UPSEE 2014 - Solved Paper

Physics

Question 1

A solid cylinder of mass M and radius R rolls without slipping down on inclined plane of length L and height h. What is the speed of its centre of mass, when the cylinder reaches its bottom?

Options:

A. $\sqrt{\frac{4}{3}gh}$

B. $\sqrt{4gh}$

C. $\sqrt{2gh}$

D. $\sqrt{\frac{3}{4}gh}$

Answer: A

Solution:

Acceleration of a rolling body down on an inclined plane is

$$a = \frac{g\sin\theta}{1+\frac{k^2}{R^2}}$$

where k is the radius of gyration, R is the radius of the cylinder.
For a solid cylinder

$$I = MK^2 = \frac{MR^2}{2}$$

$$\therefore \frac{K^2}{R^2} = \frac{1}{2}$$

$$\therefore a = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta$$

Also $v^2 = U_0^2 + 2as$

$$= (0)^2 + 2\times\frac{2}{3}g\sin\theta\times L$$

$$= \frac{4}{3}g\times\frac{h}{L}\times L = \frac{4}{3}gh$$

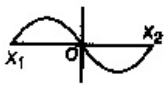
$$v = \sqrt{\frac{4}{3}gh}$$

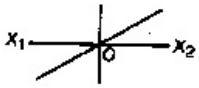
Question 2

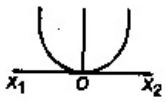
A particle of mass m oscillates with simple harmonic motion between two points x_1 and x_2, the equilibrium position being O. its potential

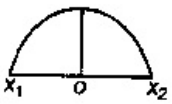
energy is potted on the graph. Which of the following curve represents the phenomenon?

Options:

A. 

B. 

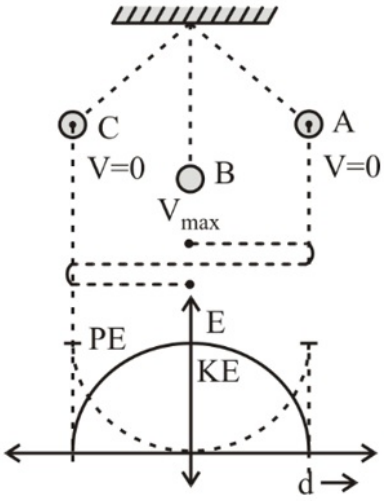
C. 

D. 

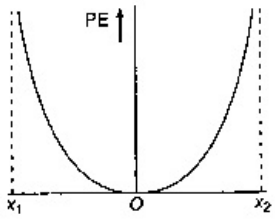
Answer: C

Solution:

Potential energy of a particle vibrating in SHM is given by PE = $\frac{1}{2}m\omega^2x^2$



So the curve of potential energy is



Further PE is zero at x =0 and maximum at x = 0

Question 3

A double slit experiments is performed with light of wavelength 500 nm. A thin film of thickness 2 μm and refractive index 1.5 is introduced in the path of the supper beam. The location of the central maximum will

Options:

A. remain unshifted

B. shift downward by nearly two fringes

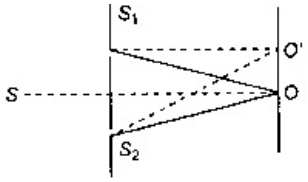
C. shift upward by nearly two fringes

D. shift downward by its fringes

Answer: C

Solution:

In young's Double slit Experiment (YDSE) be the position of central bright fringe on the screen in absence of any film, because the $S_1O=S_2O$.



When we introduce a film of thickness t and refractive index μ, an additional path difference equal to $(\mu t - t) = (\mu - 1)t$ is introduced. The optical path of upper beam becomes longer. For path difference on screen to be zero, path from lower slit S_2 should also be more. Thus, central bright fringe wire be located some what at $O' as S_2O' > S_2O.$
The fringe pattern shifts upwards.
Now as a change in path difference of λ corresponds to a change in position on the screen by

$$\beta = \frac{D}{d}\lambda$$

Change in optical path difference

$$\Delta y = (\mu - 1) \times \frac{D}{d}$$

$$\Delta y = (1.5 - 1) \times (2 \times 10^{-6}) \frac{D}{d}$$

$$\Delta y = \frac{1}{2} \times 2 \times 10^{-6} \frac{D}{d} = \frac{D}{d} \times 10^{-6} m$$

Fringe width

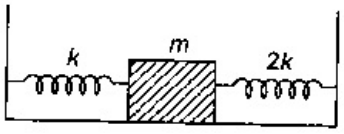
$$\frac{\Delta y}{\beta} = \frac{d}{\frac{D}{d} \times 500 \times 10^{-9}} = 2$$

$$= \frac{10^{-6}}{500 \times 10^{-9}} = \frac{10^{-6} \times 10^{9}}{500} = \frac{10^{3}}{500}$$

$$\therefore \Delta y = 2\beta$$

Question 4

Two springs of force constant k and 2k are connected to a mass shown in figure. The frequency of oscillation of the mass is



Options:

A. $-\frac{1}{2\pi}\sqrt{\frac{m}{k}}$

B. $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

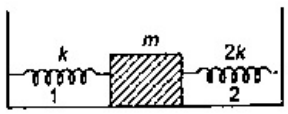
C. $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

D. $\frac{1}{2\pi}\sqrt{\frac{m}{k}}$

Answer: C

Solution:

Let the mass m be displaced toward right by a distance x. Then spring 1 will be extended and the spring 2 will be compressed by an amount x. On releasing mass m, the restoring force on m due to spring 1 is F_1=-kx and restoring force due to spring 2 is = F_2=-2kx



Total restoring force on mass m is

$f = -kx - 2kx = -3kx$

$f = \frac{md^2x}{dt^2} = -3kx$

$\frac{d^2x}{dt^2} = -\frac{3k}{m}x$

$\frac{d^2x}{dt^2} + \frac{3k}{m} \times 0$

This is similar to $\frac{d^2x}{dt^2} + \omega^2 x = 0$

Which represent equation of SHM. Comparing the two equations,

$\omega^2 = \frac{3k}{m}$

$\omega = \sqrt{\frac{3k}{m}}$

$2\pi f = \sqrt{\frac{3k}{m}}$

$f = \frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

Question 5

A diatomic molecule is formed by two atoms which may be treated as mass points m_1 and m_2 joined by a massless rod of length r. The, the moments of inertia of the molecule about an axis passing through the centre of mass and perpendicular to rod in

Options:

A. zero

B. $(m_1 + m_2)r^2$

C. $\frac{(m_1 + m_2)}{(m_1 m_2)}r^2$

D. $\left(\frac{m_1 m_2}{m_1 + m_2}\right)r^2$

Answer: D

Solution:

Let point C be the centre of mass situated at distance x from atom of mass $m_1$and at distance (r-x) from mass m_2. By definition of centre of mass, we have

$$m_1 x = m_2(r - x)$$

$$x = \frac{m_2}{(m_1 + m_2)}$$

Moment of inertia about the centre of mass is

$$I = m_1 x^2 + m_2(r - x)^2$$

$$\text{Or } I = m_1\left(\frac{m_2 r}{m_1 + m_2}\right) + m_2\left(r - \frac{m_2 r}{m_1 + m_2}\right)^2$$

$$I = m_1\left(\frac{m_2 r}{m_1 + m_2}\right)^2 + m_2\left(\frac{m_1 r}{m_1 + m_2}\right)^2$$

$$I = \frac{m_1 m_2}{m_1 + m_2}r^2$$

Question 6

A particle moves so that its acceleration a is given by a =- bn, where x is displacement from the equilibrium position and b is non -negative real constant. The time period of oscillation of the particle is

Options:

A. $2\pi\sqrt{b}$

B. $\frac{2\pi}{b}$

C. $\frac{2\pi}{\sqrt{b}}$

D. $2\sqrt{\frac{\pi}{b}}$

Answer: C

Solution:

Given that the acceleration is, a = -bn ……(1)
For simple harmonic motion

$a = -\omega^2 x$

On comparing with equation (1)

$\omega^2 = b$

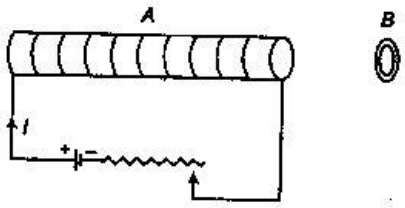
$\omega = \sqrt{b}$

$\frac{2\pi}{t} = \sqrt{b}$

Or $t = \frac{2\pi}{\sqrt{b}}$

Question 7

An aluminium ring B faces an electromagnet A. The current I through A can be altered,



Options:

A. if I increase, A will repel B

B. if I increase, A will attract B

C. if I increase, A will repelB

D. whether I increase or decreases, B will not experience any force

Answer: A

Solution:

If the current is passed through solenoid A as shown. Current entering at end A is clockwise. When seen from the side of A, so it develops south polarity at A and North polarity at B. When current l through A increase, it is like north of the magnet approaching the ring B and due to Lenz's law face of ring B facing A develops north polarity and hence there is repulsion between the two.
Hence as current I increase, A will repel B.

Question 8

In Thomson experiment of finding $\frac{e}{m}$ for electrons, beam of electrons is replaced by that of muons) particle with same charge as of electrons but mass 208 times that of electrons). No deflection condition in this case satisfied, if

Options:

A. B is increased 208 times

B. E is increased 208 times

C. B is increased 14.4 times

D. None of the above

Answer: C

Solution:

Electric potential energy is, $qV = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2qV}{m}}$$

$$qvB = qE \Rightarrow B = \frac{E}{v}$$

$$B = \frac{E}{\sqrt{\frac{2qv}{m}}} = \frac{E\sqrt{m}}{\sqrt{2qv}}$$

$$B_1 = \frac{E\sqrt{m_e}}{\sqrt{2eV}} \text{ and } B_2 = \frac{E\sqrt{m_\mu}}{\sqrt{2eV}}$$

$$\frac{B_2}{B_1} = \frac{\sqrt{m_\mu}}{\sqrt{m_e}}$$

$$B_1 = \sqrt{208} = 14.4$$

$$B_2 = 14.4\ B_1$$

Question 9

If a ladder weighing 250 N is placed against a smooth vertical wall having coefficient of friction between it and floor is 0.3, then what is the maximum force of friction available at the point of contact between the ladder and floor?

Options:

A. 75 N

B. 50 N

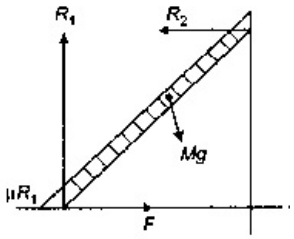
C. 35 N

D. 25 N

Answer: A

Solution:

Given that the weight of the ladder is Mg = 250 N
Reaction force on the floor in the vertical direction is



Friction force at contact point on the floor = μR_1
f = $0.3 \times 250N = 75N$

Question 10

Which of the following statements is incorrect?

Options:

A. All reversible cycles have same efficiency

B. Reversible cycle has more efficiency than an irreversible one

C. Carnot cycle is a reversible one

D. Carnot cycle has the maximum efficiency in all cycles

Answer: A

Solution:

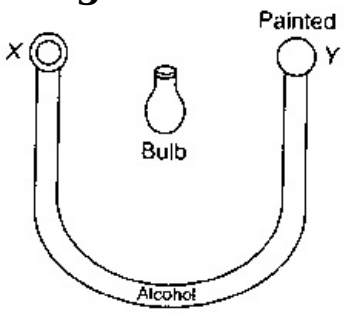
Reversible Process: A process that can be made to reverse its direction of flow by changing the physical conditions and it is considered that there is no dissipation if energy. ex. Carnot cycle.

Irreversible Process: A process that can't be made to reverse its direction of flow by changing the physical conditions known as an irreversible process.

The reversible cycle has more efficiency than an irreversible cycle. All reversible cycles have different efficiency.

Question 11

The following figure shows two air filled bulbs connected by a U-tube partly filled with alcohol. What happens to the levels of alcohol in the limbs X and Y when an electric bulb is placed midway between the bulb is lighted?



Bulb X is Black Painted and Bulb Y is white Painted

Options:

A. The level of alcohol in limb X falls while that in limb Y rises

B. The level of alcohol in limb X rises, while that the limb Y falls

C. The level of alcohol falls in both limbs

D. There is no change in the levels of alcohol in the two limbs

Answer: A

Solution:

Black bulb absorbs more heat in comparison with painted bulb. So air in black bulb expands more. Hence, the level of alcohol in limb X falls while that in limb y rises.

Question 12

The bob of a pendulum of length L is pulled a side from its equilibrium position through an angle θ and then released. The bob will, then pass through its equilibrium position with a speed v, where v equals to

Options:

A. $\sqrt{2gl(1-\sin\theta)}$

B. $\sqrt{2gl(1+\cos\theta)}$

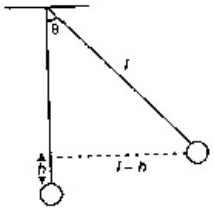
C. $\sqrt{2gl(1-\cos\theta)}$

D. $\sqrt{2gl(1+\sin\theta)}$

Answer: C

Solution:

Suppose bob rises up a height h as shown then after releasing, potential energy at extreme position becomes kinetic energy of mean position.



$mgh = \frac{1}{2}mv^2_{max}$

$V_{max} = \sqrt{2gh}$ .....(i)

From the figure,

$\cos\theta = \frac{l-h}{l}$

$\Rightarrow l\cos\theta = l-h$

$h = l(1-\cos\theta)$

From Eq. (i), $V_{max} = \sqrt{2gl(1-\cos\theta)}$

Question 13

A particle moving along the x-axis has acceleration f, at time t, given by $f=f_0\left(1-\frac{t}{T}\right)$ where f_0 and T are constant. The particle at t = 0 has zero velocity. In the time interval between t = 0 and the instant, when f = 0, the particle velocity v_x is

Options:

A. f_0T

B. $\frac{1}{2}f_0T^2$

C. $f_0T/5$

D. $\frac{1}{2}f_0T$

Answer: D

Solution:

In this problem, acceleration is variable so acceleration is

$f = \frac{dv}{dt}$

$= f_0\left(1-\frac{t}{T}\right)$

At, t = 0 , f = f_o

At t = t, f = 0

We have to calculate the velocity of the particle in the time from t=0

to t = 1 s

$$\frac{dv}{dt} = f_0\left(1 - \frac{t}{T}\right)$$

Integrating both sides

$$\int dv = \int_0^T f_0\left(1 - \frac{t}{T}\right) dt$$

$$v = f_0\left[t - \frac{t^2}{2T}\right]_0^T dt$$

$$v = f_0\left[T - \frac{T^2}{2T}\right]$$

$$\Rightarrow v = f_0\left[T - \frac{T}{2}\right]$$

$$v = f_0\frac{T}{2}$$

Question 14

A spherical condenser has inner and outer spheres of radii a and b respectively. The space between the two is filled with air. The difference between the capacities of two condensers formed when the outer sphere is earthed and when the inner sphere is earthed will be

Options:

A. Zero

B. $4\pi\varepsilon_0 a$

C. $4\pi\varepsilon_0 b$

D. $4\pi\varepsilon_0 b\left(\frac{b}{b-a}\right)$

Answer: C

Solution:

Given that the inner radius of the sphere is a and outer radius of the sphere is b.

The capacity of condenser when the outer sphere is earthed

$$C_1 = 4\pi\varepsilon_0\frac{ab}{(b-a)}$$

Capacity when the inner sphere is earthed

$$C_2 = 4\pi\varepsilon_0 b + \frac{4\pi\varepsilon_0 ab}{b-a} = 4\pi\varepsilon_0\left(\frac{b^2}{b-a}\right)$$

$$\text{Difference in capacity} = C_2 - C_1 = 4\pi\varepsilon_0 b$$

Question 15

The resistance of a wire at room temperature 30^0C is found to be 10 Ω. Now to increases the resistance by 10%, the temperature of the wire must be [The temperature coefficient of resistance of the material of the wire is 0.002 per ^{0}C].

Options:

A. 36^0C

B. 83^0C

C. 63^0C

D. 33^0C

Answer: B

Solution:

As we know that the resistance of wire at temperature t is $R_t=R_0(1+\alpha t)$

Initially the resistance of the wire is

$$R_0(1+30\alpha)=10\Omega$$

$$\text{Finally, } R_0(1+\alpha t)=11\Omega$$

$$\frac{11}{10}=\frac{1+\alpha t}{1+30\alpha}$$

$$\Rightarrow 10+(10\times 0.002\times t)=11+330\times 0.002$$

$$\Rightarrow 0.02t=1+0.66=1.66$$

$$\Rightarrow t=\frac{1.66}{0.02}=83^{\circ}C$$

Question 16

Pick out the wrong statement.

Options:

A. In a simple battery circuit, the point of lowest potential is the negative terminal of the battery

B. The resistance of an incandescent lamp is greatest when the lamp is switched off

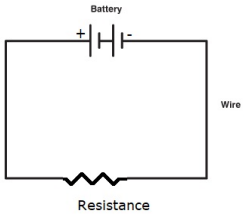
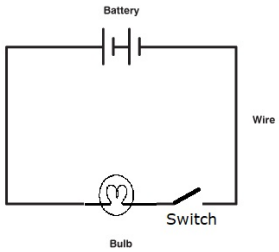
C. An ordinary 100 W lamp has less resistance than a 60 W lamp

D. At constant voltage, the heat development in a uniform wire varies inversely as the length of the wire used

Answer: B

Solution:

S.No.	Option	Circuit Diagram	Explanation
			The +ve terminal refers to the higher

1.	In a simple battery circuit, the point of lowest potential is the negative terminal of the battery		potential and -ve terminal refers to the lower potential of the battery. And the -ve terminal of the battery is the lowest potential.
2.	The resistance of an incandescent lamp is greatest when the lamp is switched off		The resistance of an incandescent lamp is increased when the temperature increases. So as the temperature increases resistance of filament also increases.
3.	An ordinary 100 W lamp has less resistance than a 60 W lamp		As we know that the power is, $P = \frac{V^2}{R}$ $P \propto \frac{1}{R}$ So if the power of the lamp is higher its resistance will be low.
4.	At constant voltage, the heat development in a uniform wire varies inversely as the length of the wire used		Heat developed in a wire is, $H = i^2Rt$ As we know that the resistivity is $\rho = \frac{RA}{l}$ $R = \frac{\rho l}{A}$ At constant voltage heat developed in wire is $H = Vit$ $H = Pt$ $H = \frac{V^2t}{R}$ $H = \frac{V^2tA}{\rho l}$ $H \propto \frac{tA}{l}$

Question 17

The current in the winding on a toroid is 2.0A. There are 400 turns and the mean circumferential length is 40 cm. If the inside magnetic field is 1.0 T, the relative permeability is near to

Options:

A. 1000

B. 2000

C. 2500

D. 4000

Answer: C

Solution:

The magnetic field inside the toroid is, B = $\frac{\mu_0\mu_r Ni}{2\pi r}$

$$1 = \frac{4\pi \times 10^{-7} \times \mu_r \times 400 \times 2}{2\pi \times 40 \times 10^{-2}}$$

$$\mu_r = \frac{2\pi \times 400 \times 10^{-2}}{4\pi \times 10^{-7} \times 800} = \frac{80 \times 10^{-2}}{4 \times 10^{-7} \times 800}$$

$$= \frac{80 \times 10^5}{4 \times 800} = 2500$$

Question 18

The magnetic moment produced in a substance of 1g is $6 \times 10^{-7} A - m^2$. If its density is 5/gcm^3, then the intensity of magnetization in A/m will be

Options:

A. 8.3×10^{-6}

B. 3

C. 1.2×10^{-7}

D. 3×10^{-6}

Answer: B

Solution:

The intensity of magnetization is, I=$\frac{M}{v} = \frac{M}{\text{mass / density}}$

Given mass = 1 g = 10^{-3}kg

And density =5g/cm^3

$$= \frac{5 \times 10^{-3} kg}{(10^{-2})^3 m^3}$$

$$= 5 \times 10^3 kg / m^3$$

$$I = \frac{6 \times 10^{-7} \times 5 \times 10^3}{10^{-3}} = 3$$

Question 19

The power factor of a yard choke is

Options:

A. nearly zero

B. exactly zero

C. nearly one

D. exactly one

Answer: B

Solution:

Power factor

$\cos\phi = \frac{R}{Z}$

In choke coil, $\Phi = 90^o$

So, $\cos\Phi = 0$

The power factor of a yard choke is zero.

Question 20

When the momentum of a proton is changed by an amount p_0, then the corresponding change in the de-Broglie wavelength is found to be 0.25%. The original momentum of the proton was

Options:

A. p_0

B. 100 p_0

C. 400 p_0

D. 4 p_0

Answer: C

Solution:

We have $\lambda = \frac{h}{p}$

$\lambda \propto \frac{1}{p}$

$\Rightarrow \frac{\Delta p}{p} = \frac{\Delta\lambda}{\lambda}$

$\Rightarrow \left|\frac{\Delta p}{p}\right| = \left|\frac{\Delta\lambda}{\lambda}\right|$

$\frac{p_0}{p} = \frac{0.25}{100}$

$\frac{p_0}{p} = \frac{1}{400}$

$p = 400p_0$

Question 21

A gamma ray photon creates and electron, positron is 0.5 Me V and the total KE of the electron, positron pair is 0.78 MeV, then the energy of the gamma ray photon must be

Options:

A. 0.78 MeV

B. 1.78 MeV

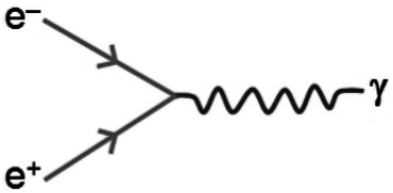
C. 1.28 MeV

D. 0.28 MeV

Answer: B

Solution:

Electron positron annihilation is



$e^- + e^+ \rightarrow \gamma$

So the energy of γ - ray photon is,

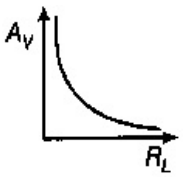
E = (0.5 + 0.5 + 0.78) MeV

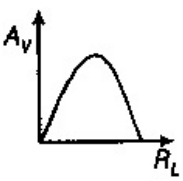
E = 1.78MeV

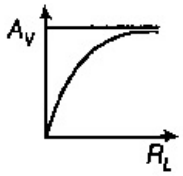
Question 22

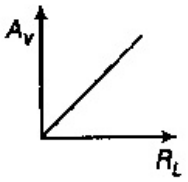
The correct curve between voltage gain (Av) and load resistance (R_L) is

Options:

A. 

B. 

C. 

D. 

Answer: C

Solution:

According to $|A_v| = \frac{\mu}{1 + \frac{r_p}{R_L}}$

As R_L increases A_V also increases, when R_L becomes too high, then A_V= maximum = μ. Hence, only C. is correct.

Question 23

Red light of wavelength 625 nm is incident normally on an optical diffraction grating with 2×10^5 lines/ m. Including central principal maxima, how many maxima may be observed on a screen which is far from the grating?

Options:

A. 15

B. 17

C. 20

D. 18

Answer: B

Solution:

For principle maxima in grating spectra

$\frac{\sin\theta}{N} = n\lambda$

Where n= (1,2,3) is the order of principle maximum and θ is the angle of diffraction.

$$n = \frac{1}{\lambda N} = \frac{1}{6.25 \times 10^{-7} \times 2 \times 10^5} = 8$$
$$\therefore \text{Number of maxima} = 2n + 1 = 2 \times 8 + 1 = 17$$

Question 24

In a series reonant L-C-R circuit, the voltage across R is 100 V and R = 1 kΩ with C = 2μF. The resonant frequency ω is 200 rads^{-1}. At resonance the voltage across L is

Options:

A. $2.5 \times 10^{-2}V$

B. $4 \times 10^{-8}V$

C. 250V

D. $4 \times 10^{-3}V$

Answer: C

Solution:

At resonance, $\omega L = \frac{1}{\omega C}$

Current flowing through the circuit,

$I = \frac{V_R}{R} = \frac{100}{1000}$

$= 0.1A$

So, voltage across L is given by

$V_L = IX_L = I\omega L$

But $\omega L = \frac{1}{\omega C}$

$\therefore V_L = \frac{1}{\omega C}$

$= \frac{0.1}{200 \times 2 \times 10^{-6}}$

$= 250V$

Question 25

The dimensions of self-inductance L are

Options:

A. $[ML^2T^{-2}A^{-2}]$

B. $[ML^2T^{-1}A^{-2}]$

C. $[ML^2T^{-1}A^{-1}]$

D. $[ML^{-2}T^{-2}A^{-2}]$

Answer: A

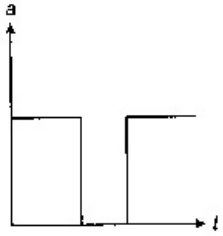
Solution:

Self-inductance L = $\frac{e}{\Delta i / \Delta t} = \frac{e\Delta t}{\Delta i}$

$$\therefore \text{Unit of } L = \frac{\text{Volt} - \text{second}}{\text{Ampere}}$$
$$= \frac{(\text{Joule / Coulomb})\,\text{second}}{\text{Ampere}}$$
$$= \frac{\text{Newton} - \text{metre} - \text{second}}{\text{Coulomb} - \text{ampere}}$$
$$\Rightarrow \text{Unit of } L = \frac{\text{Newton} - \text{metre}}{\text{Ampere}^2}$$
$$\therefore [L] = \frac{[MLT^{-2}][L]}{[A^2]} = [ML^2T^{-2}A^{-2}]$$

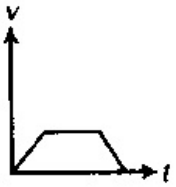
Question 26

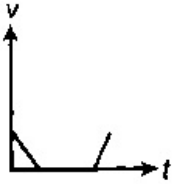
Acceleration – time graph of a body is shown below.

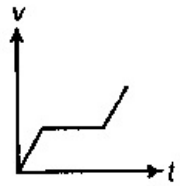


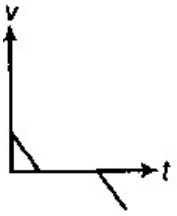
The correspond velocity-time graph of the same body is

Options:

A. 

B. 

C. 

D. 

Answer: C

Solution:

As we know that the acceleration is, $a = \frac{dv}{dt}$
In the given graph, acceleration is constant for first part of motion.

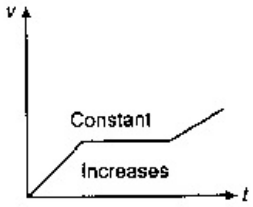
So, velocity increases uniformly.

When, a=0

i.e., $0 = \frac{dv}{dt}$

$\Rightarrow v = constant$

thus, we can draw the velocity-time graph as



Where v is velocity.

Question 27

A can filled with water is revolved in a vertical circle of radius 4 m and the water does not fall down. The time period for a revolution is about

Options:

A. 2s

B. 4s

C. 8s

D. 10s

Answer: B

Solution:

when a body is revolving in circular motion, it is acted upon by a centripetal force directed towards the centre. Water will not fall, if weight is balanced by centripetal force. Therefore,

$$mg = \frac{mv^2}{r}$$

$$v^2 = rg$$

$$v = \sqrt{rg}$$

The circumference of a circle is $2\pi r$.

Time for a revolution = $\frac{2\pi r}{v}$

$$T = \frac{2\pi r}{\sqrt{gv}} = 2r\sqrt{\frac{r}{g}}$$

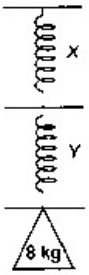
Given, r = 4m,g = 9.8 ms^{-2}

$\therefore T = 2\pi\sqrt{\frac{4}{9.8}}$

$\Rightarrow T = \frac{4\pi}{\sqrt{9.8}} = 4s.$

Question 28

A body of mass 8 kg is suspended through two light springs X and Y connected in series as shown in figure. The readings in X and Y respectively are



Options:

A. zero,8 kg

B. 6 kg, 2 kg

C. 2 kg, 6 kg

D. 8 kg, 8 kg

Answer: D

Solution:

As the springs are light in weight, therefore tension in both springs will be zero, so both spring will show same reading as 8 kg. Reading of X is 8 kg and Reading of Y is also 8 kg.

Question 29

A rocket of mass 1000 kg is to be projected vertically upwards. The gases are exhausted vertically downwards with velocity 100 $ms^{-1}$with respect to the rocket. What is the minimum rate of burning of fuel, so as to just lift the rocket upwards against, the gravitational attraction? (Take g=10ms^{-1})

Options:

A. 50 kgs^{-1}

B. 100 kgs^{-1}

C. 200 kgs^{-1}

D. 400 kgs^{-1}

Answer: B

Solution:

Given, the velocity of exhaust gases with respect to rocket = 100ms^{-1}

The minimum force on the rocket to lift it

F_{min} = mg = 1000×10 = 10000 N

Hence, the minimum rate of burning of fuel is given by

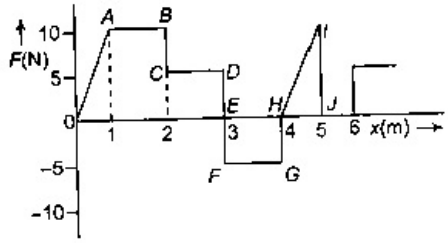
$$\frac{dm}{dt} = \frac{F_{min}}{v}$$

$$\frac{dm}{dt} = \frac{10000}{100}$$

$$\frac{dm}{dt} = 100 \text{ kgs}^{-1}$$

Question 30

The relationship between the force F and position x of a body is shown in figure. The work done in displacing the body from x= 1 m to x= 5 m will be



Options:

A. 30 J

B. 15 J

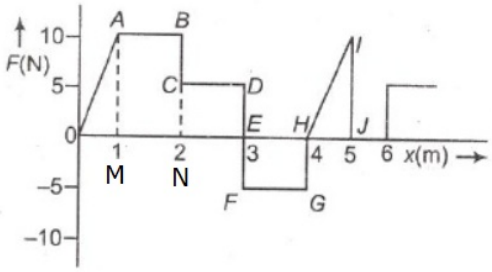
C. 25 J

D. 20 J

Answer: B

Solution:

Work done = area enclosed by F –x graph.



W = Area of ABNM + Area of CDEN – Area of EFGH + Area of HIJ

W = $1\times10+1\times5-1\times5+\frac{1}{2}\times1\times10$

W = 10 + 5 - 5 + 5 = 15 J

Question 31

There identical spheres of mass m each are placed at the corners of an equilateral triangle of side 2 m. Taking one of the corner as the origin, the position vector of the centre of mass is

Options:

A. $\sqrt{3}\left(\hat{i}-\hat{j}\right)$

B. $\frac{\hat{i}}{\sqrt{3}}+\hat{j}$

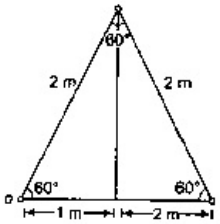
C. $\frac{\hat{i}+\hat{j}}{3}$

D. $\hat{i}+\frac{\hat{j}}{\sqrt{3}}$

Answer: D

Solution:

Given that an equilateral triangle of side 2m.



The X coordinate of center of mass is

$\bar{x}=\frac{\sum m_i x_i}{\sum m_i}=\frac{m\times0+m\times1+m\times2}{m+m+m}=1$

The Y coordinate of the centre of mass is

$$\bar{y} = \frac{\sum m_i y_i}{\sum m_i} = \frac{m \times 0 + m(2\sin 60^0) + m \times 0}{m + m + m}$$

$$\bar{y} = \frac{m\sqrt{3}}{3m} = \frac{1}{\sqrt{3}}$$

Position vector of centre of mass is $\left(\hat{i} + \frac{\hat{j}}{\sqrt{3}}\right)$.

Question 32

A particle of mass m = 5 unit is moving with a uniform speed v = $3\sqrt{2}$ unit in the XOY plane along the line y = x+4. The magnitude of the angular momentum of the particle about the origin is

Options:

A. 60 unit

B. $40\sqrt{2}$ unit

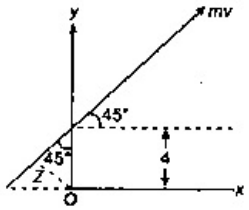
C. zero

D. 7.5 unit

Answer: A

Solution:

Momentum of the particle
p =mass$\times$ velocity
p = (5) $\times (3\sqrt{2}) = 15\sqrt{2}$
The direction of momentum in the XOY plane is given by y=x+4
Slope of the line = 1=tanθ



i.e., $\theta = 45^0$
The intercept of its strength line = 4
Length of the perpendicular z from the origin of the strength line.

$$= 4\sin 45^0 = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Angular momentum,
= momentum $\times$ perpendicular length
$= 15\sqrt{2} \times 2\sqrt{2} = 60$ unit

Question 33

The escape velocity for the earth is v_e. The escape velocity for a planet whose redius is $\frac{1}{4}$th of the radius of the earth and mass half that of the earth is

Options:

A. $\frac{V_e}{\sqrt{2}}$

B. $\sqrt{2}V_e$

C. $2V_e$

D. $\frac{V_e}{2}$

Answer: B

Solution:

Escape velocity of a body from the surface of the earth is given by

$V_e = \sqrt{\frac{2GM_e}{R_e}}$ .....(i)

so according to the question,

$M_p = \frac{M_e}{2}$ and $R_p = \frac{R_e}{4}$

Now, escape velocity of a body from the surface of the planet is given by

$V_p = \sqrt{\frac{2GM_p}{R_p}}$

or $V_p = \sqrt{\frac{2G \times M_e \times 4}{2R_e}} = \sqrt{2}\sqrt{\frac{2GM_e}{R_e}}$

$= \sqrt{2}V_e$ [from Eq.(i)]

Question 34

In case of steel wire or a metal wire, the elastic limit is reached when

Options:

A. the wire just break

B. the load is more than the weight of wire

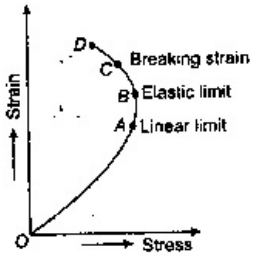
C. elongation is inversely proportional to the tension

D. None of the above

Answer: C

Solution:

According to Hooke’s law, the stress produced in a body is proportional to the strain. But this proportionality exists for small strains only.
Strain - Stress curve for a wire is



When a load is applied to a damped wire, the length of wire goes on increasing part OA of the graph is straight line, indicating that upto point A increase in length is directly proportional to load.

After point A on further increasing the load, the increases in length is no longer proportional to load, however till point B, the elastic property exists in the wire, the stress at point B is called elastic limit.

Beyond B is the wire does not return to its original position, but its length is permanently increased.

Question 35

A wire of length L metre, made of a material of specific gravity 8 is floating horizontally on the surface of water. If it is not wet by water, the maximum diameter of the wire (in millimeter) upto which it can continue to float is (surface tension of water is T = $70 \times 10^{-3} Nm^{-1}$)

Options:

A. 1.5

B. 1.1

C. 0.75

D. 0.55

Answer: B

Solution:

Let L is the length of wire and T is the surface tension of water.

Then

$$TL = V\rho g$$

$$TL = \pi r^2 L\rho g$$

$$T = \pi r^2 \rho g$$

$$70\times10^{-3} = 3.14\times r^2\times 8\times10^{-3}\times 9.8$$

$$r^2 = \frac{70\times10^{-3}}{3.14\times 8\times10^{-3}\times 9.8}$$

$$r^2 = 0.28$$

$$r = 0.53$$

So minimum diameter of the wire is, d = 2r

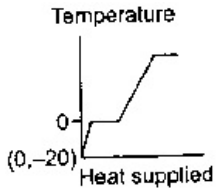
d = 2 × 0.53 m

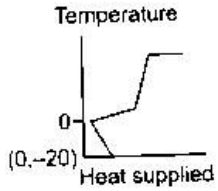
d = 1.1 m

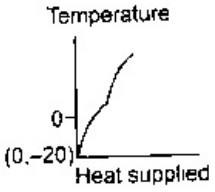
Question 36

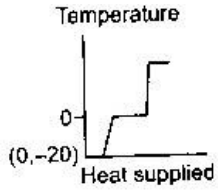
A block of ice at temperature - 20^0C is slowly heated and converted to steam at 100^0C. Which of the following diagram is most appropriate?

Options:

A. 

B. 

C. 

D. 

Answer: A

Solution:

The ice at - 20^0C converts to ice at 0^0C, the supplied heat

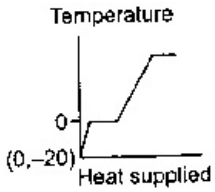
converted to change the temperature of the ice.

At 0^0C ice converts into 0^oC water- the supplied heat converted it into latent heat which changes the phase from ice to water.

Water at 0^oC converts into 100^oC water- the supplied heat converted to change the temperature of the water.

Water at 100^0C converts into steam at 100^0C - the supplied heat converted it into latent heat which changes the phase from water to steam.

So the correct variation temperature and supplied heat is



Question 37

A perfectly black body is one where

Options:

A. absorptive power is infinity

B. absorption point is 0

C. emissive power is 1

D. absorptive power is 1

Answer: D

Solution:

Black body- A black body is an object that is a perfect absorber of electromagnetic radiation. It doesn't reflect light at all.

A perfectly black body neither reflects nor transmit. (i.e.r=0,t=0) any part of the incident radiative energy but absorb whole of it or we can say its absorptive power is 1.

Question 38

A volume gas thermometer works on the principle of

Options:

A. Archimedes principle

B. pascal’s law

C. Boyle’s law

D. Charles’s

Answer: B

Solution:

A constant volume gas thermometer works on Pascal’s law.

Question 39

Two gases, carbon monoxide (CO) and nitrogen (N_2) at the same temperature, have kinetic energies E_1 and E_2respectively.Then,

Options:

A. $E_1 = E_2$

B. $E_1 > E_2$

C. $E_1 < E_2$

D. $E_1$and E_2 cannot be compared

Answer: A

Solution:

The gases carbon monoxide (CO) and nitrogen (N_2) are diatomic, so both have equal kinetic energy

$E_{CO} = E_1 = \frac{5}{2}kT$

$E_{N2} = E_2 = \frac{5}{2}kT$

Question 40

A pendulum has time period T in air when it is made of oscillate in water, it acquired a time period T’ = $\sqrt{2}$T.The density of the pendulum bob is equal to (density of water = 1)

Options:

A. $\sqrt{2}$

B. 2

C. $2\sqrt{2}$

D. None of these

Answer: B

Solution:

The effective acceleration of a bob in water $= g' = g\left[1 - \frac{\sigma}{p}\right]$ .....(i)

Where, σ and p are the densities of water and the bob respectively,

Since, the periods of oscillation of the bob in air and water are given as

$T = 2\pi\sqrt{\frac{l}{g}}$

and $T' = 2\pi\sqrt{\frac{l}{g'}}$

$\frac{T}{T'} = \sqrt{\frac{g'}{g}}$

$\frac{T}{T'} = \sqrt{1 - \frac{\sigma}{\rho}}$

$\frac{T}{T'} = \sqrt{1 - \frac{1}{\rho}} \quad (\sigma = 1)$

putting $\frac{T}{T'} = \frac{1}{\sqrt{2}}$

We obtain, $\frac{1}{2} = 1 - \frac{1}{p} \Rightarrow p = 2$

Question 41

When a stationary wave is formed, then its frequency is

Options:

A. same as that of the individual waves

B. twice that of the individual waves

C. half that of the individual waves

D. $\sqrt{2}$ that of the individual waves

Answer: A

Solution:

Stationary waves- When two waves of the same frequency, wavelength and amplitude travel in opposite directions at the same speed, their superposition gives the stationary wave.

When a stationary wave is formed, then its frequency is the same as that of the individual waves.

Question 42

The equation of a wave is y = 5 sin $\left(\frac{t}{0.04}-\frac{x}{4}\right)$ where x is in cm and t is in second. The maximum velocity of the wave will be

Options:

A. 1 ms^{-1}

B. 2 ms^{-1}

C. 1.5 ms^{-1}

D. 1.25 ms^{-1}

Answer: D

Solution:

Equation of wave y = 5 sin$\left(\frac{t}{0.04}-\frac{x}{4}\right)$.

The standard equation of a wave in the given form is $y = a\sin\left(\omega t-\frac{2\pi}{\lambda}\right)$

Comparing the given equation with the standard equation, we get

a=5

and $\omega=\frac{1}{0.04}=25$

and $\lambda = 8\pi$

Therefore maximum velocity of particles of the medium.

$V_{max} = a\omega = 5\times 25$

$= 125 cms^{-1}$

$= 1.25 ms^{-1}$

Question 43

Two tuning forks A and B having frequency of 500 Hz each are placed with B to the ring to the right of A. An observer is between the forks and is moving towards B with a speed of 25 m/s.
The speed of sound is 345 m/s and the wind speed is 5 m/s from A to B. Calculate the difference in the two frequencies heard by the observer.

Options:

A. 72.5 Hz

B. 55.6 Hz

C. 76.2 Hz

D. 80.9 Hz

Answer: A

Solution:

Let the fork at A be sound source S_1 and the fork at B be sound source S_2

For the sound from source S_1,

Apparent frequency = $f_1 = f^{\left(\frac{c - v_0}{c - v_s}\right)}$

Here, c = 345+5=350 m/s as wind and sound are in same directions.

$V_0 = +25 m/s, V_s = 0$

$\Rightarrow f_1 = 500\left(\frac{350 - 25}{350 - 0}\right) = 464.3Hz$

For the sound from source S_2,

C=345-5=340m/s as wind is opposite to sound v_s=0m/s, v_0=-25 m/s

$\Rightarrow f_2 = f\left(\frac{c - v_0}{c - v_s}\right) = 500\left(\frac{340 - (-25)}{340 - 0}\right)$

$\Rightarrow f_2 - f_1 = (536.80 - 464.30) = 72.5Hz$

As the difference is very large, the observer will not hear the beats.

Question 44

The top of the atmosphere is about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm^{-1}. Still, we do not get an electric shock as we stop out of our house into the open house, because (assume the house to be a steel cage so that there is no field inside)

Options:

A. there is a potential difference between our body and the ground

B. $100VM^{-1}$ is not a high electric field so that we do not feel the shock

C. our body and the ground forms an equipotential surface

D. the dry atmosphere is not a conductor

Answer: D

Solution:

As we know that the dry atmosphere is not a conductor of electricity. It is only conducted when the top of the atmosphere

increases to a very large value and there is moisture in the air, the lightning strikes the ground. Since 400 kV is not very high voltage so, it will not strike the ground

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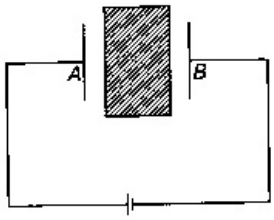
Question 45

An insulator plate is passed between the plates of a capacitor. Then, current



Options:

A. first flows from A to B and then from B to A

B. first flows from B to A and then from A to B

C. always flows from B to A

D. always flows from A to B

Answer: B

Solution:

As insulator plate is passed between the plates of the capacitor, its capacity increases first and then decreases as the plate slips out. As a result, positive charge on plate A increase first and then decreases, hence current in outer circuit flows from B to A and then from A to B.

Question 46

The mobility of free electrons (charge =e mass = m and relaxation time = τ) in a metal is proportional to

Options:

A. $\frac{e}{m}\tau$

B. $\frac{m}{e}\tau$

C. $\frac{e}{m\tau}$

D. $\frac{m}{e\tau}$

Answer: A

Solution:

Drift velocity per unit electric field is called mobility of electron, i.e.

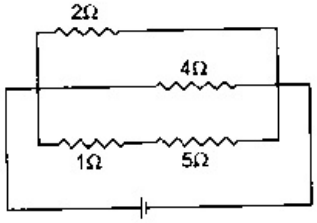
$\mu = \frac{v_d}{E}$

Now $p = \frac{m}{ne^2\tau}$

$\therefore \mu = \frac{ne^2\tau}{mne} = \frac{e\tau}{m}$

Question 47

A current of 3 A flows through the 2Ω resistor shown in the circuit. The power dissipated in the 5Ω resistor is



Options:

A. 4W

B. 2W

C. 1W

D. 5 W

Answer: D

Solution:

The voltage across 2Ωresistance.

$v = 2 \times 3 = 6v$

So voltage across the bottom arm

$V_1 = 6V$

The current passing through the bottom arm

$i = \frac{6}{1+5} = 1 \text{ Amp}$

Thus, power across 5Ω

$p = I^2R = (1)^2 \times 5 = 5w$

Question 48

A diverging meniscus lens of 1.5 refractive index has concave surface of radii 3 and 4 cm. The position of the image, if an object is placed 12 cm in front of the lens, is

Options:

A. 7 cm

B. -8 cm

C. 9 cm

D. 10 cm

Answer: B

Solution:

According to the lens maker formula, focal length of the lens is

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

For given concave lens

R_1=-3cm and R_2=-4 cm

$$\therefore \frac{1}{v}-\frac{1}{u}=(\mu-1)\left(\frac{1}{-3}+\frac{1}{4}\right)$$

$$\text{or } \frac{1}{v}-\frac{1}{(-12)}=(1.5-1)\left(\frac{-4+3}{12}\right)$$

$$\text{or } \frac{1}{v}+\frac{1}{12}=0.5\times\frac{-1}{12}=\frac{-1}{24}$$

$$\text{or } \frac{1}{v}=-\frac{1}{24}-\frac{1}{12}$$

$$=\frac{-1-2}{24}$$

$$=-\frac{3}{24}=-\frac{1}{8}$$

$$\Rightarrow v=-8\text{cm}$$

Question 49

If the focal length of a lens of a camera is 5 f and that of another is 2.5 f, what is the time of exposure for the second if for the first one is $\frac{1}{200}$ s? (where, f is focal length/unit aperture)

Options:

A. $\frac{1}{200}$

B. $\frac{1}{800}$

C. $\frac{1}{6400}$

D. $\frac{1}{3200}$

Answer: B

Solution:

The time of exposure for good quality prints depends on both f-number and E-number.

$$T \times \left(\frac{1}{\text{f}-\text{number}}\right)^2 \left(\frac{1}{\text{E}-\text{number}}\right)$$

$$\text{Where, f}-\text{number} \times \frac{1}{\text{focal length}}$$

$$\Rightarrow \frac{T_1}{T_2} = \frac{F_1^2}{F_2^2}$$

$$\text{Given, } f_1 = 5f, f_2 = 25f \text{ and } T_1 = \frac{1}{200}\text{s}$$

$$T_2 = \frac{T_1 F_2^2}{F_1^2} = \frac{1}{200} \times \frac{(2.5)^2}{(5)^2} = \frac{1}{200} \times \frac{6.25}{25}$$

$$T_2 = \frac{1}{800}\text{s}$$

Question 50

Critical angle for certain medium is $\sin^{-1}(0.6)$. the polarizing angle of that medium is

Options:

A. $\tan^{-1}(1.5)$

B. $\sin^{-1}(0.8)$

C. $\tan^{-1}(1.6667)$

D. $\tan^{-1}(0.667)$

Answer: C

Solution:

Critical angle C = $\sin^{-1}(0.6)$

$$\sin(c) = 0.6$$

$$\mu = \frac{1}{\sin c} = \frac{1}{0.6}$$

Polarizing angle $i_p = \tan^{-1}(\mu)$

$$= \tan^{-1}\left(\frac{1}{0.6}\right)$$

$$= \tan^{-1}(1.6667)$$

Chemistry

Question 51

In the reaction, $CHCL_3 + 40H^- \rightarrow HCOO^- + 3CL^- + 2H_2O$ The intermediate species formed is / are

Options:

A. CCl_2

B. CCl_3^-

C. Both A and B

D. None of the above

Answer: C

Solution:

$CHCl_3 + OH^- \rightarrow : CCl_3^-$

$: CCl_3^- \rightleftharpoons : CCl_2 + Cl^-$

$: CCl_2 \xrightarrow{OH^-} HO - C^-Cl_2 \underset{-OH^-}{\overset{H_2O}{\rightarrow}} HO - CHCl_2 \underset{-2Cl^-}{\overset{OH^-}{\rightarrow}} HCOO^-$

Question 52

Which of the following represents the correct order of the acidity in the given compound?

Options:

A. $ClCH_3COOH < BrCH_2COOH < CH_3COOH < FCH_2COOH$

B. $CH_3COOH < BrCH_2COOH < ClCH_2COOH < FCH_2COOH$

C. $< FCH_2COOH < ClCH_2COOH < BrCH_2COOH < CH_3COOH$

D. $< CH_3COOH < FCH_2COOH < ClCH_2COOH < BrCH_2COOH$

Answer: B

Solution:

The acidity of halogenated acid increases almost proportionately with the increase in electronegativity of the halogen present. Therefore, the correct order of acidity is

$CH_3COOH < BrCH_2COOH < ClCH_2COOH$

$< FCH_2COOH$

Question 53

What will be the major product when 2-amino propane is treated with nitrous acid?

Options:

A. Propane-2-0

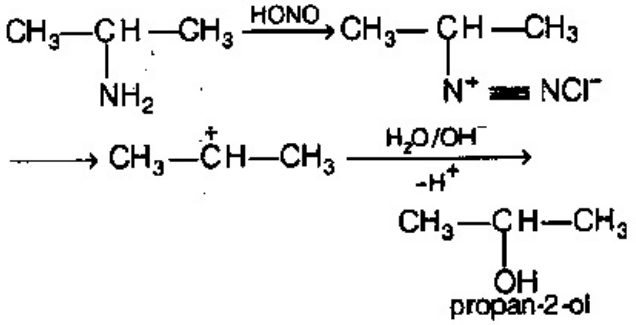
B. Cyclopropene

C. Propanol

D. 2-nitropropane

Answer: A

Solution:



Question 54

There of the following four reactions are due to one similar feature of carbonyl compounds, while the fourth one is different, Which one is fourth.?

Options:

A. Haloform reaction

B. Aldol condensation

C. Knoevenagel reaction

D. Witting reaction

Answer: D

Solution:

Halofrom reaction, aldol condensation and knoevenagel reaction involve the formation of a resonance stabilized anion, while the witting reaction involves the addition of a nucleophile on the carbonyl carbon. The driving force for the witting reaction is the

formation of a very strong P-O bond.

Question 55

The two forms of D-glucopyranose obtained from the solution of D-glucose are called

Options:

A. enantiomer

B. epimer

C. anomer

D. isomer

Answer: C

Solution:

$\alpha-D(+)-$glucopyranose and $\beta-D(+)-$glucopyranose are anomers (a pair of stereoisomers which differ in configuration only around first carbon atom).

Question 56

Which type of polymer is the Buna-S-rubber?

Options:

A. Addition polymer

B. Condensation polymer

C. Copolymer

D. None of the above

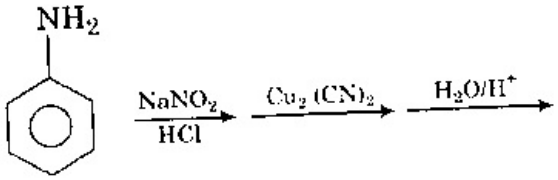
Answer: C

Solution:

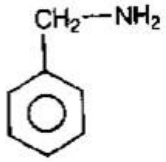
Buna-s-rubber is styrene-butadiene rubber (SBR).it is a copolymer of buta-1,3-diene and styrene.

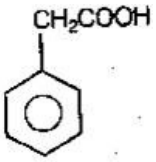
Question 57

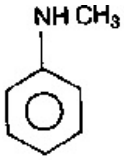
What will be the final product of following reaction sequence?

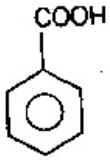


Options:

A. 

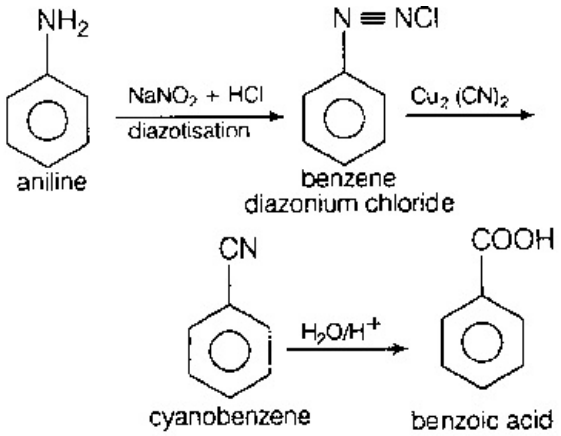
B. 

C. 

D. 

Answer: D

Solution:



Question 58

The number of possible alkynes with molecular formula C_5H_8 is

Options:

A. 1

B. 3

C. 5

D. 8

Answer: B

Solution:

C_5H_8(pentyne) shows following three isomers

$CH_3–CH_2–CH_2–C \equiv CH$ pent – 1 – yen

$CH_3–CH_2–C \equiv CH_3$ pent – 2 – yen

$CH_3 – \overset{CH_3}{CH}– C \equiv CH$ 3 – methylbut – 1 – yen

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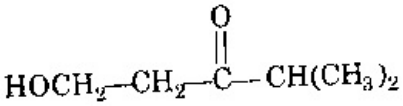
Question 59

The IUPAC name of the following compound is



Options:

A. 2-methy1-5hydroxy-3-pentane

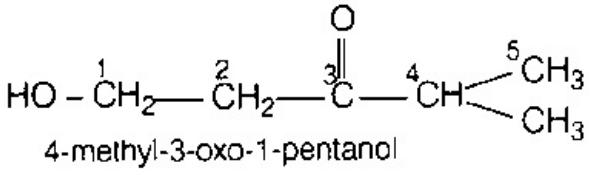
B. 4-methyl-3-oxo-1-pentanol

C. 1-hydroxy-4methyl-3-pentanone

D. 3-keto-hexan-1-0l

Answer: B

Solution:



Question 60

Which of the following orders is not Correct regarding the –I effect of the substituents

Options:

A. $–SR < –OR < –\overset{+}{O}R_2$

B. $–\overset{+}{N}R_2 < –OR < –F$

C. $–I < –Cl < –Br – F$

D. $–\overset{+}{n}R_3 < –\overset{+}{O}R_2 < –OR$

Answer: C

Solution:

Decreasing order of –l power of groups with respect to reference H is

$-NH_2 < -OR < -OH < -I < -Br < -Cl < -F < -COOR < -CHO < -NO_2$

Question 61

Which of the following types of reactions occurs when a substituent has got a double bond with evenly distributed π-electron cloud?

Options:

A. Electrophilic addition

B. Nucleophilic addition

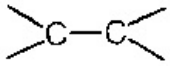
C. Electrophilic substitution

D. Nucleophilic substitution

Answer: A

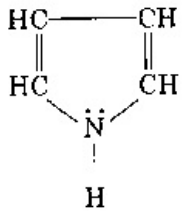
Solution:

When a substituent has got a double bond with evenly distributed π-electron cloud, electrophilic addition reaction takes place.



Question 62

How many delocalized π- electrons are there in the following compound?



Options:

A. 1

B. 2

C. 4

D. 6

Answer: D

Solution:

In the given compound, 4π-electrons of double bond and 1 lone pair on N-atom leads to delocalization of 6π- electrons.

Question 63

Which of the following is an anti-knocking compound?

Options:

A. TEL

B. LTC

C. Freon

D. Gasoline

Answer: A

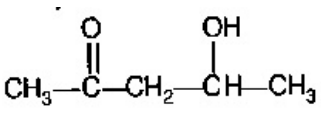
Solution:

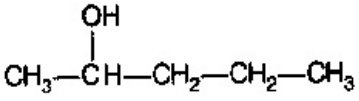
Tetra Ethyl Lead (TEL) is an antiknocking compound.it is used for raising octane number of petro.

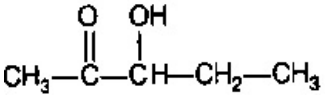
Question 64

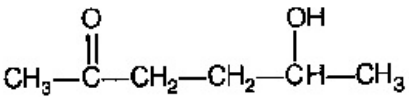
Which one of the following will most readily be dehydrated in acidic medium?

Options:

A. 

B. 

C. 

D. 

Answer: A

Solution:

Greater the conjugation, greater the stability and hence, easier the dehydration. Thus, compound (a), in acidic conditions, will most readily be dehydrated.

Question 65

Which one of the following is a Grignard reagent?

Options:

A. CH_3OMgl

B. C_6H_5MgBr

C. C_2H_5OMgBr

D. $(C_2H_5)_4pb$

Answer: B

Solution:

Grignard reagents are organometallic compounds in which one valency of Mg is saturated with organic group(alkyl, aryl etc) and other valency is saturated with halogen i.e. it is R Mg X.

Question 66

A compound contains two dissimilar asymmetric C-atoms. The number of optical isomers is

Options:

A. 1

B. 2

C. 4

D. 8

Answer: C

Solution:

Number of optical isomers = $2^n=2^2=4$ where, n = number of dissimilar asymmetric C-atoms.

Question 67

Which of the following is the strongest nucleophile?

Options:

A. OH^-

B. CH_3O^-

C. CH_3S^-

D. $C_2H_5O^-$

Answer: D

Solution:

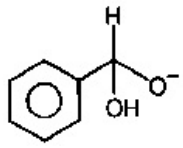
In (a), B. and D. nucleophilic site is same, so nucleophilicity follows the same order as basicity. The order of acidic character of conjugate acid of these bases is $C_2H_5OH < H_2O < CH_3OH$
So, the order of basicity of their base is of opposite order, i.e.
$C_2H_5O^- > OH^- > CH_3O^-$

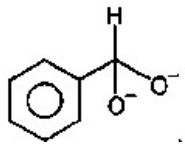
Between B. and C. , CH_3S^- is more nucleophilic as S is less electronegative as compared to O. So, the order of nucleophilicity is
$C_2H_5O^- > OH^- > CH_3S^- > CH_3O^-$

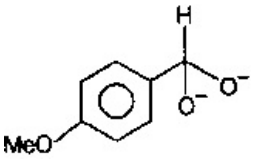
Question 68

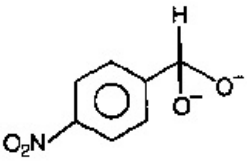
In a Cannizzaro reaction, the intermediate that will be least hydride donor is

Options:

A. 

B. 

C. 

D. 

Answer: D

Solution:

The effect of electron withdrawing substituent in the benzene ring fastens the cannizzaro reaction.

Question 69

Ni^{2+} ion (Z = 28) contains unpaired electrons

Options:

A. 1

B. 2

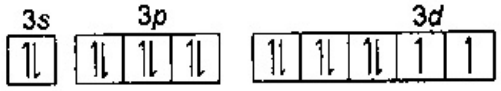
C. 3

D. 5

Answer: B

Solution:

$_{28}Ni = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 4s^2, 3d^8$ $Ni^{2+} = 1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^8$

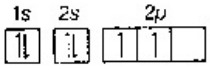


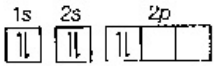
$\therefore Ni^{2+}$ ion has two unpaired electrons.

Question 70

Which of the following will violate Pauli S exclusion principle?

Options:

A. 

B. 

C. Both A and B

D. None of the above

Answer: D

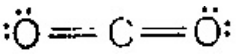
Solution:

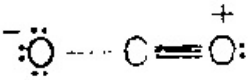
Pauli’s exclusion principle states that no two electrons in an atom can have the same set of all quantum numbers i.e the two electrons having exactly the same value of principal quantum number'n', azimuthal quantum number 'l' and magnetic quantum number 'm' and spin quantum number 's' is not possible. The spin quantum number gives an idea about the direction of spin of electron either in clockwise $\left(+\frac{1}{2}\right)$ or anticlockwise $\left(-\frac{1}{2}\right)$ direction. Thus, a maximum of two electrons can be incorporated in the orbital with same value of 'n', 'm', and 'l' but never with 's' .If the two electrons are in the same orbital with similar spin , they will occupy the same region of space within the atom. So, electrons with similar spin value can't exist in the same atom.
In option (A) it is seen that the spin of electrons are opposite in 1s and 2s orbitals.Though two electrons with similar spin are seen to be present, but they occupy different orbitals i.e p_x and p_y orbital of 2p orbital. Similarly in option(B) , the electrons with different spin values are occupying the same orbital. Thus (A) and (B) are in accordance with Pauli's principle.
Hence, option (D) is correct.

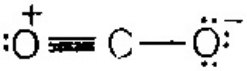
Question 71

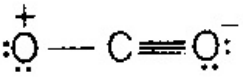
Which of the following reasonating structures in not correct for CO_2?

Options:

A. 

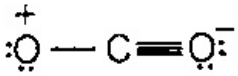
B. 

C. 

D. 

Answer: D

Solution:



is not possible as in this structure, the O-atom, which is bonded to carbon by a triple bond, has 10 electrons in its valency shell. But this situation is impossible as d-orbitals are not present in O-atom. So, O atom cannot expand its valency beyond 8.

Question 72

The pair of molecules forming strongest hydrogen bonds are

Options:

A. HCOOH and CH_3COOH

B. CH_3COCH_3 and $CHCl_3$

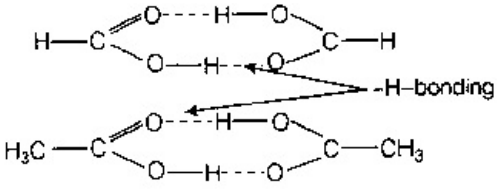
C. H_2O and H_2

D. SiF_6 and SiH_4

Answer: A

Solution:

Carboxylic acids have strong intermolecular H-bonding



Question 73

The bond order of H_2^- ion is $\frac{1}{2}$. If it has 2 bonding electrons, how many antibonding electrons it will have?

Options:

A. 4

B. 3

C. 2

D. 1

Answer: D

Solution:

Bond order $= \frac{1}{2}(N_b - N_a)$

$\frac{1}{2} = \frac{1}{2}(2 - N_a)$

$\therefore N_a = 2 - 1 = 1$

Question 74

Iodide	PI_3	ASI_3	SbI_3
Bond angle	102	100°2′	99°

The bond angle is maximum in PI_3, which is

Options:

A. due to small size of p

B. due to more bp- bp repulsion in pl_3

C. due to less electronegativity of p

D. None of the above

Answer: B

Solution:

Among p, As and Sb, p is the most electronegative element, due to this, there is more bp - bp repulsion.

Question 75

The radius of H-atom in its ground state is $0.53\,\mathring{A}$. The radius of $_3Li^{2+}$ in the similar state is

Options:

A. $0.058\,\mathring{A}$

B. $0.177\,\mathring{A}$

C. $0.706\,\mathring{A}$

D. $1.06\,\mathring{A}$

Answer: B

Solution:

According to Bohr's theory, the expression for radii of ions like He^{+}, Li^{2+} etc of their stationary states is given by:

$r_n = \frac{52.9n^2}{Z}\ pm$

$= \frac{0.53n^2}{Z}\mathring{A}$ where Z is the atomic number, n is the stationary state of principal quantum number.

As given in the question, n=1 as Li^{2+} possess the same state as hydrogen atom .

Also, Z=3 for Li^{2+}

hence,

$$r_{Li^{2+}} = \frac{0.53 \times (1)^2}{3}$$

= 0.177 Å

so, option (2) is correct.

Question 76

The incorrect statements among the following is

Options:

A. the first ionization potential of Al is less than the first ionization potential of Mg

B. the second ionization potential of Mg is lower than the second ionization potential of Na

C. the first ionization potential of Na is less than the first ionisation potential of Mg

D. the third ionization potential of Mg is greater than the third ionisatiion potential of Al

Answer: A

Solution:

lE_2 of Mg is lower that of Na because in case of Mg^{2+}, 3s electron has to be removed while in case of Na^+ an electron from the stable inert gas configuration (neon) has to be removed.

Question 77

Which one of the following arrangements represents the correct order of electron gain enthalpy (with negative sign) of the given atomic species?

Options:

A. Cl < F < S < CO

B. O < S < F < Cl

C. S < O < Cl < F

D. F < Cl < O < S

Answer: B

Solution:

Generally electron gain enthalpy increases in a period from left to

right but decreases in a group on moving down. Therefore, halogens have very high electron affinities.
Fluorine due to its smaller size has unexpectedly low electron gain enthalpy than Cl. Similar is shown in case of O and A. Thus, the order of electron gain enthalpy is O<S<F<Cl

---

Question 78

Which pair of elements has same chemical properties?

Options:

A. 13,22

B. 3,11

C. 4,24

D. 2,4

Answer: B

Solution:

The pair which belongs to some group i.e., in which both the elements have same outer electronic configuration has same chemical properties.

$\therefore 3 \Rightarrow 1s^2, 2s^1$

$11 \Rightarrow 1s^2, 2s^1, 2p^6, 3s^1$

Question 79

Acidified solution of chromic acid on treatment with H_2O_2 yields

Options:

A. $CrO_3+H_2O+O_2$

B. $Cr_2+H_2O+O_2$

C. CrO_5+H_2O

D. $H_2Cr_2O_7+H_2O+O_2$

Answer: C

Solution:

$K_2Cr_2O_7 + H_2SO_4 \rightarrow K_2SO_4 + H_2Cr_2O_7$

$H_2Cr_2O_7 + 4H_2O_2 \rightarrow 2CrO_5 + 5H_2O$

Question 80

Bleaching action of bleaching powder is due to

Options:

A. Cl

B. O

C. Ca

D. Cl_3

Answer: B

Solution:

Bleaching power, when reacts with water, forms nascent oxygen. This is responsible for bleaching action.

$2CaOCl_2 + 2H_2O \rightleftharpoons CaCl_2 + Ca(OH)_2 + 2HClO$

$HClO \rightleftharpoons HCl + [O]$

Question 81

Which of the following is an alum?

Options:

A. $NaAlO_2$

B. $Na_2SO_4.Al_2\ (SO_4)_3.24H_2O$

C. $KCl.MgCl_2.6H_2O$

D. $FeSO_4.(NH4)_2SO_4.6H_2O$

Answer: B

Solution:

Alum are double sulphate of trivalent metal like iron, aluminium chromium etc. and an univalent metal such as potassium or sodium with 24 molecules of water. e.g. $Na_2SO_4.Al_2(SO_4)_3.24H_2O$

Question 82

Sodium thiosulphate is used in photography

Options:

A. as AgBr grain is reduced to non-metallic silver

B. to convert metallic silver into silver salt

C. to remove reduced silver

D. to remove under composed AgBr in the form of Na3 $[Ag(S_2O_3)_2]$ (a complex salt)

Answer: D

Solution:

In photography, used AgBr is removed with the help of sodium thiosulphate, i.e., hypo.

$AgBr + 2Na_2S_2O_3 \rightarrow Na_3[Ag(S_2O_3)_2] + NaBr$

silver bromide	siduym thiosulphate	sodium argento thiosulphate

---

Question 83

When Lunar caustic reacts with acetylene gas, it yields

Options:

A. AG_2O

B. AgCOOH

C. Ag_2C_2

D. Ag

Answer: C

Solution:

When acetylene gas is passed into ammonical Lunar caustic (i.e., silver nitrate solution), a white precitpitate of silver acetaldehyde is obtained.

$2AgNO_3 + 2NH_4OH + C_2H_2 \rightarrow$
$2NH_4NO_3 + 2H_2O + Ag_2C_2 \downarrow$

Question 84

Calculate the ionic radius of a Cs^+ ion, assuming that the cell edge

length for CsCl is 0.4123 nm and that the ionic radius of a Cl^- ion is 0.181 nm

Options:

A. 0.176 nm

B. 0.231 nm

C. 0.357 nm

D. 0.116 nm

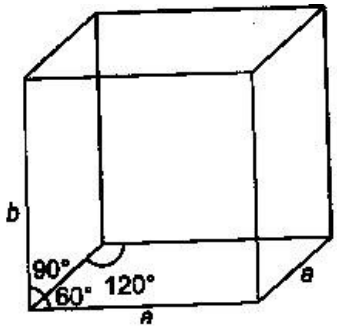
Answer: A

Solution:

$$r_{cs} + r_{cl^-} = \frac{d_{body}}{2} = \frac{0.7141}{2} = 0.3571$$
$$r_{cs} = 0.3571 - 0.181 = 0.176\,nm$$

Question 85

Ice crystallises in a hexagonal lattice. At the low temperature, the lattice constant were a= $4.53\,\mathring{A}$ and b = $7.41\,\mathring{A}$. How many H_2O molecules are contained in a unit cell? [d(ide)=0.92g/cm^3]



Options:

A. 4

B. 3

C. 2

D. 1

Answer: A

Solution:

Volume of the unit cell, V=area of rhombus base× b

$= (a^2 \sin 60^0) \times b$

$= (4.53)^2 \times \frac{\sqrt{3}}{2} \times 7.41 \times 10^{-24} cm^3$

$= 132 \times 10^{-24} cm^3$

Thus, the mass of unit cell

$= v \times d = 132 \times 10^{-24} \times 0.92 \times 6.02 \times 10^{23}$

$= 73g$

This is close to 4 times the molecular weight of H_2O. there are four molecules of H_2O. per unit.

Question 86

When 1 mole of a gas is heated at constant volume, temperature is raised from 298 K to 308 K. Heat supplied to the gas is 500 J. Then, which statement is correct?

Options:

A. $q = -w = 500J; \Delta E = 0$

B. $q = w = 500J; \Delta E = 0$

C. $q = \Delta E = 500J; w = 0$

D. $\Delta E = 0; q = w = -500J$

Answer: D

Solution:

At constant volume, $p\Delta v = 0$

$\therefore q = \Delta E$

Question 87

Heat of formation of H_2O is – 188 KJ/mol and H_2O_2 is 286 KJ/mol. The enthalpy change for the reaction; $2H_2O_2 \rightarrow 2H_2O + O_2$ is

Options:

A. 196 kJ

B. -196 KJ

C. 984 KJ

D. -984KJ

Answer: A

Solution:

$H_2 + \frac{1}{2}O_2 \rightarrow H_2O; \Delta H = -188\,KJmol^{-1}....(i)$

$H_2 + O_2 \rightarrow H_2O_2; \Delta H = -286\,KJmol^{-1}.....(ii)$

Multiply Eqs (i) and (ii) by 2

$2H_2 + O_2 \rightarrow 2H_2O; \Delta H = -376\,KJmol^{-1}.....(iii)$

$2H_2 + 2O_2 \rightarrow 2H_2O_2; \Delta H = -572\,KJmol^{-1}...(iv)$

Eq. (iii) – Eq. (iv)

$2H_2 + O_2 \rightarrow 2H_2O; \Delta Hr = +196\,KJ$

Question 88

Which of the following statements is correct. for Tyndall effect?

Options:

A. Scattering and polarizing of light by small suspended particles is called Tyndall effect

B. Tyndall effect of colloidial particles is due to dispersion of light

C. Tyndall effect is due to refraction of light

D. Tyndall effect is zig-zag motion of supended

Answer: A

Solution:

When a beam of light is passed through a colloidal solution, its path becomes visible. This is known as Tyndall effect.

Question 89

If M is molecular weight of solvent, K_b is molal elevation constant, T_b is its bolilling point, P^0 is its vapour pressure at temperature T and P_s is vapour pressure of its solution having a non-volatile solute at TK, then

Options:

A. $\frac{p^0 - p_s}{p^0} = \frac{\Delta T_b}{K_b} \times M$

B. $\frac{p^0 - p_s}{p^0} = \frac{K_b}{\Delta T_b} \times M$

C. $\frac{p^0 - p_s}{p^0} = \frac{K_b}{\Delta T_b} \times \frac{M}{1000}$

D. $\frac{p^0 - p_s}{p^0} = \frac{\Delta T_b}{K_b} \times \frac{M}{1000}$

Answer: D

Solution:

$$\frac{p^\circ - p_s}{p^\circ} = \frac{n}{N}$$

$$= \frac{\text{molality} \times M}{1000}$$

and molality $= \frac{\Delta T_b}{K_b}$ $(\because \Delta T_b = K_b \times m)$

Question 90

A 0.001 molal solution of $[pt(NH_3)_4\ Cl_4]$in water had a freezing point depression of 0.0054^0C. If K_f for water is 1.80, the correct formulation of the above molecule is

Options:

A. $[pt(NH_3)_4Cl_3]Cl$

B. $[pt(NH_3)_4Cl_2]Cl_2$

C. $[pt(NH_3)_4Cl]Cl_2$

D. $[pt(NH_3)_4Cl_4]$

Answer: B

Solution:

Suppose,

$[pt(NH_3)_4Cl_4 \xrightarrow{\text{Dissociation}} n$ number of product ions

$\therefore i = n$

But from the given data $\Delta T_f = iK_f m$

$\Rightarrow 0.0054 = n \times 1.80 \times 0.001$

$\therefore n = 3$

Hence, the formula must be the one which gives 3 ions of products.

Question 91

In Zeiger Natta polymerization of ethylene, the active species is

Options:

A. $AlCl_3$

B. Et_3Al

C. CH_2CH_2

D. Ti^{3+}

Answer: D

Solution:

$(C_2H_5)_3\,Al + TiCl_4 \rightarrow$ Active species. Ti^{3+} has one active site vacant and thus accommodate one alkyl group. [as $(C_2H_5)_3$ Al reduces $TiCl_4$ to $TiCl_3$]

Question 92

Of the given anions, the strongest Bronsted base is

Options:

A. ClO^-

B. ClO^-_3

C. ClO^-_2

D. ClO^-_4

Answer: A

Solution:

HClO is the weakest acid, hence its conjugate base, i.e., $ClO^-$is the strongest Bronsted base.

Question 93

Standard electrode potential data are useful for understanding the suitability of an oxidant in a redox titration. Some half cell reactions and their standard potentials are given below.

$MnO_4^-(aq) + 8H^+(aq) + 5e^- \rightarrow Mn^{2+}(aq) + 4H_2O(\tau); E^0 = 1.51\,V$

$Cr_2O_7^{2-}(aq) + 14H^+ + 6e^- \rightarrow 2Cr^{3+}(aq) + 7H_2O(\tau); E^0 = 1.38\,V$

$Fe^{3+}(aq) + e^- \rightarrow Fe^{2+}(aq); E^0 = 0.77V$

$Cl_2(g) + 2e^- \rightarrow 2Cl^-(aq); E^0 = 1.40V$

Identify the incorrect statement regarding the quantitative estimation of gaseous $Fe(NO_3)_2$.

Options:

A. MnO_4^- can be used in aqueous HCl

B. $Cr_2O_7^{2-}$ can be used in aqueous HCl

C. MnO_4^- can be used in aqueous H_2SO_4

D. $Cr_2O_7^{2-}$ can be used in aqueous H_2SO_4

Answer: A

Solution:

The reaction between MnO_4^- and HCl may be represented as follows

$$MnO_4^-(aq) + 16H^+ 10Cl^- \rightarrow 2Mn^{2+}(aq) + 8H_2O(l) + 5Cl_2(g)$$

Thus on the basis of this reaction following electrochemical cell will be represented

$ptCl_2(g)$ (1 atm)l $Cl^-(aq)$ ll$MnO_4^-(aq)$l$Mn^{2+}(aq)$

Since, $E^\circ_{cell} = E^\circ_{cathode} - E^\circ_{anode}$

From given data $E^\circ_{cell} = 1.51 - 140 = 0.11v$

E^-_{cell} is positive, hence ΔG° is negative. Thus, above cell reaction is feasible but MnO_4^- ion can oxidise Fe^{2+} to Fe^{3+} and Cl^- to Cl_2 in aqueous medium also. Therefore quantitative estimation of aqueous $Fe(NO_3)_2$, it is not a suitable reagent.

Question 94

When the sample of copper with the zinc impurity is to be purified by electrolysis, the appropriate electrodes are
Anode Cathode

Options:

A. pure zinc pure copper

B. impure zinc pure copper

C. impure zinc impure sample

D. impure sample pure copper

Answer: D

Solution:

Impure sample is made the anode and pure copper acts as the cathode.

Question 95

The compound that can work both as an oxidizing as well as reducing agent is

Options:

A. $KMNO_4$

B. H_2O_2

C. $Fe(So_4)_3$

D. $K_2Cr_2O_7$

Answer: B

Solution:

The oxidation number of O in H_2O_2 is -1. It can either increases to zero in O_2 or decreases to -2 in H_2O. Therefore, H_2O_2 can act both as an oxidizing as well as a reducing agent.

Question 96

The reaction,

$$10FeSO_4 + 2KMNO_4 + 8H_2SO_4 \rightarrow 2MNSO_4 + 5Fe_2(SO_4)_3 + K_2SO_4 + 8H_2O$$

Is an example of reaction of

Options:

A. disproportionation

B. intermolecular redox

C. intramolecular redox

D. None of these

Answer: B

Solution:

The reaction is an example of intermolecular redox reaction.

$$10FeSO_4 + 5Fe_2(SO_4)_3 + K_2SO_4 + 8H_2O$$

$$2Fe^{2+} \rightarrow (Fe^{3+})_2 + 2e^-$$

$$Mn^{7+} + 5e^- \rightarrow Mn^{2+}$$

Question 97

For the following equilibrium,

$N_2O_4 \rightleftharpoons 2NO_2$ in gaseous phase, NO_2 is 50% of the total volume when equilibrium is set up. Hence, percent of dissociation of N_2O_4 is

Options:

A. 50%

B. 25%

C. 66.66%

D. 33.33%

Answer: D

Solution:

$N_2O_4 \rightleftharpoons 2NO_2$

1 0

$(1-x)$ $2x$

Total moles $= 2x + (1-x)(1+x)$

$\therefore$ % of NO_2 by volume $= \frac{2x}{(1+x)} \times 100 = 50$

or $x = \frac{1}{3} = 0.33$

Hence per cent of dissociation of N_2O_4 is 33.33%

Question 98

XY_2 dissociates as $XY_2(g) \rightleftharpoons XY(g) + Y(g)$
When the initial pressure of $XY_2$is 600 mm Hg, the total equilibrium pressure is 800 mm Hg. Calculate K for the reaction assuming that the volume of the system remains unchanged.

Options:

A. 50

B. 100

C. 166.6

D. 400.0

Answer: B

Solution:

$XY_2 \rightleftharpoons XY + Y$

Initial 600mm 0 0

At eq. 600-p p p

Total pressure = (600-p) +p = 600p

Or (600+p)=800

$\Rightarrow$P=200 mm

And p due to XY_2 =400mm

K= $\frac{200 \times 200}{400} = 100$

Question 99

Hydrogen ion concentration in moL/in a solution of pH = 5.4 will be

Options:

A. 3.98×10^{8}

B. 3.88×10^{6}

C. 3.68×10^{-6}

D. 3.98×10^{-6}

Answer: D

Solution:

pH = - log[H^+]

[H^+]=Antilog (-5.4) = 3.98×10^{-6}

Question 100

The solubility of AgI in NaI solution is less than that in pure water because

Options:

A. Agl forms complex with Nal

B. of common ion effect

C. solubility product of Agl is less

D. the temperature of the solution decreases

Answer: B

Solution:

Solubility is decreased due to common ion effect.

$AgI \rightleftharpoons Ag^+ + I^-$

$NaI \rightleftharpoons Na^+ + I^-$

H^+ is common ion in both the reactions.

Maths

Question 101

Let F_1 be the set of parallelograms, F_2 be the set of rectangles, F_3 be the set of rhombuses, F_4 be the set of squares and F_5 be the set of trapeziums in a plane. Then, F_1 may be equal to

Options:

A. $F_2 \cap F_3$

B. $F_3 \cap F_4$

C. $F_2 \cup F_5$

D. $F_2 \cup F_3 \cup F_4 \cup F_1$

Answer: D

Solution:

Given, F_1=the set of parallelograms

F_2 = the set of rectangles

F_3= the set of rhombuses

F_4= the set of squares

And F_5 = the set of trapeziums

By definition of a parallelogram, opposite sides are equal and parallel. In rectangles, rhombuses, and squares, all have opposite sides equal and parallel, therefore

$F_2 \subset F_1, F_3 \subset F_1, \subset F_4 \subset F_1$

$\therefore F_1 = F_1 \cup F_2 \cup F_3 \cup F_4$

Question 102

If A = {x:x is a multiple of 4} and B = {x: x is a multiple of 6}, then $A \cap B$ consists of all multiples of

Options:

A. 16

B. 12

C. 8

D. 4

Answer: B

Solution:

Given, A = {x : x is a multiple of 4}
= {4,8,12,16,20....}
And B={x : x is a multiple of 6}
= {6,12,18,24,....}
$\therefore A \cap B = \{12, 24, ...\}$
$= \{x : x \text{ is a multiple of } 12\}$

Question 103

Let f = $\left\{\left(x, \frac{x^2}{1+x^2}\right) : x \varepsilon R\right\}$ be a function from R to R. Determine the range of f.

Options:

A. [0,1)

B. [0,1]

C. [0,2)

D. None of these

Answer: A

Solution:

Given, f= $\left\{\left(x, \frac{x^2}{1+x^2}\right) : x \in R\right\}$ and $f : R \to R$
Let $y=\frac{x^2}{1+x^2}$
$\because$ F(x) or y is positive for all values of x
And $1+x^2 > x^2$
$\Rightarrow 0 \leq y<1$
$\therefore$ Range of f = {y: y$\in$R and y$\in$[0,1)}

Question 104

If $\theta_1, \theta_2, \theta_3, ..., \theta_n$ are in AP, whose common difference is d, then sin d ($\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sin\theta_3 + ... + \sec \sec\theta_{n-1} \sec\theta_n$) is equal to

Options:

A. $\tan\theta_n - \tan\theta_2)$

B. $\tan\theta_n + \tan\theta_1)$

C. $\tan\theta_n - \tan\theta_1)$

D. None of these

Answer: C

Solution:

Given $\theta_1, \theta_2, \theta_3, \ldots\ldots\theta_n$ are in AP. $\Rightarrow \theta_2 - \theta_1 = \theta_3 - \theta_2 = \ldots = \theta_n - \theta_{n-1} = d$(i)

Now, taking only first term,

Sin d $\sec^{\theta_1}$ $\sec^{\theta_2} = \frac{\sin d}{\cos\theta_1 \cos\theta_2}$

$= \frac{\sin(\theta_2 - \theta_1)}{\cos\theta_1 \cos\theta_2}$ [from Eq.(i)

$= \frac{\sin\theta_2 \cos\theta_1 - \cos\theta_2 \sin\theta_1}{\cos\theta_1 \cos\theta_2}$

$= \frac{\sin\theta_2 \cos\theta_1}{\cos\theta_1 \cos\theta_2} - \frac{\cos\theta_2 \sin\theta_1}{\cos\theta_1 \cos\theta_2}$

$= \tan\theta_2 - \tan\theta_1$

Similarly, we can solve other terms which will be

$\tan\theta_3 - \tan\theta_2, \tan\theta_4 - \tan\theta_3, \ldots\ldots$ $\therefore \sin d\,(\sec\theta_1 \sec\theta_2 + \sec\theta_2 \sec\theta_3 + \ldots + \sec\theta_{n-1} \sec\theta_n) = \tan\theta_2 - \tan\theta_1 + \tan\theta_3 - \tan\theta_2 + \ldots + \tan\theta_n - \tan\theta_{n-1} = -\tan\theta_1 + \tan\theta_n = \tan\theta_n - \tan\theta_1$

Question 105

The probability that a man will live 10 more years, is $\frac{1}{4}$ and the probability that his wife will live 10 more years, is $\frac{1}{3}$ .Then m what is the probability that neither will be alive in 10 yr?

Options:

A. $\frac{1}{2}$

B. $\frac{3}{7}$

C. $\frac{2}{3}$

D. $\frac{1}{4}$

Answer: A

Solution:

Given, $P^{(\overline{M})} = \frac{1}{4} \Rightarrow p(\overline{M}) = 1 - \frac{1}{4} = \frac{3}{4}$ and $p(\overline{M}) = \frac{1}{3} \Rightarrow (\overline{M}) = 1 - \frac{1}{2} = \frac{2}{3}$

Here, both events are independent, so the required probability

$= p(\overline{W} \cap \overline{M})$

$= p(\overline{W}).P(\overline{M})$

$= \frac{3}{4} \times \frac{2}{3}$

$= \frac{1}{2}$

Question 106

The probability that in a year of the 22nd century chosen at random at random, there will be 53 Sunday, is

Options:

A. $\frac{3}{28}$

B. $\frac{2}{28}$

C. $\frac{7}{28}$

D. $\frac{5}{28}$

Answer: D

Solution:

We know, a leap year is fallen within 4 yr, so its probability = $\frac{25}{100} = \frac{1}{4a}$

In a century, the probability of 53 Sunday in a leap year $\frac{1}{4} \times \frac{2}{7} = \frac{2}{28}$

Non-teap year in a century = 75

Probability of selecting a non-leap year

$\frac{3}{7} \times \frac{1}{7} = \frac{3}{28}$

$\therefore$ Required probability = $\frac{2}{28} + \frac{3}{28} = \frac{5}{28}$

Question 107

The values of x,y and z for the system of equations x+2y+3z = 6, 3x – 2y +z= 2 and 4x+2y +z = 7 are respectively

Options:

A. 1,1,1

B. 1,2,3

C. 1,3,2

D. 2,3,1

Answer: A

Solution:

The given system of equations is
X + 2y + 3Z = 6 ……. (i)
3x -2y + z = 2 …… (ii)
and 4x + 2y + z = 7 …..(iii)

Here, $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} B = \begin{bmatrix} 6 \\ 2 \\ 7 \end{bmatrix} \text{and} X = \begin{bmatrix} X \\ y \\ z \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{vmatrix}$

$= 1(-2-2) - 2(3-4) + 3(6+8)$

$= -4 + 2 + 42 = 40$

$\text{Now, adj } A = \begin{bmatrix} -4 & 4 & 8 \\ 1 & -11 & 8 \\ 14 & 6 & -8 \end{bmatrix}$

$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{40} \begin{bmatrix} -4 & 4 & 8 \\ 1 & -11 & 8 \\ 14 & 6 & -8 \end{bmatrix}$

$\text{Now, } X = A^{-1}B = \frac{1}{40} \begin{bmatrix} -4 & 4 & 8 \\ 1 & -11 & 8 \\ 14 & 6 & -8 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ 7 \end{bmatrix}$

$= \frac{1}{40} \begin{bmatrix} -24+ & 8+ & 56 \\ 6 & -22+ & 56 \\ 84+ & 12 & -56 \end{bmatrix} = \frac{1}{40} \begin{bmatrix} 40 \\ 40 \\ 40 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow x = 1, y = 1, z = 1$

Question 108

The minimum force required to move a body of weight w placed on a rough horizontal plane surface is

Options:

A. $w \sin \lambda$

B. $w \cos \lambda$

C. $w\tan\lambda$

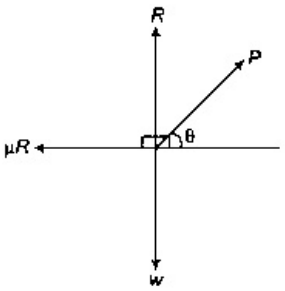
D. $w\cot\lambda$

Answer: A

Solution:

We have, $P\cos\theta = \mu R$

$R + P\sin\theta = W$



$\Rightarrow P\cos\theta = \mu(w - p\sin\theta)$

$\Rightarrow P(\cos\theta + \mu\sin\theta) = \mu w$

$\Rightarrow p = \dfrac{\mu w}{\cos\theta + \mu\sin\theta}$

$= \dfrac{w\tan\lambda}{\cos\theta + \tan\lambda\sin\theta}(\because \mu = \tan\lambda)$

$= \dfrac{w\sin\lambda}{\cos\theta + \cos\lambda + \sin\theta\sin\lambda}$

$= \dfrac{w\sin\lambda}{\cos(\theta - \lambda)}$

p is minimum when $\cos(\theta - \lambda)$ is maximum i.e.

$\text{Cos}(\theta - \lambda) = 1$

Hence, minimum $p = w\sin\lambda$.

Question 109

The resultant of two like parallel forces P and Q passes through a point O. If the resultant also passes through O when Q and R replace P and Q respectively, then

Options:

A. P, Q, R are in GP

B. Q,P,R are in GP

C. R,P,Q are in GP

D. P,Q,R are in AP

Answer: A

Solution:

Let the two like parallel forces p and q act at points A and B on the rigid body. Since, the resultant of p and q passes through O.

therefore

$$AQ = \left[\frac{AB}{P+Q}\right]Q \ldots\ldots(i)$$

The resultant of like parallel forces Q and R, acting at A and B respectively, also passes through O. Therefore,

$$AO = \left(\frac{AB}{Q+R}\right)R \ldots\ldots(ii)$$

From Eqs. (i) and (ii) ,we get

$$\left(\frac{AB}{p+Q}\right)Q = \left(\frac{AB}{Q+R}\right)R$$

$$\Rightarrow Q^2 + QR = PR + QR$$

$$\Rightarrow Q^2 = PR$$

$$\Rightarrow P, Q \text{ and } R \text{ are in Gp.}$$

Question 110

If a particle A is moving also a straight line with velocity 3 m/s and another particle B has a velocity 5 m/s at an angle of 60^0 to the path of A, then the velocity B relative to A is

Options:

A. $\sqrt{39}$ m / s

B. $\sqrt{19}$ m / s

C. 19 m / s

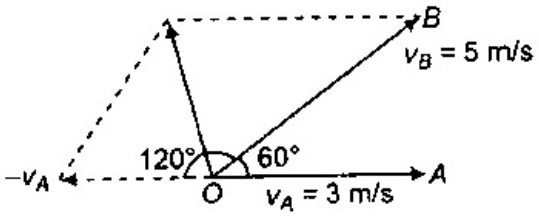
D. None of these

Answer: B

Solution:

Given, V_A=3m/s

and V_B=5m/s



$$\therefore V_{BA} = \sqrt{V_A^2 + V_B^2 + 2V_A V_B \cos\theta}$$

$$(\because \theta = 120^0)$$

$$\sqrt{9 + 25 + 2 \times 3 \times 5 \times \left(-\frac{1}{2}\right)}$$

$$= \sqrt{34 - 15} = \sqrt{19} \text{m / s}$$

Question 111

A particle is projected down on inclined plane with velocity of 21m/s at an angle $60^0$with the horizontal. Its range on the inclined plane, inclined at an angle of 30^0 with the horizontal is

Options:

A. 21 dm

B. 2.1 dm

C. 30 dm

D. 6 dm

Answer: D

Solution:

Range down the plane = $\frac{2U^2 \cos\alpha \sin(\alpha+\beta)}{g\cos^2\beta}$

$= \frac{2\times 21\times 21\cos 60^0 \sin 90^0}{10\times \cos^2 30^0}$

$= \frac{2\times 441\times \frac{1}{2}}{10\times \frac{3}{4}} = 58.8 \text{ cm}$

$= 5.88\,\text{dm} \approx 6\,\text{dm}$

Question 112

If 3 $\sin^2\theta + 2\sin^2\phi = 1 \text{ and } 3\sin 2\theta = 2\sin 2\phi, 0 < \theta < \frac{\pi}{2} \text{ and } 0 < \phi < \frac{\pi}{2},$**then the value of** $\theta + 2\phi$**is**

Options:

A. $\frac{\pi}{4}$

B. $\frac{\pi}{2}$

C. π

D. None of these

Answer: B

Solution:

Given that, $3\sin^2\theta + 2\sin^2\phi = 1$

$\Rightarrow 3\sin^2\theta = \cos 2\phi$ .....(i)

and $3\sin\theta\cos\theta = \sin 2\theta$.....(ii)

On squaring and adding Eqs. (i) and (ii), we get

$9\sin^2\theta(\sin^2\theta+\cos^2\theta)=1$

$\Rightarrow \sin\theta=\frac{1}{3}\text{ and }\cos\theta=\frac{2\sqrt{2}}{3}$

$\therefore \cos 2\phi=3\times\frac{1}{9}=\frac{1}{3}\text{ and }\sin 2\phi=\frac{2\sqrt{2}}{3}$

$\text{Now, }\cos(\theta+2\phi)=\cos\theta\cos 2\phi-\sin\theta\sin 2\phi$

$=\frac{2\sqrt{2}}{3}\cdot\frac{1}{3}-\frac{1}{3}\cdot\frac{2\sqrt{2}}{3}=0$

$\text{and }\theta+2\phi<\frac{3\pi}{2}$

$\therefore \theta+2\phi=\frac{\pi}{2}$

Question 113

The equation e^{sinx}-e^{-sinx}-4=0 has

Options:

A. no solution

B. two solutions

C. three solutions

D. None of these

Answer: A

Solution:

Given, $e^{\sin x}-e^{-\sin x}-4=0$

$\Rightarrow e^{2\sin x}-4e^{\sin x}-1=0$

$e^{\sin x}=\frac{4\pm\sqrt{16+4}}{2}=2\pm\sqrt{5}$

$\Rightarrow \sin x=\log(2+\sqrt{5})$

$[\because \log(2-\sqrt{5})\text{is not defined}]$

$\text{Since},2+\sqrt{5}>e\Rightarrow\log(2+\sqrt{5})>1$

$\Rightarrow \text{Sin}x>1,\text{which is not possible.}$

Hence, no solution exist.

Question 114

At a distance 2h m from the foot of a tower of the top of the tower subtend equal angles. Height of the pole should be

Options:

A. $\frac{5h}{3}$m

B. $\frac{4h}{3}$m

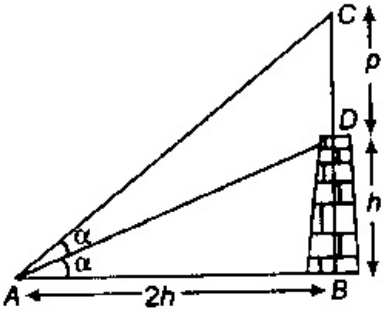
C. $\frac{7h}{5}m$

D. $\frac{3h}{2}m$

Answer: A

Solution:

In $\triangle ABD$, $\tan\alpha = \frac{h}{2h}$



$\Rightarrow \tan\alpha = \frac{1}{2}$

in ΔABC,

$\tan 2\alpha = \frac{h+p}{2h}$

$\Rightarrow \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{h+p}{2h}$

$\Rightarrow \frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2} = \frac{h+p}{2h}$

$\Rightarrow \frac{4}{3} = \frac{h+p}{2h}$

$\Rightarrow 8h = 3h + 3p$

$\Rightarrow 5h = 3p \Rightarrow p = \frac{5h}{3}m$

Question 115

The locus of a point which moves such that its distance from the point (0,0) is twice its distance from the Y-axis, is

Options:

A. $x^2 - y^2 = 0$

B. $x^2 - 3y^2 = 0$

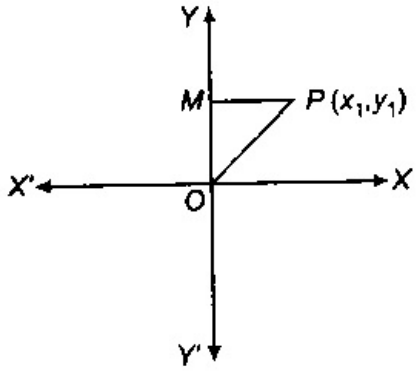
C. $3x^2 - y^2 = 0$

D. None of these

Answer: C

Solution:

By hypothesis, $|OP| = 2|PM|$



$\Rightarrow \sqrt{x_1^2 + y_1^2} = 2|x_1|$

On squaring both sides, we get

$x_1^2 + y_1^2 = 4x_1^2 \Rightarrow 3x_1^2 - y_1^2 = 0$

$\therefore$ Locus of the point is $3x^2 - y^2 = 0$.

Question 116

Find the distance of the line 4x+7y+5 = 0 from the point (1,2) along the line 2x-y =0.

Options:

A. $\frac{23}{7}\sqrt{5}$ sq units

B. $\frac{23}{18}\sqrt{5}$ sq units

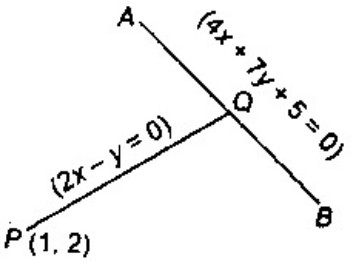
C. $\frac{23}{8}\sqrt{5}$ sq units

D. None of these

Answer: B

Solution:

The equation of line AB is 4x+7Y+5=0...(i)



And equation of line PQ is 2x-y=0.....(ii)

On solving Eqs. (i) and (ii), we get

$$x = -\frac{5}{18} \text{ and } y = -\frac{5}{9}$$

$$\therefore \text{Coordinates of } Q = \left(-\frac{5}{18}, -\frac{5}{9}\right)$$

$$\text{Length of } QP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(1 + \frac{5}{18}\right)^2 + \left(2 + \frac{5}{9}\right)^2}$$

$$= \sqrt{\left(\frac{23}{18}\right)^2 + \left(\frac{23}{9}\right)^2}$$

$$\frac{23}{9}\sqrt{\frac{1}{4} + 1} = \frac{23}{9} \times \frac{\sqrt{5}}{2}$$

$$= \frac{23}{18}\sqrt{5} \text{ squnits}$$

Question 117

The tangent at (1,7) to curve x^2=y-6 touches the circle $x^2+y^2+16x+12Y+c=0$ at

Options:

A. (6,7)

B. (-6,7)

C. (6,-7)

D. (-6,-7)

Answer: D

Solution:

The tangent at (1,7) to the parabola x^2=y-6 is

$$x = \frac{1}{2}(y + 7) - 6$$

$$\Rightarrow 2x = y + 7 - 12$$

$$\Rightarrow y = 2x + 5$$

Which is also a tangent to the circle

$$x^2 + y^2 + 16x + 12y + c = 0$$

$$\therefore x^2 + (2x + 5)^2 + 16x + 12(2x + 5) + c = 0$$

$$\Rightarrow 5x^2 + 60x + 85 + c = 0$$

Must have equal roots. Let α and β be the roots of the equation.Then,

$$\alpha + \beta = -12 \Rightarrow \alpha = -6 (\because \alpha = \beta)$$

$$\therefore x = -6 \text{ and } \quad y = 2x + 5 = -7$$

$$\Rightarrow \text{Point of contact is} (-6, -7)$$

Question 118

Find the length of the line segment joining the vertex of the parabola

$y^2=4ax$ and a point on the parabola, where the line segment makes an angle θ to the X-axis.

Options:

A. $\frac{2a\cos\theta}{\sin^2\theta}$

B. $\frac{4a\cos\theta}{\sin^2\theta}$

C. $\frac{4a\cos\theta}{3\sin^2\theta}$

D. None of these

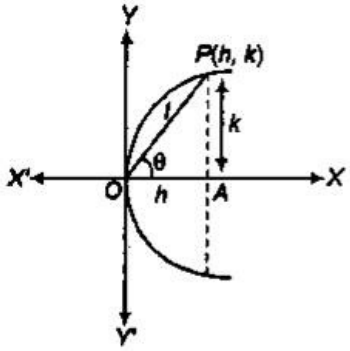
Answer: B

Solution:

Let any point (h,k) will satisfy

$Y^2=4ax$ i.e.,$k^2 = 4ah$ .....(i)

Let a line OP mkes an angleθ from the X-axis.



$\therefore$ inΔOAP,

$$\sin\theta = \frac{PA}{OP}$$

$\Rightarrow \sin\theta = l$

$\Rightarrow K = l\sin\theta$

$$\text{and}\cos\theta = \frac{OA}{OP}$$

$$\Rightarrow \cos\theta = \frac{h}{l} \Rightarrow h = l\cos\theta$$

Hence, from.(i), we get

$l^2\sin^2\theta = 4a \times l\cos\theta$

(put$k = l\sin\theta, h = l\cos\theta$)

$$\Rightarrow l = \frac{4a\cos\theta}{\sin^2\theta}$$

Question 119

The line $x=at^2$ meets the ellipse $\frac{x^2}{a^2}+\frac{y^2}{a^2}=1$ in the real points, if

Options:

A. $|t| < 2]$

B. $|t| \leq 1$

C. $|t| > 1$

D. None of these

Answer: B

Solution:

Putting x at^2 in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we get

$t^4 + \frac{y^2}{b^2} = 1$

i.e., $y^2 = b^2(1 - t^4)$

$= b^2(1 + t^2)1 - t^2)$

y is real, if $1 - t^2 \geq 0$ i, e., $|t| \leq 1$.

Question 120

The equation of tangents to the ellipse $3x^2 + 4y^2 = 5$, which are inclined at 30^0 to the x-axis, are

Options:

A. $y = \sqrt{3}x \pm \frac{5}{2}$

B. $y = \frac{1}{\sqrt{3}}x \pm \frac{5}{2}$

C. $y = \frac{1}{\sqrt{3}}x \pm 1$

D. None of these

Answer: D

Solution:

Given equation of ellipse is $\frac{x^2}{\frac{5}{3}} + \frac{y^2}{\frac{5}{4}} = 1$

The equation of tangent in slope from is $y = mx \pm \sqrt{\frac{5}{3}m^2 + \frac{5}{4}}$

Slope of tangent are $\frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$.

$\therefore y = \pm\frac{1}{\sqrt{3}}x \pm \sqrt{\frac{5}{9} + \frac{5}{4}}$

$\Rightarrow y = \pm\frac{1}{\sqrt{3}}x \pm \frac{\sqrt{65}}{6}$

Question 121

Common roots of the equations $z^3+2z^2+2z+1=0$ and $z^{1985}+z^{100}+1=0$ are

Options:

A. ω, ω^2

B. ω, ω^3

C. ω^2, ω^3

D. None of these

Answer: A

Solution:

The given equation $z^3+2z^2+2z+1=0$ can rewritten as $(z+1)(z^2+z+1)=0.$
Its roots are $-1, \omega$ and $\omega^2.$
$z^{1985}+z^{100}+1 = z^{3(661)+2}+z^{3(33)+1}+1$
$=z^2+z+1$
The roots of the equation are w and w^2

Hence, ω and ω^2 are the common roots.

Question 122

The nth term of the series 1+2+5+12+25+.....is

Options:

A. $(n-1)(n-2)$

B. $\frac{1}{3}n(n-1)(n-2)+n$

C. n

D. None of these

Answer: B

Solution:

Let nth term of the series is t_n and sum is S.
Then, S= 1+2+5+12+25+46+....t_n
S=1+2+5+12+25+....+ t_{n-1} + t_n
On subtracting, we get

$0=1+1+3+7+13+21+.....+(t_n- t_{n-1})-t_n$

$\therefore T_n = 1+\{1+3+7+13+21+....+\text{upto}(n-1)\}$

Let $(n-1)$th term and sum of the series

$1+3+7+13+21+...$ are t_{n-1} and s'.

respectively .Then

$S' = 1+3+7+13+21+....+t_{n-1}$

$S' = 1+3+7+13+....+t_{n-2}+t_{n-1}$

On subracting, we get

$0 = 1+2+4+6+8+.....$

$+(t_{n-1}-t_{n-2})-t_{n-1}$

$\therefore t_{n-1} = 1+2\{1+2+3+4+...\text{upto}(n-2)\}$

$= 1+2.\frac{1}{2}(n-2)(n-1) = n^2-3n+3$

$\Rightarrow t_n = (n+1)^2-3(n+1)+3$

$= n^2-n+1$

$\therefore t_n = 1+\{1+3+7+13+....\text{upto}(n-1)\}$

$= 1+\sum_{n=1}^{n-1}(n^2-n+1)$

$= 1\sum_{n=1}^{n-1}n^2-\sum_{n=1}^{n-1}n+\sum_{n=1}^{n-1}1$

$1+\frac{1}{6}n(n-1)(2n-1)$

$-\frac{1}{2}n(n-1)+(n-1)$

$= \frac{1}{3}n(n-1)(n-2)+n$

Hence, $t_n = \frac{1}{3}n(n-1)(n-2)+n$

Question 123

$(100)^{50}+(99)^{50}$

Options:

A. $<(101)^{50}$

B. $<(101)$

C. $>(101)^{50}$

D. $>(101)$

Answer: A

Solution:

Here, $(101)^{50} = (100+1)^{50} = 100^{50}+C_1100^{49}$

$+{}^{50}C_2100^{48}+.....+1...(i)$

and $(99)^{50} = (100-1)^{50} = 100^{50}-{}^{50}C_1100^{49}+$

${}^{50}C_2100^{48}-.....+1...(ii)$

On subtracting Eq. (ii) from Eq. (i), we get $(101)^{50}-(99)^{50} = 2\{{}^{50}C_1100^{49}+{}^{50}C_3100^{47}+...\}$

$= 2 \times {}^{50}C_1 100^{49} + (2 \times {}^{50}C_3 100^{47} + \ldots)$

$100 \times 100^{49} + A positive e number > 100^{50}$

$\Rightarrow (101)^{50} - (99)^{50} > (100)^{50}$

$\Rightarrow (101)^{50} > (100)^{50} + (99)^{50}$

or $(100)^{50} + (99)^{50} < (101)^{50}$

Question 124

If the determinant $\Delta = \begin{vmatrix} 3 & -2 & \sin 3\theta \\ -7 & 8 & \cos 2\theta \\ -11 & 14 & 2 \end{vmatrix} = 0$ **then the value of** $\sin\theta$ **is**

Options:

A. $\frac{1}{3}$ or 1

B. $\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{3}}{2}$

C. 0 or $\frac{1}{2}$

D. None of these

Answer: C

Solution:

Applying $R_2 \to R_2 + 4R_1$ and $R_3 \to R_3 + 7R_1$ we get

$$\begin{vmatrix} 3- & 2 & \sin 3\theta \\ 5 & 0 & \cos 2\theta + 4\sin 3\theta \\ 10 & 0 & 2 + 7\sin 3\theta \end{vmatrix} = 0$$

$\Rightarrow 2\left[5(2 + 7\sin 3\theta) - 10(\cos 2\theta + 4\sin 3\theta)\right] = 0$

$\Rightarrow 2 + 7\sin 3\theta - 2\cos 2\theta - 8\sin 3\theta = 0$

$\Rightarrow 2 - 2\cos 2\theta - \sin 3\theta = 0$

$\Rightarrow \sin\theta\left(4\sin^2\theta + 4\sin\theta - 3\right) = 0$

$\Rightarrow \sin\theta = 0 \text{ or } (2\sin\theta - 1) = 0 \text{ or } (2\sin\theta + 3) = 0$

$\Rightarrow \sin\theta = 0 \text{ or } \sin\theta = \frac{1}{2}$

Question 125

The relation R in R defined by R = $\{(a,b) : a \le b^3\}$, **is**

Options:

A. reflexive

B. symmetric

C. transitive

D. None of these

Answer: D

Solution:

Given,R= {a, b: a ≤b^3)

It is observed that

$\left(\frac{1}{2},\frac{1}{2}\right) \in R \text{as} \frac{1}{2} < \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$

So R is not reflexive.

Now, $(1,2) \in R\left(\text{as} 1 < 2^3 = 8\right)$

But $(2,1) \in R\left(\text{as} 2^3 > 1\right)$

So, R is not symmetric.

We have $\left(3\frac{3}{2}\right),\left(\frac{3}{2},\frac{3}{5}\right) \in R \text{as} 3 < \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$

But $\left(3,\frac{6}{5}\right) \in R \text{as} 3 > \left(\frac{6}{5}\right)^3$

Therefore, R is not transitive.

Hence, R is neither reflexive nor symmetric nor transitive.

Question 126

The value of 2 $\tan^{-1}$x- is

Options:

A. $\tan^{-1}$x

B. tan x tan$\cot^{1}$x)

C. cot x

D. cosec^{-1}x

Answer: A

Solution:

$$2\tan^{-1}\left(\text{cosec}\tan^{-1}x - \tan\cot^{-1}x\right)$$

$$= 2\tan^{-1}\left[\text{cosec}\left\{\text{cosec}^{-1}\frac{\sqrt{1+x^2}}{x}\right\} - \tan\left\{\tan^{-1}\left(\frac{1}{x}\right)\right\}\right]$$

$$= 2\tan^{-1}\left[\left\{\frac{\sqrt{1+x^2}}{x} - \frac{1}{x}\right\}\right] = 2\tan^{-1}\left[\frac{\sqrt{1+x^2}-1}{x}\right]$$

$$= 2\tan^{-1}\left[\frac{\sec\theta - 1}{\tan\theta}\right] \quad (\text{put} x = \tan\theta)$$

$= 2\tan^{-1}\left[\frac{1-\cos\theta}{\sin\theta}\right]$

$= 2\tan^{-1}\left[\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}.\cos\frac{\theta}{2}}\right]$

$= 2\tan^{-1}\tan\frac{\theta}{2} = 2.\frac{\theta}{2} = \theta = \tan^{-1}x$

Question 127

Let f (x+y) = f(x) + f (y) for all x and y. If the function f(x) is continuous at x = 0, then f(x) is continuous

Options:

A. only at x = 0

B. at x $\varepsilon R - \{0\}$

C. for all x

D. None of these

Answer: C

Solution:

Given, f(x+y) = f(x) ;for all x and y.
Since, f(x) is continuous at x=0 we have
$\lim_{x\to 0} f(x) = f(0).$

To show that f(x) is continuous at any point a, we shall prove that
$\lim_{x\to a} f(x) = f(a)$
$\lim_{h\to 0} f(a+h) = f(a)$
Indeed $f(a+h) = \lim_{h\to 0}[f(a)+f(h)]$
$= f(a) + \lim_{h\to 0} f(h) = f(a) + f(0)$
$= f(a+0) = f(a)$

Question 128

Let f(x)= $\begin{Bmatrix} x^n \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{Bmatrix}$. Then, f(x) is continuous but not differentiable at x = 0, if

Options:

A. $n\varepsilon(0,1)$

B. $n\varepsilon[1,\infty)$

C. $n\varepsilon(-\infty,0)$

D. n=0

Answer: A

Solution:

Since, f(x) is continuous at x = 0, therefore

$\lim_{x\to 0} f(x) = f(0) \Rightarrow \lim_{x\to 0} x^2 \sin\frac{1}{x} = 0, \forall n > 0$

$f(x)\text{ is differentiable at } x = 0, \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$

exists finitely.

$\Rightarrow \lim_{x\to 0}\frac{f(x)-f(0)}{x-0}\text{ exists finitely}$

$\Rightarrow \lim_{x\to 0} x^{n-1}\sin\frac{1}{x}\text{ exists finitely}$

$\Rightarrow n-1 > 0 \Rightarrow n > 1$

Hence f(x) is continuous

but NOT differentiable at $x = 0, \text{if } n \in (0,1).$

Question 129

If $\sin^2 x+\cos^2 y = 1$, then $\frac{dy}{dx}$ is equal to

Options:

A. $\frac{\sin 2x}{\sin 2y}$

B. $\frac{\sin^2 y}{\sin 2x}$

C. $\frac{\sin^2 x}{\sin^2 y}$

D. $-\frac{\sin^2 y}{\sin^2 x}$

Answer: A

Solution:

Given, $\sin^2 x + \cos^2 y = 1$

On differentiating both sides w.r.t. x, we

$\frac{d}{dx}\left(\sin^2 x + \cos^2 y\right) = \frac{d}{dx}(1)$

$\Rightarrow 2\sin x\cos x + 2\cos y\left(-\sin y\frac{dy}{dx}\right) = 0$

$\left[\text{using chain rule}, \frac{d}{dx} f\{g(x)\} = f'(x)\frac{d}{dx}g(x)\right]$

$\Rightarrow -2\sin y\cos y\frac{dy}{dx} = -2\sin x\cos x$

$\Rightarrow \frac{dy}{dx} = \frac{-\sin 2x}{-\sin 2y} = \frac{\sin 2x}{\sin 2y}$

$(\because \sin 2x = 2\sin x\cos x)$

Question 130

The attitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is

Options:

A. $\frac{r}{2}$

B. $\frac{r}{3}$

C. $\frac{3r}{4}$

D. $\frac{4r}{3}$

Answer: D

Solution:

Let R be the radius and h be the height of cone.

$\therefore OA = h - r$

in ΔOAB,

$r^2 = R^2 + (h-r)^2$

$\Rightarrow r^2 = R^2 + h^2 + r^2 - 2rh$

$\Rightarrow R^2 = 2rh - h^2$

The volume V of the cone is given by

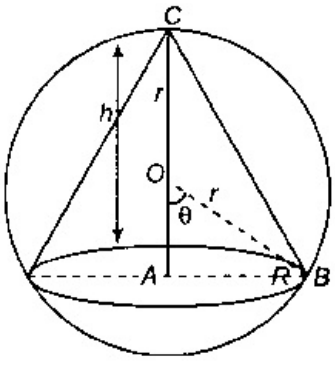
$V = \frac{1}{3}\pi R^2 h$

$= \frac{1}{3}\pi h(2rh - h^2) = \frac{1}{3}\pi(2rh^2 - h^3)$

On differentiation w. r. t. h, we get

$\frac{dv}{dh} = \frac{1}{3}\pi(4rh - 3h^2)$

For maximum and minimum, put $\frac{dv}{dh} = 0 \Rightarrow 4rh - 3h^2$
$\Rightarrow 4r = 3h$



$\Rightarrow h = \frac{4r}{3} \qquad (h \neq 0)$

Now, $\frac{d^2V}{dh^2} = \frac{1}{3}\pi(4r - 6h)$

At $h = \frac{4r}{3}$,

$\left(\frac{d^2V}{dh^2}\right)_{h=\frac{4r}{3}} = \frac{1}{3}\pi\left(4r - 6 \times \frac{4r}{3}\right)$

$= \frac{\pi}{3}(4r - 8r)$

$= \frac{-4r\pi}{3} < 0$

$\Rightarrow V$ is maximum when $h = \frac{4r}{3}$.

Hence, volume of the cone is maximum when $h\frac{4r}{3}$ When is the altitude of cone.

Question 131

$\int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}}$ **dx is equal to**

Options:

A. $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x}{x+1}\right) + c$

B. $\frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{x}{\sqrt{3}(x+1)}\right] + c$

C. $\frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{x}{(x+1)^2}\right] + c$

D. None of the above

Answer: B

Solution:

Let $I = \int \frac{x+2}{(x^2+3x+3)\sqrt{x+1}} dx$

$\text{put } x+1=t^2 \Rightarrow dx=2t\,dt$

$$\therefore I=\int \frac{\left(t^2-1\right)+2}{\left\{\left(t^2-1\right)^2+\left(t^2-1\right)+3\right\}\sqrt{t^2}}.(2t)\,dt$$

$$=2\int \frac{t^2+1}{t^4+t^2+1}dt=2\int \frac{1+\frac{1}{t^2}}{t^2+1\frac{1}{t^2}}dt$$

$$=2\int \frac{1+\frac{1}{t^2}}{\left(t-\frac{1}{t}\right)+\left(\sqrt{3}\right)^2}dt$$

$$=2\int \frac{du}{u^2+\left(\sqrt{3}\right)^2}$$

$$\left(\text{Where, } u=t-\frac{1}{t}\Rightarrow du=1+\frac{1}{t^2}dt\right)$$

$$=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{u}{\sqrt{3}}\right)+c$$

$$\therefore I=\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t^2-1}{\sqrt{3}t}\right)+c$$

$$=\frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{x}{\sqrt{3(x+1)}}\right]+c$$

Question 132

$\int \sin^{-1}\sqrt{\frac{x}{a+x}}dx$ is equal to

Options:

A. $\left[\tan^{-1}\sqrt{\frac{x}{a}}+\sqrt{\frac{x}{a}}\right]+c$

B. $a\left[\tan^{-1}\sqrt{\frac{x}{a}}-\sqrt{\frac{x}{a}}\right]+c$

C. $a\left[\tan^{-1}\sqrt{\frac{x}{a}}.\frac{(a+x)}{a}\right]+c$

D. $a\left[\tan^{-1}\sqrt{\frac{x}{a}}.\frac{(a+x)}{a}-\sqrt{\frac{x}{a}}\right]+c$

Answer: D

Solution:

$$\text{Let } I=\int \sin^{-1}\sqrt{\frac{x}{a+x}}dx$$

$\text{Put } x=a\tan^2\theta$

$\Rightarrow dx=2a\tan\theta\sec^2\theta\,d\theta$

$$\text{Now, } I=\int \sin^{-1}\sqrt{\frac{a\tan^2\theta}{a+a\tan^2\theta}}.2a\tan\theta\sec^2\theta\,d\theta$$

$$\int \sin^{-1}\sqrt{\frac{\tan^2\theta}{\sec^2\theta}}.2a\tan\theta\sec^2\theta\,d\theta$$

$\left(\because 1+\tan^2\theta=\sec^2\theta\right)$

$= \int \sin^{-1}(\sin\theta)\, 2a\tan\theta\sec^2\theta\, d\theta$

$= 2a\int \underset{I}{\theta}\left(\tan\theta\sec^2\theta\right) d\theta$

$\left[\because \sin^{-1}(\sin\theta) = \theta\right]$

$= 2a\left[\theta\int(\tan\theta\sec^2\theta)\, d\theta\right]$

$-\int\left\{\frac{d}{d\theta}(\theta)\int\tan\theta\sec^2\theta d\theta\right\} d\theta$

$= 2a\left[\theta\frac{\sec^2\theta}{0} - \int 1\frac{\sec^2\theta}{2} d\theta\right] + c$

$\left(\int\tan\theta\sec^2\theta = \int z dz = \frac{z^2}{2} = \frac{\sec^2\theta}{2}\right)$,

where, $z = \sec\theta$

$= a\left[\theta\sec^2\theta - \tan\theta + c\right]$

$= a\left[\tan^{-1}\sqrt{\frac{x}{a}}.\frac{a+x}{a} - \sqrt{\frac{x}{a}}\right] + c$

$\left(\because 1 + \tan^2\theta = \sec^2\theta \text{ and } \sec^2\theta = 1 + \frac{x}{a}\right)$

Question 133

A girl walks 4 km towards West, then she walks 3 km in a direction 30^0 East of North and stops. Then, the girl's displacement from her initial point of departures is

Options:

A. $-\frac{5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

B. $\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$

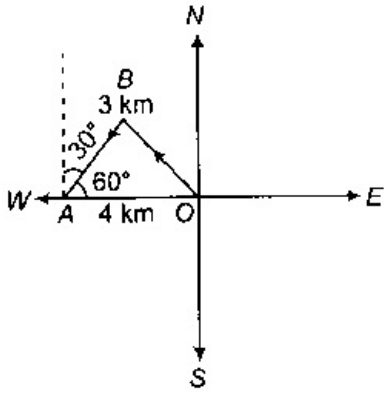
C. $-\frac{1}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

D. None of these

Answer: A

Solution:

Let O and B the initial and final position of the girl, respectively. Then the girl's position can be shown as in the figure.



Now we have $OA = 4\hat{i}$

$AB = \hat{i}|AB|\cos 60^0 + \hat{j}|AB|\sin 60^0$

($AB\cos 60^0$ is component of AB along x-axis and $AB\sin 60^0$ is component of AB along y-axis).

$\hat{i} = 3 \times \frac{1}{2} + \hat{j}3 \times \frac{\sqrt{3}}{2} = \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

By the trangle law of vector law of vector addition, we have

$OB = AO + AB$

$= (-4\hat{i}) + (\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j})$

$= (-4 + \frac{3}{2})\hat{i} + \frac{3\sqrt{3}}{2}\hat{j})$

$= (\frac{-8+3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j} = \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

Hence, the girl's displacement from her initial point of departure is $\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$.

Question 134

If $a = \hat{i} + \hat{j} + \hat{k}, b = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $c = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|c| = \sqrt{3}$, then the value of α and β are respectively

Options:

A. $\pm 1, 1$

B. $\pm 2, 1$

C. $0, \pm 1$

D. None of these

Answer: A

Solution:

$\because$ **a, b** and c are linearly dependent vectors.

$\Rightarrow [abc] = 0$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

$\Rightarrow 1(3\beta - 4\alpha) - 1(4\beta - 4) + 1(4\alpha - 3) = 0$

$\Rightarrow -\beta + 1 = 0 \Rightarrow \beta = 1$

Now $|c| = \sqrt{3}$

$\Rightarrow \sqrt{1 + \alpha^2 + \beta^2} = \sqrt{3}$

$\Rightarrow 1 + 1 + \alpha^2 = 3 \Rightarrow \alpha^2 = 1$

$\Rightarrow \alpha = \pm 1$

Question 135

The projection of the vector $a = \hat{i} - 2\hat{j} + \hat{k}$ on the vector b= $4\hat{i} - 4\hat{j} + 7\hat{k}$ is

Options:

A. $\frac{9}{19}$

B. $\frac{19}{9}$

C. 9

D. $\sqrt{19}$

Answer: B

Solution:

We know then, projection of a on b = $b = \frac{ab}{|b|}$

$\therefore ab = (\hat{i} - 2\hat{j} + k).(4\hat{i} - \hat{j} + 7\hat{k})$

$= 4 + 8 + 7 = 19$

and $|b| = \sqrt{4^2 + 4^2 + 7^2} = \sqrt{81} = 9$

Hence, projection of a and b is $\frac{19}{9}$.

Question 136

Forces of magnitude 5 and 3 units acting in the directions $6\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + 6\hat{k}$ respectively act on a particle which is displaced from the point (2,2,-1) to (4,3,1). The work done by the forces is

Options:

A. 148 units

B. $\frac{148}{7}$ units

C. $\frac{78}{7}$ units

D. None of these

Answer: B

Solution:

Let F be the resultant force and d be the displacement vector.

Then, F = $5\frac{(6\hat{i} - 2\hat{j} + 3k)}{\sqrt{63 + 4 + 9}} + 3\frac{(3\hat{i} - 2\hat{j} + 6\hat{k})}{\sqrt{9 + 4 + 36}}$

$= \frac{1}{7}\left(39\hat{i}+33\hat{k}\right)$

and $d = \left(4\hat{i}+3\hat{j}+\hat{k}\right)-\left(2\hat{i}+2\hat{j}-\hat{k}\right)$

$= \left(2\hat{i}+\hat{j}+2\hat{k}\right)$

$\therefore$ Total work done = F.d

$= \frac{1}{7}\left[\left(39\hat{i}+4\hat{j}+33\hat{k}\right).\left(2\hat{i}+\hat{j}+2\hat{k}\right)\right]$

$\frac{1}{7}[78+4+66] = \frac{148}{7}$ Units

Question 137

The solution of differential equation $\left(x^2+y^2\right)-2xy\frac{dy}{dx}=0$ is

Options:

A. $x^2+y^2=xC$

B. $x^2-y^2=xC$

C. $x^2+y^2=C$

D. $x^2-y^2=C$

Answer: B

Solution:

Given differential equation is $\left(x^2+y^2\right)-2xy\frac{dy}{dx}=0$

Which is homogeneous.

($\because$ degree of each term is same i.e.,2)

It can be rewritten as

$\frac{dy}{dx}=\frac{\left(x^2+y^2\right)}{2xy}=\frac{1}{2}\left[\frac{x}{y}+\frac{y}{x}\right]$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dy}{dx}$,

So that the differential

equation becomes

$v = x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1}{v}+v\right) \Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{1-v^2}{2v}$

$\Rightarrow \int \frac{2v}{1-v^2}dv = \int \frac{1}{x}dx$

$\Rightarrow -\log\left(1-v^2\right) = \log|x| - \log|c|$

$\Rightarrow x\left(1-v^2\right) = c$

$\Rightarrow \frac{x^2-y^2}{x} = c \quad \left(\because v = \frac{y}{x}\right)$

Hence, $x^2-y^2 = xC$ is the required solution.

Question 138

The solution of the equation $\frac{dy}{dx}=\frac{x(2\log x+1)}{\sin y+y\cos y}$ is

Options:

A. $y\sin y=x^2\log x+c$

B. $y=x^2+\log x+c$

C. $y\sin y=x^2+c$

D. None of the above

Answer: A

Solution:

Given equation is $\frac{dy}{dx}=\frac{x(2\log x+1)}{\sin y+y\cos y}$

$\Rightarrow(\sin y+y\cos y)dx=x(2\log x+1)dx$

On integrating both sides, sides, we get

$\int\sin y dy+y\sin y-\int\sin y dy$

$=x^2\log x-\int x^2.\frac{1}{x}dx+\int x dx+c$

$\Rightarrow y\sin y=x^2\log x+c$

Question 139

The area of the portion of the circle $x^2+y^2=64$ which is exterior to the parabola $y^2=12x$, is

Options:

A. $(8\pi-\sqrt{3})$ sq units

B. $\frac{16}{3}(8-\sqrt{3})$ sq units

C. $\frac{16}{3}(8\pi-\sqrt{3})$ sq units

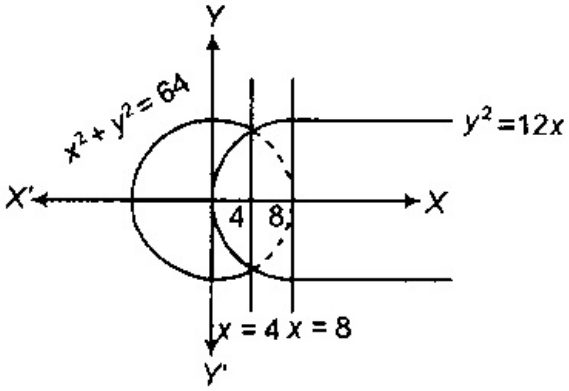
D. None of the above

Answer: C

Solution:

Required shaded area =

Area of circle $-2\left[\int_0^4 2\sqrt{3}\sqrt{x}dx+\int_4^8\sqrt{64-x^2}dx\right]$



$$=64\pi-2\left[\left(2\sqrt{3}x^{3/2}\times\frac{2}{3}\right)\right]_0^4$$
$$+\left(\frac{x}{2}\sqrt{64-x^2}+\frac{64}{2}\sin^{-1}\frac{x}{8}\right)_4^4\Bigg]$$
$$=64\pi-\frac{64}{\sqrt{3}}-32\pi+16\sqrt{3}+\frac{32\pi}{3}$$
$$=\frac{128\pi}{3}-\frac{16\sqrt{3}}{3}$$
$$=\frac{16}{3}\left(8\pi-\sqrt{3}\right)\text{sq units}$$

Question 140

$\lim_{n\to\infty}\frac{1}{n}\left(\frac{1}{n+1}+\frac{2}{n+2}+.....+\frac{3n}{4n}\right)$ **is equal to**

Options:

A. log 4

B. -log4

C. 1-log4

D. None of the above

Answer: B

Solution:

$\lim_{n\to\infty}\frac{1}{n}\left(\frac{1}{n+1}+\frac{2}{n+2}+.....+\frac{3n}{4n}\right)$

$$\lim_{n\to\infty}\frac{1}{n}\left(\frac{\frac{1}{n}}{1+\frac{1}{n}}+\frac{\frac{2}{n}}{1+\frac{2}{n}}+.....+\frac{\frac{3n}{n}}{1+\frac{3n}{n}}\right)$$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{3n}\left(\frac{\frac{r}{n}}{1+\frac{r}{n}}\right)$$

$$=\int_0^3\frac{x}{1+x}dx$$

$$=\int_0^3\frac{x+1-1}{1+x}dx$$

$$=\int_0^3 dx-\int_0^3\frac{1}{1+x}dx$$

$$=[x-\log(x+1)]_0^3$$

$$=-\log 4$$

Question 141

The value of integral $\int_0^{\pi/2}\frac{\sin^2 x}{\sin x+\cos x}dx$ is equal to

Options:

A. $\sqrt{2}\left(\log\sqrt{2}\right)$

B. $\sqrt{2}\left(\sqrt{2}+1\right)$

C. $\frac{1}{\sqrt{2}}\log\left(\sqrt{2}+1\right)$

D. None of the above

Answer: C

Solution:

Let $I=\int_0^{\pi/2}\frac{\sin^2 x}{\sin x+\cos x}dx$ .....(i)

Now, $I=\int_0^{\pi/2}\frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos x\left(\frac{\pi}{2}-x\right)}dx$

$\Rightarrow I=\int_0^{\pi/2}\frac{\cos^2 x}{\sin x+\cos x}dx$.....(ii)

On adding Eqs. (i),we get

$$2I = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x}\,dx$$

$$= \int_0^{\pi/2} \frac{1}{\sin x + \cos x}dx$$

$$= \frac{1}{\sqrt{2}} \int_0^{\pi/2} \frac{1}{\cos\left(x - \frac{\pi}{4}\right)}dx$$

$$= \frac{1}{\sqrt{2}} \int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right)dx$$

$$= \frac{1}{\sqrt{2}} \left[\log\left\{\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right\}\right]_0^{\pi/2}$$

$$= \frac{1}{\sqrt{2}} \left[\log\left(\sqrt{2} + 1\right) - \log\left(\sqrt{2} - 1\right)\right]$$

$$= \frac{1}{\sqrt{2}} \log\left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1}\right)$$

$$= \frac{1}{\sqrt{2}} \log\left(\sqrt{2} + 1\right)^2$$

$$= \sqrt{2} \log\left(\sqrt{2} + 1\right)$$

$$I = \frac{1}{\sqrt{2}} \log\left(\sqrt{2} + 1\right)$$

Question 142

The product of the perpendiculars drawn from the foci upon any tangent to an ellipse

Options:

A. depends upon foci

B. is constant

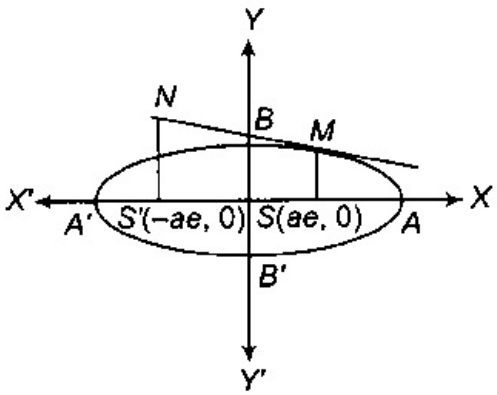
C. depends upon the tangent

D. None of the above

Answer: B

Solution:

Let equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \ldots (i)$



Its foci are S(ae,0) and (-ae,0), The equation of tangent at any point (a $\cos\theta$,bsinθ)to ellipse is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ ....(ii)

Let the perpendicular from S and S’ upon Eq. (ii) be SM and S’N.

Then, SM. S’N= $\frac{-1}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}\left(e^2\cos^2\theta-1\right) \Rightarrow SM.S'N=b^2=Constant$

Question 143

The parametric equations of the circle $x^2+y^2+mx+my=0$ **are**

Options:

A. $x=-\frac{m}{2}+\frac{m}{\sqrt{2}}\cos\theta, y=\frac{m}{2}+\frac{m}{\sqrt{2}}\sin\theta$

B. $x=-\frac{m}{2}+\frac{m}{\sqrt{2}}\cos\theta, y=-\frac{m}{2}+\frac{m}{\sqrt{2}}\sin\theta$

C. $x=0, y=0$

D. None of the above

Answer: B

Solution:

Here, $x^2+y^2+mx+my=0$

$x^2+y^2+mx+my=0$

$\Rightarrow \left(x^2+mx\right)+\left(y^2+mx\right)=0$

$\Rightarrow \left(x^2+mx+\frac{m^2}{4}\right)^2+\left(y^2+mx+\frac{m^2}{4}\right)=\frac{m^2}{2}$

$\Rightarrow \left[x-\left(-\frac{m}{2}\right)\right]^2+\left[y-\left(-\frac{m}{2}\right)\right]^2=\left(\frac{m}{\sqrt{2}}\right)^2$

So, the parametric equations of circle are

$x=-\frac{m}{2}+\frac{m}{\sqrt{2}}\cos\theta$

$and\, y=-\frac{m}{2}+\frac{m}{\sqrt{2}}\sin\theta$

Question 144

If in a $\Delta ABC, \sin^3 A+\sin^3 B+\sin^3 C=3\sin A\sin B\sin C,$ **then the value of the determinant**

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

Options:

A. 0

B. 1

C. 2

D. 3

Answer: A

Solution:

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$= (a+b+c)(bc+ca+ab-a^2-b^2-c^2)$$

$$= -(a^3+b^3+c^3-3abc)$$

$$= -8R^3(\sin^3 A+\sin^3 B+\sin^3 C-3\sin A\sin B\sin C)$$

$$= -8R^3(3\sin A\sin B\sin C-3\sin A\sin B\sin C)$$

$$= 0$$

Question 145

$\sin\frac{\pi}{n}+\sin\frac{3\pi}{n}+\sin\frac{5\pi}{n}+.....$up to n terms is equal to

Options:

A. 1

B. 2

C. 3

D. 0

Answer: D

Solution:

$$\sin\frac{\pi}{n}+\sin\frac{3\pi}{n}+\sin\frac{5\pi}{n}+....\text{upton terms}$$

$$= \frac{\sin\frac{n.2\pi}{2n}}{\sin\frac{2\pi}{2n}}.\sin\left(\frac{2.\frac{\pi}{n}+(n-1)2\frac{\pi}{n}}{2}\right)$$

$$= \frac{\sin\pi}{\sin\frac{\pi}{n}}.\sin\left(\frac{2\pi+2n\pi-2\pi}{2n}\right)$$

$$= \frac{\sin^2\pi}{\sin\frac{\pi}{n}} = 0$$

Question 146

The maximum and minimum value of $6\sin x\cos x+4\cos 2x$ are respectively

Options:

A. 5,5

B. -5,5

C. 5,-5

D. None of these

Answer: C

Solution:

$6\sin x\cos x + 4\cos 2x = 3\sin 2x + 4\cos 2x$

$\therefore -\sqrt{3^2+4^2} \le 3\sin 2x + 4\cos 2x \le \sqrt{3^2+4^2}$

$\Rightarrow -5 \le 3\sin 2x + 4\cos 2x \le 5$

Thus maximum valueis

5 and minimum value is -5.

Question 147

Let f(x) = x $(x-1)^2$, the point at which f(x) assumes maximum and minimum are respectively

Options:

A. $\frac{1}{3}, 1$

B. $1, \frac{1}{3}$

C. 3,1

D. None of these

Answer: A

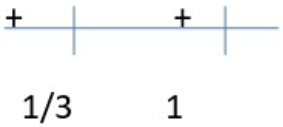
Solution:

Given, f(x) = x $(x-1)^2$

$\therefore$ F’(x)=2x(x-1)+$(x-1)^2$

$\Rightarrow$ F’(x)=(x-1)(2x+x-1)

$\Rightarrow$ F’(x)=(x-1)(3x-1)



Using number line rule for f’(x), we have adjoining figure which shows f’(x) changes sing from +ve to – ve at x $\frac{1}{3}$.Hence, at x=$\frac{1}{3}$, we have maximum and f’(x) changes sing from –ve to +ve at x = 1. Hence f(x) minimum at x = 1.

Question 148

Rectangles are inscribed in a circle of radius r. The dimensions of the rectangle which has the maximum area, are

Options:

A. r,r

B. 2r,2r

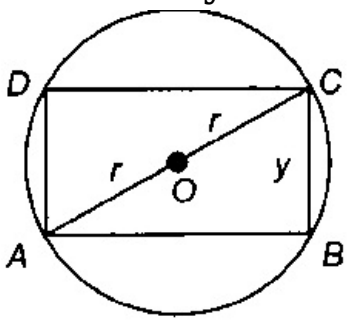
C. $\sqrt{2}r, \sqrt{2}r$

D. None of these

Answer: C

Solution:

Let ABCD be the rectangle inscribed in a circle of radius r.
Let AB= x
And BC =y



Then, $x^2+y^2=4r^2$

$\Rightarrow y = \sqrt{4r^2 - x^2}$ ...(i)

Area of rectangle,

$A = xy$

$= x\sqrt{4r^2 - x^2}$

$Let\, u = A^2 = x^2\left(4r^2 - x^2\right)$

$\Rightarrow \frac{dy}{dx} = 8r^2x - 4x^3$

$Put \frac{dy}{dx} = 0 for\, maxima\, or\, minima$

$\therefore 4x\left(2r^2 - x^2\right) = 0$

$\Rightarrow x = \sqrt{2}r$

$Also, \frac{d^2u}{dx^2} = 8r^2 - 12x^2$

$\therefore \left(\frac{d^2u}{dx^2}\right)_{x=\sqrt{2}r} = 8r^2 24r^2 < 0$

$\therefore u\, and\, A\, area\, maximum\, at\, x = \sqrt{2}r.$

$From\, Eq.(i) y = \sqrt{2}r = x$

$\therefore Dimensions\, of\, the\, rectange$

$are\, \sqrt{2}r\, and\, \sqrt{2}r.$

Question 149

The equation $4^{(x^2+2)} - 9.2^{(x^2+2)} + 8 = 0$ has the solution

Options:

A. $x = 1$

B. $x = 0$

C. $x = \sqrt{2}$

D. $x = -\sqrt{2}$

Answer: A

Solution:

$4^{(x^2+2)} - 9.2^{(x^2+2)} + 8 = 0$

$\Rightarrow \left(2^{(x^2+2)}\right)^2 - 9.2^{(x^2+2)} + 8 = 0$

Put $2^{(x^2+2)} = y$, then $y^2 - 9y + 8 = 0$

which given $y = 8$ and $y = 1$.

When $y = 8$ then $2^{x^2+2} = 8$

$\Rightarrow 2^{x^2+2} = 2^3$

$\Rightarrow x^2 + 2 = 3$

$\Rightarrow x^2 = 1$

$\Rightarrow x = 1, -1$

When $y = 1$ then $2^{x^2+2} = 1$

$\Rightarrow 2^{x^2+2} = 2^0$

$\Rightarrow x^2 + 2 = 0$

$\Rightarrow x^2 = -2$

Which is not possibe.

Question 150

If a, b, c are in GP and log a - log2b, log2b - log3c and log3c - loga are in AP, then a, b, c are the lengths of the sides of a triangle which is

Options:

A. acute angled

B. obtuse angled

C. right angled

D. equilateral

Answer: B

Solution:

As given, b^2=ac {since, a,b,c form a G.P.}
and 2(log 2b - log 3c) = [log a - log 2b] + [log 3c - log a]

So,2(log 2b - log 3c) = [log a - log 2b] + [log 3c - log a]
⇒ 2log 2b - 2log 3c = log 3c - log 2b
⇒3log2b - 3log3c = 0
⇒log(2b/3c) = 0
⇒2b=3c
⇒b/c = 3/2

Since, a,b,c are in G.P.
b/c = a/b =3/2
Thus, b=2a/3 and c=4a/9
b+c=10a/9 > a
So, a,b,c form a triangle with a as the greatest side (b & c are the fractional part of a)

The angle opposite to the greatest side is also the largest internal angle of the triangle.

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{\frac{b^2}{c^2}+1-\frac{a^2}{c^2}}{2\frac{b}{c}} = \frac{\frac{9}{4}+1-\frac{81}{16}}{2\left(\frac{3}{2}\right)} = \frac{\frac{36+16-81}{16}}{3} = \frac{-29}{48}$$

cos A <0 ⇒ A > π/2
∴The angle A is obtuse.
Hence, a, b, c are the lengths of sides of an obtuse-angled triangle.
