44. X-rays

Short Answer

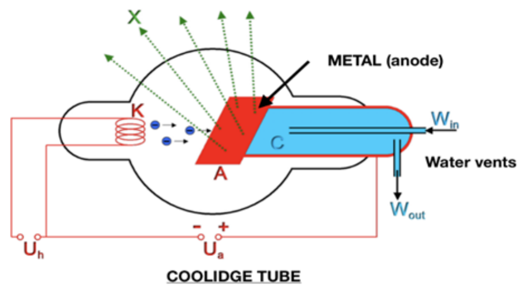
1. Question

When a Coolidge tube is operated for some time it becomes hot. Where does the heat come from?

Answer

William Coolidge refurbished the hot cathode ray tube to generate X-rays, which is now known as Coolidge tube. It contains a hot filament which generates electrons by *thermionic emission* and a metal anode where electrons loose the kinetic energy to produce x-rays. There exists an electric field between the filament and the metal to accelerate the electrons to high energies. As the electrons strike the metal, some of their energy is utilized in created in x-rays, and the leftover energy (due to collisions) is absorbed by the metal as heat. This energy along with the hot filament, raises the temperature of the Coolidge tube. So, water .

vents are provided to cool off the metal



2. Question

In a Coolidge tube, electrons strike the target and stop inside it. Does the target get more and more negatively charged as time passes?

Answer

A Coolidge tube is a complete circuit, with filament as the cathode, metal plate as the anode and a power supply to drive the circuit (as shown in the figure above). The electrons ejected from the cathode by thermionic emissions are received by the metal where they give off X-rays and knock off electrons from the target atoms till they finally come to rest. These newly generated electrons are then driven by the electric field of the circuit towards the terminals of the power supply. There are no excess electrons in the metal and thus the target does not get more negative.

3. Question

Can X-rays be used for photoelectric effect?

Answer

Photoelectric effect is the ejection of an electron from the target metal when the energy of the incoming radiation (photon) is greater than (or equal to) the work function of the metal. Thus, if the energy of the photon of the X-ray exceeds the work function, it can participate in the photoelectric process.

4. Question

Can X-rays be polarized?

Answer

Electromagnetic waves such as X-rays have an Electric field and Magnetic field components perpendicular to each other which gives it two dimensions. These are transverse waves and thus can be polarized.

5. Question

X-ray and visible light travel at the same speed in vacuum. Do they travel at the same speed in glass?

Answer

The refractive index varies slightly with wavelength which is why we have the phenomena of chromatic aberration. The formula of refractive index is as follows:

$$n = \frac{c}{v}$$

Where, n is the index of refraction, c is the speed of light in vacuum and v is the speed of light in the medium.The refractive index of x-rays is slightly less than 1 and hence less than that of visible light. Thus, the speed of X-rays is more in glass (and in other materials) than in vacuum.

6. Question

Characteristic X-rays may be used to identify the element from which they are being emitted. Can continuous X-rays be used for this purpose?

Answer

The production process of characteristic x-rays and continuous x-rays are quite different. Characteristic x-rays are created when electrons knock off inner shell electrons of the target atom. The vacancy created by this is later filled by transition of electrons from higher energy levels giving off a photon of energy equal to the energy difference between the two levels. The energy of photon is unique to each element which is why they are used to identify elements. Continuous x-ray on the other hand are created by collisions of electrons with the target. This converts a fraction of the kinetic energy of the electron to photon. The energy of photon can be anywhere between 0 to maximum energy of the incoming electron (eV). So, this cannot be used to detect elements as it depends on the incoming electron and not the target.

7. Question

Is it possible that in a Coolidge tube characteristic Lα X-rays are emitted but not Kα X-rays?

Answer

Kα X-rays are created when an electron jumps from L shell to fill the vacancy in K-shell. Lα X-rays are produced when the vacancy in L shell is filled by M shell. This can only happen when L shell gives its electron to K shell initially to create Kα X-rays. So, it's not possible to get characteristic Lα X-rays prior to Kα X-rays.

8. Question

Can Lα X-ray of one material have shorter wavelength than Kα X-ray of another?

Answer

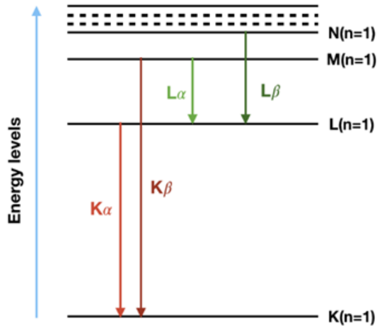
The energy gap between the L shell and the K shell is very large as compared to the gap between L shell and M shell or any such subsequent shells. The difference in energy between the shells is the emitted X-ray photon energy. Wavelength is inversely proportional to the energy. So, higher the energy, shorter is the wavelength. As the transition energy to K shell from any successive shell will always be higher than the transitions occurring to L shell, Kα X-ray will always have shorter wavelength than Lα X-ray.

9. Question

Can a hydrogen atom emit characteristic X-rays?

Answer

Hydrogen atom has very closely spaced energy levels. As the difference between two energy levels is very small, we do not get characteristic x-rays from hydrogen.



10. Question

Why is exposure to X-rays injurious to health but not exposure to visible light, when both are electromagnetic waves?

Answer

X-rays are high energy radiation with high penetration power. They can easily penetrate through human tissue and bones and can damage them. Prolong exposure can mutate cells and cause genetic defects and cancer.

Objective I

1. Question

X-ray beam can be deflected

A. by an electric field

B. by a magnetic field

C. by an electric field as well as by a magnetic field

D. neither by an electric filed nor by a magnetic field.

Answer

X-rays are packets of electromagnetic energy or photons which are uncharged. Thus, they cannot be deflected by either electric or magnetic field.

2. Question

Consider a photon of continuous X-ray coming from a Coolidge tube. Its energy comes form

A. the kinetic energy of the striking electron

B. the kinetic energy of the free electrons of the target

C. the kinetic energy of the ions of the target

D. an atomic transition in the target.

Answer

The continuous energy spectrum of X-rays is a result of electrons striking the target atoms and loosing energy. An electron has a kinetic energy (K) given by K=eV, where “e” is the charge of the electron and “V” is the potential difference applied. Now, electrons can either give all of its energy to the target or only a fraction. So the generated photon can have any energy between “0” to “e” V. This gives us the continuous spectrum of wavelengths: $\lambda = \frac{hc}{E}$ Here, λ is the wavelength of light, h is the Planck’s constant ($6.62607004 \times 10^{-34}$ m^2 kg / s), c is the speed of light in vacuum and E is the energy of the photon.

3. Question

The energy of a photon of characteristic X-ray from a Coolidge tube comes from

A. the kinetic energy of the striking electron

B. the kinetic energy of the free electrons of the target

C. the kinetic energy of the ions of the target

D. an atomic transition in the target.

Answer

The electron striking the metal can knock off electrons from the shells of the target atoms. These electrons are then filled by the subsequent energy levels. During the transition of electrons from higher energy level to the newly created vacant space, the difference of the energy between the two energy levels is given off as photons forming characteristic x-rays.

4. Question

If the potential difference applied to the tube is doubled and the separation between the filament and the target is also doubled, the cutoff wavelength

A. will remain unchanged

B. will be doubled

C. will be halved

D. will become for times the original

Answer

The cut-off wavelength is given by:

$\lambda_{min} = \frac{hc}{eV}$where, λ_{min} is the cut-off wavelength, h is the Planck's constant ($6.62607004 \times 10^{-34}$ m^2 kg / s), c is the speed of light in vacuum, e is the charge of the electron and V is the potential difference applied at the ends of the tubeSo, doubling the applied potential will reduce the cut-off wavelength by half.

5. Question

If the current in the circuit for heating the filament is increased, the cutoff wavelength

A. will increase

B. will decrease

C. will remain unchanged

D. will change

Answer

The cut-off wavelength does not depend on the current and only depends on the potential. So, it will remain unchanged. Increasing the current will increase the number of photons per unit time which is the intensity of x-rays.

6. Question

Moseley's law for characteristic X-rays is $\sqrt{v}$ = a (Z – b). In this,

A. both a and b are independent of the material

B. a is independent but b depends on the material

C. b is independent but a depends on the material

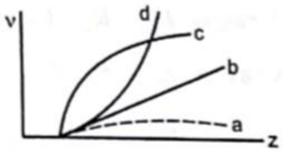
D. both a and b depend on the material.

Answer

In Moseley's law, "a" represents the proportionality constant while "b" represents the screening constant, with independent of the target material.

7. Question

Frequencies of K_α X-rays of different materials are measured. Which one of the graphs in figure may represent the relation between the frequency v and the atomic number Z.



Answer

Moseley's equation is given by:

$\sqrt{v} = a(Z - b) v = a^2(Z - b)^2$ where, "a" represents the proportionality constant while "b" represents the screening constant and Z is the atomic number of the element.Squaring both sides, we getSo the graph should resemble a parabola (y=x^2)

8. Question

The X-ray beam coming from an X-ray tube

A. is monochromatic

B. has all wavelengths smaller than a certain maximum wavelength

C. has all wavelengths greater than a certain minimum wavelength

D. has all wavelengths lying between a minimum and a maximum wavelength.

Answer

The wavelength of X-ray photon is given by:

$$\lambda = \frac{hc}{E}$$

Here, λ is the wavelength of light, h is the Planck's constant (6.62607004 × 10^{-34} m^2 kg / s), and c is the speed of light in vacuum and E" is the energy of the electron which can range from 0"to eV

$\lambda_{min} = \frac{hc}{eV}$And the minimum wavelength that the photon can have iswhere, "eV" is the kinetic energy acquired by the electron. So, in principle, x-rays can have any wavelength greater than the cut-off wavelength.

9. Question

One of the following wavelengths is absent and the rest are present in the X-rays coming from a Coolidge tube. Which one is the absent wavelength?

A. 25 pm

B. 50 pm

C. 75 pm

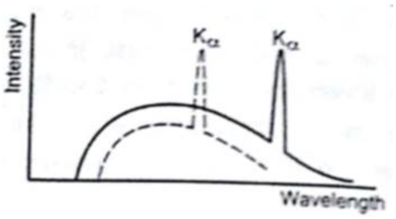
D. 100 pm

Answer

In a Coolidge tube, all wavelengths above cut-off wavelength are present. Thus, 25pm is the cut-off minimum wavelength and is absent.

10. Question

Figure shows the intensity-wavelength relations of X-rays coming from two different Coolidge tubes. The solid curve represents the relation for the tube A in which the potential difference between the target and the filament is V_A and the atomic number of the target material is Z_A. These quantities are V_B and Z_B for the other tube. Then,



A. $V_A > V_B$, $Z_A > Z_B$

B. $V_A > V_B$, $Z_A < Z_B$

C. $V_A < V_B$, $Z_A > Z_B$

D. $V_A < V_B$, $Z_A < Z_B$

Answer

The cut-off frequency is given by:

$$\frac{1}{\lambda} = a^2(Z - b)^2$$

$\lambda_{min} = \frac{hc}{eV}$where, λ_{min} is the cut-off wavelength, h is the Planck's constant ($6.62607004 \times 10^{-34}$ m^2 kg / s), c is the speed of light in vacuum, e is the charge of the electron and V is the potential difference applied at the ends of the tubeSo, greater the operating voltage, lesser would be the cut-off wavelength. As, λ_{min} is less of A than B, $V_A > V_B$ .From Moseley's law, $\sqrt{v} = a (Z - b)$, therefore in terms of wavelength(λ), we havewhere, "a" represents the proportionality constant while "b" represents the screening constant and Z is the atomic number of the element.Looking at the characteristic peaks, we can see that $\lambda_A > \lambda_B$. So from the inverse relation, $Z_A < Z_B$.

11. Question

50% of the X-ray coming from a Coolidge tube is able to pass through a 0.1 mm thick aluminum foil. If the potential difference between the target and the filament is increased, the fraction of the X-ray passing through the same foil will be

A. 0%

B. < 50%

C. 50%

D. > 50%

Answer

More the applied potential in the Coolidge tube, more will be the energy of the x-rays and thus greater will be their penetrating power. So, more x-rays will be transmitted through the foil.

12. Question

50% of the X-rays coming from a Coolidge tube is able to pass through a 0.1 mm thick aluminium foil. The potential difference between the target and the filament is increased. The thickness of aluminum foil, which will allow 50% of the X-ray to pass through, will be

A. zero

B. < 0.1 mm

C. 0.1 mm

D. > 0.1 mm

Answer

Increasing the operating potential increases the penetrating power of the x-rays. Thus to allow the same fraction of x-rays to pass for increased potential, we need to thicken the foil (to aid in the attenuation process)

13. Question

X-ray from a Coolidge tube is incident on a thin aluminium foil. The intensity of the X-ray transmitted by the foil is found to be I_0. The heating current is increased so as to increase the temperature of the filament. The intensity of the X-ray transmitted by the foil will be

A. zero

B. $< I_0$

C. I_0

D. $> I_0$.

Answer

Increasing the operating voltage increases the energy of x-rays, while increasing the current increases the intensity. Thus, if we increase the current, the intensity will increase.

14. Question

Visible light passing through a circular hole forms a diffraction disc of radius 0.1 mm on a screen. If X-ray is passed through the same set-up, the radius of the diffraction disc will be

A. zero

B. <0.1 mm

C. 0.1 mm

D. > 0.1 mm

Answer

From the theory of diffraction, the radius of diffraction varies directly with the wavelength. So, radius is smaller for smaller wavelengths (X-rays) and larger for larger wavelengths (visible light)

Objective II

1. Question

For harder X-rays,

A. the wavelength is higher

B. the intensity is higher

C. the frequency is higher

D. the photon energy is higher

Answer

Energy and wavelength decide the kind of x-ray (soft or hard) that will be generated. The hard x-rays have shorter wavelength and thus possess greater energy.

2. Question

Cutoff wavelength of X-rays coming from a Coolidge tube depends on the

A. target material

B. accelerating voltage

C. separation between the target and the filament

D. temperature of the filament

Answer

The cut-off wavelength is given by:

$\lambda_{min} = \frac{hc}{eV}$where, λ_{min} is the cut-off wavelength, h is the Planck's constant($6.62607004 \times 10^{-34}$ m^2 kg / s), c is the speed of light in vacuum, e is the charge of the electron and V is the potential difference applied at the ends of the tubeThus, it only depends on the potential and not the target or the temperature or the distance between the target and the filament.

3. Question

Mark the correct options.

A. An atom with a vacancy has smaller energy than a neutral atom.

B. K X-ray is emitted when a hole makes a jump from the K shell to some other shell.

C. The wavelength of K X-ray is smaller than the wavelength of L X-ray of the same material.

D. The wavelength of K_α X-ray of the same material.

Answer

An atom with vacancy has higher energy than a neutral atom. So, (A) is incorrect.K X-ray is created when a vacancy (hole) is filled in the K shell by an electron jumping from the successive shell to k-shell. So, the hole or vacancy is created in the shell where the electron jumps from. Hence, option (B) is correct.Characteristic K X-rays are produced when an electron from subsequent L, M, and N… shells drops to fill the vacant K shell. The energy of this transition is greater than the energy of transition form M, N, O… to L shell. Wavelength is inversely promotional to energy. So, KX rays have shorter wavelengths.The wavelengths for K and K are given as:

$\lambda_\alpha = \frac{hc}{E_K - E_L}\lambda_\beta = \frac{hc}{E_K - E_M}$where, λ_α and λ_β are the wavelength, h is the Planck's constant($6.62607004 \times 10^{-34}$ m^2 kg / s), c is the speed of light in vacuum, E_K-E_M and E_K-E_L are the difference in energy levels between the K and M shells and K and L shells.As, E_K-E_M is more than E_K-E_L, $\lambda_\alpha > \lambda_\beta$. So, (D) is incorrect.

4. Question

For a given material, the energy and wavelength of characteristic X-rays satisfy

A. E (K_α) > E (K_β) > E (K_γ)

B. E (M_α) > E (L_α) > E (K_α)

C. λ (K_α) > λ (K_β) > λ (K_γ)

D. λ (M_α) > λ (L_α) > λ (K_α)

Answer

Higher the shell from where the electron is transitioning, higher will be the energy. So energy of K_β transition is more than K_α . Energy is inversely promotional to wavelength. So, λ (K_α) > λ (K_β) > λ

(K_γ).M_α is a transition of an electron from N shell to M shell and requires much less energy than K_α transition from L shell to K shell. Thus, in terms of wavelength, $\lambda (M_\alpha) > \lambda (L_\alpha) > \lambda (K_\alpha)$.

5. Question

The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation,

A. the intensity increases

B. the minimum wavelength increases

C. the intensity remains unchanged

D. the minimum wavelength decreases.

Answer

From the equation of cut-off wavelength:

$\lambda_{min} = \frac{hc}{eV}$where, λ_{min} is the cut-off wavelength, h is the Planck's constant ($6.62607004 \times 10^{-34}$ m^2 kg / s), c is the speed of light in vacuum, e is the charge of the electron and V is the potential difference applied at the ends of the tubeIncreasing the potential (V) will decrease the minimum wavelength. Hence, option (D) is correct.Intensity depends on the heating current. As this is not changed, Intensity is unchanged. So, (C) is correct.

6. Question

When an electron strikes the target in a Coolidge tube, its entire kinetic energy

A. is converted into a photon

B. may be converted into a photon

C. is converted into heat

D. may be converted into heat

Answer

An electron striking the metal can lose its energy via collisions which generates heat and bremsstrahlung and/or can knock off electrons to produce characteristic x-rays. So, the energy may be converted to heat or to a photon.

7. Question

X-ray incident on a material

A. exerts a force on it

B. transfers energy to it

C. transfers momentum to it

D. transfers impulse to it

Answer

A X-ray can issue a force on the material as it's penetrating. Essentially they are made of photons of particular energy which can impart energy, impulse and momentum to the target atom's electron. So, all options are correct.

8. Question

Consider a photon of continuous X-ray and a photon of characteristic X-ray of the same wavelength. Which of the following is/are different for the two photons?

A. Frequency

B. Energy

C. Penetrating power

D. Method of creation

Answer

Here, the photons are of same wavelength. So they carry the same frequency from ν=c/λ.As frequency is same, the energies are same and thus their penetrating power is same. So, option (A), (B) and (C) are incorrect.Continuous x-rays are produced due to collisions of electrons in the target material whereas Characteristic x-rays are produced when the electrons knock off bounded electrons from the shells of the target atom. So the method of creation is different. Hence, option (D) is correct

Exercises

1. Question

Find the energy, the frequency and the momentum of an X-ray photon of wavelength 0.10 nm.

Answer

Given:

Wavelength of X-ray (λ)=0.10 nm or

$$\mathrm{E} = \mathrm{h}\vartheta$$

$$\vartheta = \frac{c}{\lambda}$$

$$E = \frac{hc}{\lambda}$$

$$E = \frac{(6.626 \times 10^{-34}.3 \times 10^{8})}{0.10 \times 10^{-9}}$$

E = 1.98×10^{-15} Joules/ photon

from above we can calculate, the frequency

$$\vartheta = \frac{c}{\lambda} = \frac{(3 \times 10^{8})}{0.10 \times 10^{-9}} = 3 \times 10^{18}\ hertz$$

to find the momentum, we use

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}}{(0.10 \times 10^{-9})} = 6.626 \times 10^{-24} kg.m/s$$

2. Question

Iron emits K_α X-ray of energy 6.4 keV and calcium emits K_α X-ray of energy 3.69 keV. Calculate the times taken by an iron K_α photon and a calcium K_α photon to cross through a distance of 3 km.

Answer

Given: Iron ray energy=6.4 Kev

Calcium ray of energy= 3.69 Kev

We have to calculate the time taken each of those rays to travel a distance of 3 Km

$t = \frac{Distance}{speed}$

$= \frac{3 \times 10^3}{3 \times 10^8}$

$t = 10^{-5} s \ or \ 10 \ \mu s$

Both the Ka photon and X-ray photon will take the same time that is 10μs

3. Question

Find the cutoff wavelength for the continuous X-rays coming from an X-ray tube operating at 30 kV.

Answer

Given:

X-ray tube operating at 30Kv

To calculate, cut-off wavelength we use

$\lambda = \frac{hc}{eV} = \frac{1242eV - nm}{e \times 30 \times 10^3} = 414 \times 10^{-4} nm$

At this wavelength cut-off occurs

4. Question

What potential difference should be applied across an X-ray tube to get X-ray of wavelength not less than 0.10 nm? What is the maximum energy of a photon of this X-ray in joule?

Answer

Given:

wavelength of X-ray $(\lambda) = 0.10 \ nm \ or \ 0.10 \times 10^{-9} m$

$\lambda = {}^{hC}/_{eV} \ or \ V = hc/e\lambda$

$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 10^{-10}}$

$= 12.4$ KV

To Find the Max. energy of photon of wavelength λ

$E = \frac{hc}{\lambda}$

$E = \frac{(6.626 \times 10^{-34} . 3 \times 10^8)}{0.10 \times 10^{-9}}$

Maximum energy of photon of this X-ray in joule $E = 1.98 \times 10^{-15}$ Joules/ photon

5. Question

The X-ray coming from a Coolidge tube has a cutoff wavelength of 80 pm. Find the kinetic energy of the electrons hitting the target.

Answer

Given:

Cutoff wavelength of the Coolidge tube,

λ=80pmEnergy of the electron hitting the target

E is given by

$$E = \frac{hc}{\lambda}$$

Here,

h=Planck's constant$c$ = Speed of light

λ= Wavelength of light

$$\therefore E = \frac{1242\times10^{-9}}{80\times10^{-12}}$$

$$\Rightarrow E = 15.54 \times 10^{3}\ eV$$

≈15.5 keV

6. Question

If the operating potential in an X-ray tube is increased by 1%, by what percentage does the cut-off wavelength decrease?

Answer

Given:

Let

λ be the cut off wavelength and V be the operating potential in the X-ray tube.Then,

$$\lambda = \frac{hc}{V}$$

Here,h = Planck's constant$c$ = Speed of light

If the operating voltage is increased by 1%, then the new operating voltage v' will be given by

V'=V+1/100×V= 1.01

$$V' = \frac{V+1}{100} \times V = 1.01$$

V Cut-off wavelength λ on increasing the operating voltage is given by

$$\lambda = \frac{\text{h}}{1.01}\ V = \frac{\lambda}{1.01}$$

difference in wavelength $= \ \lambda - \ \frac{\lambda}{1.01} = \frac{0.01}{\lambda 1.01}$

Percentage change in the wavelength is given by

$\frac{0.01}{\lambda 1.01} \times \lambda \times 100$ =11.01=0.9901 = 1% (approx.)

7. Question

The distance between the cathode (filament) and the target in an X-ray tube is 1.5 m. If the cutoff wavelength is 30 pm, find the electric field between the cathode and the target.

Answer

Given:

Distance between the filament and the target in the X-ray tube d=1.5m

cut-off wavelength= 30pm

Energy(E) is given by

$$E = \frac{hc}{\lambda}$$

thus,

$$E = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{30 \times 10^{-12}}\ Joules$$

$$= \frac{1242 \times 10^{-9}}{30 \times 10^{-12}}\ eV$$

$$= 41.4 \times 10^{3}\ eV$$

Now,

Electric field = V/d= $\frac{41.4 \times 10^{3}}{1.5}$

=27.6 kV/m

8. Question

The short-wavelength limit shifts by 26 pm when the operating voltage in an X-ray tube is increased to 1.5 times the original value. What was the original value of the operating voltage?

Answer

Given:

Let λ be the initial wavelength, V be the initial potential

Hence, λ' be the new wavelength and V' be the new operating voltage

When the operating voltage of the X-ray tube is increased to 1.5 times

$$\lambda' = \lambda - 26pm$$

$$V' = 1.5V$$

$$E = \frac{hc}{\lambda}$$

$$eV = \frac{hc}{\lambda}$$

V- operating potential

$$\lambda = \frac{hc}{eV}$$

$$\lambda V = \lambda' V'$$

$$\lambda V = \lambda - 26 \times 1.5V$$

$$0.5\lambda = 26 \times 1.5$$

$\lambda = 78pm$

Therefore the initial wavelength is $78 \times 10^{-12}m$

Now, the operating voltage V is given by

$$V = \frac{hc}{e\lambda}$$

$$V = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 78 \times 10^{-12}}$$

$$V = 15.9\ kV$$

9. Question

The electron beam in a color TV is accelerated through 32 kV and then strikes the screen. What is the wavelength of the most energetic X-ray photon?

Answer

Given:

potential across the X-ray Tube V= 32kV

Equation relating wavelength and energy (In terms of eV) is given by

$$\lambda = \frac{hc}{E}$$

$$\lambda = \frac{1242 \times 10^{-9}}{32 \times 10^{3}}$$

$$\lambda = 38.8\ pm$$

10. Question

When 40 kV is applied across an X-ray tube, X-ray is obtained with a maximum frequency of 9.7×10^{18} Hz. Calculate the value of Planck constant from these data.

Answer

Given:

potential applied to the X-ray tube V=40kV

Frequency of the X-ray= $9.7 \times 10^{18}hz$

wavelength of X-ray is given by,

$$\lambda = \frac{hc}{eV}$$

$$h = \frac{eV}{v}\ \because v = c/\lambda$$

$$h = \frac{40 \times 10^{3}}{9.7 \times 10^{18}} \times e$$

$$h = 4.12 \times 10^{-15}eVs$$

11. Question

An X-ray tube operates at 40 kV. Suppose the electron converts 70% of its energy into a photon at each collision. Find the lowest three wavelengths emitted from the tube. Neglect the energy imparted to the atom with which the electron collides.

Answer

Given:

Potential of the X-ray tube V= 40kv

energy of the X-ray = $40 \times 10^3 eV$

Energy utilized by the electron is given by

$$E = \frac{70}{100} \times 40 \times 10^3 eV = 28 \times 10^3 eV$$

Wavelength of the X-ray is given by

$$\lambda = \frac{hc}{eV}$$

$$\lambda = \frac{1242 \times 10^{-9}}{28 \times 10^3}$$

$$\lambda = 44.35\ pm$$

for the second wavelength

E=30%

$$E = \frac{70}{100} \times (40 - 28) \times 10^3 eV = 84 \times 10^2 eV$$

Therefore,

$$\lambda = \frac{hc}{eV}$$

$$\lambda = \frac{1242 \times 10^{-9}}{84 \times 10^2}$$

$$\lambda = 148\ pm$$

For the third wavelength,

$$E = \frac{70}{100} \times (12 - 8.4) \times 10^3 eV = 25.2 \times 10^3 eV$$

$$\lambda = \frac{1242 \times 10^{-9}}{25.2 \times 10^2}$$

$$\lambda = 493\ pm$$

12. Question

The wavelength of K_α X-ray of tungsten is 21.3 pm. It takes 11.3 keV to knock out an electron from the L shell of a tungsten atom. What should be the minimum accelerating voltage across an X-ray tube having tungsten target which allows production of K_α X-ray?

Answer

Given:

wavelength of X-ray = 21.3 pm

Energy required to knock out an electron from L-shell = 11.3 KeV

and the voltage required= 11.3 kV

The energy gap between K and L shell, is given by

$$E_k - E_L = \frac{1242 \times 10^{-9}}{21.3 \times 10^{-12}}$$

$$Ek - EL = 58.309\ KeV$$

$$EL = 11.3\ KeV$$

$$Ek = 69.609\ KeV$$

Thus the accelerating voltage across the X-ray tube for the production of kα ray is

V_k=69.609 keV

13. Question

The K_α X-ray of argon has a wavelength of 0.36 nm. The minimum energy needed to ionize an argon atom is 16 eV. Find the energy needed to knock out an electron from the K shell of an argon atom

Answer

Given:

Wavelength of Kβ X-ray of argon= 0.36 nm

Energy required to ionize an argon atom= 16eV

Energy of Kβ x-ray of argon E is given by,

$$E = \frac{1242 \times 10^{-9}}{0.36 \times\ 10^{-9}}$$

E= 3450 eV

Energy needed to knock out an electron from K shell

Ek=3450+16 eV

Ek=3.47 KeV (approx.)

14. Question

The K_α X-rays of aluminum (Z = 13) and zinc (Z = 30) have wavelengths 887 pm and 146 pm respectively. Use Moseley's law $\sqrt{v} = \alpha\ (Z - b)$ to find the wavelength of the K_α X-ray of iron (Z = 26).

Answer

Given:

Kα X-rays of zinc and aluminum have wavelengths, respectively

$$\lambda 1 = 887\ \text{pm}$$

$$\lambda 2 = 146\ pm$$

By using the formula

$v = c/\lambda$

frequencies of Kα X-rays of zinc and aluminum, respectively are

$v1 = 33.82 \times 10^{20}$

$v2 = 2.055 \times 10^{18}$

Using Moseley's law $\sqrt{v} = \alpha\ (Z - b)$,

for Aluminum,

$5.815 \times 10^{8} = a(13 - b)\ \ldots\ldots(1)$

for Zinc,

$1.4331 \times 10^{9} = a(30 - b)\ \ldots\ldots(2)$

Dividing equations 1 by 2,

$$\frac{13 - b}{30 - b} = \frac{5.815 \times 10^{8}}{1.4331 \times 10^{9}}$$

solving. we get

b= 1.3949

Therefore

a = 5×10^{7}

for Fe(Iron), Frequency is given by

$v = 5 \times 10^{7}(26 - 1.39)$

$v = 123.05 \times 10^{7}$

By using the formula

$v = c/\lambda$

wavelength of Kα X-rays given out by Iron atom is given by

$$\lambda = \frac{3 \times 10^{8}}{5.1413 \times 10^{14}} = 198\ pm$$

15. Question

A certain element emits K_{α} X-ray of energy 3.69 keV. Use the data from the previous problem to identify the element.

Answer

Given:

Energy of Kα X-rays= 3.69 KeV or 3690 eV

we're asked to identify the element which exhibits this behavior,

Wavelength is given by

$$\lambda = \frac{hc}{eV}$$

$$\lambda = \frac{1242 \times 10^{-9}}{3690} = 0.34 \times 10^{-9} m$$

Using Moseley's law $\sqrt{c/\lambda} = \alpha(Z - b)$,

$$\frac{\sqrt{3 \times 10^8}}{\sqrt{0.34 \times 10^{-9}}} = 5 \times 10^7 (Z - 1.39)$$

$$9.39 \times 10^8 = 5 \times 10^7 (Z - 1.39)$$

$$(9.39 \times 10^8) = 5 \times 10^7 (Z - 1.39)$$

$$\frac{9.39}{5} = (Z - 1.39)$$

Which gives us,

$$Z = \frac{9.39}{5} + 1.39$$

Z=20.17 (atomic number) which can be approximated to Z=20

Therefore, the element with z=20 is calcium (Ca)

16. Question

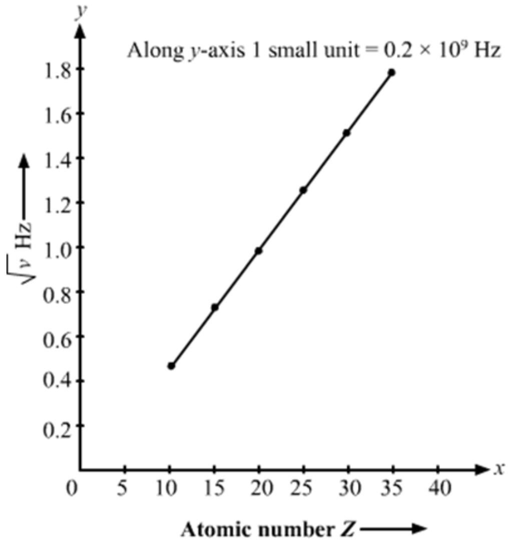
The K_β X-rays from certain elements are given below. Draw a Moseley-type plot of $\sqrt{v}$ versus Z for K_β radiation.

Answer

Given:

Element	Ne	P	Ca	Mn	Zn	Br
Energy (keV)	0.858	2.14	4.02	6.51	9.57	18.3

The required graph is as follows, the atomic number Z is plotted along x axis and $\sqrt{v}$ along y axis



17. Question

Use Moseley's law with b = 1 to find the frequency of the K_α X-ray of La (Z = 57) if the frequency of the K_α X-ray of Cu (Z = 29) is known to be 1.88×10^{16} Hz.

Answer

Given:

Using Moseley's law

$$\frac{\sqrt{c}}{\lambda} = \alpha(Z - b)\,, b = 1$$

comparing both conditions,

$$\frac{vLa}{vCu} = \left(\frac{ZLa - 1}{ZCu - 1}\right)^2$$

$$vLa = vCu\left(\frac{ZLa - 1}{ZCu - 1}\right)^2$$

$$vLa = 1.88 \times 10^{18}\left(\frac{ZLa - 1}{ZCu - 1}\right)^2$$

$$Wkt, ZLa = 57\ ZCu = 29$$

$$vLa = 1.88 \times 10^{18}\left(\frac{57 - 1}{29 - 1}\right)^2$$

$$vLa = 7.52 \times 10^{18} hz$$

18. Question

The K_α and K_β X-rays of molybdenum have wavelengths 0.71 Å and 0.63 Å respectively. Find the wavelength of L_α X-ray of molybdenum

Answer

Given:

$$K\alpha = Ek - EL\ \ldots(1)\ \lambda K\alpha = 0.71\ A^0$$

$K\beta = Ek - EM$ ... (2) $\lambda K\beta = 0.63\ A^0$

$L\alpha = EL - EM$ ... (3) $\lambda L\alpha = ??$

relating and comparing them together,

$K\alpha - K\beta = EM - EL = -L\alpha$

$$L\alpha = K\alpha - K\beta = \frac{3 \times 10^8}{0.63 \times 10^{-10}} - \frac{3 \times 10^8}{0.71 \times 10^{-10}}$$

simplifying

$L\alpha = 0.536 \times 10^{18} hz$

Therefore wavelength= $\lambda = {}^{c}/_{\vartheta} = \frac{3\times10^8}{0.536\times10^{18}}$

$= 5.6\ A^0$

19. Question

The wavelengths of K_α and L_α X-rays of a material are 21.3 pm and 141 pm respectively. Find the wavelength of K_β X-ray of the material

Answer

Given:

wavelength of $k\alpha$ rays =21.3 pm

wavelength of $k\beta$ rays =141 pm

Energy of $k\alpha$ rays E1 is given by

$$E1 = \frac{1242}{21.3 \times 10^{-3}} = 58.309 \times 10^3 eV$$

Energy of $l\alpha$ rays E2 is given by

$$E2 = \frac{1242}{141 \times 10^{-5}}$$

$= 8.8085 \times 10^3 eV$

Energy of $k\beta$ rays E3 is given by

$E3 = E1 + E2$

$E3 = (58.309 + 8.809) \times 10^3 eV$

$E3 = 67.118 \times 10^3 eV$

wavelength of $k\beta$ is given by

$$\lambda = \frac{hc}{E}$$

$$= \frac{1242}{67.118 \times 10^3}$$

$= 18.5\ pm$

20. Question

The energy of a silver atom with a vacancy in K shell is 25.31 keV, in L shell is 3.56 keV and in M shell is 0.530 keV higher than the energy of the atom with no vacancy. Find the frequency of K_α, L_β and L_α X-rays of silver.

Answer

Given:

Energy of electron in the K shell Ek= 25.31 KeV

Energy of electron in the L shell EL=3.56 Kev

Energy of the electron in the M shell Em=0.530 keV

let v1 be the frequency of $k\alpha$ x-ray , v2 and v3 be of the $k\beta$ and $l\alpha$ x-rays

kα x-ray is emitted when the transition takes place between l and k $shells$

$$K\alpha = Ek - EL = hv1v1 = \frac{Ek - EL}{h}v1 = \frac{25.31 - 3.56}{6.63 \times 10^{-34}} \times 1.6 \times 10^{-19} \times 10^3 v1 = 5.25 \times 10^{18} hz$$

kβ x-ray is emitted when the transition takes place between k and m shells

$$k\beta = Ek - Em = hv2v2 = \frac{25.31 - 0.53}{6.63 \times 10^{-34}} \times 1.6 \times 10^{-19} \times 10^3 v2 = 5.985 \times 10^{18} hz$$

Lα x-ray is emitted when the transition takes place between l and m shells

$$Kl = El - Em = hv3v3 = \frac{El - Em}{h}v3 = \frac{3.56 - 0.530}{6.63 \times 10^{-34}} \times 1.6 \times 10^{-19} \times 10^3 v3 = 7.32 \times 10^{17} hz$$

v1=5.25×10^{18} hz

v2=5.985×10^{18} hz

v3=7.32×10^{17} hz

v1,v2 and v3 are the frequencies of $K\alpha, k\beta$ and $L\alpha$ x-rays

21. Question

Find the maximum potential difference which may be applied across an X-ray tube with tungsten target without emitting any characteristic K or L X-ray. The energy levels of the tungsten atom with an electron knocked out are as follows.

Answer

Given:

Cell containing vacancy	K	L	M
Energy in keV	69.5	11.3	2.3

The potential that must be set across the X ray tube without emitting any characteristic K or l rays be 'V'

Energy of the electron relation, is given by

$E = eV$

This is the energy gap that L shells sit in. Therefore, If the voltage were to be set to a point without emitting any characteristic X rays,

The maximum potential difference that can be applied will be

11.3 eV, which corresponds to L shell.

22. Question

The electric current in an X-ray tube (from the target to the filament) operating at 40 kV is 10 mA. Assume that on an average, 1% of the total kinetic energy of the electrons hitting the target are converted into X-rays.

(a) What is the total power emitted as X-rays and

(b) how much heat is produced in the target every second?

Answer

Given:

operating voltage= 40Kv

current = 10mA

1% of total kinetic energy gives X-rays

$$i = ne \text{ or } n = \frac{10^{-2}}{1.6 \times 10^{-19}}$$

$n=0.625\times10^{17}$ (number of electrons)K.E of one electron$=eV=1.6\times10^{-19}\times40\times10^{3}=6.4\times10^{-15}$ joules$T_{ke}=0.625\times6.4\times10^{17}\times10^{-15}T_{ke}= 4\times10^{2}$ joules

b) Heat produced in the target per second

=400-4=396 Joules of heat

23. Question

Heat at the rate of 200 W is produced in an X-ray tub operating at 20 kV. Find the current in the circuit. Assume that only a small fraction of the kinetic energy of electrons is converted into X-rays.

Answer

Given:

Heat produced per second= power(P)=200W

operating potential= 20Kv

we have to find the current in the circuit

power is given by the equation, P=VI

V- potential difference and I- current

$I = \frac{ne}{t}$ ∵ since current is number of electrons passing per unit time

where, e= charge on an electron

n - no. of electrons

$P = \frac{neV}{t} 200 = \frac{neV}{t}\left(\frac{ne}{t}\right)V = 200$this gives,

$$I = \frac{200}{v}$$

$I = \frac{200}{20\times10^{3}} = 10\ mA$, is the current in the circuit.

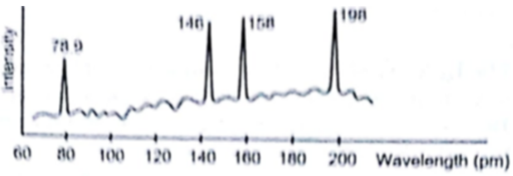
24. Question

Continuous X-rays are made to strike a tissue paper soaked with polluted water. The incoming X-rays excite the atoms of the sample by knocking out the electrons from the inner shells. Characteristic X-rays are subsequently emitted. The emitted X-rays are analysed and the intensity is plotted against the wavelength figure. Assuming that only K_{α} intensities are detected, list the elements present in the sample from the plot. Use Moseley's equation.

Answer

Given:

v = $(25 \times 10^{14}$ HZ) $(Z - 1)^{2}$.



Using Moseley's equation

v = (25 × 10^{14} HZ) (Z – 1)^2 $or \frac{c}{\lambda} = (25 \times 10^{14})(Z-1)^2$

I case

λ = 78.9× 10^{-12} mTherefore, $\frac{3 \times 10^8}{78.9 \times 10^{-12} \times 25 \times 10^{14}} = (Z-1)^2$

$(Z-1)^2 = 0.00152 \times 10^6 = 1520$

$z = 39.98 = 40(approx.)$Element with z = 40 is Zr - zirconium

II case

$\lambda = 146 \times 10^{-12} m$

$therefore,$

$$\frac{3 \times 10^8}{146 \times 10^{-12} \times 25 \times 10^{14}} = (Z-1)^2$$

$(Z-1)^2 = 0.000822 \times 10^6$

$z = 29.669 = 30(approx.)$Element with z =30 is Zn-zinc

III case

$\lambda = 158 \times 10^{-12} m$

Therefore,

$$\frac{3 \times 10^8}{158 \times 10^{-12} \times 25 \times 10^{14}} = (Z-1)^2$$

$(Z-1)^2 = 0.000759 \times 10^6$

$z = 28.5589 = 29(approx.)$Element with z = 29 is Cu-copper

VI case

$\lambda = 198 \times 10^{-12} m$

therefore,

$$\frac{3 \times 10^8}{198 \times 10^{-12} \times 25 \times 10^{14}} = (Z-1)^2$$

$(Z-1)^2 = 0.000606 \times 10^6$

Z = 25.6162 = 26(approx.)Element with Z = 26 is Fe-Iron

The elements Present in the polluted water are Zirconium, Zinc, Copper and Iron.

25. Question

A free atom of iron emits K_α X-rays of energy 6.4 keV. Calculate the recoil kinetic energy of the atom. Mass M an iron atom = 9.3 × 10^{-26} kg.

Answer

Given:

Energy of the X-ray= 6.4 keV

E= 6.4 KeV = $6.4 \times 10^3 eV$

Mass M an iron atom = 9.3 × 10^{-26} kg.

Momentum of the photon is given by

$$= \frac{E}{c} = \frac{6.4 \times 10^3}{3 \times 10^8}$$

=3.41×10^{-24} kg/sec

According to conservation of momentum,

momentum of photon= momentum of atom

Hence, momentum of the atom

p=3.41× 10^{-24} kg/sec

Recoil K.E of the atom, given by

$$= \frac{P^2}{2M}$$

$$= \frac{(3.41 \times 10^{-24})^2}{2 \times 9.3 \times 10^{-26} \times 1.6 \times 10^{-19}} \because wkt\ 1\ joule = 1.6 \times 10^{-19} eV$$

Recoil K.E of the atom = 3.9 eV

26. Question

The stopping potential in a photoelectric experiment is linearly related to the inverse of the wavelength (1/λ) of the light falling on the cathode. The potential difference inverse of the cut-off wavelength (1/λ) of the X-ray emitted. Show that the slopes of the lines in the two cases are equal and find its value.

Answer

Given:

Stopping potential is proportional to inverse of wavelength (1/λ)

let Vo be the stopping potential

v_1 and λ_1 be the operating frequency and voltage of the x-ray tubev_2 and λ_2 be the cut- off frequency and voltage of the x-ray tube$eV_0 = hv_1 - hv_2$since ,

$$\lambda 1 = \frac{hc}{eV}\text{and}$$

$$eV_0 = \frac{hc}{\lambda_2} V\lambda_1 = \frac{hc}{e}\ and\ V_0\lambda_2 = \frac{hc}{e}$$

in the above equations, slopes are the same

∴$V_0\lambda_2$=$V\lambda_1$

$$= \frac{hc}{e} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19}}$$

Value of the slope= 1.242×10^{-6} Vm

27. Question

Suppose a monochromatic X-ray beam of wavelength 100 pm is sent through a Young's double slit and the interference pattern is observed on a photographic plate placed 40 cm away from the slit. What should be the separation between the slits so that the successive maxima on the screen are separated by a distance of 0.1 mm?

Answer

wavelength of mono-chromatic beam $(\lambda) = 100 \times 10^{-12} m$

distance of screen from the slit $(D) = 40 \times 10^{-2}\ m$

Distance between successive maxima $(\beta) = 0.1 \times 10^{-3} m$

formula relating λ, D and β, given by

$$\beta = \frac{\lambda D}{d}$$

here, d is the separation between the slits

$= \frac{100 \times 10^{-12} \times 40 \times 10^{-2}}{10^{-3} \times 0.1}$ Distance between separation of each slit $= 4 \times 10^{-7} m$