Exercise 4.2

Answer 1E.

Consider the function $f(x) = 3 - \frac{1}{2}x$

Evaluate the Riemann sum for $f(x), 2 \le x \le 14$ with six subintervals:

Recall the Riemann sum:

If *f* is a function defined for $a \le x \le b$, we divide the interval $[a,b]$ into *n* subintervals of equal width $\Delta x = \frac{(b-a)}{n}$ then the Riemann sum is $\sum_{i=1}^{n} f(x_i)\Delta x$.

Here $a = 2, b = 14, n = 6$

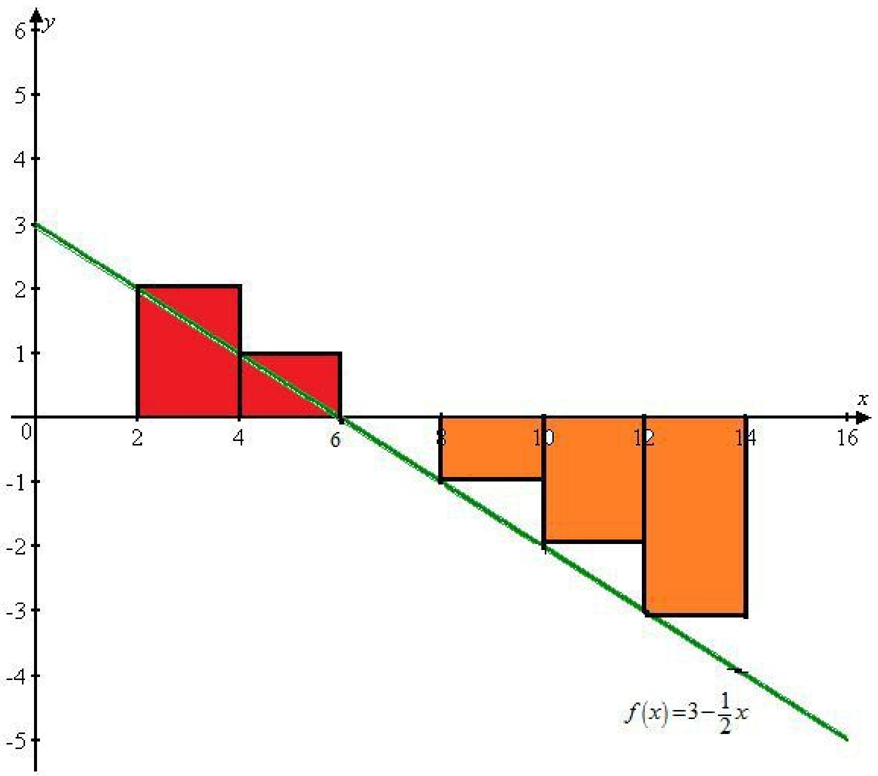
Thus, the interval width is $\Delta x = \frac{(b-a)}{n}$

$= \frac{14-2}{6}$

$= 2$

And the left end points are $x_0 = 2, x_1 = 4, x_2 = 6, x_3 = 8, x_4 = 10,$ and $x_5 = 12$

Sketch the graph of $f(x) = 3 - \frac{1}{2}x$ is as follows:



From the graph observe that

$f(2) = 2, f(4) = 1, f(6) = 0, f(8) = -1, f(10) = -2, f(12) = -3$

The Riemann sum is

$$L_6 = \sum_{i=1}^{6} f(x_{i-1})\Delta x$$
$$= \Delta x\left[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)\right]$$
$$= 2\left[f(2)+f(4)+f(6)+f(8)+f(10)+f(12)\right]$$
$$= 2[2+1+0-1-2-3]$$
$$= 2(-3)$$
$$= -6$$

Therefore, the Riemann sum is $\boxed{-6}$.

Thus, the Riemann sum represents the sum of the areas of the two rectangles above the *x*-axis minus the sum of the areas of the three rectangles below the *x*-axis.

Answer 2E.

Consider the function,

$$f(x) = x^2 - 2x,\ \ 0 \le x \le 3$$

With n=6, the interval width is,

$$\Delta x = \frac{b-a}{n}$$
$$= \frac{3-0}{6}$$
$$= \frac{1}{2}$$

Divide it into 6 sub intervals then the right endpoints of the subintervals are,

$$x_1 = 0.5, x_2 = 1, x_3 = 1.5, x_4 = 2, x_5 = 2.5, x_6 = 3$$

The Riemann sum is,

$$R_n = \sum_{i=1}^{n} f(x_i *)\Delta x$$

Therefore, the Riemann sum is,

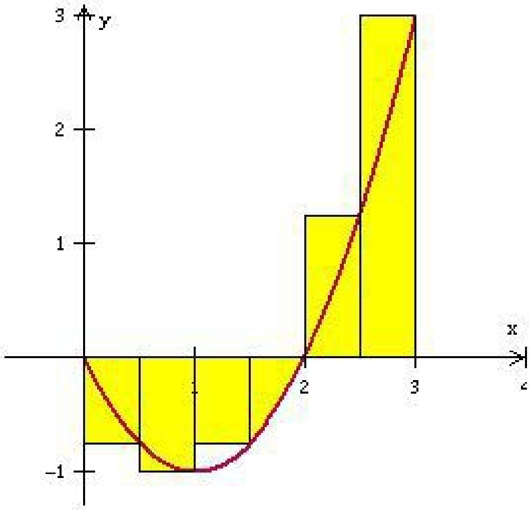
$$R_6 = \sum_{i=1}^{6} f(x_i)\Delta x$$
$$= \frac{1}{2}\left((0.5)^2 - 2(0.5)\right) + \left(1^2 - 2(1)\right) + \left((1.5)^2 - 2(1.5)\right) + \left(2^2 - 2(2)\right) +$$
$$\left((2.5)^2 - 2(2.5)\right) + \left(3^2 - 2(3)\right)$$

$$\boxed{R_6 = 0.875}$$

$$= \frac{1}{2}(-0.75 - 1 - 0.75 + 1.25 + 3)$$
$$= \frac{1}{2}(1.75)$$

Sketch the graph of the function $f(x) = x^2 - 2x,\ 0 \le x \le 3$.

$f(x) = x^2 - 2x,\ 0 \le x \le 3$



Riemann sum represents the sum of the areas of the rectangles with respect to x – axis.

Answer 3E.

We have $f(x) = \sqrt{x} - 2,\quad 1 \le x \le 6$

We divide the interval [1, 6] in to 5 subintervals then

$$\Delta x = \frac{6-1}{5} = 1$$

Subintervals are [1, 2], [2, 3], [3, 4], [4, 5] and [5, 6]

Mid points are 1.5, 2.5, 3.5, 4.5, and 5.5,

Then Riemann sum is

$$M_5 = \sum_{i=1}^{5} f(x_i^*)\Delta x \qquad [x_i^* \text{ is the mid point of the } i^{th} \text{ subinterval}]$$

$$\Rightarrow \quad M_5 = \left[f(x_1^*) + f(x_2^*) + f(x_3^*) + f(x_4^*) + f(x_5^*)\right]\Delta x$$

$$\Rightarrow \quad M_5 = \left[f(1.5) + f(2.5) + f(3.5) + f(4.5) + f(5.5)\right](1)$$

$$\Rightarrow \quad M_5 = \left[\left(\sqrt{1.5} - 2\right) + \left(\sqrt{2.5} - 2\right) + \left(\sqrt{3.5} - 2\right) + \left(\sqrt{4.5} - 2\right) + \left(\sqrt{5.5} - 2\right)\right]$$

$$\Rightarrow \quad \boxed{M_5 \approx -0.856759}$$

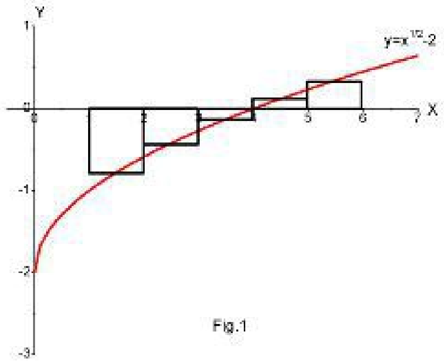


Fig.1

Riemann sum represents

$$\begin{pmatrix}\text{Sum of the areas of two rectangles}\\ \text{above x-axis}\end{pmatrix}-\begin{pmatrix}\text{Sum of the areas of three rectangles}\\ \text{below x-axis}\end{pmatrix}$$

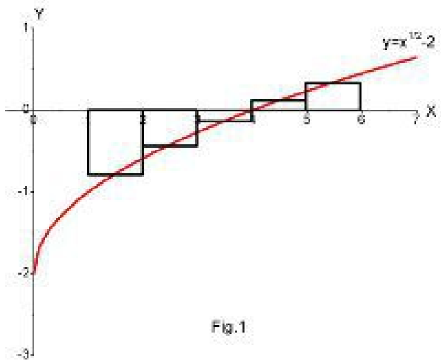


Fig.1

Riemann sum represents

$$\begin{pmatrix}\text{Sum of the areas of two rectangles}\\ \text{above x-axis}\end{pmatrix}-\begin{pmatrix}\text{Sum of the areas of three rectangles}\\ \text{below x-axis}\end{pmatrix}$$

Answer 4E.

Consider the function $f(x)=\sin x,\ 0\le x\le \frac{3\pi}{2}$.

The objective is to evaluate the Riemann sum with six subintervals taking the sample points to be right endpoints and midpoints.

(a)

Right endpoints are sample points:

Interval is $\left[0,\frac{3\pi}{2}\right]$

Divide it into 6 sub intervals then we get $\Delta x=\frac{\pi}{4}$

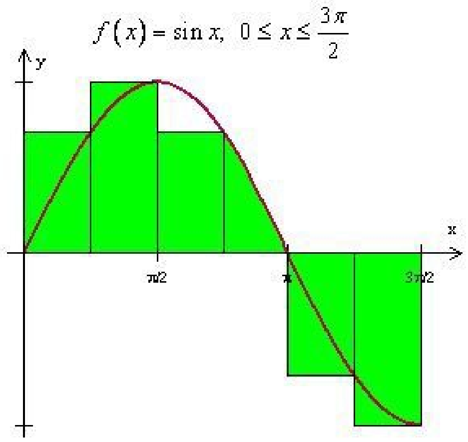
Therefore, the Riemann sum is,

$$\mathrm{R}_6=\sum_{i=1}^{6} f(x_i)\Delta x$$

$$R_6=\frac{\pi}{4}\left[f\left(\frac{\pi}{4}\right)+f\left(\frac{\pi}{2}\right)+f\left(\frac{3\pi}{4}\right)+f\left(\frac{4\pi}{4}\right)+f\left(\frac{5\pi}{4}\right)+f\left(\frac{3\pi}{2}\right)\right]$$

$$=0.55536$$

Sketch the graph of the function $f(x) = \sin x,\ 0 \le x \le \frac{3\pi}{2}$



Riemann sum represents the sum of the areas of the rectangles with respect to $x-$axis.

(b)

Midpoints are sample points:

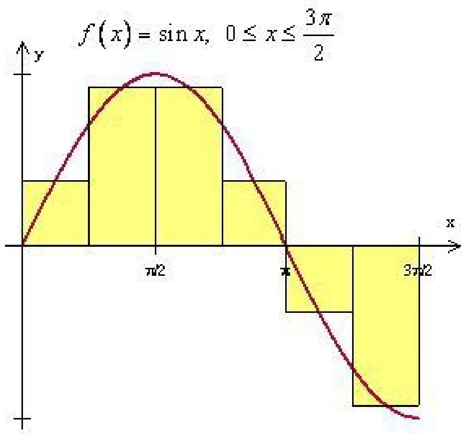
Interval is $\left[0, \frac{3\pi}{2}\right]$

Divide it into 6 sub intervals then we get $\Delta x = \frac{\pi}{4}$

Now

$$M_6 = \frac{\pi}{4}\left[f\left(\frac{\pi}{8}\right) + f\left(\frac{3\pi}{8}\right) + f\left(\frac{5\pi}{8}\right) + f\left(\frac{7\pi}{8}\right) + f\left(\frac{9\pi}{8}\right) + f\left(\frac{11\pi}{8}\right)\right]$$
$$= 1.02617$$

Sketch the graph of the function $f(x) = \sin x,\ 0 \le x \le \frac{3\pi}{2}$



Riemann sum represents the sum of the areas of the rectangles with respect to $x-$axis,

Answer 5E.

Midpoint Rule:

$$\int_a^b f(x)dx = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$$
$$= \Delta x\left[f(\bar{x}_1) + f(\bar{x}_2) + \ldots + f(\bar{x}_n)\right]$$

Where $\Delta x = \frac{b-a}{n}$

$\bar{x}_i = \frac{1}{2}(x_{i-1} + x_i)$ Midpoint of $[x_{i-1}, x_i]$

Given $\int_0^{10} f(x)dx$, $n=5$

$$\Delta x = \frac{b-a}{n} = \frac{10-0}{5} = 2$$

$$[0,2],[2,4],[4,6],[6,8],[8,10]$$

Mid points are 1, 3, 5, 7, 9.

From the graph we get the values of f

$$f(x_0) = f(0) = 3$$
$$f(x_1) = f(2) = -1$$
$$f(x_2) = f(4) = 0$$
$$f(x_3) = f(6) = -2$$
$$f(x_4) = f(8) = 2$$
$$f(x_5) = f(10) = 4$$

For midpoints:

$$f(\bar{x}_1) = f(1) = 0$$
$$f(\bar{x}_2) = f(3) = -1$$
$$f(\bar{x}_3) = f(5) = -1$$
$$f(\bar{x}_4) = f(7) = 0$$
$$f(\bar{x}_5) = f(9) = 3$$

ıght end points

$$f(x)dx = \sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \Delta x\left[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)\right]$$
$$= \Delta x\left[f(2)+f(4)+f(6)+f(8)+f(10)\right]$$
$$= 2\left[-1+0+(-2)+2+4\right]$$
$$= 2[3]$$
$$= 6$$

$$\boxed{\therefore \int_0^{10} f(x)dx = 6}$$

(b) left end points

$$\int_0^{10} f(x)dx = \sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \Delta x\left[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)\right]$$
$$= \Delta x\left[f(0)+f(2)+f(4)+f(6)+f(8)\right]$$
$$= 2\left[3+-1+0+(-2)+2\right]$$
$$= 2[2]$$
$$= 4$$

$$\boxed{\therefore \int_0^{10} f(x)dx = 4}$$

(c) mid points

$$\int_0^{10} f(x)dx = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$$
$$= \Delta x\left[f(\bar{x}_1)+f(\bar{x}_2)+f(\bar{x}_3)+f(\bar{x}_4)+f(\bar{x}_5)\right]$$
$$= \Delta x\left[f(1)+f(3)+f(5)+f(7)+f(9)\right]$$
$$= 2\left[0+(-1)+(-1)+0+3\right]$$
$$= 2[1]$$
$$= 2$$

$$\boxed{\therefore \int_0^{10} f(x)dx = 2}$$

Answer 6E.

Midpoint Rule:

$$\int_a^b f(x)dx = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$$
$$= \Delta x\left[f(\bar{x}_1)+f(\bar{x}_2)+....+f(\bar{x}_n)\right]$$

Where $\Delta x = \frac{b-a}{n}$

$\bar{x}_i = \frac{1}{2}(x_{i-1}+x_i)$ Midpoint of $[x_{i-1}, x_i]$

Given $\int_{-2}^{4} g(x)dx$, $n=6$

$$\Delta x = \frac{b-a}{n} = \frac{4-(-2)}{6} = 1$$

$[-2,-1],[-1,0],[0,1],[1,2],[2,3],[3,4]$

Mid points are -1.5, -0.5, 0.5, 1.5, 2.5, and 3.5.
From the graph we get the values of f

$$f(x_0) = f(-2) = 0$$
$$f(x_1) = f(-1) = -1.5$$
$$f(x_2) = f(0) = 0$$
$$f(x_3) = f(1) = 1.5$$
$$f(x_4) = f(2) = 0.5$$
$$f(x_5) = f(3) = -1$$
$$f(x_6) = f(4) = 0.5$$

For midpoints:

$$f(\bar{x}_1) = f(-1.5) = -1$$
$$f(\bar{x}_2) = f(-0.5) = -1$$
$$f(\bar{x}_3) = f(0.5) = 1$$
$$f(\bar{x}_4) = f(1.5) = 1$$
$$f(\bar{x}_5) = f(2.5) = 0$$
$$f(\bar{x}_6) = f(3.5) = -0.5$$

(a) right end points

$$\int_{-2}^{4} g(x)dx = \sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \Delta x\left[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)+f(x_6)\right]$$
$$= \Delta x\left[f(-1)+f(0)+f(1)+f(2)+f(3)+f(4)\right]$$
$$= 1\left[-1.5+0+1.5+0.5-1+0.5\right]$$
$$= 1[0]$$
$$= 0$$

$$\boxed{\therefore \int_{-2}^{4} g(x)dx = 0}$$

(b) left end points

$$\int_{-2}^{4} g(x)\,dx = \sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \Delta x\left[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)\right]$$
$$= \Delta x\left[f(-2)+f(-1)+f(0)+f(1)+f(2)+f(3)\right]$$
$$= 1\left[0+(-1.5)+0+1.5+0.5-1\right]$$
$$= 1[-0.5]$$
$$= -0.5$$

$$\boxed{\therefore \int_{-2}^{4} g(x)\,dx = -0.5}$$

(c) mid points

$$\int_{-2}^{4} g(x)\,dx = \sum_{i=1}^{n} f(\overline{x}_i)\Delta x$$
$$= \Delta x\left[f(\overline{x}_1)+f(\overline{x}_2)+f(\overline{x}_3)+f(\overline{x}_4)+f(\overline{x}_5)+f(\overline{x}_6)\right]$$
$$= \Delta x\left[f(-1.5)+f(-0.5)+f(0.5)+f(1.5)+f(2.5)+f(3.5)\right]$$
$$= 1\left[-1+(-1)+1+1+0+(-0.5)\right]$$
$$= 1[-0.5]$$
$$= -0.5$$

$$\boxed{\therefore \int_{-2}^{4} g(x)\,dx = -0.5}$$

Answer 7E.

Given $\int_{10}^{30} f(x)\,dx$

$$\Delta x = \frac{b-a}{n} = 4$$

Given values of $f(x)$

$$f(x_0) = f(10) = -12$$
$$f(x_1) = f(14) = -6$$
$$f(x_2) = f(18) = -2$$
$$f(x_3) = f(22) = 1$$
$$f(x_4) = f(26) = 3$$
$$f(x_5) = f(30) = 8$$

(a) Upper sum

$$\int_{10}^{30} f(x)\,dx = \sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \Delta x\left[f(x_1)+f(x_2)+f(x_3)+f(x_4)+f(x_5)\right]$$
$$= 4\left[f(14)+f(18)+f(22)+f(26)+f(30)\right]$$
$$= 4[-6-2+1+3+8]$$
$$= 4[4]$$
$$= 16$$

$$\boxed{\therefore \int_{10}^{30} f(x)\,dx = 16}$$

(b) Lower sum

$$\int_{10}^{30} f(x)\,dx = \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \Delta x\left[f(x_0)+f(x_1)+f(x_2)+f(x_3)+f(x_4)\right]$$

$$= 4\left[f(10)+f(14)+f(18)+f(22)+f(26)\right]$$

$$= 4\left[-12-6-2+1+3\right]$$

$$= 4\left[-16\right]$$

$$= -64$$

$$\therefore \int_{10}^{30} f(x)dx = -64$$

Answer 8E.

if we consider three equal subintervals in [3,9] , we get Δx = 2.

so, starting from the lowest, we have the left end points of the intervals 3 , 5, 7 and the right end points 5 , 7 , 9 while the middle points of the subintervals are 4, 6, 8.

now, the integrand

(a) using right end points is 2 { f(5) + f(7) + f (9) }

= 2 { -0.6 + 0.9+ 1.8 } = 4.2

(b) using left end points is 2 { f(3) + f(5+ f(7) } = 2 { -3.4 -0.6 +0.9} = -6. 2

(c) middle points of the intervals is 2 { f(4) + f(6) + f(8) } = 2 { -2.1 +0.3 + 1.4 } = -0.8

clearly, as the elements increase , the images with respect to the given function also increasing and we can say that the function is an increasing function on the domain.

so, the integrand with respect to left end points < integrand with the midpoints < integrand using right end points is justified.

from this discussion, we can say that the actual integrand lies between -6.2 and 4.2.

Answer 9E.

Midpoint Rule:

$$\int_{a}^{b} f(x)dx = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$$

$$= \Delta x\left[f(\bar{x}_1)+f(\bar{x}_2)+\ldots+f(\bar{x}_n)\right]$$

Where $\Delta x = \dfrac{b-a}{n}$

$\bar{x}_i = \dfrac{1}{2}(x_{i-1}+x_i)$ Midpoint of $[x_{i-1}, x_i]$

Given $\int_0^8 \sin\sqrt{x}dx$, $n=4$

$$\Delta x=\frac{b-a}{n}=\frac{8-0}{4}=2$$

$[0,2],[2,4],[4,6],[6,8]$

Mid points are 1, 3, 5, 7.

$$\int_0^8 \sin\sqrt{x}dx=\Delta x\left[f(1)+f(3)+f(5)+f(7)\right]$$
$$=2\left[\sin\sqrt{1}+\sin\sqrt{3}+\sin\sqrt{5}+\sin\sqrt{7}\right]$$
$$=2\left[0.84147+0.98703+0.7867+0.47577\right]$$
$$=2[3.091086]$$
$$=6.1820372$$

$$\boxed{\therefore \int_0^8 \sin\sqrt{x}dx=6.1820}$$

Given $\int_0^8 \sin\sqrt{x}dx$, $n=4$

$$\Delta x=\frac{b-a}{n}=\frac{8-0}{4}=2$$

$[0,2],[2,4],[4,6],[6,8]$

Mid points are 1, 3, 5, 7.

$$\int_0^8 \sin\sqrt{x}dx=\Delta x\left[f(1)+f(3)+f(5)+f(7)\right]$$
$$=2\left[\sin\sqrt{1}+\sin\sqrt{3}+\sin\sqrt{5}+\sin\sqrt{7}\right]$$
$$=2\left[0.84147+0.98703+0.7867+0.47577\right]$$
$$=2[3.091086]$$
$$=6.1820372$$

$$\boxed{\therefore \int_0^8 \sin\sqrt{x}dx=6.1820}$$

Answer 10E.

The midpoints with x_0, x_1, x_2, x_3 and x_4 are calculated as,

$$\bar{x}_1=\frac{1}{2}(x_{1-1}+x_1)$$
$$=\frac{1}{2}(x_0+x_1)$$
$$=\frac{1}{2}\left(0+\frac{\pi}{8}\right)$$
$$=\frac{\pi}{16}$$

Proceed in the same manner, the remaining points are shown below:

$$\bar{x}_2=\frac{3\pi}{16}, \bar{x}_3=\frac{5\pi}{16}, \text{ and } \bar{x}_4=\frac{7\pi}{16}.$$

Plug in $\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4$ in the midpoint rule formula, then the value of the approximate integral is,

$$M_4=\frac{\pi}{8}\left[f\left(\frac{\pi}{16}\right)+f\left(\frac{3\pi}{16}\right)+f\left(\frac{5\pi}{16}\right)+f\left(\frac{7\pi}{16}\right)\right]$$
$$=\frac{\pi}{8}\left[\cos^4\left(\frac{\pi}{16}\right)+\cos^4\left(\frac{3\pi}{16}\right)+\cos^4\left(\frac{5\pi}{16}\right)+\cos^4\left(\frac{7\pi}{16}\right)\right]$$
$$\approx 0.3927[0.9253+0.4780+0.0953+0.0014]$$
$$=0.3927[1.5]$$

$$\approx 0.5890$$

Hence, $\int_0^{\frac{\pi}{2}} \cos^4 x dx \approx \boxed{0.5890}$.

Answer 11E.

Midpoint Rule:

$$\int_a^b f(x)dx = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$$

$$= \Delta x\left[f(\bar{x}_1)+f(\bar{x}_2)+....+f(\bar{x}_n)\right]$$

Where $\Delta x = \dfrac{b-a}{n}$

$\bar{x}_i = \dfrac{1}{2}(x_{i-1}+x_i)$ Midpoint of $[x_{i-1}, x_i]$

Answer 12E.

Midpoint Rule:

$$\int_a^b f(x)dx = \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$$

$$= \Delta x\left[f(\bar{x}_1)+f(\bar{x}_2)+....+f(\bar{x}_n)\right]$$

Where $\Delta x = \dfrac{b-a}{n}$

$\bar{x}_i = \dfrac{1}{2}(x_{i-1}+x_i)$ Midpoint of $[x_{i-1}, x_i]$

Given $\int_1^4 \sqrt{x^3+1}dx$, $n=6$

$$\Delta x = \frac{b-a}{n} = \frac{4-1}{6} = 0.5$$

$[1,1.5],[1.5,2],[2,2.5],[2.5,3],[3,3.5]$

Mid points are 1.25, 1.75, 2.25, 2.75, 3.25, 3.75.

$$\int_1^4 \sqrt{x^3+1}dx = \Delta x\left[f(1.25)+f(1.75)+f(2.25)+f(2.75)+f(3.25)+f(3.75)\right]$$

$$= 0.5\left[\begin{array}{l}\sqrt{(1.25)^3+1}+\sqrt{(1.75)^3+1}+\sqrt{(2.25)^3+1}+\sqrt{(2.75)^3+1}+\sqrt{(3.25)^3+1}\\ +\sqrt{(3.75)^3+1}\end{array}\right]$$

$$= 0.5\left[\begin{array}{l}\sqrt{1.953125+1}+\sqrt{5.359375+1}+\sqrt{11.390625+1}+\sqrt{20.796875+1}\\ +\sqrt{34.328125+1}+\sqrt{52.734375+1}\end{array}\right]$$

$$= 0.5\left[\begin{array}{l}\sqrt{2.953125}+\sqrt{6.359375}+\sqrt{12.390625}+\sqrt{21.796875}+\sqrt{35.328125}\\ +\sqrt{53.734375}\end{array}\right]$$

$$= 0.5[1.718465+2.5217+3.52003+4.6687+5.9437+7.33037]$$

$$= 0.5[25.742896]$$

$$= 12.871441$$

$$\boxed{\therefore \int_1^4 \sqrt{x^3+1}dx = 12.8714}$$

Answer 13E.

Consider the integral $\int_0^2 \frac{x}{x+1}dx$

Use the Midpoint Rule with the value of $n=10$ to approximate the integral:

Use computer algebra system, Maple to find Midpoint Rule:

First load the package with (Student[Calculus1]);

with(*Student*[*Calculus1*]);

The Maple command and output:

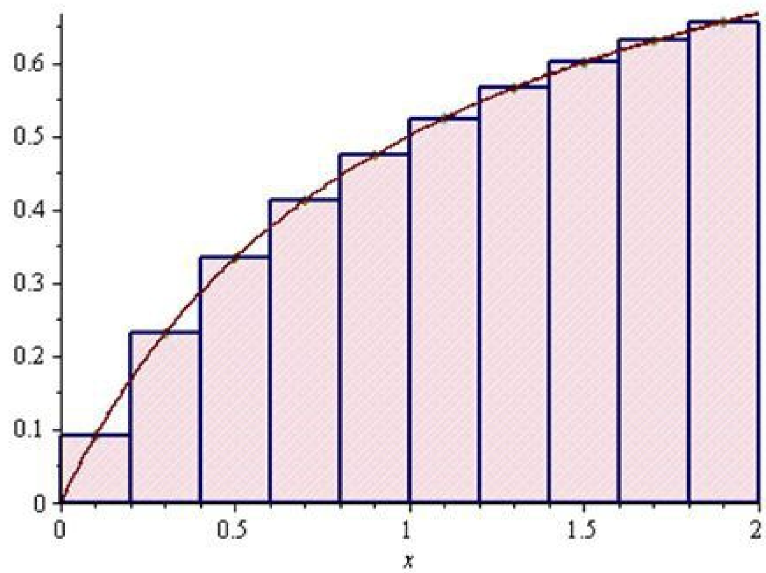
> $RiemannSum\left(\frac{x}{x+1}, x = 0.0..2.0, method = midpoint, partition = 10\right);$

0.9028579061

Sketch a graph $\int_0^2 \frac{x}{x+1}dx$ with $n=10$

The Maple command and output:

> $RiemannSum\left(\frac{x}{x+1}, x = 0.0..2.0, method = midpoint, output = plot, boxoptions = [filled = [color = pink, transparency = 0.5]]\right);$



A midpoint Riemann sum approximation of $\int_0^2 \frac{x}{x+1}dx$ and the partition are uniform. The approximate value of the integral is $\boxed{0.9029}$.

Use the Midpoint Rule with the value of $n=20$ to approximate the integral:

Use computer algebra system, Maple to find Midpoint Rule:

First load the package with (Student[Calculus1]);

with(*Student*[*Calculus1*]);

The Maple command and output:

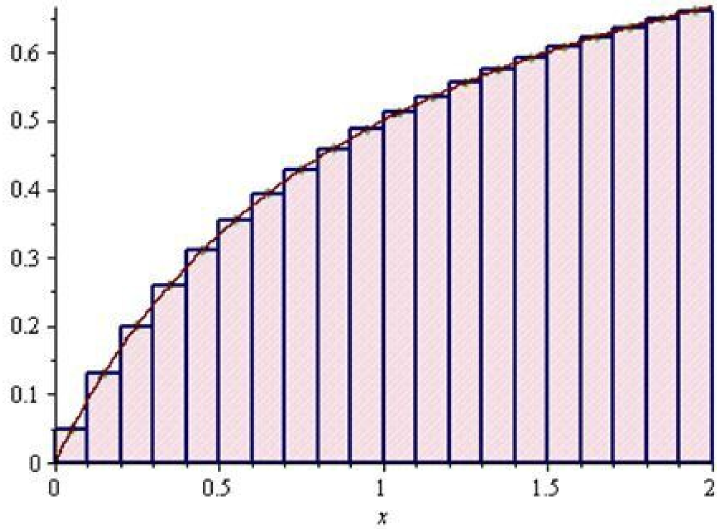
> $RiemannSum\left(\frac{x}{x+1}, x = 0.0..2.0, method = midpoint, partition = 20\right);$

0.9017573654

Sketch a graph $\int_0^2 \frac{x}{x+1}dx$ with $n=10$

The Maple command and output:

> $RiemannSum\left(\frac{x}{x+1}, x = 0.0..2.0, method = midpoint, partition = 20, output = plot, boxoptions = [filled = [color = pink, transparency = 0.5]]\right);$



A midpoint Riemann sum approximation of $\int_0^2 \frac{x}{x+1}dx$ and the partition are uniform. The approximate value of the integral is $\boxed{0.9018}$ and the number of subintervals is used 20.

Answer 14E.

Given function $f(x)=\frac{x}{x+1}$, $[0,2]$, $n=100$

Mat lab program to estimate L_n and R_n is

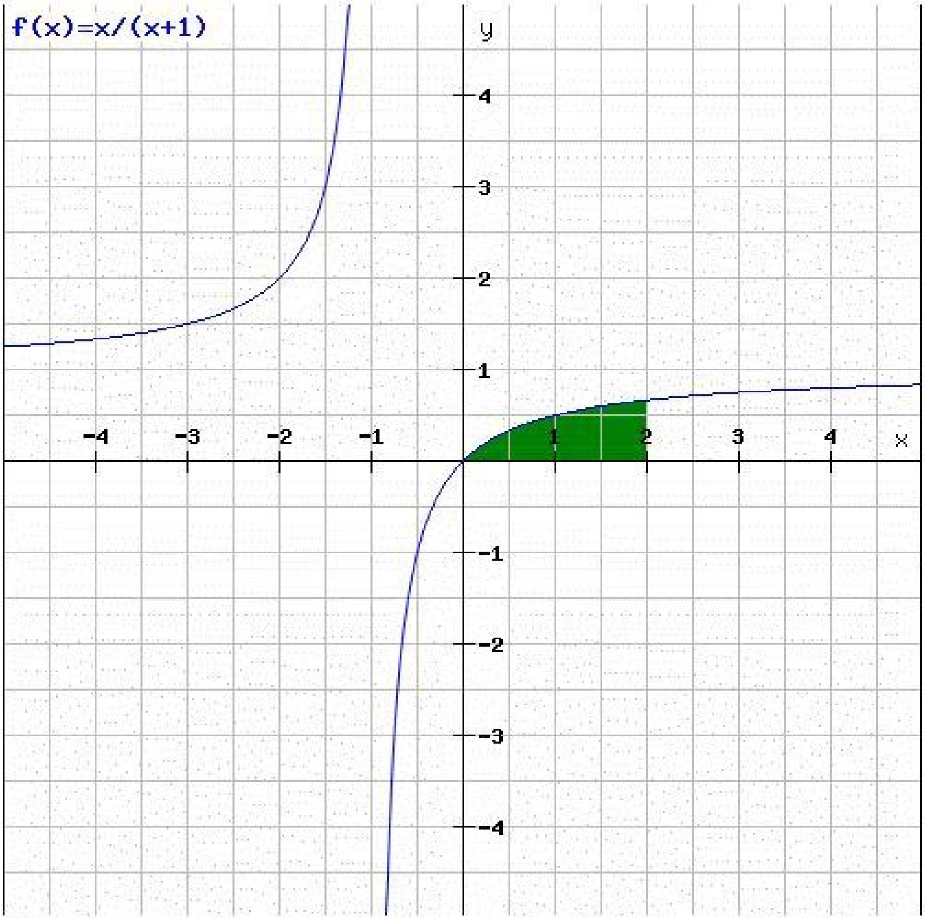
```
function Integration =intVal
clc;
a=0;
b=2;


n=100; % changeing n value 10,30,50 and 100
dx=(b-a)/n;
intVal1=0;
LValue=0;
for i=1:n

x(i)=a+i*dx;
   fx(i)=x(i)/(x(i)+1);
   intVal1=intVal1+fx(i);
    % this is for left end points


      l(i)=a+(i-1)*dx;
        gx(i)=l(i)/(l(i)+1);
        LValue=LValue+gx(i);
end
```

```
intVal=intVal1*dx
LValue=LValue*dx
Integration=intVal
return
```

Computed values are $L_n = 0.8947$ and $R_n = 0.9080$

Hence $\boxed{0.8946 < \int_0^2 \frac{x}{x+1}\,dx < 0.9081}$

Graph for the given function



Answer 15E.

$\int_0^{\pi} \sin x \, d\,x$

using the right Riemann sums , no,. of subintervals n = 5 , 10,50,100.

the following table consists of the right end points of the subintervals , respective images and their sums when multiplied with the lengths of the subintervals :

0.628571429	0.587989832	0.369593609
1.257842157	0.951428221	0.598040596
1.887112885	0.950387663	0.597386532
2.516383614	0.585266784	0.367881979
3.145654342	-0.004061678	-0.002553054
n=5		1.930349661

n=10

0.314285714	0.309137252	0.097157422
0.628571429	0.587989832	0.184796804
0.942857143	0.809239911	0.254332543
1.257142857	0.951212694	0.298952561
1.571428572	0.9999998	0.314285651
1.885714286	0.950821793	0.298829706
2.2	0.808496404	0.254098869
2.514285715	0.586966557	0.184475203
2.828571429	0.307934453	0.0967794
3.142857144	-0.00126449	-0.000397411
		1.983310749

n=50		
0.062857143	0.062815759	0.003948419
0.125781429	0.125450027	0.00788543
0.188705714	0.187587743	0.01179123
0.25163	0.248982957	0.015650357
0.314554286	0.309392657	0.019447538
0.377478571	0.368577731	0.023167743
0.440402857	0.426303916	0.026796246
0.503327143	0.482342722	0.030318685
0.566251429	0.536472339	0.033721119
0.629175714	0.588478512	0.036990078
0.6921	0.638155394	0.040112625
0.755024286	0.685306355	0.0430764
0.817948571	0.729744764	0.045869671
0.880872857	0.771294727	0.048481383
0.943797143	0.809791783	0.050901198
1.006721429	0.845083553	0.053119538
1.069645714	0.877030348	0.055127622
1.13257	0.905505717	0.056917502
1.195494286	0.93039695	0.058482094
1.258418571	0.951605524	0.059815204
1.321342857	0.969047491	0.060911557
1.384267143	0.982653813	0.061766811
1.447191429	0.992370636	0.062377583
1.510115714	0.998159496	0.062741454
1.57304	0.999997483	0.062856985
1.635964286	0.99787732	0.062723717
1.698888571	0.991807399	0.06234218
1.761812857	0.981811747	0.061713881
1.824737143	0.967929927	0.06084131
1.887661429	0.950216886	0.059727919
1.950585714	0.928742735	0.058378115
2.01351	0.903592473	0.056797241
2.076434286	0.874865647	0.054991555
2.139358571	0.842675964	0.052968204
2.202282857	0.807150834	0.050735195
2.265207143	0.768430873	0.048301369
2.328131429	0.72666934	0.045676359
2.391055714	0.682031534	0.042870554
2.45398	0.634694139	0.03989506
2.516904286	0.584844522	0.036761656
2.579828571	0.532679998	0.033482743
2.642752857	0.478407042	0.0300713
2.705677143	0.422240474	0.02654083
2.768601429	0.364402611	0.022905307
2.831525714	0.305122384	0.019179121
2.89445	0.244634435	0.015377022
2.957374286	0.183178183	0.011514057
3.020298571	0.120996883	0.007605518
3.083222857	0.058336658	0.003666876
3.146147143	-0.004554473	-0.000286281
		1.99705528

n=100	
0.031428571	0.0314
0.062857142	0.0628
0.094285713	0.0941
0.125714283	0.1253
0.157142854	0.1564
0.188571424	0.1874
0.219999995	0.2182
0.251428565	0.2487
0.282857136	0.2791
0.314285707	0.3091
0.345714277	0.338
0.377142848	0.3682
0.408571418	0.397
0.439999989	0.4259
0.47142856	0.4541
0.50285713	0.4819
0.534285701	0.509
0.565714271	0.5360
0.597142842	0.5622
0.628571412	0.5879
0.659999983	0.6131
0.691428554	0.6376
0.722857124	0.661
0.754285695	0.6847
0.785714265	0.7073
0.817142836	0.729
0.848571407	0.7503
0.879999977	0.7707
0.911428548	0.7903
0.942857118	0.8092
0.974285689	0.8273
1.005714259	0.8445
1.03714283	0.8609
1.068571401	0.8765
1.099999971	0.8912
1.131428542	0.9050
1.162857112	0.9179
1.194285683	0.9299
1.225714253	0.9410
1.257142824	0.9512
1.288571395	0.9604
1.319999965	0.9687
1.351428536	0.9760
1.382857106	0.9823
1.414285677	0.9877
1.445714248	0.9921
1.477142818	0.9956
1.508571389	0.9980
1.539999959	0.9995

1.6028571	0.9994
1.634285671	0.9979
1.665714242	0.9954
1.697142812	0.9920
1.728571383	0.9875
1.759999953	0.9821
1.791428524	0.975
1.822857094	0.9684
1.854285665	0.9600
1.885714236	0.9508
1.917142806	0.9406
1.948571377	0.9294
1.979999947	0.9174
2.011428518	0.9044
2.042857089	0.8906
2.074285659	0.8759
2.10571423	0.8603
2.1371428	0.8438
2.168571371	0.8265
2.199999941	0.8084
2.231428512	0.789
2.262857083	0.7699
2.294285653	0.7495
2.325714224	0.7283
2.357142794	0.7064
2.388571365	0.6838
2.419999936	0.660
2.451428506	0.6366
2.482857077	0.6121
2.514285647	0.5869
2.545714218	0.5612
2.577142788	0.5349
2.608571359	0.5081
2.63999993	0.4808
2.6714285	0.4530
2.702857071	0.4247
2.734285641	0.396
2.765714212	0.3670
2.797142782	0.3376
2.828571353	0.3079
2.859999924	0.2778
2.891428494	0.2475
2.922857065	0.2169
2.954285635	0.1862
2.985714206	0.1552
3.017142777	0.1241
3.048571347	0.0928
3.079999918	0.06
3.111428488	0.0301
3.142857059	-0.0012

observe that the individual tables on the left side denote n = 5 , 10 , 50 while the right represent n = 100.

the right Riemann sums in all the tables are tending to 2.

therefore $\int_0^\pi \sin x \, d\,x = 2$

Answer 16E.

$\int_0^2 \sqrt{1+x^4}\ dx =$

applying number of subintervals n = 5, 10 , 50 , 100 , the left and right end points of the subintervals are shown and the corresponding images under the function which are multiplied with the length of the subinterval are found and ultimately their sums are brought down in the tables as follows :

n=5			
0	1	0	
0.4	1.012719112	0.405087645	0.405087645
0.8	1.187265766	0.474906307	0.474906307
1.2	1.75316856	0.701267424	0.701267424
1.6	2.748381342	1.099352537	1.099352537
2	4.123105626		1.64924225
		2.680613912	4.329856162

n=10			
0	1	0.2	
0.2	1.00079968	0.200159936	0.200159936
0.4	1.012719112	0.202543822	0.202543822
0.6	1.06282642	0.212565284	0.212565284
0.8	1.187265766	0.237453153	0.237453153
1	1.414213562	0.282842712	0.282842712
1.2	1.75316856	0.350633712	0.350633712
1.4	2.200363606	0.440072721	0.440072721
1.6	2.748381342	0.549676268	0.549676268
1.8	3.390811112	0.678162222	0.678162222
2	4.123105626		0.824621125
		3.354109832	3.978730958

n=50			
0	1	0.04	
0.04	1.00000128	0.040000051	0.040000051
0.08	1.00002048	0.040000819	0.040000819
0.12	1.000103675	0.040004147	0.040004147
0.16	1.000327626	0.040013105	0.040013105
0.2	1.00079968	0.040031987	0.040031987
0.24	1.001657506	0.0400663	0.0400663
0.28	1.003068572	0.040122743	0.040122743
0.32	1.005229208	0.040209168	0.040209168
0.36	1.008363109	0.040334524	0.040334524
0.4	1.012719112	0.040508764	0.040508764
0.44	1.018568093	0.040742724	0.040742724
0.48	1.026198889	0.041047956	0.041047956
0.52	1.035913201	0.041436528	0.041436528
0.56	1.048019542	0.041920782	0.041920782
0.6	1.06282642	0.042513057	0.042513057
0.64	1.080635073	0.043225403	0.043225403
0.68	1.101732163	0.044069287	0.044069287
0.72	1.126382954	0.045055318	0.045055318
0.76	1.154825424	0.046193017	0.046193017
0.8	1.187265766	0.047490631	0.047490631
0.84	1.223875549	0.048955022	0.048955022
0.88	1.264790639	0.050591626	0.050591626
0.92	1.310111812	0.052404472	0.052404472
0.96	1.35990682	0.054396273	0.054396273
1	1.414213562	0.056568542	0.056568542
1.04	1.473043978	0.058921759	0.058921759
1.08	1.536388284	0.061455531	0.061455531
1.12	1.604219237	0.064168769	0.064168769
1.16	1.676496156	0.067059846	0.067059846
1.2	1.75316856	0.070126742	0.070126742
1.24	1.834179315	0.073367173	0.073367173
1.28	1.919467259	0.07677869	0.07677869
1.32	2.008969328	0.080358773	0.080358773
1.36	2.10262221	0.084104888	0.084104888
1.4	2.200363606	0.088014544	0.088014544
1.44	2.302133133	0.092085325	0.092085325
1.48	2.407872953	0.096314918	0.096314918
1.52	2.517528185	0.100701127	0.100701127
1.56	2.631047122	0.105241885	0.105241885
1.6	2.748381342	0.109935254	0.109935254
1.64	2.869485696	0.114779428	0.114779428
1.68	2.994318246	0.11977273	0.11977273
1.72	3.122840143	0.124913606	0.124913606
1.76	3.255015478	0.130200619	0.130200619
1.8	3.390811112	0.135632444	0.135632444
1.84	3.530196504	0.14120786	0.14120786
1.88	3.673143526	0.146925741	0.146925741
1.92	3.819626285	0.152785051	0.152785051
1.96	3.969620959	0.158784838	0.158784838
2	4.123105626		0.164924225
		3.591539791	3.716464016

n=100
0
0
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0
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1
1
1

observe that the third column shows the Left Riemann sum and the fourth column shows the right Riemann sum of each of the table.

as n increases , the refinement of partition is done and the difference between the lower Riemann sum (left Riemann sum) and the upper Riemann sum (right Riemann sum) is considerably reduced and shows that the Riemann integration tend to approximately 3.65.

when n = 5 , the lower sum is 2.6806 and upper sum is 4.3298 and as the partition points increase to n = 100, the lower sum increased to 3.62 and upper sum is reduced to 3.68.

in case of $\int_{-1}^{2}\sqrt{1+x^4}\ \mathrm{d}x = \int_{-1}^{0}\sqrt{1+x^4}\ \mathrm{d}x + \int_{0}^{2}\sqrt{1+x^4}\ \mathrm{d}x$

-1	1.386494919	0.027729898	
-0.98	1.35990682	0.027198136	0.027198136
-0.96	1.334447062	0.026688941	0.026688941
-0.94	1.310111812	0.026202236	0.026202236
-0.92	1.286895489	0.02573791	0.02573791
-0.9	1.264790639	0.025295813	0.025295813
-0.88	1.243787828	0.024875757	0.024875757
-0.86	1.223875549	0.024477511	0.024477511
-0.84	1.205040149	0.024100803	0.024100803
-0.82	1.187265766	0.023745315	0.023745315
-0.8	1.170534305	0.023410686	0.023410686
-0.78	1.154825424	0.023096508	0.023096508
-0.76	1.140116555	0.022802331	0.022802331
-0.74	1.126382954	0.022527659	0.022527659
-0.72	1.113597773	0.022271955	0.022271955
-0.7	1.101732163	0.022034643	0.022034643
-0.68	1.090755408	0.021815108	0.021815108
-0.66	1.080635073	0.021612701	0.021612701
-0.64	1.071337183	0.021426744	0.021426744
-0.62	1.06282642	0.021256528	0.021256528
-0.6	1.05506633	0.021101327	0.021101327
-0.58	1.048019542	0.020960391	0.020960391
-0.56	1.041648002	0.02083296	0.02083296
-0.54	1.035913201	0.020718264	0.020718264
-0.52	1.030776406	0.020615528	0.020615528
-0.5	1.026198889	0.020523978	0.020523978
-0.48	1.022142143	0.020442843	0.020442843
-0.46	1.018568093	0.020371362	0.020371362
-0.44	1.015439294	0.020308786	0.020308786
-0.42	1.012719112	0.020254382	0.020254382
-0.4	1.010371892	0.020207438	0.020207438
-0.38	1.008363109	0.020167262	0.020167262
-0.36	1.006659505	0.02013319	0.02013319
-0.34	1.005229208	0.020104584	0.020104584
-0.32	1.004041832	0.020080837	0.020080837
-0.3	1.003068572	0.020061371	0.020061371
-0.28	1.002282276	0.020045646	0.020045646
-0.26	1.001657506	0.02003315	0.02003315
-0.24	1.001170595	0.020023412	0.020023412
-0.22	1.00079968	0.020015994	0.020015994
-0.2	1.000524742	0.020010495	0.020010495
-0.18	1.000327626	0.020006553	0.020006553
-0.16	1.000192062	0.020003841	0.020003841
-0.14	1.000103675	0.020002073	0.020002073
-0.12	1.000049999	0.020001	0.020001
-0.1	1.00002048	0.02000041	0.02000041
-0.08	1.00000648	0.02000013	0.02000013
-0.06	1.00000128	0.020000026	0.020000026
-0.04	1.00000008	0.020000002	0.020000002
-0.02	1	0.02	0.02
0	1		0.02
		1.085334418	1.07760452

as in the previous cases , the lower Riemann sum of the function on [-1, 0] is 1.085334418

while the upper Riemann sum is 1.09760452(see the sum in the table is wrongly printed)

adding the corresponding sums in the above table, we get

the lower Riemann sum of the given function o [-1 ,2] is 4.707717207 while the upper Riemann sum is 4.782449422.

Answer 17E.

We have been given the limit as

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1-x_i^2}{4+x_i^2} \Delta x, \quad [2,6]$$

we have a = 2 , b = 6

Then $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1-x_i^2}{4+x_i^2} \Delta x = \int_2^6 \frac{1-x^2}{4+x^2}\, dx$

Answer 18E.

We have been given limit as

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos(x_i)}{x_i} \Delta x$$

we have a = π , b = 2π

then $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos(x)_i}{x_i} \Delta x = \int_\pi^{2\pi} \frac{\cos(x)}{x}\, dx$

Answer 19E.

Given $\lim_{n\to\infty} \sum_{i=1}^{n} \left[5\left(x_i^*\right)^3 - 4x_i^*\right]\Delta x$, $[2,7]$

Comparing the given limit with

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad \text{where } \Delta x = \frac{b-a}{n}, \; x_i = a + i\Delta x$$

We have $\lim_{n\to\infty} \sum_{i=1}^{n} \left[5\left(x_i^*\right)^3 - 4x_i^*\right]\Delta x = \int_2^7 \left(5x^3 - 4x\right) dx$

$$\therefore \lim_{n\to\infty} \sum_{i=1}^{n} \left[5\left(x_i^*\right)^3 - 4x_i^*\right]\Delta x = \int_2^7 \left(5x^3 - 4x\right) dx$$

Answer 20E.

Consider $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{x_i^*}{(x_i^*)^2+4}\Delta x$ in the interval $[1,3]$

Note that the following **definition of definite integral**:

If *f* is a function defined for $a \le x \le b$, we divide the interval $[a,b]$ into *n* sub intervals of equal width $\Delta x = \frac{b-a}{n}$. We let $x_0(=a), x_1, x_2, \ldots, x_n(=b)$ be end points of these subintervals and we let $x_0^*, x_1^*, x_2^*, \ldots, x_n^*$ be any sample points in these subintervals, so x_i^* lies in the ith subinterval $[x_{i-1}, x_i]$. Then the definite integral of *f* from *a* to *b* is

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x$$

provided that the limit exists and gives the same value for all possible choices of sample points. If it does exist, we say that *f* is integrable on $[a,b]$.

Let $f(x) = \frac{x}{x^2+4}$

Clearly $f(x)$ is continuous on $[1,3]$

Hence by definition of definite integral,

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{x_i^*}{(x_i^*)^2+4}\Delta x = \int_1^3\left(\frac{x}{x^2+4}\right)dx$$

Thus, $\boxed{\lim_{n\to\infty}\sum_{i=1}^{n}\frac{x_i^*}{(x_i^*)^2+4}\Delta x = \int_1^3\left(\frac{x}{x^2+4}\right)dx}$.

Answer 21E.

Consider the integral $\int_2^5 (4-2x)dx$.

Use the following form of the definition of the integral in theorem and evaluate the integral.

If f is integrable on $[a,b]$, then $\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$.

Where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$.

The limit values are $b=5$ and $a=2$.

Substitute $b=5$ and $a=2$ values in $\Delta x = \frac{b-a}{n}$.

$$\begin{aligned}\Delta x &= \frac{b-a}{n}\\ &= \frac{5-2}{n}\\ &= \frac{3}{n}\end{aligned}$$

Substitute $a=2$, $\Delta x=\frac{3}{n}$ and $i=0,1,2,3...$ values in $x_i=a+i\Delta x$.

$$\begin{aligned} x_0 &= a \\ &= 2 \\ x_1 &= a+\Delta x \\ &= 2+\frac{3}{n} \\ x_2 &= a+2\Delta x \\ &= 2+2\cdot\frac{3}{n},\dots \\ x_i &= a+i\Delta x \\ &= 2+i\cdot\frac{3}{n} \end{aligned}$$

Use the above theorem and evaluate the integral.

$$\begin{aligned} \int_2^5 (4-2x)\,dx &= \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \\ &= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(2+\frac{3i}{n}\right)\cdot\frac{3}{n} \\ &= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left(4-2\left(2+\frac{3i}{n}\right)\right) \\ &= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left(4-4-\frac{6i}{n}\right) \end{aligned}$$

$$\begin{aligned} &= \lim_{n\to\infty}\frac{3}{n}\sum_{i=1}^{n}\left(-\frac{6i}{n}\right) \\ &= \lim_{n\to\infty}\left(\frac{-18}{n^2}\right)\sum_{i=1}^{n}(i) \\ &= \lim_{n\to\infty}\left(\frac{-18}{n^2}\right)\cdot\frac{n(n+1)}{2} \\ &= -9\left(\lim_{n\to\infty}1+\lim_{n\to\infty}\frac{1}{n}\right) \end{aligned}$$

Use the following limit law:

$$\lim_{x\to a}\left[f(x)+g(x)\right]=\lim_{x\to a}f(x)+\lim_{x\to a}g(x)$$

$$\begin{aligned} -9\left(\lim_{n\to\infty}1+\lim_{n\to\infty}\frac{1}{n}\right) &= -9\lim_{n\to\infty}\left(1+\frac{1}{n}\right) \\ &= -9(1+0) \\ &= -9 \end{aligned}$$

Therefore, the integral of $\int_2^5 (4-2x)\,dx$ is $\boxed{-9}$.

Answer 22E.

To evaluate the integral $\int_1^4 \left(x^2-4x+2\right)dx$ use the following theorem.

If f is integrable on $[a,b]$, then

$$\int_a^b f(x)\,dx=\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

And, $\Delta x=\frac{b-a}{n}$, $x_i=a+i\Delta x$

Assume that,

$$f(x) = x^2 - 4x + 2$$

Consider,

$$\begin{aligned}\Delta x &= \frac{b-a}{n} \\ &= \frac{4-1}{n} \\ &= \frac{3}{n}\end{aligned}$$

$$\begin{aligned}x_0 &= a \\ &= 1\end{aligned}$$

$$\begin{aligned}x_1 &= a + \Delta x \\ &= 1 + \frac{3}{n}\end{aligned}$$

And,

$$\begin{aligned}x_2 &= a + 2\Delta x \\ &= 1 + 2 \cdot \frac{3}{n}, \ldots\end{aligned}$$

$$\begin{aligned}x_i &= a + i\Delta x \\ &= 1 + i \cdot \frac{3}{n}\end{aligned}$$

Therefore,

$$\int_1^4 \left(x^2 - 4x + 2\right) dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(1 + \frac{3i}{n}\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(\left(1 + \frac{3i}{n}\right)^2 - 4\left(1 + \frac{3i}{n}\right) + 2\right) \quad \left(\text{Since } f(x) = x^2 - 4x + 2\right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(1 + \frac{9i^2}{n^2} + \frac{6i}{n} - 4 - \frac{12i}{n} + 2\right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left(\frac{9i^2}{n^2} - \frac{6i}{n} - 1\right)$$

$$= \lim_{n\to\infty} \frac{3}{n} \left(\frac{9\sum_{i=1}^{n} i^2}{n^2} - \frac{6\sum_{i=1}^{n} i}{n} - \sum_{i=1}^{n} 1\right)$$

The known results are,

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} 1 = n$$

So,

$$\int_1^4 (x^2 - 4x + 2)\,dx = \lim_{n\to\infty} \frac{3}{n}\left(\frac{9\sum_{i=1}^{n} i^2}{n^2} - \frac{6\sum_{i=1}^{n} i}{n} - \sum_{i=1}^{n} 1 \right)$$

$$= \lim_{n\to\infty} \frac{3}{n}\left(\frac{9n(n+1)(2n+1)}{6n^2} - \frac{6n(n+1)}{2n} - n \right)$$

$$= \lim_{n\to\infty} \left(\frac{27n(n+1)(2n+1)}{6n^3} - \frac{18n(n+1)}{2n^2} - 3 \right)$$

$$= \lim_{n\to\infty} \left(\frac{27n^3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6n^3} - \frac{18n^2\left(1+\frac{1}{n}\right)}{2n^2} - 3 \right)$$

Continuation of the above,

$$\int_1^4 (x^2 - 4x + 2)\,dx = \lim_{n\to\infty} \left(\frac{9}{2}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) - 9\left(1+\frac{1}{n}\right) - 3 \right)$$

$$= \left(\frac{9}{2}\left(1+\frac{1}{\infty}\right)\left(2+\frac{1}{\infty}\right) - 9\left(1+\frac{1}{\infty}\right) - 3 \right)$$

$$= \left(\frac{9}{2}(1+0)(2+0) - 9(1+0) - 3 \right)$$

$$= 9 - 12$$

$$= \boxed{-3}$$

Answer 23E.

Consider the integral $\int_{-2}^{0} (x^2 + x)\,dx$

Note that, if f is integrable on $[a,b]$, then

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

where $\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$

Here $[a,b]=[-2,0]$

And

$$\Delta x=\frac{b-a}{n}$$
$$=\frac{0-(-2)}{n}$$
$$=\frac{2}{n}$$

Now

$$x_0=a=-2,$$
$$x_1=a+\Delta x=-2+\frac{2}{n}$$
$$x_2=a+2\Delta x=-2+2\cdot\frac{2}{n},...$$
$$x_i=a+i\Delta x=-2+i\cdot\frac{2}{n}$$

Therefore,

$$\int_{-2}^{0}\left(x^2+x\right)dx \quad =\lim_{n\to\infty}\sum_{i=1}^{n}f\left(x_i\right)\Delta x$$
$$=\lim_{n\to\infty}\sum_{i=1}^{n}f\left(-2+\frac{2i}{n}\right).\frac{2}{n}$$
$$=\lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left(\left(-2+\frac{2i}{n}\right)^2+\left(-2+\frac{2i}{n}\right)\right)$$
$$=\lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left(4+\frac{4i^2}{n^2}-\frac{8i}{n}-2+\frac{2i}{n}\right)$$
$$=\lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left(2+\frac{4i^2}{n^2}-\frac{6i}{n}\right)$$
$$=\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{4}{n}+\frac{8i^2}{n^3}-\frac{12i}{n^2}\right)$$

Note that,

$$\sum n=\frac{n(n+1)}{2} \text{ and } \sum n^2=\frac{n(n+1)(2n+1)}{6}$$

Therefore,

$$\int_{-2}^{0}\left(x^2+x\right)dx \quad =\lim_{n\to\infty}\left(\frac{4n}{n}+\frac{8n(n+1)(2n+1)}{6n^3}-\frac{12n(n+1)}{2n^2}\right)$$
$$=\lim_{n\to\infty}\left(\frac{4n}{n}+\frac{4n^3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{3n^3}-\frac{6n^2\left(1+\frac{1}{n}\right)}{n^2}\right)$$
$$=\lim_{n\to\infty}\left(4+\frac{4}{3}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)-6\left(1+\frac{1}{n}\right)\right)$$
$$=\left(4+\frac{4}{3}(1+0)(2+0)-6(1+0)\right)$$
$$=\left(4+\frac{8}{3}-6\right)$$
$$=\frac{2}{3}$$

Thus, $\int_{-2}^{0}\left(x^2+x\right)dx=\boxed{\frac{2}{3}}$.

Answer 24E.

Given $\int_0^2 (2x - x^3)\,dx$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{n}$$
$$= \frac{2}{n}$$

$$x_0 = a = 0, \qquad x_1 = a + \Delta x = 0 + \frac{2}{n}$$

$$x_2 = a + 2\Delta x = 0 + 2 \cdot \frac{2}{n}, \ldots$$

$$x_i = a + i\Delta x = 0 + i \cdot \frac{2}{n}$$

And we can use the theorem

If f is integrable on $[a,b]$, then

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad \text{where } \Delta x = \frac{b-a}{n} \quad , x_i = a + i\Delta x$$

$$\int_0^2 (2x - x^3)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} f\left(\frac{2i}{n}\right) \cdot \frac{2}{n}$$
$$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(2\left(\frac{2i}{n}\right) - \left(\frac{2i}{n}\right)^3\right)$$
$$= \lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left(\frac{4i}{n} - \frac{8i^3}{n^3}\right)$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{8i}{n^2} - \frac{16i^3}{n^4}\right)$$
$$= \lim_{n\to\infty} \left(\frac{8n(n+1)}{2n^2} - \frac{16n^2(n+1)^2}{4n^4}\right)$$
$$= \lim_{n\to\infty} \left(4\left(1+\frac{1}{n}\right) - 4\left(1 + \frac{2}{n} + \frac{1}{n^2}\right)\right)$$
$$= (4(1+0) - 4(1+0+0))$$
$$= 4 - 4$$
$$= 0$$

$$\boxed{\therefore \int_0^2 (2x - x^3)\,dx = 0}$$

Answer 25E.

Given $\int_0^1 (x^3 - 3x^2)\,dx$

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$$

$$x_0 = a = 0, x_1 = a + \Delta x = 0 + \frac{1}{n}$$

$$x_2 = a + 2\Delta x = 0 + 2.\frac{1}{n}, \ldots x_i = a + i\Delta x = 0 + i.\frac{1}{n}$$

And we can use the theorem

If f is integrable on $[a,b]$, then

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad \text{where } \Delta x = \frac{b-a}{n} \quad , x_i = a + i\Delta x$$

$$\int_0^1 (x^3 - 3x^2)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{i}{n}\right)\cdot\frac{1}{n}$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left(\left(\frac{i}{n}\right)^3 - 3\left(\frac{i}{n}\right)^2\right)$$

$$= \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^3}{n^3} - \frac{3i^2}{n^2}\right)$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i^3}{n^4} - \frac{3i^2}{n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{n^2(n+1)^2}{4n^4} - \frac{3n(n+1)(2n+1)}{6n^3}\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{4}\left(1+\frac{2}{n}+\frac{1}{n^2}\right) - \frac{1}{2}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right)$$

$$= \left(\frac{1}{4}(1+0+0) - \frac{1}{2}(1+0)(2+0)\right)$$

$$= \frac{1}{4} - 1$$

$$= \frac{-3}{4}$$

$$\boxed{\therefore \int_0^1 (x^3 - 3x^2)\,dx = \frac{-3}{4}}$$

Answer 26E.

(A) Given that $\int_0^4 (x^2 - 3x)\,dx$, where $n = 8$, $\Delta x = \frac{4}{8} = 0.5$

The right end points are

$x_1 = 0.5,\ x_2 = 1,\ x_3 = 1.5,\ x_4 = 2,\ x_5 = 2.5,\ x_6 = 3,\ x_7 = 3.5,\ x_8 = 4$

So the Riemann sum is

$$R_8 = \sum_{i=1}^{8} f(x_i)\,\Delta x$$

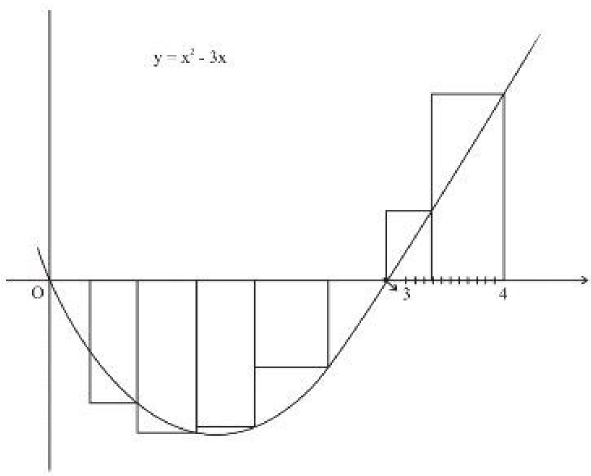
$$= f(0.5)\Delta x + f(1)\Delta x + f(1.5)\Delta x + f(2)\Delta x + f(2.5)\Delta x + f(3)\Delta x + f(3.5)\Delta x + f(4)\Delta x$$

$$= \frac{1}{2}[-1.25 - 2 - 2.25 - 2 - 1.25 - 0 + 1.75 + 4]$$

$$= \frac{1}{2}(-3)$$

$$= -1.5$$

(B) The graph for the function is as follows:



(C)

$\int_0^4 \left(x^2-3x\right)dx$,

where $a=0,\ b=4,\ \Delta x=\frac{4-0}{n}=\frac{4}{n}$

$x_0=0,\ x_1=\frac{4}{n},\ x_2=\frac{8}{n},\ x_i=\frac{4i}{n}$

$$\int_0^4 \left(x^3-3x\right)dx=\lim_{n\to\infty} f\left(x_i\right)\Delta x=\lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{4i}{n}\right)\frac{4}{n}$$

$$=\frac{4}{n}\lim_{n\to\infty}\sum_{i=1}^{n}\left\{\frac{16i^2}{n^2}-\frac{12i}{n}\right\}$$

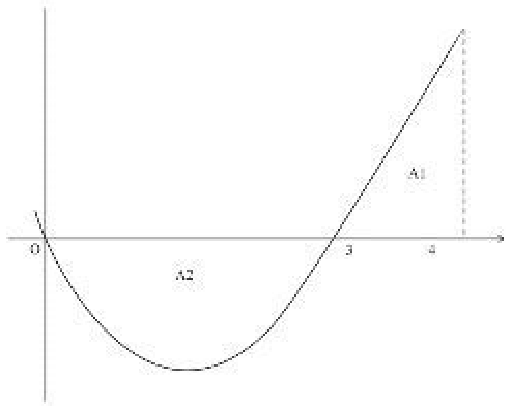
$$=\frac{4}{n}\times\frac{16}{n^2}\lim_{n\to\infty}\sum_{i=1}^{n} i^2-\frac{4}{n}\times\frac{12}{n}\lim_{n\to\infty}\sum_{i=1}^{n} i$$

$$=\frac{64}{n^3}\frac{n(n+1)(2n+1)}{6}-\frac{48}{n^2}\frac{n(n+1)}{2}$$

$$=64\times\frac{2}{6}-24$$

$$=\frac{64}{3}-24=\boxed{-2.67}$$

(D)



$$\int_0^4 \left(x^2-3x\right)dx=A_1-A_2$$

$$=\boxed{-2.67}$$

Answer 27E.

We have to prove that $\int_a^b x\,dx=\frac{b^2-a^2}{2}$

Interval is $\left[a,\ b\right]$ and $f(x)=x$

Then $\Delta x=\frac{b-a}{n}$ where n = number of subintervals

Thus $x_0=a,\ x_1=a+\frac{b-a}{n},\ x_2=a+\frac{b-a}{n}\times 2$

$x_3=a+\frac{b-a}{n}\times 3,\ldots\ldots\ x_i=a+\frac{b-a}{n}\times i$

$$\int_a^b x\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(a+\frac{b-a}{n}i\right)\frac{b-a}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(a+\frac{b-a}{n}i\right)\frac{b-a}{n}$$

$$= \lim_{n\to\infty}\left(\frac{b-a}{n}\right)\left(a\sum_{i=1}^{n}1+\frac{b-a}{n}\sum_{i=1}^{n}i\right)$$

$$= \lim_{n\to\infty}\left(\frac{b-a}{n}\right)\left(an+\frac{b-a}{n}\times\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty}\left(ab-a^2+\frac{(b-a)^2}{n}\times\frac{(n+1)}{2}\right)$$

$$= \lim_{n\to\infty}\left(ab-a^2+\frac{(b-a)^2}{2}\times(1+1/n)\right)$$

$$= \left(ab-a^2+\frac{(b-a)^2}{2}\right)$$

$$= ab-a^2+\frac{b^2}{2}+\frac{a^2}{2}-ab$$

$$\boxed{=\frac{b^2-a^2}{2}}$$

Answer 28E.

We have to prove that $\int_a^b x^2\,dx = \frac{b^3-a^3}{3}$

Where $\Delta x = \frac{b-a}{n}$, $x_1 = a+\frac{b-a}{n}$, $x_2 = a+2\frac{(b-a)}{n}$, ... $x_i = a+i\frac{(b-a)}{n}$

Thus

$$\int_a^b x^2\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(a+\frac{i(b-a)}{n}\right)\frac{b-a}{n}$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n}\left(a+\frac{i(b-a)}{n}\right)^2\frac{b-a}{n}$$

$$= \lim_{n\to\infty}\frac{b-a}{n}\sum_{i=1}^{n}\left(a^2+\frac{i^2(b-a)^2}{n^2}+2a\frac{i(b-a)}{n}\right)$$

$$= \lim_{n\to\infty}\frac{b-a}{n}\left(a^2n+\frac{(b-a)^2}{n^2}\times\frac{n(n+1)(2n+1)}{6}+2a\frac{(b-a)}{n}\times\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty}\left(a^2(b-a)+\frac{(b-a)^3}{n^3}\times\frac{n(n+1)(2n+1)}{6}+2a\frac{(b-a)^2}{n^2}\times\frac{n(n+1)}{2}\right)$$

$$= \lim_{n\to\infty}\left(a^2(b-a)+\frac{(b-a)^3}{1}\times\frac{(1+1/n)(2+1/n)}{6}+2a\frac{(b-a)^2}{1}\times\frac{(1+1/n)}{2}\right)$$

$$= \left(a^2(b-a)+\frac{(b-a)^3}{1}\times\frac{(1+0)(2+0)}{6}+2a\frac{(b-a)^2}{1}\times\frac{(1+0)}{2}\right)$$

$$= (b-a)a^2+a(b-a)^2+\frac{1}{3}(b-a)^3$$

$$= a^2b-a^3+a\left(b^2+a^2-2ab\right)+\frac{1}{3}\left(b^3-a^3-3ab^2+3a^2b\right)$$

$$= \boxed{\frac{b^3-a^3}{3}}$$

Answer 29E.

We have to express the integral $\int_2^6 \frac{x}{1+x^5}dx$ as a limit of Riemann sums.

Here the interval is [2, 6] in the form of $[a, b]$, where $a = 2$ and $b = 6$.

Let $f(x) = \frac{x}{1+x^5}$

The width of n subintervals is $\Delta x = \frac{b-a}{n} = \frac{6-2}{n} = \frac{4}{n}$

And n intervals are

$[x_0, x_1], [x_1, x_2], [x_2, x_3], --- [x_{i-1}, x_i] -- [x_{n-1}, x_n]$,

where $x_0 = 2$, $x_1 = 2 + \frac{4}{n}$, $x_2 = 2 + \frac{8}{n}$, $x_3 = 2 + \frac{12}{n} --- x_i = 2 + \frac{4i}{n}$

Since $f(x) = \frac{x}{1+x^5}$ is continuous in the interval [2, 6].

So if we use right end points of sub-intervals as sample points, then we can use the definition of the integral as follows:

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x,$$

where $\int_2^6 \frac{x}{1+x^5} = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(2+\frac{4i}{n}\right)\frac{4}{n}$

So
$$\boxed{\int_2^6 \frac{x}{1+x^5} = \lim_{n\to\infty}\sum_{i=1}^{n} \frac{\left(2+4i/n\right)}{\left[1+\left(2+4i/n\right)^5\right]}\cdot\frac{4}{n}}$$
Riemann sum

Answer 30E.

Given $\int_0^{2\pi} x^2 \sin x dx$,

If f is integrable on $[a,b]$, then

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

Where $\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$

For the given problem
$$\Delta x = \frac{b-a}{n} = \frac{2\pi - 0}{n}$$
$$E = \frac{2\pi}{n}$$
$$x_i = a + i\Delta x = 0 + i\frac{2\pi}{n} = \frac{2\pi i}{n}$$

$$\int_0^{2\pi} x^2 \sin x dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{2\pi i}{n}\right)\frac{2\pi}{n}$$
$$= \frac{2\pi}{n}\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2\pi i}{n}\right)^2 \sin\left(\frac{2\pi i}{n}\right)$$

Therefore
$$\boxed{\int_0^{2\pi} x^2 \sin x dx = \frac{2\pi}{n}\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{2\pi i}{n}\right)^2 \sin\left(\frac{2\pi i}{n}\right)}$$

Answer 31E.

(a)

Evaluate the integral $\int_0^2 g(x)\,dx$

From the graph it is clear that the interval $[0,2]$, the graph forms a triangle with base 2 units and height 4 units.

The area of the triangle formula is,

$$\begin{aligned} A &= \frac{1}{2}\cdot \text{base}\cdot \text{height} \\ &= \frac{1}{2}bh \\ &= \frac{1}{2}(2)(4) \\ &= 4 \end{aligned}$$

Therefore,

$$\int_0^2 g(x)\,dx = \boxed{4}$$

(b)

Find the integral $\int_2^6 g(x)\,dx$

In the interval $[2,6]$, the graph represents a semi- circle of diameter 4 units.

The area of the semi-circle formula is,

$$\begin{aligned} A &= \frac{\pi r^2}{2} \\ &= \frac{\pi(2)^2}{2} \qquad (\text{radius } r = 2) \\ &= 2\pi \end{aligned}$$

Observe that the semicircle is below x axis, therefore,

$$\int_2^6 g(x)\,dx = \boxed{-2\pi}$$

(c)

Find the integral $\int_0^7 g(x)\,dx$:

Observe from the graph that,

$$\int_0^7 g(x)\,dx = \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx$$

First find the integral $\int_6^7 g(x)\,dx$

In the interval $[6,7]$, the graph is a triangle with base and height one unit each.

So that,

$$\begin{aligned} A &= \frac{1}{2}\cdot \text{base}\cdot \text{height} \\ &= \frac{1}{2}bh \\ &= \frac{1}{2}(1)(1) \\ &= \frac{1}{2} \end{aligned}$$

Thus

$$\begin{aligned} \int_0^7 g(x)\,dx &= \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx \\ &= 4 - 2\pi + \frac{1}{2} \quad \left[\int_0^2 g(x)\,dx = 4, \int_2^6 g(x)\,dx = -2\pi \text{ and } \int_6^7 g(x)\,dx = \frac{1}{2}\right] \\ &= \left(\frac{9}{2} - 2\pi\right) \end{aligned}$$

Therefore, $\int_0^7 g(x)\,dx = \boxed{\left(\frac{9}{2} - 2\pi\right)}$

Answer 32E.

Given that the integral as $\int_2^{10} x^6\,dx$

Here $a = 2$ and $b = 10$

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{10-2}{n}$$

$$\Delta x = \frac{8}{n}$$

And $x_i^* = x_i$

$$= a + i\Delta x$$

$$x_i^* = 2 + \frac{8i}{n}$$

Hence, the integral becomes

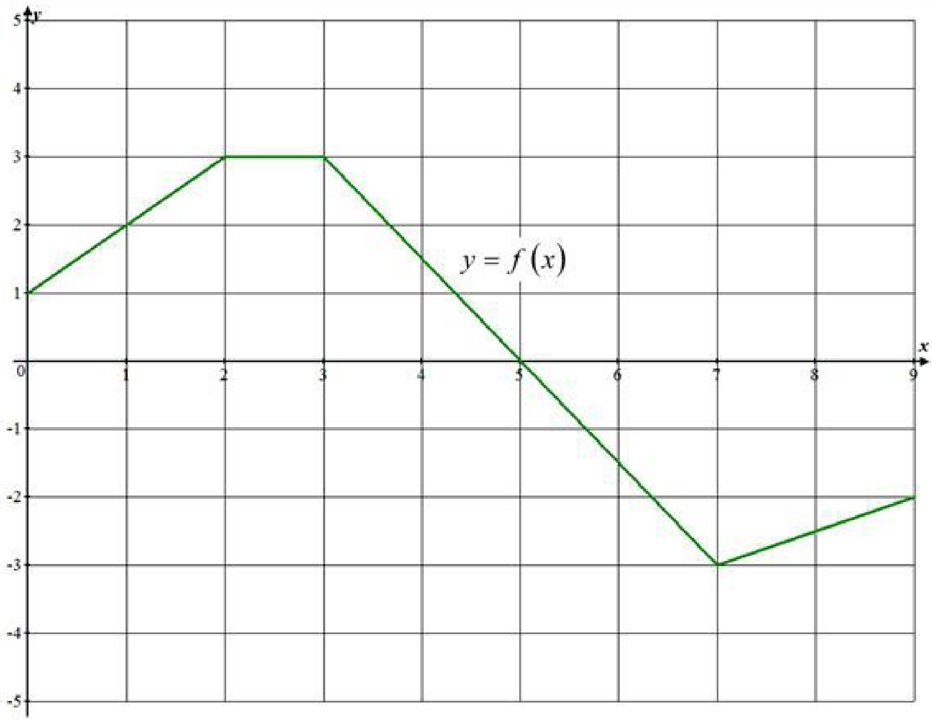
$$\begin{aligned} \int_2^{10} x^6\,dx &= \lim_{n\to\infty}\sum_{i=1}^{n}\left(2+\frac{8i}{n}\right)^6\cdot\left(\frac{8}{n}\right) \\ &= 8\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\left(2+\frac{8i}{n}\right)^6 \end{aligned}$$

Now with the help of CAS software, we solve this integral as follows:

$$\begin{aligned} \int_2^{10} x^6\,dx &= 8\lim_{n\to\infty}\frac{1}{n}\cdot\frac{64(58.593n^6 + 164.052n^5 + 131.208n^4 - 27.776n^2 + 2048)}{21n^5} \\ &= 8\left(\frac{1.249984}{7}\right) \\ &= \frac{9.999872}{7} \\ &\approx 1.4285531 \end{aligned}$$

Answer 33E.

Consider the following graph:



(a)

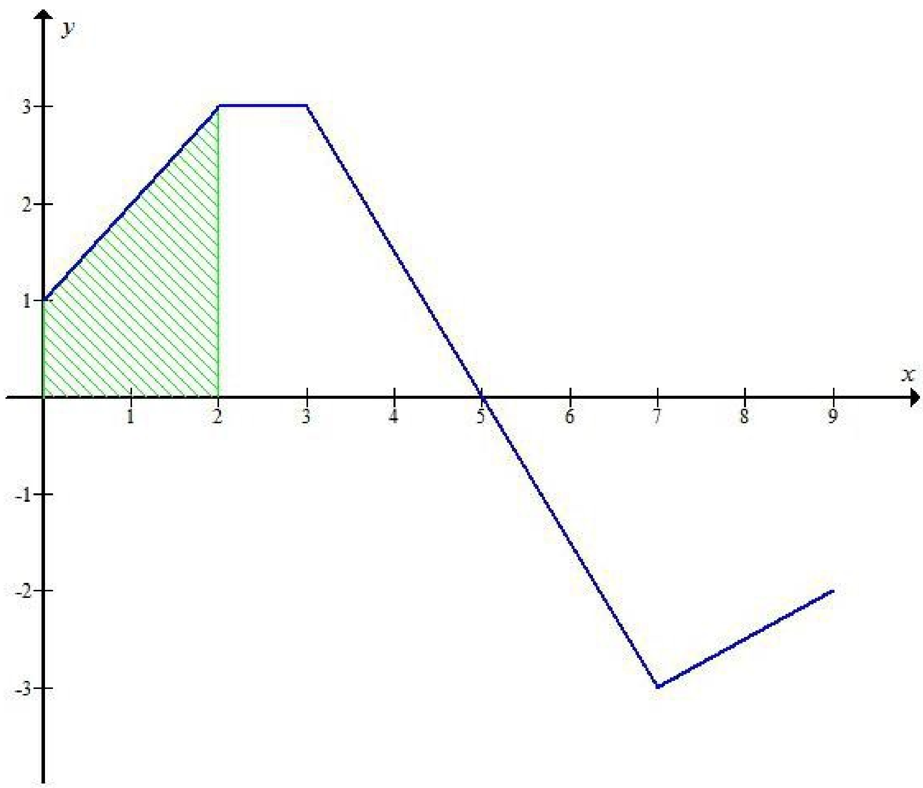
Consider the following integral:

$$\int_0^2 f(x)\,dx$$

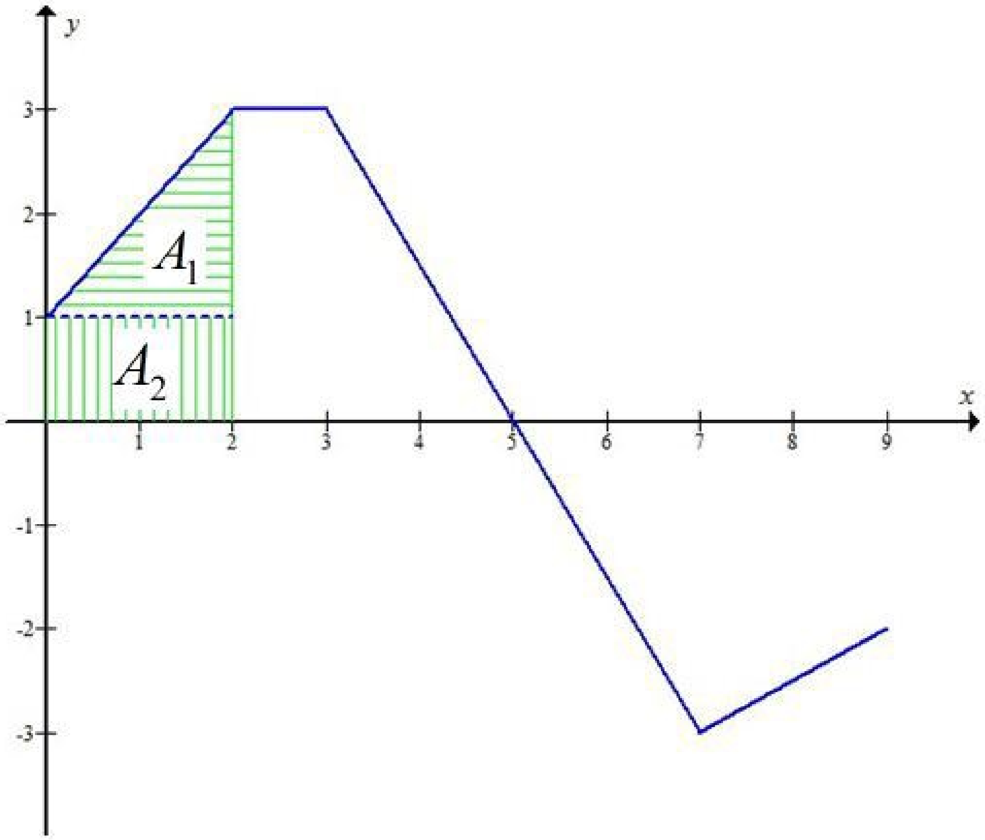
To evaluate the integral, use the above graph.

The definite integral is interpreted as the area of the region bounded by the function and the x axis over the interval $[a,b]$. The definite integral $\int_0^2 f(x)\,dx$ is evaluated over the interval $[0,2]$.

Construct a diagram showing the region for which we want the area:



The shaded region is the shape of a trapezoid. However choose to find its area by recognizing the region as the combination of a triangle and a rectangle:



The triangular region has a base length of $b = 2$ and a height of $h = 2$.

Therefore, the area of the triangular region is,

$$
\begin{aligned}
A_1 &= \frac{1}{2}bh \\
&= \frac{1}{2}(2)(2) \\
&= 2
\end{aligned}
$$

The rectangular region underneath the triangle has length $l = 2$ and height $w = 1$.

Therefore, the area of the rectangular region is,

$$
\begin{aligned}
A_2 &= l \cdot w \\
&= 2(1) \\
&= 2
\end{aligned}
$$

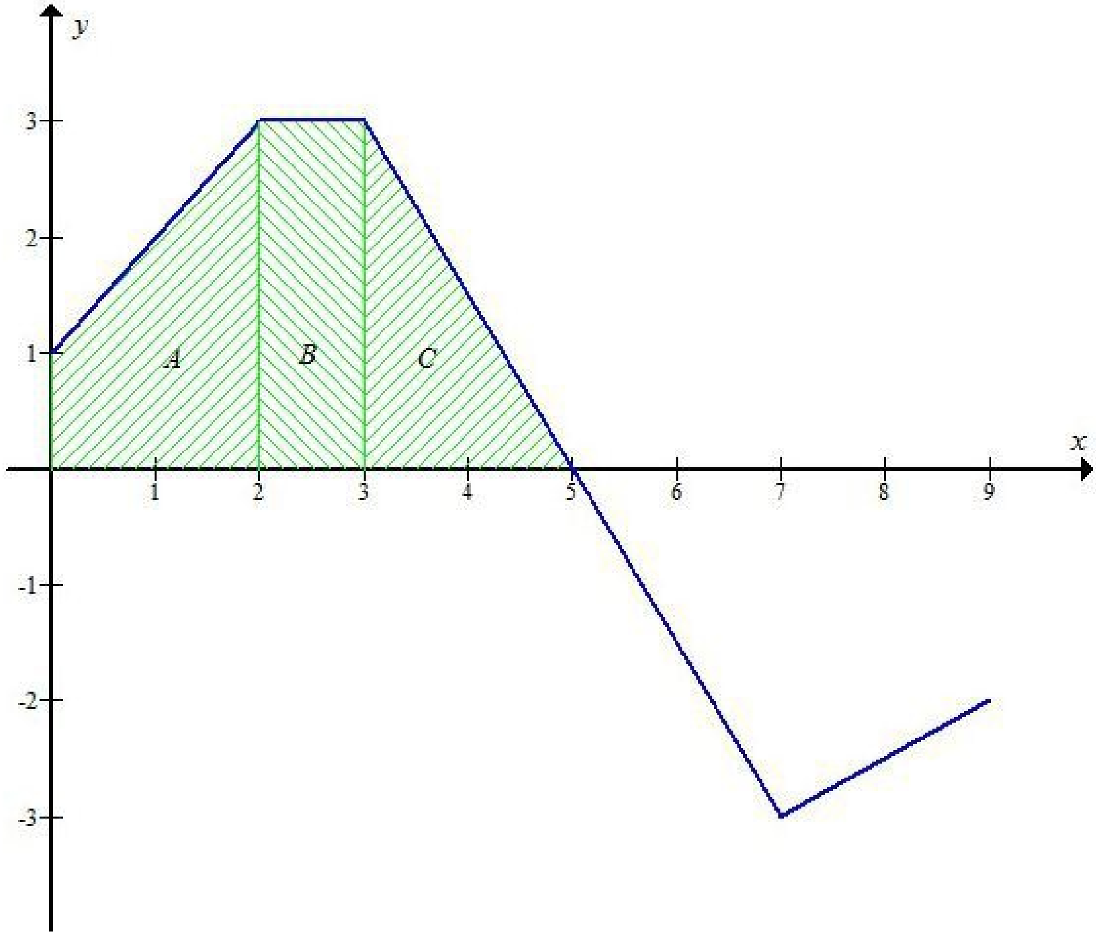
The total area of the shaded region is the sum of the triangle and the rectangle.

Hence, the integral as the sum of areas is,

$$
\begin{aligned}
\int_0^2 f(x)\,dx &= A_1 + A_2 \\
&= 2 + 2 \\
&= \boxed{4}
\end{aligned}
$$

(b)

Construct a diagram shown the full region for which the area:



The total area is the sum of regions $A, B,$ **and** C.

From part (a) the area of region A is 4.

Region B is a rectangle with length $l = 1$ and height $w = 3$.

$$\begin{aligned} B &= l \cdot w \\ &= 1(3) \\ &= 3 \end{aligned}$$

Region C is a triangle with base $b = 2$ and height $h = 3$.

Therefore,

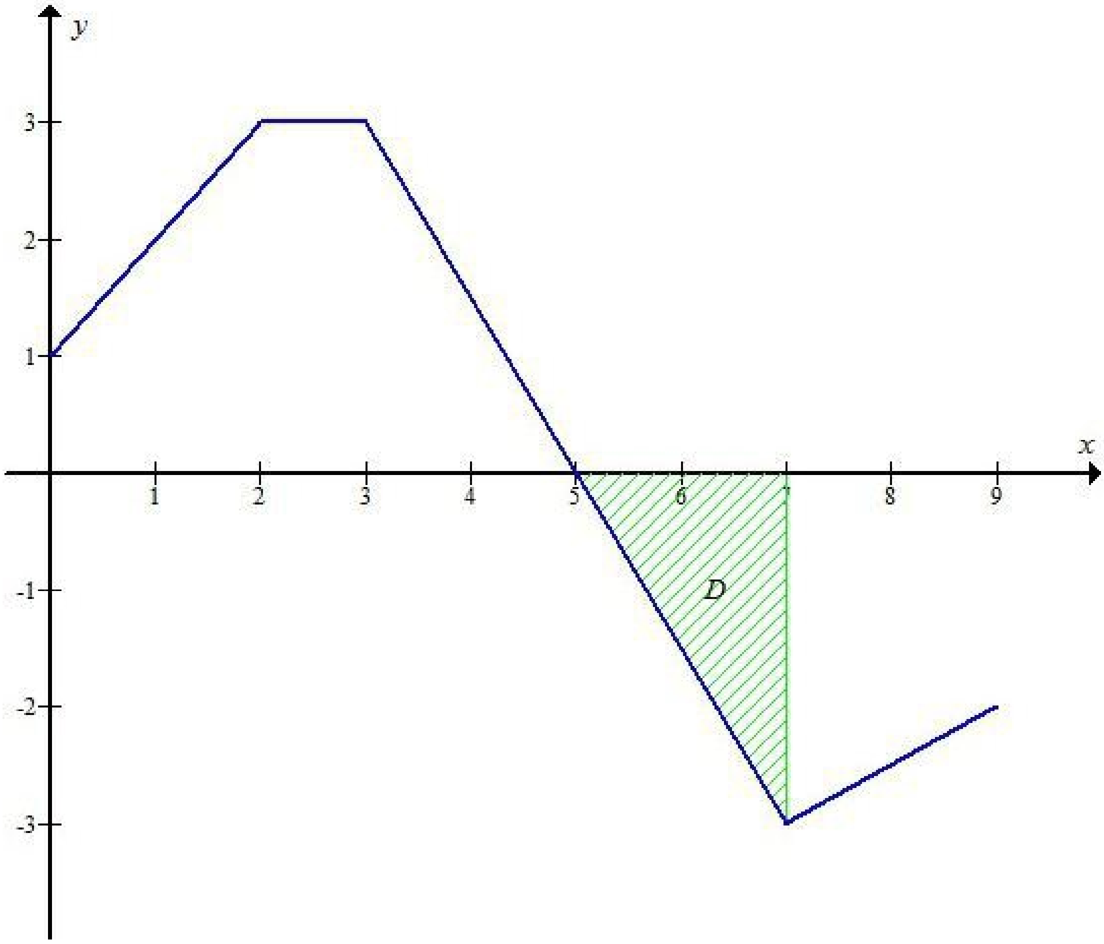
$$\begin{aligned} C &= \frac{1}{2}bh \\ &= \frac{1}{2}(2)(3) \\ &= 3 \end{aligned}$$

Hence, the integral as the sum of areas is,

$$\begin{aligned} \int_0^5 f(x)\ dx &= A + B + C \\ &= 4 + 3 + 3 \\ &= \boxed{10} \end{aligned}$$

(c)

Construct a diagram shown the full region for which the area,



Region D is a triangle with length $b=2$ and height $h=3$.

Therefore,

$$\begin{aligned} D &= \frac{1}{2}bh \\ &= \frac{1}{2}(2)(3) \\ &= 3 \end{aligned}$$

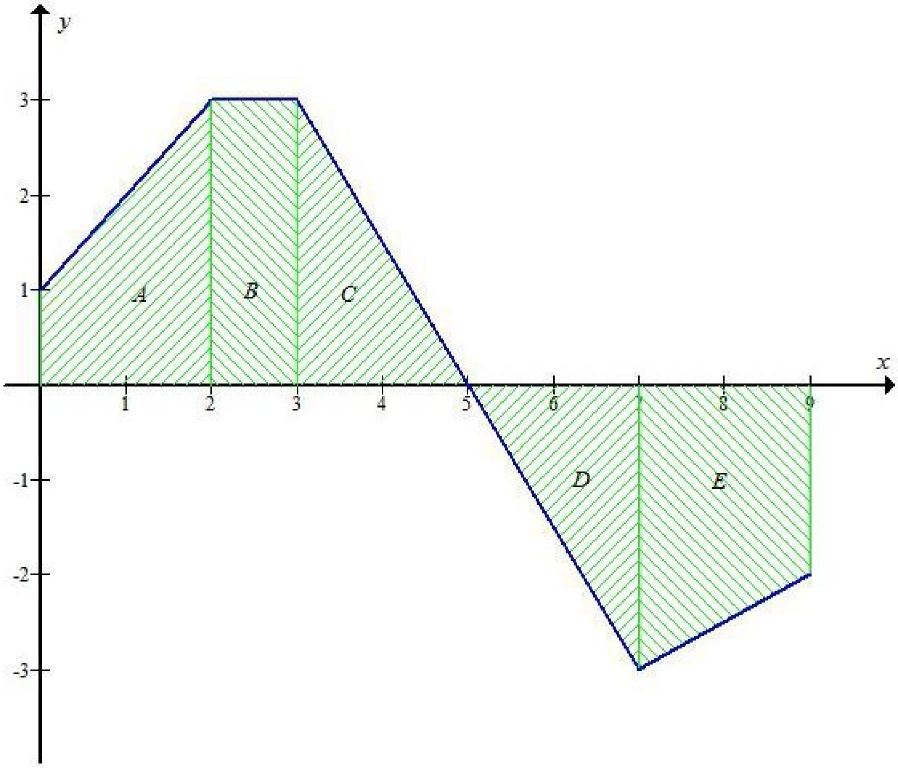
Since region D is entirely below the $x-$axis, we interpret its area as a negative quantity for the integral.

Hence, the integral as areas is,

$$\int_5^7 f(x)\,dx = \boxed{-3}$$

(d)

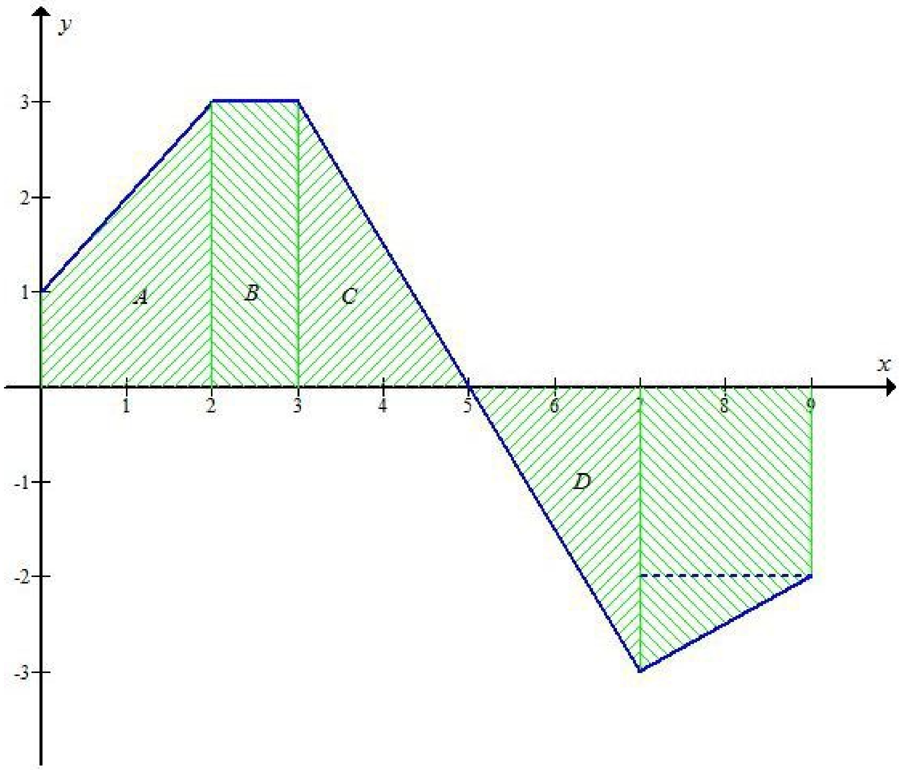
Construct a diagram showing the full region for which the area:



The total area is the sum of regions A, B, C, D and E.

Already found the areas of regions A, B, C and D.

Region E is a trapezoid whose area find as the sum of a triangle and a rectangle.



The triangular region has a base length of $b = 2$ and a height of $h = 1$.

Therefore, the area of the triangular region is,

$$\begin{aligned}\frac{1}{2}bh &= \frac{1}{2}(2)(1) \\ &= 1\end{aligned}$$

The rectangular region above the triangle has length $l = 2$ and height $w = 2$.

Therefore, the area of the rectangular region is,

$$l \cdot w = 2(2)$$
$$= 4$$

Hence, the total area of region E is the sum of the areas of the triangle and the rectangle.

$$E = 4 + 1$$
$$= 5$$

To evaluate the integral $\int_0^9 f(x)\, dx$, consider the regions $A, B,$ and C have positive areas since the regions are entirely above the $x-$axis.

However, since regions D and E are entirely below the $x-$axis, interpret their areas as negative quantities.

Hence, the required area is

$$\int_0^9 f(x)\, dx = A + B + C - D - E$$
$$= 4 + 3 + 3 - 3 - 5$$
$$= \boxed{2}$$

Answer 34E.

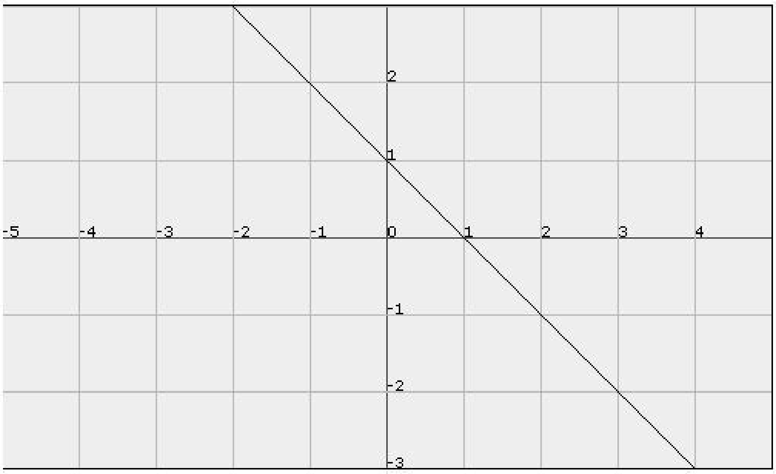
(A) $$\int_0^2 g(x)\,dx = \text{Area of triangle}$$
$$= \frac{1}{2} \times 2 \times 4$$
$$= \boxed{4}$$

(B) $$\int_2^6 g(x)\,dx = \text{Area of semicircle}$$
$$= \frac{\pi r^2}{2}$$
$$= \frac{\pi}{2} \times (-4)$$
$$\boxed{= -2\pi}$$

(C) $$\int_0^7 g(x)dx = \int_0^2 g(x)dx + \int_2^6 g(x)dx + \int_6^7 f(x)dx$$
$$= 4 - 2\pi + \frac{1}{2} \times 1 \times 1$$
$$= 4 - 2\pi + 0.5$$
$$\boxed{= 4.5 - 2\pi}$$

Answer 35E.

Given $\int_{-1}^{2} (1 - x)\,dx$



$$\int_{-1}^{2}(1-x)\,dx = \int_{-1}^{1}(1-x)\,dx + \int_{1}^{2}(1-x)\,dx$$

$\int_{-1}^{1}(1-x)\,dx$ This is a triangle with base 2 and height 2

So the area for this integral is $= \frac{1}{2}\times 2\times 2 = 2$

$\int_{1}^{2}(1-x)\,dx$ This is a triangle with base 1 and height 1

So the area for this integral is $= \frac{1}{2}\times 1\times 1 = \frac{1}{2}$

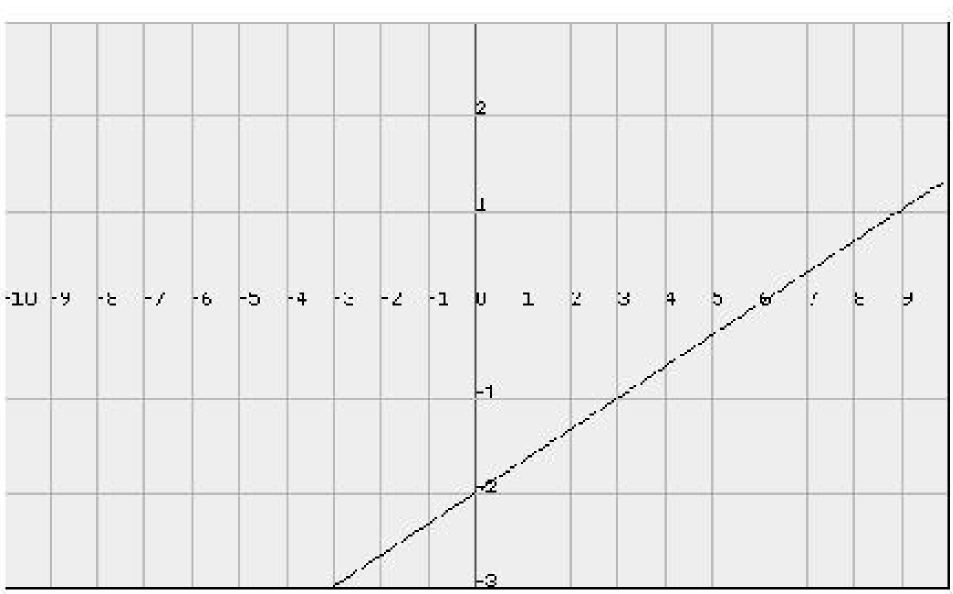
This area is negative because it is below the x-axis

$$\begin{aligned}\int_{-1}^{2}(1-x)\,dx &= \int_{-1}^{1}(1-x)\,dx + \int_{1}^{2}(1-x)\,dx \\ &= 2-\frac{1}{2} \\ &= \frac{3}{2}\end{aligned}$$

$$\boxed{\int_{-1}^{2}(1-x)\,dx = \frac{3}{2}}$$

Answer 36E.

Given $\int_{0}^{9}\left(\frac{1}{3}x-2\right)dx$



$$\int_0^9\left(\frac{1}{3}x-2\right)dx = \int_0^6\left(\frac{1}{3}x-2\right)dx+\int_6^9\left(\frac{1}{3}x-2\right)dx$$

$\int_0^6\left(\frac{1}{3}x-2\right)dx$ This is a triangle with base 6 and height 2

So the area for this integral is $=\frac{1}{2}\times 6\times 2=6$

This area is negative because it is below the x-axis

$$\int_0^6\left(\frac{1}{3}x-2\right)dx=-6$$

$\int_6^9\left(\frac{1}{3}x-2\right)dx$ This is a triangle with base 3 and height 1

So the area for this integral is $=\frac{1}{2}\times 3\times 1=\frac{3}{2}$

$$\int_6^9\left(\frac{1}{3}x-2\right)dx=\frac{3}{2}$$

$$\int_0^9\left(\frac{1}{3}x-2\right)dx = \int_0^6\left(\frac{1}{3}x-2\right)dx+\int_6^9\left(\frac{1}{3}x-2\right)dx$$

$$=-6+\frac{3}{2}$$

$$=\frac{-12+3}{2}$$

$$=\frac{-9}{2}$$

$$=-4.5$$

$$\boxed{\int_0^9\left(\frac{1}{3}x-2\right)dx=-4.5}$$

Answer 37E.

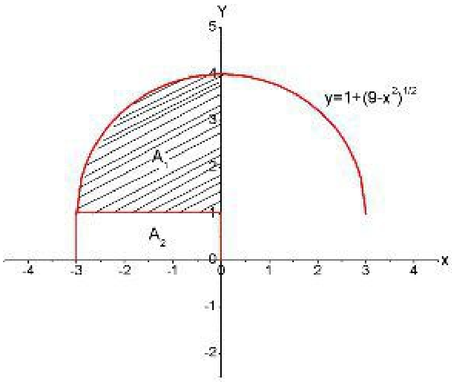


Fig.1

Now $\int_{-3}^{0}\left(1+\sqrt{9-x^2}\right)dx=\int_{-3}^{0}1dx+\int_{-3}^{0}\sqrt{9-x^2}dx$

Now, A_1, represents quarter of a circle with radius 3 and A_2 represents a rectangle with sides 1 and 3

Thus $\int_{-3}^{0}\left(1+\sqrt{9-x^2}\right)dx=A_1+A_2$

$$=\frac{1}{4}\pi r^2+1\times 3$$

$$=\frac{\pi}{4}\left(3^2\right)+3$$

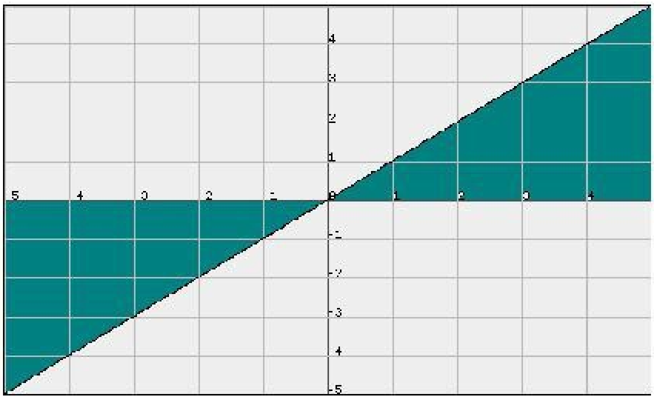
$$=\boxed{3+\frac{9\pi}{4}}$$

Answer 38E.

Given $\int_{-5}^{5}\left(x-\sqrt{25-x^2}\right)dx$

$$\int_{-5}^{5}\left(x-\sqrt{25-x^2}\right)dx=\int_{-5}^{5}xdx-\int_{-5}^{5}\left(\sqrt{25-x^2}\right)dx$$

Now taking $\int_{-5}^{5}xdx=\int_{-5}^{0}xdx+\int_{0}^{5}xdx$



$\int_{-5}^{0}xdx$ This area represents a triangle with base 5 and height 5.

Therefore the area is $=\frac{1}{2}base\times height=\frac{1}{2}\times5\times5$

$$=\frac{25}{2}$$

This area is negative because it is below the x-axis $=\frac{-25}{2}$

$\int_{0}^{5}xdx$ This area represents a triangle with base 5 and height 5.

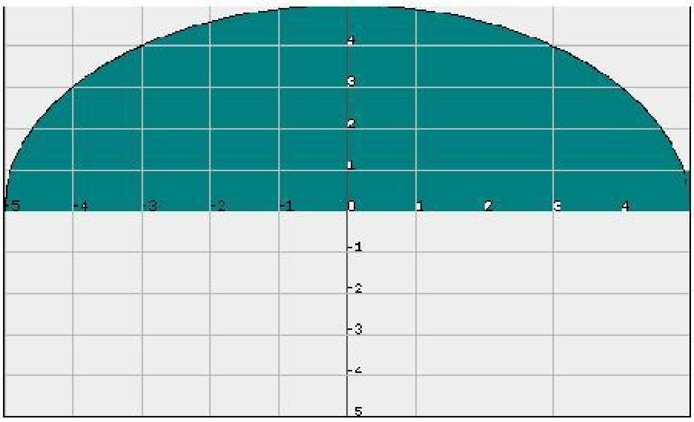
Therefore the area is $=\frac{1}{2}base\times height=\frac{1}{2}\times5\times5$

$$=\frac{25}{2}$$

$$\int_{-5}^{5}xdx=\int_{-5}^{0}xdx+\int_{0}^{5}xdx$$

$$=\frac{-25}{2}+\frac{25}{2}=0$$

Now taking $\int_{-5}^{5}\left(\sqrt{25-x^2}\right)dx$



This area represents area of the half-circle with radius 5.

$$\text{The area} = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(5)^2$$
$$= \frac{25}{2}\pi$$

$$\int_{-5}^{5}\left(x-\sqrt{25-x^2}\right)dx = \int_{-5}^{5} x\,dx - \int_{-5}^{5}\left(\sqrt{25-x^2}\right)dx$$
$$= 0 - \frac{25}{2}\pi$$
$$= \frac{25}{2}\pi$$

The required area is $\boxed{\int_{-5}^{5}\left(x-\sqrt{25-x^2}\right)dx = -\frac{25\pi}{2}}$

Answer 39E.

We sketchy the curve of $f(x) = |x|$ in the interval [-1, 2]

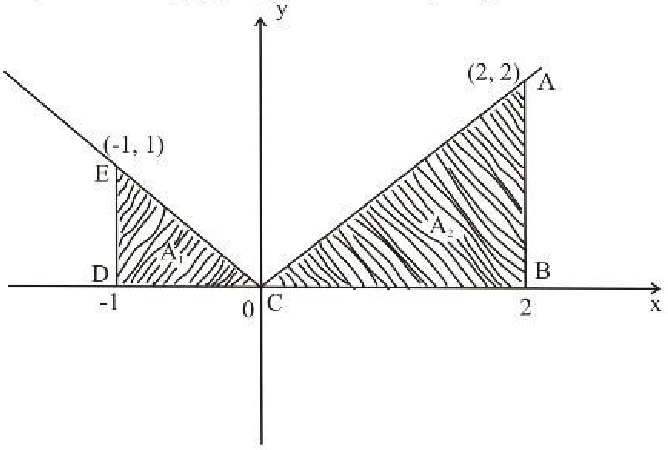


Fig. 1

This is the graph of $f(x) = |x|$ in the interval [-1, 2]

Here we get two triangle ODE and ABC and both the triangles are above the x – axis

So the value of $\int_{-1}^{2}|x|dx$ = Area of Δ ODE + Area of ΔABC

$= A_1 + A_2$ (Let)

Thus $\int_{-1}^{2}|x|dx = \frac{1}{2}\times|OD|\times|DE| + \frac{1}{2}\times|BC|\times|AB|$

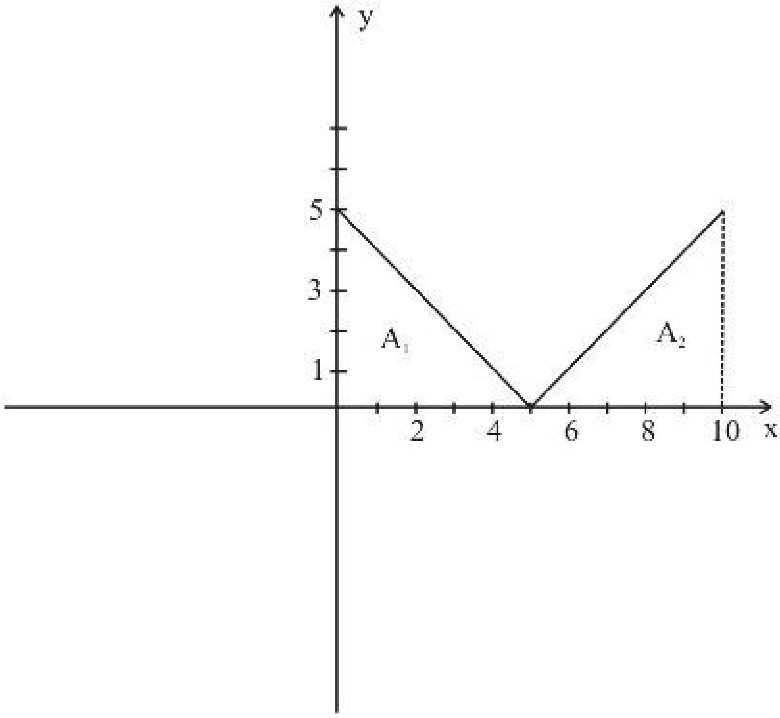
$$= \frac{1}{2}[1\times 1 + 2\times 2]$$
$$= \frac{1}{2}[1+4]$$
$$= \frac{5}{2} = 2.5$$

So $\boxed{\int_{-1}^{2}|x|dx = 2.5}$

Answer 40E.

We have to evaluate $\int_0^{10} |x-5| dx$.

We sketch the graph of $y = |x-5|$ in the interval $[0,10]$ as follows:



Then $\int_0^{10} |x-5| dx = A_1 + A_2$

$= 2A_1$

$= 2\left[\frac{1}{2} \times 5 \times 5\right]$ [From the graph $A_1 = A_2$]

$= 2\frac{25}{2}$

$= \boxed{25}$

Answer 41E.

Evaluate $\int_{\pi}^{\pi} \sin(x)^2 \cos(x)^4 \, dx$

Because the beginning and end of the interval are the same, (π), the answer is simply zero.

then $\int_{\pi}^{\pi} \sin(x)^2 \cos(x)^4 \, dx = 0$

Answer 42E.

We are given that

$$\int_0^1 3x\sqrt{x^2+4} \, dx = 5\sqrt{5} - 8$$

Then, we know that

$$\int_a^b f(x)\,\mathrm{d}x = -\int_b^a f(x)\,\mathrm{d}x$$

So,

$$\int_1^0 3x\sqrt{x^2+4}\,\mathrm{d}x = 8-5\sqrt{5}\ ....(1)$$

Put $x = u \Rightarrow \mathrm{dx} = \mathrm{du}$

When $x = 1 \Rightarrow u = 1$

When $x = 0 \Rightarrow u = 0$

Putting these values in the equation (1), we get

$$\int_1^0 3u\sqrt{u^2+4}\,\mathrm{d}u = 8-5\sqrt{5}$$

Answer 43E.

Given $\int_0^1 x^2 dx = \frac{1}{3}$

Using the properties of integrals

$$\int_0^1 \left(5-6x^2\right)dx = 5\int_0^1 1dx - 6\int_0^1 x^2 dx \qquad \text{By using property (4)}$$

$$= 5(1-0) - 6\times\frac{1}{3} \qquad \text{By the property (1)}$$

$$= 5-2 = \boxed{3}$$

Answer 44E.

We are given that an integral expression

$$\int_2^5 \left(1+3x^4\right)\mathrm{d}x$$

We know that

$$\int_a^b [f(x)+g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)dx$$

Then, we can write of the given integral expression

$$\int_2^5 \left(1+3x^4\right) dx = \int_2^5 dx + 3\int_2^5 x^4\,dx$$

$$= [x]_2^5 + 3\left(618.6\right)$$

$$= (5-2) + 3(618.6)$$

$$= 3 + 1855.8$$

$$= 1858.8$$

$$\Rightarrow \int_2^5 \left(1+3x^4\right) dx = 1858.8$$

[By example $3\int_2^5 x^4 dx = 618.6$]

So, the solution is

$$\int_2^5 \left(1+3x^4\right) dx = 1858.8$$

Answer 45E.

We have to evaluate $\int_1^4 \left(2x^2-3x+1\right)dx$

Now we use following two properties

$$\int_a^b \left[f(x)+g(x)\right] = \int_a^b f(x)dx + \int_a^b f(x)dx$$

And $$\int_a^b \left[f(x)-g(x)\right] = \int_a^b f(x)dx - \int_a^b g(x)dx$$

Then

$$\int_1^4 \left(2x^2-3x+1\right)dx = \int_1^4 2x^2dx - \int_1^4 3xdx + \int_1^4 1dx$$

Now we use the properties

$\int_a^b Cdx = C(b-a)$ Where C is any constant

And $\int_a^b Cf(x)\,dx = C\int_a^b f(x)\,dx$

So $$\int_1^4 \left(2x^2-3x+1\right)dx = 2\int_1^4 x^2dx - 3\int_1^4 xdx + 1\int_1^4 dx$$

$$= 2\int_1^4 x^2dx - 3\int_1^4 xdx + (4-1)$$

$$= 2\int_1^4 x^2dx - 3\int_1^4 xdx + 3$$

Now we use

$$\int_a^b x\,dx = \frac{b^2-a^2}{2} \text{ and } \int_a^b x^2\,dx = \frac{b^3-a^3}{3}$$

So $$\int_1^4 \left(2x^2-3x+1\right)dx = 2.\left[\frac{4^3-1^3}{3}\right] - 3\left[\frac{4^2-1^2}{2}\right] + 3$$

$$= 2.\left[\frac{64-1}{3}\right] - 3\left[\frac{16-1}{2}\right] + 3 = 2.\frac{63}{3} - 3.(7.5) + 3$$

$$= 42 - 22.5 + 3 = 22.5$$

So $$\boxed{\int_1^4 \left(2x^2-3x+1\right)dx = 22.5}$$

Answer 46E.

Evaluate the given integral:

$$\int_0^{\pi/2}(2\cos x-5x)\,dx=2\int_0^{\pi/2}\cos x\,dx-\int_0^{\pi/2}x\,dx \quad \ldots\ldots (1)$$

Now, to evaluate further use the following two results:

$$\int_a^b x\,dx=\frac{b^2-a^2}{2}$$

And

$$\int_0^{\pi/2}\cos x\,dx=1$$

Apply in (1):

$$\int_0^{\pi/2}(2\cos x-5x)\,dx=2.1-5.\frac{\left(\left(\frac{\pi}{2}\right)^2-0^2\right)}{2}$$

$$=2-\frac{5}{2}\left(\frac{\pi^2}{4}\right)$$

$$=\frac{16-5\pi^2}{8}$$

Hence the value of the integral is:

$$\boxed{\int_0^{\pi/2}(2\cos x-5x)\,dx=\frac{1}{8}\left(16-5\pi^2\right)}$$

Answer 47E.

We have by the property of integral

$$\int_a^b f(x)\,dx+\int_b^c f(x)\,dx=\int_a^c f(x)\,dx \qquad \text{--- (1)}$$

So we have

$$\int_{-2}^2 f(x)\,dx+\int_2^5 f(x)\,dx-\int_{-2}^{-1} f(x)\,dx=\int_{-2}^5 f(x)\,dx-\int_{-2}^{-1} f(x)\,dx$$

Now by the other property of integral we have

$$\int_a^b f(x)\,dx=-\int_b^a f(x)\,dx$$

So we have

$$\int_{-2}^2 f(x)\,dx+\int_2^5 f(x)\,dx-\int_{-2}^{-1} f(x)\,dx=\int_{-2}^5 f(x)\,dx+\int_{-1}^{-2} f(x)\,dx$$

Again on right hand side of the equation we can use the property (1) used above

So we have

$$\int_{-2}^2 f(x)\,dx+\int_2^5 f(x)\,dx-\int_{-2}^{-1} f(x)\,dx=\boxed{\int_{-1}^5 f(x)\,dx}$$

Answer 48E.

Given $\int_1^5 f(x)\,dx=12$

And $\int_4^5 f(x)\,dx=3.6$

Then we have to find $\int_1^4 f(x)\,dx$

Since we have by the property of integral,

$$\int_a^c f(x)\,dx+\int_c^b f(x)\,dx=\int_a^b f(x)\,dx$$

So we have

$$\int_1^4 f(x)\,dx+\int_4^5 f(x)\,dx=\int_1^5 f(x)\,dx$$

Or $\int_1^4 f(x)\,dx+3.6=12$

Or $\int_1^4 f(x)\,dx=12-3.6$

Or $\boxed{\int_1^4 f(x)\,dx=8.4}$

Answer 49E.

We have been given that $\int_0^9 f(x)dx = 37$ and $\int_0^9 g(x)dx = 16$

Now $\int_0^9 [2f(x)+3g(x)]dx = 2\int_0^9 f(x)dx + 3\int_0^9 g(x)dx$

[By property (2) and (3)]

$= 2\times 37 + 3\times 16$

$= 74+48 \boxed{=122}$

Answer 50E.

Consider the following function:

The objective is to find the integral $\int_0^5 f(x)dx$ using the formula.

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Find the integral.

$$\int_0^5 f(x)dx = \int_0^3 f(x)dx + \int_3^5 f(x)dx$$

$$= \int_0^3 3dx + \int_3^5 xdx \quad \left(\begin{array}{l}\text{Use the given conditions}\\ f(x)=3,\ x<3\\ f(x)=x, x\geq 3\end{array}\right)$$

$$= 3[x]_0^3 + \left[\frac{x^2}{2}\right]_3^5$$

$$= 3[3-0] + \frac{1}{2}[5^2-3^2]$$

$$= 9 + \frac{1}{2}[25-9]$$

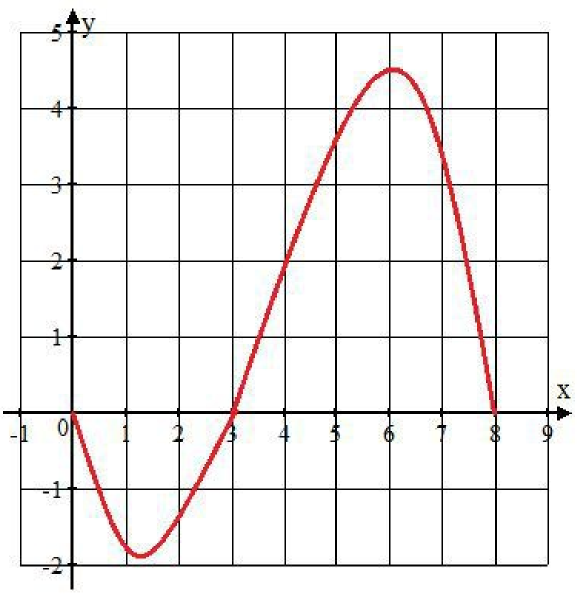
$$= 9+8$$

$$= 17$$

Therefore, the given integral is $\boxed{17}$.

Answer 51E.

Consider the following graph,



The objective is to list the following functions in increasing order.

$$(A) = \int_0^8 f(x)dx, (B) = \int_0^3 f(x)dx, (C) = \int_3^8 f(x)dx, (D) = \int_4^8 f(x)dx, \text{and } (E) = f'(1)$$

(A)

Consider the integral,

$$\int_0^8 f(x)dx$$

The above integral geometrically denotes the area under the curve $f(x)$ in the interval [0,8] with *x*-axis and it can be approximated by adding area of the unit squares in it.

$$\int_0^8 f(x)dx = -0.75 - 1 - 0.75 - 0.75 + 8(1) + 2(0.9) + 3(0.75) + 5(0.25)$$
$$= 10.05$$

(B)

Consider the integral,

$$\int_0^3 f(x)dx$$

The above integral geometrically denotes the area under the curve $f(x)$ in the interval [0,3] with *x*-axis and it can be approximated by adding area of the unit squares in it.

$$\int_0^3 f(x)dx = -0.75 - 1 - 0.75 - 0.75$$
$$= \boxed{-3.25}$$

(C)

Consider the integral,

$$\int_3^8 f(x)dx$$

The above integral geometrically denotes the area under the curve $f(x)$ in the interval [3,8] with *x*-axis and it can be approximated by adding area of the unit squares in it.

$$\int_3^8 f(x)dx = 8(1) + 2(0.9) + 3(0.75) + 4(0.25)$$
$$= 13.05$$

(D)

Consider the integral,

$$\int_4^8 f(x)dx$$

The above integral geometrically denotes the area under the curve $f(x)$ in the interval [4,8] with *x*-axis and it can be approximated by adding area of the unit squares in it.

$$\int_4^8 f(x)dx = 8(1) + 2(0.9) + 2(0.75) + 4(0.25)$$
$$= 12.3$$

(E)

Consider the derivative

$$f'(1)$$

This is a slope of the line passing through the points $(2.5,-1)$ and $(3.5,1)$

$$f'(1) = \frac{1+1}{3.5-2.5}$$
$$= 2$$

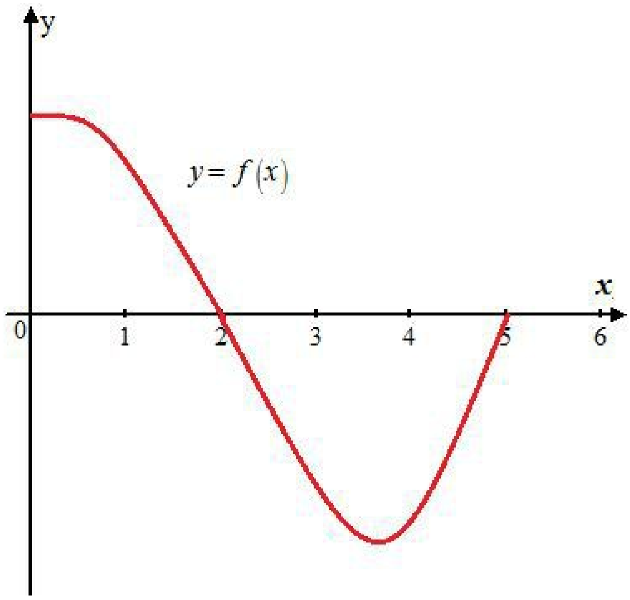
Therefore, the increasing order of the function is $\boxed{(B)<(E)<(A)<(D)<(C)}$.

Answer 52EE.

Consider the integral,

$$F(x)=\int_2^x f(t)dt \quad \text{...... (1)}$$

The objective is to find the largest value among the $F(0), F(1), F(2), F(3), F(4)$ from the following graph,



Find $F(0)$ by substituting $x=0$ in (1),

$$F(0)=\int_2^0 f(t)dt$$
$$=-\int_0^2 f(t)dt$$

The value of $F(0)$ is negative. Because in the graph within the limit value of $\int_0^2$ is positive. So in the integral there is a negative sign, so $F(0)$ will be negative.

Find $F(1)$ by substituting $x=1$ in (1),

$$F(1)=\int_2^1 f(t)dt$$
$$=-\int_1^2 f(t)dt$$

The value of $F(1)$ is negative, since value from $\int_1^2$ is positive so the whole value will be negative.

Find $F(2)$ by substituting $x=2$ in (1),

$$F(2)=\int_2^2 f(t)dt$$

The value of $F(2)$ is positive, since value from $\int_2^2$ is positive so the whole value will be positive.

Find $F(3)$ by substituting $x=3$ in (1),

$$F(3)=\int_2^3 f(t)dt$$

The value of $F(3)$ is negative, since value from $\int_2^3$ is negative so the whole value will be negative.

Find $F(4)$ by substituting $x=4$ in (1),

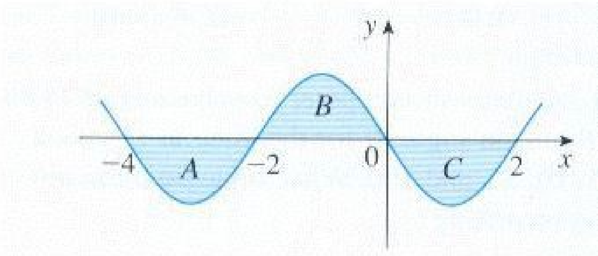
$$F(4)=\int_2^4 f(t)dt$$

The value of $F(4)$ is negative since value from $\int_2^4$ is negative so the whole value will be negative.

So, the largest value is $F(2)$ because it is the only positive value.

Answer 53E.

Given graph is shown below.



From the graph we can observe that
$A=-3, B=3$ and $C=-3$.
A and C are negative as the bounded region is below the x axis.

We need to find the value of $\int_{-4}^{2}\left[f(x)+2x+5\right]dx$

Thus the integral is

$$\int_{-4}^{2}\left[f(x)+2x+5\right]dx=\int_{-4}^{2}\left[f(x)\right]dx+\int_{-4}^{2}\left[2x+5\right]dx$$

$$=\int_{-4}^{-2}\left[f(x)\right]dx+\int_{-2}^{0}\left[f(x)\right]dx+\int_{0}^{2}\left[f(x)\right]dx+\int_{-4}^{2}\left[2x+5\right]dx$$

$$=A+B+C+\left[2\frac{x^2}{2}+5x\right]_{-4}^{2}$$

$$=A+B+C+\left(2^2+5.2\right)-\left((-4)^2+5.(-4)\right)$$

$$=A+B+C+(14)-(16-20)$$
$$=A+B+C+18$$
$$=-3+3-3+18$$
$$=15$$

Therefore $\int_{-4}^{2}\left[f(x)+2x+5\right]dx=\boxed{15}$

Answer 54E.

we have the property 8 which states that if m is the infimum of f and M is the supremum of f on [a, b] , then $m\left(b-a\right) \leq \int_a^b f(x)\, d\,x \leq M(b-a)$.

in the given problem , the absolute minimum is m means infimum of f on [0 ,2] is m and supremum is M ,. then by the above said property m (2-0) $\leq \int_0^2 f(x)\, d\,x \leq$ M(2-0)

or , $2m \leq \int_0^2 f(x)\, d\,x \leq 2M$.

Answer 55E.

Consider the inequality:

$$\int_0^4 \left(x^2-4x+4\right)dx \geq 0$$

To verify the inequality, take the left hand side of the inequality.

$$\int_0^4 \left(x^2-4x+4\right)dx = \int_0^4 (x-2)^2\,dx.$$

Use the comparison property of the integral to verify the given inequality holds true or not.

"If $f(x) \geq 0$ for $a \leq x \leq b$, then $\int_a^b f(x)dx \geq 0$."

Notice that $(x-2)^2 \geq 0$ for all $0 \leq x \leq 4$, so apply the comparison property of the integral.

$$\int_0^4 (x-2)^2\,dx \geq 0.$$

$$\int_0^4 \left(x^2-4x+4\right)dx \geq 0$$

Therefore, the inequality hold **true.**

Answer 56E.

$$\int_0^1 \sqrt{1+x^2}\; d\,x \leq \int_0^1 \sqrt{1+x}\; d\,x$$

Comparison Properties of the Integral says that

If $f(x) \geq g(x)$ for $a \leq x \leq b$,

then $\int_a^b f(x)\, d\,x \geq \int_a^b g(x)\, d\,x$

Since $\sqrt{1+x^2} \leq \sqrt{1+x}$ on [0, 1],

$\int_0^1 \sqrt{1+x^2}\; d\,x \leq \int_0^1 \sqrt{1+x}\; d\,x$ is true.

Answer 57E.

We have to show that $2 \le \int_{-1}^{1}\left(\sqrt{1+x^2}\right)dx \le 2\sqrt{2}$

Since $\quad -1 \le x \le 1$

Then $\quad 0 \le x^2 \le 1$

Adding 1 on each side, we get

$$1 \le 1+x^2 \le 2$$

Taking square roots

$$1 \le \sqrt{1+x^2} \le \sqrt{2} \qquad \text{.....(1)}$$

From (1)

$$1 \le \sqrt{1+x^2} \le \sqrt{2}$$

Then $\quad \int_{-1}^{1} 1dx \le \int_{-1}^{1} \sqrt{1+x^2}dx \le \int_{-1}^{1} \sqrt{2}dx$

Thus $\quad [x]_{-1}^{1} \le \int_{-1}^{1} \sqrt{1+x^2}dx \le \sqrt{2}[x]_{-1}^{1}$

Or $\quad [1-(-1)] \le \int_{-1}^{1} \sqrt{1+x^2}dx \le \sqrt{2}[1-(-1)]$

Or $\quad \boxed{2 \le \int_{-1}^{1} \sqrt{1+x^2}dx \le 2\sqrt{2}}$

Answer 58E.

We need to verify the inequality $\frac{\sqrt{2}\pi}{24} \le \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos x dx \le \frac{\sqrt{3}\pi}{24}$

We have the property

if $m \le f(x) \le M$ for $a \le x \le b$, then $m(b-a) \le \int_{a}^{b} f(x)dx \le M(b-a)$

Let $f(x) = \cos x$

$$f\left(\frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} = M$$

$$f\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} = m$$

And

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos x dx = [\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= \left[\sin\left(\frac{\pi}{4}\right)\right] - \left[\sin\left(\frac{\pi}{6}\right)\right]$$

$$= \left[\frac{1}{\sqrt{2}} - \frac{1}{2}\right]$$

$$= \left[\frac{\sqrt{2}-1}{2}\right]$$

So we have $m \le f(x) \le M$

That is $\frac{1}{\sqrt{2}} \le \left[\frac{\sqrt{2}-1}{2}\right] \le \frac{\sqrt{3}}{2}$

$$m(b-a) \le f(x) \le M(b-a)$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{\pi}{4}-\frac{\pi}{6}\right) \le \left[\frac{\sqrt{2}-1}{2}\right] \le \frac{\sqrt{3}}{2}\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{6\pi-4\pi}{24}\right) \le \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos x dx \le \frac{\sqrt{3}}{2}\left(\frac{6\pi-4\pi}{24}\right)$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left(\frac{2\pi}{24}\right) \le \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos x dx \le \frac{\sqrt{3}}{2}\left(\frac{2\pi}{24}\right)$$

$$\Rightarrow \frac{\sqrt{2}\pi}{24} \le \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos x dx \le \frac{\sqrt{3}\pi}{24}$$

Therefore $\boxed{\frac{\sqrt{2}\pi}{24} \le \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos x dx \le \frac{\sqrt{3}\pi}{24}}$

Answer 59E.

Property 8 states that if $m \le f(x) \le M$ for $a \le x \le b$, then

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

For this problem we must use this property to estimate the value of the integral $\int_1^4 \sqrt{x}\,dx$

So, the first thing we do is calculate the minimum and maximum value of $f(x)$ over the range from 1 to 4.

$$m = f(1) = \sqrt{1} = 1$$

$$M = f(4) = \sqrt{4} = 2$$

So, plugging these values into the formula in property 8 gives:

$$1(4-1) \le \int_1^4 \sqrt{x}\,dx \le 2(4-1)$$

$$3 \le \int_1^4 \sqrt{x}\,dx \le 6$$

So the value of this integral is somewhere between 3 and 6 inclusive.

Answer 60E.

use property 8 to estimate $\int_0^2 \frac{1}{1+x^2}\, dx$.

Since $f(x) = \frac{1}{1+x^2}$ is decreasing on [0,2], at x=0 is an absolute maximum and at x=2 is an absolute min.

$$M = f(0) = 1$$

$$m = f(2) = \frac{1}{5}$$

so by Property 8: If $m \le f(x) \le M$ & $a \le x \le b$

then $m(b-a) \le \int_a^b f(x)\, dx \le M(b-a)$.

Thus we have:

$$\frac{1}{5}(2-0) \le \int_0^2 \frac{1}{1+x^2}\, dx \le 1(2-0)$$

$$\frac{2}{5} \le \int_0^2 \frac{1}{1+x^2}\, dx \le 2$$

therefore, the exact value for $\int_0^2 \frac{1}{1+x^2}\, dx$ lies in the interval $\left[\frac{2}{5}, 2\right]$.

Answer 61E.

Since $f(x) = \tan x$ is an increasing function on the interval $\left[\frac{\pi}{4}, \frac{\pi}{3}\right]$.

So its absolute minimum is $m = f\left(\frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1$

And its absolute maximum is $M = f\left(\frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$

So by the property of the integral, we have

If $m \le f(x) \le M$ for $a \le x \le b$ then

$$m(a-b) \le \int_a^b f(x)\,dx \le M(b-a)$$

So we have

$$1\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \le \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x\, dx \le \sqrt{3}\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

Or $\boxed{\frac{\pi}{12} \le \int_{\pi/4}^{\pi/3} \tan x\, dx \le \sqrt{3}\frac{\pi}{12}}$

So the value of the interval $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x\, dx$ lies in the interval $\left[\frac{\pi}{12}, \frac{\sqrt{3}\pi}{12}\right]$.

Answer 62E.

Since $f(x) = x^3 - 3x + 3$ where $0 \le x \le 2$

First we have to find absolute minimum and maximum value of $f(x)$ in the interval [0, 2]

So we take first derivative of $f(x)$

$$f'(x) = 3x^2 - 3$$

$f'(x) = 0$ When $3x^2 - 3 = 0$ or $3x^2 = 3$

Or $x = 1$ where $0 \le x \le 2$

So there is only one critical number in the closed interval [0, 2] and $f(x)$ is continuous for all value of x . So we use closed interval method for absolute maximum and minimum, we have

$$f(0) = 3, \quad f(1) = 1, \quad f(2) = 5$$

So its absolute minimum is $m = f(1) = 1$

And its absolute maximum is $M = f(2) = 5$

By the property of the integral, we have

If $m \le f(x) \le M$ for $a \le x \le b$ then

$m(a-b) \le \int_a^b f(x)\,dx \le M(b-a)$

So we have

$$1(2-0) \le \int_0^2 (x^3 - 3x + 3)\,dx \le 5(2-0)$$

Or $\boxed{2 \le \int_0^2 (x^3 - 3x + 3)\,dx \le 10}$

Answer 63E.

Since $f(x) = \sqrt{1+x^4}$ where $-1 \le x \le 1$

First we have to find absolute minimum and maximum value of $f(x)$ in the interval [-1, 1]

So we take first derivative of $f(x)$

$$f'(x) = \frac{1}{2}\frac{4.x^3}{\sqrt{1+x^4}} = \frac{2.x^3}{\sqrt{1+x^4}}$$

$f'(x) = 0$ Where x = 0

So there is only one critical point in the closed interval [-1, 1] and $f(x)$ is continuous for all value of x in [-1, 1].So we use closed interval method

We have

$$f(0) = 1, \quad f(-1) = f(1) = \sqrt{2}$$

So its absolute minimum is $m = f(0) = 1$

And its absolute maximum is $M = f(-1) = f(1) = \sqrt{2}$

By the property of the integral, we have

If $m \le f(x) \le M$ for $a \le x \le b$ then $m(a-b) \le \int_a^b f(x)dx \le M(b-a)$

So we have $1 \le f(x) \le \sqrt{2}$ for $-1 \le x \le 1$

$$1(1-(-1)) \le \int_{-1}^1 \sqrt{1+x^4}\,dx \le \sqrt{2}(1-(-1))$$

Or $\boxed{2 \le \int_{-1}^1 \sqrt{1+x^4}\,dx \le 2\sqrt{2}}$

Answer 64E.

Given that $\int_{\pi}^{2\pi}(x-2\sin x)\,dx$

Because $f(x)=x-2\sin x$ is a increasing on $[\pi,2\pi]$, its absolute maximum value is $M=f(2\pi)=2\pi$ and its absolute minimum value is $m=f(\pi)=\pi$

Thus, by the property if $m\le f(x)\le M$ for $a\le x\le b$, then

$m(b-a)\le\int_a^b f(x)\,dx\le M(b-a)$

Applying this to the given problem, we have

$\Rightarrow \pi(2\pi-\pi)\le\int_{\pi}^{2\pi}(x-2\sin x)\,dx\le 2\pi(2\pi-\pi)$

$\Rightarrow \pi^2\le\int_{\pi}^{2\pi}(x-2\sin x)\,dx\le 2\pi^2$

Hence $\pi^2\le\int_{\pi}^{2\pi}(x-2\sin x)\,dx\le 2\pi^2$

Answer 65E.

We know that $\sqrt{x^4+1}\ge\sqrt{x^4}=x^2$ for all x

Then

$$\int_1^3\sqrt{x^4+1}\,dx\ge\int_1^3 x^2\,dx$$

Using the property $\int_a^b x^2\,dx=\dfrac{b^3-a^3}{3}$

We have $\int_1^3\sqrt{x^4+1}\,dx\ge\dfrac{3^3-1^3}{3}$

Or $\boxed{\int_1^3\sqrt{x^4+1}\,dx\ge\dfrac{26}{3}}$

Answer 66E.

We know that for every real value of x,

$$\sin x\le 1$$

Then $x\sin x\le x$

And so $\int_0^{\pi/2}x\sin x\,dx\le\int_0^{\pi/2}x\,dx$

Using the property $\int_a^b x\,dx=\dfrac{b^2-a^2}{2}$

We have $\int_0^{\pi/2}x\sin x\,dx\le\dfrac{(\pi/2)^2-0^2}{2}$

Or $\boxed{\int_0^{\pi/2}x\sin x\,dx\le\dfrac{\pi^2}{8}}$

Answer 67E.

Let $g(x)=cf(x)$

We divide the interval [a, b] into n subintervals

The width of the subintervals is Δx

Right end point of the i^{th} subinterval is x_i

Then by definition of integral

$$\int_a^b g(x)\,dx=\int_a^b cf(x)\,dx=\lim_{n\to\infty}\sum_{i=1}^{n}g(x_i)\Delta x$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n}cf(x_i)\Delta x$$

$$=\lim_{n\to\infty}c\sum_{i=1}^{n}f(x_i)\Delta x$$

$$=c\lim_{n\to\infty}\sum_{i=1}^{n}f(x_i)\Delta x$$

$$=c\int_a^b f(x)\,dx \qquad \text{[By the definition]}$$

Thus $\int_a^b cf(x)\,dx=c\int_a^b f(x)\,dx$

Answer 69E.

if f is discontinuous at coutable number of points and continuous on the remaining subintervals on [a , b] , then f is Riemann integrable.

but f(x) = 0 when x is rational and f(x) = 1 when x is irrational is a function which is continuous nowhere on R.

so, f is not Riemann integrable on any interval of R.

----

consider the partition on [0 ,1] such that || P|| --> 0 , there are only two points in each subinterval one is rational and the other is irrational , by the hypothesis if p is the rational then f (p) = 0 and so, the lower Riemann sum is 0 (1-0) = 0 while q is an irrational ==> f(q) = 1 and the upper Riemann integral is 1(1-0) = 1.

since the lower Riemann integral not equal to upper Riemann integral , f is not Riemann integrable on [0 ,1].

Answer 70E.

Consider the function, $f(x)=\begin{cases} 0 & x=0 \\ \frac{1}{x} & 0<x\leq 1 \end{cases}$.

Write the formula to find integral of the function using the Riemann sum of a function $f(x)$ over an interval $[a,b]$.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{\infty} f(x_i)\Delta x$$

Here, $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x.$.

Using this formula, write the following.

$$\int_0^1 f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{\infty} f(x_i)\Delta x$$

Here, $f(x)=\begin{cases} 0 & x=0 \\ \frac{1}{x} & 0<x\leq 1 \end{cases}$, $\Delta x = \frac{1-0}{n} = \frac{1}{n}, x_i = 0 + i\cdot\frac{1}{n} = \frac{i}{n}$

Now, the integral becomes the following:

$$\begin{aligned}
\int_0^1 f(x)\,dx &= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(\frac{i}{n}\right)\frac{i}{n} \\
&= \lim_{n\to\infty}\left(f\left(\frac{1}{n}\right)\frac{1}{n} + f\left(\frac{2}{n}\right)\frac{2}{n} + f\left(\frac{3}{n}\right)\frac{3}{n} + f\left(\frac{3}{n}\right)\frac{3}{n} + \dots + f\left(\frac{n}{n}\right)\frac{n}{n}\right) \\
&= \lim_{n\to\infty}\left(n\cdot\frac{1}{n} + \frac{n}{2}\cdot\frac{2}{n} + \frac{n}{3}\cdot\frac{3}{n} + \frac{n}{3}\cdot\frac{3}{n} + \dots + \frac{n}{n}\cdot\frac{n}{n}\right) \\
&= \lim_{n\to\infty}(1+1+1+1+\dots+1) \\
&= \lim_{n\to\infty}(n) \\
&= \infty
\end{aligned}$$

This follows that the function, $f(x)=\begin{cases} 0 & x=0 \\ \frac{1}{x} & 0<x\leq 1 \end{cases}$ is not integrable over $[0,1]$.

Answer 71E.

Write $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^4}{n^5}$ in the form $\int_a^b f(x)\,dx$

If f can be integrated on $[a,\ b]$, then $\int_a^b f(x)\,dx=\lim_{n\to\infty}\sum_{i=1}^{n}f(x_i)\Delta x$

Where $\Delta x=\frac{b-a}{n}$ and $x_i=a+i\Delta x$

Consider the sum $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^4}{n^5}$.

It can rewritten as $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^4\left(\frac{1}{n}\right)$

Now compare $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^4\left(\frac{1}{n}\right)$ with $\lim_{n\to\infty}\sum_{i=1}^{n}f(x_i)\Delta x$.

Then $f(x_i)=\left(\frac{i}{n}\right)^4$ and $\Delta x=\frac{1}{n}$

Thus, if we let $f(x)=x^4$, we get

$$\begin{aligned}x_i&=\frac{i}{n}\\a+i\Delta x&=\frac{i}{n}\\a+\frac{i}{n}&=\frac{i}{n}\\a&=0\end{aligned}$$

Since $\Delta x=\frac{1}{n}$, this implies $b-a=1$.

Put $a=0$ in $b-a=1$ to obtain the value of *b*.

$$\begin{aligned}b-0&=1\\b&=1\end{aligned}$$

Therefore $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^4}{n^5}=\boxed{\int_0^1 x^4\,dx}$

Answer 72E.

Given that $\frac{1}{n}\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{1+\left(\frac{i}{n}\right)^2}$

$$\frac{1}{n}\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{1+\left(\frac{i}{n}\right)^2}=\frac{1-0}{n}\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{1+\left(\frac{i}{n}\right)^2}$$

$$=\frac{1-0}{n}\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^4}{n^4}$$

Here considering $\Delta x=\frac{b-a}{n}=\frac{1-0}{n}$ and $f(x)=\frac{1}{1+x^2}$

Then $x_i=\frac{i}{n}$

$$\lim_{n\to\infty}\sum_{i=1}^{n}f(x_i)\Delta x=\lim_{n\to\infty}\sum_{i=1}^{n}f\left(\frac{i}{n}\right)\left(\frac{1-0}{n}\right)$$

Hence the definite integral is $\int_0^1\frac{1}{1+x^2}dx$

Therefore $\boxed{\frac{1}{n}\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{1+\left(\frac{i}{n}\right)^2}=\int_0^1\frac{1}{1+x^2}dx}$

Answer 73E.

we know that as x increases , x -2 decreases.

so, f (x) = x -2 is a decreasing function on [1,2].

so, consider the arbitrary partition on [1,2] then the kth subinterval is $\left[1+\frac{k-1}{n}, 1+\frac{k}{n}\right] \forall 1 \leq k \leq n$.

using the antiderivative F of f on [1, 2] wher F(x) = $\frac{-1}{x}$ and by fundamental theorem of calculus we get $\int_1^2 f(x)\,\mathrm{d}\,x = F(2) - F(1) = \frac{-1}{2} + \frac{1}{1} = \frac{1}{2}$