Nuclei

Case Study Based Questions

Case Study 1

The nucleus of an atom consists of a tightly packed arrangement of protons and neutrons. These are the two heavy particles in an atom and hence 99.9% of the mass is concentrated in the nucleus. Of the two, the protons possess a net positive charge and hence the nucleus of an atom is positively charged on the whole and the negatively charged electrons revolve around the central nucleus. Since, the mass concentration at the nucleus of an atom is immense the nuclear forces holding the protons and the neutrons together are also large.

Read the given passage carefully and give the answer of the following questions:

Q 1. The nuclide $^{238}_{92}$ U has all the following except:

a. 92 b. 146

c. 238 d. O

Q2. The density of a nucleus is of the order of:

a. 10^{15} kg m⁻³ b. 10^{18} kg m⁻³

c. $10^{17} \text{ kg m}^{-3}$ d. $10^{16} \text{ kg m}^{-3}$

Q3. Nuclear force is:

a. strong, short range and charge independent force

b. charge independent, attractive and long range force

c. strong, charge dependent and short range attractive force

d. long range, charge dependent and attractive force

Q4. The mass number of a nucleus is M and its atomic number is Z. The number of neutrons in the nucleus is:

a. M – Z b. M

c. Z d. M + Z

Solutions

1. (d) 0

The nuclide $^{238}_{92}$ U has number of protons 92, number of neutrons 146 (238 – 92) and number of nucleons 238 (146 + 92). Hence, nuclide $^{238}_{92}$ U has all *i.e.*, 92, 146 and 238 except 0.

- **2.** (c) 10^{17} kg m⁻³
- **3.** (a) strong, short range and charge independent force
- **4.** (a) M Z

Case Study 2

Apsara is the oldest of India's research reactors. The reactor was designed by the Bhabha Atomic Research Centre (BARC) and built with assistance from the United Kingdom. A nuclear reactor, formerly known as an atomic pile, is a device used to initiate and control a self-sustained nuclear chain reaction. Nuclear reactors are used at nuclear power plants for electricity generation and in nuclear marine propulsion. Heat from nuclear fission is passed to a working fluid (water or gas), which in turn runs through steam turbines.



Read the given passage carefully and give the answer of the following questions:

Q1. The splitting of a nucleus into smaller nuclei is:

a. fusion b. fission

c. half-life d. gamma radiation

Q2. Name the moderator used in the nuclear reactor.

a. Plutonium b. Thorium

c. Graphite d. Beryllium

- Q3. Which isotope of uranium (U) has the capacity to sustain the chain reaction?
- a. U-230

b. U-235

c. U-245

d. U-225

- Q4. What is the beneficial aspect of nuclear fission?
- a. The ability to absorb energy
- b. The ability to produce more energy than nuclear fusion
- c. The ability to release tremendous amount of energy
- d. There are no beneficial aspects of nuclear fission

Solutions

- 1. (b) fission
- 2. (c) Graphite
- 3. (b) U-235
- **4.** (c) The ability to release tremendous amount of energy.

Case Study 3

Neutrons and protons are identical particle in the sense that their masses are nearly the same and the force, called nuclear force, does into distinguish them. Nuclear force is the strongest force. Stability of nucleus is determined by the neutron-proton ratio or mass defect or packing fraction. Shape of nucleus is calculated by quadrupole moment and spin of nucleus depends on even or odd mass number. Volume of nucleus depends on the mass number. Whole mass of the atom (nearly 99%) is centered at the nucleus.

Read the given passage carefully and give the answer of the following questions:

- Q1. What is the range of nuclear force?
- Q2. A force between two protons is same as the force between proton and neutron. What is the nature of the force?
- Q3. Two protons are kept at a separation of 40 Å. If F_n is the nuclear force and F_e is the electrostatic force between them, then what is the relation between F_n and F_e ?
- Q4. All the nucleons in an atom are held by which forces?

Solutions

- **1.** The nuclear force is of short range and the <u>range of nuclear force</u> is the order of 1.4×10^{-15} m.
- **2.** The nature of the force is strong nuclear force.
- **3.** Nuclear force is much stronger than the electrostatic force inside the nucleus i.e., at distances of the order of fermi. At 40 Å, nuclear force is ineffective and only electrostatic force of repulsion is present. This is very high at this distance because nuclear force is not acting now and the gravitational force is very feeble. Thus,

 $F_{\text{nuclear}} \ll F_{\text{electrostatic}}$ in this case.

4. All the nucleons in an atom are held by nuclear forces.

Case Study 4

When subatomic particles undergo reactions, energy is conserved, but mass is not necessarily conserved. However, a particle's mass contributes to its total energy, in accordance with Einstein's famous equation, $E = mc^2$. In this equation, E denotes the energy carried by a particle because of its mass. The particle can also have additional energy due to its motion and its interactions with other particles. Consider a neutron at rest and well separated from other particles. It decays into a proton, an electron and an undetected third particle as given here:

Neutron \rightarrow Proton + Electron +?

The given table summarizes some data from a single neutron decay, Electron volt is a unit of energy. Column 2 shows the rest mass of the particle times the speed of light squared.

Particle	$\frac{\text{Mass} \times c^2}{(\text{MeV})}$	Kinetic energy (MeV)
Neutron	940.97	0.00
Proton	939.67	0.01
Electron	0.51	0.39

Read the given passage carefully and give the answer of the following questions:

Q1. From the given table, which properties of the undetected third particle can be calculate?

- Q2. Assuming the table contains no major errors, what can we conclude about the (mass c^2) of the undetected third particle?
- Q3. Could this reaction occur?

Proton → Neutron + Other particles

Q4. How much mass has to be converted into energy to produced electric power of 500 MW for one hour?

Solutions

- **1.** As just shown, energy conservation allows us to calculate the third particle's total energy. But we do not know what percentage of that total energy is mass energy.
- **2.** According to the given information, subatomic reactions do not conserve mass. So, we cannot find the third particle's mass by setting

$$m_{neutron} = m_{proton} + m_{electron} + m_{third particle}$$

The neutron has energy 940.97 MeV. The proton has energy 939.67 MeV + 0.01 MeV = 939.68 MeV. The electron has energy 0.51 MeV + 0.39 MeV

= 0.90 MeV. Therefore, the third particle has energy

$$E_{third\ particle} = E_{neutron} - E_{proton} - E_{electron}$$

We just found the third particle's total energy, the sum of its mass energy and kinetic energy. Without more information, we cannot figure out how much of that energy is mass energy.

3. Yes, but only if the proton has potential energy (due to interactions with other particles).

4. Given,
$$P = 500 \text{ MW} = 5 \times 10^8 \text{ W}$$
, $t = 1 \text{ h} = 3600 \text{ s}$

Energy produced,

$$E = P \times t = 5 \times 10^8 \times 3600 = 18 \times 10^{11} J$$

As
$$E = \Delta mc^2$$

$$\Delta m = \frac{E}{c^2} = \frac{18 \times 10^{11}}{(3 \times 10^8)^2} = \frac{18 \times 10^{11}}{9 \times 10^{16}} = 2 \times 10^{-5} \text{ kg}$$

Case Study 5

In the year 1939, German scientist Otto Hahn and Strassmann discovered that when an uranium isotope was bombarded with a neutron, it breaks into two intermediate mass fragments. It was observed that, the sum of the masses of new fragments formed were less than the mass of the original nuclei. This difference in the mass appeared as the energy released in the process. Thus, the phenomenon of splitting of a heavy nucleus (usually A > 230) into two or more lighter nuclei by the bombardment of proton, neutron, α -particle, etc. with liberation of energy is called nuclear fission.

$$^{235}_{92}\text{U} + ^{1}_{0}n \rightarrow ^{236}_{92}\text{U} \rightarrow ^{144}_{56}\text{Ba} + ^{89}_{36}\text{Kr} + 3^{1}_{0}n + Q$$
Unstable nucleus

Read the given passage carefully and give the answer of the following questions:

- Q1. Nuclear fission can be explained on which basis?
- Q2. Which of the following is/are fission reaction(s)?

(i)
$${}^1_0n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{133}_{51}Sb + {}^{99}_{41}Nb + 4{}^1_0n$$

(ii)
$${}^1_0n + {}^{235}_{92}U \rightarrow {}^{1.40}_{54}Xe + {}^{94}_{38}Sr + 2{}^1_0n$$

(iii)
$${}_{1}^{2}H + {}_{1}^{2}H \rightarrow {}_{2}^{6}He + {}_{0}^{1}n$$

- Q3. What is the energy of a neutron released per fission of a uranium atom?
- Q4. In any fission process, what is the ratio of mass of daughter nucleus to mass of parent nucleus?

Solutions

- 1. Nuclear fission can be explained on the basis of <u>liquid drop model</u>.
- **2.** Reactions I and II represent fission of uranium isotope $^{235}_{92}$ U, when bombarded with neutrons that breaks

it into two intermediate mass nuclear fragments. However, reaction III represents two deuterons fuses together to form the light isotope of helium.

- 3. The energy of the neutron released per fission of the uranium atom is 2 MeV.
- **4.** In fission process, when a parent nucleus breaks into daughter products, then some mass is lost in the form of energy. Thus,

Mass of fission products < Mass of parent nucleus.

$$\Rightarrow \frac{\text{Mass of fission products}}{\text{Mass of parent nucleus}} < 1$$

Solutions for Questions 6 to 11 are Given Below

Case Study 6

Nuclear Fission

In the year 1939, German scientist Otto Hahn and Strassmann discovered that when an uranium isotope was bombarded with a neutron, it breaks into two intermediate mass fragments. It was observed that, the sum of the masses of new fragments formed were less than the mass of the original nuclei. This difference in the mass appeared as the energy released in the process. Thus, the phenomenon of splitting of a heavy nucleus (usually A > 230) into two or more lighter nuclei by the bombardment of proton, neutron, α -particle, etc with liberation of energy is called nuclear fission.

$${}_{92}\mathrm{U}^{235} + {}_{0}n^{1} \rightarrow {}_{92}\mathrm{U}^{236} \rightarrow {}_{56}\mathrm{Ba}^{144} + {}_{36}\mathrm{Kr}^{89} + 3{}_{0}n^{1} + Q$$
Unstable nucleus

- (i) Nuclear fission can be explained on the basis of
 - (a) Millikan's oil drop method
 - (b) Liquid drop model
 - (c) Shell model
 - (d) Bohr's model.
- (ii) For sustaining the nuclear fission chain reaction in a sample (of small size) of ²³⁵₉₂U, it
 is desirable to slow down fast neutrons by
 - (a) friction

(b) elastic damping/scattering

(c) absorption

(d) none of these.

(iii) Which of the following is/are fission reaction(s)?

(I)
$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{236}_{92}U \rightarrow {}^{133}_{51}Sb + {}^{99}_{41}Nb + {}^{1}_{0}n$$

(II)
$${}_{0}^{1}n + {}_{92}^{235}U \rightarrow {}_{54}^{1.40}Xe + {}_{38}^{94}Sr + 2{}_{0}^{1}n$$

$$(III)_{1}^{2}H + {}_{1}^{2}H \rightarrow {}_{2}^{3}He + {}_{0}^{1}n$$

(a) Both II and III

(b) Both I and III

(c) Only II

(d) Both I and II

(iv)	On an average, the number of neutrons and the energy of a neutron released per fission of a uranium atom are respectively					
	(a) 2.5 and 2 keV	(b) 3 and 1 keV	(c)	2.5 and 2 MeV	(d)	2 and 2 keV
(v)	In any fission process, ratio (a) less than 1 (c) equal to 1	of mass of daughter nucle	(b)	mass of parent nucleu greater than 1 depends o the mass		rent nucleus.
Cas	se Study 7					
Dis	covery of Nucleus					
The nucleus was first discovered in 1911 by Lord Rutherford and his associates by experiments on scattering of α -particles by atoms. He found that the scattering results could be explained, if atoms consist of a small, central, massive and positive core surrounded by orbiting electrons. The experimental results indicated that the size of the nucleus is of the order of 10^{-14} m and is thus 10000 times smaller than the size of atom.						
(i)	Ratio of mass of nucleus w (a) 1	ith mass of atom is approximate (b) 10		ly 10 ³	(d)	10^{10}
(ii)	Masses of nuclei of hydrog (a) 1:2:3	en, deuterium and tritium a (b) 1:1:1		n ratio 1:1:2	(d)	1:2:4
(iii)	Nuclides with same neutro (a) isobars	n number but different ator (b) isotopes		number are isotones	(d)	none of these
(iv)	If <i>R</i> is the radius and <i>A</i> is to (a) a straight line	he mass number, then log <i>R</i> (b) a parabola		sus log A graph will be an ellipse		none of these.
(v)	The ratio of the nuclear rac (a) 1.23	lii of the gold isotope ¹⁹⁷ ₇₉ Au (b) 0.216		silver isotope $^{107}_{47}$ Au is 2.13		3.46
Cas	e Study 8					
Nuclear Energy A heavy nucleus breaks into comparatively lighter nuclei which are more stable compared to the original heavy nucleus. When a heavy nucleus like uranium is bombarded by slow moving neutrons, it splits into two parts releasing large amount of energy. The typical fission reaction of $_{92}U^{235}$. $_{92}U^{235} + _{0}n^{1} \rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3_{0}n^{1} + 200 \text{ MeV}$						
The	fission of 92U235 approxima	tely released 200 MeV of en	ergy			
(i)	If 200 MeV energy is relead produce a power of 1 kW is (a) 3.125 × 10 ¹³					
(ii)	ii) The release in energy in nuclear fission is consistent with the fact that uranium has (a) more mass per nucleon than either of the two fragments (b) more mass per nucleon as the two fragment (c) exactly the same mass per nucleon as the two fragments (d) less mass per nucleon than either of two fragments.					
(iii)	(iii) When $_{92}$ U ²³⁵ undergoes fission, about 0.1% of the original mass is converted into energy. The energy released when 1 kg of $_{92}$ U ²³⁵ undergoes fission is					y. The energy released
	(a) $9 \times 10^{11} \text{ J}$	(b) $9 \times 10^{13} \text{ J}$	(c)	$9\times 10^{15}J$	(d)	$9\times10^{18}J$

(iv) A nuclear fission is said to be critical when multiplication factor or K (b) K > 1 (a) K = 1(c) K < 1</p> (d) K = 0(v) Einstein's mass-energy conversion relation $E = mc^2$ is illustrated by

(b) β-decay

Case Study 9

(a) nuclear fission

Nuclear Force

Neutrons and protons are identical particle in the sense that their masses are nearly the same and the force, called nuclear force, does into distinguish them. Nuclear force is the strongest force. Stability of nucleus is determined by the neutron proton ratio or mass defect or packing fraction. Shape of nucleus is calculated by quadrupole moment and spin of nucleus depends on even or odd mass number. Volume of nucleus depends on the mass number. Whole mass of the atom (nearly 99%) is centred at the nucleus.

- (i) The correct statements about the nuclear force is/are
 - (a) change independent

(b) short range force

(c) rocket propulsion (d) steam engine

(c) non-conservative force

- (d) all of these.
- (ii) The range of nuclear force is the order of
 - (a) 2×10^{-10} m
- (b) 1.5×10^{-20} m
- (c) 1.2×10^{-4} m (d) 1.4×10^{-15} m
- (iii) A force between two protons is same as the force between proton and neutron. The nature of the force is
 - (a) electrical force
- (b) weak nuclear force
- (c) gravitational force (d) strong nuclear force.
- (iv) Two protons are kept at a separation of 40 Å. F_n is the nuclear force and F_e is the electrostatic force between them. Then
 - (a) $F_n \ll F_e$
- (b) $F_n = F_o$
- (c) $F_n >> F_o$ (d) $F_n \approx F_o$

- (v) All the nucleons in an atom are held by
 - (a) nuclear forces

(b) van der Waal's forces

(c) tensor forces

(d) coulomb forces

Case Study 10

Nuclear Density

The density of nuclear matter is the ratio of the mass of a nucleus to its volume. As the volume of a nucleus is directly proportional to its mass number A, so the density of nuclear matter is independent of the size of the nucleus. Thus, the nuclear matter behaves like a liquid of constant density. Different nuclei are like drops of this liquid, of different sizes but of same density.

Let A be the mass number and R be the radius of a nucleus. If m is the average mass of a nucleon, then Mass of nucleus = mA

Volume of nucleus
$$=\frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

... Nuclear density,
$$\rho_{\text{nu}} = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$$
 or $\rho_{\text{nu}} = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$

Clearly, nuclear density is independent of mass number A or the size of the nucleus.

The nuclear mass density is of the order 10¹⁷ kg m⁻³. This density is very large as compared to the density of ordinary matter, say water, for which $\rho = 1.0 \times 10^3 \text{ kg m}^{-3}$.

- (i) The nuclear radius of $^{16}_{8}$ O is 3×10^{-15} m. The density of nuclear matter is
 - (a) $2.9 \times 10^{34} \text{ kg m}^{-3}$ (b) $1.2 \times 10^{17} \text{ kg m}^{-3}$ (c) $16 \times 10^{27} \text{ kg m}^{-3}$ (d) $2.4 \times 10^{17} \text{ kg m}^{-3}$

- (ii) What is the density of hydrogen nucleus in SI units? Given $R_0 = 1.1$ fermi and $m_p = 1.007825$ amu.
 - (a) $2.98 \times 10^{17} \text{ kg m}^{-3}$
- (b) $3.0 \times 10^{34} \text{ kg m}^{-3}$ (c) $1.99 \times 10^{11} \text{ kg m}^{-3}$ (d) $7.85 \times 10^{17} \text{ kg m}^{-3}$

- (iii) Density of a nucleus is
 - (a) more for lighter elements and less for heavier elements
 - (b) more for heavier elements and less for lighter elements
 - (c) very less compared to ordinary matter
 - (d) a constant.
- (iv) The nuclear mass of ${}^{56}_{26}$ Fe is 55.85 amu. The its nuclear density is
- (a) $5.0 \times 10^{19} \text{ kg m}^{-3}$ (b) $1.5 \times 10^{19} \text{ kg m}^{-3}$ (c) $2.9 \times 10^{17} \text{ kg m}^{-3}$ (d) $9.2 \times 10^{26} \text{ kg m}^{-3}$
- (v) If the nucleus of $^{27}_{13}$ Al has a nuclear radius of about 3.6 fm, then $^{125}_{52}$ Te would have its radius approximately as
 - (a) 9.6 fm
- (b) 12 fm
- (c) 4.8 fm
- (d) 6 fm

Case Study 11

Mass-Energy

When subatomic particles undergo reactions, energy is conserved, but mass is not necessarily conserved. However, a particle's mass "contributes" to its total energy, in accordance with Einstein's famous equation, $E = mc^2$. In this equation, E denotes the energy carried by a particle because of its mass. The particle can also have additional energy due to its motion and its interactions with other particles. Consider a neutron at rest and well separated from other particles. It decays into a proton, an electron and an undetected third particle as given here: Neutron \rightarrow proton + electron + ???

The given table summarizes some data from a single neutron decay. Electron volt is a unit of energy. Column 2 shows the rest mass of the particle times the speed of light squared.

Particle	$Mass \times c^2 (MeV)$	Kinetic energy (MeV)
Neutron	Neutron 940.97	
Proton	939.67	0.01
Electron	0.51	0.39

- (i) From the given table, which properties of the undetected third particle can be calculate?
 - (a) Total energy, but not kinetic energy
- (b) Kinetic energy, but not total energy
- (c) Both total energy and kinetic energy
- (d) Neither total energy nor kinetic energy
- (ii) Assuming the table contains no major errors, what can we conclude about the (mass \times c^2) of the undetected third particle?
 - (a) It is 0.79 MeV
 - (b) It is 0.39 MeV
 - (c) It is less than or equal to 0.79 MeV; but we cannot be more precise.
 - (d) It is less than or equal to 0.40 MeV; but we cannot be more precise.
- (iii) Could this reaction occur?

Proton \rightarrow neutron + other particles

- (a) Yes, if the other particles have much more kinetic energy than mass energy.
- (b) Yes, but only if the proton has potential energy (due to interactions with other particles).
- (c) No, because a neutron is more massive than a proton.
- (d) No, because a proton is positively charged while a neutron is electrically neutral.
- (iv) How much mass has to be converted into energy to produce electric power of 500 MW for one hour?
 - (a) $2 \times 10^{-5} \text{ kg}$
- (b) $1 \times 10^{-5} \text{ kg}$
- (c) $3 \times 10^{-5} \text{ kg}$
- (d) $4 \times 10^{-5} \text{ kg}$

- (v) The equivalent energy of 1 g of substance is
 - (a) $9 \times 10^{13} \text{ J}$
- (b) $6 \times 10^{12} \text{ J}$
- (c) $3 \times 10^{13} \text{ J}$
- (d) $6 \times 10^{13} \text{ J}$

HINTS & EXPLANATIONS

6. (i) (b)

- (ii) (b): Fast neutrons are slowed down by elastic scattering with light nuclei as each collision takes away nearly 50% of energy.
- (iii) (d): Reactions I and II represent fission of uranium isotope $^{235}_{92}$ U, when bombarded with neutrons that breaks it into two intermediate mass nuclear fragments. However, reaction III represents two deuterons fuses together to from the light isotope of helium.
- (iv) (c): On an average 2.5 neutrons are released per fission of the uranium atom.

The energy of the neutron released per fission of the uranium atom is 2 MeV.

(v) (a): In fission process, when a parent nucleus breaks into daughter products, then some mass is lost in the form of energy. Thus, mass of fission products < mass of parent nucleus.

 $\Rightarrow \frac{\text{Mass of fission products}}{\text{Mass of parent nucleus}} < 1$

7. (i) (a): As nearly 99.9% mass of atom is in nucleus

$$\therefore \frac{\text{Mass of nucleus}}{\text{Mass of atom}} = \frac{99.9}{100} = 0.99 \approx 1$$

- (ii) (a): Since, the nuclei of deuterium and tritium are isotopes of hydrogen, they must contain only one proton each. But the masses of the nuclei of hydrogen, deuterium and tritium are in the ration of 1:2:3, because of presence of neutral matter in deuterium and tritium nuclei.
- (iii) (c)

(iv) (a):
$$R = R_0 A^{1/3}$$

$$\log R = \log R_0 + \frac{1}{3} \log A$$

On comparing the above equation of straight line; y = mx + c. So, the graph between log A and log R is a straight line also.

- (v) (a): Here, $A_1 = 197$ and $A_2 = 107$
- $\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{197}{107}\right)^{1/3} = 1.225 \approx 1.23$
- **8.** (i) (a): Let the number of fissions per second be n. Energy released per second

$$= n \times 200 \text{ MeV} = n \times 200 \times 1.6 \times 10^{-13} \text{ J}$$

Energy required per second = power \times time

$$= 1 \text{ kW} \times 1 \text{ s} = 1000 \text{ J}$$

$$\therefore n \times 200 \times 1.6 \times 10^{-13} = 1000$$

or
$$n = \frac{1000}{3.2 \times 10^{-11}} = \frac{10}{3.2} \times 10^{13} = 3.125 \times 10^{13}$$

- (ii) (a)
- (iii) (b): As only 0.1% of the original mass is converted into energy, hence out of 1 kg mass 1 g is converted into energy.
- :. Energy released during fission, $E = \Delta mc^2$ = 1 g × (3 × 10⁸ m s⁻¹)² = 10⁻³ × 9 × 10¹⁶ J = 9 × 10¹³ J
- (iv) (a)
- (v) (a)
- (i) (d): All options are basic properties of nuclear forces. So, all options are correct.

(ii) (d): The nuclear force is of short range and the range of nuclear force is the order of 1.4×10^{-15} m. Now, volume $\propto R^3 \propto A$

(iii) (d)

(iv) (a): Nuclear force is much stronger than the electrostatic force inside the nucleus *i.e.*, at distances of the order of fermi. At 40 Å, nuclear force is ineffective and only electrostatic force of repulsion is present. This is very high at this distance because nuclear force is not acting now and the gravitational force is very feeble. $F_{\text{nuclear}} << F_{\text{electrostatic}}$ in this case.

(v) (a)

10. (i) (d): Here
$$R = 3 \times 10^{-15}$$
 m
Nuclear mass = 16 amu = $16 \times 1.66 \times 10^{-27}$ kg

$$\rho_{nu} = \frac{Nuclear\ mass}{Nuclear\ volume} = \frac{16 \times 1.66 \times 10^{-27}}{\frac{4}{3}\pi (3 \times 10^{-15})^3}$$

=
$$2.359 \times 10^{17} \text{ kg m}^{-3} \approx 2.4 \times 10^{17} \text{ kg m}^{-3}$$

(ii) (a): Density,

$$\rho = \frac{3m_p}{4\pi R_0^3} = \frac{3 \times 1.007825 \times 1.66 \times 10^{-27}}{4 \times \frac{22}{7} \times (1.1 \times 10^{-15})^3}$$
$$= 2.98 \times 10^{17} \text{ kg m}^{-3}$$

(iii) (d): Density =
$$\frac{\text{Mass}}{\text{Volume}} = \frac{Am_p}{\frac{4}{3}\pi(R_oA^{1/3})}$$
$$= \frac{m_p}{\frac{4}{3}\pi R_0^3},$$

where $m_p = 1.6 \times 10^{-27} \text{ kg}^3$ = $2.3 \times 10^{17} \text{ kg m}^{-3}$, which is a constant.

(iv) (c): Given, mass of $m_{\text{Fe}} = 55.85 \text{ amu}$ = $55.85 \times 1.66 \times 10^{-27} \text{ kg} = 9.27 \times 10^{-26} \text{ kg}$

Nuclear radius = $R_0 A^{1/3} = 1.1 \times 10^{-15} \times (56)^{1/3} \text{ m}$

$$\rho_{\text{nu}} = \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{m_{\text{Fe}}}{\frac{4}{3}\pi R^3}$$

$$= \frac{9.27 \times 10^{-26}}{\frac{4\pi}{3} \times (1.1 \times 10^{-15})^3 \times 56} = 2.9 \times 10^{17} \text{ kg m}^{-3}$$

(v) (d): Here, $A_1 = 27$, $A_2 = 125$, $R_1 = 3.6$ fm

As,
$$\frac{R_2}{R_1} = \left(\frac{A_2}{A_1}\right)^{1/3} = \left(\frac{125}{27}\right)^{1/3} = \frac{5}{3}$$

$$\therefore R_2 = \frac{5}{3}R_1 = \frac{5}{3} \times 3.6 = 6 \text{ fm}$$

11. (i) (a): As just shown, energy conservation allows us to calculate the third particle's total energy. But we do not know what percentage of that total is mass energy.

(ii) (d): According to the passage, subatomic reactions do not conserve mass. So, we cannot find the third particle's mass by setting $m_{\rm neutron}$ equal to $m_{\rm proton}$ + $m_{\rm electron}$ + $m_{\rm third\ particle}$.

The neutron has energy 940.97 MeV. The proton has energy 939.67 MeV + 0.01 MeV = 939.69 MeV. The electron has energy 0.51 MeV + 0.39 MeV = 0.90 MeV. Therefore, the third particle has energy

$$\begin{split} E_{\text{third particle}} &= E_{\text{neutron}} - E_{\text{proton}} - E_{\text{Electron}} \\ &= 940.97 - 939.67 - 0.90 = 0.40 \text{ MeV} \end{split}$$

(iv) (a): Here, $P = 500 \text{ MW} = 5 \times 10^8 \text{ W}$,

We just found the third particle's total energy, the sum of its mass energy and kinetic energy. Without more information, we cannot figure out how much of that energy is mass energy.

(iii) (b)

[:: A = 56]

t = 1 h = 3600 s
Energy produced,
$$E = P \times t = 5 \times 10^8 \times 3600$$

= 18 × 10¹¹ J
As $E = \Delta mc^2$

$$\therefore \Delta m = \frac{E}{c^2} = \frac{18 \times 10^{11}}{(3 \times 10^8)^2} = \frac{18 \times 10^{11}}{9 \times 10^{16}} = 2 \times 10^{-5} \text{ kg}$$

(v) (a): Using,
$$E = mc^2$$

Here, $m = 1$ g = 1×10^{-3} kg, $c = 3 \times 10^8$ m s⁻¹
 $\therefore E = 10^{-3} \times 9 \times 10^{16} = 9 \times 10^{13}$ J