Sample Paper 14

Class IX 2022-23

Mathematics

Time: 3 Hours **Max. Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 3 Qs of 5 marks, 3 Qs of 3 marks and 2 Questions of 2 marks has been provided.
8. Draw neat figures wherever required. Take $\pi = \frac{22}{7}$ wherever required if not stated.

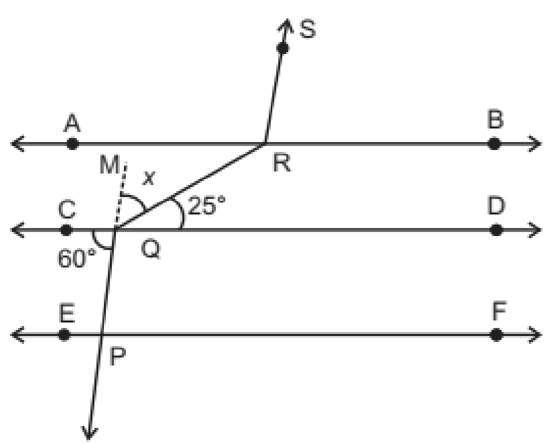
SECTION - A

(Section A consists of 20 questions of 1 mark each.)

1. Ram, Shyam and Sonu told their positions as (1, –2), (3, –1) and (1, 2) respectively. These points lie in:
(a) I quadrant
(b) II quadrant
(c) III quadrant
(d) Do not lie in the same quadrant **1**

2. If the length and breadth of a rectangle are 12 cm and 8 cm, respectively and if the perimeters of the square and rectangle are same, then the area of the square is:
(a) 64 cm^2 (b) 36 cm^2
(c) 100 cm^2 (d) 200 cm^2 **1**

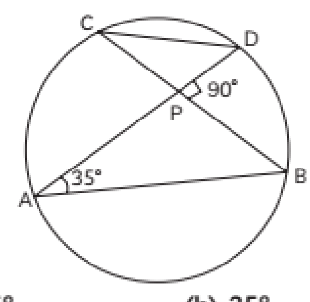
3. In the given figure, if AB || CD || EF, PQ || RS, $\angle$RQD = 25° and $\angle$CQP = 60°, then $\angle$QRS is equal to:



(a) 85° (b) 135°
(c) 145° (d) 110° **1**

4. The value of $1.\overline{79}$ in $\frac{p}{q}$ form, where p and q are integers, is:
(a) $\frac{179}{99}$ (b) $\frac{178}{99}$
(c) $\frac{176}{99}$ (d) $\frac{179}{99}$ **1**

5. Renu's mathematics teacher drew a circle on the board and make an angle $\angle$BAP = 35° and ask her students to find the $\angle$CDP. In the given figure, $\angle$CDP is:



(a) 55° (b) 35°
(c) 25° (d) 125° **1**

6. Which of the linear equations satisfies the values of $x = 1$, $y = 3.5$?
(a) $x + y = 7$ (b) $x - 2y = 8$
(c) $2y - x = 6$ (d) $y - x = 3$ **1**

7. How many square meters of the canvas is required for a conical tent whose height is 3.5 m and the radius of the base is 12 m?
(a) 474.12 m^2 (b) 470.42 m^2
(c) 471.42 m^2 (d) 417.42 cm^2 **1**

8. The remainder when $x^4 + 8x + 5$ is divided by $x + 4$ is:

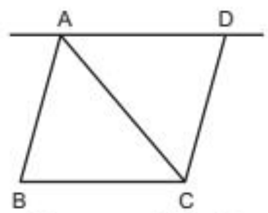
(a) 229 (b) 134
(c) –229 (d) –234 1

9. The class size of the grouped frequency table given below is:

5 – 5.2	5.2 – 5.4	5.4 – 5.6	5.6 – 5.8	5.8 – 6.0
34	4	4	4	6

(a) 0.2 (b) 1.5
(c) 0.9 (d) 1.2 1

10. In a triangle (as shown in fig) AB = CD, AD = BC and AC is the angle bisector of $\angle A$, then find which among the following conditions is true for congruence of ΔABC and ΔCDA by SAS rule?



(a) $\angle A = \angle D$ (b) $\angle B = \angle A$
(c) $\angle B = \angle D$ (d) $\angle C = \angle A$ 1

11. The perimeter of an isosceles triangle is 32 cm and the ratio of the equal side to its base is 3 : 2. The area of the isosceles triangle is:

(a) $64\sqrt{2}$ cm^2 (b) 32 cm^2
(c) $32\sqrt{2}$ cm^2 (d) $32\sqrt{3}$ cm^2 1

12. Ayush went to the store which sells trendy home decor items. He saw beautiful multicoloured round lampshades and bought two of them for the room.

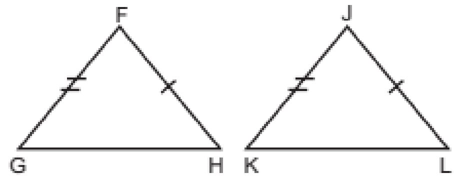
The radius of a sphere (in cm) whose volumes is 12π cm^3, is:

(a) 3 (b) $3\sqrt{3}$
(c) $3^{\frac{2}{3}}$ (d) $3^{\frac{1}{3}}$ 1

13. The supplement of .... degrees is the same as the sum of two complementary angles.

(a) 45 (b) 30
(c) 90 (d) 60 1

14. Shreyansh wants to prove that $\Delta FGH \cong \Delta JKL$ using SAS rule. He knows that FG = JK and FH = JL. What additional piece of information does he need?

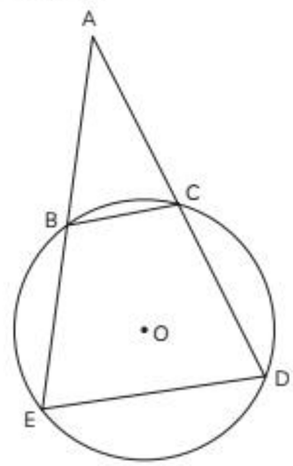


(a) $\angle F = \angle J$ (b) $\angle H = \angle L$
(c) $\angle G = \angle K$ (d) $\angle F = \angle G$ 1

15. If abscissa and ordinate of a point $P(a + 4, 2a - 1)$ are equal, then the value of a is:

(a) 5 (b) 1
(c) 4 (d) 6 1

16. In the given figure, AC = 8 cm, BC || DE. Also, EB and DC when produced meet at A. The length of AB is:



(a) 4 (b) 8
(c) 7 (d) 4 1

17. If the opposite angle of the parallelogram is 22 less, than thrice the other angle. Then the opposite angle is:

(a) 22° (b) 25°
(c) 28° (d) 30° 1

18. Which option shows $5y - 8x = 7(x + y) - 9$ expressed in the form of $ax + by + c = 0$?

(a) $-x + 6y - 9 = 0$ (b) $-x + 12y - 9 = 0$
(c) $15x + 2y - 9 = 0$ (d) $15x - 4y - 9 = 0$ 1

Direction: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of reason (R).

Choose the correct option.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

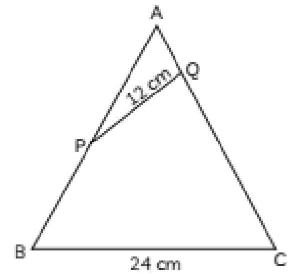
(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. Statement A (Assertion): The $\sqrt{2} \times \sqrt{7} = \sqrt{14}$, is an irrational number.

Statement R (Reason): The product of two Irrational numbers is an Irrational number. 1

20. Statement A (Assertion): In a triangle ABC, two points P and Q are on the sides AB and AC respectively such that PQ = 12 cm. If the length of BC is 24 cm, then PQ must be parallel to BC.



Statement R (Reason): The line segment joining the mid-points of two sides of a triangle is parallel to the third side. 1

SECTION - B

(Section B consists of 5 questions of 2 marks each.)

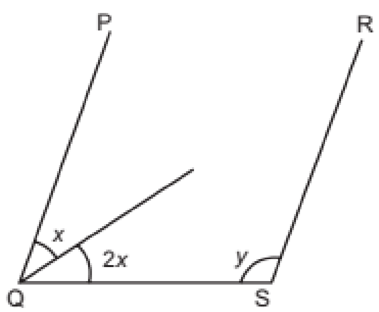
21. Find the value of k, if $x - 1$ is a factor of $p(x)$.

$$p(x) = 2x^2 + kx + \sqrt{2}$$

OR

Factorise $2x^2 + 7x + 3$. Using the middle term split method. 2

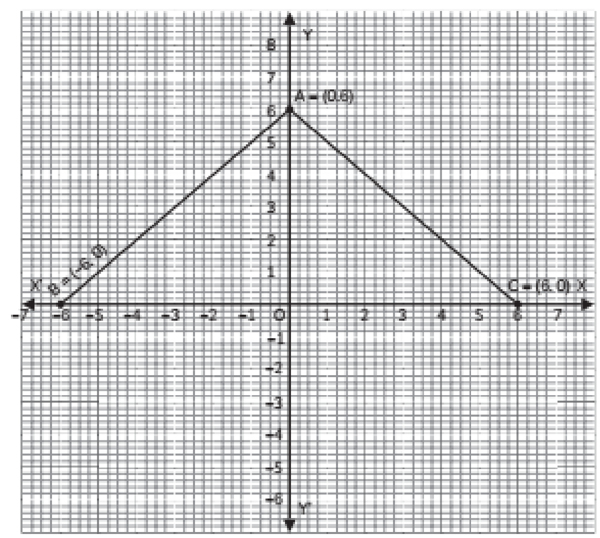
22. In the given figure, PQ || RS, and $x : y = 2 : 3$, then find the value of y.



2

23. Ram and Ravi have the same weight. If they each gain weight by 2 kg, how will their new weights be compared? 2

24. Fatima wants to find the area of triangle ABC. 2



25. A hemispherical bowl made of brass has an inner radius 5.25 cm. What is the total cost of tin-plating it on the inside at the rate of 4 per cm^2?

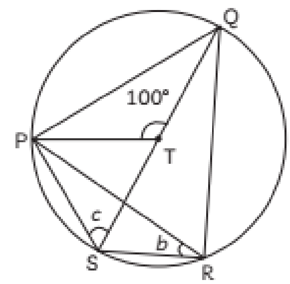
OR

The radius of a spherical balloon increases from 7 cm to 14 cm as air is being pumped into it. Find the ratio of surface areas of the balloon in the two cases. 2

SECTION - C

(Section C consists of 6 questions of 3 marks each.)

26. **In the quadrilateral PQRS inscribed in a circle with centre T, find the value of b and c.**



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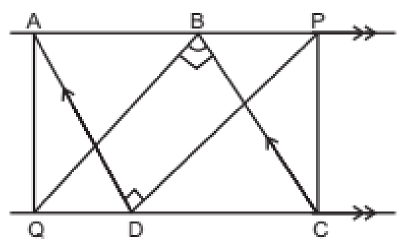
27. **Add $1.\overline{32}$ and $2.\overline{31}$, and find the result obtained in $\frac{p}{q}$ form.** 3

28. **ABCD is a parallelogram where $\angle ADP = \angle CBQ = 90°$ also ABP and QDC are straight lines. Prove that triangle ADP is congruent to CBQ. Also, explain why AQ is parallel to PC?**



3

29. **Electricity companies generate bills which are a combination of fixed monthly charges of ₹300 and charges based on the consumption, Atul's March bill is ₹1230. Now find a linear equation of the electricity bill (say y), for March month and the total bill of April and February (Non-leap). [Assuming the consumption depends upon the number of days]**

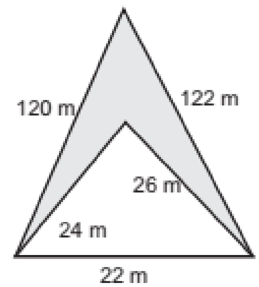
OR

In the equation $10x - 3y = 15$, Seema wants three different solutions. Find them. 3

30. **Factorise the following:**

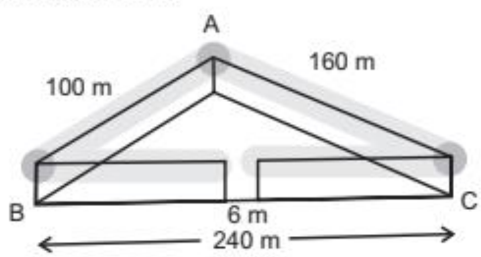
$2x^2 + y^2 + 8z^2 - 2\sqrt{2}\, xy + 4\sqrt{2}\, yz - 8xz.$ 3

31. **Calculate the area of the shaded region in the figure.**



OR

A triangular park ABC with sides 240 m, 160 m and 100 m situated in the Gokul Dham society as shown in the figure. A gardener, Nathulal has to put a fence all around it and also plant grass inside. How much area does he need to plant and also find the cost of fencing it with barbed wire at the rate of ₹20 per meter leaving a space 6 m wide for a gate on one side.



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SECTION - D

(Section D consists of 4 questions of 5 marks each.)

32. **Write any 5 solutions for the linear equation $2x + 3y = 7$.** 5

33. **Following is the frequency distribution of total marks obtained by the students of different sections of class VIII.**

Marks Obtained	Number of Students
100 – 150	60
150 – 200	100
200 – 300	100
300 – 500	80
500 – 800	180

Draw a histogram for the distribution above. **5**

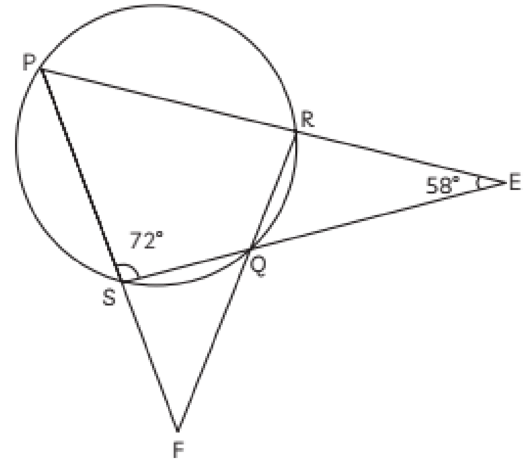
34. **If $a = \sqrt{3} + 2$, $b = 2 - \sqrt{3}$ then find the value of $\frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\left(\frac{a}{b}\right) + 2\left(\frac{b}{a}\right) - 222$**

OR

Simplify the following:

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}.$$ **5**

35. **In the figure shown below, side PR and SQ of a cyclic quadrilateral PRQS are produced to meet at E and sides PS and RQ are produced to meet at F. Find ∠RQS and ∠PFR, if ∠PSQ = 72° and ∠PES = 58°.**



OR

If non-parallel sides of a trapezium are equal, prove it is cyclic. **5**

SECTION - E

(Case study based questions are compulsory.)

36. *Swastik is a bright student in his class and he is selected to participate in the 'Inter-School Quiz Show'. In the show, the anchor gives him a situation that 3 years ago, the mother's age was 20 years more than the son's age. The sum of their present ages is 30 years. The anchor asks him to answer the following questions based on the given situation.*

(A) What could be the linear equation of the sum of present ages of the son and the mother in variables x and y respectively? **1**

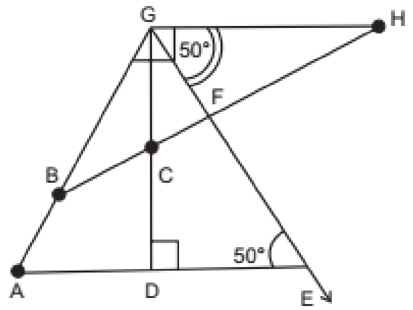
(B) What is the present age of the son?

OR

If the sister of the mother is 3 years younger than the mother, then find the age of the mother's sister. **2**

(C) Find the present age of the mother. **1**

37. *In the game period, the teacher of class IX divides the students of the class into groups of 8 students and instructs each student of one of the groups to stand at a particular spot and mark it. And then the teacher connects those positions with a rope as shown in the figure below. In the figure, points A, B, C, D, E, F, G, and H represent the positions of the students at certain points making certain angles with each other.*



(A) Find a linear pair of angles on the line AG. 1

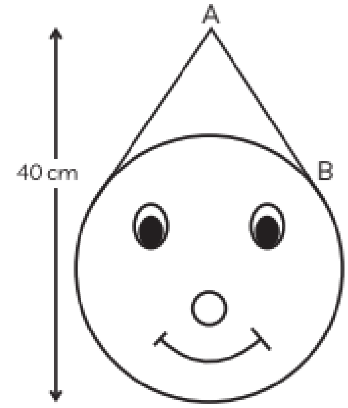
(B) Find an angle not adjacent to, but complementary to ∠FGC.

OR

If GF = GC, then what is the angle ∠DCF? 2

(C) Find an angle which is adjacent and supplementary to ∠DCB. 1

38. *On her birthday, Nisha brings a toy in the shape a sphere on one end and a cone on the other for her 5-year-old son. The spherical part's radius is 9 cm, respectively. The spherical and conical pieces have the same radii and the conical part is 40 cm tall.*



(A) Find the slant height of the conical part. 1

(B) Find the surface area of the speherical part, if the radius of sphere is 7 cm. 1

(C) Find the surface area of the toy.

OR

If Nisha wants to gift wrap the toy by using gift paper. Find the cost of paper used if 9 cm^2 paper ₹7. 2

SOLUTION

SAMPLE PAPER - 4

SECTION - A

1. *(d) Do not lie in the same quadrant.*

Explanation: Point (1, –2) and (3, –1), x-coordinate is positive and y coordinate is negative.

So, they lie in IV Quadrant

Point (1, 2), x and y coordinates are positive. So, they lie in the I Quadrant. So, given points do not lie in the same Quadrant.

Caution

➥ *The coordinates describe a point in the plane uniquely point (x, y) is not the same as (y, x), i.e., (x, y) ≠ (y, x).*

2. *(c) 100 cm².*

Explanation: Given, length of the rectangle,

$l = 12$ cm

Breadth of the rectangle,

$b = 8$ cm

We have, perimeter of the rectangle

$= 2(l + b)$

$= 2(12 + 8)$ cm

$= 40$ cm

Since, Perimeter of a square

= Perimeter of rectangle

$4 \times \text{side} = 40$ cm

$\text{side} = \frac{40}{4}$ cm

$\text{side} = 10$ cm

Now, area of the square $= (\text{side})^2$

$= (10)^2$ cm^2

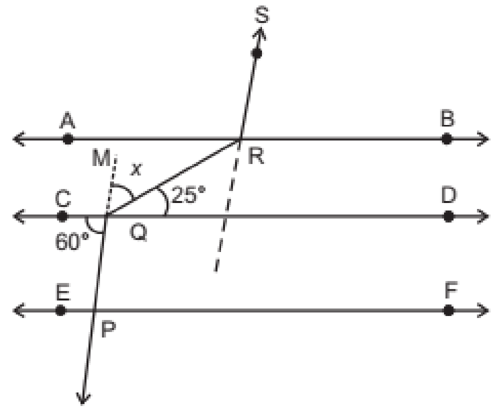
Area of square $= 100$ cm^2.

3. *(c) 145°.*

Explanation: According to question,

PQ || RS, produce PQ to M

∠CQP = ∠MQD



$$60° = x + 25°$$
$$x = 60° - 25°$$
$$x = 35°$$

QM ∥ RS and QR is transversal

$$\angle ARQ = \angle RQD = 25°$$ [Alternate angle]

$$x + [\angle ARQ + \angle ARS] = 180°$$
$$35° + [25° + \angle ARS] = 180°$$
$$60° + \angle ARS = 180°$$
$$\angle ARS = 180° - 60°$$
$$\angle ARS = 120°$$
$$\Rightarrow \quad \angle QRS = \angle ARQ + \angle ARS$$
$$\Rightarrow \quad = 25° + 120° = 145°$$

4. *(b)* $\frac{178}{99}$

Explanation: Let

$$x = 1.\overline{79} \quad ...(i)$$

Multiply equation (i) by 100, we get

$$100x = 179.79...$$
$$100x = 178 + 1.79...$$
$$100x = 178 + x \quad \text{[From (i)]}$$
$$99x = 178$$
$$x = \frac{178}{99}$$

Caution

➥ *Students get confused in identifying the number of digits that should be multiplied. So remember, the multiplication depends upon the number of digits containing the bar. If 2 digits contain a bar multiply by 100, if four digits contain a bar then by 10000.*

5. *(a) 55°*

Explanation: $\angle DCB = \angle BAD$

[Angles made on the same segment]

$$\angle DCB = 35°$$

Now, in ΔCDP

$$\angle C + \angle D + \angle P = 180°$$ [Angle sum property]

$$35° + \angle D + 90° = 180°$$ $[\angle DPB = \angle DPC = 90°]$

$$\angle D = 180° - 35° - 90°$$
$$\angle D = 55°$$

6. *(c) $2y - x = 6$*

Explanation: Here, given that $x = 1$, $y = 3.5$. Now, we will check one by one.

Option (a) $x + y = 7$

$$1 + 3.5 = 4.5 \neq 7$$

Option (b) $x - 2y = 8$

$$1 - 2 \times 3.5 = 1 - 7 = -6 \neq 8$$

Option (c) $2y - x = 6$

$$2 \times 3.5 - 1 = 7 - 1 = 6 = 6$$

Option (d) $y - x = 3$

$$3.5 - 1 = 3.5 - 1 = 2.5 \neq 3$$

Clearly, from the above, $x = 1$, $y = 3.5$ is a solution of $2y - x = 6$.

7. *(c) 471.42 m^2*

Explanation: Height of conical tent,

$$h = 3.5 \text{ m}$$

The radius of the base,

$$b = 12 \text{ m}$$

Slant height, $l^2 = r^2 + h^2$

$$\Rightarrow \quad l = \sqrt{(12)^2 + (3.5)^2}$$
$$\Rightarrow \quad l = \sqrt{144 + 12.25}$$
$$\Rightarrow \quad l = \sqrt{156.25}$$
$$\Rightarrow \quad l = 12.5 \text{ m}$$

The canvas required for a conical tent is equal to the volume of a cone.

Volume of the cone $= \pi r l$

$$V = \frac{22}{7} \times 12 \times 12.5$$
$$V = 471.42 \text{ m}^2.$$

8. *(a) 229*

Explanation: Let given polynomials be:

$$p(x) = x^4 + 8x + 5 \quad ...(i)$$
$$g(x) = x + 4$$

Then, $p(x)$ is divided by $g(x)$,

For finding the zero of $g(x)$, put $g(x) = 0$

$$x + 4 = 0$$
$$\Rightarrow \quad x = -4$$

So, it is the zero of $g(x)$.

On putting $x = -4$ in equation (i), we get

$$p(-4) = (-4)^4 + 8(-4) + 5$$
$$= 256 - 32 + 5$$

$\Rightarrow \qquad p(-4) = 229$

Hence, the value of $p(-4)$ is 229, which is the required remainder obtained by dividing $x^4 + 8x + 5$ by $x + 4$.

9. *(a)* 0.2

Explanation: Class size is the difference between the upper and lower limit of a class in a frequency distribution.

Therefore, for any given class,

$$5.2 - 5 = 5.4 - 5.2 = 5.6 - 5.4 = 5.8 - 5.6$$
$$= 6.0 - 5.8$$
$$= 0.2.$$

10. *(c)* $\angle B = \angle D$

Explanation: As In ΔABC and ΔCDA,

$$AB = CD$$
$$AD = BC$$

For SAS Rule, if two sides and the included angles of one triangle are equal to the two sides and the included angle of the other triangle, then triangles are congruent.

Therefore,

For $\qquad \Delta ABC \cong \Delta CDA$

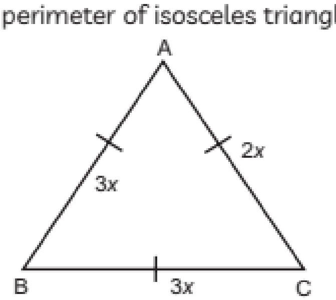
by SAS, $\angle B$ must be equal to $\angle D$

11. *(c)* $32\sqrt{2}$ cm^2

Explanation: Given, the ratio of the equal side to the base of the isosceles triangle is 3 : 2.

Let the equal sides and the base of the isosceles triangle be $3x$, $3x$ and $2x$ respectively.

Since, perimeter of isosceles triangle = 32 cm



$\Rightarrow \qquad 3x + 3x + 2x = 32$ cm

$\Rightarrow \qquad 8x = 32$ cm

$\Rightarrow \qquad x = 4$ cm

Therefore, the sides of the isosceles triangle are 12 cm, 12 cm and 8 cm.

Let $a = 12$ cm, $b = 12$ cm and $c = 8$ cm

Now, semi-perimeter (s) of the triangle

$$= \frac{a+b+c}{2} = \frac{12+12+8}{2}$$
$$= 16 \text{ cm}$$

We have, area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{16(16-12)(16-12)(16-8)}$$
$$= \sqrt{16 \times 4 \times 4 \times 8}$$
$$= \sqrt{2048}$$
$$= 32\sqrt{2} \text{ cm}^2$$

12. *(c)* $3^{\frac{2}{3}}$

Explanation: Let 'r' cm be the radius of the sphere.

Then, $\qquad \frac{4}{3}\pi r^3 = 12\pi$

$\Rightarrow \qquad r^3 = 9$

i.e., $\qquad r^3 = 3^2$

$\Rightarrow \qquad r = (3^2)^{\frac{1}{3}}$

$\Rightarrow \qquad r = 3^{\frac{2}{3}}$

13. *(c)* 90

Explanation: We have, the sum of two supplementary angles is 180°.

If one the supplementary angles is 90°, then other supplementary angle = 180° – 90° = 90°

Thus, the supplement of 90° is same as the sum of two complementary angles.

14. *(a)* $\angle F = \angle J$

Explanation: We know for SAS, if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle then the triangles are congruent.

So, $\angle F = \angle J$

15. *(a)* 5

Explanation:

Abscissa of P = x-coordinate = $a + 4$

Ordinate of P = y-coordinate = $2a - 1$

Since, $\qquad a + 4 = 2a - 1$

$\Rightarrow \qquad a - 2a = -1 - 4$

$\Rightarrow \qquad -a = -5$

$\Rightarrow \qquad a = 5$

16. *(b)* 8

Explanation: Given that BC || ED so,

$\qquad \angle ABC = \angle AED$

and $\qquad \angle ACB = \angle ADE$

[Corresponding angles]

But, BCDE is a cyclic quadrilateral

$\therefore \qquad \angle ABC = \angle ADE$

and $\qquad \angle ACB = \angle AED \Rightarrow \angle ABC = \angle ACB$

Thus, in ΔABC,

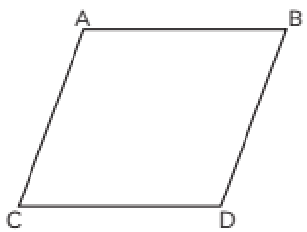
$$\angle ABC = \angle ACB$$
$$AB = AC$$
[Sides of isosceles triangle]
$$AB = 8 \text{ cm}$$
Hence, $\quad AC = 8$ cm.

17. *(a) 22°*

Explanation: Let the other angle be x.

$\therefore$ The opposite angle is $2x - 22$.

We know, the opposite angles of the parallelogram are equal.



$\therefore \quad x = 2x - 22°$

$\Rightarrow \quad x - 2x = -22°$

$\Rightarrow \quad -x = -22°$

$\Rightarrow \quad x = 22°$

Then, opposite angle $= 2x - 22°$

$= 2 \times 22° - 22°$

$= 22°$.

18. *(c) $15x + 2y - 9 = 0$*

Explanation: Here,

$5y - 8x = 7(x + y) - 9$

$\Rightarrow \quad 5y - 8x = 7x + 7y - 9$

$\Rightarrow \quad 5y - 8x - 7x - 7y + 9 = 0$

$\Rightarrow \quad -15x - 2y + 9 = 0$

Multiply both side by -1, we get

$15x + 2y - 9 = 0$

Hence, it is the standard form of the given equation.

19. *(c) Assertion (A) is true but reason (R) is false.*

Explanation: Given,

$$\sqrt{2} \times \sqrt{7} = \sqrt{2 \times 7} = \sqrt{14} \quad (\because \sqrt{a}\sqrt{b} = \sqrt{ab})$$

Clearly, $\sqrt{2} \times \sqrt{7} = \sqrt{14}$ is an irrational number. The product of two irrational numbers can be a rational or an irrational number depending upon two numbers.

20. *(d) Assertion (A) is false but reason (R) is true.*

Explanation: PQ is parallel to BC only if P and Q are mid-points of AB and AC respectively.

SECTION - B

21. Let $\quad g(x) = x - 1$

$g(x) = 0 = x - 1 \Rightarrow x = 1$

So, $\quad p(x) = 2x^2 + kx + \sqrt{2}$

$\Rightarrow \quad p(1) = 2(1)^2 + k(1) + \sqrt{2}$

$\Rightarrow \quad = 2 + k + \sqrt{2} \quad$ ...(i)

As $(x - 1)$ is a factor of $p(x)$

$\therefore \quad p(x) = 0$

$2 + k + \sqrt{2} = 0 \quad$ [From Equation (i)]

$k = -[2 + \sqrt{2}]$.

OR

$2x^2 + 7x + 3$

Here $a = 2$, $b = 7$ and $c = 3$

We need to split $7x$ into two parts $7x = 6x + x$

The product of the coefficients of terms is 2×3 is 6.

The sum of the coefficents terms is 7

So, we get

$f(x) = 2x^2 + 7x + 3$

$= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$f(x) = (2x + 1)(x + 3)$

22. In the given figure, PQ || RS, then,

$\angle PQS + \angle RSQ = 180°$

[Sum of interior angles]

$(x + 2x) + y = 180°$

$3x + y = 180°$

Since, $\quad x : y = 2 : 3$

let $x = 2a$, $y = 3a$

$\therefore \quad 3 \times (2a) + 3a = 180°$

$\Rightarrow \quad 9a = 180°$

$\Rightarrow \quad a = 20°$

Thus, the value of $y = 3 \times 20° = 60°$.

23. **Given:** Ram and Ravi have the same weight.

To find: New weights of Ram and Ravi.

Let the weight of Ravi and Ram be x kg.

After gaining 2 kg, their weight becomes $(x + 2)$ kg.

From Euclid's second axiom, when equals are added to equals, the wholes will be equal.

Therefore, the weight of Ram and Ravi will be equal.

24. As per the given graph, points are A(0, 6), B(–6, 0) and (6, 0)

$$\text{Area of triangle} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times BC \times OA$$

As $BC = 6 -(-6)$

$= 12$ units

$OA = 6$ units.

$\therefore$ Area of Triangle $= \frac{1}{2} \times 12 \times 6$

$= 36$ sq. units.

25. The radius of hemispherical bowl

$= 5.25$ cm

The surface area of hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times (5.25)^2$$

$= 173.25 \text{ cm}^2$

Cost of tin-plating 1 cm^2

$= ₹4$

Cost of tin-plating 173.25 cm^2

$= ₹4 \times 173.25$

$= ₹693.$

OR

Given,

Radius of balloon

$= r = 7$ cm

Radius of pumped balloon

$= R = 14$ cm

Ratio of surface area

$$= \frac{\text{TSA of balloon with } r = 7 \text{ cm}}{\text{TSA of balloon with } R = 14 \text{ cm}}$$

$$\Rightarrow \quad = \frac{(4\pi r^2)}{(4\pi R^2)}$$

$$\Rightarrow \quad = \frac{r^2}{R^2}$$

$$\Rightarrow \quad = \frac{(7)^2}{(14)^2}$$

$$\Rightarrow \quad = \frac{49}{196}$$

$$\Rightarrow \quad = \frac{1}{4}$$

Hence, the ratio of surface areas of the balloon in the two cases is 1 : 4.

SECTION - C

26. On-line SQ,

$\angle PTQ + \angle PTS = 180°$ [Linear pair]

$100° + \angle PTS = 180°$

$\angle PTS = 180° - 100°$

$\angle PTS = 80°$ ...(i)

Now, from segment PS, the angle made on the centre is double the angle by the same segment on the circumference.

$\angle PTS = 2\angle PRS$

$\Rightarrow \quad 80° = 2 \times b$

$\Rightarrow \quad b = 40°$

Similarly, $\angle PTQ = 2\angle PSQ$

[PQ is common]

$\Rightarrow \quad 100° = 2 \times c$

$$\Rightarrow \quad c = \frac{100°}{2}$$

$\Rightarrow \quad c = 50°.$

27. Convert into $\frac{p}{q}$ form, we get

Let $x = 1.\overline{32}$ ...(i)

Multiply by 100 on both side, we get

$100x = 132.\overline{32}$. ...(ii)

Subtract (i) from (ii), we get

$100x - x = 132.\overline{32} - 1.\overline{32}$

$\Rightarrow \quad 99x = 131$

$$\Rightarrow \quad x = \frac{131}{99}$$

Similarly, convert into $2.\overline{31}$ form, we get

Let $y = 2.\overline{31}$ ...(iii)

Multiply by 100

$100y = 231.\overline{31}$ ...(iv)

Subtract (iii) from (iv), we get

$100y - y = 231.\overline{31} - 2.\overline{31}$

$\Rightarrow \quad 99y = 229$

$$\Rightarrow \quad y = \frac{229}{99}$$

Now, $$x + y = \frac{131 + 229}{99}$$

$\Rightarrow \quad = \dfrac{360}{99}$

$\Rightarrow \quad = \dfrac{40}{11}$

28. In ΔADP and ΔCBQ,

$\angle ADP = \angle CBQ = 90^\circ$ [Given]

$AD = CB$

[Opposite sides of parallelogram are equal]

$\angle DAB = \angle BCD$

[Opposite angles of parallelogram are equal]

So, by ASA Congruence Rule,

$\Delta ADP \cong \Delta CBQ$,

Since ΔADP and ΔCBQ are congruent

$AP = CQ$ and also parallel

$\therefore$ APCQ is a parallelogram, so

$AQ = PC$.

29. Since, the bill consists of fixed and variable charges.

Therefore, ₹1230 is the March bill, where ₹300 is fixed charge in it.

$\therefore$ Bill = Fixed charge + variable charge

= ₹300 + ₹930

Since, March having 31 days. Therefore,

Bill for 31 days = ₹930

$\therefore$ Bill for 1 day = $\dfrac{930}{31}$ = ₹30

Thus, the required linear equation is

$y = 300 + 30x$

or, $30x - y + 300 = 0$,

where y = bill amount and x = number of days.

Now, for April, number of days = 30

Bill amount, $y = 300 + 30 \times 30$

$= 300 + 900$

y = ₹1200

For February, number of days = 28

Bill amount, $y = 300 + 30 \times 28$

$= 300 + 840$

y = ₹1140

Thus, the total bill for April and February

= ₹1200 + ₹1140

= ₹2340.

OR

The given equation is

$10x - 3y = 15$ ...(i)

Put $x = 0$ in equation (i), we get

$10(0) - 3y = 15$

$\Rightarrow \quad 0 - 3y = 15$

$\Rightarrow \quad y = \dfrac{15}{-3}$

$\therefore \quad y = -5$

So, the solution set is $(0, -5)$.

Now, put $y = 0$ in equation (i), we get

$10x - 3(0) = 15$

$10x = 15$

$\Rightarrow \quad x = \dfrac{15}{10}$

$\therefore \quad x = \dfrac{3}{2}$

So, the other solution is $\left(\dfrac{3}{2}, 0\right)$

Now, put $x = 3$ in equation (i), we get

$10(3) - 3y = 15$

$\Rightarrow \quad 30 - 3y = 15$

$\Rightarrow \quad -3y = 15 - 30$

$\Rightarrow \quad -3y = -15$

$\Rightarrow \quad y = \dfrac{15}{3}$

$\therefore \quad y = 5$

So, one more solution set is $(3, 5)$.

Thus, the required three different solutions of the given equation are $(0, -5)$ $\left(\dfrac{3}{2}, 0\right)$ and $(3, 5)$.

30. $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

$= \left(\sqrt{2}\right)^2 x^2 + y^2 + \left(2\sqrt{2}\right)^2 z^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

$= \left(-\sqrt{2}x\right)^2 + y^2 + \left(2\sqrt{2}z\right)^2 - 2\sqrt{2}\,xy + 4\sqrt{2}\,yz - 8xz$

$= \left(-\sqrt{2}x\right)^2 + y^2 + \left(2\sqrt{2}z\right)^2 - 2\left(\sqrt{2}x\right)(y) + 2(y)\left(2\sqrt{2}z\right) + 2\left(-\sqrt{2}x\right)\left(2\sqrt{2}z\right)$

Using $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

Putting $a = -\sqrt{2}\,x$, $b = y$, $c = 2\sqrt{2}\,z$

$= \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)^2$

$= \left(-\sqrt{2}x + y + 2\sqrt{2}z\right)\left(-\sqrt{2}x + y + 2\sqrt{2}z\right)$

31. For triangle, having the sides 122 m, 120 m and 22 m:

$$s = \frac{a+b+c}{2} = \frac{122+120+22}{2}$$
$$= \frac{264}{2} = 132 \text{ m}$$

Area of triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{132(132-122)(132-120)(132-22)}$$
$$= \sqrt{132 \times 10 \times 12 \times 110}$$
$$= 1320 \text{ m}^2$$

For the triangle having the sides 22 m, 24 m and 26 m:

$$s = \frac{a+b+c}{2} = \frac{22+24+26}{2} = \frac{72}{2}$$
$$= 36 \text{ m}$$

Area of the triangle

$$= \sqrt{36(36-22)(36-24)(36-26)}$$
$$= \sqrt{36 \times 14 \times 12 \times 10}$$
$$= 24\sqrt{105} = 24 \times 10.25 = 246 \text{ m}^2$$

(approx)

Therefore, area of the shaded region

$$= (1320 - 246) \text{ m}^2$$
$$= 1074 \text{ m}^2.$$

OR

Here $a = 240$, $b = 160$, $c = 100$

$$s = \frac{240+160+100}{2} = 250$$

Area of triangular park

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{250(250-240)(250-160)(250-100)}$$
$$= \sqrt{250 \times 10 \times 90 \times 150}$$
$$= \sqrt{2500 \times 3 \times 3 \times 3 \times 5 \times 10 \times 10}$$
$$= 50 \times 3 \times 10\sqrt{15} = 1500\sqrt{15} \text{ m}^2$$

Therefore, area of needed to plant

$$= 1500\sqrt{15} \text{ m}^2$$

Now, to find the cost of fencing, we need to calculate the total required length of fence.

Length of fence to be required

$$= 100 + 160 + 240 - 6$$
$$= 500 - 6 = 494 \text{ m}$$

Since, the cost of fencing

= ₹20 per meter

Total cost of fencing park

= ₹20 × 494

= ₹9880.

SECTION - D

32. Given equation is

$$2x + 3y = 7$$

Putting $x = 0$

$\Rightarrow \quad 2(0) + 3y = 7$

$\Rightarrow \quad 0 + 3y = 7$

$\Rightarrow \quad 3y = 7$

$\Rightarrow \quad y = \frac{7}{3}$

So, $\left(0, \frac{7}{3}\right)$ is a solution.

Putting $y = 0$

$\Rightarrow \quad 2x + 3(0) = 7$

$\Rightarrow \quad 2x + 0 = 7$

$\Rightarrow \quad 2x = 7$

$\Rightarrow \quad x = \frac{7}{2}$

So, $\left(\frac{7}{2}, 0\right)$ is a solution.

Putting $x = 1$

$\Rightarrow \quad 2(1) + 3y = 7$

$\Rightarrow \quad 2 + 3y = 7$

$\Rightarrow \quad 3y = 7 - 2$

$\Rightarrow \quad 3y = 5$

$\Rightarrow \quad y = \frac{5}{3}$

So, $\left(1, \frac{5}{3}\right)$ is a solution.

Putting $y = 1$

$\Rightarrow \quad 2x + 3(1) = 7$

$\Rightarrow \quad 2x + 3 = 7$

$\Rightarrow \quad 2x = 7 - 3$

$\Rightarrow \quad 2x = 4$

$\Rightarrow \quad x = 2$

So, (2, 1) is a solution.

Putting $x = 3$

$\Rightarrow \quad 2(3) + 3y = 7$

$\Rightarrow \quad 6 + 3y = 7$

$\Rightarrow \quad 3y = 7 - 6$

$\Rightarrow \quad 3y = 1$

$\Rightarrow \quad y = \frac{1}{3}$

So, $\left(3, \frac{1}{3}\right)$ is a solution.

Thus, the five solutions of the given equation are:

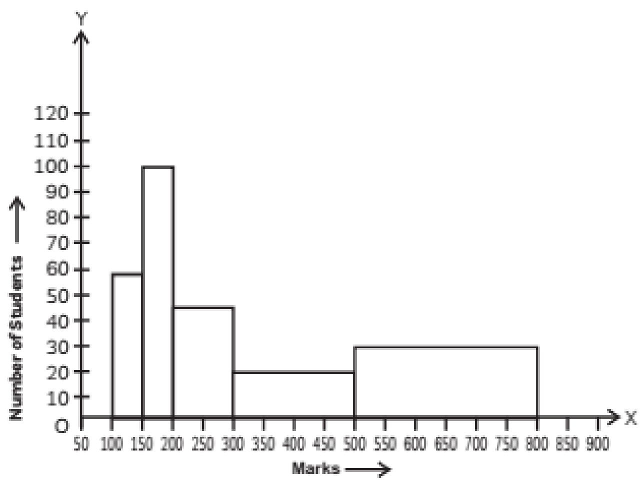
x	0	7/2	1	2	3
y	7/3	0	5/3	1	1/3

33. The widths of the class intervals vary for each data in the table given. These widths serve as the width of the rectangle in the histograms.

So, before drawing histogram, the length of the rectangles is to be found in each case. This is due to the property of histogram that the area of the rectangles should be proportional to the frequencies.

Length of each rectangle is given as $\frac{c}{C} \times f$, where c is the minimum class width, C is the class width of the particular class and f is the frequency. In the above case $c = 50$. Consider the following table.

Marks Obtained	Frequency	Width of the Class (C)	Length of the Rectangle
100 – 150	60	50	$\frac{50}{50} \times 60 = 60$
150 – 200	100	50	$\frac{50}{50} \times 100 = 100$
200 – 300	100	100	$\frac{50}{100} \times 100 = 50$
300 – 500	80	200	$\frac{50}{200} \times 80 = 20$
500 – 800	180	300	$\frac{50}{300} \times 180 = 30$



34.

$a = 2 + \sqrt{3}$

$b = 2 - \sqrt{3}$

$$\frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} \qquad \text{[Rationalise]}$$

$$\frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$[(a+b)(a-b) = a^2 - b^2]$$

$$\frac{a}{b} = \frac{7+4\sqrt{3}}{1} \qquad [(a+b)^2 = a^2 + b^2 + 2ab]$$

$$= 7 + 4\sqrt{3}$$

$$\frac{a^2}{b^2} = (49 + 48 + 56\sqrt{3}) = 97 + 56\sqrt{3}$$

Now if $\frac{a}{b} = 7 + 4\sqrt{3}$

Then $\frac{b}{a} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$

$$= \frac{7-4\sqrt{3}}{(7)^2 - (4\sqrt{3})^2}$$

$$= \frac{7-4\sqrt{3}}{49-48}$$

$$= \frac{7-4\sqrt{3}}{1} = 7 - 4\sqrt{3}$$

$$\frac{b^2}{a^2} = (7)^2 + (4\sqrt{3})^2 - 2 \times 7 \times 4\sqrt{3}$$

$$= 49 + 48 - 56\sqrt{3}$$

$$= 97 - 56\sqrt{3}$$

$$2\left(\frac{a}{b}\right) = 2(7 + 4\sqrt{3}) = 14 + 8\sqrt{3}.$$

$$2\left(\frac{b}{a}\right) = 2(7 - 4\sqrt{3}) = 14 - 8\sqrt{3}.$$

So, $= \frac{a^2}{b^2} + \frac{b^2}{a^2} + 2\left(\frac{a}{b}\right) + 2\left(\frac{b}{a}\right) - 222$

$= 97 + 56\sqrt{3} + 97 - 56\sqrt{3} + 14 + 8\sqrt{3} + 14 - 8\sqrt{3} - 222$

$= 194 + 28 - 222$

$= 222 - 222 = 0$

OR

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \quad ...(i)$$

Suppose,

$$P = \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}},\ Q = \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \text{ and}$$

$$R = \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

Now,

$$P = \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} \times \frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}}$$

$$= \frac{7\sqrt{30}-21}{10-3} = \frac{(7\sqrt{30}-3)}{7} = \sqrt{30}-3$$

$$Q = \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}}$$

$$= \frac{2\sqrt{30}-10}{6-5}$$

$= 2\sqrt{30} - 10$

$$R = \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \times \frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}$$

$$= \frac{3\sqrt{30}-18}{-3}$$

$= -\sqrt{30} + 6$

$\therefore P - Q - R$

$= (\sqrt{30} - 3) - (2\sqrt{30} - 10) - (-\sqrt{30} + 6)$

$= \sqrt{30} - 3 - 2\sqrt{30} + 10 + \sqrt{30} - 6$

$= 2\sqrt{30} - 2\sqrt{30} - 3 + 10 - 6$

$= 1.$

35. Here, in ΔPES,

$\angle EPS + \angle PES + \angle ESP = 180^\circ$

[Angle sum property]

$\angle EPS + 58^\circ + 72^\circ = 180^\circ$

$\angle EPS = 180^\circ - 130^\circ$

$\angle EPS = 50^\circ$

Or, $\angle RPS = 50^\circ$

Now, $\angle PSQ + \angle PRQ = 180^\circ$

[Opposite angles of a cyclic quadrilateral are supplementary]

$\Rightarrow \quad 72^\circ + \angle PRQ = 180^\circ$

$\Rightarrow \quad \angle PRQ = 180^\circ - 72^\circ$

$\Rightarrow \quad \angle PRQ = 108^\circ$

Similarly,

$\angle RPS + \angle RQS = 180^\circ$

$\Rightarrow \quad 50^\circ + \angle RQS = 180^\circ$

$\Rightarrow \quad \angle RQS = 180^\circ - 50^\circ$

$\Rightarrow \quad \angle RQS = 130^\circ$

Now, In ΔPRF,

$\angle RPF + \angle PRF + \angle PFR = 180^\circ$

[Angle sum property]

$108^\circ + 50^\circ + \angle PFR = 180^\circ$

$\Rightarrow \quad 158^\circ + \angle PFR = 180^\circ$

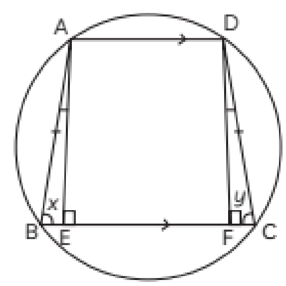
$\Rightarrow \quad \angle PFR = 180^\circ - 158^\circ$

$\Rightarrow \quad \angle PFR = 22^\circ$

Hence, $\angle PFR = 22^\circ$

and $\angle RQS = 130^\circ$

OR



ABCD is a trapezium in which AD ∥ BC and its non-parallel sides AB and DC are equal.

It is an isosceles trapezium.

To prove: ABCD is a cyclic quadrilateral

Construction: Draw AE ⊥ BC and DF ⊥ BC

Proof: In ΔABE and ΔDCF

$\angle AEB = \angle DFC$ [Each 90°]

$AB = DC$ [Given]

$AE = DF$

[Perpendicular distance between 2 parallel lines are same]

$\Delta ABE \cong \Delta DCF$

[By RHS congruence Rule]

So, $\angle B = \angle C$ [By CPCT] ...(i)

And $x = y$ ...(ii)

Now, $\angle BAD = \angle x + \angle EAD$

$= \angle x + 90^\circ$ ...(iii)

Similarly, $\angle CDA = \angle y + 90^\circ$ ...(iv)

[From (ii)]

In Quadrilateral ABCD,

$\angle B + \angle C + \angle CDA + \angle BAD = 360^\circ$

$\angle B + \angle B + \angle CDA + \angle CDA = 360^\circ$

[From (i), (iii) and (iv)]

$2[\angle B + \angle CDA] = 360^\circ$

$\angle B + \angle CDA = 180^\circ$

So, opposite angles of trapezium ABCD are supplementary.

Hence, trapezium ABCD is cyclic.

SECTION - E

36. (A) Let, the present age of son be x years, and the present age of mother be y years.

Since, the sum of present ages of the son and the mother is 30.

$\therefore \quad x + y = 30$

Thus, the required linear equation will be

$x + y = 30.$

(B) Let, the present age of son be x years, and the present age of the mother be y years.

3 years ago,

Son's age = $(x - 3)$ years

Mother's age = $(y - 3)$ years

According to question,

$(y - 3) = (x - 3) + 20$

$\Rightarrow (y - 3) - (x - 3) = 20$

$\Rightarrow \quad y - 3 - x + 3 = 20$

$\Rightarrow \quad y - x = 20$

$\Rightarrow \quad y = 20 + x \quad$ ...(i)

Also, we have, the sum of their present ages is 30 years.

$x + y = 30$

Putting the value of y from the equation (i), we get,

$x + 20 + x = 30$

$\Rightarrow \quad 2x = 30 - 20$

$\Rightarrow \quad x = \frac{10}{2}$

$x = 5$ years

Thus, the present age of the son is 5 years.

OR

Present age of the mother

= 25 years [From (B)]

Since, the mother's sister is 3 years younger than mother.

$\therefore$ Her mother's sister age

= 25 – 3

= 22

Thus, the age of the mother's sister is 22 years.

(C) $x = 5$ years [From (B)]

$y = 20 + x$ [From (i)]

So, we get

$y = 5 + 20$

$\therefore \quad y = 25$ years

Thus, the present age of the mother is 25 years.

37. (A) We have, the sum of the linear pair is 180°.

From the figure, $\angle ABC$ and $\angle GBC$ make a linear pair at vertex (B)

$\therefore \angle ABC + \angle GBC = 180^\circ$

(B) From the figure,

$\angle FGC = \angle CGH - \angle FGH$

$= 90^\circ - 50^\circ$

$\angle FGC = 40^\circ$

Complementary of

$\angle FGC = 90^\circ - 40^\circ = 50^\circ$

$\angle GED = 50^\circ$

OR

From the figure, in ΔGCF,

$\angle C + \angle G + \angle F = 180^\circ$

But GC = GF

So, by isosceles angle property,

$\angle C = \angle F$

Now, $\angle C + \angle G + \angle C = 180^\circ$

$\angle C + 90^\circ - 50^\circ + \angle C = 180^\circ$

$2\angle C + 40^\circ = 180^\circ$

$2\angle C = 140^\circ$

$\angle C = 70^\circ$

$\angle GCF = 70^\circ$

Clearly, $\angle DCF$ is linear pair of $\angle GCF$.

$\angle DCF = 180^\circ - 70^\circ$

$= 110^\circ.$

(C) Adjacent angles have a common arm and the sum of supplementary angles is 180°. From the figure,

$\angle BCG + \angle DCB = 180^\circ$

Thus, the required angle is $\angle BCG$.

38. (A) AB is the slant height of the cone.

Here, $r = 9$ cm, $h = 40$ cm

We know, $l^2 = r^2 + h^2$

$l^2 = (9)^2 + (40)^2$

$l^2 = 81 + 1600$

$l^2 = 1681$

$l = \sqrt{1681}$

$l = 41$ cm

Hence, the start height of the cone is 41 cm.

(B) The curved surface area of sphere

$= 4\pi r^2$

Here, $r = 7\ cm$

Curved surface area $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 7 \times 7$

$= 616$ cm^2

(C) Let r cm be the radius and h cm be the height of the conical part.

Then $r = 9$ cm, $h = 40$ cm

Also, $l = 41$ cm [as part A]

Now,

The surface area of the toy

= surface area of spherical part + curved surface area of the conical part

$= 616 + \pi rl$

$\Rightarrow \quad 616 + \frac{22}{7} \times 9 \times 41$

$= 616 + 1159.71$

$= 1775.71$ cm^2

OR

To wrap the toy we need to focus on the curved surface area of the toy.

As in the last question, we calculated the curved surface area of toy

The curved surface area of toy

$= 1775.71$ cm^2

Cost of wrapping paper for 9 cm^2 = ₹7

Cost of wrapping paper for 1775.71 cm^2

$= \frac{1775.71 \times 7}{9}$

= ₹1381.10

The total cost of wrapping paper is ₹1381.10.