Sample Paper-03
Mathematics
Class – XI

Time allowed: 3 hours **M. M: 100**

General Instructions:

(i) All questions are compulsory.
(ii) This question paper contains 29 questions.
(iii) Question 1- 4 in Section A are very short-answer type questions carrying 1 mark each.
(iv) Question 5-12 in Section B are short-answer type questions carrying 2 marks each.
(v) Question 13-23 in Section C are long-answer-I type questions carrying 4 marks each.
(vi) Question 24-29 in Section D are long-answer-II type questions carrying 6 marks each.

Section A

1. Find the domain of the function $f(x)=\dfrac{1}{\sqrt{2-x^2}}$

2. If $A=\{y=\sin x, 0\le x<\dfrac{\pi}{4}\}$ *and* $B=\{y=\cos x, 0\le x<\dfrac{\pi}{4}\}$ then what is $(A\cap B)$

3. What is the maximum value of a if $a=1-\sin x$

4. Name the locus of points (M), the sum of whose distance from two given points is a constant

Section B

5. Check whether the three points (2, 0), (5, 3), (2, 6) are collinear.

6. Write the condition so that the equation $ax^2+ay^2+bx+cy+d=0$ represents a circle.

7. Solve $\cos 3x=-\dfrac{1}{2}$

8. Prove by mathematical induction that $1+2+3+........+n=\dfrac{n(n+1)}{2}$

9. Find the square root of $\sqrt{-8i}$

10. Solve the inequality $\dfrac{2x+5}{x-2}\ge 3$

11. Find the value of x if ${}^{12}C_x={}^{12}C_{x+4}$

12. Three cars are there in a race. Car A is 3times as likely to win as car B. Car B is twice as likely to win as car C. What is the probability of winning each car.

Section C

13. If $f(x)$ is a function that contains 3 in its domain and range and satisfy the relation

$f(f(x)).(1+f(x))=-f(x)$ find f(3)

14. If $\tan A = \frac{1}{3}$ *and* $\tan B = \frac{1}{2}$ prove that $\sin 2(A+B) = 1$

15. Find two numbers such that their arithmetic mean is 15 and Geometric mean is 9 without using the identity $(a+b)^2 = (a-b)^2 + 4ab$

16. Let $f: R \to R$ be a function given by $f(x) = x^2 + 2$ find $f^{-1}(27)$

17. Find the domain and range of the function $f(x) = \frac{x-a}{a+1-x}$ where a is a positive integer.

18. Find the limit of $\lim_{x \to 0} \frac{\sqrt{a+x} - \sqrt{a}}{x}$

19. Find the sign and value of the expression $\sin 75^0 + \cos 75^0$

20. In how many ways can 3 students from Class 12, 4 from class 11, 4 from class 10 and 2 from class 9 be seated in a row so that those of the same classes sit together. Also find the number of ways they can be arranged in at a round table

21. A circle represented by the equation $(x-a)^2 + (y-b)^2 = r^2$

This makes two complete revolutions along the positive direction of the x axis. Find the equation of the circle in the new position

22. Show that the equation $x^2 + 4y^2 + 4x + 16y + 16 = 0$ represents an ellipse.

23. Calculate the mean deviation about the mean from the following data

x_i	2	15	17	23	27
f_i	12	6	12	9	5

Section D

24. If the ratio of the roots of the equation $x^2 + px + q = 0$ is the same as $x^2 + p_1x + q_1 = 0$ then prove that $p^2q_1 = p_1^2q$

25. Prove that $a.a^{\frac{1}{2}}.a^{\frac{1}{4}}.a^{\frac{1}{8}}\ldots\ldots\infty = a^2$

26. In a survey of 700 students in a medical college 200 went for regular entrance coaching, 295 attended only correspondence coaching, 115 attended both regular and correspondence coaching. Find how many got admission without any entrance coaching.

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Answer

Section A

1. **Solution:**
Domain of is in the open interval $(-2, 2)$
2. **Solution:**
$(A \cap B) = \{\phi\}$
3. **Solution**
Max value is 2
4. **Solution:** Ellipse

Section B

5. **Solution**
Condition for co-linearity is not satisfied here since
$\begin{vmatrix} 2-2 & 0-6 \\ 5-2 & 3-6 \end{vmatrix} = \begin{vmatrix} 0 & -6 \\ 3 & -3 \end{vmatrix} \neq 0$
6. **Solution:**
$b^2 + c^2 - 4ad > 0$
7. **Solution:**

$$\cos 3x = \cos \frac{2\pi}{3}$$

$$3x = 2n\pi \pm \frac{2\pi}{3}$$

$$x = \frac{2n\pi}{3} \pm \frac{2\pi}{9}, n \in Z$$

8. **Solution:**

Let $P(n)$ be the statement given by $1+2+3+........+n = \frac{n(n+1)}{2}$

$$P(1) = \frac{1(1+1)}{2}$$

=1, True
Let it be true for n=m

$$1+2+3+....:.+m = \frac{m(m+1)}{2}$$

$$1+2+3+....:.+m+(m+1) = \frac{m(m+1)}{2} + (m+1)$$

$$P(m+1) = \frac{m(m+1)}{2} + (m+1)$$

$$P(m+1) = \frac{m^2+3m+2}{2}$$

$$P(m+1)=\frac{(m+1)(m+2)}{2}$$

$Thus\ P(m)\ is\ true \Rightarrow P(m+1)\ is\ True$

9. Solution:

Let $\sqrt{z}\ =\ \sqrt{-8i}$

$$\sqrt{z}=\pm\{\frac{\sqrt{|z|-Re(z)}}{\sqrt{2}}\}-i\{\frac{\sqrt{|z|-Re(z)}}{\sqrt{2}}\},\ Im(z)<0$$

$$\sqrt{-8i}=\pm\{\frac{\sqrt{8+0}}{\sqrt{2}}-i\frac{\sqrt{8-0}}{\sqrt{2}}\},\ Im(z)<0$$

$=\pm(2-2i)$

10. Solution

$$\frac{2x+5}{x-2}-3\geq 0$$

$$=\frac{2x+5-3x+6}{x-2}\geq 0$$

$$=\frac{-x+11}{x-2}\geq 0$$

$$=\frac{x-11}{x-2}\leq 0$$

$=(x-11)(x-2)\leq 0$

$x\in(2,11]$

11. Solution

$x+x+4=12$

$2x=8$

$x=4$

12. Solution

Let p be the probability of winning Car C, $P(C)$

P(C) = p

P(B) = 2p

P(A) = 6p

P(A) + P(B) + P(C) = 1

p + 2p + 6p = 1

9p = 1

$$p=\frac{1}{9}$$

$$P(C)=\frac{1}{9}$$

$$P(B)=\frac{2}{9}$$

$$P(A)=\frac{6}{9}$$

Section C

13. Solution :

Let a satisfy the relation $f(a)=3$

$f(f(a)).(1+f(a))=-f(a)$

$f(3).(4)=-3$

$f(3)=-\frac{3}{4}$

14. Solution:

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$=\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}.\frac{1}{2}}$$

$$=1$$

$A+B=45$

$2(A+B)=90$

$\sin 90=1$

15. Solution:

Form a quadratic equation sum of whose roots are 30 and product of the roots is 81

$x^2-x(30)+81=0$

$x^2-3x-27x+81=0$

$x(x-3)-27(x-3)$

$(x-3)(x-27)=0$

Hence the numbers are 3 and 27

16. Solution:

Let $f:R\to R$ be a function given by $f(x)=x^2+2$ find $f^{-1}(27)$

$f(x)=x^2+2$

$x^2+2=27$

$x^2=25$

$x=\pm 5$

$f^{-1}(27)=\{-5,5\}$

17. Solution:

The function is defined for all values of x where the denominator is not equal to zero

$a+1-x\neq 0$

Hence domain =

$R-\{(a+1)\}$

Range of f

Let $y=f(x)$

$$y=\frac{x-a}{a+1-x}$$

$(a+1)y-xy=x-a$

$x(y+1)=(a+1)y+a$

$$x=\frac{(a+1)y+2}{y+1}$$

Range of $f = R-\{-1\}$

18. Solution

Rationalize the numerator

$$\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a}}{x}$$

$$=\lim_{x\to 0}\frac{(\sqrt{a+x}-\sqrt{a})(\sqrt{a+x}+\sqrt{a})}{x(\sqrt{a+x}+\sqrt{a})}$$

$$=\lim_{x\to 0}\frac{x}{x(\sqrt{a+x}+\sqrt{a})}$$

$$=\frac{1}{2\sqrt{a}}$$

19. Solution:

$$\sin 75^0+\cos 75^0$$

$$=\sqrt{2}(\frac{1}{\sqrt{2}}\sin 75^0+\frac{1}{\sqrt{2}}\cos 75^0$$

$$=\sqrt{2}(\cos 45^0 \sin 75^0+\sin 45^0\cos 75^0)$$

$$=\sqrt{2}\sin(75^0+45^0)$$

$$=\sqrt{2}\sin 120^0$$

Hence sign is positive and value is $\frac{\sqrt{2}.\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

20.Solution:

There are 4 groups and four groups can be arranged in $4!$ ways. Class 12 can be arranged in $3!$ ways , Class 11 can be arranged in $4!$ Class 10 can be arranged in $4!$. Class 9 can be arranged in $2!$ ways

Hence Total number of ways that they can be arranged in a row $4!\times 3!\times 4!\times 4!\times 2!=165888$

In a circular seating arrangement the four groups can be arranged only in $3!$ ways only.

Hence the total number of ways that they can be seated at a round table =
$3!\times 3!\times 4!\times 4!\times 2!=41472$

21. Solution

The new coordinates of the centre in the new position are
$(a+4\pi r\ ,b)$

$$\{x-(a+4\pi r)\}^2+(y-b)^2=r^2$$

22. Solution

$$x^2+4y^2+4x+16y+16=0$$

$$x^2+4x+4+4y^2+16y+16=4$$

$$(x+2)^2+4(y+2)^2=4$$

$$\frac{(x+2)^2}{2^2}+\frac{(y+2)^2}{1^2}=1$$

This equation represents an ellipse.

23. Solution

x_i	f_i	$f_i\,x_i$	$\lvert x_i-15\rvert$	$f_i\lvert x_i-15\rvert$
2	12	24	13	156
15	6	90	0	0
17	12	204	2	24

23	9	207	8	72
27	5	135	12	60
	$N=\Sigma f_i=44$	$\Sigma f_i\ x_i=660$		$f_i\Sigma\lvert x_i-15\rvert=312$

$$Mean=\overline{X}=\frac{1}{N}(\Sigma f_i\ x_i)=\frac{660}{44}=15$$

$$MeanDeviation=M.D=\frac{1}{N}(\Sigma f_i\ \lvert x_i-15\rvert)=\frac{312}{44}=7.0909$$

Section D

24. Solution

Let the ratios be

$a\ :\ b$

$x^2+px+q=0$

$a\alpha+b\alpha=-p$

$a\beta+b\beta=-p_1$

$a\alpha\times b\alpha=q$

$a\beta\times b\beta=q_1$

$(a+b)\alpha=-p$

$(a+b)\beta=-p_1$

$ab\alpha^2=q$

$ab\beta^2=q_1$

$$\frac{(a+b)^2\alpha^2}{(a+b)^2\beta^2}=\frac{p^2}{p_1^2}$$

$$\frac{\alpha^2}{\beta^2}=\frac{p^2}{p_1^2}$$

$$\frac{\alpha^2}{\beta^2}=\frac{q}{q_1}$$

$$\frac{p^2}{p_1^2}=\frac{q}{q_1}$$

$p^2q_1=p_1^2q$

25. Solution :

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\ldots\infty=\frac{1}{1-\frac{1}{2}}=\frac{1}{\frac{1}{2}}=2$$

$$a.a^{\frac{1}{2}}.a^{\frac{1}{4}}.a^{\frac{1}{8}}\ldots\ldots\infty=a^2$$

26. Solution

It is given that

$n(U)=700\ ,n(A)=200\ ,n(B)=295\ ,n(A\cap B)=115$

We need to find out

$n(A'\cap B')$

$n(A' \cap B') = n(A \cup B)'$

$= n(U) - n(A \cup B)$

$= n(U) - \{n(A) + n(B) - n(A \cap B)\}$

= 700 – {200 + 295 - 115} = 320