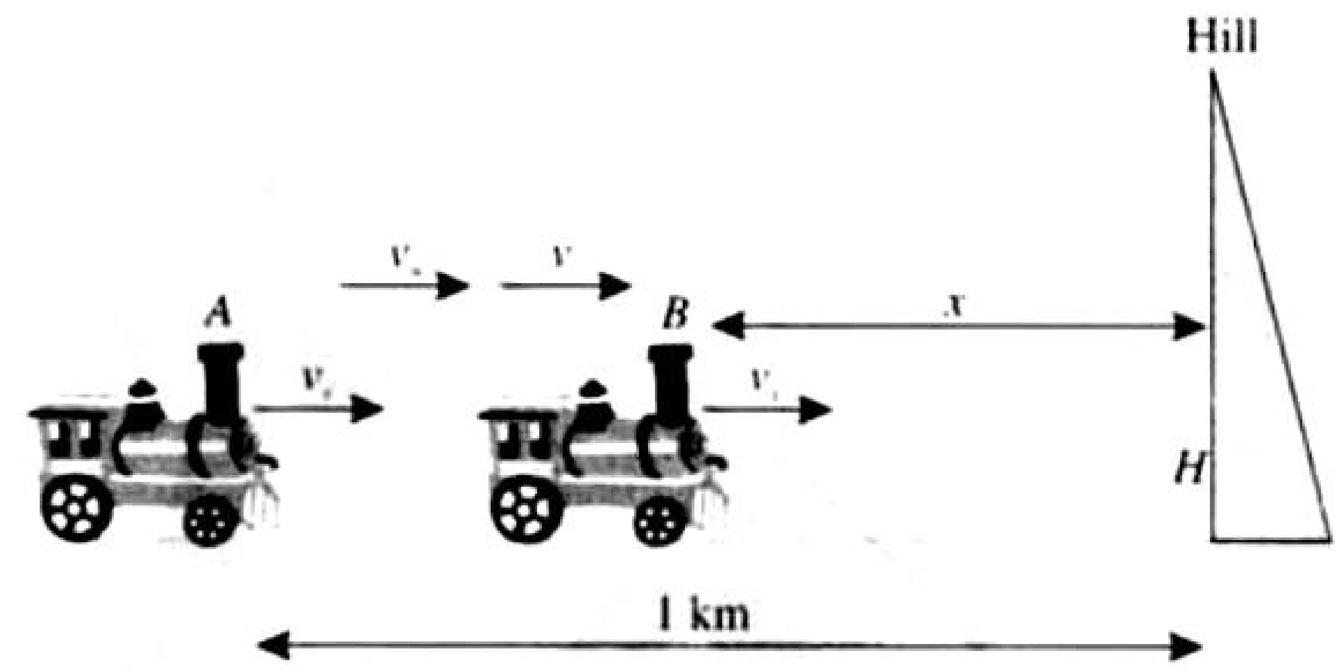
$$\Rightarrow \quad (x-y) = \frac{(2n+1)}{k}\pi$$

10. i. The frequency of the whistle as heard by observer on the hill

$$f' = f_0\left[\frac{(v+v_w)-v_o}{(v+v_w)-v_s}\right]$$

$$= 580\left[\frac{1200+40}{1200+40-40}\right] = 599 \text{ Hz}$$

ii. Let echo from the hill is heard by the driver at B which is at a distance x from the hill. The sound was produced when the source was at a distance 1 km from hill.



The time taken by the driver to reach from A to B

$$t_1 = \frac{1-x}{40} \qquad \text{(i)}$$

The time taken by the echo to reach from hill

$t_2 = t_{AH} + t_{HB}$

$$t_2 = \frac{1}{(1200+40)} + \frac{x}{(1200-40)} \qquad \text{(ii)}$$

where t_{AH} = time taken by sound from A to H with velocity $(1200 + 40)$

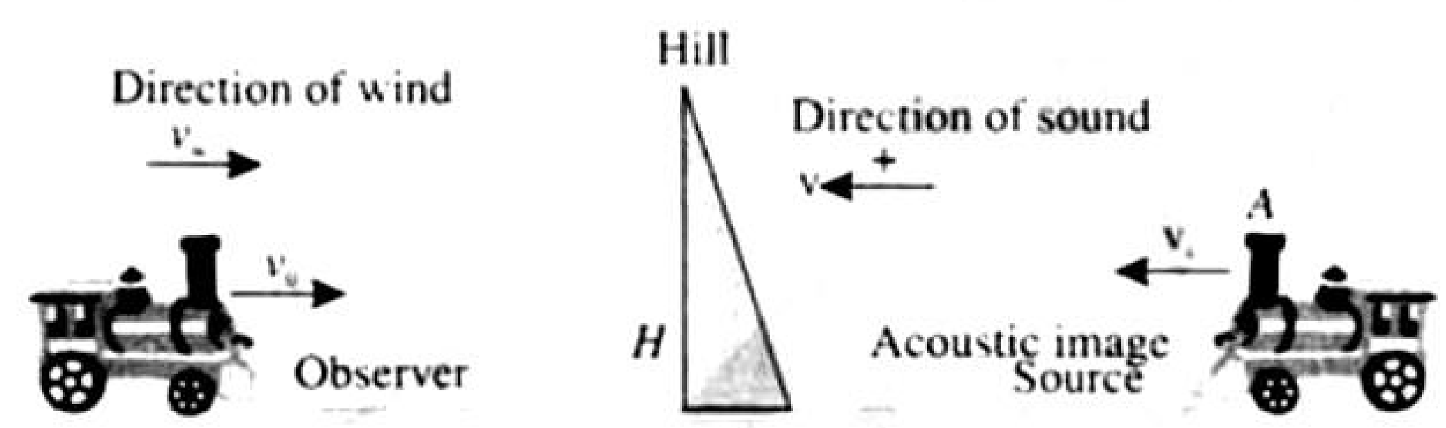
t_{HB} = time taken by sound from H to B with velocity $(1200 - 40)$

From (i) and (ii) $t_1 = t_2$

$$\Rightarrow \quad \frac{1-x}{40} = \frac{1}{(1200+40)} + \frac{x}{(1200-40)}$$

Which gives $x = 0.902$ m

The frequency of echo as heard by the driver can be calculated by considering that the source is the acoustic image.



$$f'' = f_0\left[\frac{(v-v_w)-(-v_0)}{(v-v_w)-v_s}\right] = f_0\left[\frac{(v-v_w)+v_0}{(v-v_w)-v_s}\right]$$

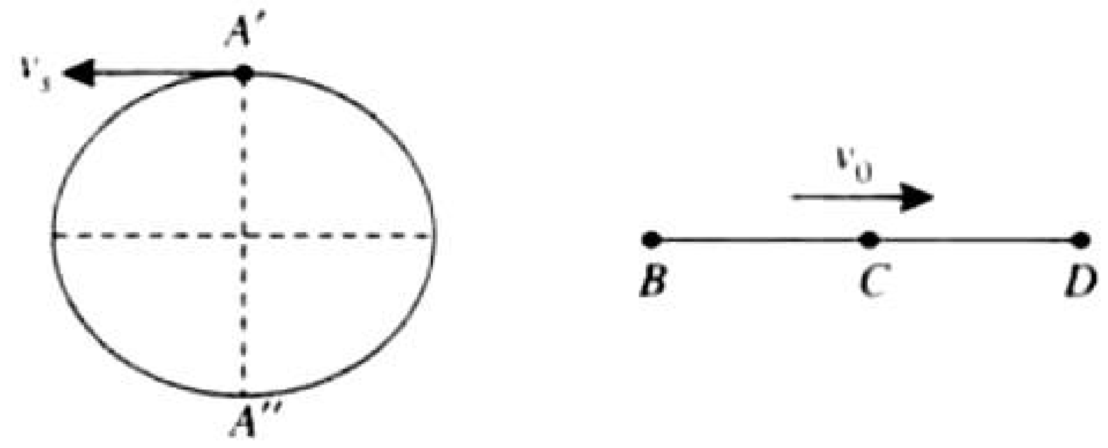
$$= 580\left[\frac{(1200-40)+40}{(1200-40)-40}\right] = 621 \text{ Hz}$$

11. The angular frequency of the detector

$$2\pi f = 2\pi\left(\frac{5}{\pi}\right) = 10 \text{ rad/s}$$

The angular frequency of the detector source is equal, hence their time periods will also be the same.

⇒ When the detector is at C moving towards D, the source is at A' moving left wards. In this situation that the frequency heard is minimum

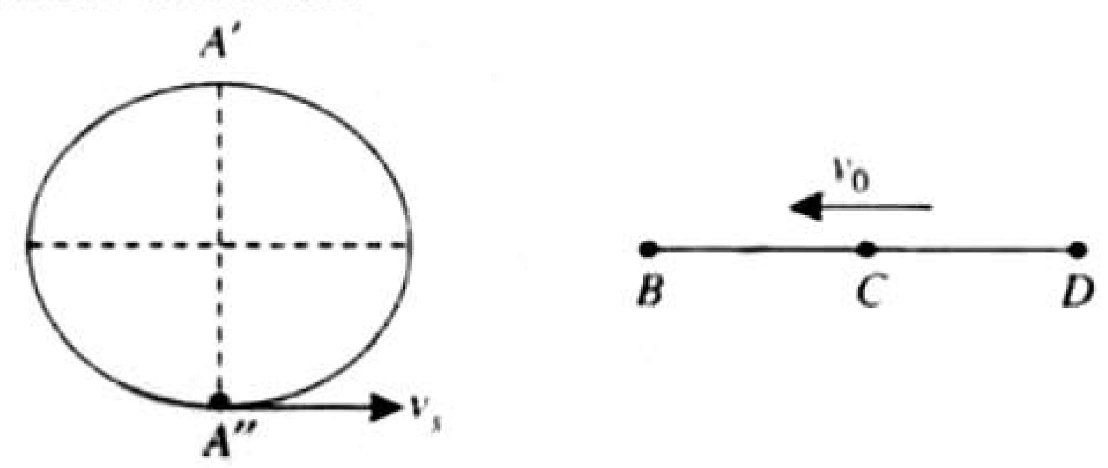


$$f' = f_0\left[\frac{v - v_0}{v - (-v_s)}\right] = f_0\left[\frac{v - v_0}{v + v_s}\right]$$

Velocity of source $v_s = \omega A = 60$ m/s
Velocity of detector $v_o = R\omega = 30$ m/s

$$\Rightarrow \quad f' = 340 \times \frac{(340 - 60)}{(340 + 30)} = 257.3 \text{ Hz}$$

Again when the detector is at C moving towards B, the source is at A'' moving rightwards. In this situation that the frequency heard is maximum



$$f'' = f_0\left[\frac{v - (-v_0)}{v - v_s}\right] = f_0\left[\frac{v + v_0}{v - v_s}\right]$$

$$\Rightarrow \quad f' = 340 \times \frac{(340 + 60)}{(340 - 30)} = 485.7 \text{ Hz}$$

12. a. The standard equation of a plane progressive wave is

$$y = A\cos\left(\frac{2\pi}{\lambda}x + 2\pi f t\right) \quad \text{(i)}$$

The given is equation is $y_1 = A\cos(ax + bt)$ (ii)

Comparing with (ii) we get $\frac{2\pi}{\lambda} = a \Rightarrow \lambda = \frac{2\pi}{a}$

and $2\pi f = b \Rightarrow f = \frac{b}{2\pi}$

b. Since the wave reflects by an obstacle, it will suffer a phase difference of π. The intensity of the reflected wave is 0.64 times of the incident wave.
Intensity of incident wave $I \propto A^2$
Intensity of reflected wave $I' = 0.64I \Rightarrow I' \propto A'^2$
$\Rightarrow \quad 0.64I \propto A'^2 \Rightarrow 0.64A^2 \propto A'^2$
$\Rightarrow \quad A' \propto 0.8A$
Hence the equation of resultant wave will be

$$y_2 = 0.8\,A\cos(ax - bt + \pi) = -0.8\,A\cos(ax + bt) \quad \text{(iii)}$$

c. The resultant wave equation can be found by superposition principle

$$y = y_1 + y_2 = A\cos(ax + bt) + [-0.8\,A\cos(ax + bt)] \quad \text{(iv)}$$

The particle velocity can be found by differentiating (iv) equation

$$v_p = \frac{dy}{dt} = -Ab\sin(ax + bt) - 0.8\,Ab\sin(ax - bt)$$

$$= -Ab\,[\sin(ax + bt) + 0.8\,Ab\sin(ax - bt)]$$

$$= -Ab[\sin ax\cos bt + \cos ax\sin bt + 0.8\sin ax\cos bt - 0.8\cos ax\sin bt]$$

$$v_p = -Ab[1.8\sin ax\cos bt + 0.2\cos ax\sin bt]$$

The maximum velocity will occur when $\sin(ax) = 1$ and $\cos bt = 1$ under these condition $\cos ax = 0$ and $\sin bt = 0$

$\therefore \quad |v_{max}| = 1.8Ab$ and $|V_{min}| = 0$

d. $y = [A\cos(ax + bt)] - [0.8\,A\cos(ax - bt)]$

$= [0.8\,A\cos(ax + bt) + 0.2\,A\cos(ax + bt)] - [0.8\,A\cos(ax - bt)]$

$= [0.8\,A\cos(ax + bt) + 0.8\,A\cos(ax + bt)] + 0.2\,A\cos(ax + bt)$

$$= 0.8A\left[-2\sin\left\{\frac{(ax + bt) + (ax - bt)}{2}\right\}\sin\left\{\frac{(ax + bt) - (ax - bt)}{2}\right\}\right]$$

$0.2\,A\cos(ax + bt)]$

$\Rightarrow \quad y = -1.6A\sin ax\sin bt + 0.2A\cos(ax + bt)$

where $(-1.6\,A\sin ax\sin bt)$ is the equation of a standing wave and $0.2\,A\cos(ax + bt)$ is the equation of traveling wave.
Antinodes of the standing waves are the positions where the displacement amplitude is maximum.

i.e., $\quad \sin ax = 1 = \sin\left[n\pi + (-1)^n\frac{\pi}{2}\right]$

$$\Rightarrow \quad x = \left[n + \frac{(-1)^n}{2}\right]\frac{\pi}{a}$$

The equation of a traveling wave is $0.2\,A\cos(ax + bt)$. This is the equation of traveling wave moving in $-X$ direction.

13. i. Let the signal waves be given by

$$y_1 = A\sin 2\pi\omega_1 t, \quad y_2 = A\sin 2\pi\omega_2 t.$$

The resultant disturbance is given by

$$y = y_1 + y_2 = A\sin 2\pi\omega_1 t + A\sin 2\pi\omega_2 t$$

$$= 2A\sin\frac{2\pi(\omega_1 + \omega_2)t}{2}\cos\frac{2\pi(\omega_2 - \omega_1)t}{2}$$

$$= 2A\cos\pi(\omega_2 - \omega_1)t\sin 2\pi\frac{(\omega_1 + \omega_2)t}{2}$$

Let $\quad \omega_1 = \omega,\ \omega_2 = \omega + \Delta\omega$
Therefore, $\quad \omega_1 + \omega_2 \approx 2\omega$

$$y = 2A\cos\pi(\omega_2 - \omega_1)t\sin 2\pi\omega t$$

Thus, the resultant disturbance has amplitude $2A\cos\pi(\omega_2 - \omega_1)t$

For maxima: $\cos\pi(\omega_2 - \omega_1)t = \pm 1$

or $\quad \pi(\omega_2 - \omega_1)t = r\pi,\ r = 0, 1, 2, 3$

$$t = \frac{r}{\omega_2 - \omega_1} = 0, \frac{1}{\omega_2 - \omega_1}, \frac{2}{\omega_2 - \omega_1}, \ldots$$

Clearly time interval between successive maxima

$$= \frac{1}{\omega_2 - \omega_1} = \frac{1}{10^3} = 10^{-3}\text{ s}$$

ii. The resultant intensity is given by

$$I = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$$

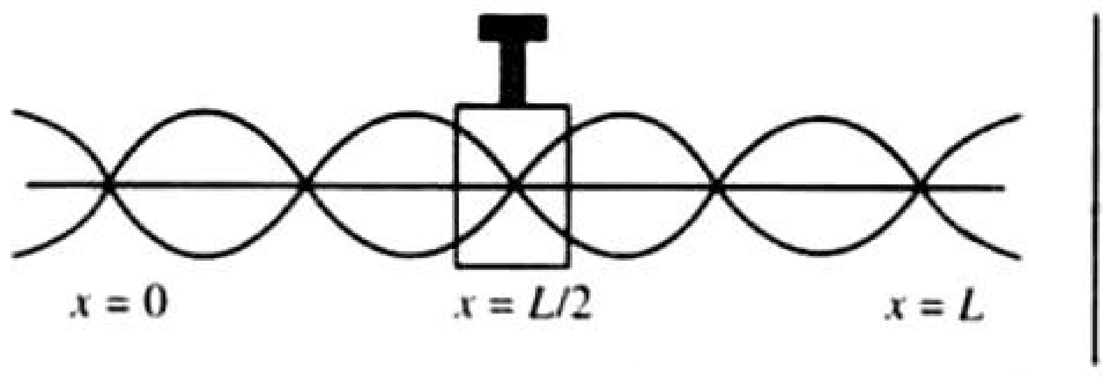
When $\delta = 0$, intensity is maximum $I_{max} = 4A^2$
When $\Delta = \pi/2$, intensity $I = 2A^2$
When $\Delta = \pi$, intensity $I_{min} = 0$
When $\Delta = 3\pi/2$, intensity $I = 2A^2$

When $\Delta = 2\pi$, intensity $I_{max} = 4A^2$

The detector remaining idle from $\theta = \pi/2$ to $3\pi/2$ or in each half cycle. Hence the required time

$$t = \frac{T}{2} = \frac{10^{-3}}{2} = 5 \times 10^{-4} \text{ s}$$

14. As found in case of strings, in case of rods also the clamped point behaves as a node while the free end antinode. The situation is shown in the figure.



Because the distance between two consecutive nodes is ($\lambda/2$) while between a node and antinode is $\lambda/4$, hence

$$4 \times \left[\frac{\lambda}{2}\right] + 2\left[\frac{\lambda}{4}\right] = L \quad \text{or} \quad \lambda = \frac{2 \times 1}{5} = 0.4 \text{ m}$$

Further, it is given that

$$Y = 2 \times 10^{11} \text{ N/m}^2 \quad \text{and} \quad \rho = 8 \times 10^3 \text{ kg/m}^3$$

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{8 \times 10^3}} = 5000 \text{ m/s}$$

Hence, from $v = n\lambda$,

$$n = \frac{v}{\lambda} = \frac{5000}{0.4} = 12500 \text{ Hz}$$

Now if incident and reflected waves along the rod are

$$y_1 = A \sin(\omega t - kx) \quad \text{and} \quad y_2 = A \sin(\omega t + kx + \phi)$$

The resultant wave will be

$$y = y_1 + y_2 = A[\sin(\omega t - kx) + \sin(\omega t + kx + \phi)]$$

$$= 2A\cos\left(kx + \frac{\phi}{2}\right)\sin\left(\omega t + \frac{\phi}{2}\right)$$

Because there is an antinode at the free end of the rod, hence amplitude is maximum at $x = 0$. So

$$\cos\left(k \times 0 + \frac{\phi}{2}\right) = \text{Maximum} = 1 \quad \text{i.e.,} \quad \phi = 0$$

And

$$A_{max} = 2A = 2 \times 10^{-6} \text{ m (given)}$$

$$y = 2 \times 10^{-6} \cos kx \sin \omega t$$

$$y = 2 \times 10^{-6} \cos\left[\frac{2\pi x}{\lambda}\right]\sin(2\pi n t)$$

Putting values of λ and n, we get

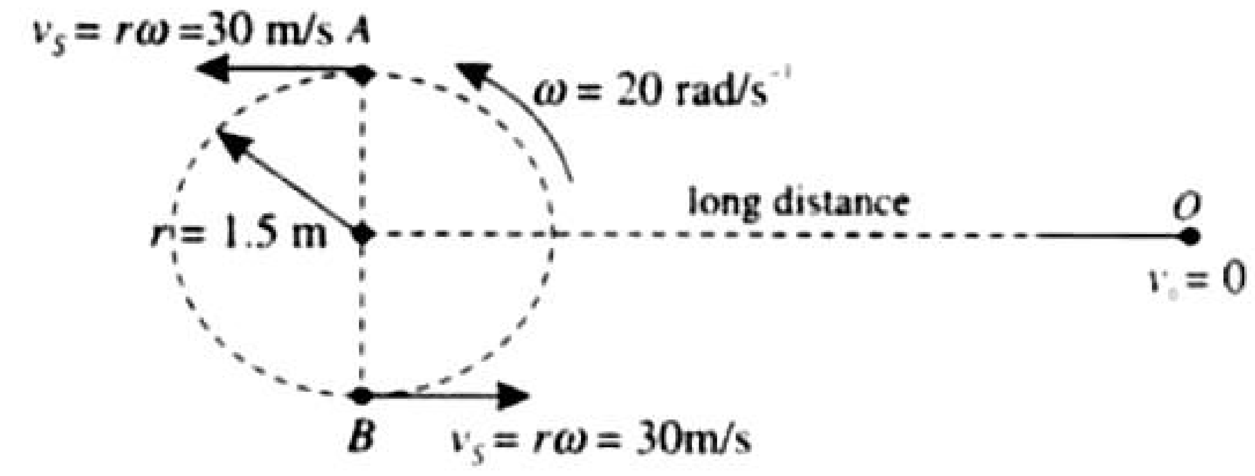
$$y = 2 \times 10^{-6} \cos 5\pi x \sin 25000\pi t$$

Now, because for a point 2 cm from the midpoint $x = (0.50 \pm 0.02)$, hence

$$y = 2 \times 10^{-6} \cos 5\pi(0.5 \pm 0.02) \sin 25000\pi t$$

15. The speed of whistle

$$v_s = r\omega = 1.5 \times 20 = 30 \text{ ms}^{-1}$$



When the source is instantaneously at the position A, then the frequency heard by the observer will be

$$f' = f_0\left[\frac{v}{v - v_s}\right] = 440\left[\frac{330}{330 - 30}\right] = 484 \text{ Hz}$$

When the source is instantaneously at the position B, then the frequency heard by the observer will be

$$f'' = f_0\left[\frac{v}{v - v_s}\right] = 440\left[\frac{330}{330 - 30}\right] = 403.3 \text{ Hz}$$

Hence the range of frequencies heard by the observer is 403.3 Hz to 484 Hz.

16. Let l_1 and l_2 be lengths of open organ pipe and closed organ pipe respectively.

First overtone of open organ pipe $= 2n_1 = 2 \times \frac{v}{2l_1} = \frac{v}{l_1}$

First overtone of closed organ pipe $= 3n_2 = 3 \times \frac{v}{4l_2}$

According to question, $\frac{v}{l_1} - \frac{3v}{4l_2} = \pm 2.2$ (i)

As n_2 is the fundamental frequency of closed organ pipe

$$n_2 = \frac{v}{4l_2}$$

$$l_2 = \frac{v}{4n_2} = \frac{330}{4 \times 110} = 0.75 \text{ m}$$

From Eq. (i),

$$\frac{v}{l_1} = \frac{3v}{4l_2} \pm 2.2$$

$$\frac{v}{l_1} = 3n_2 \pm 2.2$$

Taking positive sign

$$\frac{v}{l_1} = 3 \times 110 + 2.2 = 332.2$$

$$\therefore \quad l_1 = \frac{v}{332.2} = \frac{330}{332.2} \text{ m} = 0.993 \text{ m} = 99.3 \text{ cm}$$

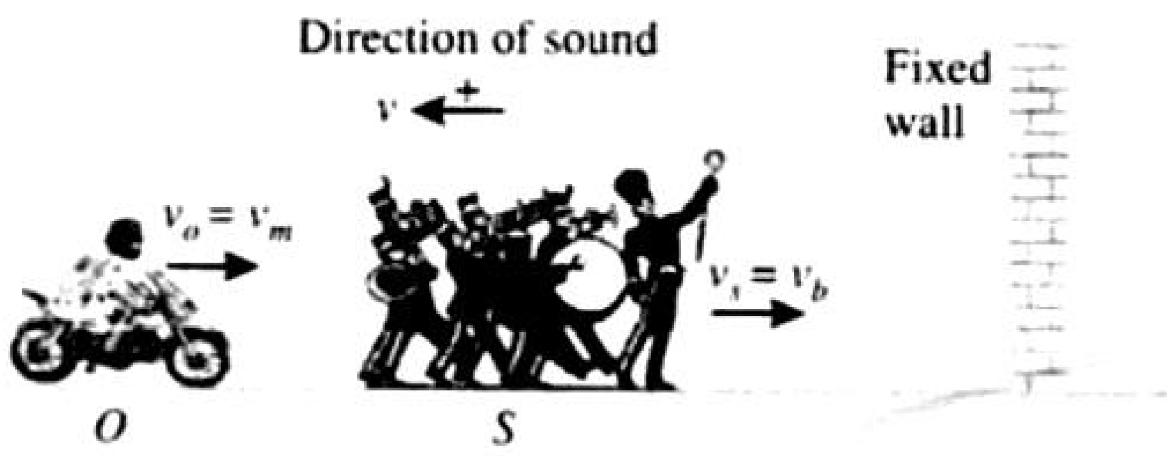
Taking negative sign

$$\frac{v}{l_1} = 3n_2 - 2.2 = 3 \times 110 - 2.2 = 327.8$$

$$\therefore \quad l_1 = \frac{v}{327.8} = \frac{330}{327.8} = 1.006 \text{ m} = 100.6 \text{ cm}$$

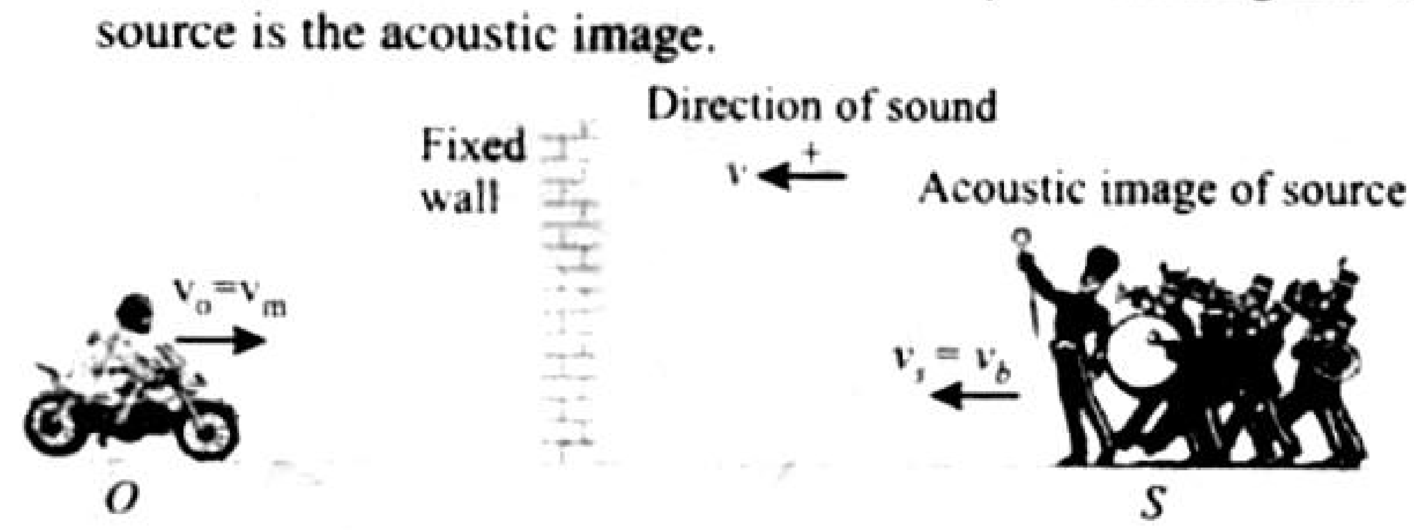
17. Motorist receives two sound waves one directly from the sound source and other reflected from the fixed wall. Let the apparent frequencies of these two waves as received by motorist are f' and f'' respectively.

For Direct Sound: The apparent frequency received by motorist.



$$f' = \left[\frac{v - v_o}{v - v_s}\right] = f\left[\frac{v - (-v_m)}{v - (-v_s)}\right] = f\left[\frac{v + v_m}{v + v_s}\right] \quad \text{(i)}$$

For reflected sound: The frequency of reflected sound as heard by the motorist can be calculated by considering that the source is the acoustic image.



$$f'' = \left[\frac{v - v_o}{v - v_s}\right] = f\left[\frac{v - (-v_m)}{v - v_s}\right] = f\left[\frac{v + v_m}{v - v_s}\right] \quad \text{(ii)}$$

Hence, beat frequency as heard by the motorist

$$\Delta f = f'' - f' = \left(\frac{v + v_m}{v - v_b} - \frac{v + v_m}{v + v_b}\right)f$$

or $$\Delta f = \frac{2v_b(v + v_m)}{(v^2 - v_b^2)}f$$

18. a. The fundamental frequency of the closed organ pipe = $v/4l$.
In closed organ pipe only odd harmonics are present.

Second overtone of pipe = $5v/4L$

Given $5v/4L = 440$

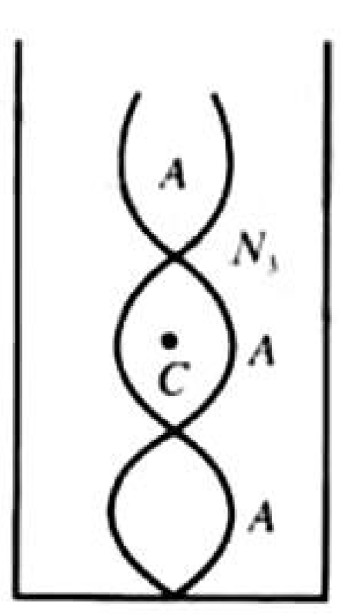
On solving, we get

$$L = \frac{5v}{4 \times 440} = \frac{5 \times 330}{4 \times 440} = \frac{15}{16}\ \text{m} = 0.9375\ \text{m} = 93.75\ \text{cm}$$

b. The equation of variation of pressure amplitude at any distance x from the node is

$$\Delta P = \Delta P_0 \cos kx$$

Pressure variation is maximum at a node and minimum (zero) at antinode.



N_1

Distance of centre C from N_2 is $\lambda/8$

$$\therefore \quad \Delta P = \Delta P_0 \cos \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \Delta P_0 \frac{\pi}{4} = \frac{\Delta P_0}{\sqrt{2}}$$

c. At antinode, the pressure variation is minimum (zero), therefore at antinode pressure remains equal to P_0 (always).
Therefore, at antinode $P_{max} = P_{min} = P_0$.

19. Given that $m_1 = 0.06$ kg, $m_2 = 0.2$ kg
Let m' be the mass per unit length then
$m'_1 = 0.0125$ kg/m, $m'_2 = 0.078125$ kg/m
Wire PQR is under a tension of 80 N = T_0. A sinusoidal wave pulse is sent from P.

a. v_1 = velocity of wave on PQ

$$= \sqrt{\frac{T}{m_1}} = \sqrt{\frac{80}{0.0125}} = 80\ \text{m/s}$$

v_2 = velocity of wave on QR

$$= \sqrt{\frac{T}{m_2}} = \sqrt{\frac{80}{0.078125}} = 32\ \text{m/s}$$

Total time taken for wave pulse to reach from P to R

$$= \frac{PQ}{v_1} + \frac{QR}{v_2} = \left(\frac{4.8}{80} + \frac{2.56}{32}\right)\text{s} = 0.06 + 0.08 = 0.14\ \text{s}$$

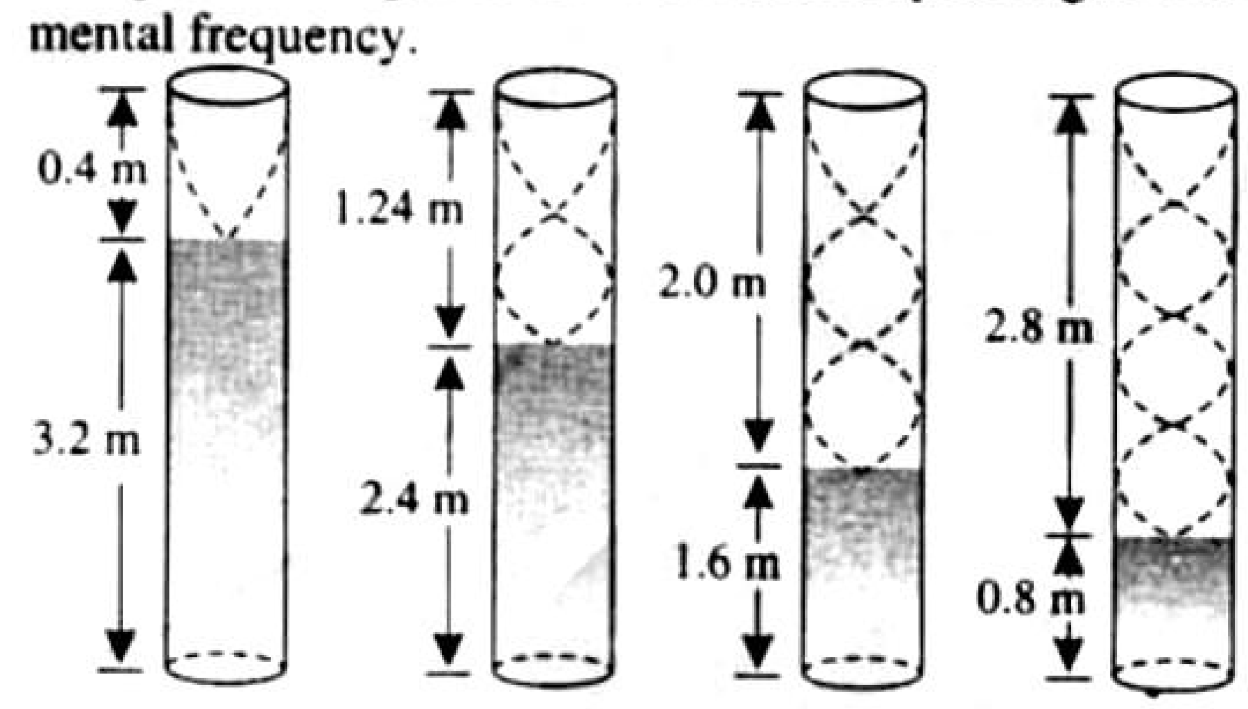
b. Amplitude of reflected wave: $A_r = \left(\frac{v_2 - v_1}{v_2 + v_1}\right)A_i$

$$= \left(\frac{32 - 80}{32 + 80}\right)3.5 = -1.5\ \text{cm}$$

A_r is –ve, so reflected wave is inverted
Amplitude of transmitted wave:

$$A_t = \left(\frac{2v_2}{v_2 + v_1}\right)A_i = \left(\frac{2 \times 32}{32 + 80}\right)3.5 = 2\ \text{cm}$$

20. Let l_0 be the length of air column corresponding to the fundamental frequency.



$$\frac{l_0}{4} = \frac{\lambda}{4} \text{ or } \lambda = 4l_0$$

Also $v = f\lambda$

Hence $f_0 = \frac{v}{\lambda}$

Then $\frac{v}{4l_0} = 212.5$

or $$l_0 = \frac{v}{4(212.5)} = \frac{340}{4(212.5)} = 0.4\ \text{m.}$$

In closed pipe only odd harmonics are obtained. Now let l_1, l_2, l_3, l_4, etc., be the lengths corresponding to the 3rd harmonic, 5th harmonic, 7th harmonic etc. Then

3rd harmonic; $3\left(\frac{v}{4l_1}\right) = 212.5 \Rightarrow l_1 = 1.2$ m

5th harmonic; $5\left(\frac{v}{4l_2}\right) = 212.5 \Rightarrow l_2 = 2.0$ m

7th harmonic; $7\left(\frac{v}{4l_3}\right) = 212.5 \Rightarrow l_3 = 2.8$ m

9th harmonic; $9\left(\frac{v}{4l_4}\right) = 212.5 \Rightarrow l_4 = 3.6$ m

or heights of water level are (3.6 – 0.4) m, (3.6 – 1.2) m, (3.6 – 2.0) m and (3.6 – 2.8) m.

Therefore heights of water level are 3.2 m, 2.4 m, 2.4 m, 1.6 m and 0.8 m from bottom.

Let A and a be the area of cross-sections of the pipe and hole respectively.

Then $A = \pi(2 \times 10^{-2})^2 = 1.26 \times 10^{-3}\ \text{m}^{-2}$

and $a = \pi(10^{-3})^2 = 3.14 \times 10^{-6}\ \text{m}^2$

Velocity of efflux, $v_1 = \sqrt{2gH}$

Continuity equation at top of the tube and orifice

$$a\sqrt{2gH} = A\left(\frac{-dH}{dt}\right)$$

Therefore rate of fall of water level in the pipe,

$$\left(\frac{-dH}{dt}\right) = \frac{a}{A}\sqrt{2gH}$$

Substituting the values, we get

$$\frac{-dH}{dt} = \frac{3.14 \times 10^{-6}}{1.26 \times 10^{-3}}\sqrt{2 \times 10 \times H}$$

$$\Rightarrow \quad \frac{-dH}{dt} = (1.11 \times 10^{-2})\sqrt{H}$$

Between first two resonances, the water level falls from 3.2 m to 2.4 m.

$$\therefore \quad \frac{dH}{\sqrt{H}} = -1.11 \times 10^{-2}\, dt \Rightarrow \int_{3.2}^{2.4} \frac{dH}{\sqrt{H}} = -(1.11 \times 10^{-2})\int_0^t dt$$

$$\Rightarrow \quad 2[\sqrt{2.4} - \sqrt{3.2}] = -(1.11 \times 10^{-2}) \cdot t \Rightarrow t = 43\ \text{sec.}$$

21. Situation 1.

Speed of sound inside the water

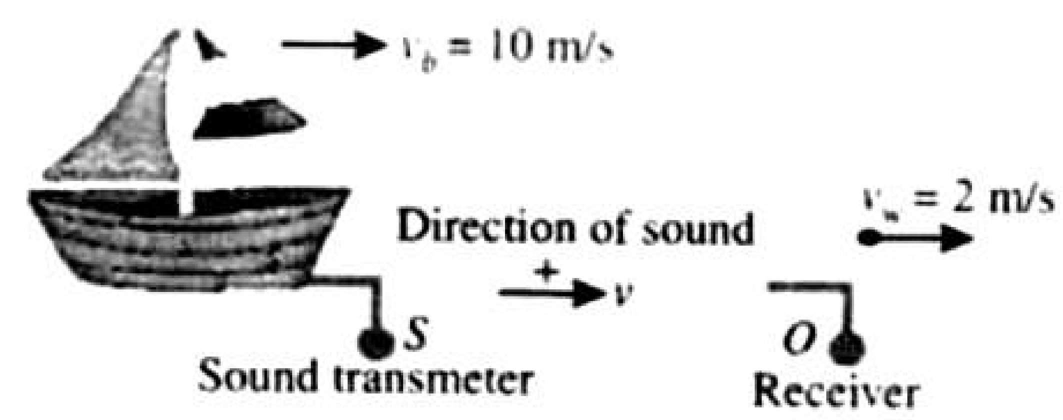
$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.088 \times 10^9}{10^3}} = 1445\ \text{m/s}$$

The frequency of sound inside the water

$$f = \frac{v}{\lambda} = \frac{1445}{14.45 \times 10^{-3}} = 10^5\ \text{Hz}$$

In this case the medium through which the sound is traveling is also in motion. So we will use the relation of Doppler's formula

Apparent frequency $\quad f' = f\left[\frac{(v + v_m) - v_0}{(v + v_m) - v_s}\right] \quad$ (i)



$v_m = 2$ m/s, $v_0 = 0$, $v_s = 10$ m/s

The frequency detected by a receiver

$$f' = 10^5\left[\frac{1445 + 2 - 0}{1445 + 2 - 10}\right] = 1.007 \times 10^5\ \text{Hz}$$

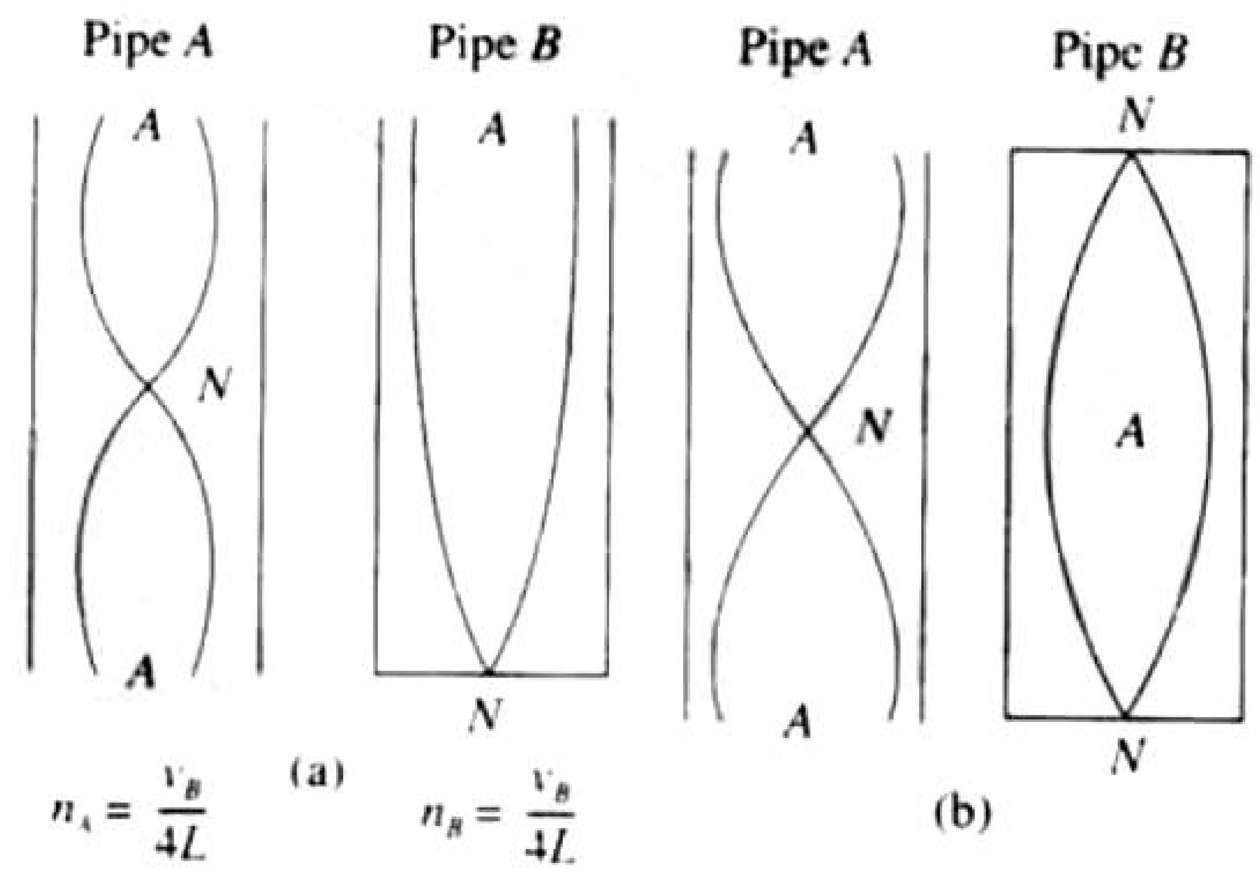
b. Situation 2. In air

$$C = \sqrt{\frac{\gamma RT}{M}} = 344\ \text{m/s}$$

Again applying equation (i)

$$f' = 10^5\left[\frac{344 - 5 - 0}{344 - 5 - 10}\right] = 1.03 \times 10^5\ \text{Hz}$$

22. a. If L is the length of each pipe A and B, then fundamental frequency of pipe A (open at both ends)



$$n_A = \frac{v_A}{2L} \quad \text{(i)}$$

Fundamental frequency of pipe B (closed at one end)

$$n_B = \frac{v_B}{4L} \quad \text{(ii)}$$

Given $2n_A = 3n_B$

$$2\left(\frac{v_A}{2L}\right) = 3\left(\frac{v_B}{4L}\right)$$

$$\Rightarrow \frac{v_A}{v_B} = \frac{3}{4} \quad \text{(iii)}$$

But $\quad v_A = \sqrt{\frac{\gamma_A RT_A}{M_A}}$; $v_B = \sqrt{\frac{\gamma_B RT_B}{M_B}}$

Given $T_A = T_B$

For monoatomic get

$$\gamma_A = \frac{5}{3}$$

For diatomic get

$$\gamma_B = \frac{7}{5}$$

$$\therefore \quad \frac{v_A}{v_B} = \sqrt{\left\{\frac{\gamma_A}{\gamma_B}\frac{M_B}{M_A}\right\}}$$

$$= \sqrt{\frac{(5/3)\,M_B}{(7/5)\,M_A}} =$$

$$= \sqrt{\frac{25\,M_B}{21\,M_A}} \qquad \text{(iv)}$$

From Eqs. (iii) and (iv)

$$\sqrt{\frac{25\,M_B}{21\,M_A}} = \frac{3}{4}$$

$$\frac{M_A}{M_B} = \left(\frac{4}{3}\right)^2 \times \frac{25}{21} = \frac{400}{189}$$

b. When pipe B is closed at both ends, fundamental frequency of pipe B becomes

$$n_B = \frac{v_B}{2L} \qquad \text{(v)}$$

Using Eqs. (i), (iii) and (v), we get

$$\frac{n_A}{n_B} = \frac{v_A}{v_B} = \frac{3}{4}$$

23. Fundamental frequency of air column closed at one end is

$$n = \frac{v}{4(l+e)} = \frac{v}{4(l+0.3D)}$$

Given $n = 480$ Hz, $D = 5$ cm $= 5 \times 10^{-2}$ m

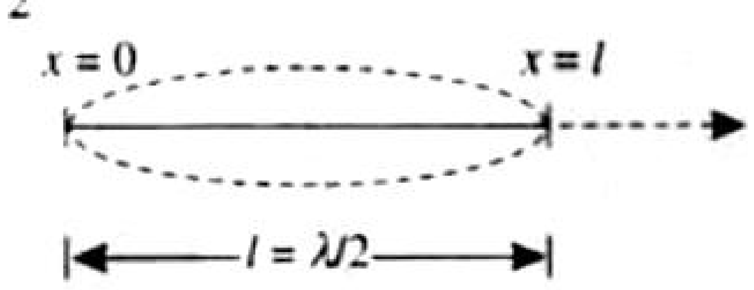
$l = 16$ cm $= 16 \times 10^{-2}$ m

$v = 4n\,(l + 0.3D)$

$= 4 \times 480\,[16 \times 10^{-2} + 0.3 \times 5 \times 10^{-2}]$ m/s

$= 4 \times 480 \times 17.5 \times 10^{-2}$ m/s $= 336$ m/s

24. Here $l = \dfrac{\lambda}{2}$ or $\lambda = 2l$



Since $k = \dfrac{2\pi}{\lambda} \quad \therefore k = \dfrac{2\pi}{2l} = \dfrac{\pi}{l}$

The amplitude of vibration at a distance x from $x = 0$ given by $A = a \sin kx$

Mechanical energy at x of length dx is

$$dE = \frac{1}{2}(dm)A^2\omega^2 = \frac{1}{2}(\mu dx)(a \sin kx)^2(2\pi f)^2$$

$$= 2\pi^2 \mu f^2 a^2 \sin^2 kx\, dx$$

But $\quad v = f\lambda \quad \therefore f = \dfrac{v}{\lambda} \Rightarrow f^2 = \dfrac{v^2}{\lambda^2} = \dfrac{T/\mu}{4l^2}$

$$\therefore \quad dF = 2\pi^2\mu\left(\frac{T/\mu}{4l^2}\right)a^2 \sin^2\left\{\left(\frac{\pi}{l}\right)x\right\}dx$$

$\therefore$ Total energy of the string

$$E = \int dE = \int_0^l 2\pi^2\mu \frac{T/\mu}{4l^2} a^2 \sin^2\left(\frac{\pi x}{l}\right)dx$$

$$= \frac{\pi^2 T a^2}{4l}$$

25. Let the speed of the train be v_r

Case I: The train is approaching

Let f_0 be the natural frequency of the whistle. Using Doppler's effect relation

Apparent frequency

$$f' = f_0\left(\frac{v_S}{v_S - v_T}\right)$$

where v_s = Speed of sound = 300 m/s (given)

$f = 2.2$ KHz $= 2200$ Hz (given)

$$\therefore \quad 2200 = f_0\left(\frac{300}{300 - v_T}\right) \qquad \text{(i)}$$

Case II: The train is receding

$$f'' = f_0\left(\frac{v_S}{v_S - v_T}\right)$$

Here, $v'' = 1.8$ KHz $= 1800$ Hz (given)

$$1800 = f_0\left(\frac{300}{300 - v_T}\right) \qquad \text{(ii)}$$

Dividing (i) and (ii)

$$= \frac{2200}{1800} = \frac{300}{300 - v_T} \times \frac{300 + v_T}{300}$$

$$\Rightarrow \quad \frac{11}{9} = \frac{300 + v_T}{300 - v_T}$$

$$\Rightarrow \quad 3300 - 11v_T = 2700 - 9v_T$$

$$\Rightarrow \quad 600 = 20v_T \Rightarrow v_T = 30 \text{ m/s}$$

26. The wave form of a transverse harmonic disturbance $y = a \sin(\omega t \pm kx \pm \phi)$

Given $v_{max} = a\omega = 3$ m/s (i)

$A_{max} = a\omega^2 = 90$ m/s^2 (ii)

Velocity of wave $v = 20$ m/s (iii)

Dividing (ii) by (i) $\dfrac{a\omega^2}{a\omega} = \dfrac{90}{3}$

$$\Rightarrow \quad \omega = 30 \text{ rad/s} \qquad \text{(iv)}$$

Substituting the value of ω in (i) we get

$$a = \frac{3}{30} = 0.1 \text{ m} \qquad \text{(v)}$$

$$\text{Now } k = \frac{2\pi}{\lambda} = \frac{2\pi}{v/v} = \frac{2\pi f}{v} = \frac{\omega}{v} = \frac{30}{20} = \frac{3}{2} \qquad \text{(vi)}$$

From (iv), (v) and (vi) the wave form is

$$y = 0.1 \sin\left[30t \pm \frac{3}{2}x \pm \phi\right]$$

$$y = (0.1 \text{ m})\sin[(30 \text{ rad/s})t \pm (1.5 \text{ m}^{-1})x + \phi]$$