Exercise 4.4

Answer 1E.

Given $\int \frac{1}{x^2\sqrt{1+x^2}}dx = -\frac{\sqrt{1+x^2}}{x}+C$

We need to verify that the formula is correct.

$$\frac{d}{dx}\left(-\frac{\sqrt{1+x^2}}{x}+C\right)$$

$$= -\left(\frac{x.\frac{d}{dx}\sqrt{1+x^2}-\sqrt{1+x^2}\frac{d}{dx}x}{x^2}-\frac{d}{dx}C\right)\left(\because \frac{d}{dx}\frac{U}{V}=\frac{V\frac{dU}{dx}-U\frac{dV}{dx}}{V^2}\right)$$

$$= -\left(\frac{x.\frac{1}{2\sqrt{1+x^2}}\frac{d}{dx}x^2-\sqrt{1+x^2}.1}{x^2}-0\right)$$

$$= -\left(\frac{x.\frac{1}{2\sqrt{1+x^2}}2x-\sqrt{1+x^2}}{x^2}\right)$$

$$= -\left(\frac{\frac{x^2-(1+x^2)}{\sqrt{1+x^2}}}{x^2}\right)$$

$$= -\left(\frac{\frac{-1}{\sqrt{1+x^2}}}{x^2} \right)$$

$$= \left(\frac{1}{x^2\sqrt{1+x^2}} \right)$$

$$\therefore \int \frac{1}{x^2\sqrt{1+x^2}} dx = -\frac{\sqrt{1+x^2}}{x} + C$$

Answer 2E.

Given $\int \cos^2 dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$

We need to verify that the formula is correct.

Now $\frac{d}{dx}\left(\frac{1}{2}x + \frac{1}{4}\sin 2x + C\right)$

$$= \left(\frac{1}{2}\frac{d}{dx}x + \frac{1}{4}\frac{d}{dx}\sin 2x + \frac{d}{dx}C\right)$$

$$= \left(\frac{1}{2}.1 + \frac{1}{4}.\cos 2x \frac{d}{dx} 2x + 0\right)$$

$$= \left(\frac{1}{2} + \frac{1}{4}.\cos 2x.2\right)$$

$$= \left(\frac{1}{2} + \frac{1}{2}.\cos 2x\right)$$

$$= \frac{1}{2}(1 + \cos 2x)$$

$$= \frac{1}{2}(2\cos^2 x)$$

$$= \cos^2 x$$

$$\therefore \int \cos^2 dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

Answer 3E.

Let us start the question from differentiation of right hand side.

$$\frac{d}{dx}\left(\text{sinx} - \frac{1}{3}\sin^3 x + c\right) = \text{cosx} - \frac{1}{3}\cdot 3\sin^2 \text{xcosx} + 0$$

$$= \text{cosx} - \left(1 - \cos^2 x\right).\text{cosx}$$

$$= \text{cosx} - \text{cosx} + \cos^3 x$$

$$= \cos^3 x$$

Integrating both sides with respect to x,

$$\text{sinx} - \frac{1}{3}\sin^3 x + c = \int \cos^3 x \, d\, x$$

Hence verified.

Answer 4E.

Let us start the question by differentiation on right hand side,

$$\frac{d}{dx}\left[\frac{2}{3b^2}\left(bx-2a\right)\sqrt{a+bx}+C\right]=\frac{2}{3b^2}\left[\left(bx-2a\right)\frac{1}{2\sqrt{a+bx}}\cdot b+b\cdot\sqrt{a+bx}\right]$$

$$=\frac{2}{3b^2}\left[\frac{b(bx-2a)+2b(a+bx)}{2\sqrt{a+bx}}\right]$$

$$=\frac{b^2x-2ab+2ab+2b^2x}{3b^2\sqrt{a+bx}}$$

$$=\frac{3b^2x}{3b^2\sqrt{a+bx}}$$

$$=\frac{x}{\sqrt{a+bx}}$$

Integrating both sides

$$\frac{2}{3b^2}\left(bx-2a\right)\sqrt{a+bx}+c=\int\frac{x}{\sqrt{a+bx}}\,dx$$

Hence verified

Answer 5E.

Consider the indefinite integral,

$$\int\left(x^2+x^{-2}\right)dx$$

The objective is to find the general indefinite integral.

Rewrite the indefinite integral as,

$$\int\left(x^2+\frac{1}{x^2}\right)dx$$

Therefore, the general solution is,

$$\int\left(x^2+\frac{1}{x^2}\right)dx=\int\left(x^2\right)dx+\int\frac{1}{x^2}dx$$

$$=\boxed{\frac{x^3}{3}-\frac{1}{x}+C}$$

Answer 6E.

$$\int\sqrt{x^3}+\sqrt[3]{x^2}\,dx=\int x^{\frac{3}{2}}+x^{\frac{2}{3}}\,dx$$

$$=\frac{x^{\frac{5}{2}}}{\frac{5}{2}}+\frac{x^{\frac{5}{3}}}{\frac{5}{3}}+c$$

$$=\frac{2}{5}x^{\frac{5}{2}}+\frac{3}{5}x^{\frac{5}{3}}+c$$

Answer 7E.

Consider the integral, $\int\left(x^4-\frac{1}{2}x^3+\frac{1}{4}x-2\right)dx$

Find the general indefinite integral.

Recall the formula for the indefinite integral.

$$\int x^n dx=\frac{x^{n+1}}{n+1}+C \quad ...... (1)$$

$$\int[f(x)+g(x)]dx=\int f(x)dx+\int g(x)dx \quad ...... (2)$$

$$\int cf(x)dx=c\int f(x)dx \quad ...... (3)$$

Thus, calculate the value of the integral.

$$\int\left(x^4-\frac{1}{2}x^3+\frac{1}{4}x-2\right)dx=\int x^4dx-\int\frac{1}{2}x^3dx+\int\frac{1}{4}xdx-\int 2dx \quad \text{By using formula (2)}$$

$$=\int x^4dx-\frac{1}{2}\int x^3dx+\frac{1}{4}\int xdx-2\int dx \quad \text{By using formula (3)}$$

$$=\frac{x^5}{5}-\frac{1}{2}\cdot\frac{x^4}{4}+\frac{1}{4}\cdot\frac{x^2}{2}-2x+C \quad \text{By using formula (1)}$$

$$=\frac{x^5}{5}-\frac{x^4}{8}+\frac{x^2}{8}-2x+C$$

Therefore, $\int\left(x^4-\frac{1}{2}x^3+\frac{1}{4}x-2\right)dx=\boxed{\frac{x^5}{5}-\frac{x^4}{8}+\frac{x^2}{8}-2x+C}$

Answer 8E.

$$\int\left(y^3+1.8y^2-2.4y\right)\,dy=\frac{y^4}{4}+1.8\frac{y^3}{3}-2.4\frac{y^2}{2}+c$$

$$=\frac{y^4}{4}+0.6y^3-1.2y^2+c$$

Here we used the formula $\int x^n\,dx=\frac{x^{n+1}}{n+1}+C$

Answer 9E.

Consider the following integral:

$$\int(u+4)(2u+1)du$$

Evaluate the indefinite integral.

$$\int(u+4)(2u+1)du=\int\left(2u^2+9u+4\right)du$$

$=\int 2u^2du+\int 9u\,du+\int 4\,du$ Separate all integrals

$=2\int u^2du+9\int u\,du+4\int du$ Using $\int c\,f(x)dx=c\int f(x)dx$

$=2\frac{u^{2+1}}{2+1}+9\frac{u^{1+1}}{1+1}+4u+C$ Using $\left(\int x^n dx=\frac{x^{n+1}}{n+1}+C, n\neq-1\right)$

$=2\frac{u^3}{3}+9\frac{u^2}{2}+4u+C$ Simplify

Therefore, $\boxed{\int(u+4)(2u+1)du=2\frac{u^3}{3}+9\frac{u^2}{2}+4u+C}$

Check: To check this solution, differentiate it.

$$\frac{d}{du}\left(2\frac{u^3}{3}+9\frac{u^2}{2}+4u\right)=\frac{2}{3}\frac{d}{du}(u^3)+\frac{9}{2}\frac{d}{du}(u^2)+4\frac{d}{du}(u)$$ Apply differentiation.

$$=\frac{2}{3}\cdot 3u^2+\frac{9}{2}\cdot 2u+4(1)$$

$$=2u^2+9u+4$$
$$=(u+4)(2u+1)$$ Simplify and Factor

Hence, the check is verified.

Answer 10E.

$$\int v\left(v^2+2\right)^2 \mathrm{d}\,v=\int v\left(v^4+4v^2+4\right)\mathrm{d}\,v$$

$$=\int v^5+4v^3+4v\,\mathrm{d}\,v$$

$$=\frac{v^6}{6}+4\frac{v^4}{4}+4\frac{v^2}{2}+c$$

$$=\frac{v^6}{6}+v^4+2v^2+c$$

Here we used the formula $\int x^n\,\mathrm{d}\,x=\frac{x^{n+1}}{n+1}+C$

Answer 12E.

$$\int\left(u^2+1+\frac{1}{u^2}\right)du=\int u^2du+\int 1du+\int u^{-2}du$$

$$=\frac{u^3}{3}+u+\frac{u^{-1}}{-1}+C$$

$$=\frac{u^3}{3}+u-\frac{1}{u}+C$$

Answer 13E.

$$\int\left(\theta-\csc\theta\cot\theta\right)\mathrm{d}\,\theta=\frac{\theta^2}{2}+\csc\theta+c$$

Here we used the formula $\int x^n\,\mathrm{d}\,x=\frac{x^{n+1}}{n+1}+C$

and the formula $\int \csc x \cot x\,\mathrm{d}\,x=-\csc x+C$

Answer 14E.

$$\int \text{sect}(\text{sect} + \text{tant})\, d\, t = \int \left[\sec^2 t + \sec t \tan t\right] d\, t$$

$$= \text{tant} + \text{sect} + c$$

Here we used the formula $\int \sec^2 x\, d\, x = \text{tanx} + C$

And $\int \text{secxtanx}\, d\, x = \text{secx} + C$

Answer 15E.

$$\int \left(1 + \tan^2 \alpha\right) d\, \alpha$$

Rewrite using the fundamental identity: $1 + \tan^2 x = \sec^2 x$

$$\int \sec^2 \alpha\, d\, \alpha = \tan\alpha + c$$

Answer 16E.

$$\int \frac{\sin 2x}{\sin x} dx = \int \frac{2 \sin x \cos x}{\sin x} dx$$

$$= 2\int \cos x\, dx$$

$$= 2 \sin x + C$$

$$\boxed{\int \frac{\sin 2x}{\sin x} dx = 2 \sin x + C}$$

Answer 17E.

We are given that

$$\int \left(\text{cosx} + \frac{x}{2}\right) d\, x$$

We know that

$$\int [f(x) + g(x)]\, dx = \int f(x)\, dx + \int g(x)\, dx$$

Then, we can write

$$\int \left(\cos x + \frac{x}{2} \right) dx = \int \cos x\, dx + \int \frac{x}{2}\, dx$$

$$= \sin x + \frac{1}{2} \times \frac{x^2}{2} + C$$

$$= \sin x + \frac{x^2}{4} + C$$

$$\int \left(\cos x + \frac{x}{2} \right) dx = \sin x + \frac{x^2}{4} + C$$

Here C is a constant.

Take $C = 0$, we get

$$\int \left(\cos x + \frac{x}{2} \right) dx = \sin x + \frac{x^2}{4}$$

Take $C = 1$, we get

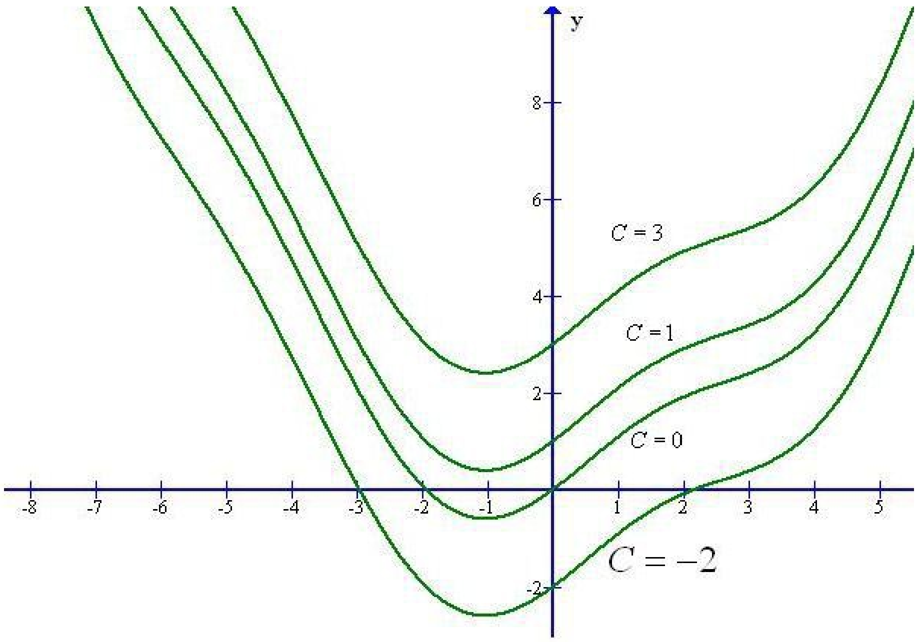
$$\int \left(\cos x + \frac{x}{2} \right) dx = \sin x + \frac{x^2}{4} + 1$$

Take $C = 3$, we get

$$\int \left(\cos x + \frac{x}{2} \right) dx = \sin x + \frac{x^2}{4} + 3$$

Take $C = -2$, we get

$$\int \left(\cos x + \frac{x}{2} \right) dx = \sin x + \frac{x^2}{4} - 2$$



Answer 18E.

We are given that

$$\int\left(1-x^2\right)^2 \, dx$$

Here, $\left(1-x^2\right)^2 = 1+x^4-2x^2$

Then,

$$\int\left(1-x^2\right)^2 \, dx = \int\left(1-2x^2+x^4\right) dx$$

$$= \int 1\cdot dx - 2\int x^2 \, dx + \int x^4 \, dx$$

$$= x - 2\cdot\frac{x^3}{3} + \frac{x^5}{5} + C$$

$$\int\left(1-x^2\right)^2 \, dx = x - \frac{2x^3}{3} + \frac{x^5}{5} + C$$

Here C is a constant.

Take $C=0$, we get

$$\int\left(1-x^2\right)^2 \, dx = x - \frac{2x^3}{3} + \frac{x^5}{5}$$

Take $C=1$, we get

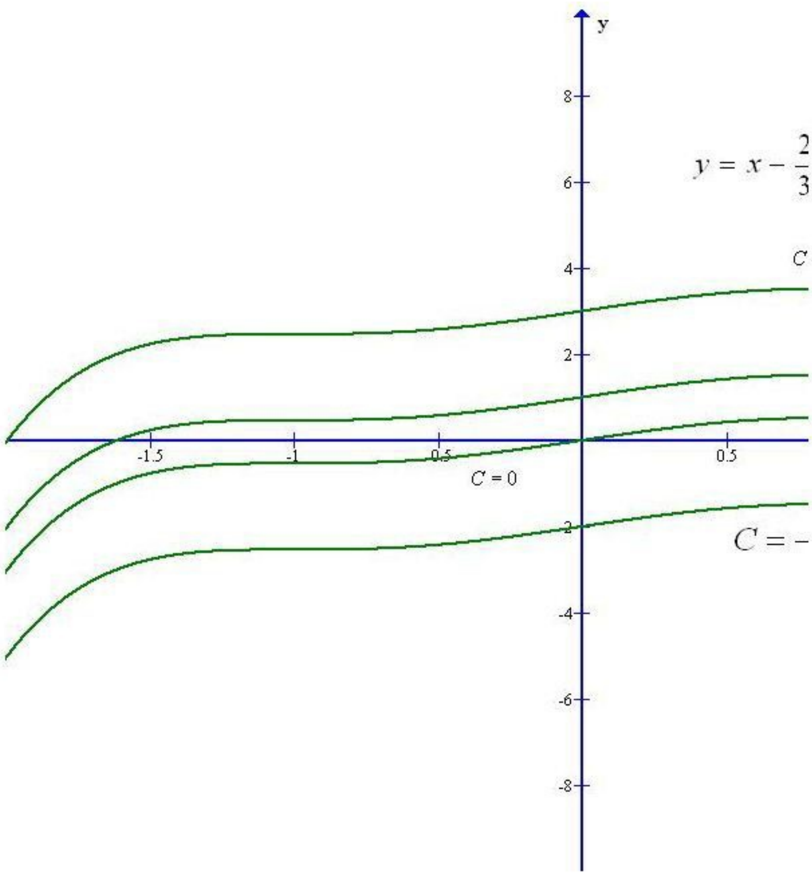
$$\int \left(1-x^2\right)^2 \, dx = x - \frac{2x^3}{3} + \frac{x^5}{5} + 1$$

Take $C = 3$, we get

$$\int \left(1-x^2\right)^2 \, dx = x - \frac{2x^3}{3} + \frac{x^5}{5} + 3$$

Take $C = -2$, we get

$$\int \left(1-x^2\right)^2 \, dx = x - \frac{2x^3}{3} + \frac{x^5}{5} - 2$$



Answer 19E.

Given $\int_{-2}^{3}(x^2-3)dx$

$$\int_{-2}^{3}(x^2-3)dx = \int_{-2}^{3}(x^2-3)dx$$

$$= \left(\frac{x^{2+1}}{2+1}-3x\right)_{-2}^{3} \quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1}+C, n \neq -1\right)$$

$$= \left(\frac{x^3}{3}-3x\right)_{-2}^{3}$$

$$= \left(\frac{3^3}{3}-3.3\right)-\left(\frac{(-2)^3}{3}-3.(-2)\right)$$

$$= (9-9)-\left(-\frac{8}{3}+6\right)$$

$$= (0)-\left(\frac{-8+18}{3}\right)$$

$$= -\frac{10}{3}$$

$$\boxed{\therefore \int_{-2}^{3}(x^2-3)dx = -\frac{10}{3}}$$

Answer 20E.

Given $\int_{1}^{2}(4x^3-3x^2+2x)dx$

$$\int_{1}^{2}(4x^3-3x^2+2x)dx = \int_{1}^{2}(4x^3-3x^2+2x)dx$$

$$= \left(4\frac{x^{3+1}}{3+1}-3\frac{x^{2+1}}{2+1}+2\frac{x^{1+1}}{1+1}\right)_{1}^{2} \quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1}+C, n \neq -1\right)$$

$$= \left(4\frac{x^4}{4}-3\frac{x^3}{3}+2\frac{x^2}{2}\right)_{1}^{2}$$

$$= (x^4-x^3+x^2)_{1}^{2}$$

$$= (2^4-2^3+2^2)-(1^4-1^3+1^2)$$

$$= (16-8+4)-(1-1+1)$$

$$= (12)-(1)$$

$$= 11$$

$$\boxed{\therefore \int_{1}^{2}(4x^3-3x^2+2x)dx = 11}$$

Answer 21E.

Given $\int_{-2}^{0}\left(\frac{1}{2}t^4+\frac{1}{4}t^3-t\right)dt$

$$\int_{-2}^{0}\left(\frac{1}{2}t^4+\frac{1}{4}t^3-t\right)dt = \int_{-2}^{0}\left(\frac{1}{2}t^4+\frac{1}{4}t^3-t\right)dt$$

$$= \left(\frac{1}{2}\frac{t^{4+1}}{4+1}+\frac{1}{4}\frac{t^{3+1}}{3+1}-\frac{t^{1+1}}{1+1}\right)_{-2}^{0} \quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1}+C, n \neq -1\right)$$

$$= \left(\frac{1}{2}.\frac{t^5}{5}+\frac{1}{4}.\frac{t^4}{4}-\frac{t^2}{2}\right)_{-2}^{0}$$

$$= \left(\frac{t^5}{10}+\frac{t^4}{16}-\frac{t^2}{2}\right)_{-2}^{0}$$

$$=\left(\frac{0^5}{10}+\frac{0^4}{16}-\frac{0^2}{2}\right)-\left(\frac{(-2)^5}{10}+\frac{(-2)^4}{16}-\frac{(-2)^2}{2}\right)$$
$$=(0)-\left(\frac{-32}{10}+\frac{16}{16}-\frac{4}{2}\right)$$
$$=-(-3.2+1-2)$$
$$=4.2=\frac{42}{10}=\frac{21}{5}$$

$$\therefore \int_{-2}^{0}\left(\frac{1}{2}t^4+\frac{1}{4}t^3-t\right)dt=\frac{21}{5}$$

Answer 22E.

Given $\int_0^3\left(1+6\varpi^2-10\varpi^4\right)d\varpi$

$$\int_0^3\left(1+6\varpi^2-10\varpi^4\right)d\varpi=\int_0^3\left(1+6\varpi^2-10\varpi^4\right)d\varpi$$
$$=\left(\varpi+\frac{6\varpi^{2+1}}{2+1}-\frac{10\varpi^{4+1}}{4+1}\right)_0^3 \quad \left(\because \int x^n dx=\frac{x^{n+1}}{n+1}+C, n\neq -1\right)$$
$$=\left(\varpi+\frac{6\varpi^3}{3}-\frac{10\varpi^5}{5}\right)_0^3$$

$$=\left(\varpi+2\varpi^3-2\varpi^5\right)_0^3$$
$$=\left(3+2.3^3-2.3^5\right)-\left(0+2.0^3-2.0^5\right)$$
$$=(3+54-486)-(0)$$
$$=-429$$

$$\therefore \int_0^3\left(1+6\varpi^2-10\varpi^4\right)d\varpi=-429$$

Answer 23E.

Given $\int_0^2(2x-3)\left(4x^2+1\right)dx$

$$\int_0^2(2x-3)\left(4x^2+1\right)dx=\int_0^2\left(8x^3-12x^2+2x-3\right)dx$$
$$=\left(8.\frac{x^{3+1}}{3+1}-12\frac{x^{2+1}}{2+1}+2\frac{x^{1+1}}{1+1}-3x\right)_0^2 \quad \left(\because \int x^n dx=\frac{x^{n+1}}{n+1}+C, n\neq -1\right)$$
$$=\left(8.\frac{x^4}{4}-12\frac{x^3}{3}+2\frac{x^2}{2}-3x\right)_0^2$$

$$=\left(2x^4-4x^3+x^2-3x\right)_0^2$$
$$=\left(2.2^4-4.2^3+2^2-3.2\right)-\left(2.0^4-4.0^3+0^2-3.0\right)$$
$$=(32-32+4-6)-(0)$$
$$=-2$$

$$\therefore \int_0^2(2x-3)\left(4x^2+1\right)dx=-2$$

Answer 24E.

Given $\int_{-1}^{1} t(1-t)^2 dt$

$$\int_{-1}^{1} t(1-t)^2 dt = \int_{-1}^{1} t\left(1+t^2-2t\right)dt$$
$$= \int_{-1}^{1}\left(t^3-2t^2+t\right)dt$$
$$= \left(\frac{t^{3+1}}{3+1}-2\frac{t^{2+1}}{2+1}+\frac{t^{1+1}}{1+1}\right)_{-1}^{1} \quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1}+C, n \neq -1\right)$$

$$= \left(\frac{t^4}{4}-2\frac{t^3}{3}+\frac{t^2}{2}\right)_{-1}^{1}$$
$$= \left(\frac{1^4}{4}-2\frac{1^3}{3}+\frac{1^2}{2}\right)-\left(\frac{(-1)^4}{4}-2\frac{(-1)^3}{3}+\frac{(-1)^2}{2}\right)$$
$$= \left(\frac{1}{4}-\frac{2}{3}+\frac{1}{2}\right)-\left(\frac{1}{4}+\frac{2}{3}+\frac{1}{2}\right)$$
$$= \left(\frac{3-8+6}{12}\right)-\left(\frac{3+8+6}{12}\right)$$
$$= \left(\frac{1}{12}\right)-\left(\frac{17}{12}\right)$$
$$= \frac{-16}{12}$$
$$= \frac{-4}{3}$$

$$\boxed{\therefore \int_{-1}^{1} t(1-t)^2 dt = \frac{-4}{3}}$$

Answer 25E.

$$\int_{0}^{\pi}(4\sin\theta-3\cos\theta)d\theta$$
$$= -4\cos\theta-3\sin\theta\Big|_0^{\pi}$$
$$= -4\cos\pi-3\sin\pi+4\cos 0+3\sin 0$$
$$= -4(-1)-3(0)+4(1)+3(0)$$
$$\boxed{=8}$$

Answer 26E.

$$= \left(-\frac{2^{-1}}{1}+2\frac{2^{-2}}{1}\right)-\left(-\frac{1^{-1}}{1}+2\frac{1^{-2}}{1}\right)$$
$$= \left(-\frac{1}{2}+2\frac{1}{4}\right)-(-1+2)$$
$$= \left(-\frac{1}{2}+\frac{1}{2}\right)-(1)$$
$$= (0)-(1)$$
$$= -1$$

$$\boxed{\therefore \int_{1}^{2}\left(\frac{1}{x^2}-\frac{4}{x^3}\right)dx = -1}$$

Answer 27E.

Given $\int_1^4\left(\frac{4+6u}{\sqrt{u}}\right)du$

$$\int_1^4\left(\frac{4+6u}{\sqrt{u}}\right)du=\int_1^4\left(4.u^{\frac{-1}{2}}+6u^{1-\frac{1}{2}}\right)du$$

$$=\int_1^4\left(4.u^{\frac{-1}{2}}+6u^{\frac{1}{2}}\right)du$$

$$=\left(4.\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+6.\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right)_1^4 \quad \left(\because \int x^n dx=\frac{x^{n+1}}{n+1}+C, n\neq -1\right)$$

$$=\left(4.\frac{u^{\frac{1}{2}}}{\frac{1}{2}}+6.\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right)_1^4$$

$$=\left(8u^{\frac{1}{2}}+4u^{\frac{3}{2}}\right)_1^4$$

$$=\left(8.4^{\frac{1}{2}}+4.4^{\frac{3}{2}}\right)-\left(8.1^{\frac{1}{2}}+4.1^{\frac{3}{2}}\right)$$

$$=(8.2+4.8)-(8.1+4.1)$$

$$=(48)-(12)$$

$$=36$$

$$\boxed{\therefore \int_1^4\left(\frac{4+6u}{\sqrt{u}}\right)du=36}$$

Answer 29E.

We have $\int_1^4\sqrt{\frac{5}{x}}dx=\sqrt{5}\int_1^4 x^{-1/2}dx$

$$=2\sqrt{5}\,x^{1/2}\Big]_1^4$$

$$=2\sqrt{5}\left((4)^{1/2}-(1)^{1/2}\right)$$

$$=2\sqrt{5}(1)$$

$$=\boxed{2\sqrt{5}}$$

Answer 30E.

We have $\int_1^9\frac{3x-2}{\sqrt{x}}dx=\int_1^9\left(3x^{1/2}-2x^{-1/2}\right)dx$

$$=\frac{3x^{3/2}}{3/2}-\frac{2x^{1/2}}{1/2}\Big]_1^9$$

$$=2x^{3/2}-4x^{1/2}\Big]_1^9$$

$$=\left[2(9)^{3/2}-4(9)^{1/2}\right]-\left[2(1)^{3/2}-4(1)^{1/2}\right]$$

$$=54-12+2$$

$$=\boxed{44}$$

Answer 31E.

Given $\int_1^4 \sqrt{t}(1+t)dt$

$$\int_1^4 \sqrt{t}(1+t)dt = \int_1^4 \left(t^{\frac{1}{2}} + t^{\frac{3}{2}}\right)dt$$

$$= \left(\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right)_1^4 \quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right)$$

$$= \left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + \frac{t^{\frac{5}{2}}}{\frac{5}{2}}\right)_1^4$$

$$= \left(\frac{2}{3}t^{\frac{3}{2}} + \frac{2}{5}t^{\frac{5}{2}}\right)_1^4$$

$$= \left(\frac{2}{3}.4^{\frac{3}{2}} + \frac{2}{5}.4^{\frac{5}{2}}\right) - \left(\frac{2}{3}.1^{\frac{3}{2}} + \frac{2}{5}.1^{\frac{5}{2}}\right)$$

$$= \left(\frac{2}{3}.8 + \frac{2}{5}.32\right) - \left(\frac{2}{3} + \frac{2}{5}\right)$$

$$= \left(\frac{16}{3} + \frac{64}{5}\right) - \left(\frac{10+6}{15}\right)$$

$$= \left(\frac{80+192}{15}\right) - \left(\frac{10+6}{15}\right)$$

$$= \frac{256}{15}$$

$$\therefore \int_1^4 \sqrt{t}(1+t)dt = \frac{256}{15}$$

Answer 32E.

Given $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \csc^2\theta d\theta$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \csc^2\theta d\theta = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \csc^2\theta d\theta$$

$$= (-\cot\theta)_{\frac{\pi}{4}}^{\frac{\pi}{3}} \quad \left(\because \int \csc^2\theta d\theta = -\cot\theta + C\right)$$

$$= \left(-\cot\frac{\pi}{3}\right) - \left(-\cot\frac{\pi}{4}\right)$$

$$= \left(\frac{-1}{\sqrt{3}}\right) - (-1)$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}}$$

$$\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \csc^2\theta d\theta = \frac{\sqrt{3}-1}{\sqrt{3}}$$

Answer 33E.

$$\int_0^{\frac{\pi}{4}} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta = \int_0^{\frac{\pi}{4}} \left(\frac{1}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta}\right) d\theta$$
$$= \int_0^{\frac{\pi}{4}} \left(\sec^2\theta + 1\right) d\theta$$
$$= \tan\theta + \theta\Big]_0^{\frac{\pi}{4}}$$
$$= \left(\tan\frac{\pi}{4} + \frac{\pi}{4}\right) - (\tan 0 + 0)$$
$$= \boxed{1+\frac{\pi}{4}}$$

Answer 34E.

$$\int_0^{\pi/3} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\,d\theta$$
$$= \int_0^{\pi/3} \frac{\sin\theta\left(1+\tan^2\theta\right)}{\sec^2\theta}\,d\theta$$
$$= \int_0^{\pi/3} \frac{\sin\theta \times \sec^2\theta}{\sec^2\theta}\,d\theta$$
$$= \int_0^{\pi/3} \sin\theta\, d\theta = -\cos\theta\Big]_0^{\pi/3}$$
$$= -\cos\frac{\pi}{3} + \cos 0$$
$$= -\frac{1}{2} + 1 = \boxed{\frac{1}{2}}$$

Answer 35E.

We have $\displaystyle\int_1^{64} \frac{1+\sqrt[3]{x}}{\sqrt{x}}\,dx = \int_1^{64} \left(x^{-1/2} + x^{1/3-1/2}\right) dx$

$$= \int_1^{64} \left(x^{-1/2} + x^{-1/6}\right) dx$$
$$= \frac{x^{1/2}}{1/2} + \frac{x^{5/6}}{5/6}\Bigg|_1^{64}$$
$$= 2x^{1/2} + \frac{6}{5}x^{5/6}\Bigg|_1^{64}$$

Therefore,

$$\int_1^{64} \frac{1+\sqrt[3]{x}}{\sqrt{x}}\,dx = \left[2(64)^{1/2} + \frac{6}{5}(64)^{5/6}\right] - \left[2(1)^{1/2} + \frac{6}{5}(1)^{5/6}\right]$$
$$= 16 + \frac{6\times 32}{5} - 2 - \frac{6}{5}$$
$$= 14 + \frac{186}{5}$$
$$= \boxed{\frac{256}{5}}$$

Answer 36E.

Given $\int_1^8 \left(\frac{x-1}{\sqrt[3]{x^2}}\right) du$

$$\int_1^8 \left(\frac{x-1}{\sqrt[3]{x^2}}\right) du = \int_1^8 (x-1)x^{\frac{-2}{3}}\,du$$

$$= \int_1^8 \left(x^{1-\frac{2}{3}} - x^{\frac{-2}{3}}\right) du$$

$$= \int_1^8 \left(x^{\frac{1}{3}} - x^{\frac{-2}{3}}\right) du$$

$$= \left[\frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} - \frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1}\right]_1^8 \quad \left(\because \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1\right)$$

$$= \left[\frac{3}{4}x^{\frac{4}{3}} - 3x^{\frac{1}{3}}\right]_1^8$$

$$= \left[\left(\frac{3}{4}.8^{\frac{4}{3}} - 3.8^{\frac{1}{3}}\right) - \left(\frac{3}{4}.1^{\frac{4}{3}} - 3.1^{\frac{1}{3}}\right)\right]$$

$$= \left[\left(\frac{3}{4}.16 - 3.2\right) - \left(\frac{3}{4} - 3\right)\right]$$

$$= \left[(6) - \left(\frac{-9}{4}\right)\right]$$

$$= \frac{31}{4}$$

$$\boxed{\therefore \int_1^8 \left(\frac{x-1}{\sqrt[3]{x^2}}\right) du = \frac{31}{4}}$$

Answer 37E.

$$\int_0^1 \left(\sqrt[4]{x^5} + \sqrt[5]{x^4}\right) dx = \int_0^1 \left(x^{5/4} + x^{4/5}\right) dx$$

$$= \left[\frac{x^{5/4+1}}{\frac{5}{4}+1} + \frac{x^{4/5+1}}{\frac{4}{5}+1}\right]_0^1$$

$$= \left[\frac{4}{9}x^{9/4} + \frac{5}{9}x^{9/5}\right]_0^1$$

$$= \left(\frac{4}{9}(1)^{9/4} + \frac{5}{9}(1)^{9/5}\right) - (0)$$

$$= \frac{4}{9} + \frac{5}{9}$$

$$= \boxed{1}$$

Answer 38E.

$$\int_0^1 (1+x^2)^3\,dx = \int_0^1 (1+3x^2+3x^4+x^6)\,dx$$
$$= x+\frac{3x^3}{3}+\frac{3x^5}{5}+\frac{x^7}{7}\Big|_0^1$$
$$= x+x^3+\frac{3x^5}{5}+\frac{x^7}{7}\Big|_0^1$$
$$= 1+1+\frac{3}{5}+\frac{1}{7}$$
$$= \frac{70+21+5}{35}$$
$$= \boxed{\frac{96}{35}}$$

Answer 39E.

$$= \left(-\frac{3^2}{2}+3.3\right)-\left(-\frac{2^2}{2}+3.2\right)+\left(\frac{5^2}{2}-3.5\right)-\left(\frac{3^2}{2}-3.3\right)$$
$$= \left(-\frac{9}{2}+9\right)-\left(-\frac{4}{2}+6\right)+\left(\frac{25}{2}-15\right)-\left(\frac{9}{2}-9\right)$$
$$= \frac{-9+4+25-9}{2}+9-6-15+9$$
$$= \frac{11}{2}-3$$
$$= \frac{5}{2}$$
$$\boxed{\therefore \int_2^5 |x-3|\,dx = \frac{5}{2}}$$

Answer 40E.

Given $\int_0^2 |2x-1|\,dx$

$$\int_0^2 |2x-1|\,dx = \int_0^{\frac{1}{2}} -(2x-1)\,dx + \int_{\frac{1}{2}}^2 (2x-1)\,dx$$
$$= \int_0^{\frac{1}{2}} (-2x+1)\,dx + \int_{\frac{1}{2}}^2 (2x-1)\,dx$$
$$= \left(-2\frac{x^{1+1}}{1+1}+x\right)_0^{\frac{1}{2}} + \left(2\frac{x^{1+1}}{1+1}-x\right)_{\frac{1}{2}}^2 \qquad \left(\because \int x^n\,dx = \frac{x^{n+1}}{n+1}+C, n\neq -1\right)$$

$$=\left(-2\frac{x^2}{2}+x\right)_0^{\frac{1}{2}}+\left(2\frac{x^2}{2}-x\right)_{\frac{1}{2}}^2$$

$$=\left(-x^2+x\right)_0^{\frac{1}{2}}+\left(x^2-x\right)_{\frac{1}{2}}^2$$

$$=\left(-\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)\right)-\left(-(0)^2+(0)\right)+\left(2^2-2\right)-\left(\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)\right)$$

$$=\left(-\left(\frac{1}{4}\right)+\left(\frac{1}{2}\right)\right)-(0)+(2)-\left(\left(\frac{1}{4}\right)-\left(\frac{1}{2}\right)\right)$$

$$=\left(\frac{1}{4}\right)+(2)-\left(\frac{-1}{4}\right)$$

$$=\frac{5}{2}$$

$$\boxed{\therefore \int_0^2 |2x-1|\,dx=\frac{5}{2}}$$

Answer 41E.

We have $\int_{-1}^{2}(x-2|x|)\,dx=\int_{-1}^{0}(x-2|x|)\,dx+\int_{0}^{2}(x-2|x|)\,dx$

$$=\int_{-1}^{0}(x+2x)\,dx+\int_{0}^{2}(x-2x)\,dx \quad (\textit{if } x<0,\ |x|=-x \text{ and } \textit{if } x\geq 0,\ |x|=x)$$

$$=\int_{-1}^{0}3x\,dx+\int_{0}^{2}(-x)\,dx$$

$$=\frac{3x^2}{2}\Big]_{-1}^{0}-\frac{x^2}{2}\Big]_{0}^{2}$$

$$=0-\frac{3(-1)^2}{2}-\frac{4}{2}$$

$$=\boxed{-\frac{7}{2}}$$

Answer 42E.

$$\int_0^{3\pi/2}|\sin x|\,dx=\int_0^{\pi}|\sin x|\,dx+\int_{\pi}^{3\pi/2}|\sin x|\,dx$$

$$=\int_0^{\pi}\sin x\,dx-\int_{\pi}^{3\pi/2}\sin x\,dx$$

$$\left(\begin{array}{l}\textit{if } x\in(0,\pi),\sin x>0\Rightarrow|\sin x|=\sin x,\\ \textit{And if } x\in\left(\pi,\frac{3\pi}{2}\right),\sin x<0\Rightarrow|\sin x|=-\sin x\end{array}\right)$$

$$=-\cos x\Big]_0^{\pi}+\cos x\Big]_{\pi}^{3\pi/2}$$

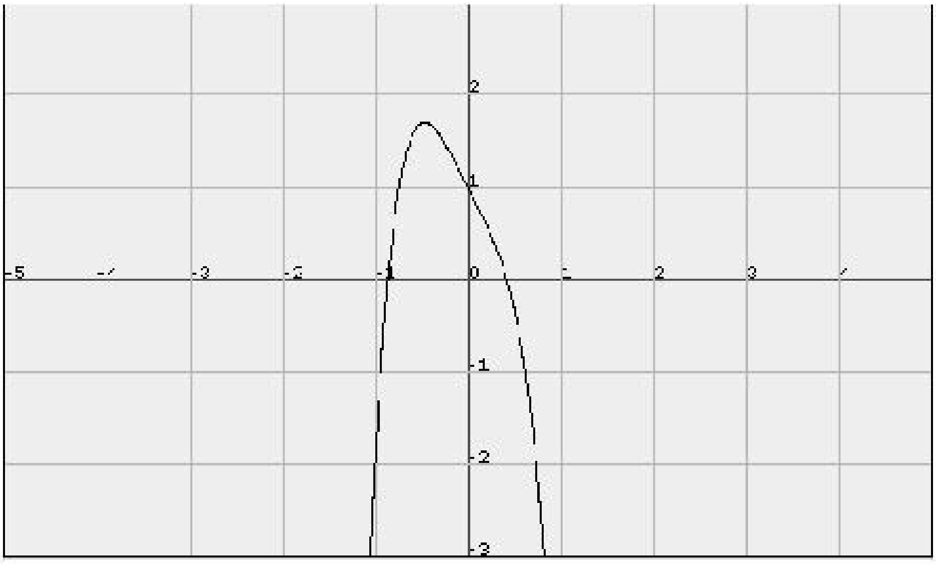
$$=-\cos\pi+\cos 0+\cos\frac{3\pi}{2}-\cos\pi$$

$$=1+1+0+1$$

$$=\boxed{3}$$

Answer 43E.

Given $y = 1 - 2x - 5x^4$



x intercepts are -0.85 and 0.4 (approximately)

The area under the curve and above the x axis is

$$\int_{-0.85}^{0.4} \left(1 - 2x - 5x^4\right) dx$$

$$\approx \left[x - 2\frac{x^{1+1}}{1+1} - 5\frac{x^{4+1}}{4+1}\right]_{-0.85}^{0.3}$$

$$\approx \left[x - x^2 - x^5\right]_{-0.85}^{0.4}$$

$$\approx \left[(0.4) - (0.4)^2 - (0.4)^5\right] - \left[(-0.85) - (-0.85)^2 - (-0.85)^5\right]$$

$$\approx \left[(0.3) - 0.09 - 0.00243\right] - \left[-0.85 - 0.7225 + 0.4437053125\right]$$

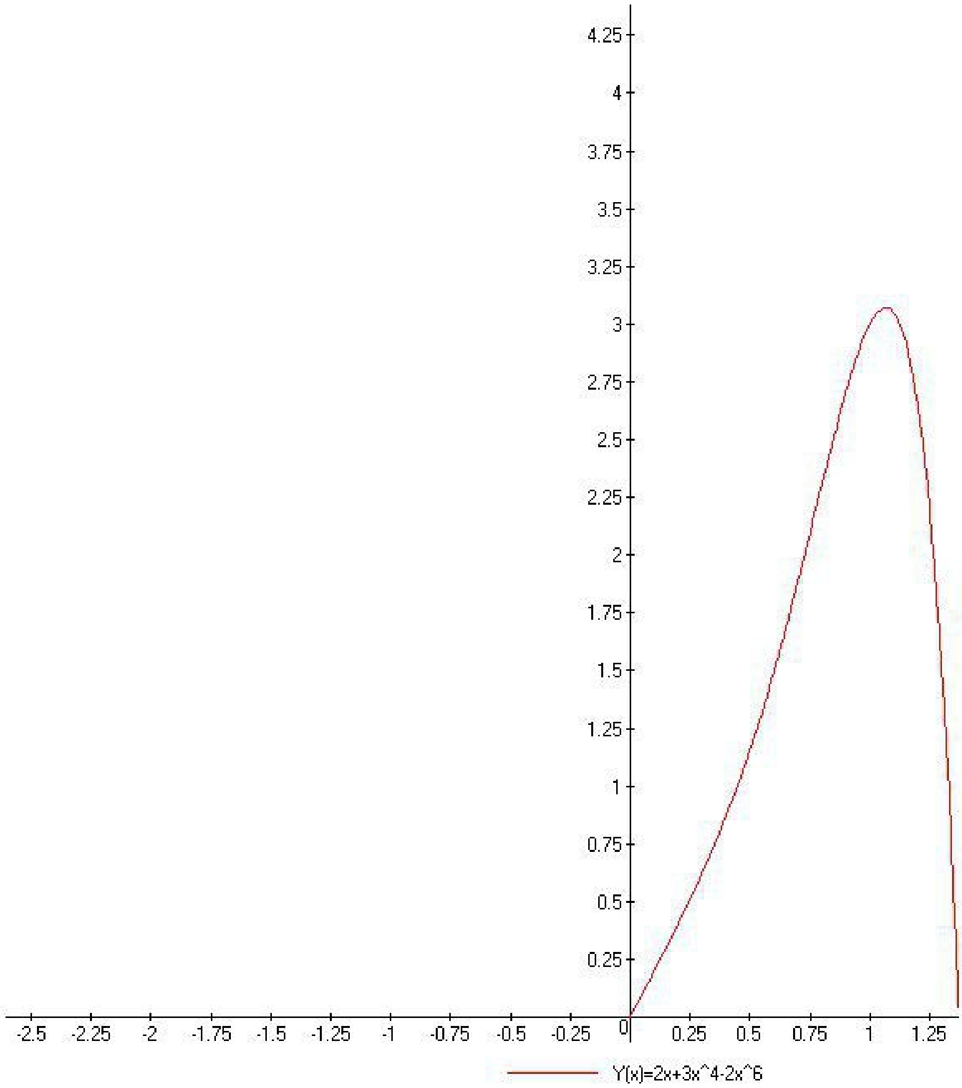
$$\approx 1.349704$$

The required area is $\boxed{A \approx 1.35}$

Answer 44E.

y = 2x + 3x4 - 2x6

the graph of the curve is :



observe that all the enclosed region is above x axis and the curve becomes parallel to y axis below y axis.

so the function has assymptotes x = -3, 3.

now, using the fundamental theorem , we find the area of the curve between x = 0 , 1.372 which are the intersection points of the curve with x axis.

i.e. $\int_0^{1.372}\left(2x+3x^4-2x^6\right)dx = x^2+\frac{3x^5}{5}-\frac{2x^7}{7}\Big\}_0^{1.372}$ =3.556652

Answer 45E.

The graph is as follows:

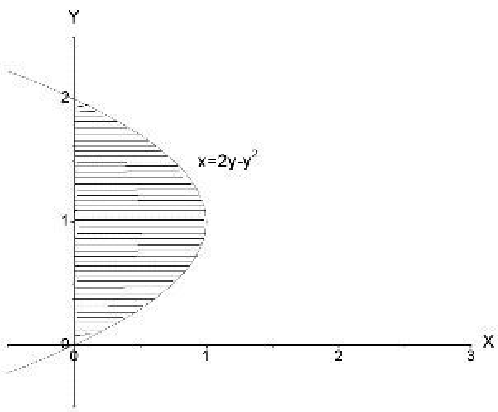


FIG.1

Since area of the region is given by the integral $\int_0^2 (2y - y^2) dy$. So we have to evaluate this integral with respect to y for getting the area of shaded region.
We rewrite the integral as

$$\int_0^2 (2y - y^2) dy = \int_0^2 2y \, dy - \int_0^2 y^2 \, dy$$

We used the property, $\int_a^b [f(y) - g(y)] dy = \int_a^b f(y) dy - \int_a^b g(y) dy$..
And using fundamental theorem of calculus, since an anti-derivative of $2y$ is y^2
And anti derivative of y^2 is $\frac{y^3}{3}$.

So we have

$$\int_0^2 (2y - y^2) dy = y^2 \Big]_0^2 - \frac{y^3}{3}\Big]_0^2$$

$$= y^2 - \frac{y^3}{3}\Big]_0^2$$

$$= \left(2^2 - \frac{2^3}{3}\right) - (0 - 0)$$

$$= 4 - \frac{8}{3}$$

$$= \frac{4}{3}$$

So area of the region is $\boxed{= \frac{4}{3}}$.

Answer 46E.

The graph is as follows:

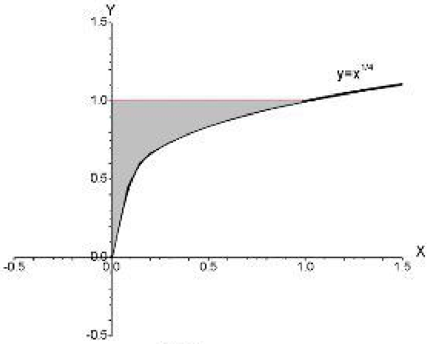


FIG.1

Since $y = \sqrt[4]{x}$ then $y^4 = x$

So the area of shaded region will be $\int_0^1 y^4 dy$.

We have to evaluate this integral with respect to y for getting the area of shaded region.

We rewrite the integral as $\int_0^1 y^4 dy$

Using fundamental theorem of calculus, since an anti-derivative of y^4 is $\frac{y^5}{5}$.

So we have

$$\int_0^1 y^4 dy = \frac{y^5}{5}\Big]_0^1$$

$$= \frac{1^5}{5} - 0$$

$$= \frac{1}{5}$$

So the area of shaded region is $\boxed{= \frac{1}{5}}$.

Answer 47E.

Since $w'(t)$ is the rate of growth of a child in pounds per year

So $\int_5^{10} w'(t)\,dt$ will represent the increase in the child's weight between the ages from 5 to 10 years

Means $w(10)-w(5)=\int_5^{10} w'(t)\,dt$

Answer 48E.

The current in a wire is defined as the derivative of the charge.

$I(t)=Q'(t)$

Then $\int_a^b I(t)dt=\int_a^b Q'(t)dt$

By the net change theorem, $\int_a^b F'(x)dx=F(b)-F(a)$

Then $\int_a^b Q'(t)dt=Q(b)-Q(a)$

Therefore, $\int_a^b I(t)dt$ represent the change in the charge Q from time $t=a$ to $t=b$.

Answer 49E.

Since $r(t)$ is rate at which the oil leaks in gallons per minute at time t.

And $r(t)=-V'(t)$ where $V(t)$ is the volume of oil at time t

Here negative sign indicates that the volume is decreasing.

Then $\int_0^{120} r(t)\,dt=-\int_0^{120} V'(t)\,dt=-V(120)+V(0)$

Thus $\int_0^{120} r(t)\,dt$ will respect the total amount of oil (in gallon) that leaked in first 120 minutes.

Or $\int_0^{120} r(t)\,dt$ will represent the number of gallons of oil that leaked in first 2 hours.

Answer 50E.

A honey bee population starts with 100 bees and increases at rate of $n'(t)$ bees per week.

Then $100+\int_0^{15} n'(t)\,dt$ will represent total number of bees after 15 weeks.

Answer 51E.

Since $\int_{1000}^{5000} R'(x)\,dx=R(5000)-R(1000)$

So this integral represents the increase in revenue when number of units is increased form 1000 units to 5000 units

Or in other words

$\int_{1000}^{5000} R'(x)\,dx$ Represents increase in revenue when production level is increased from 1000 to 5000 units

Answer 52E.

Consider that $f(x)$ is the slope of a trail at a distance of x miles from the start of the trail.

Since, the first derivative of the elevation E with respect to the distance x, is the slope of the trail.

Then $f(x) = E'(x) = \frac{d}{dx}E(x)$

Therefore, $\int_3^5 f(x)\,dx = \int_3^5 E'(x)\,dx$

$= E(5) - E(3)$ [By net change theorem]

Therefore, $\int_3^5 f(x)\,dx$ represents the change in the elevation E between $x = 3$ miles to 5 miles.

Answer 53E.

Since x is measured in meters and $f(x)$ is measured in Newton (N).

And $\int_0^{100} f(x)dx$ represent the area under the curve of $f(x)$ from $x = 0$ to $x = 100$.

So unit of $\int_0^{100} f(x)dx$ = unit of $f(x)$ × unit of x

$= \text{Newton} \times \text{meter}$

$\boxed{\begin{array}{l} = \text{newton meter (Nm)} \\ = \text{Joules (J)} \end{array}}$

Answer 54E.

Given that units for x are feet and units for $a(x)$ are pounds per foot

Then units for $\frac{da}{dx} = \frac{\left(\text{pounds}/\text{foot}\right)}{\text{feet}} = \frac{\text{pounds}}{(\text{foot})^2}$

And since $\int_2^8 a(x)dx$ represent the area of the region under the curve from $x = 2$ to 8 feet

So unit of area $\int_2^8 a(x)dx$ = unit of a(x) × unit of x

$= \text{pounds per foot} \times \text{feet}$

$\boxed{= \text{pounds}}$

Answer 55EE.

Consider the following velocity function.

$v(t)=3t-5$, $0\le t\le 3.$

The objective is to find the displacement of the particle.

The velocity $v(t)$ is the derivative of a position function $s(t)$ and then the

Integral $\int_{t_1}^{t_2} v(t)dt = s(t_2)-s(t_1).$

This integral gives the change of position of the object in the time interval $[t_1,t_2]$.

Use this formula find the displacement of the particle whose velocity (in meters per second) is $v(t)=3t-5$ in the time interval $0\le t\le 3.$

So, evaluate the integral $\int_0^3 (3t-5)dt.$

$$\begin{aligned}\int_0^3 (3t-5)dt &= \int_0^3 3t\,dt - \int_0^3 5\,dt\\ &= \left[\frac{3t^2}{2}\right]_0^3 - [5t]_0^3\\ &= \left[\frac{3\cdot 9}{2}-\frac{3\cdot 0}{2}\right]-[5\cdot 3-5\cdot 0]\\ &= \frac{27}{2}-15\\ &= \frac{-3}{2}\end{aligned}$$

Hence, the displacement of the particle is $\boxed{-\frac{3}{2}\text{ meters}}$.

Find the total distance traveled we need to consider intervals when velocity is positive as well as intervals when velocity is negative. We sum the distances traveled throughout each of these intervals. To ensure distances are positive, we integrate the absolute value of the velocity function. Hence, total distance is calculated:

$$\int_{t_1}^{t_2} |v(t)|\,dt$$

So on any subinterval when velocity is negative, the integrand becomes $-v(t)$.

On the interval $0\le t\le 3$, the velocity function $v(t)=3t-5$ is positive if:

$$\begin{aligned}3t-5&>0\\ 3t&>5\\ t&>\frac{5}{3}\end{aligned}$$

Hence, velocity is positive on the interval $\frac{5}{3}<t\le 3$.

Then, the velocity is negative on the interval $0\le t<\frac{5}{3}$.

Rewrite the total integral by separating it over these subintervals:

$$\begin{aligned}\int_{t_1}^{t_2} |v(t)|\,dt &= \int_0^3 |3t-5|\,dt\\ &= \int_0^{5/3} -(3t-5)\,dt + \int_{5/3}^3 (3t-5)\,dt\end{aligned}$$

The first integrand is made negative because the velocity is negative on the interval $0\le t<\frac{5}{3}$.

Evaluate the integrals using the antiderivative we found in part (a):

$$\int_0^{5/3} -(3t-5)\,dt + \int_{5/3}^{3}(3t-5)\,dt$$

$$= -\left[\frac{3}{2}t^2 - 5t\right]_0^{5/3} + \left[\frac{3}{2}t^2 - 5t\right]_{5/3}^{3}$$

$$= \begin{cases} -\left[\left(\frac{3}{2}\left(\frac{5}{3}\right)^2 - 5\left(\frac{5}{3}\right)\right) - \left(\frac{3}{2}(0)^2 - 5(0)\right)\right] \\ +\left[\left(\frac{3}{2}(3)^2 - 5(3)\right) - \left(\frac{3}{2}\left(\frac{5}{3}\right)^2 - 5\left(\frac{5}{3}\right)\right)\right] \end{cases}$$

$$= -\left[\frac{25}{6} - \frac{25}{3}\right] + \left[\left(\frac{27}{2} - 15\right) - \left(\frac{25}{6} - \frac{25}{3}\right)\right]$$

$$= -\left(-\frac{25}{6}\right) + \left[-\frac{3}{2} - \left(-\frac{25}{6}\right)\right]$$

$$= \frac{41}{6}$$

Therefore, the total distance traveled over the interval $0 \le t \le 3$ is $\frac{41}{6}$, or approximately, $\boxed{6.83 \text{ meters}}$.

Answer 56E.

(A) Given $v(t) = t^2 - 2t - 8, \quad 1 \le t \le 6$

By the equation $\int_{t_1}^{t_2} v(t)\,dt = S(t_2) - S(t_1)$

The displacement is (in the time interval [1, 6])

$$S(t_2) - S(t_1) = \int_1^6 v(t)\,dt = \int_1^6 \left(t^2 - 2t - 8\right)dt$$

$$= \left[\frac{t^3}{3} - \frac{2t^2}{2} - 8t\right]_1^6 \quad \text{(By FTC - 2)}$$

$$= \left[\frac{t^3}{3} - t^2 - 8t\right]_1^6$$

$$\Rightarrow \int_1^6 v(t)\,dt = \left[\frac{6^3}{3} - 6^2 - 8(6)\right] - \left[\frac{1^3}{3} - 1^2 - 8(1)\right]$$

$$= [72 - 36 - 48] - \left[\frac{1}{3} - 1 - 8\right]$$

$$= -12 + \frac{26}{3} = \boxed{-\frac{10}{3}} \text{ m}$$

This means that the particle moves $\boxed{\approx 3.33 \text{ m}}$ toward the left.

(B) $v(t) = t^2 - 2t - 8$

$$= t^2 - 4t + 2t - 8$$

$$= (t-4)(t+2)$$

And so $v(t) \le 0$ on the interval [1, 4].

And $v(t) \ge 0$ on [4, 6], thus total distance traveled

$$= \int_1^6 |v(t)|\,dt = \int_1^4 [-v(t)]\,dt + \int_4^6 v(t)\,dt$$

$$= \int_1^4 \left(-t^2 + 2t + 8\right)dt + \int_4^6 \left(t^2 - 2t - 8\right)dt$$

$$\Rightarrow \int_1^6 |v(t)|dt = \left[-\frac{t^3}{3}+t^2+8t\right]_1^4 + \left[\frac{t^3}{3}-t^2-8t\right]_4^6$$

$$= \left[-\frac{64}{3}+16+32+\frac{1}{3}-1-8\right]+\left[\frac{216}{3}-36-48-\frac{64}{3}+16+32\right]$$

$$= (-21+39)+\left(\frac{152}{3}-36\right)$$

$$= \frac{98}{3} \quad \boxed{\approx 32.67 \text{ m}}$$

Answer 57E.

(A) Given acceleration $a(t) = t+4 \quad (\text{in m/s}^2)$

Since acceleration $a(t) = v'(t)$, where $v(t)$ is the velocity at time t.

The general anti-derivative of $a(t)$ is

$$v(t) = \frac{t^2}{2}+4t+C \text{, where } C \text{ is any constant.}$$

But given condition is $v(0) = 5$

So, $5 = 0+0+C$ *or* $\boxed{C = 5}$

Then we have velocity at time t is

$$\boxed{v(t) = \frac{t^2}{2}+4t+5} \text{ m/s}$$

(B) Since velocity at time t is $v(t) = \frac{t^2}{2}+4t+5$

So distance traveled during the time interval $0 \le t \le 10$ is

$$S(10)-S(0) = \int_0^{10} v(t)dt$$

$$= \int_0^{10}\left(\frac{t^2}{2}+4t+5\right)dt$$

Using fundamental theorem of calculus and properties of integral, we have

$$S(10)-S(0) = \left[\frac{t^3}{6}+2t^2+5t\right]_0^{10}$$

$$= \left[\frac{1000}{6}+200+50\right]_0^{10}$$

$$= \frac{2500}{6} = \frac{1250}{3}$$

$$= 416\frac{2}{3} \text{ m}$$

So distance traveled during the time interval $0 \le t \le 10$ is $\boxed{416\frac{2}{3} \text{ m}}$.

Answer 58E.

(A) Given, acceleration $a(t) = 2t+3 \quad (\text{in m/s}^2)$

Since acceleration $a(t) = v'(t)$

Where $v(t)$ is the velocity at time t

The general anti derivative of $a(t)$ is $v(t) = t^2+3t+C$ where C is any constant

We have the given condition $v(0) = -4$

So $\quad -4 = 0+0+C$ *or* $\boxed{C = -4}$

Then we have velocity at time t is

$$\boxed{v(t) = t^2+3t-4}$$

(B) Since velocity at time t is $v(t) = t^2 + 3t - 4$

And $v(t) < 0$ on [0, 1] and $v(t) > 0$ on [1, 3]

So distance traveled during the time interval $0 \le t \le 3$ is

$$S(3) - S(0) = \int_0^1 |v(t)|dt + \int_1^3 |v(t)|dt$$
$$= \int_0^1 (-t^2 - 3t + 4)dt + \int_1^3 (t^2 + 3t - 4)dt$$

Using fundamental theorem of calculus and properties of integrals, we have

$$S(3) - S(0) = \left[-\frac{t^3}{3} - \frac{3}{2}t^2 + 4t\right]_0^1 + \left[\frac{t^3}{3} + \frac{3}{2}t^2 - 4t\right]_1^3$$
$$= \left[-\frac{1}{3} - \frac{3}{2} + 4\right] + \left[9 + \frac{27}{2} - 12\right] - \left[\frac{1}{3} + \frac{3}{2} - 4\right]$$
$$= \frac{89}{6} \text{ meters}$$

So distance traveled during the time interval [0, 3] is $= \boxed{\frac{89}{6} \text{ m}}$

Answer 59E.

We have been given that linear density $\rho(x) = 9 + 2\sqrt{x}$ (in kg/m)

Where x is length of rod measured in meters from one end of the rod

Since total length of the rod is 4m (given)

So we have to find total mass of the rod from $x = 0$ to $x = 4$

If $m(x)$ is the mass of the rod then linear density is $\rho(x) = m'(x)$

So mass of the rod between $x = 0$ and $x = 4$ is

$$m(4) - m(0) = \int_0^4 \rho(x)dx$$
$$= \int_0^4 (9 + 2\sqrt{x})dx$$

Using fundamental theorem of calculus and properties of integrals

$$m(4) - m(0) = \left[9x + 2.\frac{x^{3/2}}{3/2}\right]_0^4 \qquad \left[\frac{x^{3/2}}{3/2} \text{ is an anti derivative of } \sqrt{x}\right.$$
$$= \left[9x + \frac{4}{3}x^{3/2}\right]_0^4$$
$$= \left[9 \times 4 + \frac{4}{3}(4)^{3/2}\right]$$
$$= \left[36 + \frac{32}{3}\right] = \frac{140}{3} \text{ kg}$$
$$= 46\frac{2}{3} \text{ kg}$$

So total mass of the rod is $= \boxed{46\frac{2}{3} \text{ kg}}$

Answer 60E.

Given water flows from a storage tank at a rate of

$r(t) = 200 - 4t$ Liters per minute, where $0 \le t \le 50$.

We need to find the amount of water that flows from the tank during the first 10 minutes,

That is we need to find $\int_0^{10} r(t)dt = \int_0^{10} (200 - 4t)dt$

$$= \left(200t - 4\frac{t^{1+1}}{1+1}\right)_0^{10} \quad \left(\text{Since } \int x^n dx = \frac{x^{n+1}}{n+1} + C, n \ne -1\right)$$
$$= \left(200t - 4\frac{t^2}{2}\right)_0^{10}$$
$$= \left(200t - 2t^2\right)_0^{10}$$

Applying limits

$$= \left(200(10) - 2\left(10^2\right)\right) - \left(200(0) - 2\left(0^2\right)\right)$$
$$= (2000 - 200) - (0)$$
$$= 1800$$

The amount of water that flows from the tank during the first 10 minutes is $\boxed{= 1800}$ liters

Answer 61E.

The displacement of the car over the interval $[0,100]$ is found by integrating the velocity:

$$\int_0^{100} v(t)\,dt = s(100) - s(0)$$

Here, $s(t)$ is the position of the car at time *t*. Hence, the integral is the difference in position from start to finish.

The total distance is to be estimated over the interval $[0,100]$. To use midpoints, subdivide the interval into 5 subintervals:

$$[0,20],\ [20,40],\ [40,60], [60,80]\ ,[80,100]$$

Then, use the midpoints $10, 30, 50, 70, 90$ to evaluate the velocity in the Riemann sum. The Riemann sum is as follows:

$$\sum_{i=1}^{5} v\left(t_i^*\right)\Delta t_i = \Delta t\left[v(10) + v(30) + v(50) + v(70) + v(90)\right]$$
$$= \Delta t[38 + 58 + 51 + 53 + 47]$$
$$= \Delta t[247]$$

The length of each subinterval is 20 seconds. However, since velocity is measured in miles per hour, convert the time interval to hours:

$$20\,\text{sec} = 20\,\text{sec}\left(\frac{1\,\text{min}}{60\,\text{sec}}\right)\left(\frac{1\text{ hour}}{60\,\text{min}}\right)$$
$$= \frac{20\text{ hour}}{3600}$$
$$= \frac{1}{180}\text{hour}$$

Therefore, $\Delta t = \dfrac{1}{180}$ hours.

Then, the Riemann sum becomes the following:

$$\Delta t[247] = \frac{1}{180}(247)$$
$$\approx 1.37$$

Therefore, the total distance traveled by the car is approximately $\boxed{1.37\text{ miles}}$.

Answer 62E.

(a) The total quantity of material over the time interval $[0,6]$ is found by integrating the rate function:

$$\int_0^6 r(t)\,dt = Q(6) - Q(0)$$

Where $Q(t)$ is the material spewed at time *t*. Hence the integral is the difference in material in the atmosphere from start to finish.

We can approximate the definite integral with the Riemann sum. Expressed in sigma notation, the Riemann sum over a partition with *n* subintervals is:

$$\sum_{i=1}^{n} f(x_i^*)\Delta x_i = f(x_1^*)\Delta x_1 + f(x_2^*)\Delta x_2 + \cdots + f(x_n^*)\Delta x_n$$

Where each x_i^* is a sample point in the *i*th subinterval and Δx_i is the length of the *i*th subinterval. Often, we will use subintervals which each have the same length. When the subintervals each have the same length, the length is calculated:

$$\Delta x = \frac{b-a}{n}$$

Where the interval is $[a,b]$ and the partition contains *n* subintervals

Then the Riemann sum becomes:

$$\sum_{i=1}^{n} f(x_i^*)\Delta x_i = \Delta x\left[f(x_1^*) + f(x_2^*) + \cdots + f(x_n^*)\right]$$

Since the amount of material increases over time, we can find an upper estimate by using the right endpoints. We can subdivide the time interval into 6 sub-intervals:

$[0,1]$, $[1,2]$, $[2,3]$, $[3,4]$,$[4,5]$, $[5,6]$

Each sub-interval has length $\Delta t = 1$ second. With the values $r(t)$ from the table we can use the right endpoints $1, 2, 3, 4, 5, 6$ to evaluate the amount of material in the Riemann sum. The Riemann sum is:

$$\begin{aligned}\sum_{i=1}^{6} r(t_i^*)\Delta t_i &= \Delta t\left[r(1) + r(2) + r(3) + r(4) + r(5) + r(6)\right] \\ &= 1[10 + 24 + 36 + 46 + 54 + 60] \\ &= 1[230]\end{aligned}$$

Therefore, an upper estimate for the amount of material spewed is $\boxed{230 \text{ tonnes}}$.

Similarly, we can find a lower estimate by using the left endpoints of the sub-intervals $0, 1, 2, 3, 4, 5$. Then the Riemann sum becomes:

$$\begin{aligned}\sum_{i=1}^{6} r(t_i^*)\Delta t_i &= \Delta t\left[r(0) + r(1) + r(2) + r(3) + r(4) + r(5)\right] \\ &= 1[2 + 10 + 24 + 36 + 46 + 54] \\ &= 1[172]\end{aligned}$$

Therefore, a lower estimate for the amount of material spewed is $\boxed{172 \text{ tonnes}}$.

(b) To use midpoints, we subdivide the interval $[0,6]$ into 3 subintervals:

$[0,2]$, $[2,4]$, $[4,6]$

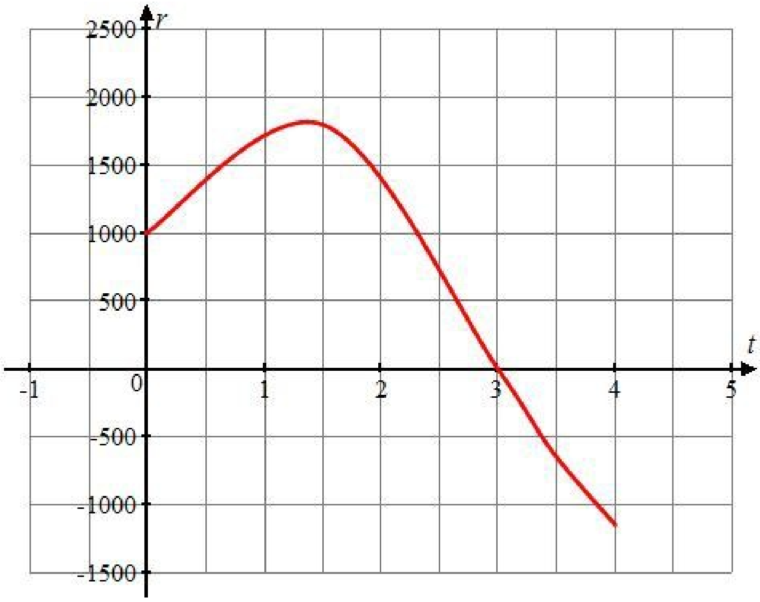
Each of these sub-intervals has length $\Delta t = 2$ seconds. We can use the sub-interval midpoints $1, 3, 5$ to evaluate the amount of material in the Riemann sum. The Riemann sum using midpoints is:

$$\begin{aligned}\sum_{i=1}^{3} r(t_i^*)\Delta t_i &= \Delta t\left[r(1)+r(3)+r(5)\right]\\ &= 2[10+36+54]\\ &= 2[100]\\ &= 200\end{aligned}$$

Therefore, using the Midpoint Rule, the amount of material spewed is approximately $\boxed{200 \text{ tonnes}}$.

Answer 63E.

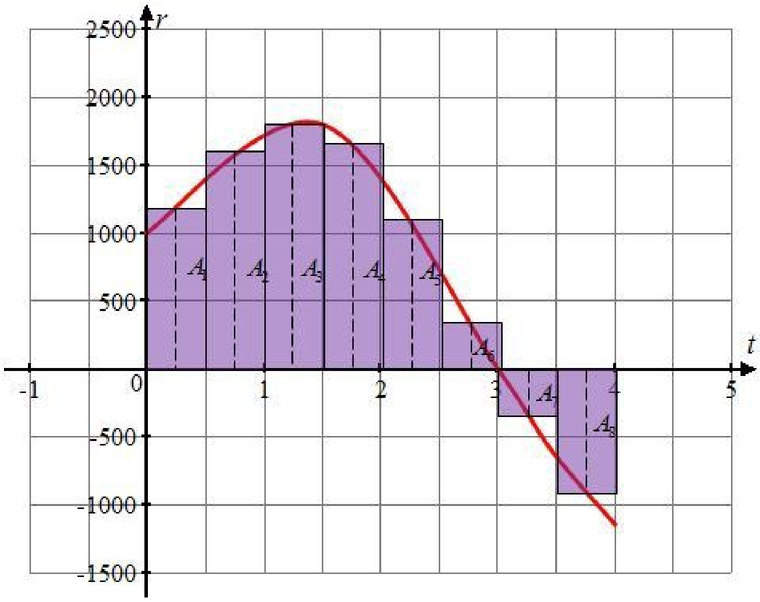
The following diagram shows the rate of the change of volume of the water in a tank with respect to time.



Use the midpoint rule to find the amount of water four days later.

The more rectangles we use, the better our estimation of the distance traveled will be.

In this case, we will use midpoints to estimate the graph using eight rectangles:



Find the area of the each rectangle using the formula $A_n = wh$, width multiplied by height.

From the figure it is observed that the all the rectangle have same width 0.5 unit.

$$A_1 = 0.5(1200)$$
$$A_2 = 0.5(1600)$$
$$A_3 = 0.5(1800)$$
$$A_4 = 0.5(1700)$$
$$A_5 = 0.5(1200)$$
$$A_6 = 0.5(300)$$
$$A_7 = 0.5(300)$$
$$A_8 = 0.5(900)$$

Write the integral to estimate the amount of water flown out from storage tank.

Amount of water $= \int_0^4 |r(t)|\,dt = A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8$

Evaluate the amount of water flown out from the storage tank.

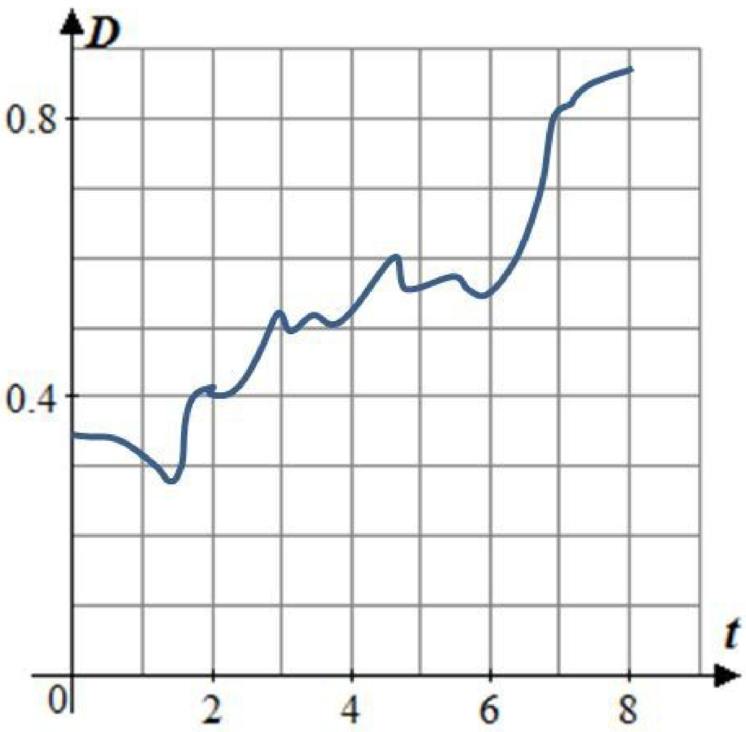
$$\begin{aligned}\int_0^4 |r(t)|\,dt &= A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 \\ &= \begin{cases} 0.5(1200) + 0.5(1600) + 0.5(1800) + 0.5(1700) \\ +0.5(1200) + 0.5(300) + 0.5(300) + 0.5(900) \end{cases} \\ &= 4500\end{aligned}$$

Hence Amount of water flown out from the storage tank is 4500 L.

The amount of water initially is 25,000 L so the amount of remain in tank four days later is
$25,000 \text{ L} - 4,5000 \text{ L} = \boxed{20,500 \text{ L}}$.

Answer 64E.

Given the graph of traffic on an Internet service provider's T1 data line from midnight to 8:00 AM is



The objective is to use Midpoint Rule to estimate the total amount of data transmitted during that time period.

Midpoint Rule:

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$$
$$= \Delta x\left[f(\bar{x}_1)+f(\bar{x}_2)+...+f(\bar{x}_n)\right]$$

Where $\Delta x = \frac{b-a}{n}$ and

$\bar{x}_i = \frac{1}{2}(x_{i-1}+x_i) = \text{midpoint of } [x_{i-1}, x_i]$

Divide the interval $[0,8]$ into four subintervals

Then

$$\Delta x = \frac{8-0}{4}$$
$$= 2$$

Intervals are $[0,2],[2,4],[4,6],[6,8]$

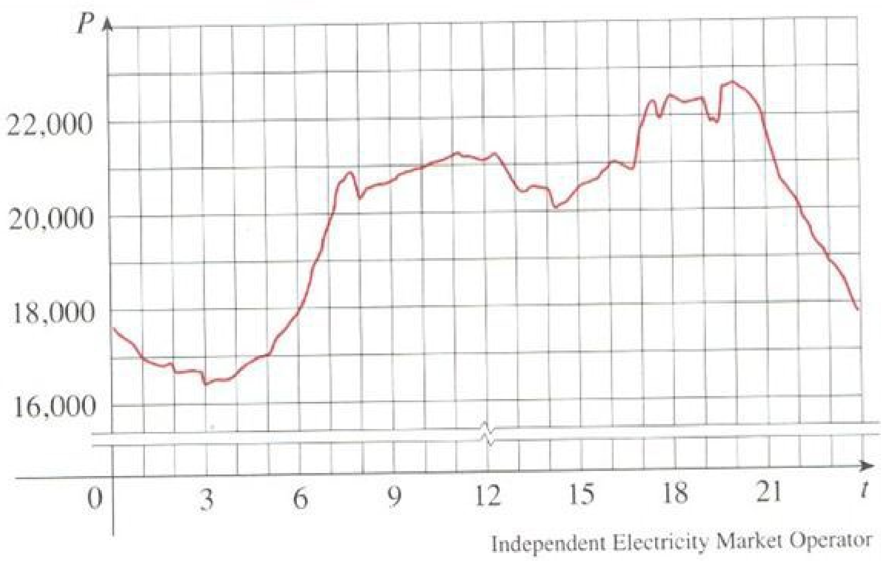
Midpoints are $1,3,5,7$.

Therefore, the total amount of data transmitted during the time period t is,

$$\int_0^8 Ddt = 2\left[f(1)+f(3)+f(5)+f(7)\right]$$
$$= 2[0.3+0.5+0.55+0.8] \qquad \text{(from the graph)}$$
$$= \boxed{4.30 \text{ megabits/second}}.$$

Answer 65E.

The graph of the power consumption in the province of Ontario, Canada, for December 9, 2004 (P is measured in megawatts; t is measured in hours starting at midnight) is shown below.



The objective is to by using the above graph estimate the total consumption $\int_0^{24} Pdt$.

Midpoint Rule:

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(\bar{x}_i)\Delta x$$
$$= \Delta x\left[f(\bar{x}_1)+f(\bar{x}_2)+...+f(\bar{x}_n)\right]$$

Where $\Delta x = \frac{b-a}{n}$ and

$\bar{x}_i = \frac{1}{2}(x_{i-1}+x_i) = \text{midpoint of } [x_{i-1}, x_i]$

Divide the interval $[0,24]$ into four subintervals

Then

$$\Delta x = \frac{24-0}{6}$$
$$= 4$$

Intervals are $[0,6],[6,12],[12,18],[18,24]$.

Midpoints are $3,9,15,21$.

Now by midpoint rule

$$\int_0^{24} P dt = 6\left[P(3)+P(9)+P(15)+P(21)\right]$$
$$= 6[16500+20800+20500+21900] \quad (\text{from the graph})$$
$$= 478200 \text{ megawatt-hours}$$

Hence, the total consumption on that day is $\boxed{478200 \text{ megawatt-hours}}$

Answer 66E.

a)

The equation the calculator gave for this graph is

$$V(t) = 0.00146t^3 - 0.11553t^2 + 24.98169t - 21.26872$$

b)

$$h(125) - h(0) = \int_0^{125} v(t)\, dt = \left[0.000365t^4 - 0.03851t^3 + 12.490845t^2\right.$$

$$\left. - 21.26872t\right]_0^{12} \approx 206.407\text{ft}$$

Answer 67E.

We are given that

$$\int (\text{sinx} + \text{sinhx})\, d\, x$$

$$\text{sinhx} = \frac{e^x + e^{-x}}{2}$$

Then

$$\int(\text{sinx} + \text{sinhx})\, dx = \int\left(\text{sinx} + \frac{e^x + e^{-x}}{2}\right) dx$$

$$= \int \text{sinx}\, dx + \frac{1}{2}\int\left(e^x + e^{-x}\right) dx$$

$$= -\text{cosx} + \frac{1}{2}\left[e^x - e^{-x}\right] + C$$

$$= -\text{cosx} + \frac{e^x - e^{-x}}{2} + C$$

$$= -\text{cosx} + \text{coshx} + C$$

$$\Rightarrow \int(\text{sinx} + \text{sinhx})\, dx = -\text{cosx} + \text{coshx} + C$$

Where C is any constant.

So, the required solution is

$$\int(\text{sinx} + \text{sinhx})\, dx = -\text{cosx} + \text{coshx} + C$$

Answer 68E.

We are given that

$$\int_{-10}^{10} \frac{2e^x}{\text{sinhx} + \text{coshx}} \cdot dx \ldots\ldots\left(1\right)$$

We know that

$$\text{sinhx} = \frac{e^x + e^{-x}}{2} \text{ and } \text{coshx} = \frac{e^x - e^{-x}}{2}$$

Let

$$f\left(x\right) = \frac{2e^x}{\text{sinhx} + \text{coshx}}$$

$$= \frac{2e^x}{\frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2}}$$

$$= \frac{2e^x}{\frac{e^x + e^{-x} + e^x - e^{-x}}{2}}$$

$$= \frac{2e^x}{\frac{2e^x}{2}}$$

$$= \frac{4e^x}{2e^x}$$

$$= 2$$

Then, we can write

$$\int_{-10}^{10} \frac{2e^x}{\sinh x + \cosh x} \cdot dx = \int_{-10}^{10} 2\, dx$$

$$= 2\int_{-10}^{10} dx$$

$$= 2[x]_{-10}^{10}$$

$$= 2[10 + 10]$$

$$= 2[20]$$

$$= 40$$

$$\int_{-10}^{10} \frac{2e^x}{\sinh x + \cosh x} \cdot dx = 40$$

Therefore the solution is 40.

Answer 69E.

We have to evaluate $\int \left(x^2 + 1 + \frac{1}{x^2+1} \right) dx$

We can rewrite the integral

$$\int \left(x^2 + 1 + \frac{1}{x^2+1} \right) dx = \int x^2 dx + \int 1 dx + \int \frac{1}{x^2+1} dx$$

Us following formula

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C_1 \text{ Where } C_1 \text{ is any constant}$$

$$\int k dx = kx + C_2 \text{ Where } C_2 \text{ and k are constant}$$

$$\int \frac{1}{x^2+1} = \tan^{-1} x + C_3 \text{ Where } C_3 \text{ is any constant}$$

So we have

$$\int \left(x^2 + 1 + \frac{1}{x^2+1} \right) dx = \left(\frac{x^3}{3} + C_1 \right) + \left(x + C_2 \right) + \left(\tan^{-1} x + C_3 \right)$$

$$= \frac{x^3}{3} + x + \tan^{-1} x + \left(C_1 + C_2 + C_3 \right)$$

Since $C_1, C_2 and\, C_3$ are constants so $C_1 + C_2 + C_3 = C$ is also constant

So we have

$$\boxed{\int \left(x^2 + 1 + \frac{1}{x^2+1} \right) dx = \frac{x^3}{3} + x + \tan^{-1} x + C} \text{ Where C is any constant}$$

Answer 70E.

We are given that

$$\int_1^2 \frac{(x-1)^3}{x^2}\, dx$$

Let $f\left(x\right) = \frac{(x-1)^3}{x^2}$

$$f\left(x\right) = \frac{x^3 - 3x^2 + 3x - 1}{x^2}$$

$$= \frac{x^3}{x^2} - \frac{3x^2}{x^2} + \frac{3x}{x^2} - \frac{1}{x^2}$$

$$= x - 3 + \frac{3}{x} - \frac{1}{x^2}$$

$$\Rightarrow f\left(x\right) = x - 3 + \frac{3}{x} - \frac{1}{x^2}$$

By using the formulas

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C_1$$

$$\int \frac{1}{x} \, dx = \ln x + C_2$$

$$\int k \, dx = kx + C_3$$

Where C_1, C_2 and C_3 are constants.

$$\int_1^2 \frac{(x-1)^3}{x^2} \, dx = \int_1^2 \left[x - 3 + \frac{3}{x} - \frac{1}{x^2}\right] dx$$

$$= \int_1^2 x \, dx - 3\int_1^2 1 \, dx + 3\int_1^2 \frac{1}{x} \, dx - \int_1^2 \frac{1}{x^2} \, dx$$

$$= \left[\frac{x^2}{2}\right]_1^2 - 3[x]_1^2 + 3[\ln x]_1^2 - \left[-\frac{1}{x}\right]_1^2$$

$$= \left[\frac{4}{2} - \frac{1}{2}\right] - 3\left[2 - 1\right] + 3\left[\ln 2 - \ln 1\right] + \left[\frac{1}{2} - 1\right]$$

$$= \left[2 - \frac{1}{2}\right] - 3\left[1\right] + 3\left[\ln 2 - 0\right] + \left[\frac{-1}{2}\right]$$

$$= \frac{3}{2} - 3 + 3\ln 2 - \frac{1}{2}$$

$$= \ln 8 - 2$$

$$\int_1^2 \frac{(x-1)^3}{x^2} \, dx = \ln 8 - 2$$

So, the required solution is $\ln 8 - 2$

Answer 71E.

We are given that

$$\int_0^{\frac{1}{\sqrt{3}}} \frac{t^2 - 1}{t^4 - 1} \, dt$$

Let, $f\left(t\right) = \frac{t^2-1}{t^4-1}$

We can write

$$f\left(t\right) = \frac{t^2-1}{\left(t^2+1\right)\left(t^2-1\right)} = \frac{1}{t^2+1}$$

$$\Rightarrow f\left(t\right) = \frac{1}{t^2+1}$$

So,

$$\int_0^{\frac{1}{\sqrt{3}}} \frac{t^2-1}{t^4-1}\, d\,t = \int_0^{\frac{1}{\sqrt{3}}} \frac{1}{t^2+1}\, d\,t$$

By using formula

$$\int \frac{1}{1+x^2}\, d\,x = \tan^{-1} x + C$$

Then

$$\int_0^{\frac{1}{\sqrt{3}}} \frac{1}{t^2+1}\, d\,t = \left[\tan^{-1} t\right]_0^{\frac{1}{\sqrt{3}}}$$

$$= \tan^{-1}\frac{1}{\sqrt{3}} - \tan^{-1} 0$$

$$= \frac{\pi}{6} - 0$$

$$= \frac{\pi}{6}$$

$$\Rightarrow \int_0^{\frac{1}{\sqrt{3}}} \frac{t^2-1}{t^4-1}\, d\,t = \frac{\pi}{6}$$

So, the required solution is $\frac{\pi}{6}$

Answer 72E.

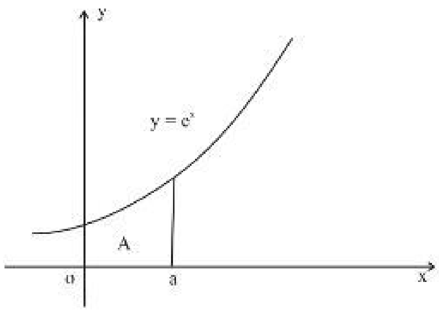


Fig. 1

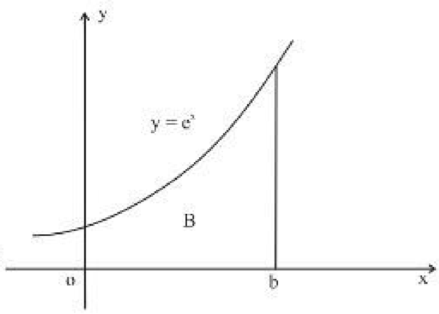


Fig. 2

Given that $B = 3A$ --- (1)

We can find the area A by integrating the function e^x in the interval [0, a]

So $\quad A = \int_0^a e^x dx$

An anti derivative of e^x is e^x, so by the fundamental theorem of calculus part 2, we have $\int_a^b f(x)dx = F(x)\Big]_a^b$ where $F' = f$

So $\quad A = e^x\Big]_0^a$

Or $\quad A = e^a - e^0$ or $\boxed{A = e^a - 1}$ Since $\left(k^0 = 1\right)$

If we integrate the function $y = e^x$ from 0 to b then we can find the area labeled B

So $B = \int_0^b e^x dx$ or $\boxed{B = e^b - 1}$ $\left(k^0 = 1\right)$

Since B = 3A

So $\quad e^b - 1 = 3\left(e^a - 1\right)$

Or $\quad e^b - 1 = 3e^a - 3$

Or $\quad e^b = 3e^a - 2$ taking ln of both sides

$\boxed{b = \ln\left(3e^a - 2\right)}$ Since $\left[\ln e^n = n\right]$