Analog Communication

* Modulation (Time domain):

→ Modulation is defined as the process in which the peak amplitude of carrier is varied according to the message signal.

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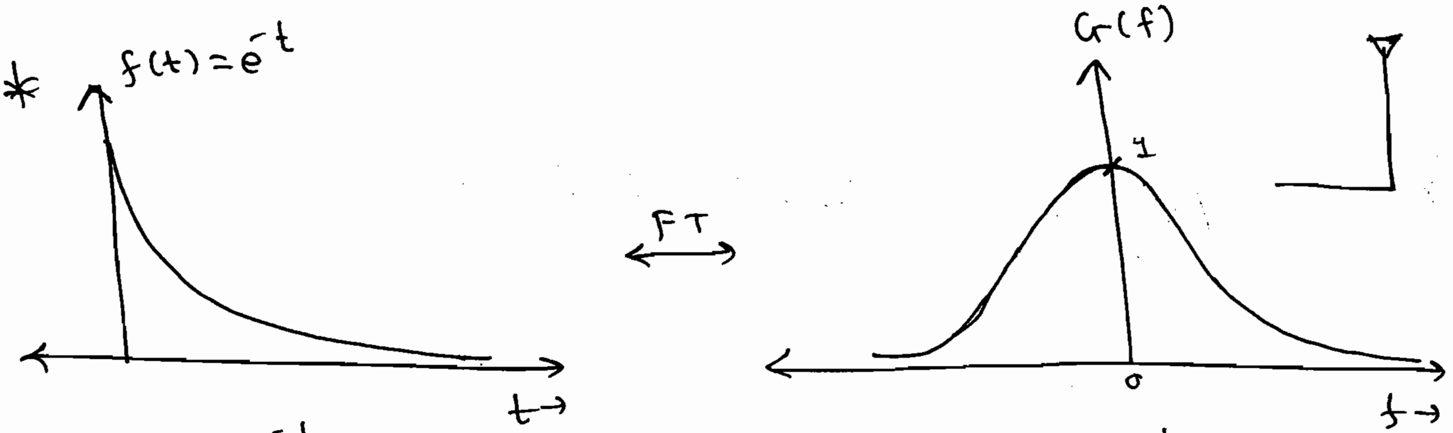

$f(t) = e^{-t}$ $\quad\quad$ $F(f) = \dfrac{1}{1+j2\pi f}$

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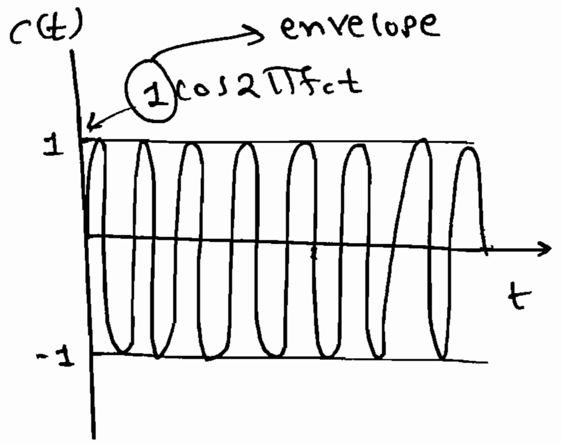
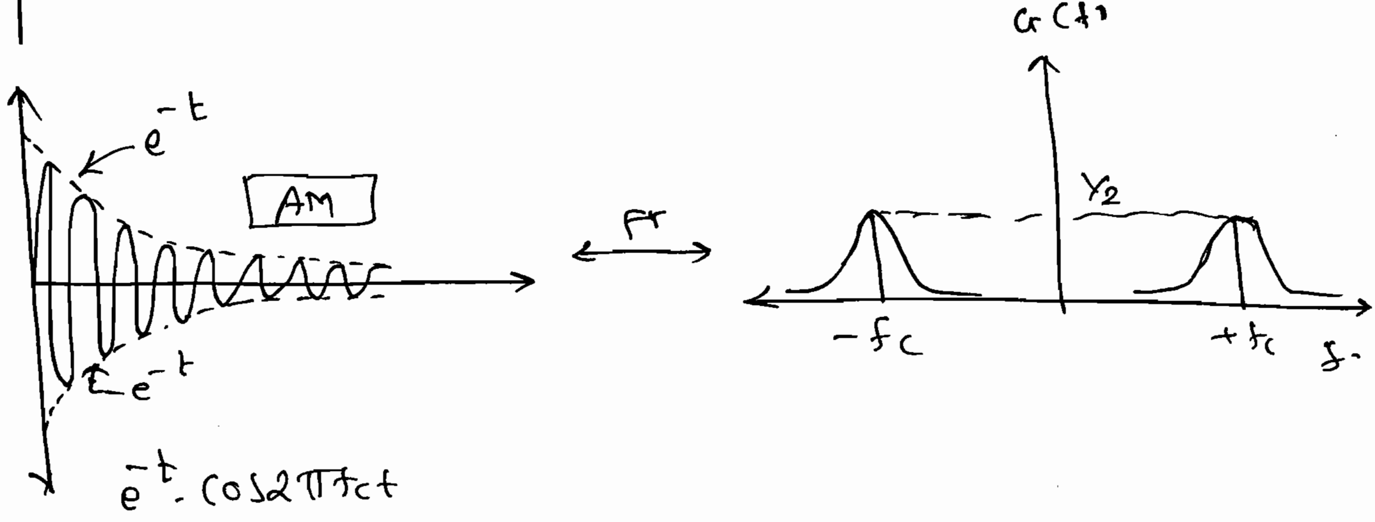




⇒ Three parameters of the carrier which can be varied according to the message signal are Amplitude, Freq. and phase.

→ So, three analog communication techniques are, $\boxed{\text{AM, FM \& PM.}}$

* H.B

$$\underbrace{A_c}_{\downarrow\ \text{AM}} \cos(2\pi \underbrace{f_c}_{\downarrow\ \text{FM}} t + \underbrace{\phi}_{\downarrow\ \text{PM}}).$$

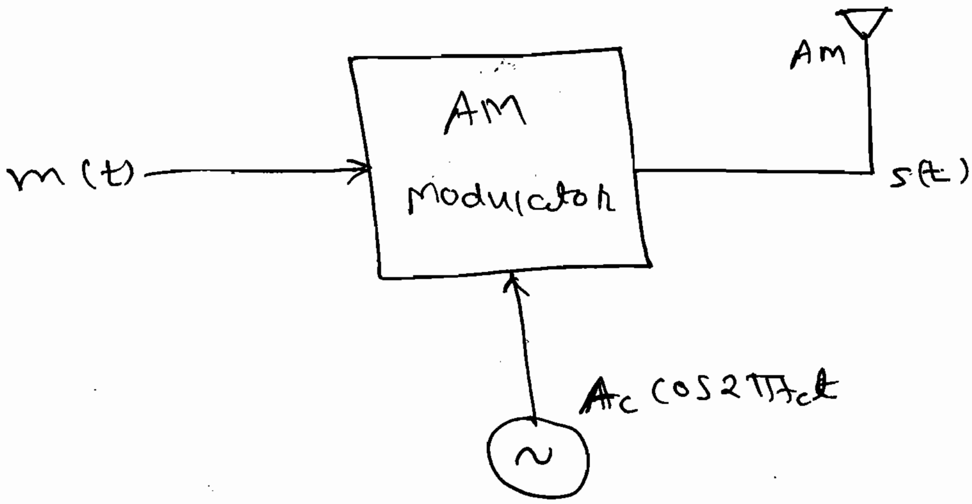
AM	FM	PM
① AM	① NBFM	① NBPM
② DSB	② WBFM	② WBPM.
③ SSB		
④ VSB		

(1) Amplitude Modulation (AM):

→ Amplitude Modulation is define as the process in which amplitude of the carrier varying according to the message signal.

→ Assume that the message signal (m(t)) and carrier signal is applied to the AM modulator.

⇒



→ The time domain eqn of the AM wave is

$$S(t) = A_c \cos 2\pi f_c t + K_a m(t) \cdot A_c \cos 2\pi f_c t$$

K_a = amplitude sensitivity of the modulator.

H.B.

$$\therefore \boxed{S(t) = A_c [1 + K_a m(t)] \cos 2\pi f_c t.}$$

This is the standard form.

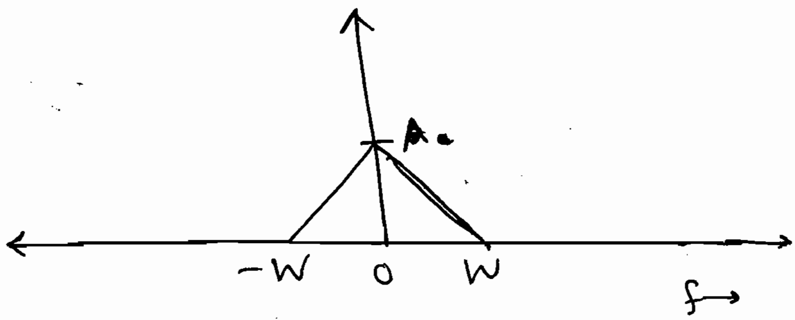
→ The peak amplitude of the carrier modulation is A_c and after modulation $A_c[1+K_a m(t)]$.

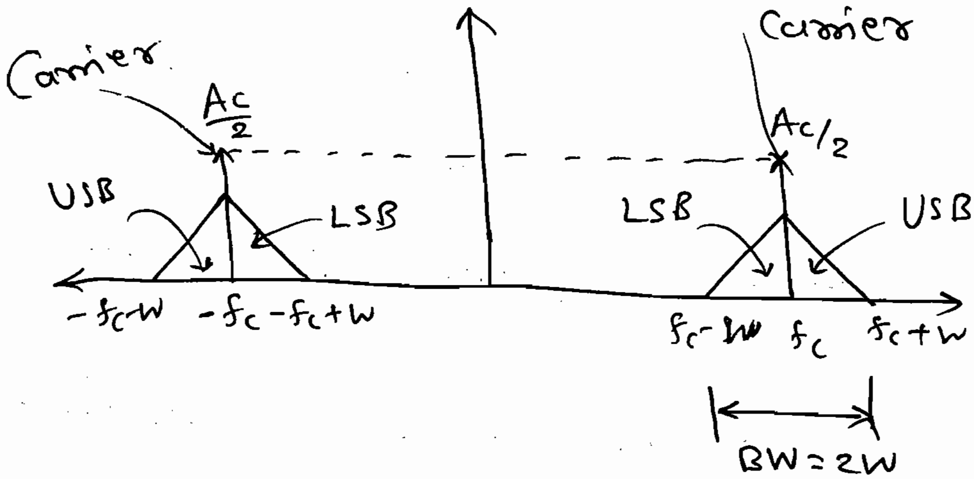
→ Now, Freq. domain eqn is,

H.B

$$\boxed{S(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{A_c K_a}{2}\left[M(f-f_c) + M(f+f_c)\right].}$$

→





→ $BW = f_H - f_L$

$BW = f_c + W - (f_c - W)$.

$\therefore \boxed{BW = 2W.}$

$\therefore$ $\boxed{\text{BW} = 2 \times \text{heighest freq. of } m(t).}$

→ AM spectrum consist of the following freqs.

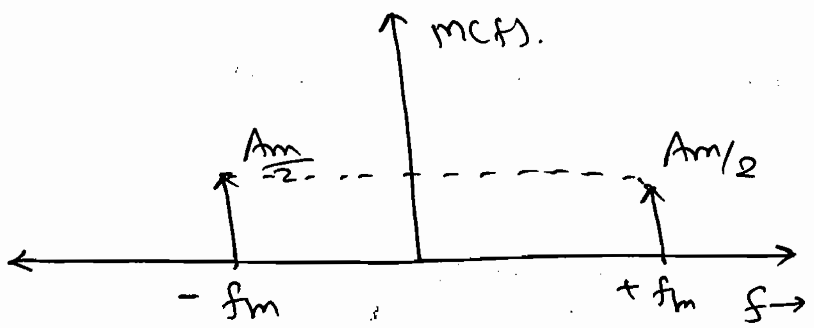
1. Carrier frequency 'f_c'.
2. Upper sideband freq. 'f_c' to 'f_c+W'.
3. Lower sideband freq. 'f_c-W' to 'f_c'.

* Single tone Modulation of AM:-

→ In the singletone modulation message is the sinusoidal signal.

$$m(t) = A_m \cos 2\pi f_m t$$

$$M(f) = \frac{A_m}{2}\left[\delta(f-f_m) + \delta(f+f_m)\right].$$



⇒ Time domain eqⁿ of the AM signal for singletone modulation is.

H.B.

$$\boxed{\begin{aligned} s(t) &= A_m \cos 2\pi f_c t + A_c K_a A_m \cos 2\pi f_m t \cdot \cos 2\pi f_c t \\ K_a A_m &= \mu = m \\ &= \text{modulation index} \\ &= \text{depth of modulation.} \end{aligned}}$$

⇒ The modulation index concept is required for the demodulation of AM.

→ Practically the modulation index lies bet^n 0 to 1.

$$\boxed{0 < \mu, m \leq 1.}$$

→ When $\mu = 0$ the carrier is not modulated.

→ When $\mu > 1$ the ~~carrier~~ AM signal is said to be over modulated.

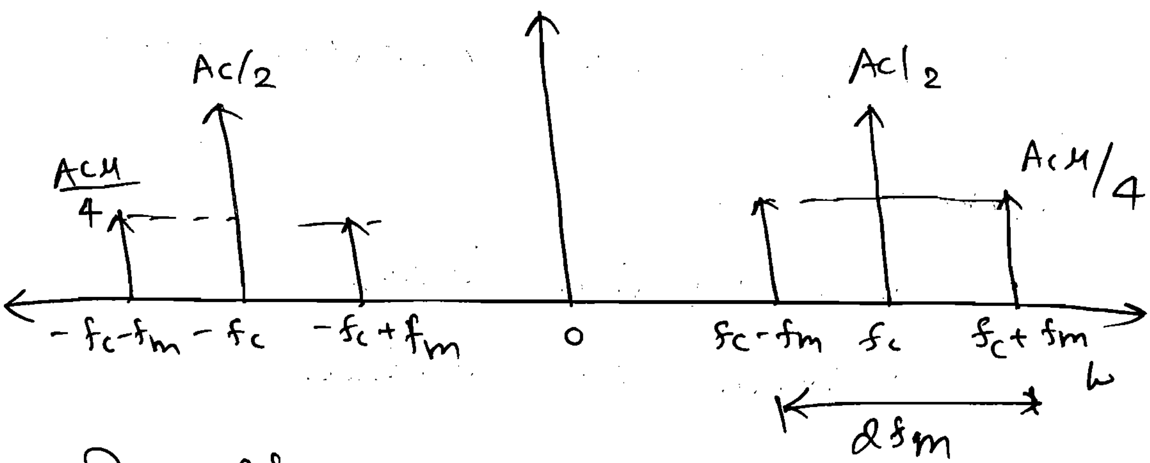
H.B.

$$\therefore \boxed{s(t) = A_c \left[1 + \mu \cos 2\pi f_m t \right] \cos 2\pi f_c t.}$$

$$\therefore s(t) = \underbrace{A_c \cos 2\pi f_c t}_{\text{Carrier}} + \underbrace{\frac{A_c \mu}{2} \cos 2\pi (f_c + f_m)t}_{\text{USB}}$$

H.B.

$$+ \underbrace{\frac{A_c \mu}{2} \cos 2\pi (f_c - f_m)t}_{\text{LSB.}}$$



Bw = 2fm.

* Power Calculation:

$\rightarrow \quad P_t = P_c + P_{USB} + P_{LSB}. \leftarrow$ H.B.

$\rightarrow \quad P_c = \frac{V_{rms}^2}{R} = \frac{\left(\frac{V_m}{\sqrt{2}}\right)^2}{R}.$

$\therefore \quad P_c = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R}$

H.B. $\therefore \quad \boxed{P_c = \frac{A_c^2}{2R}.}$

R = Antenna Resistance (Ω)

it not given then take

$R = 1\Omega.$

$\rightarrow \quad P_{USB} = \frac{\left(\frac{A_c \mu}{2\sqrt{2}}\right)^2}{R}$

$\therefore \quad P_{USB} = \frac{A_c^2 \mu^2}{8R} \leftarrow$ H.B.

$\therefore \quad P_{LSB} = \frac{A_c^2 \mu^2}{8R} \leftarrow$ H.B.

$\therefore \quad P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu^2}{8R} + \frac{A_c^2 \mu^2}{8R}.$

$\therefore \quad P_t = \frac{A_c^2}{2R}\left[1 + \frac{\mu^2}{2}\right].$

$\therefore \quad \boxed{P_t = P_c\left[1 + \frac{\mu^2}{2}\right].} \leftarrow$ H.B.

$\rightarrow \quad P_t = P_c + \frac{P_c \mu^2}{2}.$

side Band power.

Carrier power

→ Total Power $\boxed{P_t = P_c + P_{SB}.}$

→ Practically sideBand power is more important than the Carrier power because ~~strgon~~ signal to noise ratio depends only on the SideBand Power.

→ To determine the sideband power w.r.t. to the total power modulation efficiency (or) power efficiency is used.

$$\eta = \frac{P_{SB}}{P_t} \leftarrow \underline{\underline{H.B.}}$$

$$\therefore \eta = \frac{\frac{\cancel{P_c}\mu^2}{2}}{\cancel{P_c}\left(1+\frac{\mu^2}{2}\right)}$$

$$\therefore \boxed{\eta = \frac{\mu^2}{2+\mu^2}.} \leftarrow \underline{\underline{H.B.}}$$

When $\mu = 1 \Rightarrow \eta = 1/3$ or 33.33%

→ $P_{SB} = \frac{1}{3} P_t.$

(or) $\boxed{P_{SB} = 33.33\% \; P_t.}$

$\rightarrow$ $P_t = P_c + P_{SB}$.

$\mu = 1 \Rightarrow P_t = \frac{2}{3} P_t + \frac{1}{3} P_t$.

$P_t = 66.67\% P_t + 33.33\% P_t$.

$\mu = 0.707 \Rightarrow \mu = 1/\sqrt{2}$.

$\therefore$ $P_t = 80\% P_t + 20\% P_t$.

$\rightarrow$ In AM carrier takes more power than the sideband power. The maximum efficiency possible in the case of single tone modulation is 33.33 %.

$\rightarrow$ The maximum efficiency possible in AM is 50 %. (When a message is a square wave).

Ex-1 Consider an AM signal $s(t) = 20[1 + 0.9 \cos 2\pi 10^4 t] \cos 2\pi 10^6 t$. The signal is radiated into free space using an antenna having a resistance of $5\,\Omega$. Sketch the spectrum and calculate the B.W., power and modulation efficiency.

Sol^n: Here,

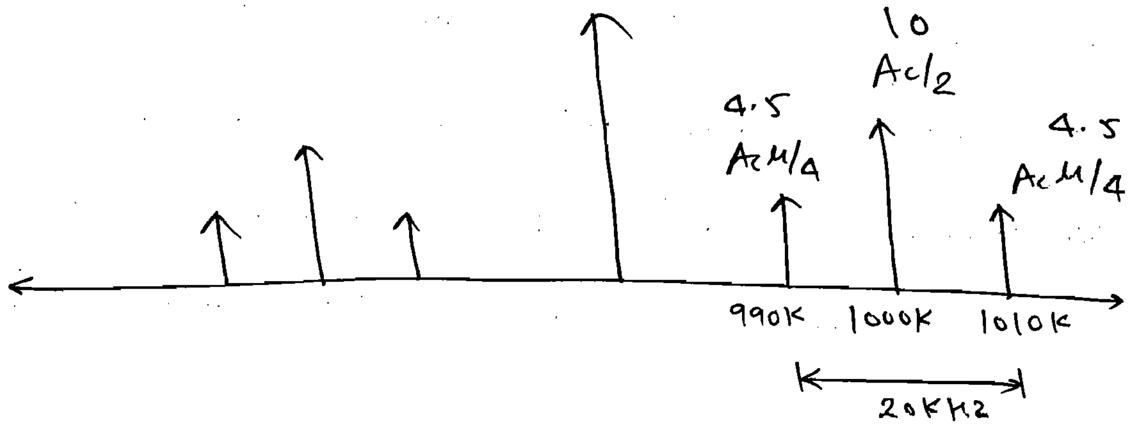
$s(t) = A_c[1 + \mu \cos 2\pi f_m t] \cos 2\pi f_c t.$

$\therefore \quad s(t) = 20[1 + 0.9 \cos 2\pi 10^4 t] \cos 2\pi 10^6 t.$

$\therefore \quad \mu = 0.9 \qquad A_c = 20.$

$f_m = 10 \text{ KHz}$

$f_c = 1000 \text{ KHz}.$



$\therefore \quad BW = 1010K - 990K$

$\boxed{BW = 20 \text{ KHz}}$

(OR)

$BW = 2 f_m.$

$= 2 \times 10$

$\boxed{BW = 20 \text{ KHz}}$

$P_c = \frac{A_c^2}{2R}.$

$\therefore \quad P_c = \frac{400}{2 \times 5}$

$\boxed{P_c = 40 \text{ watts}}$

$\therefore \quad P_t = P_c\left[1 + \frac{\mu^2}{2}\right].$

$\therefore \quad P_t = 40\left[1 + \frac{0.81}{2}\right]$

$\boxed{P_t = 56.2. \text{ watts.}}$

$$P_t = P_c + P_{SB}.$$

$$\therefore \quad P_{SB} = 56.2 - 40$$

$$\therefore \quad P_{SB} = 16.2 \text{ watts.}$$

$$\therefore \quad P_{USB} = P_{LSB} = 8.1 \text{ watts.}$$

$$\therefore \quad \eta = \frac{P_{SB}}{P_t} \qquad (\text{or}) \qquad \eta = \frac{\mu^2}{2+\mu^2}$$

$$\therefore \quad \eta = \frac{16.2}{56.2} \qquad \therefore \eta = \frac{0.81}{2.81}$$

$$\boxed{\eta = 28.8\,\%} \qquad \therefore \boxed{\eta = 28.8\,\%}$$

Ex. 2 Consider an AM signal

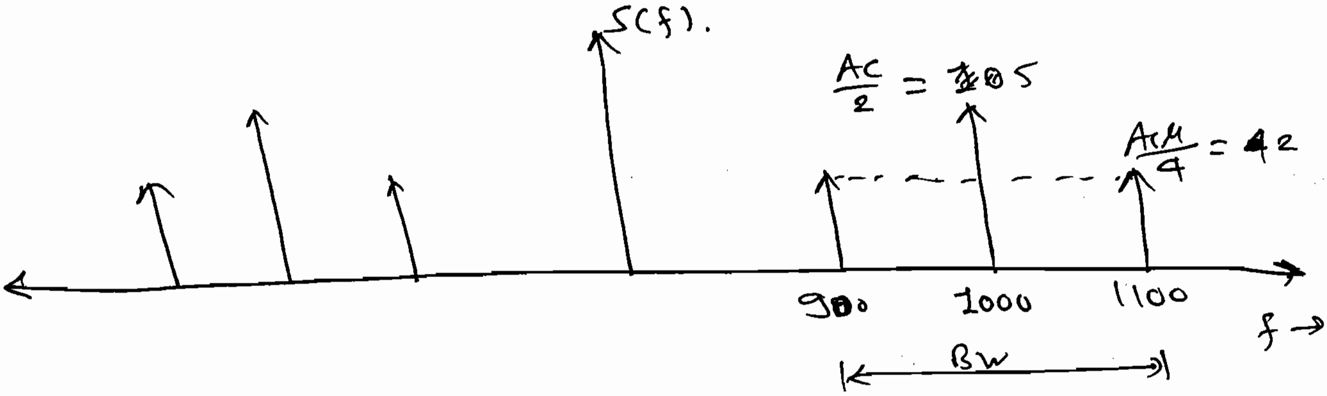
$$s(t) = 4\cos 1800\pi t + 10\cos 2000\pi t + 4\cos 2200\pi t.$$

Sketch the spectrum and calculate the B.W., power & modulation efficiency.

Soln:

$$s(t) = A_c \cos 2\pi f_c t + \frac{A_c\mu}{2}\cos 2\pi(f_c + f_m) + \frac{A_c\mu}{2}\cos 2\pi(f_c - f_m).$$

$$\rightarrow \quad s(t) = 10\cos 2000\pi t + 4\cos 1800\pi t + 4\cos 2200\pi t.$$



$\rightarrow \quad BW = 1100 - 900$

$$\boxed{BW = 200\text{Hz}}$$

$\rightarrow$ ~~Ac/2~~ $\times 10$

$$\boxed{A_c = 10}$$

$$\frac{A_c \mu}{2} = 4$$

$$\therefore \mu = \frac{4 \times 2}{10}$$

$$\boxed{\mu = 0.8}$$

$$P_c = \frac{A_c^2}{2R}$$

$$R = 1\Omega$$

$$\therefore P_c = \frac{100}{2}$$

$$\boxed{P_c = 50 \text{ watts}}$$

$$\Rightarrow \quad P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$$

$$\therefore P_t = 50\left[1 + \frac{0.64}{2}\right]$$

$$\therefore \boxed{P_t = 66\text{W}}$$

$$\therefore \eta = \frac{\mu^2}{2+\mu^2} = \frac{0.64}{2.64}$$

$$\boxed{\eta = 24.24\ \%}$$

Ex-3 A Carrier signal $c(t) = 5\cos 2\pi 10^6 t$ is modulated by message signal $m(t) = 4\cos 8\pi \times 10^3 t$. to generate an Am signal. Calculate the B.W. and power.

Ans: $c(t) = 5\cos 2\pi \times 10^6 t$.

$A_c = 5\text{ V}$

$f_c = 1000 \text{ KHz}$

$m(t) = 4\cos 8\pi 10^3 t.$

$\therefore \quad A_m = 4, \quad f_m = 4\ KHz.$

$\therefore \quad B_W = 2 f_m.$

$B_W = 2 \times 4\ K$

$\boxed{B_W = 8\ K\ Hz}$

$\rightarrow \quad P_c = \frac{A_c^2}{2R} = \frac{25}{2 \times 1} = 12.5\ W.$

$\therefore \quad \boxed{P_c = 12.5\ W.}$

* when μ is not given take

H.B

$K_a = \frac{1}{AC} \Rightarrow \boxed{\mu = \frac{A_m}{A_c}}.$

$\therefore \quad \mu = A_m / A_c$

$\therefore \quad \mu = \frac{4}{5} = 0.8.$

$\therefore \quad P_t = P_c \left[1 + \frac{\mu^2}{2}\right].$

$\therefore \quad P_t = 12.5 \left[1 + \frac{0.64}{2}\right].$

$\therefore \quad \boxed{P_t = 16.5\ \text{watts.}}$

$\eta = \frac{\mu^2}{2 + \mu^2}.$

$= \frac{0.64}{2.64}$

$\therefore \quad \boxed{\eta = 24.24\ \%}$

Ex-4 A carrier signal $c(t) = 5\cos 2\pi 10^6 t$ is modulated by a message signal $m(t) = \cos 4\pi 10^3 t$ to generate an AM signal with an modulation index of 0.5.

① Calculate B.W. and power.

② Determine the quantity $\left(P_{SB}/P_C\right)$

Gate: 2003

Ans:

→ $A_c = 5V$, $f_c = 1000$ kHz

$A_m = 1V$, $f_m = 2$ kHz.

$\mu = 0.5$.

$\therefore BW = 2 f_m$.

$\therefore \boxed{BW = 4 \text{ kHz}}$

$P_c = \dfrac{A_c^2}{2R}$

$\therefore P_c = \dfrac{25}{2}$

$\boxed{P_c = 12.5 \text{ W}}$

$\therefore P_t = P_c\left[1 + \dfrac{\mu^2}{2}\right]$.

$\therefore P_t = 12.5\left[1 + \dfrac{0.25}{2}\right]$

$\therefore \boxed{P_t = 14.0625 \text{ Watts}}$

i.e. $P_t = P_c + P_{SB}$.

$\therefore P_{SB} = 14.0625 - 12.5$

$\therefore P_{SB} = 1.5625$

$\therefore P_{SB}/P_c = \dfrac{1.5625}{12.5} = 0.125$.

$\therefore \boxed{P_{SB} = 12.5\% \ P_c}$

(or)

$\longrightarrow$ quantity $\frac{P_{SB}}{P_C} = \frac{P_c \mu^2/2}{P_c}$.

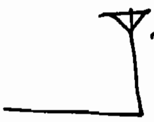
$\therefore \frac{P_{SB}}{P_c} = \frac{\mu^2}{2} = 0.125$.

$\therefore \boxed{\frac{P_{SB}}{P_c} = 12.5\ \%}$

[expressing SB Power w.r.t. Carrier Power].

Ex-5 An Am transmitter radiates 80 watts when the carrier is not modulated. Determine the total power radiated if the carrier is modulated and modulated index is 1.

Ans:



$P_t = P_c \left[1 + \frac{\mu^2}{2} \right]$.

when carrier is not modulated then $\mu = 0$. H.B

$\therefore P_t = P_c [1 + 0]$.

$\therefore \boxed{P_t = P_c}$

$\therefore P_t = P_c = 80$ Watts.

$\rightarrow$ when $\mu = 1$,

$\therefore P_t = 80 [1 + 1/2]$.

$\therefore P_t = 80 \times 1.5 = 120$ W

$\therefore \quad P_t = 120W = 80W + 40W.$

$\therefore \quad P_t = P_c + (0.5 P_c)$

$P_t = P_c + (50\% \text{ of } P_c)$

$50W \rightarrow 75W$

$200W \rightarrow 300W.$

NOTE: H.B

→ when modulation index is changed from 0 to 1 the power increases by 50%.

$\rightarrow \quad P_t = P_c\left[1 + \frac{\mu^2}{2}\right].$

$$\left[\begin{array}{ll} \mu = 0 \rightarrow & P_t = P_c \\ \mu = 1 \rightarrow & P_t = 1.5 P_c. \end{array}\right] \leftarrow \text{H.B}$$

Ex-6 An Am transmitter radiates 50 watts when the carrier is modulated by sinusoidal signal and the modulation index 0.707.

① Determine the modulation efficiency, carrier power & sideband power.

② Determine the peak Amplitude of the carrier before modulation and after modulation.

$\Rightarrow$ $P_t = 50W$; $\mu = 0.707$.

$\therefore \eta = \frac{\mu^2}{2+\mu^2}$.

$\therefore \eta = \frac{1/2}{2+1/2} = 1/5$.

$\therefore \boxed{\eta = 20\%}$ $\qquad \eta = \frac{P_{SB}}{P_t}$.

$\therefore P_c = 80\% \; P_t$

$P_{SB} = 20\% \; P_t$

$\therefore P_c = 0.8 \times 50 = 40W$.

$P_t = 0.2 \times 50 = 10W$.

$\rightarrow$ Peak Amp. ~~modulation~~ before modulation $= A_c$.

$\therefore P_c = \frac{A_c^2}{2\times 1} = A_c^2$.

$\therefore A_c^2 = 80$.

$\therefore \boxed{A_c = 8.944 V}$

$\rightarrow$ Peak Amp. after modulation $= A_c[1 + \mu \cos 2\pi f_m t]$

$\swarrow$ HB

$\therefore \boxed{V_{max} = A_c[1+\mu]. \quad (\theta = 0) \qquad V_{min} = A_c[1-\mu]. \quad (\theta = \pi)}$

$\Rightarrow V_{max} = 8.944 \; [1 + 0.707]$.

$\boxed{V_{max} = 15.27 V}$

$V_{min} = 8.944 \; [1 - 0.707] \Rightarrow \boxed{V_{min} = 6.323V}$

$$\Rightarrow \quad \frac{V_{max}}{V_{min}} = \frac{1+\mu}{1-\mu}.$$

$$\therefore V_{max}[1-\mu] = [1+\mu] V_{min}.$$

$$\therefore \boxed{\mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}} \leftarrow \underline{\underline{H.B}}$$

→ Formula to determine μ practically.

$$\therefore V_{max} + V_{min} = 2A_c.$$

$$\therefore \boxed{A_c = \frac{V_{max} + V_{min}}{2}}$$

<u>Ex-1</u> An amplitude of an AM signal varies from 5V to 15V. Determine the modulation index, carrier power, sideband power & total power.

<u>Ans:</u> $V_{min} = 5V$, $V_{max} = 15V$.

$$\therefore \mu = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}.$$

$$P_c = \frac{A_c^2}{2R}.$$

$$\therefore \mu = \frac{15-5}{20}$$

$$A_c = \frac{V_{max} + V_{min}}{2}$$

$$\mu = \frac{1}{2}.$$

$$A_c = \frac{20}{2}$$

$$\therefore \boxed{\mu = 0.5}$$

$$\boxed{A_c = 10V}$$

$\therefore\ P_c = \frac{100}{2}$

$\boxed{P_c = 50\ W}$

$\therefore\ P_{SB} = \frac{P_c \mu^2}{2}.$

$\therefore\ P_{SB} = \frac{50 \times 0.25}{2}$

$\therefore\ \boxed{P_{SB} = 6.25\ W}$

$\therefore\ P_t = P_c + P_{SB}$

$\therefore\ \boxed{P_t = 56.5\ W}$

* Antenna Current :-

→ Consider an antenna having a resistance of $R\Omega$ and 'I_t' is the antenna current when AM signal is radiated into free space.

$$P_t = P_c \left[1 + \frac{\mu^2}{2} \right].$$

$$\therefore\ I_t^2 . R = I_c^2 R \left[1 + \frac{\mu^2}{2} \right]$$

$$\therefore\ I_t^2 = I_c^2 \left[1 + \frac{\mu^2}{2} \right].$$

$$\therefore\ \boxed{I_t = I_c \sqrt{1 + \frac{\mu^2}{2}}.} \leftarrow \text{H.B.}$$

→ I_c is the antenna current before modulation & I_t is the antenna current after modulation.

Ex-2 The antenna current of an AM transmitter is 8A before modulation and 8.5 A after modulation. calculate the modulation index and modulation efficiency.

Ans: $I_c = 8A$, $I_t = 8.5 A$.

$\therefore I_t^2 = I_c^2 \left[1 + \frac{m^2}{2}\right]$.

$\therefore (8.5)^2 = (8)^2 \left[1 + \frac{m^2}{2}\right]$.

$1.128 = 1 + \frac{m^2}{2}$.

$\therefore \boxed{m = 0.5}$

$\therefore \eta = \frac{0.25}{2.25}$.

$\therefore \boxed{\eta = 11.11\%}$

* Multitone Modulation :-

$\Rightarrow$ Generalized Am signal is,

$\rightarrow S(t) = A_c \cos 2\pi f_c t + A_c K_a \boxed{m(t)} \cos 2\pi f_c t.$

$\Rightarrow$ m(t) is whatever sin or cosine $\Rightarrow$ no change in B.W. & power.

Let, $m(t) = A_{m_1} \cos 2\pi f_{m_1} t \underset{=}{\pm} A_{m_2} \cos 2\pi f_{m_2} t$

$f_{m_2} > f_{m_1}$ with -ve sign no change in B.W. & power.

$\Rightarrow S(t) = A_c \cos 2\pi f_c t + A_c K_a [A_{m_1} \cos 2\pi f_{m_1} t + A_{m_2} \cos 2\pi f_{m_2} t] \cos 2\pi f_c t.$

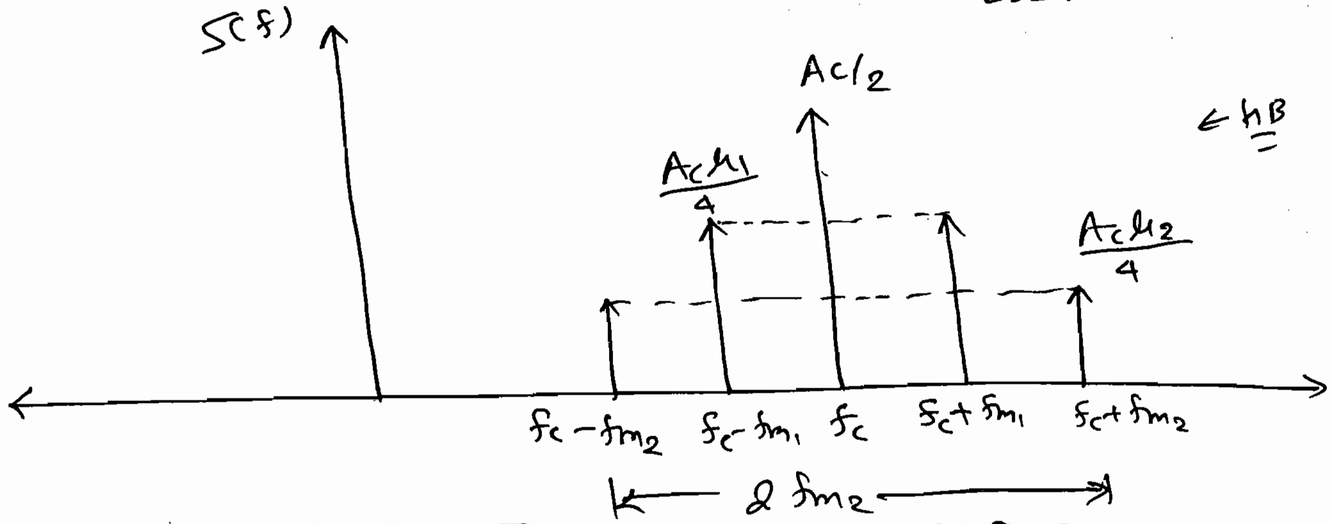
H.B

$\therefore S(t) = \underset{\text{carrier}}{A_c \cos 2\pi f_c t} + \frac{A_c \mu_1}{2} \underset{\text{USB}}{\cos 2\pi (f_c + f_{m_1}) t}$

H.B.

$+ \frac{A_c \mu_1}{2} \underset{\text{LSB}}{\cos 2\pi (f_c - f_{m_1}) t} + \frac{A_c \mu_2}{2} \underset{\text{USB}}{\cos 2\pi (f_c + f_{m_2}) t}$

$+ \frac{A_c \mu_2}{2} \underset{\text{LSB.}}{\cos 2\pi (f_c - f_{m_2}) t}$



B.W = 2 × Maximum Frequency.

$P_t = P_c + P_{USB} + P_{LSB}$.

$$\therefore P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu_1^2}{8R} + \frac{A_c^2 \mu_2^2}{8R} + \frac{A_c^2 \mu_1^2}{8R} + \frac{A_c^2 \mu_2^2}{8R}.$$

$$\therefore P_t = \frac{A_c^2}{2R}\left[1 + \frac{\mu_1^2 + \mu_2^2}{2}\right].$$

$$\therefore \boxed{P_t = P_c\left[1 + \frac{\mu_1^2 + \mu_2^2}{2}\right]}$$

let, $\mu_t^2 = \mu_1^2 + \mu_2^2 + \ldots$

$\mu_t = \sqrt{\mu_1^2 + \mu_2^2 + \cdots}$ = Total modulation index.

$$\therefore \boxed{P_t = P_c\left[1 + \frac{\mu_t^2}{2}\right].} \leftarrow \text{H.B.}$$

$$\therefore P_t = P_c + \frac{P_c \mu_t^2}{2}.$$

$$\therefore \boxed{\eta = \frac{\mu_t^2}{2 + \mu_t^2}.} \leftarrow \text{H.B.}$$

Note:

⇒ The B.W. & Power is depends only on the magnitude spectrum. (not on phase spectrum).

Ex-1 A Carrier signal $c(t) = 20\cos 2\pi 10^6 t$ is modulated by a message signal having 3 frequencies 5 KHz, 10 KHz & 20 KHz. The Corresponding Modulation indices are 0.4, 0.5, 0.6. Sketch the and calculate the B.W., Power & Modulation efficiency.

Soln:

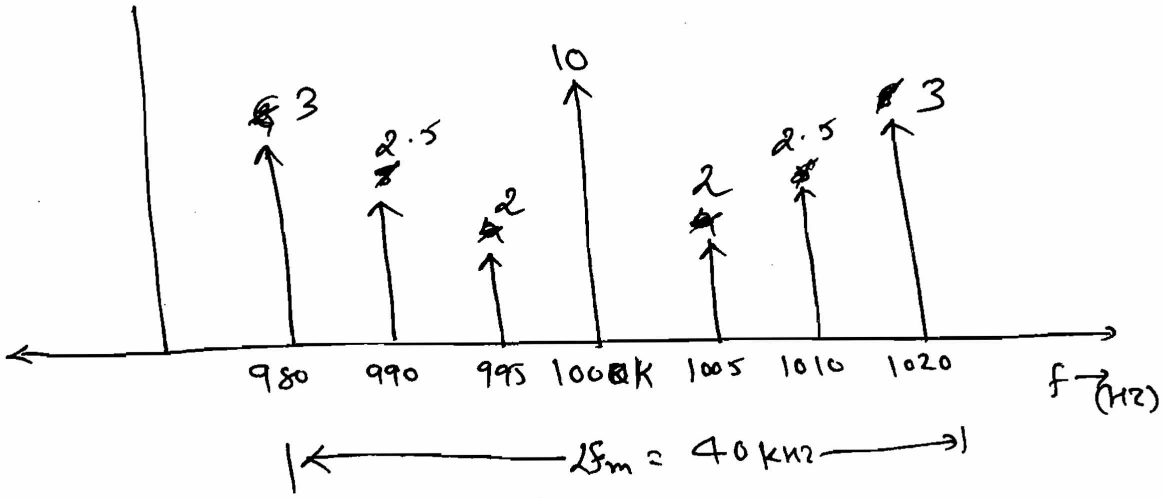
$f_c = 1000$ KHz $\quad \mu_1 = 0.4 \quad A_c = 20.$

$f_{m_1} = 5$ KHz $\quad \mu_2 = 0.5.$

$f_{m_2} = 10$ KHz $\quad \mu_3 = 0.6.$

$f_{m_3} = 20$ KHz

A ⇒



→ $B_w = 2 f_{m_3}$

$= 2 \times 20$

$B_w = 40$ KHz.

$\therefore \quad P_c = \dfrac{A_c^2}{2R}$.

$\therefore \quad P_c = \dfrac{400}{2(1)}$

$\therefore \quad \boxed{P_c = 200 \text{ watts}}$

$\therefore \quad P_t = P_c \left[1 + \dfrac{\mu_t^2}{2} \right]$.

$\therefore \quad \mu_t^2 = \mu_1^2 + \mu_2^2 + \mu_3^2$.

$\mu_t^2 = 0.16 + 0.25 + 0.36$

$\mu_t^2 = 0.77$.

$\therefore \quad P_t = 200 \left[1 + \dfrac{0.77}{2} \right]$

$\therefore \boxed{P_t = 277 \text{ W}}$

$\therefore \quad \eta = \dfrac{\mu_t^2}{2 + \mu_t^2} = \dfrac{0.77}{2.77}$.

$\therefore \boxed{\eta = 27.8}$

Ex. 2 Consider an AM signal
$s(t) = [10 + 4 \sin 2\pi 10^3 t - 8 \cos 2\pi 10^4 t] \times \cos 2\pi f_c t$.
Determine the B.W. & power.

Soln:
$s(t) = 10 \left[1 + 0.4 \sin 2\pi 10^3 t - 0.8 \cos 2\pi 10^4 t \right] \times \cos 2\pi f_c t$.

$\therefore \quad A_c = 10, \quad \mu_1 = 0.4, \quad \mu_2 = -0.8$.

$f_{m_1} = 1 \text{ kHz}, \quad f_{m_2} = 10 \text{ kHz}$.

$\therefore \quad BW = 2 f_{m_2}$

$$\boxed{BW = 20\,kHz}$$

$\mu_t^2 = 0.16 + 0.64$

$\mu_t^2 = 0.80$

$\rightarrow \quad P_c = \dfrac{A_c^2}{2R}$

$$\boxed{\mu = |K_a A_m|}$$

$P_c = \dfrac{100}{2}$

$$\therefore \quad \boxed{P_c = 50\,W}$$

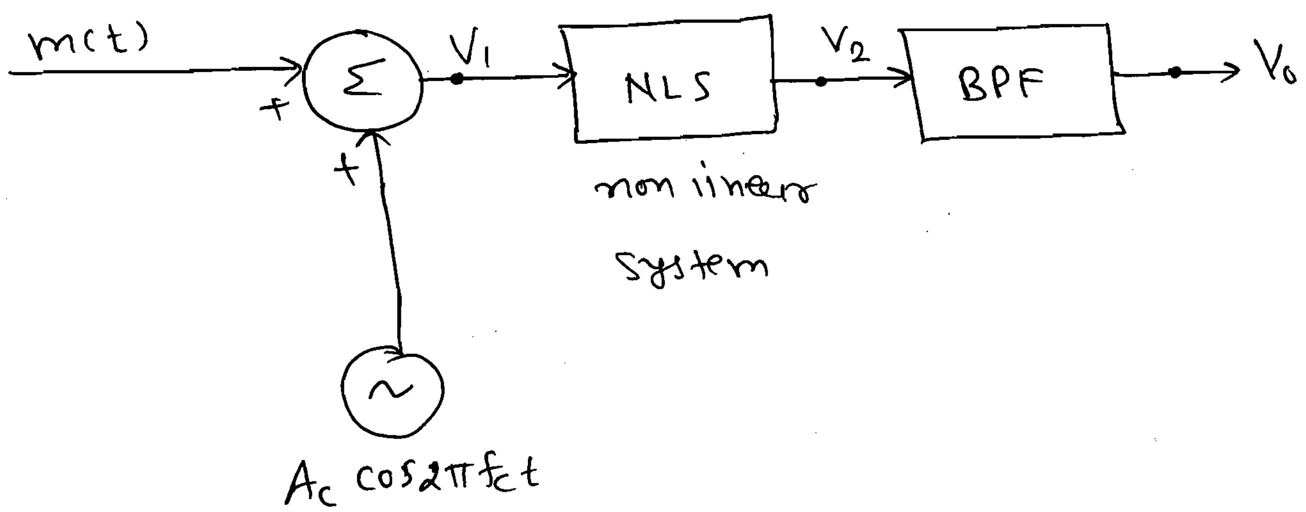
$$\therefore \quad P_t = 50\left[1 + \frac{0.80}{2}\right].$$

$$\therefore \quad \boxed{P_t = 70\,W.}$$

* Generation of AM Signal :-

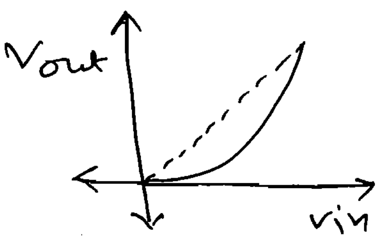
⊙ Square Law Modulator:

H.B.



→ The generalized characteristic of NLS:

H.B.



$$\boxed{V_{out} = aV_{in} + bV_{in}^2}$$

$\Rightarrow \quad V_1 = [m(t) + A_c \cos 2\pi f_c t]$.

$\therefore \quad V_2 = a V_1 + b V_1^2$.

$\therefore \quad V_2 = a m(t) + a A_c \cos 2\pi f_c t$

$+ b m^2(t) + 2 b A_c m(t) \cdot \cos 2\pi f_c t$

$+ b A_c^2 \cos^2 2\pi f_c t$.

$\rightarrow$ The bandpass Filter is used to select the freq. Component of Am signal.

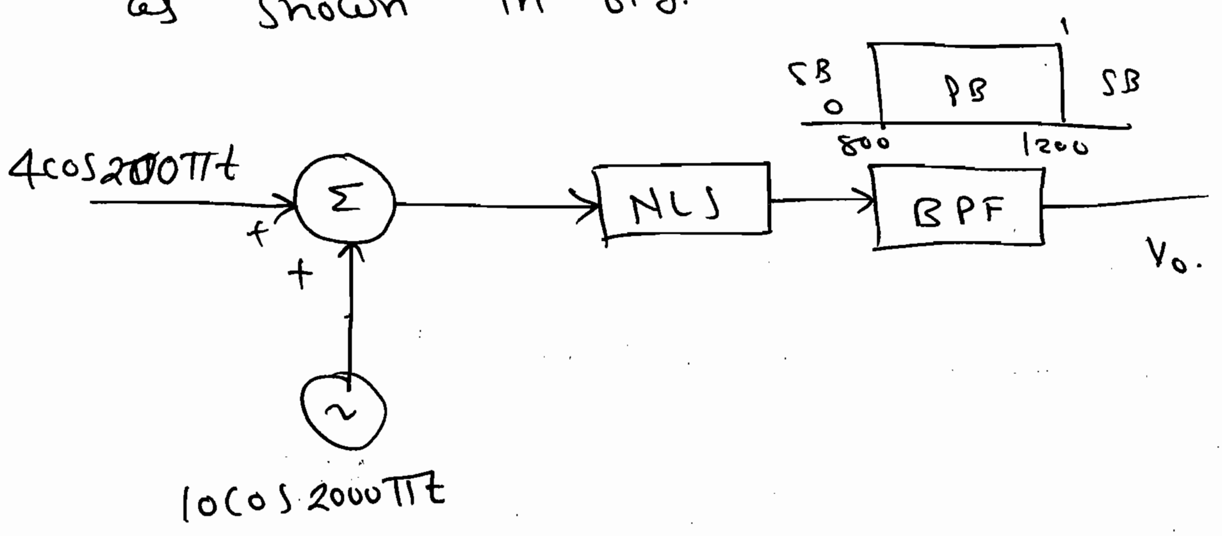
$\Rightarrow$ Output of bandpass filter.

$\therefore \quad V_0 = a A_c \cos 2\pi f_c t + 2 b A_c m(t) \cdot \cos 2\pi f_c t$

$\therefore \boxed{V_0 = a A_c \left[1 + \left(\frac{2b}{a}\right) m(t) \right] \cos 2\pi f_c t}$

$K_a = \frac{2b}{a}$.

Ex-1 Consider a square law modulator as shown in fig.



The BPF is ideal filter having passband from 800 Hz to 1200 Hz. Det. modulation index, B.W. & power of AM signal.

$V_2 = V_1 + 0.1 V_1^2$

<u>Ans</u>:

$V_1 = 4\cos 200\pi t + 10\cos 2000\pi t.$

$\therefore V_2 = V_1 + 0.1 V_1^2.$

$\therefore V_2 = [4\cos 200\pi t + 10\cos 2000\pi t] + 0.1[4\cos 200\pi t + 10\cos 2000\pi t]^2.$

$\therefore V_2 = 4\cos 200\pi t + 10\cos 2000\pi t + 0.16\cos^2 200\pi t$ ✗
$+ 8\cos 200\pi t \cdot \cos 2000\pi t - 1\cdot \cos^2 2000\pi t.$ ✗

$\therefore V_2 = 10\cos 2000\pi t + 8\cos 200\pi t \cdot \cos 2000 t\pi.$

$V_2 = 10[1 + 0.8\cos 200\pi t]\cos 2000\pi t.$

$\therefore V_2 = A_c[1 + \mu\cos 2\pi f_m t]\cos 2\pi f_c t.$

$\therefore \boxed{A_c = 10V}$ $\quad f_m = 100$ Hz

$\therefore \mu = 0.8$ $\quad f_c = 1000$ Hz.

$\therefore B_W = 2\times 100$

$\therefore \boxed{BW = 200 \text{ Hz}}$

$P_c = \dfrac{A_c^2}{2R}$

$P_c = \dfrac{100}{2} = 50$ watts

$P_t = P_c[1 + \frac{\mu^2}{2}].$

$\therefore P_t = 50[1 + \frac{0.64}{2}].$

$\boxed{P_t = 66 \text{ W}}$

* Cute point of view .!

↙ H.B.

$\mu = K_a A_m.$

$\therefore \boxed{\mu = (2b/a) A_m.} = 0.2 \times 4 = 0.8.$

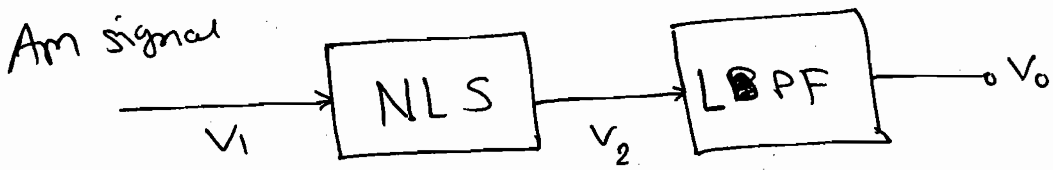
$\therefore \boxed{K_a = 2b/a.}$

$\therefore K_a = 2 \times 0.1 = 0.2$

* Demodulation of AM signal:

1. Square law demodulator
2. Envelope detector
3. Synchronous detector.

1 Square Law demodulator:



$V_1 = A_c \cos 2\pi f_c t + K_a m(t) A_c \cos 2\pi f_c t.$

$\therefore V_2 = aV_1 + bV_1^2$

$\therefore V_2 = aA_c \cancel{\cos 2\pi f_c t} + aK_a m(t) A_c \cancel{\cos 2\pi f_c t}$

$+ bA_c^2 \cancel{\cos^2 2\pi f_c t} + 2bA_c^2 K_a m(t) . \cos^2 2\pi f_c t$

$\Downarrow \quad \frac{m(t)}{2} + \cancel{\frac{m(t) \cos 2\pi (2f_c) t}{2}}$

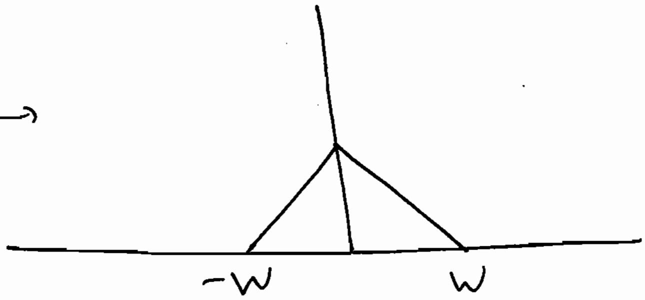
$+ b K_a^2 m^2(t) A_c^2 \cos^2 2\pi f_c t.$

$\Downarrow \quad \frac{m^2(t)}{2} + \cancel{\frac{m^2(t) . \cos 2\pi f_c (2) t}{2}}$

$\therefore V_o = bA_c^2 m(t) K_a \cos. + \frac{bK_a^2 m^2(t) A_c^2}{2}$

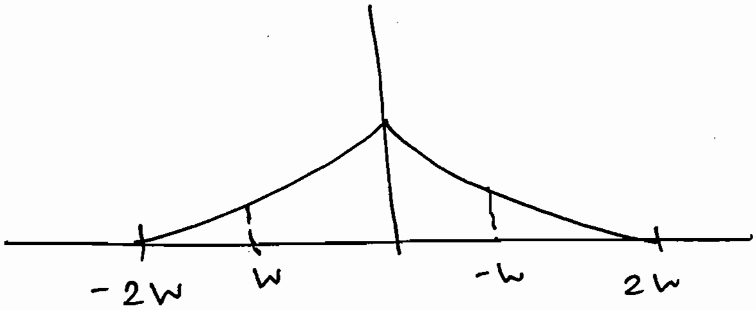
$$V_o = \underbrace{bA_c^2 K_a [m(t)]}_{\text{signal}} + \underbrace{\frac{bK_a^2 A_c^2}{2} [m^2(t)]}_{\text{noise}}.$$

$m(t) \longleftrightarrow$



$m^2(t) \longleftrightarrow$

$= M(f) * M(f).$



⇒ The output of demodulator consist of $m(t)$ and $m^2(t)$.

⇒ $m^2(t)$ is also having significant low frequencies so it is not eliminated.

$$\rightarrow \frac{\text{Signal Component}}{\text{Noise Component}} = \frac{2}{K_a m(t)}.$$

$$\therefore \boxed{\frac{S}{N} = \frac{2}{K_a A_m \cos 2\pi f_m t}} \leftarrow \text{H.B}$$

$$\therefore \left(\frac{S}{N}\right)_{min} = \frac{2}{K_a A_m}$$

$$= 2/\mu.$$

$$\left(\frac{S}{N}\right)_{min} = \frac{2}{\mu} = 0.2 \ ; \ \mu = 10$$

$$= 20 \ ; \ \mu = 0.1$$

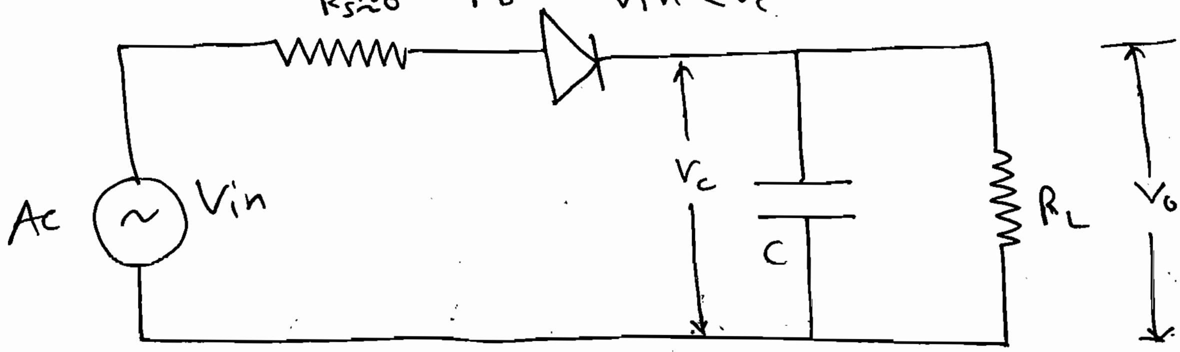
$$= 1 \ ; \ \mu = 2.$$

(μ should be < 2 in this case).

(2) Envelope Detector:

FB $\rightarrow$ $V_{in} > V_c$

RB $\rightarrow$ $V_{in} < V_c$

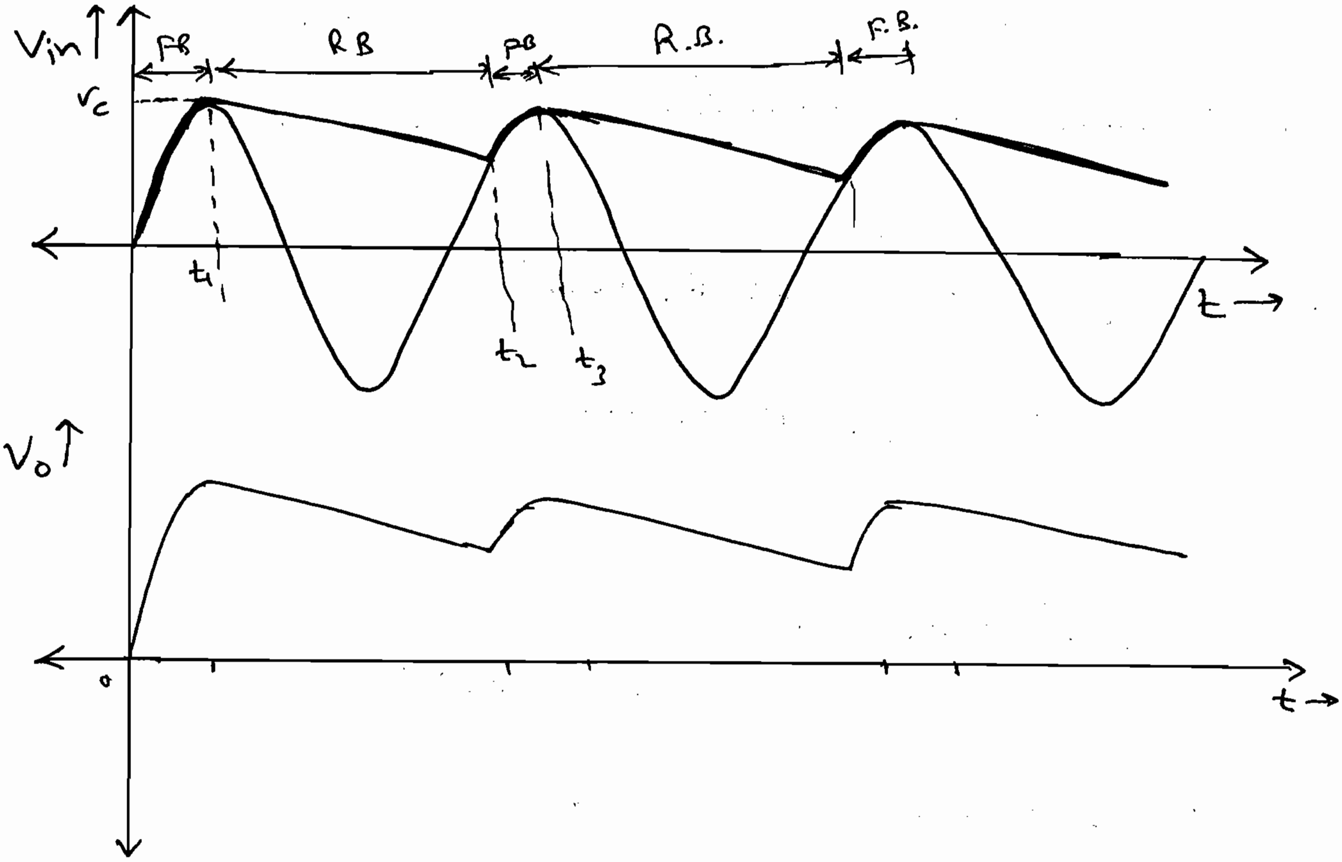


$\Rightarrow$ Hardware implementation is very simple. ($\mu \leq 1$).

$\rightarrow$ In AM signal the peak amplitude of the carrier which is also called as envelope is varied according to the message signal. So, the envelope of the AM signal represent the message signal. Envelope detector is used to track the peak amplitude of an AM signal.

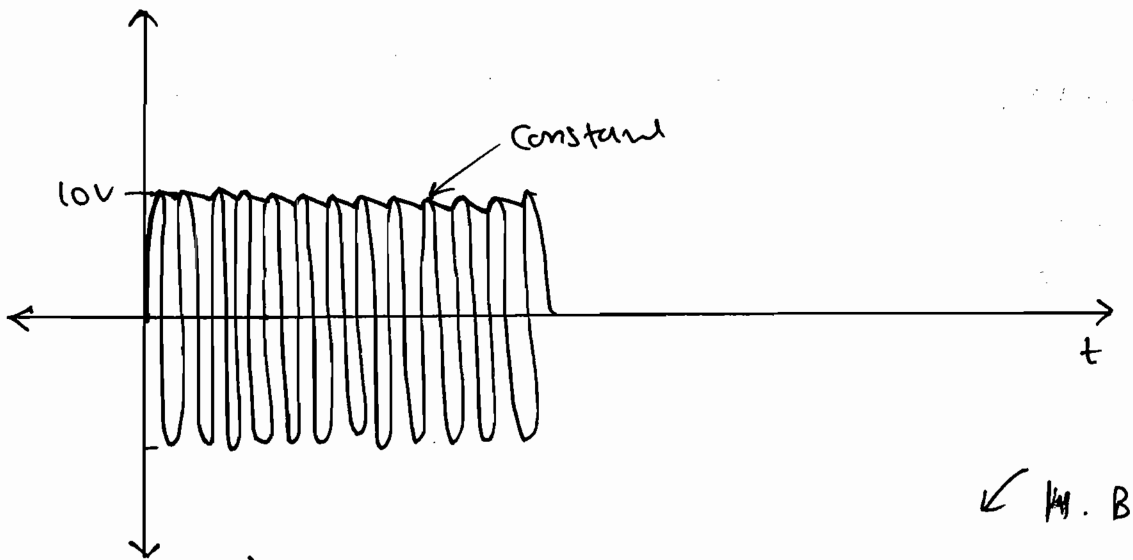
$\Rightarrow$ $R_s C \rightarrow$ very low (charging time const^n)

$R_L C \rightarrow$ very high (Discharging time con^n)



⇒ Am is very high freq. signal means time period is very very less. means V_o seen like as follow:

eg. if Am is $10 \cos 2\pi f_c t$ then V_o is ~~taken~~ 10V const.



↙ M. B.

$\boxed{\text{IMP}}$

i/p	o/p
→ $A_c[1+k_a m(t)]\cos 2\pi f_c t$	$V_o = A_c[1+k_a m(t)]$.
→ $A_c \cos 2\pi f_c t$	$V_o \simeq A_c$
→ $e^{-t}\cos 2\pi f_c t$	$V_o = e^{-t}$.

i/p o/p

$\rightarrow A_c \cos 2\pi f_c t + B \sin 2\pi f_c t$ $\quad V_o = \sqrt{A_c^2 + B^2}$

$$\Rightarrow V_o = A_c [1 + K_a m(t)].$$

$$V_o = \underbrace{A_c}_{\downarrow} + A_c K_a m(t)$$

DC can be eliminated by filter.

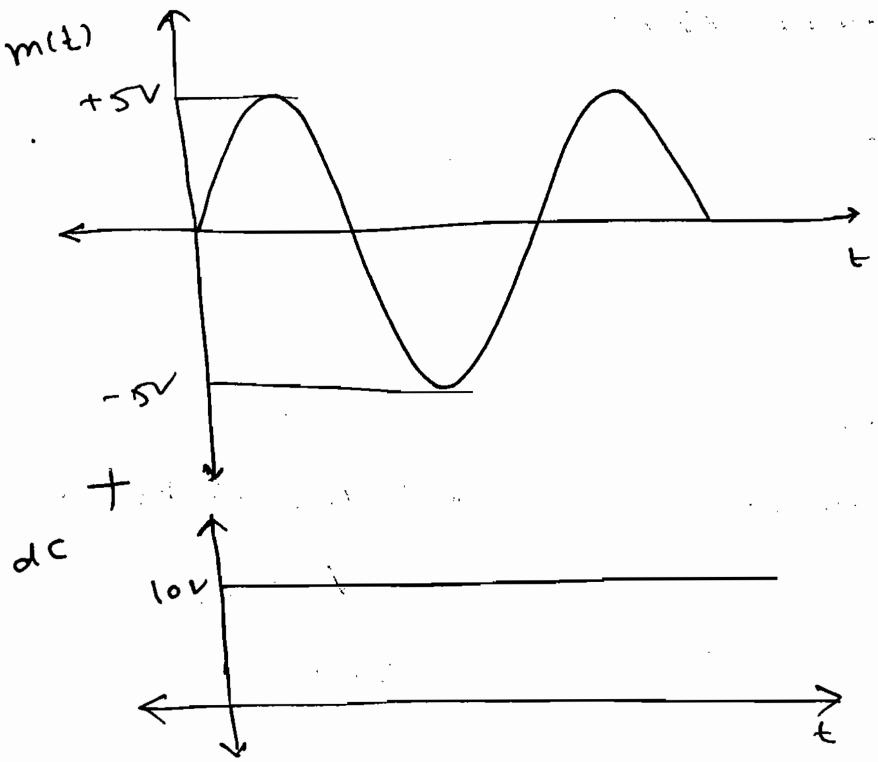
$$V_o = A_c K_a m(t)$$

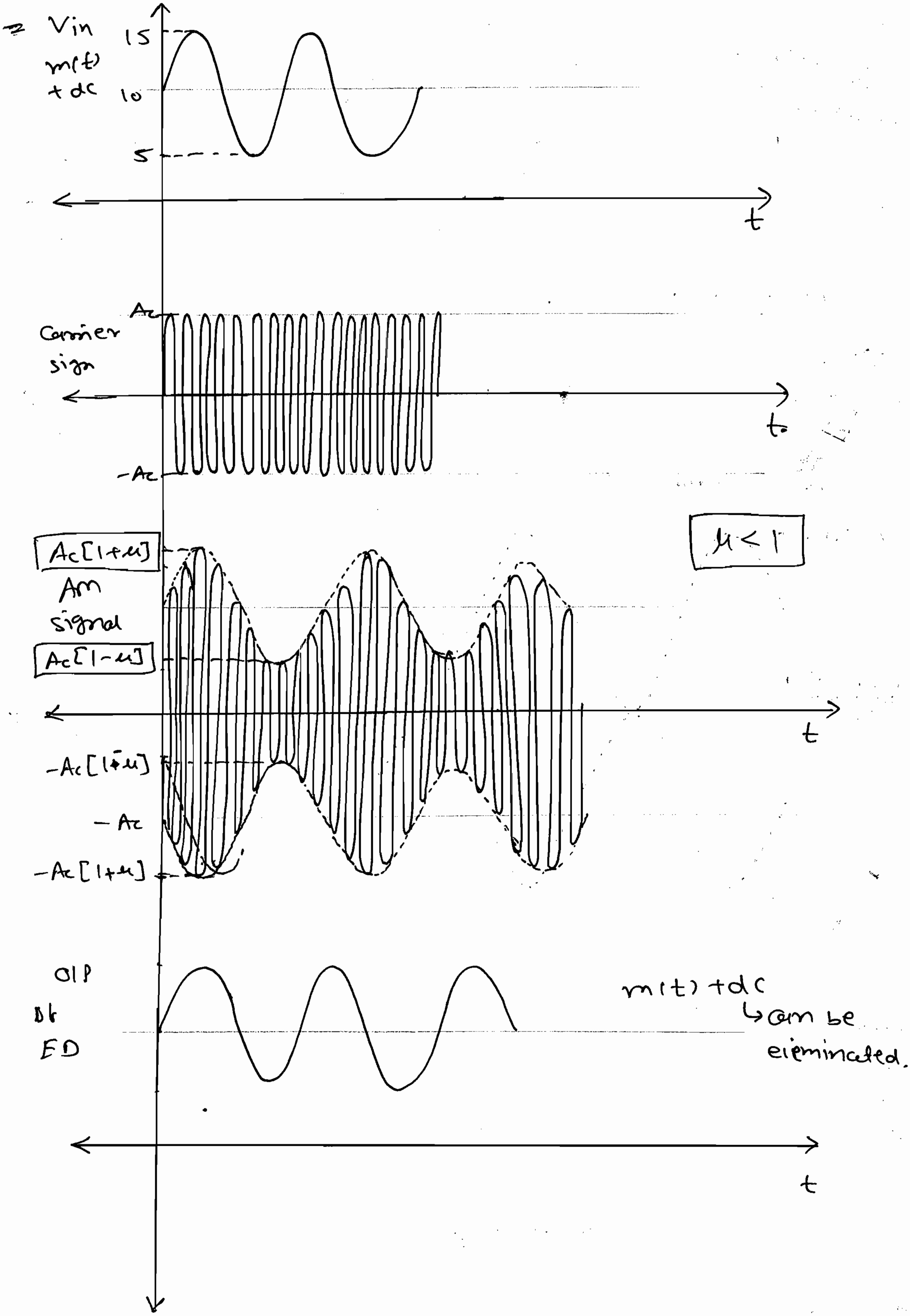
if $\boxed{K_a = 1/A_c}$

$$\therefore \boxed{V_o = m(t).}$$

* Limitation of envelope Detector:

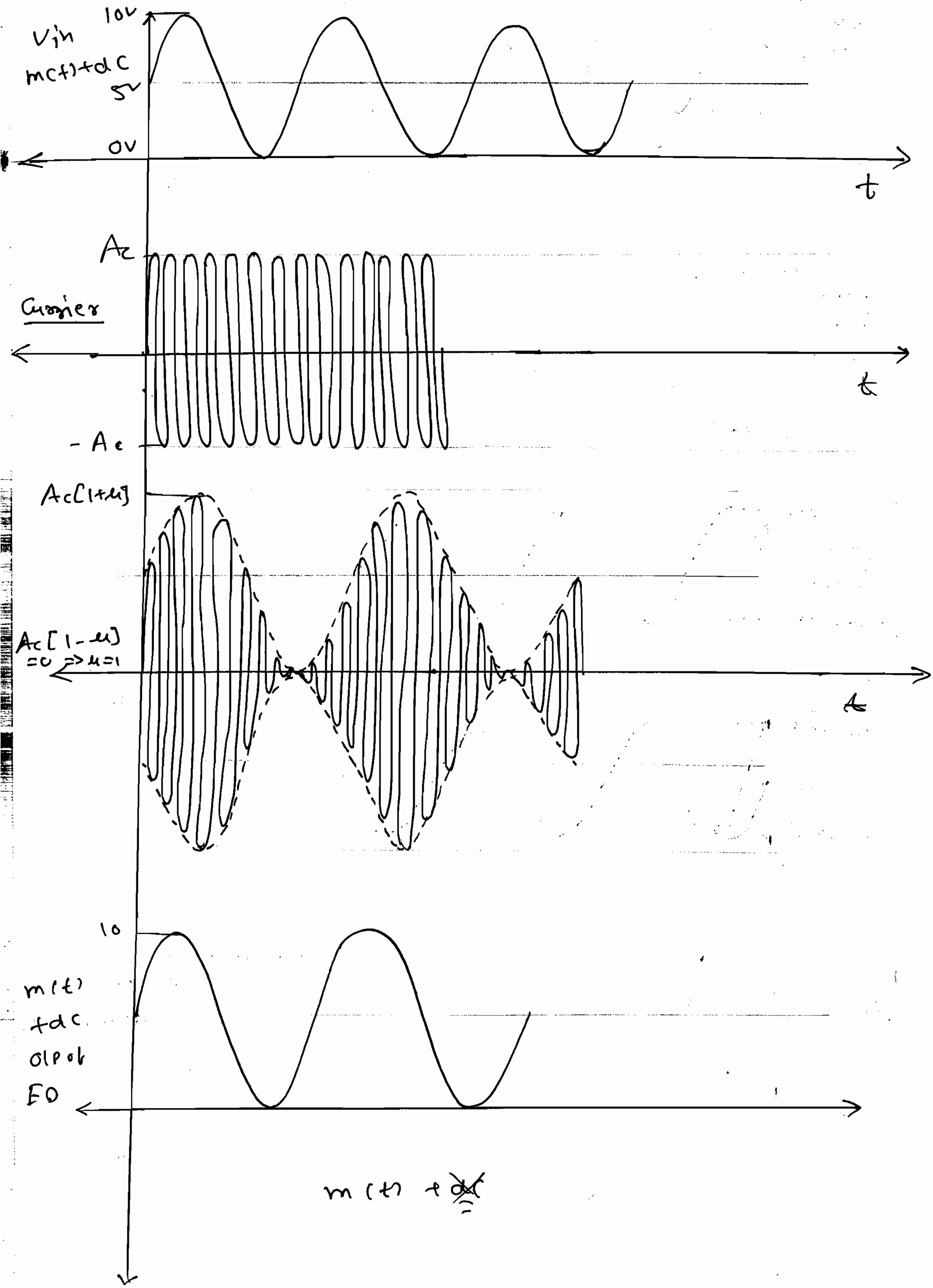
$\rightarrow$ Consider a sine signal whose amplitude varies from +5 to -5. A dc component of 10V is added to the message signal before multiplying with carrier.



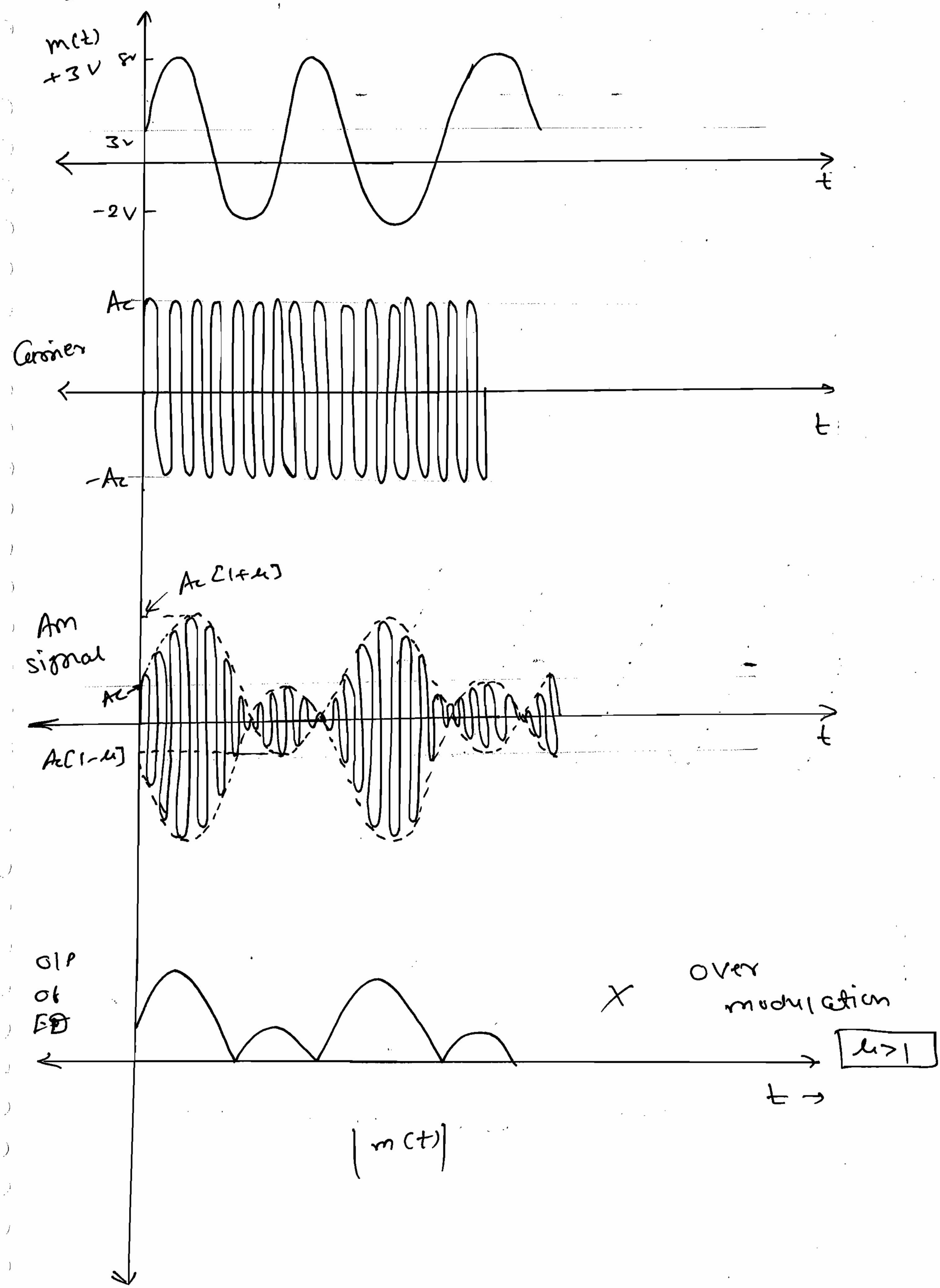


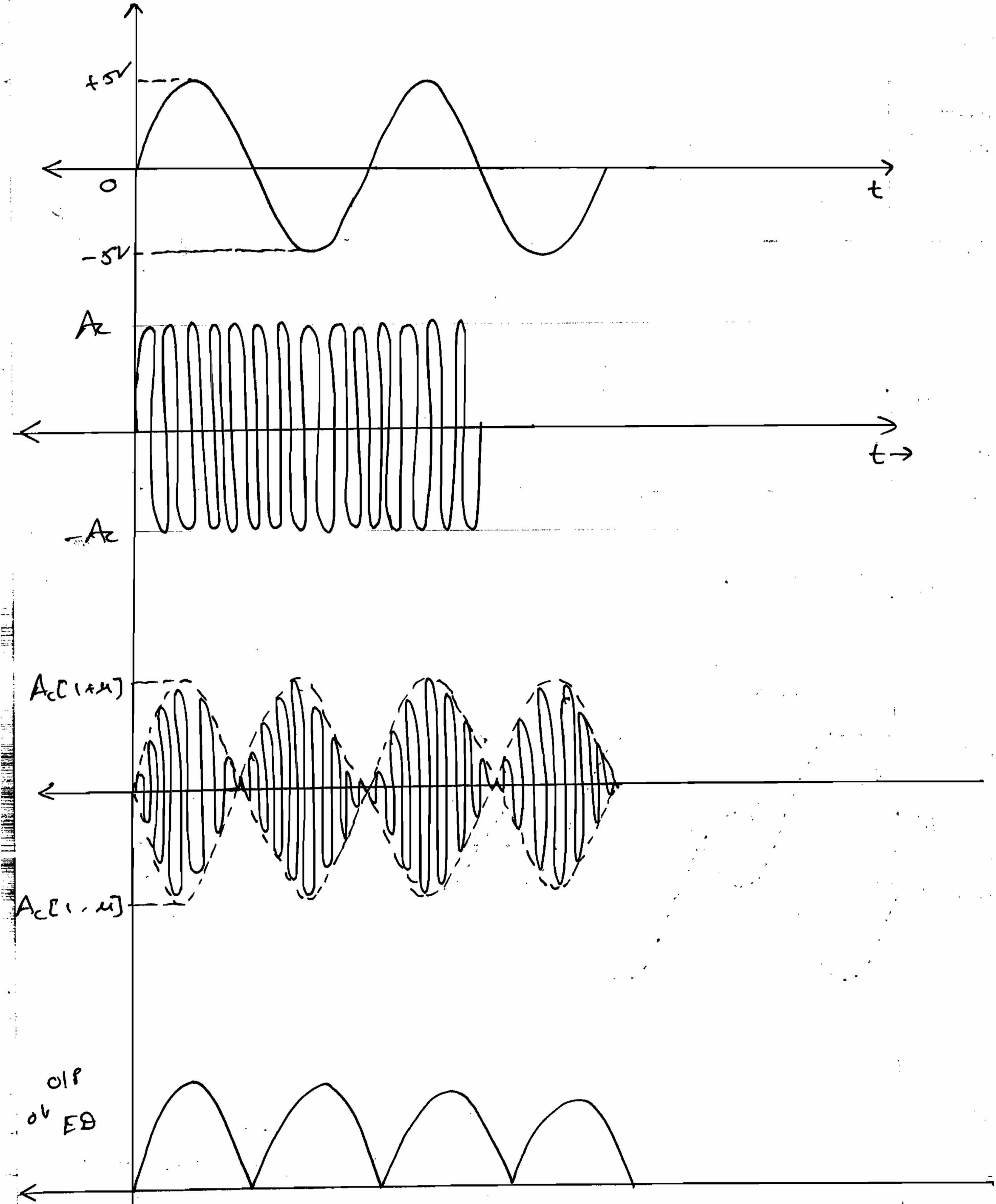
Vin
m(t)
+dc
15
10
5
t
Ac
Carrier
sign
t
-Ac
Ac[1+μ]
Am
signal
Ac[1-μ]
μ<1
t
-Ac[1-μ]
-Ac
-Ac[1+μ]
O/P
of
ED
m(t) +dc
↳ can be
eleminated.
t

* $\boxed{\mu=1}$


10V
Vin
m(t)+dc
5V
0V
t
Ac
Carrier
t
-Ac
Ac[1+μ]
Ac[1-μ]
=0 => μ=1
t
10
m(t)
+dc
o/p of
ED
m(t) + ~~dc~~

* $\boxed{\mu > 1}$


m(t)
+3V
8V
3V
-2V
t
Ac
Carrier
-Ac
t
Ac [1+μ]
Am signal
Ac
Ac[1-μ]
t
O/P of ED
over modulation
μ>1
t →
|m(t)|



$|m(t)|$ It is not an AM signal.

O/P

① $\mu \le 1 \longrightarrow V_o = A_c[1 + K_a m(t)]$. ↙ H.B

② $\mu > 1 \longrightarrow V_o = |A_c[1 + k_a m(t)]|$

③ $\rightarrow [dc + m(t)] \cos 2\pi f_c t$

$\rightarrow dc \cos 2\pi f_c t + m(t) \cos 2\pi f_c t$.

A dc component is added to the message signal & multiplied with carrier to generate the Am signal but the demodulation is possible only when $\underline{\underline{m(t) + dc}}$ is positive.

④ K_a is used in analysis to normalized the peak amplitude of the message signal so that the modulation index is ≤ 1.

$\mu = K_a A_m$.

when, $K_a = 1$ there is possibility of over modulation.

$\therefore \mu = A_m \quad m(t) = 2 \cos 2\pi \times 10^3 t$

$\therefore \mu = 2$

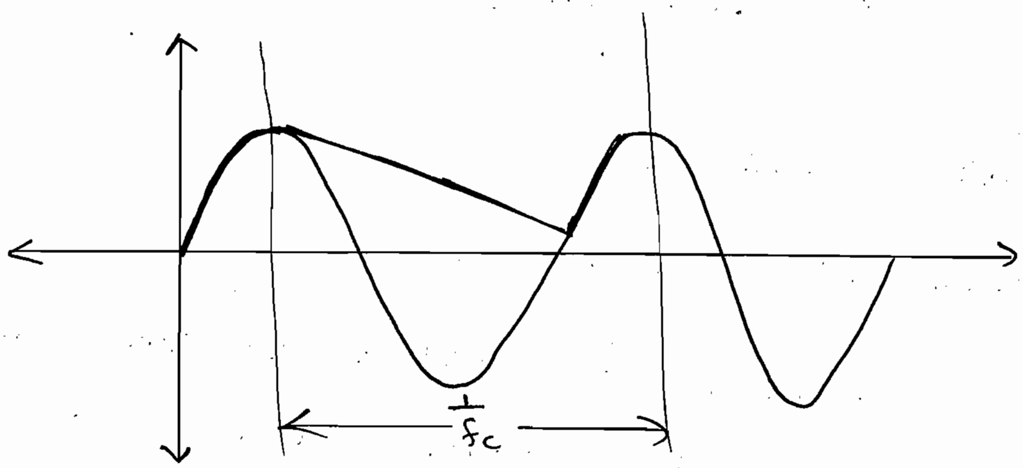
⑤ The time constant of envelope detector should satisfy following conditions.

① $R_L C >> \frac{1}{f_c}$.

② $R_L C << \frac{1}{f_m}$.

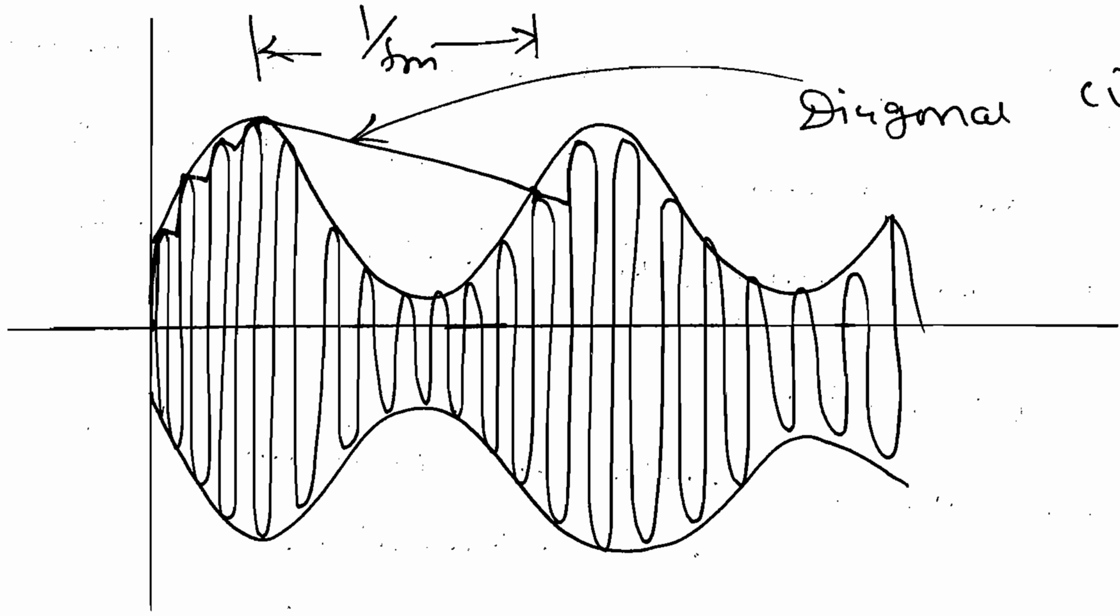
$$\therefore \boxed{\frac{1}{f_c} << R_L C << \frac{1}{f_m}} \leftarrow \underline{\underline{H.B.}}$$

(1)



In this case we can not able to trace the output (or) envelope.

(2) *



Gate-2004

Ex-1 : Am signal is demodulated using an envelop detector the carrier freq. is 1 MHZ and the msg. freq. is 2 KHZ. A suitable value for the time constant of the envelop detector is,

(A) 0.2 μs (C) 20 μs ✓

(B) 1 μs (D) 500 μs.

Ans:

$$\frac{1}{f_c} << R_L C << \frac{1}{f_m}.$$

$$\therefore \frac{1}{1\,MHz} << R_L C << \frac{1}{2\,kHz}.$$

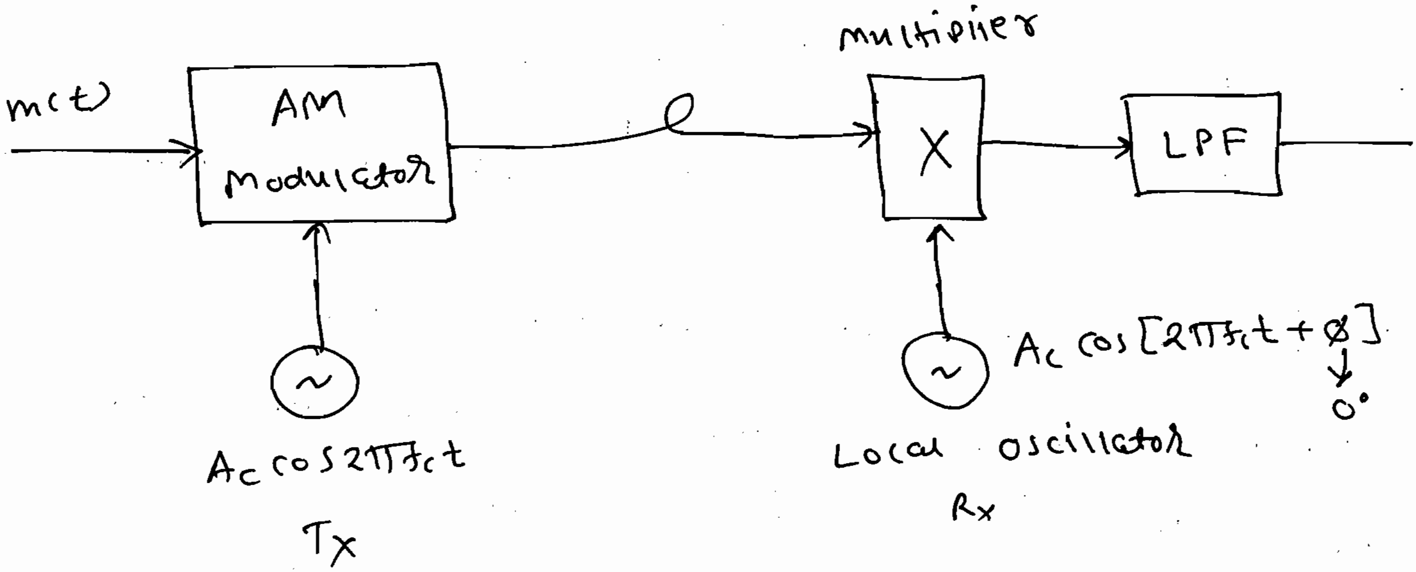
$$\therefore \frac{1}{10^6} << R_L C << \frac{1}{2\times 10^3}.$$

$$\therefore \boxed{1\mu s << R_L C << 500\mu s.}$$

So, valid ans is (c) 20 μsec.

(3) Synchronous Detector:-

⇒ Synchronous detector is capable of demodulating the overmodulating AM signal also



⇒ output of multiplier,

→ (Am) (LO)$_o$.

$$= [A_c \cos 2\pi f_c t + A_c m(t) \cos 2\pi f_c t] A_c \cos 2\pi f_c t.$$

$$= A_c^2 \cos^2 2\pi f_c t + A_c^2 m(t) \cos^2 2\pi f_c t.$$

$$= \frac{A_c^2}{2} + \frac{A_c^2 \cos 2\pi (2f_c) t}{2} + \frac{A_c^2}{2} m(t) + \frac{A_c^2 m(t)}{2} \cos 2\pi (2f_c) t.$$

$\therefore$ o/p of LPF = $\frac{A_c^2}{2} m(t)$.

⇒ Assume that the phase shift is existing b/w carrier used at transmitter and carrier generated at receiver.

⇒ Then o/p of multiplier,

$$[A_c \cos 2\pi f_c t + A_c m(t) \cos 2\pi f_c t] \times A_c \cos[2\pi f_c t + \phi]$$

⇒ finally o/p of LPF is

$$\left[\frac{A_c^2}{2} m(t)\right] \cos \phi.$$

if $\phi = 0^\circ \longrightarrow \frac{A_c^2}{2} m(t)$.

$\phi = 90^\circ \longrightarrow 0 \rightarrow$ $\boxed{\text{Quadrature null effect}}$

→ To overcome the Quadrature null effect Synchronization should be maintained b/w T_x & R_x. So, Additional Hardware is required to maintain Synchronization. The Hardware complexity of the synchronization detector is very high when compared with envelop detector, so envelope detector is used as Demodulator for AM.