SYMMETRY

7
CHAPTER

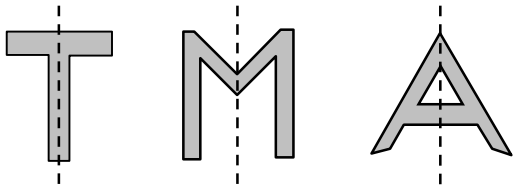
CONTENTS

➢ INTRODUCTION

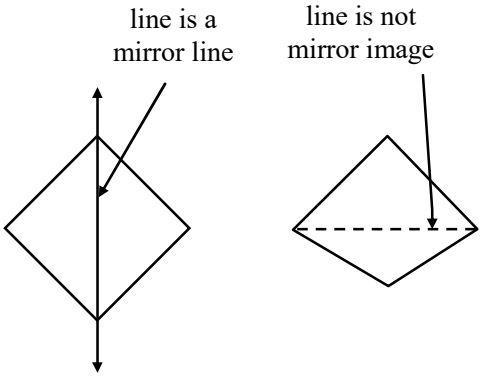
Reflectional symmetry of a figure is always about a line called the line or axis of symmetry. There is another form of symmetry called rotational symmetry. Symmetry abounds in many naturally occurring objects for example, if we start from our body we can see our hands (length), legs (length) with evenly balanced proportion, we say, ‘‘they are symmetrical’’. Other naturally occurring objects involving the concept of symmetry are : The flowers, the tree leaves etc. In the field of engineering, concept of symmetry is used, in making cars, buses etc.

➢ LINE SYMMETRY

A figure has a line symmetry if there is a line about which the figure may be folded so that the two parts of the figure will coincide. All objects and all alphabets below figure have line symmetry.



The concept of line symmetry is closely related to mirror reflection.



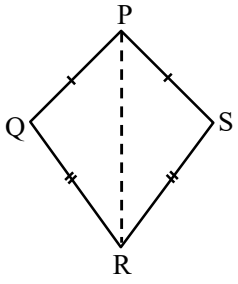
Note :

If one half completely covers the other, we say that the line is the line of symmetry otherwise not.

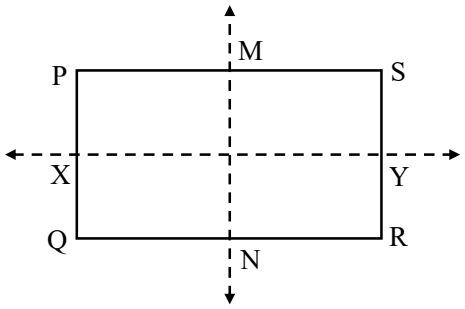
❖ EXAMPLES ❖

Ex.1 Let PQRS be a kite in which PQ = PS and QR = SR.

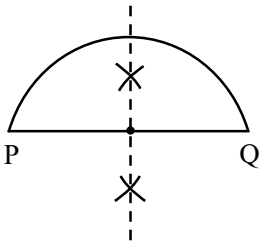
Then kite PQRS is symmetrical about diagonal PR.



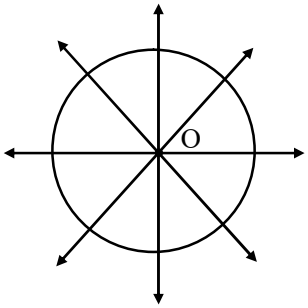
Ex.2 A rectangle has two lines of symmetry, each one of which is the line joining the mid point of opposite sides.



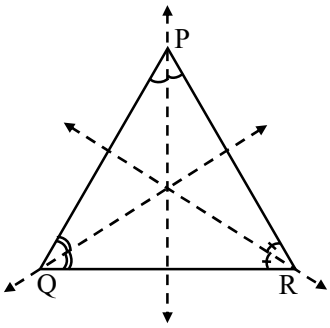
Ex.3 A semicircle has one line of symmetry, namely the perpendicular bisector of diameter PQ.



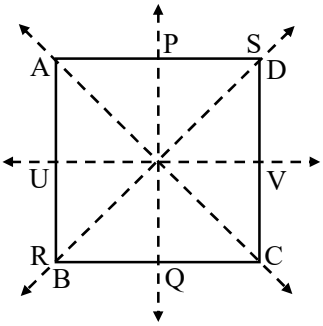
Ex.4 A circle is symmetrical about each one of its diameters.



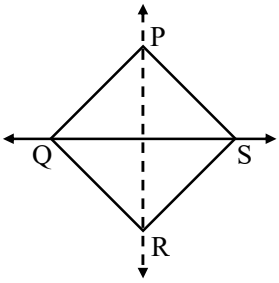
Ex.5 An equilateral triangle is symmetrical about each one of the bisectors of its interior angles.



Ex.6 A square has 4 lines of symmetry, namely the diagonals and lines joining the mid-points of its opposite sides.



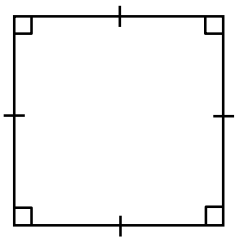
Ex.7 A rhombus is symmetrical about each of its diagonals.



➢ LINE OF SYMMETRY IN REGULAR POLYGONS

◈ Regular Polygon

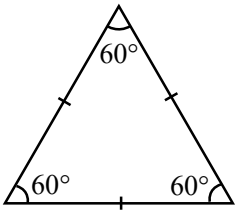
A closed figure bounded by straight line segments of equal length is called a **Regular Polygon.** For example, a square is a regular quadrilateral figure.



Let us now discuss a method of finding lines of symmetry of a regular polygon.

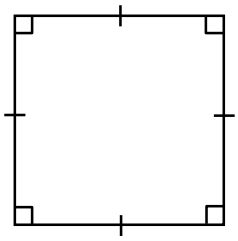
1. An Equilateral Triangle

A triangle whose all sides are of same length and each of its angles measures 60°, is called an equilateral triangle shown in figure.



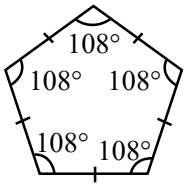
2. A Square

We have already discussed that a square is also a regular polygon because its all sides are of equal length and each of its angles is a right angle (of measure 90°) as in figure.



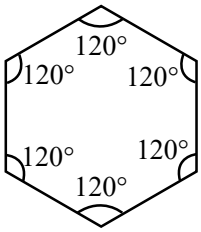
3. A Regular Pentagon

A figure bounded by five sides of equal length is called a regular pentagon. Each angle of a regular pentagon is 108°.

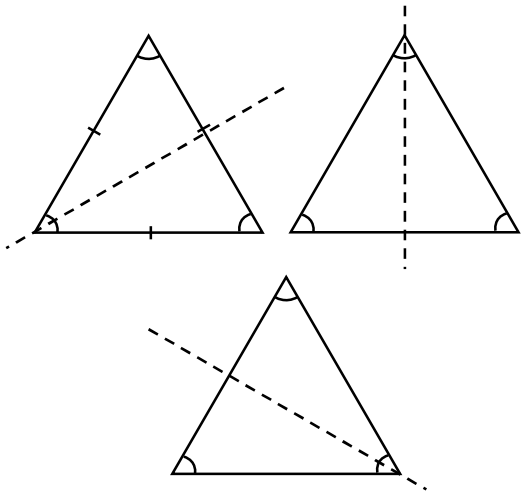


4. A Regular Hexagon

A figure bounded by six sides of equal length is called a regular hexagon. Each of its angle measures 120°.

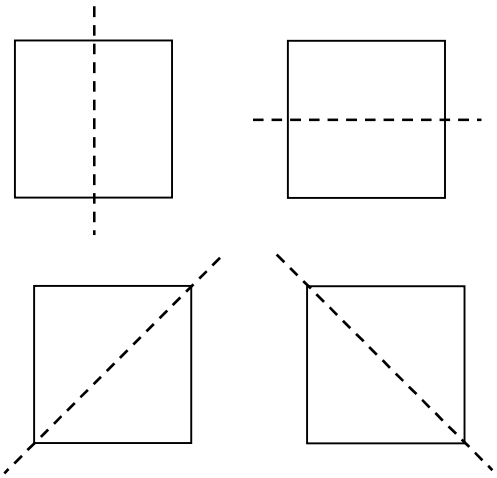


Trace an equilateral triangle on a tracing paper and find its lines of symmetry by folding.



there are three lines of symmetry for an equilateral triangle.

Draw a square on a tracing paper.

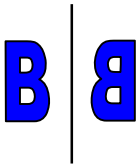


A square has four lines of symmetry.

Note :

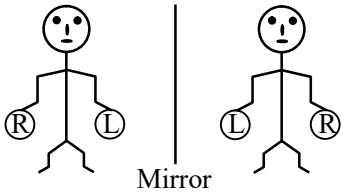
(1) We can also find the line of symmetry of a figure by putting a mirror. When the half part of a figure becomes same as its mirror image, we call the line along with mirror as the line of symmetry.

(2) In mirror reflection, the orientation of the figure changes into left-right orientation as shown in figure



Mirror

(i)



Mirror

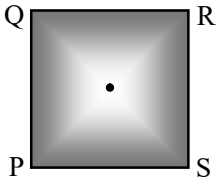
(ii)

➢ ROTATIONAL SYMMETRY

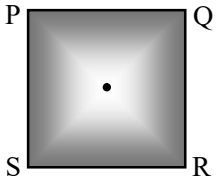
If a shape can fit exactly into itself after a certain rotation (not a full one) about a fixed point (called centre of rotation) then it is said to have rotational symmetry.

The angle of turning during rotation is called the angle of rotation.

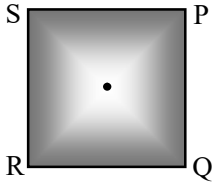
For example, a square has a rotational symmetry.



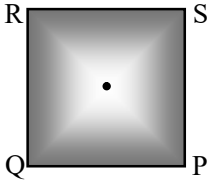
Original position



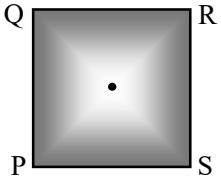
Rotated through 90°



Rotated through 180°



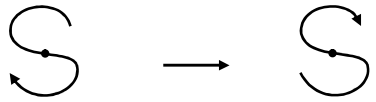
Rotated through 270°



Rotated through 360°

◈ Point Symmetry

Some figures appear to be in the same position when rotated through half turn (180°) about a fixed point, they are said to have point symmetry.



Original position Rotated through 180°

Note :

1. A square has line symmetry as well as rotational symmetry.
2. An equilateral triangle has rotational symmetry as well as line symmetry
3. A full rotation does not mean that a figure has rotational symmetry as every shape could fit exactly into itself after a full rotation.

◈ Order of Rotational Symmetry

The number of times a shape fits onto itself in one complete turn is called the order of rotational symmetry.

or

Thus, we say that the order of the rotational symmetry of a figure with a point marked (say P), is the number of rotations about a fixed point required to bring back the marked point P to its original position.

For example, an equilateral triangle has rotational symmetry of order 3 as there are three positions where it appears not to have moved.

Note :

The figures which do not have rotational symmetry are said to have rotational symmetry of order 1.

◈ Direction of Rotation

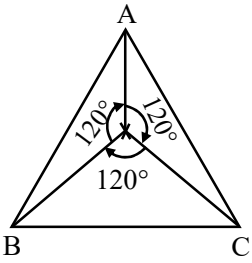
The body moving in the direction of hands of clock is said to have **clockwise rotation** and if it moves in reverse direction of hands of clock is said to have **anticlock rotation.**

Thus, we can find the order of rotation of a figure by dividing 360° by the measure of the angle rotated by the original figure when it looks just the same as before.

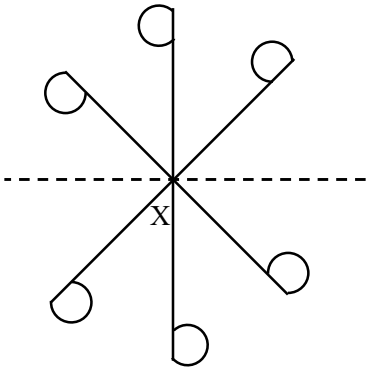
For example, for an equilateral triangle ABC, when it is rotated about point X, will take the same shape after a rotation of angle 120° as in figure.

Thus, order of rotational symmetry = $\frac{360°}{120°} = 3$.

Note that we found the same order when we rotated figure



Ex.8 Find the order of rotational symmetry about point X of the following figure:



Sol. Draw a dotted line passing through the point X.

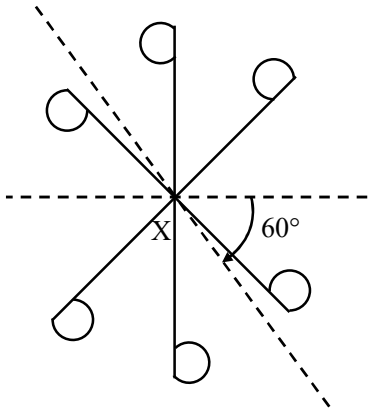
Make a copy of the figure on a tracing paper.

Put the traced copy on the figure and pin up by a thumb-tack at point X.

Rotate the figure (clockwise or anticlockwise). Stop when the traced copy completely overlaps the original figure first time during rotation.

Now put a protractor with its zero at X and measure the angle formed by the dotted lines (one line on the figure and other on the traced copy).

We find that the measure of the angle thus formed is 60° as shown in



Similarly, again rotate about X and measure the angle formed between the two dotted lines, when the original figure and the traced copy again look just the same.

We find that the measure of the angle is 120°.

i.e., we get the same figure after a rotation of 120° – 60° = (60°) about X.

And this will happen six times upto a complete rotation.

Therefore, the order of rotational symmetry of the figure about point X is 6.

Also, angle of a complete rotation = 360°

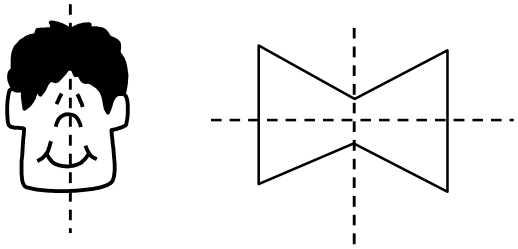
Angle after first overlapping of the original figure and the traced copy = 60°

and $\frac{360°}{60°} = 6 =$ Rotational order.

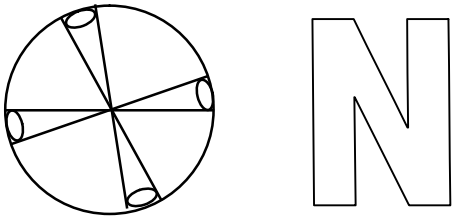
Note :

Some shapes have both lines (line symmetry & Rotational symmetry)

Eg.



(i) (ii)



(iii) (iv)

Fig. No.	Line Symmetry	Number of Lines of Symmetry	Rotational Symmetry	Order of Rotational Symmetry
(i)	Yes	1	Yes	1
(ii)	Yes	2	Yes	2
(iii)	Yes	4	Yes	4
(iv)	No	0	Yes	2

IMPORTANT POINTS TO BE REMEMBERED

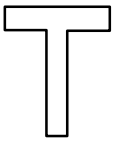
1. A figure is said to have line symmetry if there is at least one line about which when the figure is folded, the two parts of the figure coincide.
2. A polygon having equal sides and equal angles is called a regular polygon.
3. A regular polygon has as many lines of symmetry as it has sides.
4. Each regular polygon has as many lines of symmetry as it has sides

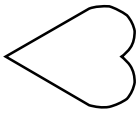
Regular Polygon	Number of lines of Symmetry
Regular hexagon	6
Regular pentagon	5
Square	4
Equilateral triangle	3

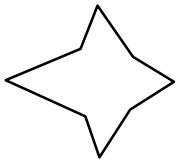
5. If, after a rotation, an object looks exactly the same, we say that it has a rotational symmetry.
6. In a complete turn, the number of times an object looks exactly the same is called the order of rotational symmetry.
7. Some figures have only one line symmetry, some have only rotational symmetry and some have both.
8. Every object has a rotational symmetry of order 1, as it occupies same position after a rotation of 360° (one complete revolution).

EXERCISE

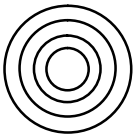
Q.1 Which of the following have reflection symmetry? Also find the lines of symmetry, if any.

(i) 

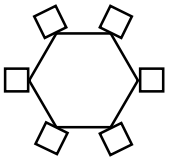
(ii) 

(iii) 

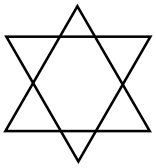
(iv) 

(v) 

Q.2 Draw all lines of symmetry in the following figures :

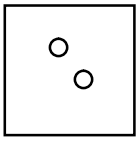
(i) 

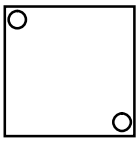
(ii) 

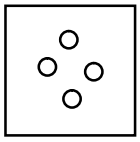
(iii) 

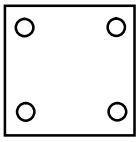
(iv) 

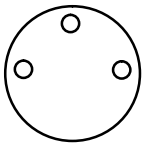
Q.3 Take paper sheets and a punching machine. Fold and punch so as to obtain each of the following figures. Then find the axis of symmetry for each of the following :

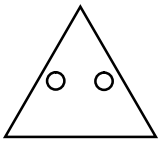
(i) 

(ii) 

(iii) 

(iv) 

(v) 

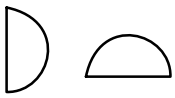
(vi) 

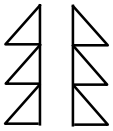
Q.4 Give any 5 examples of shapes with no line of symmetry.

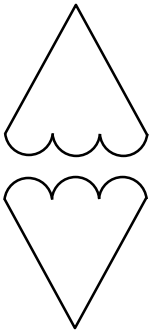
Q.5 Find the number of lines of symmetry for the following :

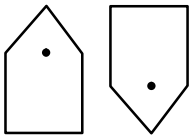
(i) A circle
(ii) A square
(iii) A rhombus
(iv) An equilateral triangle
(v) An isosceles triangle
(vi) A rectangle
(vii) A parallelogram
(viii) A regular octagon.

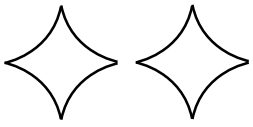
Q.6 Two symmetrical figures are given in each of the following. State which of them are reflection and which are rotation.

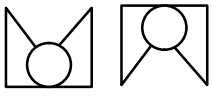
(i) 

(ii) 

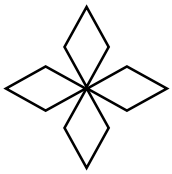
(iii) 

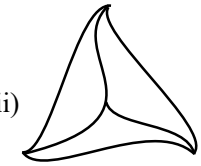
(iv) 

(v) 

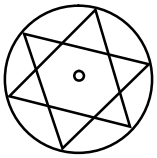
(vi) 

Q.7 Which of the following figures have rotational symmetry of order more than 1

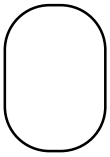
(i) 

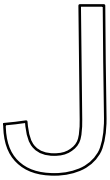
(ii) 

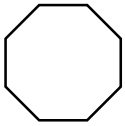
(iii) 

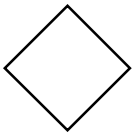
(iv) 

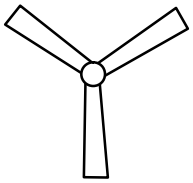
Q.8 Find the order of rotational symmetry of the following.

(i) 

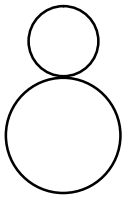
(ii) 

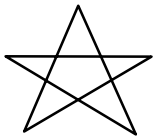
(iii) 

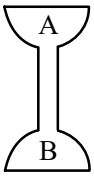
(iv) 

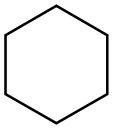
(v) 

Q.9 Write down the order of rotational symmetry and also the number of lines of symmetry (where possible)

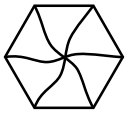
(i) 

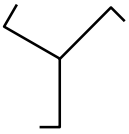
(ii) 

(iii) 

(iv) 

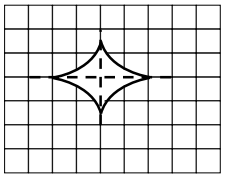
Q.10 Make up three shapes that have rotational symmetry only. Give the order of

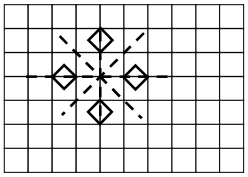
(i) 

(ii) 

(iii) 

(iv) 

(v) 

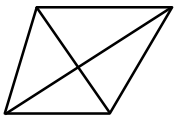
(vi) 

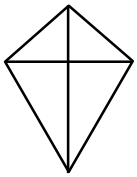
Q.11 Which of the following shapes have :

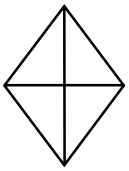
(a) rotational symmetry of order greater than 1.

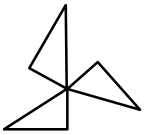
(b) line symmetry

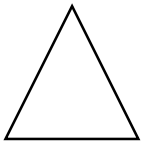
(c) both the rotational symmetry of order greater than 1 and the line symmetry.

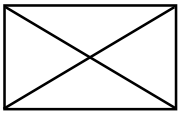
(i) 

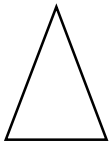
(ii) 

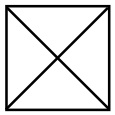
(iii) 

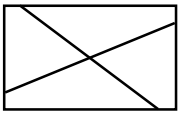
(iv) 

(v) 

(vi) 

(vii) 

(viii) 

(ix) 

Q.12 Can we have a rotational symmetry of order more than 1 whose angle of rotation is :
(i) 45°
(ii) 17°

Q.13 Name the quadrilateral which have both line and rotational symmetry of order more than 1.

ANSWER KEY

1. Reflection symmetry : (ii), (iii) & (iv) ; Line symmetry : (i), (ii), (iii), (v)

5. (i) Infinite lines (ii) Four lines (iii) Two lines (iv) Three lines (v) One line (vi) Two lines
(vii) No line of symmetry in case of parallelogram (viii) Eight lines

6. (i) Rotation (ii) Reflection (iii) Reflection (iv) Rotation (v) Rotation (vi) Rotation

7. (i) Order 4 (ii) Order 3 (iv) Order 6

8. (i) 2 (ii) 1 (iii) 8 (iv) 4 (v) 3

9. (i) Order = 1, Lines of symmetry = 1 (ii) Order = 5, Lines of symmetry = 3
(iii) Order = 1, Lines of symmetry = no (Θ B is not in symmetrical)
(iv) Order = 6, Lines of symmetry = 6

11. (a) (i) No (ii) No (iii) Yes, 4 (iv) Yes, 3 (v) Yes, 3 (vi) Yes, 2 (vii) No
(viii) Yes, 4 (ix) Yes, 2
(b) (i) No (ii) Yes, 1 (iii) Yes, 4 (iv) No (v) Yes, 3 (vi) Yes, 2 (vii) Yes, 1
(viii) Yes, 4 (ix) No
(c) (iii), (v), (vi), (viii)

12. (i) Yes (ii) No

13. Square.