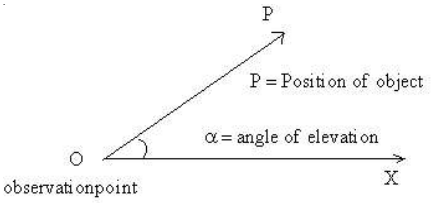
HEIGHTS AND DISTANCES

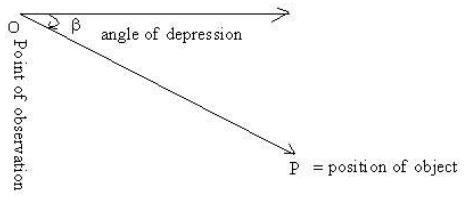
ANGLE OF ELEVATION

- If the position of the object is above the position of the observation then the angle made by the line joining object and observation point with the horizontal line drawn at the observation point is called angle of elevation.



ANGLE OF DEPRESSION:

- If the position of the object is below the position of the observation the angle made by the line joining object and observation point with the horizontal line drawn at the observation point is called angle of depression.

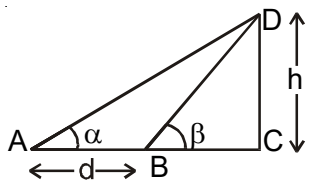


- a. The angle of elevation of the top of a tower, standing on a horizontal plane, from a point A is α. After walking a distance 'd' metres towards the foot of the tower, the angle of elevation is found to be β.

The height of the tower h = $\frac{d\sin\beta\sin\alpha}{\text{Sin}(\beta-\alpha)}$

(or) $h=\frac{d}{\text{Cot}\alpha-\cot\beta}$

Where $\overline{AB}=d$

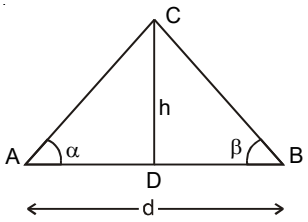


- If the Points of observation A and B lie on either side of the tower, then height of the tower

h = $\frac{d\sin\alpha\sin\beta}{\text{Sin}(\alpha+\beta)}$

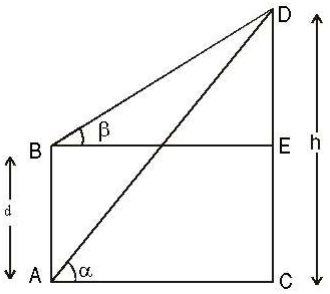
Where $\overline{AB}=d$

(or) $h=\frac{d}{\text{Cot}\alpha+\cot\beta}$



- The angles of elevation of the top of a tower from the bottom and top of a building of height 'd' metres are β and α respectively. The height of the tower is

h = $\frac{d\sin\beta\cos\alpha}{\sin(\beta-\alpha)}$ metres (or) $h=\frac{d\cot\alpha}{\cot\alpha-\cot\beta}$

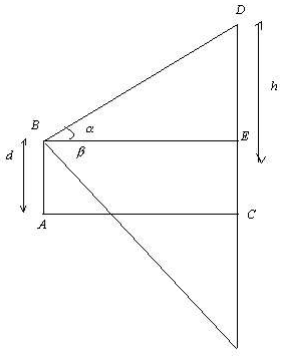


- The angle of elevation of a cloud from a height 'd' metres above the level of water in a lake is 'α' and the anlge of depression of its image in the lake is β. The height of the cloud from the water level in metres is

$h=\frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$ **(or)**

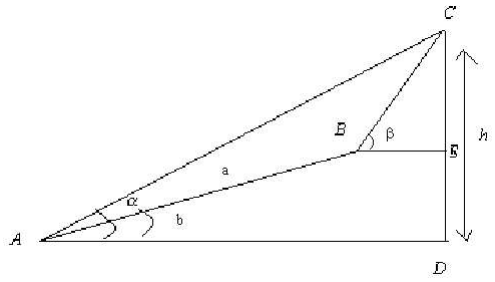
$h=\left[\frac{d(\tan\beta+\tan\alpha)}{(\tan\beta-\tan\alpha)}\right]$ **(or)**

$h=d\left[\frac{\text{Cot}\alpha+\text{Cot}\beta}{\text{Cot}\alpha-\text{Cot}\beta}\right]$



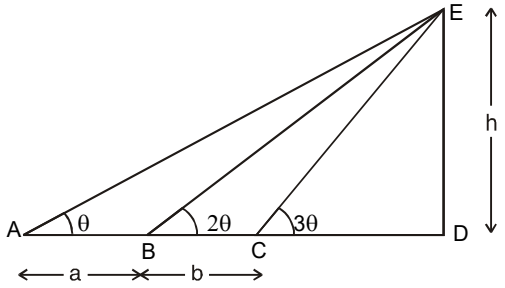
- The angle of elevation of a hill from a point A is $'\alpha'$. After walking to some point B at a distance 'a' metres from A on a slope inclined at $'\gamma'$ to the horizon, the angle of elevation was found to be β.

Height of the hill $h = \dfrac{a \operatorname{Sin}\alpha \operatorname{Sin}(\beta-\gamma)}{\operatorname{Sin}(\beta-\alpha)}$



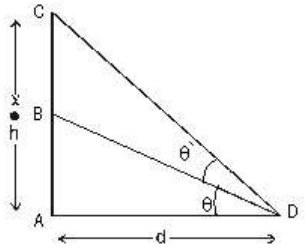
- A balloon is observed simultaneously from the three points A, B, C on a straight road directly beneath it. The angular elevation at B is twice that at A and the angular elevation at 'C' is thrice that at A. If AB=a and BC=b then the height of the balloon h in terms of a and b is,

$$h = \frac{a}{2b}\cdot\sqrt{(3b-a)(a+b)}$$



- A flag staff stands on the top of a tower of height h metres. If the tower and flag staff subtend equal angles at a distance 'd' metres from the foot of the tower, then the height the flag - staff in metres is

$$h\left[\frac{d^2+h^2}{d^2-h^2}\right]$$



MULTIPLE CHOICE QUESTIONS

LEVEL - 1

1. The angle of elevation of the top of a flag - staff when observed from a point, distant 60 meteres from its foot, is 30^0 the height of the flag-staff in metres is
   1) $20\sqrt{3}$ 2) $10\sqrt{3}$ 3) $60\sqrt{3}$ 4) $30\sqrt{3}$
2. A kite is flying with the string inclined at 30^0 to the horizon. The height of the kite above the ground, when the string is 15 mt long is
   1) 15 mt 2) 30 mt
   3) 15/2 mt 4) $15\sqrt{3}$ mt
3. If the shadow of a tower is $\sqrt{3}$ times of its height, the altitude of the sun is
   1) 15^0 2) 30^0 3) 45^0 4) 60^0
4. From the top of a cliff 24 mt height, a man observes the angle of depresion of a boat is to be 60^0. The distance of the boat from the foot of the cliff is
   1) $8\sqrt{3}$ mt 2) $8\sqrt{2}$ mt
   3) $8\sqrt{5}$ mt 4) 8 mt
5. From the top of the tree, a man observes the angle of depression of a point which is at a distance of 40 mt from the foot is 75^0. The height of the tree is
   1) $40\sqrt{3}$ mt 2) $40(2+\sqrt{3})$ mt
   3) $21\sqrt{3}$ mt 4) $3\sqrt{21}$ mt
6. The tops of two poles of heights 18 metres and 12 metres are connected by a rope. If the rope makes an angle 30^0 with the horizontal, the length of the rope in metres is
   1) 12 2) 18 3) 24 4) 30
7. The upper part of a tree broken over by the wind makes an angle of 60^0 with the ground and touches the ground at a distance of 50 metres from the foot. The height of the tree in metres is
   1) 124.2 2) 186.6 3) 243.2 4) 164.2
8. A person walking 50 metres towards a chimney in a horizontal line, through its base observes that its angle of elevation of its top changes from 30^0 to 45^0. Height of the chimney in metres is
   1) $25(3+\sqrt{3})$ 2) $50(\sqrt{3}+1)$
   3) $25(\sqrt{3}+1)$ 4) $25(\sqrt{3}-1)$
9. The angle of elevation of an electric pole from a point A on the ground is 60^0 and from a Point B towards the pole on the line joining the foot of the pole to the point A, is 75^0. If the distance AB = a, then the height of the pole is
   1) $\frac{a(3+2\sqrt{3})}{2}$ 2) $a(4+2\sqrt{3})$
   3) $\dfrac{a(2+\sqrt{3})}{2}$ 4) $\dfrac{a(2\sqrt{3}-3)}{2}$
10. At a certain point the angle of elevation of a tower is found to be $\operatorname{Cot}^{-1}(3/5)$. On walking 32 metres directly towards the tower its angle of elevation is $\operatorname{Cot}^{-1}(2/5)$. The height of the tower in metres is
    1) 32 2) 160 3) 320 4) 340

11. From the top of a cliff 200 metres above the Sea-level the angles of depression of two boats, in the same vertical plane as the observer, are 45^0 and 30^0 respectively. The distance between the boats in metres is

1) $200(\sqrt{3}+1)$ 2) $200(\sqrt{3}-1)$

3) $400(\sqrt{3}-1)$ 4) $400(\sqrt{3}+1)$

12. A person standing one the bank of a river observes that the angle subtended by a tree on the opposite bank is 60^0. When he retired 40 metres from the bank perpendicular to it. He finds the angle to be 30^0. The breadth of river and the height of the tree in metres are

1) 18, $18\sqrt{3}$ 2) 22, $22\sqrt{3}$

3) 20, $20\sqrt{3}$ 4) 12, $12\sqrt{3}$

13. The shadow of a tower on a level plane is found to be 60 metres longer when the sun's altitude is 30^0 than that when it is 45^0. The height of the tower in metres is

1) $30(\sqrt{3}+1)$ 2) $30(\sqrt{3}-1)$

3) $30(3+\sqrt{3})$ 4) $30(3-\sqrt{3})$

14. At a point A, the angle of a tower is found to be $\tan^{-1}(5/12)$ on walking 240 metres nearer the tower, the elevation is found to be $\tan^{-1}(3/4)$. The height of the tower in metres is

1) 175 2) 225 3) 275 4) 300

15. The angle of elevation of the top of an unfinished tower at a point distant 120 metres from its base is 45^0. The height of the tower that must be raised so that the angle of elevation at the same point may be 60^0 is

1) $120(\sqrt{3}-1)$m 2) $120(\sqrt{3}+1)$m

3) $120\sqrt{3}$ m 4) $140\sqrt{3}$ m

16. From the top of a tower 80 metres high, the angles of depression of two points P and Q in the same vertical plane with the tower are 45^0 and 75^0 respectively, PQ =

1) $80(\sqrt{3}+1)$mts 2) $80(\sqrt{3}-1)$mts

3) $160(\sqrt{3}+1)$mts 4) $160(\sqrt{3}-1)$mts

17. The angle of elevation of the top of a tower from two points distant a and b from the bottom and in the same stright line with it are complementary. The height of the tower is

1) $\sqrt{\frac{a}{b}}$ 2) $\sqrt{ab}$ 3) ab 4) $\frac{a}{b}$

18. The angles of elevation of top of a pole from two points A and B on the horizontal line lying on either side of the pole are observed to be 30^0 and 60^0. If AB = 100 m then height of the pole is

1) $20\sqrt{3}$ m 2) $15\sqrt{3}$ m

3) $10\sqrt{3}$ m 4) $25\sqrt{3}$ m

19. From the top of a light house of height 200 metres the angles of depression of two Ships on either side of it are observed to be 30^0 and 45^0 then the distance between the ships in metres is

1) $100(\sqrt{2}+1)$ 2) $100(\sqrt{2}-1)$

3) $100(\sqrt{3}+1)$ 4) $200(\sqrt{3}+1)$

20. A and B are two stations due north and south of a tower of height 25 m. The angles of depression of the stations from the top of the tower are observed to be 30^0 and 45^0 respectively. The distance between the two stations is

1) $25(\sqrt{3}+1)$ 2) $25(\sqrt{3}-1)$ m

3) $25\sqrt{3}$ m 4) $25(2+\sqrt{3})$m

21. Two towers are standing on a level ground. From a point on the ground mid-way between them, the angles of elevation of their tops are 60^0 and 30^0 respectively. If the height of the first tower is 100 metres, the height of the second tower is ---

1) 5/3 2) 100/3 3) 80/3 4) 135/3

22. Two pillars of equal height stand at a distance of 100 metres. At a point between them the elevation of their tops are found to be 30^0 and 60^0. Then height of the each pillar in metres is

1) $25\sqrt{3}$ 2) $20\sqrt{3}$ 3) $50\sqrt{3}$ 4) $35\sqrt{3}$

23. Two pillars are 120 ft apart and the height of one is double that of the other. From the middle point of the line joining their feet, an observer finds that the angular elevations of their tops are complementary. The height of the longer tower is

1) $35\sqrt{2}$ mt 2) $60\sqrt{2}$ mt

3) $50\sqrt{2}$ mt 4) $40\sqrt{2}$ mt

24. A flag staff of height 10 metres is placed on the top of a tower of height 30 metres. At the top of a tower of height 40 metres, the flag staff and the tower subtend equal angles then the distance between the two towers in metres is

1) $10\sqrt{2}$ 2) $20\sqrt{2}$ 3) $30\sqrt{2}$ 4) $40\sqrt{2}$

25. An aeroplane flying at a height of 300 meters above the ground passes vertically above another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 60^0 and 45^0 respectively. Then the height of the lower plane from the ground, in meters is

1) $100\sqrt{3}$ 2) $\frac{100}{\sqrt{3}}$

3) 50 4) $150\left(\sqrt{3}+1\right)$

26. A man observes the elevation of a ballon to be 30^0 at a point A. He then walks towards the balloon and at a certain place B, find the elevation to be 60^0. He further walks in the direction of the balloon and finds it to be directly over him at a height of 1/2 km then AB =

1) $\frac{1}{\sqrt{2}}$km 2) $\frac{1}{\sqrt{3}}$km

3) $\frac{1}{\sqrt{4}}$km 4) $\frac{1}{\sqrt{5}}$km

27. A flag-staff stands on a tower and the angles of elevation of the top and bottom of the flag-staff, from a point on the ground are 75^0 and 45^0 respectively. The ratio between the height of the tower and flag-staff is

1) $\sqrt{3}+1:1$ 2) $\sqrt{3}-1:2$

3) $\sqrt{3}-1:1$ 4) $\sqrt{3}:1$

28. A flag staff stands upon the top of a building. At a distance of 40 m. the angles of elevation of the tops of the flag staff and building are 60^0 and 30^0 then the height of the flag staff in metres is

1) $40\sqrt{3}$ 2) $\frac{40}{\sqrt{3}}$ 3) $\frac{160}{\sqrt{3}}$ 4) $\frac{80}{\sqrt{3}}$

29. If the angle of elevation of the top of a tower from a point is 60^0 and 40 metres vertically above this point the angle of elevation is 45^0. The height of the tower in metres is

1) 64.64 2) 94.64 3) 54.64 4) 74.64

30. The angles of depression of the foot and the top of a pole at the top of a tower of height 100 metres are 45^0 and 30^0 respectively. The height of the pole is

1) $\frac{100(3+\sqrt{3})}{2}$ 2) $\frac{100(3-\sqrt{3})}{3}$

3) $100(\sqrt{3}+1)$ 4) $100(\sqrt{3}-1)$

31. At a particular instant the height of the tower is equal to the length of its shadow after some time the length of the shadow is $\sqrt{3}$ times of the height of hte otwer, then the time lapsed between the two observations in hours is

1. 1 2. $\frac{1}{2}$ 3. $\frac{1}{4}$ 4. 24

KEY

01. 1	02. 3	03. 2	04. 1	05. 2
06. 1	07. 2	08. 3	09.1	10. 2
11. 2	12. 3	13. 1	14. 2	15. 1
16. 2	17. 2	18. 4	19. 4	20. 1
21. 2	22. 1	23. 2	24. 1	25. 1
26.2	27.2.	28.4	29.2	30.2
31.1				

HINTS

1. $\Rightarrow \text{Tan } 30^0 = \frac{h}{60}$

find h.

2. $\text{Sin } 30^0 = \frac{h}{15}$ find h

3. $\text{Tan } \theta = \frac{h}{\sqrt{3}h}$ Find h

4. $\text{Tan } \theta = \frac{24}{d}$ find d

$\text{Tan } 60^0 = \frac{24}{d}$ Find d

5. $\text{Tan } 75^0 = \frac{h}{40}$

find h

6. $\text{Sin } 30^0 = \frac{6}{x}$

find x

7. $\text{Tan } 60^0 = \frac{h_1}{50^0}$

$h_1 = 50 \text{ Tan } 60^0$

$\text{Cos } 60^0 = \frac{50}{h_2}$

$h_2 = \frac{50}{\cos 60}$

Tree = $h_1 + h_2$ -----

8. $\text{Tan } 30^0 = \frac{x}{x+50}$

find x

9. $\alpha = 60^0 \quad \beta = 75^0$

d = a

Apply

$h = \frac{d}{\cot\alpha - \cot\beta}$

find h

10. $\alpha = \text{Cot}^{-1}\frac{3}{5}$

$\text{Cot } \alpha = \frac{3}{5}$

$\beta = \text{Cot}^{-1}\frac{2}{5}$

find h

Apply $h = \frac{d}{\cot\alpha - \cot\beta}$

11. $Tan\ 30^0 = \frac{200}{200+x}$

find x

12. $\therefore h = \dfrac{d}{\cot\alpha - \cot\beta} \rightarrow (1)$

$\alpha = 30^0,\ \beta = 60^0 \qquad d = 40m$

$\therefore \text{Tan}60^0 = \dfrac{h}{x} \Rightarrow h = x\tan 60^0$

(sub 'h' value from (1)) find x

13. $\text{Tan}\,30^0 = \dfrac{x}{x+60}$ find x

14. $\alpha = \text{Tan}^{-1}\,5/12$

$\beta = \text{Tan}^{-1}\,3/4$

d = 240 find h

Apply h = $\dfrac{d}{\cot\alpha - \cot\beta}$

15. $\text{Tan}\,60^0 = \dfrac{h+120}{120}$

find h

16. h = 80^0

$\alpha = 45$

$\beta = 75^0$

find d

h = $\dfrac{d}{\cot\alpha - \cot\beta}$

17. Tan $\alpha = h/a \rightarrow (1)$

$\text{Tan}(90-\alpha) = h/b \rightarrow (2)$

(1) x (2)

find h

18. $\alpha = 60$

$\beta = 30^0$

d = 100

$h = \dfrac{d}{\cot\alpha + \cot\beta}$

find h

19. $h = 200$

$\alpha = 30$

$\beta = 45^0$

$h = \dfrac{d}{\cot\alpha + \cot\beta}$

find 'd'

20. h = 25

$\alpha = 30^0$

$\beta = 45^0$

$h = \dfrac{d}{\cot\alpha + \cot\beta}$ find 'd'

21. $\text{Tan}\,60^0 = \dfrac{100}{x}$

x = 100 cot 60^0

$\text{Tan}\,30^0 = \dfrac{h}{x}$

(sub x value)

h = x tan 30^0

find h

22. $\text{Tan}\,30^0 = \dfrac{h}{x}$

x = h cot 30^0

$\text{Tan}\,60^0 = \dfrac{h}{100-x}$

sub x value

find h

23. $\text{Tan}\,\alpha = \dfrac{2h}{60}$ ---- (1)

$\text{Tan}\,(90-\alpha) = \dfrac{h}{60}$ (2)

(1) x (2)

find h

24. $\text{Tan}\,\alpha = \dfrac{10}{d}$

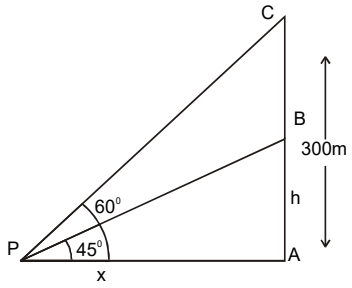
$\text{Tan}\,2\alpha = \dfrac{40}{d}$

$\dfrac{2\tan\alpha}{1-\tan^2\alpha} = \dfrac{40}{d}$

sun tan α = 10/d

find d = ?

25.



$\tan 60^0 = \dfrac{300}{x}$

$\Rightarrow x = 100\sqrt{3}$

$\tan 45^0 = \dfrac{h}{x} \Rightarrow h = x$

h = $100\sqrt{3}$ mtr.

26. Tan 60^0 = 1/2x $\Rightarrow x = \dfrac{1}{2}\cot 60^0$

$\text{Tan}\,30^0 = \dfrac{1}{2(x+d)}$

sub 'x' value and find d

27. Tan $75^0 = \frac{h_1 + h_2}{h_2}$

Towers height = h_2

Flag staff height = h_1

$Tan75^0 = 1+\frac{h_1}{h_2}$

$\frac{h_1}{h_2} = Tan75^0 - 1$

$\frac{h_1}{h_2} = \frac{2+\sqrt{3}-1}{1} = \frac{1+\sqrt{3}}{1}$

$\frac{h_2}{h_1} = \frac{1}{1+\sqrt{3}} = \frac{\sqrt{3}-1}{2}$

28. Tan $30^0 = \frac{h_1}{40}$

$h_1 = 40(\tan 30^0)$

$\text{Tan } 60^0 = \frac{h_1 + h_2}{40}$

substitude 'h_1' value & find h_2.

29. $\alpha = 60^0$

$\beta = 45^0$

d = 40

apply h = $\frac{d\cot\alpha}{\cot\beta - \cot\alpha}$

30. $\alpha = 45$

$\beta = 30^0$

h = 100

apply h = $\frac{d\cot\alpha}{\cot\beta - \cot\alpha}$

LEVEL - 2

1. A person in a balloon, who has ascended vertically from flat land at the sea level, observes the angle of depression of a ship at anchor to be 30^0. After descending vertically 600 metres, he finds the angle of depression to be 15^0. The horizontal distance of the ship from the foot of ascent in metres is

1) $300(3+\sqrt{3})$ 2) $300(3+2\sqrt{3})$

3) $150(3+2\sqrt{3})$ 4) $150(3-2\sqrt{3})$

2. In a prison wall there is a window of 1 metre height, 14 metres from the ground. An observer standing at a distance from the wall finds the angle of elevation of the top of the window and the top of the wall to be 45^0 and 60^0 respectively. The height of the wall above the window is

1) $15\sqrt{3}$ 2) $15\left(1-\frac{1}{\sqrt{3}}\right)$

3) $15(\sqrt{3}-1)$ 4) $14(\sqrt{3}-1)$

3. A man standing on a level plane observes the elevation of the top of a pole to be α. He then walks a distance equal to double the height of the pole and then finds that the elevation is now 2α. Then α =

1) 30^0 2) 15^0 3) 60^0 4) 45^0

4. There is a flag-staff at the top of 10 metres high tower. If the flag-staff makes an angle $\text{Tan}^{-1}(1/8)$ at a point 24 metres away from the tower, then the height of the flag staff in metres is

1) 26/7 2) 27/8 3) 27/6 4) 26/3

5. A tower of height 50 metres stands on a level ground. A flag-staff standing on the tower subtends an angle of $\text{Tan}^{-1}(1/3)$ at a point 100 metres away from the tower on the ground. The length of the falg-staff in metres is

1) 50 2) 75 3) 100 4) 125

6. A flag-staff stands on a tower which is on level ground. The total height of the flag-staff and tower taken together is 300 metres. The flag-staff subtends an angle of $\text{Tan}^{-1}(1/5)$ at a point P on the level ground at a distance 300 metres from the foot of the tower. The height of the tower is

1) 100 metres 2) 200 metres

3) 250 metres 4) 300 metres

7. The angular depression of the top and the foot of a tower as seen from the top of a second tower which is 150 m high and standing on the same level as the first are α and β respectively. If $\tan\alpha = 4/3$ tan $\tan\beta = 5/2$, the distance between their tops is

1) 100 m 2) 120 m 3) 110 m 4) 130 m

8. A man observes a tower AB of height h from a point P on the ground. He moves a distance 'd' towards the foot of the tower and finds that the angle of elevation is doubled. He further moves a distance 3d/4 in the same direction and the angle of elevation is three times that at P. Then $\frac{h^2}{d^2}$ =

1) 35/9 2) 35/36 3) 36/5 4) 36/35

9. The horizontal distance between two towers is 30 meters From the foot of the first tower the angle of elevation of the top of the second tower is 60^0. From the top of the second tower the angle of depression of the top of the first is 30^0. The height of the small tower is

1) $20(\sqrt{3}+1)$ mts 2) $20(\sqrt{3}-1)$ mts

3) $20\sqrt{3}$ mts 4) 20 mts

10. A flag - staff 20 metres long standing on a wall 10 metres high subtends an angle whose tangent is 0.5 at a point on the ground. If θ is the angle subtended by wall at that point then $\tan\theta$ = -----

1) 1 2) 1/3 3) 1 or 1/3 4) 2

11. A tower subtends an angle α at a point A on the same level as the foot of the tower B is a point vertically above A and AB = h metres. The angle of depression of the foot of the tower from B is β. The height of the tower is

1) $h\tan\alpha\cot\beta$ 2) $h\tan\alpha\tan\beta$

3) $h\cot\alpha\cot\beta$ 4) $h\cot\alpha\tan\beta$

12. From the top of a tree on one side of a street the angles of elevation and depression of the top and foot of a tower on the opposite side are respectively found to be α and β. If h is the height of the tree, then the height of the tower is

1) $\frac{h\sin(\alpha+\beta)}{\cos\alpha\sin\beta}$ 2) $\frac{h\sin(\alpha+\beta)}{\sin\alpha\cos\beta}$

3) $\frac{h\cos(\alpha-\beta)}{\cos\alpha\cos\beta}$ 4) $\frac{h\cos(\alpha+\beta)}{\cos\alpha\cos\beta}$

13. Flag-staff of lendgth 'd' stands on a tower of height 'h'. If at a point on the ground the angles of elevation of the tower and the top of the flag-staff be α, β respectively, then h =

1) $\frac{d\cot\beta}{\text{Cot}\alpha-\text{Cot}\beta}$ 2) $\frac{d\tan\beta}{\tan\alpha-\tan\beta}$

3) $d\left[\frac{\tan\alpha+\tan\beta}{\text{Cot}\alpha-\text{Cot}\beta}\right]$ 4) $\frac{d\tan\alpha}{\tan\beta}$

14. A vertical pole more than 100ft high consists of two portions, the lower being 1/3 of the whole. If the upper portion subtends an angle $\tan^{-1}(1/2)$ at a point distant 40 ft. from the foot of the pole, the height of the pole is

1) 105 2) 120 3) 135 4) 150

15. A straight pole A subtends a right angle at a point B of another pole at a distance of 30 meters from A, the top of A being 60^0 above the horizontal line joining the point B to the pole A. The length of the pole A is, in meters

1) $20\sqrt{3}$ 2) $40\sqrt{3}$ 3) $60\sqrt{3}$ 4) $\frac{40}{\sqrt{3}}$

16. A tower stands at the top of a hill whose height is three times the height of the tower. The tower is found to subtend an angle of $\text{Tan}^{-1}\frac{1}{7}$ at a point 2 KM away on the horizontal throught the foot of the hill. Then the height of the tower is

1) $\frac{1}{2}$KM or $\frac{1}{3}$KM 2) $\frac{1}{3}$KM or $\frac{2}{3}$KM

3) $\frac{2}{3}$KM or $\frac{1}{2}$KM 4) $\frac{3}{4}$KM or $\frac{1}{2}$KM

17. A tower of height 30 meters casts a shadow of length 40 meters at a certain instant. When the sun's elevation increases by $\text{Tan}^{-1}\frac{1}{7}$, the length of the shadow cast by the tower, in meters is

1) 20 2) 25 3) 30 4) 45

18. C is the mid point of the line joining two pionts A,B on the ground. A tower at C slightly leans towards B. If the angles of elevation of the top of the tower from A and B are 30^0, 60^0 respectvely, the angle made by the tower with the horizontal is

1) 45^0 2) 60^0 3) 75^0 4) 30^0

19. A ladder 20 meters long reaches a point 20 meters below the top of a flag. If the angle of elevation of the top of the flag at the foot of the ladder is 60^0, the length of the flagstaff, in meters is

1) 40 2) 20

3) 30 4) $10\left(2+\sqrt{3}\right)$

20. The upper $\frac{3}{4}$th portion of a vertical pole subtends an angle $\tan^{-1}\left(\frac{3}{5}\right)$ at the point in the horizontal plane through its foot. The tangent of the angle subtended by the pole at the same point is

1. 1 or 2 2. 2 or 3 3. 3 or 4 4. 4 or 1

KEY

01. 2	02. 3	03. 2	04. 1	05. 1
06. 2	07. 1	08. 2	09. 3	10. 3
11. 1	12. 1	13. 1	14. 2	15. 2
16. 3	17. 3	18. 2	19. 3	20. 4

HINTS

1. $\text{Tan}30^0=\frac{60^0+x}{d}$

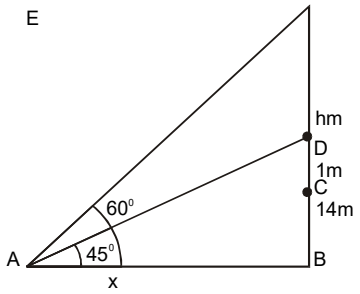
x = d (tan 30) - 600 ------ (1)

$\tan 15^0 = x/d$

x = d tan 15 ------- (2)

(1) = (2) find 'd'

2.



$\tan 45^0=\frac{15}{x}\Rightarrow x=15m$

$\tan 60^0=\frac{15+h}{x}$

4. $\alpha = \text{Tan}^{-1}\frac{1}{8}$

$\text{Tan}\,\alpha = \frac{1}{8}$

$\text{Tan}\,\beta = \frac{10}{24}$

$\text{Tan}\ (\alpha+\beta) = \frac{10+h}{24}$

apply Tan $(\alpha+\beta)$
formula. find h

5. Tan α = 1/3

Tan β = 50/100 = 1/2

$\text{Tan}\,(\alpha+\beta) = \frac{h+50}{100}$

apply Tan $(\alpha+\beta)$ = formulae find h

6. $Tan\alpha = \frac{x}{300}$

$Tan(\alpha+\beta) = 1$

$\frac{\frac{x}{300}+\frac{1}{5}}{1-\frac{x}{1500}} = 1$

$x = 200mt$

7. $\text{Tan}\,\alpha = 4/3$

$\text{Tan}\,\beta = 5/2$

h = 150

apply h = $\frac{d\cot\alpha}{\cot\beta - \cot\alpha}$

find 'd'.
Find DE, apply DE = CE - CD = 150 - d
In right angled triangle BDE apply

$\sin\beta = \frac{DE}{BE} \Rightarrow$ find BE

8. Apply $\therefore h = \frac{a}{2b}\sqrt{(a+b)(3b-a)}$

a = d; b = 3d/4, h = h

$\frac{h^2}{d^2} = \frac{35}{36}$

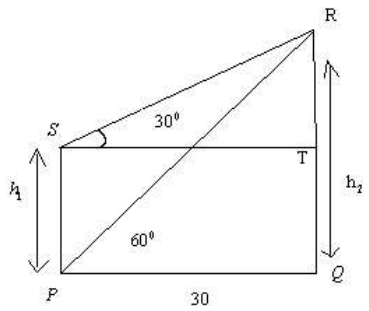
9. $\text{Tan}\,60^0 = \frac{h_1}{30}$

$h^1 = 30\ \text{Tan}\ 60^0$

$= 30\sqrt{3}$

$\text{Tan}\,30^0 = \frac{h_1 - h_2}{30}$

substitute h_1, find h_2



10. $\text{Tan}\alpha = 0.5$

= 1/2 ---- (1)

$\text{Tan}\theta = \frac{10}{d}$ ----- (2)

Tan $(\alpha+\theta) = \frac{30}{d}$

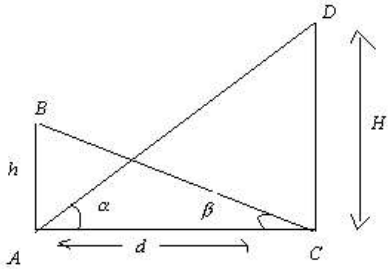
Apply Tan $(\alpha+\theta)$ formulae
find d & substitute in (2)

11. $\text{Tan}\,\beta = \frac{h}{d}$

d = h cot β

$\text{Tan}\,\alpha = \frac{H}{d}$

$H = (h\cot\beta)\text{Tan}\alpha$



12. $\text{Tan}\,\beta = \frac{h}{d}$

d = h cot β

$\text{Tan}\,\alpha = \frac{H-h}{d} \rightarrow (1)$

Substitute value of d in (1) and find H

13. Tan $\alpha = h/x$

x = h $\cot\alpha$

$\text{Tan}\,\beta = \frac{h+d}{x}$

$\text{Tan}\,\beta = \frac{h+d}{h\cot\alpha}$

find h

14. $\text{Tan}\,\alpha = \frac{1}{2}$

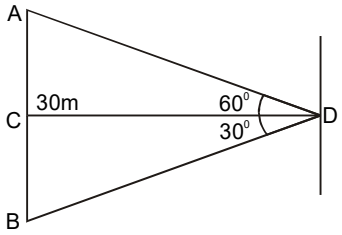
$\text{Tan}\,\beta = \frac{h}{120}$

$\text{Tan}\,(\alpha+\beta) = \frac{h}{40}$

apply $\text{Tan}\,(\alpha+\beta)$ formulae

find 'h'

15.



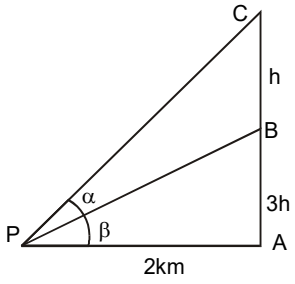
$\tan 60^0 = \frac{AC}{30}$

$AC = 30\sqrt{3}$

$\tan 30^0 = \frac{CD}{30}$

$BC = \frac{30}{\sqrt{3}}$

16.

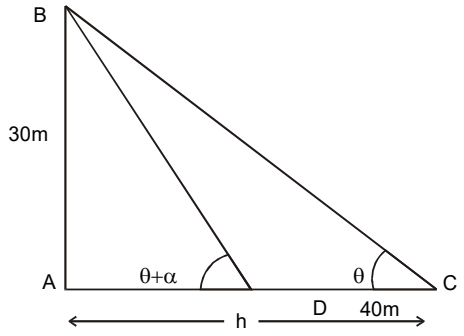


$\tan\alpha = \frac{1}{7};\ \tan\beta = \frac{3h}{2}$

$\tan(\alpha+\beta) = \frac{4h}{2} = 2h$

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 2h$

17.

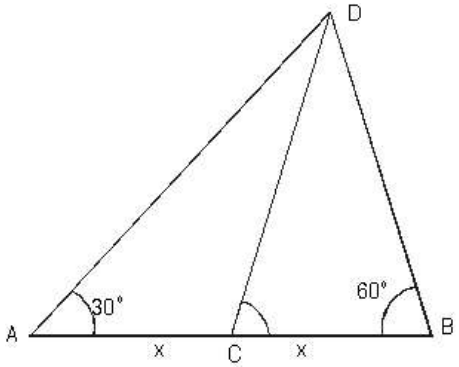


$\alpha = \tan^{-1}\left(\frac{1}{7}\right)$

$\tan\theta = \frac{30}{40} = \frac{3}{4}$

$\tan(\theta+\alpha) = \frac{30}{h}$

18.

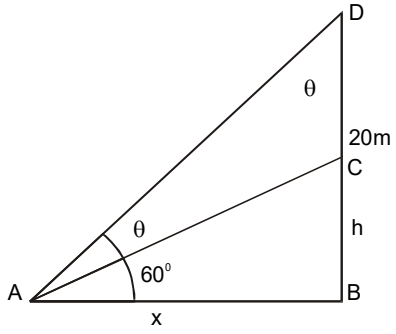


Formula :

$(m + n)\cot\theta = m\cot A - n\cot B$

$(x + x)\cot = x\cot 30^0 - x\cot 60^0$

19.



$\tan 60^0 = \frac{h+20}{x}$

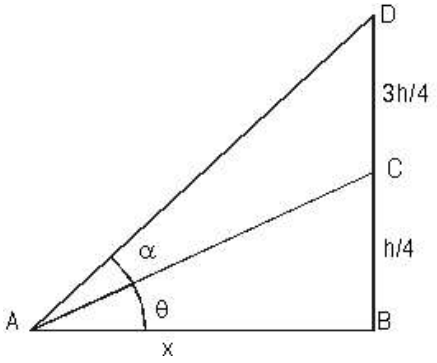
$\tan\theta = \frac{x}{h+20}$

$\tan 60^0 \cdot \tan\theta = 1$

$\tan 60^0 = \cot\theta$

$\Rightarrow \theta = 30^0$

20.



$$\alpha = \tan^{-1}\left(\frac{3}{5}\right)$$

$$\tan\theta = \frac{h}{4x}$$

$$\tan(\theta+\alpha) = \frac{h}{x}$$

$$\frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha} = \frac{4x\tan\theta}{x} = 4\tan\theta$$

LEVEL - 3

1. At a point on a level plane, a tower subtends an angle α, and a flag-staff of height 'a' metres standing on its top subtends on angle β. The height of the tower in metres is

1) $\frac{a\sin\alpha\sin(\alpha+\beta)}{\sin\beta}$ 2) $\frac{a\cos\alpha\sin(\alpha+\beta)}{\sin\beta}$

3) $\frac{a\sin\alpha\cos(\alpha-\beta)}{\sin\beta}$ 4) $\frac{a\sin\alpha\cos(\alpha+\beta)}{\sin\beta}$

2. AB is a vertical pole. The end A is on the level ground. 'C" is the mid point of AB. P is a point on the level ground. The portion CB subtends an angle β at P. If AP = nAB then tan β =

1) $\frac{n}{n^2+1}$ 2) $\frac{n}{2n^2+1}$ 3) $\frac{n}{n^2-1}$ 4) $\frac{n}{2n^2-1}$

3. The angle of elevation of an aeroplane from a point 30metres above the water level is 45^0, and the angle of depression of its reflection in the lake is 60^0. The height of the aeroplane above the lake in metres is

1) $30(2+\sqrt{3})$ 2) $30(2-\sqrt{3})$

3) $30(\sqrt{5}+1)$ 4) $30(\sqrt{5}-1)$

4. The angle of elevation of a cloud from a point h metres above a lake is θ. The angle of depression of its reflection in lake is 45^0. The height of the cloud is

1) $h\tan(45^0+\theta)$ 2) $h\cot(45^0-\theta)$

3) $h\tan(45^0-\theta)$ 4) $h\cot(45^0+\theta)$

5. The angle of elevation of the top of a tower is 45^0 from a point 10mt the water level of a lake. The angle of depression of its image in the lake is 60^0. The height of the tower is

1) $10(2+\sqrt{3})$ mt 2) $27(2+\sqrt{3})$ mt

3) $10(2-\sqrt{3})$ mt 4) $75(3+\sqrt{3})$ mt

6. The angle of elevation of the top of a hill from a point on the ground is 45^0. After walking a distance 2 km on a slope inclined at 30^0 to the horizon, the angle of elevation was found to be 60^0. Height of the hill is

1) $(\sqrt{3}+1)$ km 2) $(\sqrt{3}-1)$ km

3) $(2+\sqrt{3})$ km 4) $(2-\sqrt{3})$ km

7. The angle of elevation of the summit of a mountain at a point A is 45^0. After walking 200 mt from A towards the mountain along a road inclined at 15^0, it is observed that the angle of elevation of the summit is 60^0. The height of the mountain is

1) $100(\sqrt{6}+\sqrt{2})$ mt 2) $100(\sqrt{6}-\sqrt{2})$ mt

3) $\frac{100}{\sqrt{6}}$ mt 4) $\frac{100}{\sqrt{2}}$ mt

8. At the foot of a mountain the angle of elevation of a summit is found to be 45^0. After ascending 1 km towards the mountain up. On a slope of inclination 300, the angle of elevation is found to be 60^0. The height of the mountain is

1) $400(\sqrt{3}+2)$ mt 2) $500(\sqrt{3}+1)$ mt

3) $200(\sqrt{2}+1)$ mt 4) $100(\sqrt{6}+\sqrt{2})$ mt

9. The angles of elevation of a flying kite at three points, A, B, C on a horizontal line (lying in the same vertical plane with the kite) are in the ratio 1 : 2 : 3 if AB = a, BC = b then the height of the kite is

1) $\frac{a}{2b}\sqrt{(a+b)(3b-a)}$

2) $\frac{a}{b}\sqrt{(a-b)(3b+a)}$

3) $\frac{2a}{b}\sqrt{(a-b)(3b+a)}$

4) $\frac{a}{b}\sqrt{(a-b)(3b+a)}$

10. A balloon is observed simultaneously from three points A, B, C due west of it on a horizontal line passing directly underneath it. If the angular elevations at B and C are respectively twice and thrice that at A and if AB = 220 metres and BC = 100 metres, then the height of the balloon from the ground is

1) 167 mt 2) 176 mt 3) 184 mt 4)192 mt

11. Two light posts are of equal height. A person standing mid-way between the line joining their feet observes the elevation of the posts to be 30^0. After walking 12 metres towards one of them, he observes that the same post now subtends an angle of 60^0. Distance between them is
1) 18 m 2) 26 m 3) 36 m 4) 24 m

12. A chimney leans towards north. At equal distances due north and south of it in a horizontal plane, the elevations of the tops are α and β respectively. The inclination of the top of the chimney to the vertical is

1) $\text{Tan}^{-1}\left[\frac{\text{Sin}(\alpha-\beta)}{\sin\alpha\sin\beta}\right]$

2) $\text{Tan}^{-1}\left[\frac{2\text{Sin}(\alpha-\beta)}{\sin\alpha\sin\beta}\right]$

3) $\text{Tan}^{-1}\left[\frac{\text{Sin}(\alpha-\beta)}{2\sin\alpha\sin\beta}\right]$

4) $\text{Tan}^{-1}\left[\frac{\text{Sin}(\alpha-\beta)}{2\cos\alpha\cos\beta}\right]$

13. A pole is slightly inclined towards the east. At two points due west of it at distances a and b, the angles of elevation of the top of the pole are α and β respectively. The inclination of the pole to the horizon is

1) $\text{Tan}^{-1}\left[\frac{a+b}{b\text{Cot}\alpha - a\text{Cot}\beta}\right]$

2) $Tan^{-1}\left[\frac{b-a}{bCot\alpha - aCot\beta}\right]$

3) $\text{Cos}^{-1}\left[\frac{a-b}{b\text{Cos}\alpha - a\text{Cos}\beta}\right]$

4) $\text{Sin}^{-1}\left[\frac{a-b}{b\text{Cot}\alpha - a\text{Cot}\beta}\right]$

14. A flag staff of height (a - b) stands on the top of a tower subtends the same angle at point on the horizontal plane through the foot of the tower which are at distant a and b form the tower. The height of the tower is
1) b 2) a + b c) a 4) a - b

15. A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance 'a' so that it slides a distance 'b' down the wall making an angle β will the horizontal.

Then $\left(\frac{\alpha+\beta}{2}\right)$ =
1) b/a 2) a/b 3) 2/ab 4) 2a/b

16. A sperical balloon of radius r subtends an angle α at the eye of an observer, while the angle of elevation of its centre is β. The height of the centre of the ballon is

1) $r\cos\left(\frac{\beta}{2}\right)\sin\alpha$ 2) $r\cos ec\beta\sin\left(\frac{\alpha}{2}\right)$

3) $r\cos ec\beta\left(\frac{\alpha}{2}\right)\sin\beta$ 4) $r\cos ec\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)$

17. A balloon moving in a straight line passeses vertically above two points A and B, on a horizontal plane, 1000 ft apart. When above A it has an altitude of 60^0 as seen from B and when above B it has an altitude of 45^0 as seen from A. The distance from A of the point at which it will touch the plane is
1) 2266 ft 2) 2466 ft 3) 2566 ft 4) 2366 ft

18. On one side of a road of width 'd' metres there is a point of observation P at a height 'h' metres from the ground. If a tree on the other side of the road, makes a right angle at P, height of the tree in metres is

1) $\frac{h^2-d^2}{h}$ 2) $\frac{h^2+d^2}{h}$

3) $\frac{d^2-h^2}{h}$ 4) $\frac{2d^2+h^2}{h}$

19. A person observes that the top of three poles standing infront of him are in a line. If the height of the poles are in A.P. then the horizontal distances of the poles from the person are in
1) A.P. 2) G.P. 3) H.P. 4) AGP

20. An observer finds that the angular elevation of a tower is θ. On advancing 'a' metres towards the tower, the elevation is 45^0 and on advancing 'b' metres nearer the elevation is $90^0-\theta$ then the height of the tower in metres is

1) $\frac{ab}{a+b}$ 2) $\frac{ab}{a-b}$ 3) $\frac{2ab}{a+b}$ 4) $\frac{2ab}{a-b}$

21. PQ is a vertical tower. A, B, C are three points in a horizontal line through Q, the foot of the tower. If the angles of elevation of the top of the tower from A, B, C are α, β, γ representively. Then

$BC\cot\alpha - CA\cot\beta + AB\,Cot\gamma$ =
1) 3 2) 1 3) 2 4) 0

22. A man on the top of the light house observes a boat comming towards it. If it takes 10 minutes for the angle of depresion to change from 30^0 to 45^0, the time in minutes taken by the boat t reach the shore is
1) 13.66 2) 23.66 3) 33.66 4) 44.66

23. A man on a cliff observes a boat at an angle of depression 30^0 which is sailing towards the shore to the point immediately beneath him. 3 minutes later the angle of drpression of the boat is found to be 60^0. Assuming that the boat sails at a uniform speed, the time taken by the boat to reach the shore is
1) 1.5 min 2) 4.5 min 3) 6 min 4) 6.5 min

24. Three points A, B, C are on the straight bank of a river such that AB = BC = 2 Km. A boat is moving towards B on a line at right angles to the bank. At some point AC subtends an angle of 60^0. After moving for 10 minutes in the same direction AC subtends an angle of 120^0. The speed of boat is

1) $4\sqrt{3}\text{km/hr}$ 2) $16\sqrt{3}\text{km/hr}$

3) $8\sqrt{3}\text{km/hr}$ 4) $24\sqrt{3}\text{km/hr}$

25. The height of a hill is 3,300 metres. From the point P on the ground the angle of elevation of the top of the hill is 60^0. A balloon is moving with constant speed vertically upwards from P. After 5 minutes of its movement a person sitting in it observes the angle of elevation of the top of the hill as 30^0. The speed of the balloon is

1) 2.64 km / hr 2) 26.4 km / hr
3) 22.4 km / hr 4) 2.24 km / hr

26. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60^0. If after 10 secs the elevation is observed to be 30^0. Then the uniform speed per hour of the aeroplane is

1) $20\sqrt{3}\text{km}$ 2) $240\sqrt{3}\text{km}$

3) $256\sqrt{3}\text{km}$ 4) $250\sqrt{3}\text{km}$

27. From a light house L. two ships P and Q are observed in directions southwest and 15^0 east of south respectively. At the same time Q is observed from P in the southeast direction. If LP=4KM, the distance between the ships is

1) 4KM 2) $4\sqrt{3}$ KM 3) $5\sqrt{3}$ KM 4) $6\sqrt{3}$ KM

28. The angles oif elevation of the top of a tower standing on a horizontal plane from the two points lying on a line passing through the foot of the tower at distances a and b respectively are complementary angles. If the line joining the two points subtends an angle θ at the top of the tower, then sin θ is

1) $\frac{a+b}{a-b}$ 2) $\frac{a-b}{a+b}$ 3) $\frac{b-a}{a+b}$ 4) $\frac{a}{b}$

29. The top O o f a mountain is observed from A and B at the seal level. If N is the point vertically below P and $\angle NAB = \alpha, \angle NBA = \beta, \angle NAP = \theta, \angle NBP = \phi$ then the value of $\cot\phi \sin\beta$ is

1) $\sin\phi\cot\beta$ 2) $\cot\beta\sin\alpha$

3) $\sin\theta\sin\alpha$ 4) $\sin\phi\sin\alpha$

30. A tower is observed from two stations A and B where B is East of A at a distance 100m. The tower is due north of A and the due North-West of B. The angles of elevation of the tower from A and B are complementary. The height of the tower (in meters) is

1) $100\sqrt{2}$ 2) $100(2)^{1/4}$
3) $100(2)^{-1/4}$ 4) $100(2)^{1/3}$

31. A vertical tower of height 50 meters high stands on a sloping ground. The bottom of the tower is at the same level as the middle point of a vertical flagpole. From the top of the tower, the angles of depression of the top and bottom of the flagpole are 15^0 and 45^0 respectively. The height of the flagpole is

1) $\frac{50}{\sqrt{3}}$m 2) $50\sqrt{3}$m 3) $\frac{100}{\sqrt{3}}$m 4) $100\sqrt{3}$m

32. The width of a road is 'b' feet on one side of which there is a window 'h' feet height. A building infront of it subtends an angle θ at it. Then the height of the building is

1) $\frac{(b^2+h^2)}{b\cos\theta+h\sin\theta}$ 2) $\frac{(b^2+h^2)\sin\theta}{b\cos\theta+h\sin\theta}$

3) $\frac{(b^2+h^2)\sin\theta}{h}$ 4) $\frac{(b^2+h^2)\sin\theta}{b\cos\theta-h\sin\theta}$

33. A man observes that when he moves up a distance 'c' meters on a slope, the angle of depression of a point on the horizontal plane from the base of the slope is 30^0 when he moves up further a distance 'c' meters the angle of depression of that point is 45^0. Then the angle of inclination of the slope with the horizontal is

1) 45^0 2) 35^0 3) 75^0 4) 60^0

34. A tower 51 mts high has a mark at a height of 25 mts from the ground. At what distance the two parts subtend equal angle to an eye at the height of 5 mts from the ground.

1) 20 mts 2) 30 mts 3) 15 mts 4) 160 mts

35. A sphere of radius 'a' subtends an angle 60^0 at a point P. Then the distance of P from the center of the sphere is

1) $\frac{a}{\sqrt{3}}$ 2) 2a 3) $\frac{a\sqrt{3}}{2}$ 4) $\frac{2a}{\sqrt{3}}$

36. A spherical balloon of radius 5 subtends an angle 60^0 at a point on the horizontal. If the angle of elevation of its centre is 30^0 , the height of the centre of the balloon is

1) 5units 2) 10 units 3) 15 units 4) 20 units

KEY

01. 4	02. 2	03. 2	04. 1	05. 1
06. 1	07. 1	08. 2	09. 1	10. 2
11. 3	12. 3	13. 2	14. 1	15. 2
16. 3	17. 4	18. 2	19. 1	20. 2
21. 4	22. 2	23. 2	24. 3	25. 2
26. 2	27. 2	28. 2	29. 2	30. 2
31. 3	32. 2	33. 3	34. 4	
35.2	36.1			

HINTS

1. $\text{Tan}\beta = \frac{h}{d}$

d = h cot β

$\text{Tan}\alpha = \frac{h+a}{d}$

sub d = h cot β

$\therefore \text{Tan}\alpha = \frac{h+a}{h\cot\beta}$

find h

2. AB = x
AP = nAB = nx

$$\text{Tan}\,\alpha = \frac{x}{2nx} = \frac{1}{2n}$$

$$\text{Tan}\,(\alpha+\beta) = \frac{x}{nx} = \frac{1}{n}$$

Apply $\text{Tan}\,(\alpha+\beta)$
formulae substitute
Tan α and find tan β

3. Apply $x = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$
d = 30^0
$\beta = 60^0$
$\alpha = 45^0$

4. Apply $x = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$
d = h
$\beta = 45^0$
$\alpha = \theta$

5. d = 10
$\alpha = 45$
$\beta = 60$

$$x = \frac{d\sin(\beta+\alpha)}{\sin(\beta-\alpha)}$$

6. a = 2
$\alpha = 45^0$
$\beta = 60^0$
$\gamma = 30^0$

$$h = \frac{a\sin\alpha\sin(\beta-\gamma)}{\sin(\beta-\alpha)}$$

7. d = 200
$\alpha = 45^0$
$\beta = 60^0$
$\gamma = 15^0$

$$h = \frac{a\sin\alpha\sin(\beta-\gamma)}{\sin(\beta-\alpha)}$$

8. d = 1000 m
$\alpha = 45^0$
$\beta = 60^0$
$\gamma = 30^0$

$$h = \frac{a\sin\alpha\sin(\beta-\gamma)}{\sin(\beta-\alpha)}$$

9. Angles are in the ration 1 : 2 : 3

$$\therefore h = \frac{a}{2b}\sqrt{(a+b)(3b-a)}$$

10. a = 220
b = 100

Apply $\therefore h = \frac{a}{2b}\sqrt{(a+b)(3b-a)}$

12. Tan 30^0 = h/d
d = h cot 30^0

Tan 60^0 = $\frac{h}{d-12}$ .. (1)

substitute d = h cot 30^0
in (1) & find 'h'

13. $\text{Tan}\alpha = \frac{h}{x+d} \rightarrow (1)$

$\text{Tan}\beta = \frac{h}{x-d} \rightarrow (2)$

$\text{Tan}(90-\theta) = \frac{h}{d} \rightarrow (3)$

solve (1), (2) & (3) to find h

14. $\text{Tan}\,\theta = \frac{h}{d} \rightarrow (1)$

$\text{Tan}\,\alpha = \frac{h}{a+d} \rightarrow (2)$

$\text{Tan}\,\beta = \frac{h}{b+d} \rightarrow (3)$

(i) solve (2) & (3) to find h & d
(ii) substitude h, d values in (1)

15. a = AC - AB; b = AE - AD

$$\frac{a}{b} = \frac{AC-AB}{AE-AD} = \frac{1\cos\beta - 1\cos\alpha}{1\sin\alpha - 1\sin\beta}$$

apply transformations

16. Let BD = h
In right angled triange ACD

$$\sin\frac{\alpha}{2} = \frac{r}{AD}$$

AD = r cosec $\alpha/2$
In right angled triangle

ABD $\Rightarrow \sin\beta = \frac{h}{r\,\text{cosec}\,\alpha/2}$

$$\therefore h = r\,\text{cosec}\frac{\alpha}{2}\sin\beta$$

17. In right angled triangle ABD

$$\text{Tan}60^0 = \frac{AD}{1000}$$

AD = 1000 $\sqrt{3}$

Apply similar triangle properties to triangles ABC & BEC

$$\frac{AD}{BE} = \frac{AC}{BC}$$

18. $(AP)^2 = (AB)^2 + (BP)^2$
$= h^2 + d^2$
$(PC)^2 = (H - h)^2 + d^2$
$(AC)^2 = (AP)^2 + (PC)^2$
$H^2 = h^2 + d^2 + (H - h)^2 + d^2$
$H^2 = h^2 + d^2 + h^2 - 2Hh + h^2 + d^2$

$$H = \frac{h^2 + d^2}{h}$$

19. $\text{Tan}\,\theta = \frac{h}{x}$

$$x = h\cot\theta$$

$$\text{Tan}\,\theta = \frac{2h}{y}$$

$$y = 2h\cot\theta$$

$$\text{Tan}\theta = \frac{3h}{z}$$

$$z = 3h\cot\theta$$

20. $\Rightarrow \frac{x+z}{2} = y$

Tan $\theta = \frac{h}{h+a}$ ---------- (1)

Tan $(90^0 - \theta) = \frac{h}{h-b}$ ---------- (2)

using (1) x (2) find h

21. CQ = x
BC = y
AB = z

$$\text{Cot}\alpha = \frac{x+y+z}{h}$$

Cot $\beta = \frac{x+y}{h}$

$$\cot\gamma = \frac{x}{h}$$

Sub cot $\alpha, \cot\beta, \cot\gamma$ values in

$$BC\cot\alpha - CA\cot\beta + AB\cot\gamma$$

22. Tan $30^0 = \frac{h}{h+x}$

$$\frac{1}{\sqrt{3}} = \frac{h}{h+x}$$

h + x = $\sqrt{3}$ h

x = h($\sqrt{3}$ -1)

Distance	time
x	10

$\Rightarrow h(\sqrt{3}-1)$ in 10 min

$$h = \frac{10}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

h = $5(\sqrt{3}+1)$

$\therefore$ Time taken to reach the shore
$\Rightarrow$ time taken to cover x + time taken to cover h

23. Same as problem 22

24. $\Rightarrow$ S is circumcentre
SA = R

$$\Rightarrow \frac{4}{\sin 60} = 2R$$

Speed = $\frac{\text{distance}}{\text{time}}$

$$\Rightarrow \frac{4}{\sqrt{3}} = R$$

Speed = $\frac{\frac{4}{\sqrt{3}}}{10 \text{ m}}$

convert time into hrs

25. Tan $60^0 = \frac{3300}{d} \Rightarrow d = \frac{3300}{\sqrt{3}} x \frac{\sqrt{3}}{\sqrt{3}} = 1100\sqrt{3}$

$$\text{Tan}30^0 = \frac{3300-h}{1100\sqrt{3}}$$

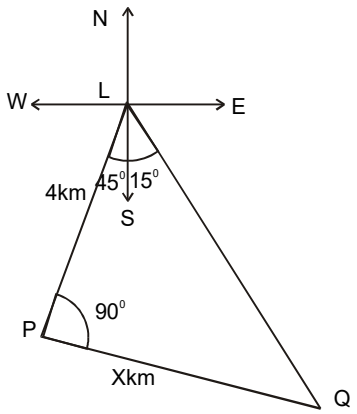
$$\frac{1}{\sqrt{3}} = \frac{3300-h}{1100\sqrt{3}}$$

h = 2200
h = 2200 m in 5 minutes

speed = $\frac{\text{distance}}{\text{time}}$

Distance to be converted meters of kilometers
Time to be converted interms of hrs.

28.



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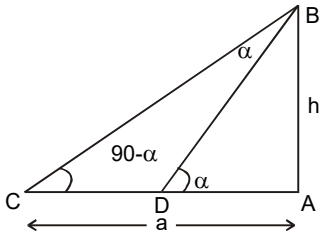


$$\tan\alpha = \frac{h}{b}$$

$$\tan\left(90^0 - \alpha\right) = \frac{h}{a}$$

$$\cot\alpha = \frac{h}{a}$$

$$\tan\alpha\cot\alpha = \frac{h}{b}, \frac{h}{a}$$

$$\Rightarrow h = \sqrt{ab}$$

$$a = 90^0 - \alpha + \theta$$

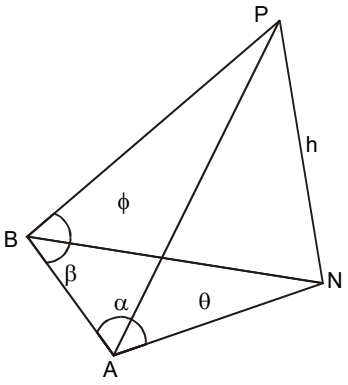
$$2\alpha = 90 + \theta$$

$$\cos 2\alpha = \cos\left(90^0 + \theta\right) = -\sin\theta$$

$$\sin\theta - -\left[\frac{1-\tan^2\alpha}{1+\tan^2\alpha}\right] = -\left[\frac{1-\frac{h^2}{b^2}}{1+\frac{h^2}{b^2}}\right],$$

$$= -\left(\frac{b-a}{b+a}\right) = \frac{a-b}{a+b}$$

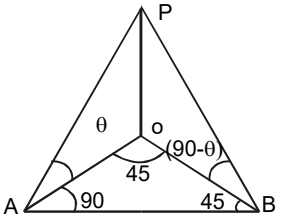
30.



AN = h cot θ, BN = h cot ϕ

$$\frac{AN}{\sin\beta} = \frac{BN}{\sin\alpha} \text{ simplify}$$

31.



AOP is a right angled triangle

OA = $h\cot\theta, OB = h\cot(90-\theta)$

$$\tan\theta = \frac{h}{100}, OB = h\tan\theta$$

$$\tan\theta = \frac{100\sqrt{2}}{h}$$

$$\frac{h}{100} = \frac{100\sqrt{2}}{h} \Rightarrow h = 100(2)^{\frac{1}{4}}$$

32. $$\tan 45^0 = \frac{50+x}{FG}$$

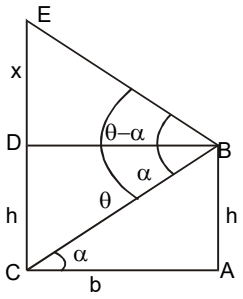
$$FG = 50 + x$$

$$\tan 15^0 = \frac{50-x}{50+x}$$

$$\Rightarrow x = \frac{50}{\sqrt{3}}$$

$$2x = \frac{100}{\sqrt{3}} \text{mtr}$$

33.

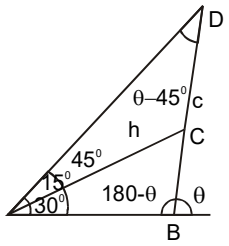


$$\tan\alpha = \frac{h}{b}, \tan(\theta-\alpha) = \frac{x}{b}$$

$$\therefore h + x = h + b.\tan(\theta-\alpha)$$

$$= h + b\left[\frac{\tan\theta - \frac{h}{b}}{1+\tan\theta.\frac{h}{b}}\right] = \frac{\tan\theta.(h^2+b^2)}{b+h\tan\theta}$$

34.



$$\text{From } \Delta ACD;\ \frac{h}{\sin(\theta-45^0)}$$

$$= \frac{c}{\sin 15^0}$$

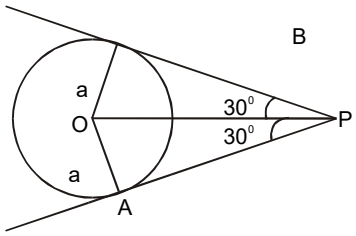
$$\Rightarrow \frac{h}{c} = \frac{\sin(\theta-45^0)}{\sin 15^0} \to (1)$$

$$\text{From } \Delta ABC;\ \frac{h}{\sin(180-\theta)} = \frac{C}{\sin 30^0}$$

$$\Rightarrow \frac{h}{c} = \frac{\sin\theta}{\sin 30^0} \to (2)$$

$$\text{from (1) \& (2)} \Rightarrow \sin\theta = \frac{\sqrt{3}+1}{2\sqrt{2}} \Rightarrow \theta = 75^0$$

35.



$$\cos ec30^0 = \frac{OP}{OB}$$

OP = a $\times$ 2

36. Height of center of the balloon = $r\cos ec\frac{\alpha}{2}\sin\beta$ where $\alpha = 60^0, \beta = 30^0$

$$\beta = \text{Tan}^{-1}\frac{1}{9}$$

$$\alpha+\beta = \text{Tan}^{-1}\frac{n}{n+1} + \text{Tan}^{-1}\frac{1}{2n+1}$$

$$\alpha+\beta = \frac{\pi}{4}$$

New type Questions

Assertion and Reasion :

1) Assertion (A) : The angles of elevation of a tower from two points which are at distances 12 mt and 64 mts from the foot of the tower on the opposite sides are complementary. The height of the tower is 24 mts.
Reasion : The angles of elevation of a tower from two points which are at distences 'a' and 'b' from the foot of the tower are Complementary then the height of the tower is $\sqrt{ab}$.
1. Both A and R are ture and 'R' is the correct explanation of A
2. Both A and R are true and 'R' is not correct explanation of A
3. A is true but 'R' is false
4. 'A' is false but 'R' is true.

2) Assertion :(A) : The shadow of a tower on a level plane is found to be 60 meters longer when sun's altitude is 30^0 than that when it is 45^0. Then the height of the tower is $30\left(\sqrt{3}+1\right) meters$
Reason : The angle of elevation of a top of a tower standing an horizontal plane from a Point 'A' is 'α'. After wlaking a distance 'd' meters towards the foot of the tower, the angle elveation is found to be β then the height of the tower is $h = \frac{d}{\cot\alpha - \cot\beta}$
1. Both A and R are ture and 'R' is the correct explanation of A
2. Both A and R are true and 'R' is not correct explanation of A
3. A is true but 'R' is false
4. 'A' is false but 'R' is true.

3) A ladder rests against a wall at an angle 30^0 to the horizontal. Its foot is pulled away through a distance x' so that it slides a distence 'y' down the wall finally making an angle 60^0 with the horizontal then x=y.

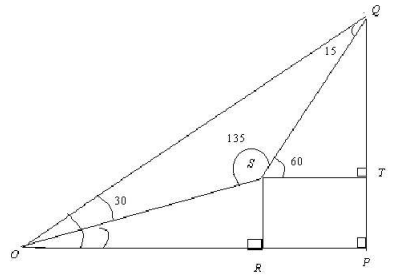
Reasion : A ladder rests against a wall at angle $'\alpha'$ to the horizontal. Its foot is pulled a way through a distence 'a' so that it slides a distence 'b' down the wall, finally making an angle β with the horizonal then $\tan\left(\frac{\alpha+\beta}{2}\right)=b/a$.

1. Both A and R are ture and 'R' is the correct explanation of A
2. Both A and R are true and 'R' is not correct explanation of A
3. A is true but 'R' is false
4. 'A' is false but 'R' is true.

4. Statement 1: A kite is flying with the string inclined at 30^0 to the horizanl. The height of the kite above the ground when the string is 15 mt long is 7.5 mts.
Statement 2: The altitude of the sun is 45^0, When the length of the shadow of a pole is same as height of the pole.which of the above statement is true ?

1. Only I 2. Only II
3. Both I and II 4. Neither I nor II

5.



$\angle POQ = 45^0 \quad \angle SOR = 15^0 \quad \angle TSQ = 60^0$

then the correct match fo the list I from list II

A) $\angle QOS$ 1) $\frac{5\Pi}{12}$

B) $\angle QSO$ 2) $\frac{\Pi}{2}$

C) $\angle SQO$ 3) $\frac{3\Pi}{4}$

D) $\angle QSR$ 4) $\frac{\Pi}{12}$

5) $\frac{\Pi}{6}$

	A	B	C	D
1)	5	3	4	1
2)	2	3	5	4
3)	1	4	3	5
4)	3	5	2	4

6) (i)The angle of the elevation of the top of a tower of height 100 mts from a point to its foot is $Tan^{-1}\frac{4}{5}$ then the distance from the point to its foot is 'a'.
(ii) From the ship mast head 200 mts height, the angle of depression of the boat is observed to be 45^0. Then the distance of the boat from the ship is 'b'.
(iii) From the top of a cliff 240 mts height, a man observes the anlge of depression of a boat is to be 60^0, then the distance of the boat from the foot of the cliff is 'c'.
The ascending order of a,b,c is

1. a b c 2. b c a 3. c a b 4. a c b

7. (1) The angle of elevation of the sun, when the length of the shadow of a pole is $\sqrt{3}$ times the height of the pole is $'\alpha'$.
(2) The angle of elevation of the sun, when the length of the shadow of a pole is euqals to theheight of the pole is $'\beta'$.
(3) The angle of elevation of the sun, when the length of the shadow of a pole is $\frac{1}{\sqrt{3}}$ times the height of the pole is $'\gamma'$.

Then descending order of α,β,γ is

1) α,β,γ 2) β,α,γ
3) γ,β,α 4) β,γ,α s

KEY

1)1 2)1 3)3 4)3 5)1
6)4 7)3

QUESTIONS FROM PREVIOUS EAMCET

1. A tower of x meters high has a flagstaff at its top. The tower and the flagstaff subtend equal angles at a point distant y meters from the foot of the tower then the length of the flagstaff in meters is **(2005)**

1. $\frac{y\left(x^2-y^2\right)}{x^2+y^2}$ 2. $\frac{x\left(y^2+x^2\right)}{y^2-x^2}$
3. $\frac{x\left(x^2+y^2\right)}{x^2-y^2}$ 4. $\frac{x\left(x^2-y^2\right)}{x^2+y^2}$

2. An aeroplane flying with uniform speed horizontally one kilometer above the ground is observed at an elevation of 60^0. After 10 seconds if the elevation is observed to be 30^0 then the speed of the plane (in km/hr) is **(2004)**

1. $\frac{240}{\sqrt{3}}$ 2. $200\sqrt{3}$ 3. $240\sqrt{3}$ 4. $\frac{120}{\sqrt{3}}$

3. A tower substends angles $\alpha, 2\alpha, 3\alpha$ at A,B,C all lying on the horizontal line through the foot of the tower then $\frac{AB}{BC} =$ **(2003)**

1. $\frac{Sin3\alpha}{Sin2\alpha}$ 2. $1+2Cos2\alpha$

3. $2+2Cos2\alpha$ 4. $\frac{Sin2\alpha}{Sin\alpha}$

4. From a point on the level ground, the angle of elevation of the top of a pole is 30^0. On moving 20 meters nearer, the angle of elevation is 45^0. Then the height of the pole in meters is : **(2002)**

1) 10 $(\sqrt{3}-1)$ 2) 10 $(\sqrt{3}+1)$

3) 15 4) 20

5. The shadow of a tower standing on a level plane is found to be 60m. longer when the sun's altitude is 30^0 than that when the sun's altitude is 45^0. The height of the toweer is **(2001)**

1. $30\left(\sqrt{3}+1\right)$ 2. $30\left(\sqrt{3}-1\right)$

3. $30\left(3+\sqrt{3}\right)$ 4. $30\left(3-\sqrt{3}\right)$

6. Two towers of heights h_1 and h_2 subtend angles 60^0 and 30^0 respectively at the midpoint of the line joining their feet then $h_1 : h_2 =$ **(2000)**

1) 1 : 2 2) 1 : 3 3) 2 : 1 4) 3 : 1

7. From the top of a building h metres, the angle of depression of an object on the ground is α, the distance of the object from the foot of the building is **(1999)**

1) h cot α 2) h tan α

3) h cos α 4) h sin α

8. The angles of elevation of a tower from two points which are at distances h_1 and h_2 from the foot of the tower on the same side are complementary. The height of the tower is **(1997)**

1) $\sqrt{h_1h_2}$ 2) h_1h_2 3) h_1 / h_2 4) $\sqrt{\frac{h_1}{h_2}}$

9. From the top of a building 60 m. high, the angle of elevation of the top of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower in meters is **(1997)**

1) 100 2) 90 3) 120 4) 105

10. There are two towers on a horizontal line. From the midpoint of the line joining their feet, the tops of the towers appears at angles of elevation of 60^0 and 30^0 respectively. The first tower has a height of 100 m. The height of the second tower is **(1996)**

1) $\frac{100}{\sqrt{3}}$ mt 2) $\frac{100}{5}$ mt

3) $100\sqrt{2}$mt 4) $\frac{100}{3}$ mt

11. The horizontal distance between two towers is 60 m and angular depression of the top of the first as seen from the second, which is 150 m height is 30^0. The height of the first tower is **(1996)**

1) $(150+20\sqrt{3})$ mt 2) $(150+15\sqrt{3})$ mt

3) $(150-20\sqrt{5})$ mt 4) $(150-20\sqrt{3})$ mt

12. The angle of elevation of the top of a tower from a point in the same level as the foot of the tower is 15^0. On moving 100 meters towards the tower, the angle of elevation increases to 30^0. The height of the tower is **(1996C)**

1) 50 mt 2) 100 mt 3) 150 mt 4) 200 mt

13. A person walking along a straight road towards a hill observes at two points distance $\sqrt{3}$ km, the angle of elevation of the hill to be 30^0 and 60^0. The height of the hill is **(1995)**

1) 3/2 km 2) $\sqrt{\frac{2}{3}}$

3) $\frac{\sqrt{3}+1}{2}$ km 4) $\sqrt{3}$ km

14. The angle of elevation of the top of the tower observed from each of the three points A, B, C on the ground, forming a triangle is the same angle α. If R is the circum radius of the ΔABC, then the height of` the tower is **(1994)**

1) R Sin α 2) R Cos α

3) R Cot α 4) R tan α

15. A flag staff of 5 mts high stands on a building of 25 mt high. At an observer at a height of 30 mt the flag staff and the building subtend equal angles. The distance of the observer from the top of the flag staff is **(1993)**

1) $5\sqrt{3}$ 2) $5\sqrt{2}$ 3) $5\sqrt{6}$ 4) $\frac{5\sqrt{3}}{\sqrt{2}}$

16. ABC is a triangular park with AB = AC = 100 cm. A clock tower is situated at the midpoint of BC. The angles of elevation of the top of the tower A and B are Cot^{-1} 3.2 and $Cosec^{-1}$ 2.6. The height of the tower is **(1992)**

1) 25 mt 2) 50 mt

3) 100 mt 4) $50\sqrt{2}$ mt

17. If a flag staff of 6 mt high placed on the top of a tower throws a shadow of $2\sqrt{3}$ mt along the ground then the angle that the sun makes with the ground is **(1990)**

1) 45^0 2) 30^0 3) 75^0 4) 60^0

18. The top of hill observed from the top bottom of a building of height h is at the angles of elevation p and q respectively. The height of the hill is **(1989)**

1) $\frac{h\cot p}{Cotp-Cotq}$ 2) $\frac{h\cot q}{Cotp-Cotq}$

3) $\frac{hcotp}{Cotq-Cotp}$ 4) $\frac{h\cot q}{CotP+Cotq}$

19. From the top of cliff 60 mt high, the angles of depression of top and bottom of a tower are 30^0 and 60^0. The height of the tower is **(1988)**

1) 40 mt 2) 50 mt 3) 60 mt 4) 30 mt

20. The angles of elevation of the tops of two vertical towers as seen from the middle point of the line joining the foot of the towers are 60^0 and 30^0 respectively. The ratio of the height of the towers is **(1987)**

1) 2 : 1 2) $\sqrt{3}$: 1 3) 3 : 2 4) 3 : 1

21. The angles of elevation of a cliff at a point A on the ground and at a point B 100 mt vertically above A and α and β respectively. The height of the cliff in metres is **(1986)**

1) $\frac{100\tan\beta}{\text{Cot}\beta-\text{Cot}\alpha}$ 2) $\frac{100\text{Cot}\beta}{\text{Cot}\beta-\text{Cot}\alpha}$

3) $\frac{100\tan\beta}{\text{Cot}\alpha+\text{Cot}\beta}$ 4) $\frac{100\cot\beta}{\text{Cot}\alpha+\text{Cot}\beta}$

22. The angle of elevation of a cloud from a point h metres above the level of water in a lake is θ and the angle of depresion of its image in the lake is ϕ. The height of the Cloud is **(1985)**

1) $\frac{h\text{Sin}(\phi+\theta)}{\text{Sin}(\phi-\theta)}$ 2) $\frac{h\text{Sin}(\phi-\theta)}{\text{Sin}(\phi+\theta)}$

3) $\frac{h\text{Sin}(\theta+\phi)}{\text{Sin}(\theta-\phi)}$ 4) $\frac{h\text{Sin}(\theta-\phi)}{\text{Sin}(\theta+\phi)}$

23. On the level ground the angle of elevation of the top of a tower is 30^0. On moving 20 metres nearer the tower, the angle of elevation is found to be 60^0. The height of the tower in metres is **(1984)**

1) $10\sqrt{3}$ 2) $8\sqrt{3}$ 3) $6\sqrt{3}$ 4) $5\sqrt{3}$

24. A man on the top of an observation tower finds an object at an angle of depression 30^0. After the object was moved 30 metres in a straight line towards the tower, he finds the angle of depression to be 45^0. The distance of the object now from the foot of the tower in metres is **(1983)**

1) $15\sqrt{3}$ 2) $15(\sqrt{3}+1)$

3) $15(\sqrt{3}-1)$ 4) $15(2+\sqrt{3})$

25. A flag - staff on the top of a tower 80 metres high subtends an angle $\tan^{-1}(\frac{1}{9})$ at a point on the ground 100 metres from the foot of the tower. The height of the flag - staff in metres is **(1982)**

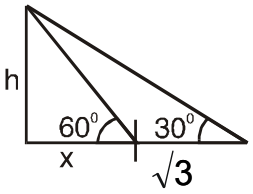
1) 10 2) 20 3) 30 4) 40

KEY

1. 2	2. 3	3.2	4. 2	5. 1
6. 4	7. 1	8. 1	9. 3	10. 4
11. 4	12. 1	13. 1	14. 4	15. 4
16. 1	17. 4	18. 1	19. 1	20. 4
21. 2	22. 1	23. 1	24. 2	25. 2

HINTS

4. $\text{Tan}30^0=\frac{h}{h+20}$ find h.



5. $\text{Tan}30^0=\frac{h}{h+60}$

find h

6. $\text{Tan}60^0=\frac{h_1}{x}$

$\text{Tan}30^0=\frac{h_2}{x}$

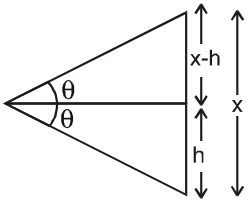
$\therefore h_1:h_2=\frac{h_1}{h_2}=\frac{x\tan 60^0}{x\tan 30^0}$

7. $\tan\alpha=\frac{h}{d}$

8. $\tan\alpha=\frac{H}{h_1}\rightarrow(1)$

$\tan(90-\alpha)=\frac{H}{h_2}\rightarrow(2)$

$\therefore(1)\times(2)$

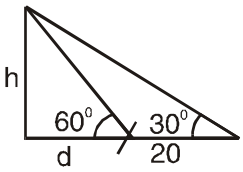
9. 

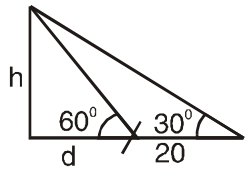
10. $\tan 60=\frac{100}{x}$

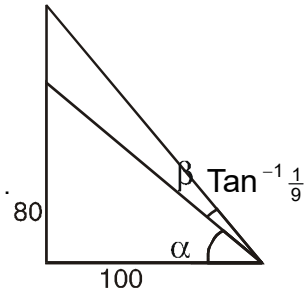
$x=\frac{100}{\tan 60}$

$\tan 30^0 = h/x$

$\Rightarrow \text{Tan}30^0=\frac{150-h}{60}$



11. 

12 

$$\text{Tan}15^0 = \frac{h}{x+100} = \frac{h}{h\cot 30+100}$$

13. $\text{Tan}\,60^0 = \frac{h}{x}$

x = h cot 60^0

$$\text{Tan}\,30^0 = \frac{h}{x+\sqrt{3}} = \frac{h}{h\cot 60+\sqrt{3}}$$

14. Since the tower subtends same α at the vertices of ΔABC the foot of the tower is at circumcentre of triangle

Height of the tower = R sin α

15. $\text{Tan}\,\theta = \frac{5}{d}$

$$\text{Tan}\,2\theta = \frac{30}{d}$$

$$\Rightarrow \frac{2\tan\theta}{1-\tan^2\theta} = \frac{30}{d}$$

17. From ΔABD $\text{Tan}\theta = h/x$

From ΔACE $\text{Tan}\,\theta = \frac{h+6}{x+2\sqrt{3}}$

$$\Rightarrow \frac{h}{x} = \frac{h+6}{x+2\sqrt{3}} \Rightarrow 2\sqrt{3}h = 6x$$

$$\text{Tan}\,\theta = \frac{h}{x} = \frac{6}{2\sqrt{3}} = \sqrt{3}$$

$\theta = 60^0$

18. Tan q = H/d

d = H cot q

$$\Rightarrow \text{TanP} = \frac{H-h}{d}$$

$$= \frac{H-h}{H\cot q}$$

19. Tan 60^0 = 60/d

d = 60 cot 60^0

$$\text{Tan}\,30^0 = \frac{60-h}{d}$$

$$\text{Tan}\,30^0 = \frac{60-h}{60\cot 60^0}$$

20. $\left.\begin{aligned}\tan 60^0 &= \frac{h_1}{x} \Rightarrow h_1 = x\tan 60^0\\ \tan 30^0 &= \frac{h_2}{x} \Rightarrow h_2 = x\tan 30^0\end{aligned}\right\}$ find $\frac{h_1}{h_2}$

21. $\text{Tan}\,\alpha = h/d$

$$d = h\cot\alpha$$

$$\tan\beta = \frac{h-100}{d}$$

$$\tan\beta = \frac{h-100}{h\cot\alpha}$$

22. $\text{Tan}\,\theta = \frac{x-h}{d}$

$$d = (x-h)\cot\theta \rightarrow (1)$$

$$\text{Tan}\,\phi = \frac{x+h}{d}$$

$$d = (x+h)\cot\phi \rightarrow (2)$$

(1) = (2) $\Rightarrow$ find x

23. $\text{Tan}\,60^0 = \frac{h}{d}$

d = h cot 60^0

$$\text{Tan}\,30^0 = \frac{h}{20+d}$$

$$\text{Tan}\,30^0 = \frac{h}{20+h\cot 60^0}$$

24. $\Rightarrow \text{Tan}30^0 = \frac{x}{x+30^0}$

25. $\text{Tan}\,\alpha = \frac{80}{100} = \frac{4}{5}$

$$\alpha = \text{Tan}^{-1}\frac{4}{5}$$

$$\text{Tan}30^0 = \frac{h}{x} \Rightarrow x = h\cot 30$$