CHAPTER 16

TRIGONOMETRIC IDENTITIES

A trigonometric identity is a trigonometric equation which is true for all values of the variable (s).

Important Trigonometrical Identities :

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\sin\theta = \sqrt{1-\cos^2\theta}$
3. $\cos\theta = \sqrt{1-\sin^2\theta}$
4. $\sec^2\theta - \tan^2\theta = 1$
5. $\sec^2\theta = 1 + \tan^2\theta$
6. $\tan^2\theta = \sec^2\theta - 1$
7. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$
8. $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$
9. $\cot^2\theta = \operatorname{cosec}^2\theta - 1$
10. $\sin\theta.\operatorname{cosec}\theta = 1$
11. $\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$
12. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$
13. $\cos\theta.\sec\theta = 1$
14. $\cos\theta = \dfrac{1}{\sec\theta}$
15. $\sec\theta = \dfrac{1}{\cos\theta}$
16. $\tan\theta.\cot\theta = 1$
17. $\tan\theta = \dfrac{1}{\cot\theta}$
18. $\cot\theta = \dfrac{1}{\cot\theta}$
19. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
20. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Some useful Results :

1. If $ax + by = m$ and $ay - bx = n$ then $(a^2+b^2)(x^2+y^2) = m^2 + n^2$
   Where, x and y can be any trigonometric ratio i.e $\sin\theta$, $\cos\theta$, $\tan\theta$ etc.
2. If $a\sin\theta + b\cos\theta = c$, then
   Base = b, perpendicular = a
   and hypotenuse = c

Note: Always try to solve this type of questions with the help of taking any value of θ (i.e. 0°, 30°, 45°, 60°, 90° etc.) such that there should not be any contradictions in the option i.e. θ must be satisfy only one option.

QUESTIONS
LEVEL-I

1. If $\alpha+\beta=90^\circ$, then :
   (a) $\sin^2\alpha+\sin^2\beta=1$
   (b) $\text{cosec}^2\alpha-\tan^2\beta=1$
   (c) $\sec^2\alpha-\cot^2\beta=1$
   (d) All of the above are true

2. If $\sec^2 A=3,\ 0<A<\frac{\pi}{2}$, then the value of $\frac{\tan^2 A-\text{cosec}^2 A}{\tan^2 A+\text{cosec}^2 A}$ is :
   (a) $\frac{4}{7}$ (b) $\frac{2}{7}$
   (c) $\frac{1}{7}$ (d) $\frac{3}{7}$

3. If $\cot^2\theta=\frac{7}{8}$ and $0<\theta<90^\circ$, then the value of $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ is :
   (a) $\frac{7}{8}$ (b) $\frac{7}{6}$
   (c) $\frac{7}{5}$ (d) $\frac{7}{4}$

4. If $\sin x+\tan x=\frac{3}{2}$ and $0\le x\le\frac{\pi}{2}$, then the value of $\sin x$ is :
   (a) $\frac{12}{13}$ (b) $\frac{5}{13}$
   (c) $\frac{2}{13}$ (d) $\frac{5}{12}$

5. $(\sin A+\cos A)^2+(\sin A-\cos A)^2$ is equal to :
   (a) 1 (b) $3\cos A$
   (c) 2 (d) $4\sin A$

6. $(\sec^4 A-\sec^2 A)$ is equal to :
   (a) $\tan^2A+\tan^4A$ (b) $\tan^4A-\tan^2A$
   (c) $\tan^2A-\tan^4A$ (d) Non of these

7. $(1-\tan\theta)^2+(1+\tan\theta)^2$ is equal to :
   (a) $2\tan^2\theta$ (b) $2\tan\theta$
   (c) $4\tan\theta$ (d) $2\sec^2\theta$

8. $(\cos^4\theta-\sin^4\theta)$ is equal to :
   (a) $2\sin^2\theta-1$
   (b) $1-2\cos^2\theta$
   (c) $\sin^2\theta-\cos^2\theta$
   (d) None of these

9. $\left(\frac{\tan A}{\sec A-1}+\frac{\tan A}{\sec A+1}\right)$ is equal to
   (a) 2 tanA (b) 2tanA.secA
   (c) 2cosecA (d) 2secA

10. The value of $(\cot^4\theta-\text{cosec}^4\theta+\cot^2\theta+\text{cosec}^2\theta)$ is :
   (a) 1 (b) 0
   (c) -1 (d) 2

11. $\left(\frac{\sin A+\sin B}{\cos A+\cos B}\right)+\left(\frac{\cos A-\cos B}{\sin A-\sin B}\right)$ is equal to :
   (a) 0 (b) 1
   (c) tanA.tanB (d) tanA+tanB

12. If a and b are real numbers such that:
$a\cos\theta + b\sin\theta = 4$ and $a\sin\theta - b\cos\theta = 3$ then (a^2+b^2) is :
(a) 7
(b) 12
(c) 25
(d) $\sqrt{12}$

13. $(1-\sin^2\alpha)\tan^2\alpha$ is equal to :
(a) $\cos^2\alpha$
(b) $\sin^2\alpha$
(c) $\cos\alpha$
(d) $\tan\alpha$

14. The value of $(\cos^4\theta - \sin^4\theta + 2\sin^2\theta)$ is equal to :
(a) $\cos^2\theta\sin^2\theta$
(b) $\sin^2\theta.\cos^4\theta$
(c) $\cos^2\theta.\sin^4\theta$
(d) 1

15. $\frac{1}{1+\sin A} + \frac{1}{1-\sin A}$ is equal to :
(a) 1
(b) $\sec^2 A$
(c) $2\sec^2 A$
(d) $2\cos^2 A$

16. $\cos^2 A(1+\tan^2 A)$ is equal to :
(a) 1
(b) $\sin^2 A - \cos^2 A$
(c) $\cos^2 A$
(d) $\sin^2 A$

17. $\cos^2\theta + \frac{1}{1+\cot^2\theta}$ is equal to :
(a) $\cos^2\theta$
(b) 1
(c) $\sin^2\theta$
(d) 2

18. $\operatorname{cosec}^2\theta + \sec^2\theta = ?$
(a) $\sec^2\theta$
(b) $\operatorname{cosec}^2\theta$
(c) $\operatorname{cosec}^2\theta + \tan^2\theta$
(d) $\operatorname{cosec}^2\theta . \sec^2\theta$

19. $(1+\cot^2\theta)(1-\cos\theta)(1+\cos\theta) = ?$
(a) 1
(b) −1
(c) 0
(d) 2

20. $\cot^2\theta - \frac{1}{\sin^2\theta} = ?$
(a) 1
(b) −1
(c) 0
(d) 2

21. $\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = ?$
(a) −1
(b) 1
(c) 0
(d) sinA.sinB

22. $\frac{1-\cos\theta}{1+\cos\theta} = ?$
(a) $(\operatorname{cosec}\theta - \cot\theta)^2$
(b) $(\operatorname{cosec}\theta + \cot\theta)^2$
(c) $\sin^2\theta$
(d) $\sin^2\theta - \cos^2\theta$

23. What is the value of $\operatorname{cosec}\theta\sqrt{1-\cos^2\theta}$:
(a) 0
(b) −2
(c) $-\sin\theta$
(d) 1

24. $(\sec\theta - 1)^2 - (\tan\theta - \sin\theta)^2$ is equal to :
(a) $\sec\theta$
(b) $(1-\cos\theta)^2$
(c) $(1-\tan\theta)^2$
(d) None of these

25. If θ is acute and $\tan\theta + \cot\theta = 2$, then $\tan^7\theta + \cot^9\theta$ is equal to :
(a) $\sqrt{3}$
(b) 3
(c) 2
(d) 4

26. $\sin\theta + \operatorname{cosec}\theta = 2$ and $0° \le \theta \le 90°$, then the value of $\sin^5\theta + \operatorname{cosec}^5\theta$ is :
(a) 0
(b) 1
(c) 10
(d) 2

27. If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$, then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = ?$

(a) 0
(b) 1
(c) 2
(d) 3

28. $(\sec\theta + \tan\theta)(1 - \sin\theta) = ?$
(a) $\sec\theta$
(b) $\cos\theta$
(c) $\sin\theta$
(d) $\text{cosec}\theta$

29. $[(\sec A + \tan A)(\sec A - \tan A)] +$
$[(\text{cosec}A - \cot A)(\text{cosec}A + \cot A)] = ?$
(a) 1
(b) 0
(c) ½
(d) 2

30. If cos(A+B) = 0, then sin(A - B) = ?
(a) cosB
(b) sinA
(c) cos2B
(d) sin2A

31. If sinA + sin2A = 1, then the value of cos 2 A + cos 4 A is :
(a) 1
(b) 0
(c) 2
(d) ½

32. If $u = a\sin A$ and $v = b\tan A$,
then $\frac{a^2}{u^2} - \frac{b^2}{v^2} = ?$
(a) 1
(b) −1
(c) 0
(d) 2

33. $(\sin A+\text{cosec}A)^2+(\cos A+\sec A)^2=?$
(a) $7+\cot^2 A$
(b) $5+\tan^2 A+\cot^2 A$
(c) $7+\tan^2 A$
(d) $7+\tan^2 A+\cot^2 A$

34. $\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = ?$
(a) 2sec2A
(b) sec2A
(c) −2sec2A
(d) −cos2A

35. $\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} = ?$
(a) $\text{cosec}\theta$
(b) $2\text{cosec}\,\theta$
(c) $\sec\theta$
(d) $2\sec\theta$

36. $\frac{\cot A + \cot B}{\tan A + \tan B} = ?$
(a) cotA
(b) 2cotB
(c) cotA+4
(d) cotA.cotB

37. If sin p + cosec p = 2, then the value of sin p + cosec p is :
(a) 2^7
(b) 0
(c) 1
(d) 2

38. If $\sec\theta - \text{cosec}\,\theta = 0$, then the value of $(\sec\theta + \text{cosec}\,\theta)$ is :
(a) $\frac{\sqrt{3}}{2}$
(b) $\frac{2}{\sqrt{3}}$
(c) 0
(d) $2\sqrt{2}$

39. If $p\sin\theta = \sqrt{3}$ and $p\cos\theta = 1$, then the value of p is :
(a) $\frac{1}{2}$
(b) $\frac{2}{\sqrt{3}}$
(c) $\frac{-1}{\sqrt{3}}$
(d) 2

40. If $u_n = \cos_n\alpha$, then the value of $2u_6 - 3u_4 + 1$ is :
(a) 1
(b) 4
(c) 6
(d) 0

41. If $0 \le \alpha \le \frac{\pi}{2}$ and $2\sin\alpha + 15\cos^2\alpha = 7$ then the value of $\cot\alpha$ is :
(a) $\frac{1}{2}$
(b) $\frac{5}{4}$
(c) $\frac{3}{4}$
(d) $\frac{1}{4}$

42. If $3\sin^2\alpha + 7\cos^2\alpha = 4$, then the value of tan α is (where $0 < \alpha < 90^\circ$):
(a) $\sqrt{2}$
(b) $\sqrt{5}$
(c) $\sqrt{3}$
(d) $\sqrt{6}$

43. The value of $(\sec\theta - \cos\theta)(\text{cosec}\theta - \sin\theta)(\tan\theta + \cot\theta)$ is:

(a) 1
(b) $\frac{3}{2}$
(c) 2
(d) 0

44. If $\tan\theta = 1$, then the value of $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^2\theta + 7\cos\theta}$ is :

(a) 2
(b) $2\frac{1}{2}$
(c) 3
(d) $\frac{4}{5}$

45. The simplified value of $(\sec A - \cos A)^2 + (\text{cosec } A - \sin A)^2 - (\cot A - \tan A)^2$ is :

(a) 0
(b) $\frac{1}{2}$
(c) 1
(d) 2

46. If θ be an acute amg;e amd $7\sin^2\theta + 3\cos^2\theta = 4$, then the value of $\tan\theta$ is :

(a) $\sqrt{3}$
(b) $\frac{1}{\sqrt{3}}$
(c) 1
(d) 0

47. If $\sin\theta + \text{cosec}\,\theta = 2$, then the value of $\sin^5\theta + \text{cosec}^5\theta$ when $0° \le \theta \le 90°$, is :

(a) 0
(b) 1
(c) 10
(d) 2

48. If $\sec^2\theta + \tan^2\theta = 7$, then the value of θ when $0° \le \theta \le 90°$ is :

(a) 60°
(b) 30°
(c) 0°
(d) 90°

49. If $\cos^2\alpha + \cos^2\beta = 2$, then the value of $\tan^3\alpha + \sin^5\beta$ is :

(a) – 1
(b) 0
(c) 1
(d) $\frac{1}{\sqrt{3}}$

50. The simplified value of $(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2$ is :

(a) – 1
(b) 0
(c) $\sec^2 x$
(d) 1

51. If $\sin\theta + \text{cose}\,\theta = 2$, then value of $\sin^{100}\theta + \text{cosec}^{100}\theta$ is equal to :

(a) 1
(b) 2
(c) 3
(d) 100

52. The value of $(\sec\theta - \cos)(\text{cosec}\,\theta - \sin\theta)(\tan\theta + \cot\theta)$ is :

(a) 1
(b) $\frac{3}{2}$
(c) 2
(d) 0

53. If θ be acute angle and $\cos\theta = \frac{15}{17}$, then the value of $\cot(90° - \theta)$ is :

(a) $\frac{2\sqrt{8}}{15}$
(b) $\frac{8}{15}$
(c) $\frac{\sqrt{2}}{17}$
(d) $\frac{8\sqrt{2}}{17}$

54. If $\sec^2\theta + \tan^2\theta = \frac{7}{12}$, then $\sec^4\theta - \tan^4\theta =$

(a) $\frac{7}{12}$
(b) $\frac{1}{2}$
(c) $\frac{5}{12}$
(d) 1

55. If $\cos x + \cos y = 2$, the value of $\sin x + \sin y$ is :

(a) 0
(b) 1
(c) 2
(d) –1

56. If $\tan 7\theta \tan 2\theta = 1$, then the value of $\tan 3\theta$ is :

(a) $\sqrt{3}$
(b) $-\frac{1}{\sqrt{3}}$
(c) $\frac{1}{\sqrt{3}}$
(d) $-\sqrt{3}$

57. If $\tan\theta = 2$, then the value of $\frac{8\sin\theta + 5\cos\theta}{\sin^3\theta + 2\cos^3\theta + 3\cos\theta}$ is :

(a) $\frac{21}{5}$ (b) $\frac{8}{5}$

(c) $\frac{7}{5}$ (d) $\frac{16}{5}$

58. If $\cos\theta + \sec\theta = 2$, the value of $\cos^6\theta + \sec^6\theta$ is :

(a) 4 (b) 8

(c) 1 (d) 2

59. The numerical value of $\frac{5}{\sec^2\theta} + \frac{2}{1+\cot^2\theta} + 3\sin^2\theta$ is :

(a) 5 (b) 2

(c) 3 (d) 4

60. The value of $\cot\theta.\tan(\tan 90°-\theta) - \sec(90°-\theta)\operatorname{cosec}\theta + (\sin^2 25° + \sin^2 65°) + \sqrt{3}(\tan 5°.\tan 30°.\tan 75°.\tan 85°.\tan 85°)$ is :

(a) 1 (b) −1

(c) 2 (d) 0

61. If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{5}{4}$, the value of $\frac{\tan^2\theta + 1}{\tan^2\theta - 1}$ is :

(a) $\frac{25}{16}$ (b) $\frac{41}{9}$

(c) $\frac{41}{40}$ (d) $\frac{40}{41}$

62. If $\tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \sqrt{3}$ the value of $\cos\theta$ is :

(a) 0 (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{2}$ (d) 1

QUESTIONS
LEVEL-II

1. If $\alpha + \beta = 90°$ and $\alpha = 2\beta$, then $\cos^2\alpha + \sin^2\beta$ is equal to :-
   (a) 1
   (b) ½
   (c) 0
   (d) 2

2. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then the value of $\cos\theta - \sin\theta$ will be :
   (a) $\sqrt{2}\cos\theta$
   (b) $\sqrt{2}(\cos\theta + \sin\theta)$
   (c) $\sqrt{2}.\sin\theta$
   (d) None of these

3. The equation $\tan^2\phi + \tan^6\phi = \tan^2\phi \, . \sec^2\phi$ is :
   (a) identity for only one value of ϕ
   (b) not an identity
   (c) identity for all values of ϕ
   (d) none of the above

4. If $7\sin^2\theta + 3\cos^2\theta = 4$ and $0 \le \theta \le \frac{\pi}{2}$, then the value of $\tan\theta$ is:-
   (a) $\sqrt{\frac{3}{7}}$
   (b) $\sqrt{\frac{2}{7}}$
   (c) $\frac{1}{\sqrt{7}}$
   (d) $\frac{1}{\sqrt{3}}$

5. If $3\sin\theta + 4\cos\theta = 5$, then the value of $\sin\theta$ is :
   (a) $\frac{3}{4}$
   (b) $\frac{3}{5}$
   (c) $\frac{4}{5}$
   (d) None of these

6. $\sqrt{\frac{1+\sin A}{1-\sin A}}$ is equal to :
   (a) secA + tanA
   (b) $\sec^2A + \tan^2A$
   (c) $\sec^2A - \tan^2A$
   (d) secA. tanA

7. $\sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$ is equal to :-
   (a) $\sec x + \tan x$
   (b) $\sec x - \tan x$
   (c) $\text{cosec}\,x + \cot x$
   (d) $\text{cose}\,x - \cot x$

8. $\frac{\sin\theta}{(1+\cos\theta)}$ is equal to :
   (a) $\frac{\sin\theta}{\cos\theta}$
   (b) $\frac{\cos\theta - 1}{\sin\theta}$
   (c) $\frac{1-\cos\theta}{\sin\theta}$
   (d) $\frac{\sin\theta + 1}{\cos\theta}$

9. If $x = r\sin\theta.\cos\theta$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$, then
   (a) $x^2 + y^2 + z^2 = r^2$
   (b) $x^2 - y^2 + z^2 = r^2$
   (c) $x^2 + y^2 - z^2 = r^2$
   (d) $-x^2 + y^2 + z^2 = y^2$

10. Which of the following is an identity?
    (a) $\sin^4\theta.\cos^4\theta = \sin^2\theta - \cos^2\theta$
    (b) $\tan^2\theta - \sin^2\theta = \tan^2\theta.\sin^2\theta$
    (c) $\frac{1-\tan^2\theta}{1+\tan^2\theta} = 1 - 2\sin^2\theta$
    (d) $\sin^6\theta + \cos^6\theta - 1 - 3\sin^2\theta.\cos^2\theta = 0$

11. If $A + B + C = \pi$, then tanA+tanB+tanC equals :
    (a) cotA.tanB.tanC
    (b) tanA.tanB.tanC
    (c) tanA.cotB.tanC
    (d) tanA.tanB.cotC

12. Evaluate : $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$

(a) $\operatorname{cosec}\theta - \cot\theta$
(b) $2\operatorname{cosec}\theta + \cot\theta$
(c) $2\operatorname{cosec}\theta$
(d) $\operatorname{cosec}\theta + \cot\theta$

13. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then
$m^2 - n^2 = ?$

(a) $4\sqrt{mn}$ (b) mn
(c) m^2n^2 (d) m^3n^3

14. $(1+\cot\theta - \operatorname{cosec}\theta)(1+\tan\theta+\sec\theta) = ?$

(a) 1 (b) −1
(c) 2 (d) −2

15. $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = ?$

(a) $\tan^2\theta + \cot^2\theta$
(b) $\tan\theta + \cot\theta$
(c) $\sin\theta + \cos\theta$
(d) None of these

16. $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta) = ?$

(a) $\tan\theta + \cot\theta$
(b) $\tan\theta - \cot\theta$
(c) $\frac{1}{\tan\theta - \cot\theta}$
(d) $\frac{1}{\tan\theta + \cot\theta}$

17. If $\sin\theta + \sin^2\theta = 1$, then $\cos^2\theta + \cos^4\theta = ?$

(a) $\cos^2\theta$ (b) $\sin^2\theta$
(c) 1 (d) $\sin\theta.\cos\theta$

18. $\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta} = ?$

(a) $\tan^2\theta - \cot^2\theta$
(b) $\tan^2\theta + \cot^2\theta$
(c) $\sec^2\theta + \operatorname{cosec}^2\theta$
(d) None of these

19. $(\tan A + \tan B)^2 + (\tan A - \tan B)^2$ is equal to :

(a) $\sec^2 A.\tan^2 B$
(b) $\sec^2 A.\sec^2 B$
(c) $\tan^2 A\tan^2 B$
(d) $\cos^2 A\cos^2 B$

20. If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, then what is the value of 2p:

(a) $p(q^2-1)$ (b) $p(1-q^2)$
(c) $q(1-p^2)$ (d) $q(p^2-1)$

21. Find the value of :
$1 + 2\sec^2 A.\tan^2 A - \sec^4 - \tan^4 A$

(a) 0 (b) 1
(c) $\sec^2A.\tan^2A$
(d) None of these

22. $\sin\alpha + \cos\beta = 2$ $(0° \le \beta < \alpha \le 90°)$, then
$\sin\left(\frac{2\alpha+\beta}{3}\right)$ is equal to :-

(a) $\sin\left(\frac{\alpha}{2}\right)$ (b) $\cos\left(\frac{\alpha}{3}\right)$
(c) $\sin\left(\frac{\alpha}{3}\right)$ (d) $\cos\left(\frac{2\alpha}{3}\right)$

23. $(\sec A - \cos A)^2 + (\operatorname{cosec}A - \sin A)^2 - (\cot A - \tan A)^2 = ?$

(a) 0 (b) ½
(c) 1 (d) 2

24. If $\sin x + \sin^2 x = 1$, then $\cos^8 x + 2\cos^6 x + \cos^4 x = ?$

(a) 0 (b) 1
(c) $1/2$ (d) $1/3$

25. If $x=\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$, then $\dfrac{2x}{1-x^2}=?$

(a) $\sec\theta$ (b) $\tan\theta$

(c) $\cot\theta$ (d) $\cos\theta$

26. If $x=\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$, then

$\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=?$

(a) x (b) $1-x$

(c) $1+x$ (d) $1/x$

27. $\dfrac{\sin x-\sin 3x}{\sin^2 x-\cos^2 x}=?$

(a) $\sin 2x$ (b) $2\tan x$

(c) $\tan 2x$ (d) $2\sin x$

28. If $\dfrac{\cos A}{\sin B}=m$ and $\dfrac{\cos A}{\cos B}=n$, then (m^2+n^2) $\cos^2 B=?$

(a) n (b) 2n

(c) n^2 (d) $2n^2$

29. $\dfrac{\sin\theta}{1-\cos\theta}+\dfrac{\tan\theta}{1+\cos\theta}=?$

(a) $\sec\theta.\text{cosec}\,\theta+\cot\theta$

(b) $\sec\theta.\text{cosec}\theta-\cot\theta$

(c) $\sec\theta.\text{cosec}\,\theta+\tan\theta$

(d) $\sec\theta.\text{cosec}\,\theta-\tan\theta$

30. If $\sin x+\sin^2 x=1$, then the value of $\cos^{12}x+3\cos^{10}x+3\cos^8 x+\cos^6 x-1$ is equal to :

(a) 2 (b) 1

(c) 0 (d) −1

31. If $m=\text{cosec}\theta-\sin\theta$ and $n=\sec\theta-\cos\theta$, then $m^{2/3}+n^{2/3}=?$

(a) $(mn)^{2/3}$ (b) $(mn)^{-2/3}$

(c) $(mn)^{-1/3}$ (d) $(mn)^{1/3}$

32. $\sqrt{\dfrac{1+\cos A}{1-\cos A}}+\sqrt{\dfrac{1-\cos A}{1+\cos A}}=?$

(a) $2\sin A$ (b) $2\cos A$

(c) $2\sec A$ (d) $2\text{cosec}A$

33. $\dfrac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=?$

(a) $\sec\theta+\tan\theta$

(b) $\sec\theta-\tan\theta$

(c) $2\sec\theta$

(d) $2\tan\theta$

34. $\dfrac{\cos\theta-\sin\theta+1}{\cos\theta+\sin\theta+1}=?$

(a) $\sec\theta+\tan\theta$

(b) $\sec\theta-\tan\theta$

(c) $2\text{cosec}\theta$

(d) $2\sec\theta$

35. $\left(\dfrac{1+\sin A-\cos A}{1+\sin A+\cos A}\right)^2=?$

(a) $\dfrac{1-\cos A}{1+\cos A}$ (b) $\dfrac{1+\cos A}{1-\cos A}$

(c) $\dfrac{\cos A}{1-\cos A}$ (d) $\dfrac{\cos A}{1+\cos A}$

36. If $\cos\theta+\sec\theta=\sqrt{3}$, then the value of $\cos^3\theta+\sec^3\theta$ is :

(a) 0 (b) 1

(c) −1 (d) $\sqrt{3}$

37. If $\sec\theta+\tan\theta=2$, then $\sec\theta$ is equal to :

(a) $\dfrac{7}{4}$ (b) $\dfrac{7}{2}$

(c) $\dfrac{5}{2}$ (d) $\dfrac{5}{4}$

38. The simplied value of

$1-\frac{\sin^2 A}{1+\cos A}+\frac{1+\cos A}{\sin A}-\frac{\sin A}{1-\cos A}$ is :

(a) cos A
(b) 0
(c) 1
(d) Sin A

39. If $\sin^2\alpha + \sin^2\beta = 2$, then the value of $\cos\left(\frac{\alpha+\beta}{2}\right)$ is :

(a) 1
(b) −1
(c) 0
(d) 0.5

40. If $\tan\theta.\tan 2\theta = 1$, then the value of $\sin^2 2\theta + \tan^2 2\theta$ is equal to :

(a) $\frac{3}{4}$
(b) $\frac{10}{3}$
(c) $3\frac{3}{4}$
(d) 3

41. If $x = \text{cosec}\,\theta - \sin\theta$ and $y = \sec\theta - \cos\theta$, then the value of $x^2y^2(x^2 + y^2 + 3)$ is :

(a) 0
(b) 1
(c) 2
(d) 3

42. If $0 \le \theta \le \frac{\pi}{2}$, $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - \text{cosec}\,\theta = 3$, then the value of $x^2 + 4y^2$ is :

(a) 1
(b) 2
(c) 3
(d) 4

43. If $\sin\theta + \sin^2\theta = 1$, then the value of $\cos^{12}\theta + 3\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1$ is :

(a) 0
(b) 1
(c) −1
(d) 2

44. If $\sec x + \cos x = 3$, then $\tan^2 x - \sin^2 x$ *is* :

(a) 5
(b) 13
(c) 9
(d) 4

45. Value of $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$ is :

(a) 4
(b) 0
(c) 1
(d) 2

46. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ an $x\sin\theta = y\cos\theta$, $\sin\theta \ne \theta$, $\cos\theta \ne 0$, then $x^2 + y^2$ is :

(a) $\frac{1}{\sqrt{2}}$
(b) $\frac{1}{2}$
(c) 1
(d) $\sqrt{2}$

47. If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A$ is equal to :

(a) 1
(b) $\frac{1}{2}$
(c) 0
(d) −1

48. If $\cos^4\theta - \sin^4\theta = \frac{2}{3}$, then the value of $2\cos^2\theta - 1$ is :

(a) 0
(b) 1
(c) $\frac{2}{3}$
(d) $\frac{3}{2}$

49. If θ be a positive acute angle satisfying $\cos^2\theta + \cos^4\theta = 1$, then the value of $\sin^2\theta + \sin\theta$ is :

(a) $\frac{3}{2}$
(b) 1
(c) $\frac{1}{2}$
(d) 0

50. If $2\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$, $(0° < \theta < 90°)$ the value of $2\sin\theta + \cos\theta$ is :

(a) $\frac{1}{\sqrt{2}}$
(b) $\sqrt{2}$
(c) $\frac{3}{\sqrt{2}}$
(d) $\frac{\sqrt{2}}{3}$

51. If $\dfrac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta}=3$, then the value of $\sin^4\theta-\cos^4\theta$ is :

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$

(c) $\frac{3}{5}$ (d) $\frac{4}{5}$

52. If $\sin\theta-\cos\theta=\dfrac{7}{13}$ and $0<\theta<90°$, then the value of $\sin\theta+\cos\theta$ is :

(a) $\frac{17}{13}$ (b) $\frac{13}{17}$

(c) $\frac{1}{13}$ (d) $\frac{1}{17}$

53. If $\sec\theta+\tan\theta=2$, the value of $\sin x+\sin y$ is :

(a) $\frac{4}{5}$ (b) 5

(c) $\frac{5}{4}$ (d) $\sqrt{2}$

54. If $\text{cosec}\,\theta-\sin\theta=l$ and $\sec\theta-\text{cosec}\,\theta=m$, then the value of $l^2m^2(l^2+m^2+3)$ is :

(a) –1 (b) 0

(c) 1 (d) 2

55. The value of $(2\cos^2\theta-1)\left(\dfrac{1+\tan\theta}{1-\tan\theta}+\dfrac{1-\tan\theta}{1+\tan\theta}\right)$ is :

(a) 4 (b) 1

(c) 3 (d) 2

QUESTION
LEVLE-III

1. If $\tan^2\theta = 2\tan^2\phi + 1$, then which of the following is correct ?
   (a) $\cos 2\theta = \cos 2\phi - 1$
   (b) $\cos 2\theta = \cos 2\phi + 1$
   (c) $\cos 2\theta = \frac{1}{2}[\cos 2\phi - 1]$
   (d) $\cos 2\theta = \frac{1}{2}[\cos 2\phi + 1]$

2. $\frac{\cos A + \text{cosec}A - 1}{\cot A - \text{cosec}A + 1}$ is equal to :-
   (a) cosecAa+cotA (b) secA+cotA
   (c) cosecA+tanA (d) cosecA–cotA

3. If $\sec\theta + \tan\theta = p$, then $\frac{p^2-1}{p^2+1}$ =?
   (a) $\cos\theta$ (b) $\sin\theta$
   (c) $\sec\theta$ (d) $\tan\theta$

4. $2\sec^2\theta - \sec^4 - 2\text{cosec}^2\theta + \text{cosec}^4\theta = ?$
   (a) $\tan^4\theta - \cot^4\theta$
   (b) $\tan^4\theta + \cot^4\theta$
   (c) $\cot^6\theta - \cot^4\theta$
   (d) $\cot^4\theta - \tan^4\theta$

5. If $a\cos^3 + 3a\cos\theta.\sin^2\theta = m$ and $a\sin^3 + 3a\sin\theta.\cos^2\theta = n$, then $(m+n)^{2/3} + (m-n)^{2/3} = ?$
   (a) $2a^{1/3}$ (b) $2a^{2/3}$
   (c) $a^{2/3}$ (d) $a^{1/3}$

6. If $\frac{\sin A}{\sin B} = p$ and $\frac{\cos A}{\cos B} = q$, then tan A = ?
   (a) $\pm\frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}}$ (b) $\pm\frac{q}{p}\sqrt{\frac{q^2-1}{1-p^2}}$
   (c) $\pm\frac{q}{p}\sqrt{\frac{1-q^2}{1-p^2}}$ (d) $\pm\frac{q}{p}\sqrt{\frac{q^2+1}{1+p^2}}$

7. If $\frac{3\pi}{\cdot} < \alpha < \pi$, then $\sqrt{2\cot\pi + \frac{1}{\sin^2\alpha}} = ?$
   (a) $1-\cot\alpha$ (b) $1+\cot\alpha$
   (c) $-1-\cot\alpha$ (d) $-1+\cot\alpha$

8. If $\text{cosec}\theta = x + \frac{1}{4x}$, then the value of $\text{cosec}\theta + \cot\theta$ is :-
   (a) $-2x$ (b) $2x$
   (c) $\frac{1}{2x}$ (d) $-\frac{1}{2x}$

9. If $\tan^2\theta = 1 - e^2$, then $\sec\theta + \tan^3\theta.\text{cosec}\theta = ?$
   (a) $(2-e^2)^{3/2}$ (b) $(2-e^2)^{3/2}$
   (c) $(2-e^2)^{1/2}$ (d) None of these

10. If $\sin\theta + \cos\theta = m$ and $\sec\theta + \text{cosec}\theta = n$, then,
    (a) $2n = m(n^2-1)$
    (b) $2m = n(m^2-1)$
    (c) $2n = m(m^2-1)$
    (d) None of these

11. If $T_n = \sin^n\theta + \cos^n\theta$, then $\frac{T_3 - T_5}{T_1} = ?$

(a) $\sin\theta.\cos\theta$
(b) $\sin^2\theta.\cos\theta$
(c) $\sin^2\theta.\cos^2\theta$
(d) $\sin\theta.\cos^2\theta$

12. If $10\sin^4\alpha + 15\cos^4\alpha = 6$, then $27\sec^6\alpha + 8\sec^6\alpha = ?$

(a) 240
(b) 250
(c) 245
(d) 235

13. If $\cot\theta(1 + \sin\theta) = 4m$ and $\cot\theta(1 - \sin\theta) = 4n$, then $(m^2 - n^2)^2 = ?$

(a) m^2n^2
(b) m^3n^3
(c) mn
(d) m^2n

14. If tan A = n tan B and sin A = m sin B, then $\frac{m^2 - 1}{n^2 - 1} = ?$

(a) cos A
(b) $2\cos^2A$
(c) $\cos^3 A$
(d) $\cos^2A$

HINTS AND SOLUTIONS

LEVEL - I

1.(d) $\sin^2\alpha + \sin^2\beta =$

$\sin^2\alpha + \sin^2(90° - \alpha) =$

$\sin^2\alpha + \cos^2\alpha = 1$ ...(1)

$\text{cosec}^2\alpha - \tan^2\beta =$

$\text{cosec}^2\alpha - \tan^2(90° - \alpha)$

$= \text{cosec}^2\alpha - \cot^2\alpha = 1$ ....(2)

$\sec^2\alpha - \cot^2\beta =$

$\sec^2\alpha - \cot^2(90° - \alpha) =$

$\sec^2\alpha - \tan^2\alpha = 1$ ....(3)

2.(c) $\tan^2A = \sec^2A - 1 = 3 - 1 = 2$, $\cot^2A = \frac{1}{2}$

5.(c) Using $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$, we get : $(\sin A + \cos A)^2 + (\sin A - \cos A)^2$

$= 2(\sin^2A + \cos^2A)$

$= 2(1)$

$= 2$

6.(a) $(\sec^4 A - \sec^2 A) = \sec^2A\,(\sec^2A - 1)$

$= \sec^2A.\tan^2A$

$= (1 + \tan^2A)\tan^2A$

$= \tan^2A + \tan^4A$

7.(d) Using,

$(a-b)^2 + (a+b)^2 = 2\,(a^2 + b^2)$

$(1 - \tan\theta)^2 + (1 + \tan\theta)^2 = 2(1 + \tan^2\theta)$

$= 2\sec^2\theta$

8.(d) $(\cos^4\theta - \sin^4\theta) = (\cos^2\theta + \sin^2\theta)$

$(\cos^2\theta - \sin^2\theta) = (1)(\cos^2\theta - \sin^2\theta)$

$= \cos^2\theta - \sin^2\theta$

$$= \frac{(\sin^2 A - \sin^2 B) + (\cos^2 A - \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{1 - 1}{(\cos A + \cos B)(\sin A - \sin B} = 0$$

12.(c) Squaring and adding, we get :

$a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$= 4^2 + 3^2$

$\Rightarrow a^2 + b^2 = 25$

13.(b) $(1 - \sin^2\alpha)\tan^2\alpha = \cos^2\alpha \dfrac{\sin^2\alpha}{\cos^2\alpha}$

$= \sin^2\alpha$

14.(d) $(\cos^4\theta - \sin^4\theta) + 2\sin^2\theta$

$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$+ 2\sin^2\theta$

$= (\cos^2\theta - \sin^2\theta) + 2\sin^2\theta$

$= \cos^2\theta + \sin^2\theta$

$= 1$

15.(c) $\dfrac{1}{1 + \sin A} + \dfrac{1}{1 - \sin A}$

$= \dfrac{2}{1 - \sin^2 A} = \dfrac{2}{\cos^2 A} = 2\sec^2 A$

16.(a) $\cos^2 A(1 + \tan^2 A)$

$= \cos^2 A\left(1 + \dfrac{\sin^2 A}{\cos^2 A}\right)$

$= \cos^2 A \dfrac{\cos^2 A + \sin^2 A}{\cos^2 A}$

$= 1$

17.(b) Given Exp.

$= \cos^2\theta + \dfrac{1}{1 + \dfrac{\cos^2\theta}{\sin^2\theta}}$

$= \cos^2\theta + \dfrac{\sin^2\theta}{\sin^2\theta + \cos^2\theta}$

$= \cos^2\theta + \sin^2\theta$

$= 1$

18.(d) Given exp.

$= \dfrac{1}{\sin^2\theta} + \dfrac{1}{\cos^2\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta.\cos^2\theta}$

$= \dfrac{1}{\sin^2\theta.\cos^2\theta} = \text{cosec}^2\theta.\sec^2\theta$

Alternatively. take $\theta = 45°$

$\text{cosec}^2\theta + \sec^2\theta = 2 + 2 = 4$

& option (D)= $\text{cosec}^2\theta.\sec^2\theta = 2 \times 2 = 4$

$\therefore$ option (D) is correct.

19.(a) Given Exp.= $\left(1 + \dfrac{\cos^2\theta}{\sin^2\theta}\right)(1 - \cos^2\theta)$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2}.\sin^2\theta$

$= 1$

Alternatively :-

$(1 + \cot^2\theta)(1 - \cos^2\theta)$

$= \text{cosec}^2\theta.\sin^2\theta$

$= \dfrac{1}{\sin^2\theta}.\sin^2\theta$

$= 1$

20.(b) Given Exp.

$= \cot^2\theta - \text{cosec}^2\theta$

$= (\text{cosec}^2\theta - 1) - \text{cosec}^2\theta = -1$

21.(c) Given Exp.

$\dfrac{\sin A - \sin B}{\cos A + \cos B} + \dfrac{\cos A - \cos B}{\sin A + \sin B}$

$$= \frac{(\sin^2 A - \sin^2 B) + (\cos^2 A - \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$

$$= \frac{1-1}{(\cos A+\cos B)(\sin A+\sin B)} = 0$$

22.(a) Given Exp $= \frac{1-\cos\theta}{1+\cos\theta} \times \frac{(1-\cos\theta)}{(1-\cos\theta)}$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2 = (\operatorname{cosec}\theta - \cot\theta)^2$$

Alternatively : take $\theta = 45°$

$$\text{Given Exp.} = \frac{1-\left(\frac{1}{\sqrt{2}}\right)}{1+\left(\frac{1}{\sqrt{2}}\right)} = \frac{\sqrt{2}-1}{\sqrt{2}+1} = (\sqrt{2}-1)^2$$

& option (A)

$=(\text{cosec}45° - \cot 45°) = (\sqrt{2}-1)^2$

hence, (A) is correct option.

23.(d) $\text{cosec}\theta\sqrt{1-\cos^2\theta} = \operatorname{cosec}\theta.\sin\theta$

$$= \frac{1}{\sin\theta}.\sin\theta = 1$$

24.(b) Given Exp.

$$= \left(\frac{1-\cos\theta}{\cos\theta}\right)^2 - \sin^2\theta\left(\frac{1-\cos\theta}{\cos\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{\cos^2\theta}(1-\sin^2\theta)$$

$$= \frac{(1-\cos\theta)^2}{\cos^2\theta}.\cos^2\theta = (1-\cos^2\theta)^2$$

Alternatively : Let $\theta = 60°$

$\therefore$ Given Exp.

$$= (2-1)^2 - \left(\sqrt{3}-\frac{\sqrt{3}}{2}\right)^2 = 1 - 3\times\frac{1}{4} = \frac{1}{4}$$

& option (B) $= (1-\cos 60°)^2$

$$= \left(1-\frac{1}{2}\right)^2 = \frac{1}{4}$$

Hence, option (B) is correct.

Note: at $\theta = 45°$, option (A) and (B) contradicts so don't take $\theta = 45°$.

25.(c) $\tan\theta + \cot\theta = 2 \Rightarrow \theta = 45°$

or $\frac{\sin^2\theta+\cos^2\theta}{\sin\theta.\cos\theta} = 2$

$\Rightarrow 1 = \sin 2\theta \Rightarrow 2\theta = 90° \Rightarrow \theta = 45°$

$\therefore \tan^7\theta + \cot^9\theta = 1 + 1 = 2$

26.(d) $\sin\theta + \text{cosec}\theta = 2 \Rightarrow \theta = 90°$

$\sin^5\theta + \text{cosec}^5\theta = 1 + 1 = 2$

$\therefore x + \frac{1}{x} = 2$ ($x = 1$ satisfying)

So, $\sin^5\theta + \operatorname{cosec}^5\theta$

$$= x^5 + \frac{1}{x^5} = 1 + 1 = 2$$

Alternatively :

$\sin\theta + \text{cosec}\,\theta = 2$

or $\sin\theta + \frac{1}{\sin\theta} = 2$

27.(a) $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

$\Rightarrow \sin\theta_1 = \sin\theta_2 = \sin\theta_3$

$\Rightarrow \theta_1 = \theta_2 = \theta_3 = \frac{\pi}{2}$

$\therefore \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$

28.(b) Given Exp.

$$= \frac{1+\sin\theta}{\cos\theta}.(1-\sin\theta)$$

$$= \frac{1-\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta}$$

$$= \cos\theta$$

Alternatively : Let $\theta = 60°$

$\therefore$ Given Exp.

$= (2+\sqrt{3})\left(\frac{2-\sqrt{3}}{2}\right) = \frac{1}{2} = \cos 60°$

i.e. $\cos\theta$

29.(d) Given Exp.

$= (\sec^2 A - \tan^2 A) + (\operatorname{cosec}^2 A - \cot^2 A)$

$= 1+1$

$= 2$

30.(c) Let $A = 60°$ & $B = 30°$

$\because \cos(A+B) = 0 \Rightarrow A+B = 90°$

$\therefore \sin(A-B) = \sin 30° = \cos 60° = \cos 2B$

Alternatively:-

$\cos(A+B) = 90° \Rightarrow A + B = 90°$

$\Rightarrow A = 90° - B$

So, $\sin(90° - B - B) = \sin(90° - 2B) = \cos 2B$

31.(b) $\sin A + \sin 2A = 1$

$A = \frac{\pi}{2} = 90°$ satisfying the above equation

$\Rightarrow \cos\pi + \cos 2\pi = -1 + 1 = 0$

32.(a) $\frac{a^2}{u^2} - \frac{b^2}{v^2} = \frac{a^2}{a^2 \sin^2 A} - \frac{b^2}{b^2 \tan^2 A}$

$= \operatorname{cosec}^2 A - \cot^2 A$

$= 1$

Alternatively ; Let A=45°, then

$u = \frac{a}{\sqrt{2}}$ and $v = b$

$\therefore \frac{a^2}{u^2} - \frac{b^2}{v^2} = \frac{a^2}{\left(\frac{a}{\sqrt{2}}\right)^2} - \frac{b^2}{b^2} = 2 - 1 = 1$

$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$

$= \frac{\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$

$= \frac{1-1}{(\cos A + \cos B)(\sin A + \sin B)} = 0$

33.(d) Let $A = 45°$

$\therefore$ Given Exp.

$= \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2 + \left(\frac{1}{\sqrt{2}} + \sqrt{2}\right)^2$

$= 2\left(\frac{1+2}{\sqrt{2}}\right)^2 = 9$

& $7 + \tan^2 A + \cot^2 A$

$= 7 + 1 + 1$

$= 9$

hence (D) is correct option.

Alternatively ;

Given Exp :

$(\sin^2 A + \operatorname{cosec}^2 A + 2) + (\cos^2 A + \sec^2 A + 2)$

$= (\sin^2 A + \operatorname{cosec}^2 A) + \cos^2 A + \sec^2 A + 4$

$= 1 + 1 + \cot^2 A + 1 + \tan^2 A + 4$

$= 7 + \tan^2 A + \cot^2 A$

34.(c) Let A=0°, then

Given Exp. $= \frac{0+1}{0-1} + \frac{0-1}{0+1} = -2$

(which is equal to $-2 \sec A$).

Alternatively:

Given Exp. =

$\frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{\sin^2 A - \cos^2 A}$

$= \frac{2(\sin^2 A + \cos^2 A)}{-(\cos^2 A - \sin^2 A)}$

$[\because (a+b)^2 + (a-b)^2 = 2(a^2+b^2)]$

$= \frac{2 \times 1}{-\cos 2A} = -2\sec 2A$

35.(b) Given Exp.

$= \frac{\sin^2 A + (1+\cos A)^2}{\sin A(1+\cos A)}$

$$= \frac{\sin^2 A + 1 + \cos^2 A + 2\cos A}{\sin A(1+\cos A)}$$

$$= \frac{2 + 2\cos A}{\sin A(1+\cos A)}$$

$[\because \sin^2 A + \cos^2 A = 1]$

$$= \frac{2(1+\cos A)}{\sin A(1+\cos A)}$$

$$= \frac{2}{\sin A} = 2\operatorname{cosec} A$$

Alternatively : Let $\theta = 30°$

Given Exp.=

$$\frac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}} + \frac{\frac{2+\sqrt{3}}{2}}{\frac{1}{2}} = \frac{1}{2+\sqrt{3}} + (2+\sqrt{3})$$

$$= 2-\sqrt{3}+2+\sqrt{3} = 4$$

and $2\operatorname{cosec}\theta = 2 \operatorname{cosec} 30° = 2 \times 2 = 4$

hence (B) is correct option.

36.(d) Given Exp.= $\dfrac{\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}}{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}$

$$= \frac{\frac{\sin B.\cos A + \sin A.\cos B}{\sin A.\sin B}}{\frac{\sin A.\cos B + \cos A.\sin B}{\cos A.\cos B}}$$

$$= \frac{\cos A.\cos B}{\sin A.\sin B}$$

$= \cot A.\cot B$

37.(d) $\sin p + \operatorname{cosec} p = 2$

or $\sin p + \dfrac{1}{\sin p} = 2$

$\therefore$ $\sin p = 1$ satisfies it.

$\therefore$ $\operatorname{cosec} p = 1$

$\therefore$ $\sin^7 p + \operatorname{cosec}^7 p = 1^7 + 1^7 = 2$

38.(d) $\sec\theta - \operatorname{cosec}\theta = 0$

$\Rightarrow \sec\theta = \operatorname{cosec}\theta$

$\Rightarrow \dfrac{1}{\cos\theta} = \dfrac{1}{\sin\theta}$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \tan\Rightarrow = 1 = \tan 45°$

$\Rightarrow \theta = 45°$

$\therefore$ $\sec\theta + \operatorname{cosec}\theta = \sec 45° + \operatorname{cosec} 45°$

$= \sqrt{2} + \sqrt{2} = 2\sqrt{2}$

39.(d) $p \sin\theta = \sqrt{3}$

$p \cos\theta = 1$

On squaring and adding

$p^2 \sin^2\theta + p^2\cos^2\theta = 3 + 1$

$\Rightarrow p^2(\sin^2\theta + \cos^2\theta) = 4$

$\Rightarrow p^2 = 4$

$\Rightarrow p = 2$

40.(d) $u_n = \cos^2\alpha + \sin^n\alpha$

$\therefore$ $u_6 = \cos^6\alpha + \sin^6\alpha$

$(\cos^2\alpha)^3 + (\sin^2\alpha)^3$

$= (\cos^2\alpha + \sin^2\alpha)^3 - 3 \cos^2\alpha.\sin^2\alpha(\sin^2\alpha + \cos^2\alpha)$

$[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)]$

$= 1 - 3\cos^2\alpha.\sin^2\alpha$

$u_4 = 1 - 3\cos^2\alpha.\sin^2\alpha$

$(\cos^2\alpha)^2 + (\sin^2\alpha)^2$

$= (\cos^2\alpha + \sin^2\alpha)^2 - 2\cos^2\alpha.\sin^2\alpha = 1 - 2\cos^2\alpha.\sin^2\alpha$

$\therefore$ $2u_6 - 3u_4 + 1$

$= 2(1 - 3\sin^2\alpha\cos^2\alpha)^2 - 3(1 - 2\sin^2\alpha\cos^2\alpha) + 1$

$= 2 - 3 + 1 = 0$

41.(c) $2\sin\alpha + 15\cos^2\alpha = 7$

$\Rightarrow 2\sin\alpha + 15(1 - \sin^2\alpha) = 7$

$\Rightarrow 2\sin\alpha + 15 - 15\sin^2\alpha = 7$

$\Rightarrow 15\sin^2\alpha - 2\sin\alpha - 8 = 0$

$\Rightarrow 15\sin^2\alpha - 12\sin\alpha + 10\sin\alpha - 8 = 0$

$\Rightarrow 3\sin\alpha\,(5\sin\alpha - 4) + 2(5\sin\alpha - 4) = 0$

$\Rightarrow (3\sin\alpha + 2)(5\sin\alpha - 4) = 0$

$\Rightarrow \sin\alpha = \frac{4}{5} \qquad \left(0 \le \alpha \le \frac{\pi}{2}\right)$

because $\sin\alpha \ne -\frac{2}{3}$

$\cos\alpha = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

$\therefore \cot\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$

42.(a) $3\sin^2\alpha + 7(1 - \sin^2\alpha) = 4$

$\Rightarrow 3\sin^2\alpha + 7 - 7\sin^2\alpha = 4$

$\Rightarrow 7 - 4\sin^2\alpha = 4$

$\Rightarrow 4\sin^2\alpha = 3 \Rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos\alpha = \sqrt{1 - \sin^2\alpha} = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

$\therefore \tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \sqrt{3}$

Alternatively :

let $\theta = 45°$

$\therefore$ Given Exp. =

$\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)(1+1)$

$= \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot 2 = \frac{1}{2} \times 2 = 1$

43.(a)

44.(a) $\tan\theta = 1 \Rightarrow \theta = 45°$

$\therefore \frac{8\sin\theta + 5\cos\theta}{\sin^3\theta - 2\cos^3\theta + 7\cos\theta}$

$= \frac{8 \times \frac{1}{\sqrt{2}} + \frac{5}{\sqrt{2}}}{\frac{1}{2\sqrt{2}} - \frac{2}{2\sqrt{2}} + \frac{7}{\sqrt{2}}}$

$= \frac{\frac{13}{\sqrt{2}}}{\frac{13}{2\sqrt{2}}} = \frac{13}{\sqrt{2}} \times \frac{2\sqrt{2}}{13} = 2$

45.(c) $(\sec A - \cos A)^2 + (\operatorname{cosec} A - \sin A)^2 + (\cot A - \tan A)^2$

$= \sec^2 A + \cos^2 A - 2\sec A \cos A + \operatorname{cosec}^2 A + \sin^2 A - 2\operatorname{cosec} A.\sin A - \cot^2 A - \tan^2 A + 2\cot A.\tan A$

$= \sec^2 A - \tan^2 A + \cos^2 A + \sin^2 A + \operatorname{cosec}^2 A - \cot^2 A - 2$

$= 3 - 1 = 2$

[$\because$ secA. cosA = 1; sinA. cosec A = 1; tan A.cotA = 1 etc]

Alternatively ;

take $\theta = 45°$

$\therefore$ Given Exp. $\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)(1+1)$

$= \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \cdot 2 = \frac{1}{2} \times 2 = 1$

46.(b)

$7\sin^2\theta + 3\cos^2\theta$

$\Rightarrow 7\frac{\sin^2\theta}{\cos^2\theta} + 3 = \frac{4}{\cos^2\theta} = 4\sec^2\theta$

$\Rightarrow 7\tan^2\theta + 3 = 4(1 + \tan^2\theta)$

$\Rightarrow 7\tan^2\theta + 3 = 4 + 4\tan^2\theta$

$\Rightarrow 3\tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \frac{1}{3}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

47.(d) $\sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 1$

$\Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$

$\therefore \sin^5\theta + \operatorname{cosec}^5\theta = 1 + 1 = 2$

48.(a) $\sec^2\theta + \tan^2\theta = 7$

$\Rightarrow 1 + \tan^2\theta + \tan^2\theta = 7$

$\Rightarrow 2\tan^2\theta = 7 - 1 = 6$

$\Rightarrow \tan^2\theta = 3 \Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = 60°$

49.(b) $\cos^2\alpha + \cos^2\beta = 2$

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta = 2$

$\Rightarrow \sin^2\alpha + \sin^2\beta = 0$

$\Rightarrow \sin\alpha = \sin\beta = 0$

$\Rightarrow \alpha = \beta = 0$

$\therefore \tan^3\alpha + \sin^5\beta = 0$

50.(d) Given Exp ;

$$= \left(\frac{1}{\cos x}\cdot\frac{1}{\cos y} + \frac{\sin x}{\cos y}\cdot\frac{\sin y}{\cos y}\right)^2$$

$$-\left(\frac{1}{\cos x}\cdot\frac{\sin y}{\cos y} + \frac{\sin x}{\cos x}\cdot\frac{1}{\cos y}\right)^2$$

$$= \frac{1}{\cos 2x.\cos 2y}\left[\begin{matrix}(1+\sin x.\sin y)^2 \\ -(\sin x + \sin y)^2\end{matrix}\right]$$

$$= \frac{1}{\cos^2 x \cos^2 y}\left(1 + \sin^2 x \sin^2 y - \sin^2 x - \sin^2 y\right)$$

$$= \frac{1}{\cos^2 x.\cos^2 y}\left[\cos^2 y\left(1 - \sin^2 x\right)\right]$$

$$= \frac{1}{\cos^2 x.\cos^2 y}.\cos^2 y.\cos^2 x = 1$$

51.(b) $\sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 1$

$\Rightarrow \operatorname{cosec}\theta = 1$

$\therefore \sin^{100}\theta + \operatorname{cosec}^{100}\theta = 1 + 1 = 2$

52.(a) $(\sec\theta - \cos\theta)(\operatorname{cosec}\theta - \sin\theta)$
$(\tan\theta + \cot\theta)$

$$= \left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)$$

$$= \left(\frac{1-\cos 2\theta}{\cos\theta}\right)\left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta}\right)$$

$$= \left(\frac{1-\cos 2\theta}{\cos\theta}\right)\left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta}\right)$$

$$= \frac{\sin^2\theta}{\cos\theta}.\frac{\cos^2\theta}{\sin\theta}.\frac{1}{\sin\theta.\cos\theta} = 1$$

Alternatively ; take $\theta = 45°$

Given Exp:

$$= \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)(1+1)$$

$$= \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}.2$$

$$= \frac{1}{2} \times 2 = 1$$

53.(b) $\cos\theta = \frac{15}{17} = \frac{\text{Base}}{\text{hypotenuse}}$

$\therefore$ Perpendicular $= \sqrt{(17)^2 - (15)^2}$

$= \sqrt{289 - 225} = \sqrt{64} = 8$

$\therefore \cot(90° - \theta) = \tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{8}{15}$

54.(a) $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \sec^2\theta + \tan^2\theta = \frac{7}{12}$

$\therefore \sec^4\theta - \tan^4\theta$

$= (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$

$= 1 \times \frac{7}{12} = \frac{7}{12}$

55.(a) $\cos x + \cos y = 2$

$\because \cos x \le 1$

$\Rightarrow \cos x = 1;\ \cos y = 1$

$\Rightarrow x = y = 0°$

$\therefore \sin x + \sin y = 0 + 0 = 0$

56.(c) As we know, if tan A .tan B = 1 $\Rightarrow$ A + B = 90°

$\therefore \tan 7\theta . \tan 2\theta = 1$ (given)

$\therefore 7\theta + 2\theta = 90 \Rightarrow 9\theta$

$= 90° \Rightarrow \theta = 10°$

$\therefore \tan 3\theta = \tan 30° = \frac{1}{\sqrt{3}}$

57.(a) Expression

$= \frac{8\sin\theta + 5\cos\theta}{\sin^3\theta + 2\cos^2\theta + 3\cos\theta}$

$= \frac{8\tan\theta + 5}{\tan\theta . \sin^2\theta + 2\cos^2\theta + 3}$

$= \frac{8\tan\theta + 5}{2\sin^2\theta + 2\cos^2\theta + 3}$

$= \frac{8\tan\theta + 5}{2(\sin^2\theta + \cos^2\theta) + 3}$

$= \frac{8 \times 2 + 5}{5} = \frac{21}{5}$

58.(d) $\cos\theta + \sec\theta = 2$

$\Rightarrow \cos\theta + \frac{1}{\cos\theta} = 2$

$\Rightarrow \cos^2\theta - 2\cos\theta + 1 = 0$

$\Rightarrow (\cos\theta - 1)^2 = 0$

$\Rightarrow \cos\theta = 1$

$\therefore \sec\theta = 1$

$\therefore \cos^6\theta + \sec^6\theta$

$= 1 + 1 = 2$

59.(a) Expression

$= \frac{5}{\sec^2\theta} + \frac{2}{1+\cot^2\theta} + 3\sin^2\theta$

$= 5\cos^2\theta + \frac{2}{\mathrm{cosec}^2\theta} + 3\sin^2\theta$

$= 5\cos^2\theta + 2\sin^2\theta + 3\sin^2\theta$

$= 5(\cos^2\theta + \sin^2\theta) = 5$

$\left[\because 1 + \cot^2\theta = \mathrm{cosec}^2\theta \frac{1}{\sec\theta} = \cos\theta . \frac{1}{\mathrm{cosec}\theta} = \sin\theta\right]$

60.(c) Given Exp ;

$\cot\theta . \cot\theta - \mathrm{cosec}\theta . \mathrm{cosec}\theta +$ $(\sin^2 25° + \cos^2 25°) + \sqrt{3}$ [(tan 5°.tan 85°).(tan 15°.tan75°). tan 30°]

$= \cot^2\theta - \mathrm{cosec}^2\theta + 1 +$

$\sqrt{3}\left(1 \times 1 \times \frac{1}{\sqrt{3}}\right)$

$= -1 + 1 + 1$

$= 1$

[$\because \sin(90° - \theta) = \cos\theta$; $\mathrm{cosec}^2\theta - \cot^2\theta = 1$; $\tan(90° - \theta) = \cot\theta$; $\sec(90° - \theta) = \mathrm{cosec}\,\theta$; and tan A.tanB = 1 if A + B = 90°]

61.(c) Given Exp :

$4\sin\theta + 4\cos\theta = 5\sin\theta - 5\cos\theta$

$\sin\theta = 9\cos\theta \Rightarrow \tan\theta = 9$

$\therefore \frac{\tan^2\theta + 1}{\tan^2\theta - 1} = \frac{81+1}{81-1} = \frac{82}{80} = \frac{41}{40}$

62.(c) $\tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \sqrt{3}$

55. The value of $(2\cos^2\theta - 1)$ $\left(\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}\right)$ is :

(a) 4 (b) 1
(c) 3 (d) 2

LEVEL - II

1.(b) $\alpha + \beta = 90°$, $\alpha = 2\beta$

$\therefore\ 2\beta + \beta = 90° \Rightarrow \beta = 30°$

and $\alpha = 2 \times 30° = 60°$

$\therefore\ \cos^2\alpha + \sin^2\beta = \cos^2 60° + \sin^2 30°$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

2.(c) $1.\sin\theta + 1.\cos\theta = \sqrt{2}\cos\theta$ ...... (i) given

Let $1.\cos\theta - 1.\sin\theta = p$ ......(ii)
{i.e. $ax + by = m$ and $bx - ay = n$}

$\therefore$ form (i) and (ii),

$(\sin^2\theta + \cos^2\theta)(1^2 + 1^2) = (\sqrt{2}\cos\theta)^2 + p^2$

$\Rightarrow 1(2) = 2\cos^2\theta + p^2$

$\Rightarrow p^2 = 2(1 - \cos^2\theta) = 2\sin^2\theta$

$\Rightarrow p = \sqrt{2}\sin\theta$

Alternatively : $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$

Squaring both sides,

$\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta = 2\cos^2\theta$

$\Rightarrow 2\sin\theta.\cos\theta = 2\cos^2\theta - 1$ .... (i)

$[\because \sin^2\theta + \cos^2\theta = 1]$

Now, $(\cos\theta - \sin\theta)^2 =$

$\cos^2\theta + \sin^2\theta - 2\sin\theta.\cos\theta$

$= 1 - (2\cos^2\theta - 1)$ [form (i)]

$= 2[1 - \cos^2\theta] = 2\sin^2\theta$

$\therefore \cos\theta - \sin\theta = \sqrt{2}.\sin\theta$

3.(B) Let $\phi = 30°$

Then,

$\text{L.H.S.} = \tan^2\phi + \tan^6\phi = \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^6$

$= \frac{1}{3} + \frac{1}{27} = \frac{10}{27}$

R.H.S. $= \tan^2\phi.\sec^2\phi$

$= \left(\frac{1}{\sqrt{3}}\right)^2 \cdot \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{1}{3} \times \frac{4}{9} = \frac{4}{27}$

L.H.S. $\neq$ R.H.S.

so it is not an identity.

4.(d) $7\sin^2\theta + 3\cos^2\theta = 4$

on dividing both sides by $\cos^2\theta$

$7\tan^2\theta + 3 = \frac{4}{\cos^2\theta} = 4\sec^2\theta$

$\Rightarrow 7\tan^2\theta + 3 = 4(1 + \tan^2\theta)$

$\Rightarrow 3\tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \frac{1}{3}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

5.(b) Given Exp.

$\Rightarrow 4\cos\theta = 5 - 3\sin\theta$

$\Rightarrow 4\sqrt{1 - \sin^2\theta} = 5 - 3\sin\theta$

$\Rightarrow 16(1 - \sin^2\theta) = (5 - 3\sin\theta)^2$

$\Rightarrow 25\sin^2\theta - 30\sin\theta + 9 = 0$

$\Rightarrow (5\sin\theta - 3)^3 = 0 \Rightarrow \sin\theta = \frac{3}{5}$

Alternatively:-

$3\sin\theta + 4\cos\theta = 5$

$\therefore$ perpendicular = 3, base = 4 and

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hypetenuse = 5

$\therefore \sin\theta = \frac{3}{5}$

6.(a) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{(1+\sin A)}{(1+\sin A)}}$

$= \frac{1+\sin A}{\sqrt{1-\sin^2 A}} = \frac{1+\sin A}{\cos A}$

$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$

7.(b) $\sqrt{\frac{\sec x - \tan x}{\sec x + \tan x}}$

$= \sqrt{\frac{\sec x - \tan x}{\sec x + \tan} \times \frac{\sec x - \tan x}{\sec x - \tan x}}$

$= \frac{\sec x - \tan x}{\sqrt{\sec^2 x - \tan^2 x}}$

$= \sec x - \tan x$

8.(c) Let $\theta = 45°$

$\therefore \frac{\sin\theta}{1+\cos\theta} = \frac{1/\sqrt{2}}{1+\left(1/\sqrt{2}\right)}$

$= \frac{1}{\sqrt{2}+1} = \sqrt{2}-1$

in option (C) put $\theta = 45°$

$\frac{1-\cos\theta}{\sin\theta} = \frac{1-\left(1/\sqrt{2}\right)}{\left(1/\sqrt{2}\right)} = \sqrt{2}-1$

hence, option (C) is correct.

Alternatively:-

$\frac{\sin\theta}{1+\cos\theta} = \frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{\sin\theta(1-\cos\theta)}{(1-\cos^2\theta)}$

$= \frac{\sin\theta(1-\cos\theta)}{\sin^2\theta} = \frac{1-\cos\theta}{\sin\theta}$

9.(a) $x^2 + y^2 + z^2 =$

$r^2\sin^2\theta.\cos^2\phi + r^2\sin^2\theta.\sin^2\phi + r^2\cos^2\theta$

$= r^2\sin^2\theta(\cos^2\phi + \sin^2\phi) + r^2\cos^2\theta$

$= r^2\sin^2\theta + r^2\cos^2\theta = r^2(\sin^2\theta + \cos^2\theta)$

$= r^2$

10.(c) L.H.S. $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{\frac{1-\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$

$= \cos^2\theta - \sin^2\theta = (1-2\sin^2\theta) =$ R.H.S.

$\therefore$ (C) is an identity.

Alternatively :- take $\theta = 45°$

$\therefore \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-1}{1+1} = 0$

and $1-2\sin^2\theta = 1-2\times\frac{1}{2} = 0$

i.e. L.H.S. = R.H.S.

Hence, (C) is an identity.

11.(b) Let A = B = C = 60°

$\therefore$ tanA+tanB+tanC= $3\sqrt{3}$

& option (B) = tanA.

tanB.tanC= $3\sqrt{3}$

$\therefore$ option (B) is correct.

Alernatively:-

A+B+C= $\pi \Rightarrow$ A+B = π – C

$\therefore$ tan(A+B) = tan (π–C) = –tanC

$\Rightarrow \frac{\tan A + \tan B}{1-\tan A.\tan B} = -\tan C$

$\Rightarrow \tan A + \tan B = -\tan C + \tan A \cdot \tan B \cdot \tan C$

$\Rightarrow$ tan A+ tan B + tanC = tanA.tan B. tan C

12.(d) Given Exp.

$$= \sqrt{\frac{(1+\cos\theta)}{(1-\cos\theta)} \times \frac{(1+\cos\theta)}{(1+\cos\theta)}}$$

$$= \frac{1+\cos\theta}{\sqrt{1-\cos^2\theta}} = \frac{1+\cos\theta}{\sin\theta}$$

$$= \operatorname{cosec}\theta + \cot\theta$$

13.(a) $m^2 - n^2 = (\tan\theta - \sin)^2 - (\tan\theta - \sin\theta)^2$

$$= 4\tan\theta \,.\, \sin\theta$$

$[\because (a+b)^2 - (a-b)^2 = 4ab]$

and $4\sqrt{mn}$

$$= 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta} = 4\sqrt{\frac{\sin^2\theta}{\cos\theta} - \sin^2\theta}$$

$$= 4\sqrt{\frac{\sin^2\theta - \sin^2\theta.\cos^2\theta}{\cos^2\theta}}$$

$$= 4\sqrt{\frac{\sin^2\theta\left(1-\cos^2\theta\right)}{\cos^2\theta}}$$

$$= 4\sqrt{\frac{\sin^4\theta}{\cos^2\theta}} = 4.\frac{\sin^2\theta}{\cos\theta}$$

$$= 4\frac{\sin\theta}{\cos\theta} \,.\, \sin\theta = 4\tan\theta.\sin\theta$$

$$\therefore m^2 - n^2 = 4\sqrt{mn}$$

Alternatively :- Let $\theta = 45°$

$$\therefore m = \tan\theta + \sin\theta = \frac{\sqrt{2}+1}{\sqrt{2}}$$

and $n = \tan\theta - \sin\theta = \dfrac{\sqrt{2}-1}{\sqrt{2}}$

$$\therefore m^2 - n^2 = \frac{1}{2}(3 + 2\sqrt{2} - 3 + 2\sqrt{2}) = 2\sqrt{2}$$

$$\& \ 4\sqrt{mn} = 4\sqrt{\frac{1}{2}} = 2\sqrt{2}$$

$$\therefore m^2 - n^2 = 4\sqrt{mn}$$

14.(c) Given Exp.

$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$

$$= \frac{\sin\theta + \cos\theta - 1}{\sin\theta} \cdot \frac{\sin\theta + \cos\theta - 1}{\cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta \,.\, \cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta - 1}{\sin\theta \,.\, \cos\theta}$$

$$= \frac{2\sin\theta \,.\, \cos\theta}{\sin\theta \,.\, \cos\theta} = 2$$

15.(b) Given Exp.

$$= \sqrt{(1 + \tan^2\theta) + (1 + \cot^2\theta)}$$

$$= \sqrt{\tan^2\theta + \cot^2\theta + 2.\tan\theta.\cot\theta}$$

$$= \sqrt{(\tan\theta + \cot\theta)^2}$$

$$= \tan\theta + \cot\theta$$

16.(d) Given Exp.

$$= \left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{1}{\cos\theta} - \cos\theta\right)$$

$$= \frac{1 - \sin^2\theta}{\sin\theta} \cdot \frac{1 - \cos^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos\theta} = \sin\theta.\cos\theta$$

$$\& \ \frac{1}{\tan\theta + \cot\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\sin\theta.\cos\theta}{\sin^2\theta + \cos^2\theta} = \sin\theta.\cos\theta$$

hence (D) is correct option.

17.(c) $\sin\theta + \sin^2\theta = 1 \Rightarrow \sin\theta = 1 - \sin^2\theta$

$$\Rightarrow \sin\theta = \cos^2\theta$$

$\therefore\ \sin^2\theta = \cos^4\theta$

$\therefore\ \cos^2\theta + \cos^4\theta = \sin\theta + \sin^2\theta = 1$ (given)

18.(a) $\dfrac{\tan\theta - \cot\theta}{\sin\theta.\cos\theta} = \dfrac{\dfrac{\sin\theta}{\cos\theta} - \dfrac{\cos\theta}{\sin\theta}}{\sin\theta.\cos\theta}$

$= \dfrac{\sin^2\theta - \cos^2\theta}{\sin^2\theta.\cos^2\theta}$

$= \dfrac{1}{\cos^2\theta} - \dfrac{1}{\sin^2\theta} = \sec^2\theta - \operatorname{cosec}^2\theta$

$= (1+\tan^2\theta) - (1+\cot^2\theta) = \tan^2\theta - \cot^2\theta$

Alternatively:- take $\theta = 45°$

$\therefore$ Given Exp.$= \dfrac{1-1}{\left(1/\sqrt{2}\right).\left(1/\sqrt{2}\right)} = 0$

& option (A)$=\tan^2 45° - \cot^2 45°$

$= 1-1=0$

hence, (A) is correct option.

19.(b) Given Exp.

$\dfrac{(\cos A.\cos B + \sin A.\sin B)^2 + (\sin A.\cos B - \sin B.\cos A)^2}{\cos^2 A\ .\ \cos^2 B}$

$= \dfrac{\cos^2(A-B) + \sin^2(A-B)}{\cos^2 A.\cos^2 B}$

$= \dfrac{1}{\cos^2 A.\cos^2 B} = \sec^2 A.\sec^2 B$

Alternatively:- take $A = 45° = B$

Given Exp.$=(1+1\times1)^2 + (1-1)^2$

$= 2^2$

$= 4$

& option (B) $= \sec^2 45°.\sec^2 45° = (\sqrt{2})^2(\sqrt{2})^2 = 4$

20.(d) Let $\theta = 45°$

then $p = \sin45° + \cos45° = \sqrt{2}$ and

$q = \sec 45° + \operatorname{cosec} 45° = 2\sqrt{2}$

$\therefore\ q(p^2-1) = 2\sqrt{2}\left[(\sqrt{2})^2 - 1\right] = 2\sqrt{2} = 2p$

hence, option (D) is correct.

21.(a) Given Exp.

$= 1 - (\sec^4 A + \tan^4 A - 2\sec^2 A.\tan^2 A)$

$= 1 - (\sec^2 A - \tan^2 A)^2$

$= 1 - 1 = 0$

Alternatively:- take $A = 45°$, then

Given Exp.

$= 1 + 2\sec^2 45°.\tan^2 45° - \sec^4 45° - \tan^4 45°$

$= 1 + 2(\sqrt{2})^2 \times 1 - (\sqrt{2})^4 - 1$

$= 0$

22.(b) $\sin\alpha + \cos\beta = 2$

$\Rightarrow \alpha = 90°\ \&\ \beta = 0°$

$\Rightarrow \sin\left(\dfrac{2\alpha+\beta}{3}\right) = \sin 60°$

$= \cos 30° = \cos\dfrac{\alpha}{3}$

23.(c) Given Exp.

$= \sec^2 A + \cos^2 A - 2 + \operatorname{cosec}^2 A + \sin^2 A - 2 - \cot^2 A - \tan^2 A + 2$

$= (\sec^2 A - \tan^2 A) + (\cos^2 A + \sin^2 A) + (\operatorname{cosec}^2 A - \cot^2 A) - 2$

$= 1 + 1 + 1 - 2$

$= 1$

Alternatively ; take $\theta = 45°$, then given Exp.$=1$

24.(b) Given Exp.

$\sin x = 1 - \sin^2 x = \cos^2 x$

$\Rightarrow \sin^2 x = \cos^4 x$

$\therefore\ \cos^8 x + 2\cos^6 x + \cos^4 x$

$= \left(\cos^4 x + \cos^2 x\right)^2$

$= \left(\sin^2 x + \sin x\right)^2$

$= 1^2 = 1$

25.(b) Let $\theta = 60°$

$\therefore \; x = \sqrt{\dfrac{1-\frac{1}{2}}{1+\frac{1}{2}}} = \dfrac{1}{\sqrt{3}}$

$\therefore \; \dfrac{2x}{1-x^2} = \dfrac{2/\sqrt{3}}{1-\frac{1}{3}} = \sqrt{3}$

and option (B). $\tan\theta = \tan 60°$

$= \sqrt{3}$

& option (B) is correct.

Note:- at $\theta = 0°$ $\cot\theta = \infty$

$\therefore$ θ can't be 0° & at $\theta = 45°$ option (B) and (C) contradicts.

26.(a) Let $\alpha = 0°$, then $x = \dfrac{1-1+0}{1+0} = 0$

& $\dfrac{2\sin 0°}{1+\cos 0° + \sin 0°} = 0 = x$

27.(d) Given Exp.

$\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = \dfrac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x}$

$= \dfrac{\sin 3x - \sin x}{\cos 2x}$

$= \dfrac{3\sin x - 4\sin^3 x - \sin x}{\cos 2x}$

$= \dfrac{2\sin x\left(1 - 2\sin^2 x\right)}{\cos 2x}$

$= \dfrac{2\sin x . \cos 2x}{\cos 2x}$

$= 2\sin x$

Alternatively:- Let $x = 30°$

$\therefore$ Given Exp

$= \dfrac{\sin 90° - \sin 30°}{\cos 60°} = \dfrac{1-\frac{1}{2}}{\frac{1}{2}} = 1 = 2\sin x$

28.(c) Let A = B = 60°, then

$m = \dfrac{1/2}{1/2} = 1$ and $n = \dfrac{1/2}{\sqrt{3}/2} = \dfrac{1}{\sqrt{3}}$

$\therefore \; \left(m^2 + n^2\right)\cos^2 B = \left(1 + \dfrac{1}{3}\right) \times \left(\dfrac{1}{2}\right)$

$= \dfrac{4}{3} \times \dfrac{1}{4} = \dfrac{1}{3} = n^2$

Alternatively:-

$\left(m^2 + n^2\right)\cos^2 B = \left(\dfrac{\cos^2 A}{\cos^2 B} + \dfrac{\cos^2 A}{\sin^2 B}\right)\cos^2 B$

$= \cos^2 A \dfrac{\sin^2 B + \cos^2 B}{\cos^2 B . \sin^2 B} . \cos^2 B$

$= \dfrac{\cos^2 A}{\sin^2 B}$

$= n^2$

29.(a) Given Exp.

$= \sin\theta \left[\dfrac{\cos^2\theta + \cos\theta + 1 - \cos\theta}{\cos\theta(1 - \cos^2\theta}\right]$

$= \sin\theta \dfrac{1 + \cos^2\theta}{\cos\theta . \sin^2\theta}$

$= \dfrac{1}{\cos\theta . \sin\theta} + \dfrac{\cos^2}{\cos\theta . \sin\theta}$

$= \sec\theta . \mathrm{cosec}\theta + \cot\theta$

Alternatively :- Let $\theta = 60°$, then

Given Exp.

$$= \frac{\sqrt{3}/2}{1-1/2} + \frac{\sqrt{3}}{1+1/2} = \sqrt{3} + \frac{2}{\sqrt{3}} = \frac{5}{\sqrt{3}}$$

& option (A) = sec60°.cosec60°

$$= 2.\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{5}{\sqrt{3}}$$

hence option (A) is correct.

Note : at $\theta = 45°$, option (A) and (C) and (B), (D) controdicts

30.(c) $\sin x + \sin^2 x = 1$

$\Rightarrow \sin x = 1 - \sin^2 x = \cos^2 x$

$\therefore \sin^2 x = \cos^4 x$

Now, $\cos^{12}x + 3\cos^{10}x + 3\cos^8 x + \cos^6 x) - 1$

$= (\cos^4 x + \cos^2 x)^3 - 1$

$= (\sin^2 x + \sin x)^3 - 1$

$= 1^3 - 1$

$= 0$

31.(b) $m = \operatorname{cosec}\theta - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta}$

$$= \frac{\cos^2\theta}{\sin\theta}$$

and $n = \sec\theta - \cos\theta = \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$

$$\therefore\ mn = \frac{\cos^2\theta}{\sin\theta}.\frac{\sin^2\theta}{\cos\theta} = \sin\theta.\cos\theta$$

and $m^{2/3} + n^{2/3} = \left(\frac{\cos^2\theta}{\sin\theta}\right)^{2/3} + \left(\frac{\sin^2\theta}{\cos\theta}\right)^{2/3}$

$$= \frac{\cos^{4/3}\theta}{\sin^{2/3}\theta} + \frac{\sin^{4/3}\theta}{\cos^{2/3}\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{(\sin\theta.\cos\theta)^{2/3}} = \frac{1}{(mn)^{2/3}}$$

$$= (mn)^{-2/3}$$

Alternatively:- Let $\theta = 45°$, then

$m = \frac{1}{\sqrt{2}}$ and $n = \frac{1}{\sqrt{2}}$

$$\therefore\ m^{2/3} + n^{2/3} = \left(\frac{1}{\sqrt{2}}\right)^{2/3} + \left(\frac{1}{\sqrt{2}}\right)$$

$$= 2.\left(\frac{1}{\sqrt{2}}\right)^{2/3} = 2.\left(\frac{1}{2}\right)^{1/3}$$

$$= 2.2^{-1/3} = 2^{2/3}$$

$$\& \ (mn)^{-2/3} = \left(\frac{1}{2}\right)^{-2/3} = 2^{2/3}$$

$$\Rightarrow m^{2/3} + n^{2/3} = (mn)^{-2/3}$$

32.(d) Given Exp.

$$= \frac{1+\cos A + 1 - \cos A}{\sqrt{1-\cos^2 A}} = \frac{2}{\sqrt{\sin^2 A}}$$

$$= 2 \operatorname{cosec} A$$

33.(a) Let $\theta = 45°$, then

$$\text{Given Exp.} = \frac{\left(1/\sqrt{2}\right) - \left(1/\sqrt{2}\right) + 1}{\left(1/\sqrt{2}\right) + \left(1/\sqrt{2}\right) - 1}$$

$$= \frac{1}{\sqrt{2}-1} = \sqrt{2} + 1$$

Now go through option

Option (A) $\sec 45° + \tan 45° = \sqrt{2} + 1$

Given Exp.

hence option (A) is correct

34.(b) Let $\theta = 45°$, then

= Given Exp.

$$= \frac{\left(1/\sqrt{2}\right) - \left(1/\sqrt{2}\right) + 1}{\left(1/\sqrt{2}\right) + \left(1/\sqrt{2}\right) + 1} = \frac{1}{\sqrt{2}+1} = \sqrt{2} - 1$$

Now option (B) = sec45°−

tan45° = $\sqrt{2}-1$

given exp.

hence option (B) is correct.

35.(a) Let A = 45°, then

Given Exp.$=\left(\frac{1}{\sqrt{2}+1}\right)^2=\left(\sqrt{2}-1\right)^2$

Now go through options.

Option (A) $=\frac{1-\cos 45^0}{1+\cos 45^0}=\frac{\sqrt{2}-1}{\sqrt{2}+1}$

$=\frac{\left(\sqrt{2}-1\right)}{\left(\sqrt{2}+1\right)}\times\frac{\left(\sqrt{2}-1\right)}{\left(\sqrt{2}-1\right)}=\left(\sqrt{2}-1\right)^2$

Hence, option (A) is correct

36.(a) $\cos\theta + \sec\theta = \sqrt{3}$

On cubing,

$(\cos\theta+\sec\theta)^3=\left(\sqrt{3}\right)^3=3\sqrt{3}$

$\Rightarrow \cos^3\theta + \sec^3\theta + 3\cos\theta.\sec\theta(\cos\theta + \sec\theta)$

$= 3\sqrt{3}$

$\Rightarrow \cos^3\theta + \sec^3\theta = 0$

37.(d) $\sec\theta + \tan\theta = 2$ ____(i)

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{2}$ ____(ii)

On adding (i) & (ii),

$\sec\theta + \tan\theta + \sec\theta - \tan\theta$

$= 2+\frac{1}{2}$

$\Rightarrow 2\sec\theta = \frac{5}{4}$

38.(a) take A = 30°

∴ Given Exp

$= 1-\frac{1/4}{1+\left(\frac{\sqrt{3}}{2}\right)}+\frac{1+\left(\frac{\sqrt{3}}{2}\right)}{1/2}-\frac{1/2}{1-\left(\frac{\sqrt{3}}{2}\right)}$

$=1-\frac{1}{2\left(2+\sqrt{3}\right)}+\left(2+\sqrt{3}\right)-\frac{1}{2-\sqrt{3}}$

$=1-\frac{2-\sqrt{3}}{2}+2+\sqrt{3}-\left(2+\sqrt{3}\right)$

$=1-1+\frac{\sqrt{3}}{2}+2+\sqrt{3}-2-\sqrt{3}$

$=\frac{\sqrt{3}}{2}=\cos 30°$ *i.e* cos A.

[**Note** : at A = 45° option (A) & (D) contradicts.]

39.(c) $\sin^2\alpha + \sin^2\beta = 2$

$\because \sin\theta \le 1$

$\therefore \sin\alpha = \sin\beta = 1$

$\therefore \alpha = \beta = 90°$

$\therefore \cos\left(\frac{\alpha+\beta}{2}\right)=\cos 90°=0$

40.(c) As we know,

if tan A.tan B = 1 ⇒ A + B = 90°

$\therefore \tan\theta.\tan 2\theta = 1$

$\Rightarrow \theta + 2\theta = 90° \Rightarrow \theta = 30°$

$\therefore \sin^2 2\theta + \tan^2\theta = \sin^2 60° + \tan^2 60°$

$=\left(\frac{\sqrt{3}}{2}\right)^2+\left(\sqrt{3}\right)^2$

$=\frac{3}{4}+3=3\frac{3}{4}$

41.(b) take $\theta = 45°$

$\therefore x=\sqrt{2}-\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$ & $y=\sqrt{2}-\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

$\therefore x^2y^2(x^2y^2+3)=\frac{1}{2}\times\frac{1}{2}\left(\frac{1}{2}+\frac{1}{2}+3\right)$

$=\frac{1}{4}(4)=1$

42.(d) take $\theta = 45°$

$\therefore 2y.\frac{1}{\sqrt{2}}=\frac{x}{\sqrt{2}}$ or $x = 2y$ ....(i)

and $2x(\sqrt{2})-y(\sqrt{2})=3$

or $2x-y=\frac{3}{12}$ ....(ii)

From (i) & (ii)

$4y-y=\frac{3}{\sqrt{2}} \Rightarrow 3y=\frac{3}{\sqrt{2}}$ or $\frac{1}{\sqrt{2}}$

$\therefore x=\frac{2}{\sqrt{2}}=\sqrt{2}$

$\therefore x^2+4y^2 = 2+4\times\left(\frac{1}{2}\right)=4$

43.(a) $\sin\theta + \sin^2\theta = 1$

$\Rightarrow \sin\theta = 1-\sin^2\theta$

$\Rightarrow \sin\theta = \cos^2\theta$

$\therefore \cos^{12}\theta + 2\cos^{10}\theta + 3\cos^8\theta + \cos^6\theta - 1$

$= (\cos^4\theta + \cos^2\theta)^3 - 1$

$= (\sin^2\theta + \cos^2\theta)^3 - 1 = 1 - 1 = 0$

44.(a) $\sec x + \cos x = 3$

On squaring bothe sides,

$\sec^2 x + \cos^2 x = 9 - 2 = 7$

$[\because \sec x.\cos x = 1]$

Now, $\tan^2 x - \sin^2 x = \sec^2 x - 1 - (1 - \cos^2 x)$

$[\because \sec^2 x - \tan^2 x = 1]$

$= \sec^2 x + \cos^2 x - 2 = 7 - 2 = 5$

[From equation (i)]

45.(b) take $\theta = 0°$

$\therefore$ Given Exp : $= 2(0 + 1) - 3(0 + 1) + 1$

$= 2 - 3 + 1$

$= 0$

Alternatively :

$2(\sin^6\theta + \cos^6) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2[(\sin^2\theta)^3 + (\cos^2\theta)^3] - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2[(\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)] - 3(\sin^4\theta + \cos^4\theta)^2 + 1$

$[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= 2\sin^4\theta + 2\cos^4\theta - 2\sin^2\theta.\cos^2\theta - 3\sin^4\theta - 3\cos^4\theta + 1$

$= -\sin^4\theta - \cos^4\theta + 2\sin^2\theta.\cos^2\theta + 1$

$= (-\sin^4\theta + \cos^4\theta + 2\sin^2\theta.\cos^2\theta) + 1$

$= -(\sin^2\theta + \cos^2\theta) + 1$

$= -1 + 1 = 0$

46.(c) take $\theta = 45°$

$\therefore x.\left(\frac{1}{\sqrt{2}}\right)^3 + y\left(\frac{1}{\sqrt{2}}\right)^3 = \frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}$

or $x + y = \sqrt{2}$ ....(i)

and $\frac{x}{\sqrt{2}}=\frac{y}{\sqrt{2}}$ or $x = y$ ....(ii)

$\therefore$ from (i) & (ii)

$x + x = \sqrt{2}$ or $2x = \sqrt{2} \Rightarrow x = \frac{1}{\sqrt{2}}$

$\therefore y = \frac{1}{\sqrt{2}}$

$\therefore x^2 + y^2 = \frac{1}{2}+\frac{1}{2}=1$

47.(a) $\cos A = 1 - \cos^2 A = \sin^2 A$

$\therefore \sin^2 A + \sin^4 A = \sin^2 A + \sin^2 A = 1$

48.(c) $\cos^4\theta - \sin^4\theta = \frac{2}{3}$

$\Rightarrow (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \frac{2}{3}$

$\Rightarrow \cos^2\theta - \sin^2\theta = \frac{2}{3}$

$\Rightarrow \cos^2\theta - (1 - \cos^2\theta) = \frac{2}{3}$

$\Rightarrow 2\cos^2\theta - 1 = \frac{2}{3}$

49.(b) $\cos^2\theta + \cos^4\theta = 1$

$\Rightarrow \cos^4\theta = 1 - \cos^2\theta = \sin^2\theta$

$\therefore \sin^2\theta + \sin\theta = \cos^4\theta + \cos^2\theta$

$= 1$

50.(c) $2.\cos\theta - 1.\sin\theta = \frac{1}{\sqrt{2}}$ ....(i)

$1.\cos\theta + 2.\sin\theta = k$ (let) ......(ii)

(ii) and (i) are of the form $ax + by = m$ & $bx - ay = n$ respectively.

then - $(a^2 + b^2)(x^2 + y^2) = m^2 + n^2$

$\Rightarrow (2^2 + 1^2)(\sin^2\theta + \cos^2\theta)$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + k^2$

$\Rightarrow 5 \times 1 = \frac{1}{2} + k^2$

$\Rightarrow k^2 = 5 - \frac{1}{2} = \frac{9}{2}$ or $= k = \frac{3}{\sqrt{2}}$

51.(c) $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 3$

$\Rightarrow \sin\theta + \cos\theta = 3\sin\theta - 3\cos\theta$

$\Rightarrow 4\cos\theta = 2\sin\theta \Rightarrow \tan\theta = 2$

$\therefore \sin^4\theta - \cos^4\theta$

$= (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$

$= \sin^2\theta - \cos^2\theta$

$= \cos^2\theta\,(\tan^2\theta - 1) = \frac{\tan^2\theta - 1}{\sec^2\theta}$

$= \frac{\tan^2\theta - 1}{1 + \tan^2\theta} = \frac{4-1}{1+4} = \frac{3}{5}$

52.(a) $\sin\theta - \cos\theta = \frac{7}{13}$ ....(i)

$\sin\theta + \cos\theta = x$ ....(ii)

On squaring both equations and adding,

$2(\sin^2\theta + \cos^2\theta) = \frac{49}{169} + x^2$

$\Rightarrow x^2 = 2 - \frac{49}{169} = \frac{338-49}{169} = \frac{289}{169}$

$\Rightarrow x = \frac{17}{13}$

53.(c) $\sec\theta = \tan\theta = 2$

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow (\sec\theta - \tan\theta) = \frac{1}{2}$

By equations (i) & (ii)

$\therefore \sec\theta + \tan\theta + \sec\theta - \tan\theta$

$= 2 + \frac{1}{2} = \frac{5}{2}$

$\Rightarrow 2\sec\theta = \frac{5}{2} \Rightarrow \sec\theta = \frac{5}{4}$

54.(c) take $\theta = 45°$

$\therefore \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$ & $m = \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\therefore l^2m^2(l^2 + m^2 + 3) = \frac{1}{2} \times \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2} + 3\right)$

$= \frac{1}{4} \times 4 = 1$

Hints and Solutions :

LEVEL - III

1.(c) $\tan^2\theta = 2\tan^2\phi + 1$ (given)

$\therefore 1 - \tan^2\phi = -2\tan^2\phi$ ....(i)

Now, $\cos 2\phi - 1 = \dfrac{1-\tan^2\phi}{1+\tan^2\phi} - 1 = \dfrac{-2\tan^2\phi}{1+\tan^2\phi}$

$= \dfrac{1-\tan^2\phi}{1+\dfrac{\tan^2\theta - 1}{2}}$ [From (i)]

$= \dfrac{1-\tan^2\theta}{1+\tan^2\theta} \times 2 = 2\ \cos\theta$

Thus, $\cos 2\theta = \dfrac{1}{2}[\cos 2\phi - 1]$

2.(a) Given Exp.

$= \dfrac{\cot A + (\text{cosec } A - 1)}{\cot A - (\text{cosec } A - 1)} \times \dfrac{\cot A + (\text{cosec } A - 1)}{\cot A + (\text{cosec } A - 1)}$

$= \dfrac{\cot^2 A + (\text{cosec} A - 1)^2 + 2\cot A(\text{cosec} A - 1)}{\cot^2 A - (\text{cosec} A - 1)^2}$

$= \dfrac{(\text{cosec}^2 A - 1) + (\text{cosec} A - 1)^2 + (\text{cosec} A - 1)}{(\text{cosec}^2 A - 1) - (\text{cosec} A - 1)^2}$

$= \dfrac{2(\text{cosec}^2 A - 1)(\text{cosecec } A + \cot A)}{2(\text{cosec } A - 1)}$

$= \text{cosec } A + \cot A$

Alternatively,

take $\theta = 30°$

$\dfrac{\cot A + \text{cosece } A - 1}{\cot A - \text{cosec } A + 1} = \dfrac{\sqrt{3} + 2 - 1}{\sqrt{3} - 2 + 1} = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \dfrac{(\sqrt{3}+1)^2}{2} = \dfrac{4+2\sqrt{3}}{2} = 2 + \sqrt{3}$ ....(i)

and option (a) $= \text{cosec } A + \cot A$

$= \text{cosec } 30° + \cot 30° = 2 + \sqrt{3}$ ....(ii)

i.e. (i) = (ii)

hence option (a) is correct.

Note : at $\theta = 45°$, option (a) & (c) contradicts

$\therefore$ do'nt take $\theta = 45°$

3.(b) $\dfrac{P^2+1}{P^2+1} = \dfrac{(\sec\theta + \tan\theta)^2 - 1}{(\sec\theta + \tan\theta)^2 + 1}$

$= \dfrac{\sec^2\theta + \tan^2\theta + 2\sec\theta\sec\theta. - 1}{\sec^2\theta + \tan^2\theta + 2\sec\theta.\tan\theta + 1}$

$= \dfrac{(\sec^2 - 1) + \tan^2\theta + 2\sec\theta\sec\theta.}{\sec^2\theta + 2\sec\theta\sec\theta. + (1 + \tan^2\theta)}$

$= \dfrac{\tan^2\theta + \tan^2\theta + 2\sec\theta.\tan\theta}{\sec^2\theta + 2\sec\theta s\tan\theta + \sec^2\theta}$

$= \dfrac{2\tan^2\theta + 2\tan\theta\tan\theta.}{2\sec^2\theta + 2\tan\theta\tan\theta.}$

$= \dfrac{2\tan\theta(\tan\theta + \sec\theta)}{2\sec\theta + (\tan\theta + \sec\theta)}$

$= \dfrac{\tan\theta}{\sec\theta} = \dfrac{\sin\theta}{\cos\theta os\theta.s} = \sin\theta$

Alternatively ,

Let $\theta = 60°$

then, $P = (2+\sqrt{3})$

$\therefore \dfrac{P^2-1}{P^2+1} = \dfrac{(2+\sqrt{3})^2 - 1}{(2+\sqrt{3})^2 + 1} = \dfrac{6+4\sqrt{3}}{8+4\sqrt{3}} = \dfrac{3+2\sqrt{3}}{4+2\sqrt{3}}$

$= \dfrac{\sqrt{3}}{2}\dfrac{(2+\sqrt{3})}{(2+\sqrt{3})}$

$= \dfrac{\sqrt{3}}{2} = \sin 60°$ i.e. $\sin\theta$

4.(d) Given Exp :

$2\sec^2\theta - (\sec^2\theta) - 2\text{cosec}^2\theta + (\text{cosec}^2\theta)^2$

$= 2(1+\tan^2\theta) - (1+\tan^2\theta)^2 - 2(1+\cot^2\theta) + (1+\cot^2\theta)^2$

$= 2 + 2\tan^2\theta - 1 - \tan^4\theta - 2\tan^2\theta - 2 - 2\cot^2\theta + 1 + \cot^4\theta - \tan^4\theta$

Alternatively ; let $\theta = 60°$

$\therefore$ Given Exp ;

$$2\times 4 - 16 - 2\times\frac{4}{3} + \frac{16}{9}$$

$$= -8 - \frac{8}{3} + \frac{16}{3} = \frac{-72-24+16}{9} = -\frac{80}{6}$$

and $\cot^4\theta - \tan^4\theta = \frac{1}{9} - 9 = -\frac{80}{9}$

hence, (d) is correct option.

Note : at 45°, option (a), (c) and (d) contradicts.

5.(b) Let $\theta = 0°$, then

$m = a$ and $n = 0$

$(m+n)^{2/3} + (m-n)^{2/3} = a^{2/3} + a^{2/3}$

$= 2a^{2/3}$

Alternatively ;

$m + n = a\cos^3\theta + 3a\cos\theta.\sin^2\theta + a\sin^3\theta + 3a\sin\theta.\cos^2\theta$

$= a(\cos\theta + \sin\theta)^3$

Similarly,

$m - n = a(\cos\theta - \sin\theta)^3$

$\therefore \cos\theta + \sin\theta = \left(\frac{m+n}{a}\right)^{1/3}$ and

$$\cos\theta - \sin\theta = \left(\frac{m-n}{a}\right)^{1/3}$$

$$\therefore \left(\frac{m+n}{a}\right)^{2/3} + \left(\frac{m-n}{a}\right)^{2/3}$$

$$= (\cos\theta + \sin\theta)^2 (\cos\theta - \sin\theta)^2$$

$= 2(\cos^2\theta + \sin^2\theta) = 2$

$\Rightarrow (m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$

6.(a) $\frac{\text{Sin A}}{\text{Sin B}} = p$ and $\frac{\cos A}{\cos B} = q$

$$\Rightarrow \frac{\sin A}{\sin B}.\frac{\cos B}{\cos A} = \frac{p}{q} \Rightarrow \frac{\tan A}{\tan B} = \frac{p}{q}$$

$$\Rightarrow \frac{\tan A}{p} = \frac{\tan B}{q} = K$$

$\therefore \tan A = kp$ and $\tan B = kq$

Now, $\sin A = p \sin B$

$$\Rightarrow \frac{\tan A}{\sqrt{1+\tan^2 A}} = p\frac{\tan B}{\sqrt{1+\tan^2 B}}$$

$$\Rightarrow \frac{pk}{\sqrt{1+p^2k^2}} = p\frac{kq}{\sqrt{1+q^2k^2}}$$

$\Rightarrow p^2(1+q^2k^2) = p^2q^2(1+p^2k^2)$

$\Rightarrow k^2(q^2 - p^2q^2) = q^2 - 1$

$$\Rightarrow k = \pm\frac{1}{q}\sqrt{\frac{q^2-1}{1-p^2}}$$

$$\therefore \tan A = \pm\frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}}$$

Alternatively :

Let $A = 30°$, $B = 45°$, then

$$p = \frac{1}{\sqrt{2}} \text{ and } q = \frac{\sqrt{3}}{\sqrt{2}} \text{ and } \tan A = \frac{1}{\sqrt{3}}$$

$$\text{Now, } \frac{p}{q}\sqrt{\frac{q^2-1}{1-p^2}} = \frac{1/\sqrt{2}}{\sqrt{3}/\sqrt{2}}\sqrt{\frac{1/2}{1/2}} = \frac{1}{\sqrt{3}} = \tan A$$

hence, option (a) is correct

7.(c) Given Exp.

$= \sqrt{2\cot\alpha + \text{cosec}\,\alpha}$

$= \sqrt{2\cot\alpha + 1 + \cot^2\alpha}$

$= |1 + \cot\alpha|$

$= -1 - \cot\alpha$

$= \sqrt{2\cot\alpha + \text{cosec}\,\alpha}$

$= \sqrt{2\cot\alpha + 1 + \cot^2\alpha}$

$= |1 + \cot\alpha|$

$= -1 - \cot\alpha$

$= \left[\begin{array}{l} \text{since } \cot\alpha < -x \text{ when } \frac{3\pi}{4} < \alpha < \pi, \\ \therefore |1 + \cot\alpha| = -1 - \cot\alpha \end{array}\right]$

8.(b) Let $\operatorname{cosec}\theta + \cot\theta = k$

$\Rightarrow \operatorname{cosec}\theta - \cot\theta = \frac{1}{k}$

On adding, we get

$2\operatorname{cosec}\theta = k + \frac{1}{k} \Rightarrow 2\left(x + \frac{1}{x}\right) = k + \frac{1}{k}$

$\Rightarrow 2x + \frac{1}{2x} = k + \frac{1}{k} \Rightarrow k = 2x$

9.(a) Let $\theta = 45°$, then

$\tan^2 45° = 1 - e^2 \Rightarrow e = 0$

& $\sec\theta + \tan^3\theta.\operatorname{cosec}\theta$

$= \sqrt{2} + 1 \times \sqrt{2}$ ......(i)

& $(2 - e^2)^{3/2} = (2 - 0)^{3/2} = 2\sqrt{2}$ ....(ii)

i.e. $\sec\theta + \tan\theta.\operatorname{cosec}\theta = (2 - e^2)^{3/2}$

Alternatively :

we have, $\sec\theta + \tan^3\theta.\operatorname{cosec}\theta$

$= \sec\theta\ (1 + \tan^3\theta \frac{\operatorname{cosec}\theta}{\sec\theta})$

$= \sec\theta\ (1 + \tan^2\theta)$

$= \sec\theta.\sec^2\theta$

$= \sec^3\theta = (\sec^2\theta)^{3/2} = (1 + \tan^2\theta)^{3/2}$

$= (1 + 1 - e^2)^{3/2} \quad \because \tan^2\theta = 1 - e^2$

$= (2 - e^2)^{3/2}$

10.(b) Let $\theta = 45°$, then

$m = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$ and $n = 2\sqrt{2}$

$\therefore 2m = n(m^2 - 1) = 2\sqrt{2}(2 - 1) = 2\sqrt{2}$

$\Rightarrow 2m = n(m^2 - 1)$

Alternatively ;

We have, $\sin\theta + \cos\theta = m$ ....(i)

and $\sec\theta + \operatorname{cosec}\theta = n$

$\Rightarrow \frac{1}{\cos\theta} + \frac{1}{\sin\theta} = n \Rightarrow \frac{\sin\theta + \cos\theta}{\sin\theta.\cos\theta} = n$

$\Rightarrow \frac{m}{\sin\theta.\cos\theta} = n$ [From (i)]

$\Rightarrow \sin\theta.\cos\theta = \frac{m}{n}$ ....(ii)

squaring (i), we get,

$\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta = m^2$

$\Rightarrow 2m = n(m^2 - 1)$

11.(c)

$\frac{T_3 - T_5}{T_1} = \frac{\sin^3\theta + \cos^3\theta - (\sin^5\theta + \cos^5\theta)}{\sin\theta + \cos\theta}$

$= \frac{(\sin^3\theta - \sin^5\theta) + (\sin^2\theta - \cos^5\theta)}{\sin\theta + \cos\theta}$

$= \frac{\sin^3\theta(1 - \sin^2\theta) + \cos^3\theta(1 - \cos^2\theta)}{\sin\theta + \cos\theta}$

$= \frac{\sin^3\theta.\cos^2\theta + \cos^3\theta.\sin^2\theta}{\sin\theta + \cos\theta}$

$= \frac{\sin^2\theta.\cos^2\theta(\sin\theta + \cos\theta)}{(\sin\theta + \cos\theta)}$

$= \sin^2\theta.\cos^2\theta$

Alternatively ;

Let $\theta = 45°$, then

$T_n = \left(\frac{1}{\sqrt{2}}\right)^n + \left(\frac{1}{\sqrt{2}}\right)^n = 2\left(\frac{1}{\sqrt{2}}\right)^n = 2^{\left(\frac{2-n}{2}\right)}$

$\therefore \frac{T_3 - T_5}{T_1} = \frac{2^{-1/2} - 2^{-3/2}}{2^{1/2}} = \frac{\frac{1}{\sqrt{2}} - \frac{1}{2\sqrt{2}}}{\sqrt{2}}$

$$= \frac{\frac{1}{2\sqrt{2}}}{\sqrt{2}} = \frac{1}{4}$$

i.e. $\frac{T_3 - T_5}{T_1} = \sin^2\theta.\cos^2\theta$

12.(b) $10\sin^4\alpha + 15\cos^4\alpha = 6 = 6(\sin^2\alpha + \cos^2\alpha)^2$

$\Rightarrow 10\sin^4\alpha + 15 = 6(\tan^2\alpha + 1)^2$

[Dividing by $\cos^4\alpha$]

$\Rightarrow (2\tan^2\alpha - 3)^2 = 0$

$\Rightarrow 2\tan^2\alpha - 3 = 0$

$\Rightarrow \tan^2\alpha = \frac{3}{2}$

$\therefore 27\operatorname{cosec}^6\alpha + 8\sec^6\alpha$

$= 27(1 + \cot^2\alpha)^3 + 8(1 + \tan^2\alpha)^3$

$= 27\left(1 + \frac{2}{3}\right)^3 + 8\left(1 + \frac{3}{2}\right)^3$

$= 27 \times \frac{125}{27} + 8 \times \frac{125}{8}$

$= 250$

13.(c) Let $\theta = 45°$, then

$= 4m \; 1 \times \left(1 + \frac{1}{\sqrt{2}}\right) \Rightarrow m$

$= \frac{\sqrt{2}+1}{4\sqrt{2}}$ and $n = \frac{\sqrt{2}-1}{4\sqrt{2}}$

$\therefore m^2 - n^2 = \frac{1}{32}\left[\left(\sqrt{2}+1\right)^2 - \left(\sqrt{2}-1\right)^2\right]$

$= \left[\frac{1}{32}\left(4\sqrt{2}\right)\right]$

$[\because (a+b)^2 - (a-b)^2 = 4ab]$

$\Rightarrow m^2 - n^2 = \frac{1}{4\sqrt{2}}$

$\therefore \left(m^2 - n^2\right) = \frac{1}{32}$

now go through options.

option (c) $= mn = \frac{\sqrt{2}+1}{4\sqrt{2}} \cdot \frac{\sqrt{2}-1}{4\sqrt{2}} = \frac{1}{32}$

hence, $\left(m^2 - n^2\right)^2 = mn.$

14.(d) We have to eliminate angle B because all options ar in terms of angle A, Now,

$\tan A = n\tan B \Rightarrow \tan B \Rightarrow \frac{1}{n}\tan A$

$\Rightarrow \cot B = \frac{n}{\tan A}$

and $\sin A = m\sin B$

$\Rightarrow \sin B = \frac{1}{m}\sin A$

$\Rightarrow \operatorname{cosec} B = \frac{m}{\sin A}$

Substituting the values of cot B in $\operatorname{cosec}^2 B - \cot^2 B = 1$, we get.

$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$

$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow m^2 - n^2\cos^2 A = \sin^2 A$

$\Rightarrow m^2 - n^2\cos^2 A = 1 - \cos^2 A$

$\Rightarrow (m^2 - 1) = (n^2 - 1)\cos^2 A$

$\Rightarrow \frac{m^2 - 1}{n^2 - 1} = \cos^2 A$