

WITH GRAPH PAPER

केन्द्रीय माध्यमिक शिक्षा बोर्ड, दिल्ली
सीनियर स्कूल सर्टिफिकेट परीक्षा (नका बारम्बरी)
परीक्षार्थी प्रवेश-पत्र के अनुसार भरे

विषय Subject : MATHEMATICS

विषय कोड Subject Code : 041

परीक्षा का दिन एवं तिथि

Day & Date of the Examination : MONDAY 14.03.2016

उत्तर देने का माध्यम

Medium of answering the paper : ENGLISH

प्रश्न पत्र के ऊपर निम्न

कोड को दर्शाए :

Code Number

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① ② ③ ④

अतिरिक्त उत्तर-पुस्तिका (अ) की संख्या

No. of supplementary answer sheet(s) used

NIL

विकलांग व्यक्ति

हाँ / नहीं

Person with Disabilities

Yes / No

No

किसी शारीरिक अवस्था से प्रभावित होने से संबंधित वर्ग में ☒ का चिह्न लगाएं।
If physically challenged, tick the category

B D H S C A

B = बुद्धिमान D = दृष्टादृष्ट H = शारीरिक अवस्था से प्रभावित S = श्रवण
C = विकलांगता A = अक्षमता

B = Visually Impaired D = Hearing Impaired H = Physically Challenged
S = Speech C = Disability A = Aural

क्या लेखक - लिखित उत्तरों पर हस्ताक्षर करेंगे : हाँ / नहीं

Whether writer provided :

Yes / No

No

यदि बुद्धिमान हैं तो अवस्था से उत्तर देंगे

If visually challenged, name of scribe used :

प्रश्न पत्र में एक खाल स्थान है जहाँ आप प्रश्न पत्र के साथ एक नाम लिख सकते हैं। यदि आपकी अवस्था से प्रभावित हैं, तो प्रश्न पत्र के साथ एक नाम लिखें।

Blank letter box is provided in question paper and one has to fill blank between each part of the name. If one Candidate's Name exceeds 30 letters, write first 21 letters.

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कार्यालय उपयोग के लिए
Space for office use

Section-A

$$\vec{r} \cdot \hat{n} = d$$

$\hat{n}$ - Unit vector perpendicular to the plane

d - Distance of the plane from origin.

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} + 6\hat{k}) = 4$$

$$\vec{r} \cdot \frac{(2\hat{i} + 3\hat{j} + 6\hat{k})}{\sqrt{2^2 + 3^2 + 6^2}} = \frac{4}{\sqrt{2^2 + 3^2 + 6^2}} \Rightarrow \vec{r} \cdot \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = \frac{4}{7} \quad (\text{Distance of plane from origin} = \frac{4}{7})$$

$$\vec{r} \cdot (6\hat{i} - 9\hat{j} + 18\hat{k}) = -30$$

$$\vec{r} \cdot \frac{(6\hat{i} - 9\hat{j} + 18\hat{k})}{\sqrt{6^2 + 9^2 + 18^2}} = \frac{-30}{\sqrt{6^2 + 9^2 + 18^2}} \Rightarrow \vec{r} \cdot \left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = \frac{-10}{7} \quad (\text{Distance of plane from origin} = \frac{10}{7} \text{ in the direction opposite to the unit vector})$$

$$\therefore \text{Distance between the planes} = \frac{4}{7} - \left(\frac{-10}{7}\right) = \frac{14}{7} = \underline{\underline{2 \text{ units}}}$$

2

$\vec{a} - \sqrt{2}\vec{b}$ is a unit vector

$$|\vec{a} - \sqrt{2}\vec{b}| = 1$$

$$|\vec{a} - \sqrt{2}\vec{b}|^2 = 1$$

$$|\vec{a}|^2 + |\sqrt{2}\vec{b}|^2 - 2\sqrt{2}\vec{a} \cdot \vec{b} = 1$$

$$1 + 2 - 2\sqrt{2}\vec{a} \cdot \vec{b} = 1$$

$$(|\vec{a}| = |\vec{b}| = 1)$$

$$\Rightarrow |\vec{a}| |\vec{b}| \cos \theta = \frac{1}{\sqrt{2}} \quad (\theta = \text{Angle between vectors } \vec{a} \text{ \& } \vec{b})$$

$$\vec{a} \cdot \vec{b} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow |\vec{a}| |\vec{b}| \cos \theta = \frac{1}{\sqrt{2}}$$

$$\therefore \theta = 45^\circ$$

(Angle between $\vec{a}$ & $\vec{b}$ is 45°)

$$3. \quad |\vec{a}| = \frac{1}{2} \quad |\vec{b}| = \frac{4}{\sqrt{3}} \quad |\vec{a} \times \vec{b}| = \frac{1}{\sqrt{3}} \quad |\vec{a} \cdot \vec{b}| = ?$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta \quad (\theta: \text{Angle between vectors } \vec{a} \text{ \& } \vec{b})$$

$$\frac{1}{\sqrt{3}} = \frac{1}{2} \times \frac{4}{\sqrt{3}} \sin \theta$$

$$\sin \theta = \frac{1}{2} \quad \theta = 30^\circ$$

$$|\vec{a} \cdot \vec{b}| = |\vec{a}| |\vec{b}| \cos \theta$$

$$= \frac{1}{2} \times \frac{4}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 1$$

$$|\vec{a} \cdot \vec{b}| = 1$$

$$4. \quad A = \begin{bmatrix} 0 & 3 \\ 2 & -5 \end{bmatrix}$$

$$kA = \begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix}$$

$$\text{But given } kA = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$$

$$\begin{bmatrix} 0 & 3k \\ 2k & -5k \end{bmatrix} = \begin{bmatrix} 0 & 4a \\ -8 & 5b \end{bmatrix}$$

Equating individual terms,

$$2k = -8$$

$$k = -4$$

$$3k = 4a$$

$$3 \times (-4) = 4a$$

$$a = \underline{\underline{-3}}$$

$$-5k = 5b$$

$$b = -k = \underline{\underline{4}}$$

5.

$$|AB| = |A||B| \quad (\text{Provided } A \text{ \& } B \text{ are square matrices})$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = 1 \times (-1) - 2 \times 3 = -7$$

$$|B| = \begin{vmatrix} 1 & -4 \\ 3 & -2 \end{vmatrix} = 1 \times (-2) - 3 \times (-4) = 10$$

$$\therefore |AB| = -7 \times 10 = \underline{\underline{-70}}$$

6.

$$|A| = 5$$

$$|AA^T| = |A||A^T|$$

(As A is a square matrix)

$$= |A||A|$$

(As $|A| = |A^T|$)

$$= |A|^2 = \underline{\underline{25}}$$

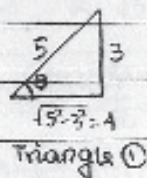
Section-B

7. To prove: $2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$

Proof: let $\sin^{-1}\left(\frac{3}{5}\right) = \theta$

$$\therefore \sin \theta = \frac{3}{5}$$

$$\therefore \tan \theta = \frac{3}{4} \quad (\text{From triangle ①})$$



$$\therefore \theta = \tan^{-1}\left(\frac{3}{4}\right)$$

Now $2\tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{2 \times \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}\right)$

$$\left\{ 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) \right\}$$

$$= \tan^{-1}\left(\frac{\frac{3}{2}}{1 - \frac{9}{16}}\right) = \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right) = \tan^{-1}\left(\frac{3 \times 8}{7}\right) = \tan^{-1}\left(\frac{24}{7}\right)$$

$$\therefore 2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7} \times \frac{17}{31}}\right)$$

$$\left\{ \tan^{-1}a - \tan^{-1}b = \tan^{-1}\left(\frac{a-b}{1+ab}\right) \right\}$$

$$= \tan^{-1}\left(\frac{24 \times 31 - 17 \times 7}{7 \times 31 + 24 \times 17}\right) = \tan^{-1}\left(\frac{625}{625}\right) = \tan^{-1}1 = \frac{\pi}{4} \quad \text{Hence Proved}$$

Let $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$ ①

$I = \int_0^{\frac{\pi}{2}} \frac{\sin^2(\frac{\pi}{2} - x)}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)} dx$

$$\left\{ \int_0^a f(x) dx = \int_0^a f(a-x) dx \right\}$$

$I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin x + \cos x} dx$ ②

Adding ① & ②

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx = \int_0^{\frac{\pi}{2}} \frac{dx}{\sin x + \cos x}$$

$$\left\{ \cos(A-B) = \cos A \cos B + \sin A \sin B \right\}$$

$$I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{dx}{\sin x + \cos x} = \frac{1}{2\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{dx}{\cos x \cdot \frac{1}{\sqrt{2}} + \sin x \cdot \frac{1}{\sqrt{2}}} = \frac{1}{2\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{dx}{\cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4}}$$

$$= \frac{1}{2\sqrt{2}} \int_0^{\frac{\pi}{2}} \frac{dx}{\cos(x - \frac{\pi}{4})}$$

Put $x - \frac{\pi}{4} = t$
 $dx = dt$

For $x=0$, $t = -\frac{\pi}{4}$, for $x = \frac{\pi}{2}$, $t = \frac{\pi}{4}$

$$\begin{aligned}
 I &= \frac{1}{2\sqrt{2}} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec t \, dt \\
 &= \frac{1}{2\sqrt{2}} \left[\log \left| \sec t + \tan t \right| \right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \\
 &= \frac{1}{2\sqrt{2}} \left(\log \left| \sec \frac{\pi}{4} + \tan \frac{\pi}{4} \right| - \log \left| \sec \left(-\frac{\pi}{4} \right) + \tan \left(-\frac{\pi}{4} \right) \right| \right) \\
 &= \frac{1}{2\sqrt{2}} \log \left| \frac{\sqrt{2}+1}{\sqrt{2}-1} \right| \\
 &= \frac{1}{2\sqrt{2}} \log \left| \frac{(\sqrt{2}+1)^2}{2-1} \right| = \frac{1}{\sqrt{2}} \log |\sqrt{2}+1|
 \end{aligned}$$

9. Let $I = \int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx$

$$\begin{aligned}
 &= \int \log(\log x) \, dx + \int \frac{dx}{(\log x)^2} \\
 &= I_1 + I_2 + C \text{ (say)} \quad \{C = \text{Arbitrary constant}\} \\
 I_1 &= \int \log(\log x) \, dx \quad \& \quad I_2 = \int \frac{dx}{(\log x)^2}
 \end{aligned}$$

Consider $I_1 = \int \log(\log x) \cdot 1 \, dx$

$$= x \log(\log x) - \int x \cdot \frac{1}{\log x} \times \frac{1}{x} \, dx \quad \left\{ \text{Applying Integration by parts} \right\}$$

$$= x \log(\log x) - \int \frac{dx}{\log x}$$

$$= x \log(\log x) - \left[\frac{x}{\log x} - \int x \cdot \frac{-1}{(\log x)^2} \times \frac{1}{x} \, dx \right] \quad \left\{ \text{Applying Integration by parts} \right\}$$

$$I_1 = x \log(\log x) - \frac{x}{\log x} - \int \frac{dx}{(\log x)^2}$$

But $\int \frac{dx}{(\log x)^2} = I_2$

$$\therefore I = I_1 + I_2 + C = x \log(\log x) - \frac{x}{\log x} + C$$

10. let $I = \int \frac{(1-\sin x) dx}{\sin x (1+\sin x)}$

$= \int \frac{(1-\sin x)(1-\sin x) dx}{\sin x (1+\sin x)(1-\sin x)}$

{ Multiplying numerator & Denominator by $1-\sin x$ }

$= \int \frac{(1-\sin x)^2 dx}{\sin x (1-\sin^2 x)} = \int \frac{(1+\sin^2 x - 2\sin x) dx}{\sin x \cos^2 x}$

$= \int \frac{dx}{\sin x \cos^2 x} + \int \frac{\sin x dx}{\cos^2 x} = 2 \int \frac{dx}{\cos^2 x}$

$= \int \frac{\sin x dx}{(1-\cos^2 x) \cos^2 x} + \int \frac{\sin x dx}{\cos^2 x} = 2 \tan x + C$ { $\int \sec^2 x$ }

$\overset{I_1}{\underbrace{\int \frac{\sin x dx}{(1-\cos^2 x) \cos^2 x}} + \overset{I_2}{\underbrace{\int \frac{\sin x dx}{\cos^2 x}}} \text{ (say)}$

$I_2 = \int \frac{\sin x dx}{\cos^2 x}$

Put $\cos x = t$

$-\sin x dx = dt$

$$\therefore I_2 = \int \frac{-dt}{t^2} = \frac{1}{t} + C_1 = \sec x + C_1$$

$$I_1 = \int \frac{\sin x \, dx}{(1 - \cos^2 x)(\cos^2 x)}$$

Put $\cos x = u$

$$-\sin x \, dx = du$$

$$I_1 = \int \frac{-du}{(1-u^2)u^2} = \int \frac{du}{(u^2-1)u^2} = \int \frac{(u^2 - (u^2-1))du}{(u^2-1)u^2}$$

$$= \int \frac{du}{u^2-1} - \int \frac{du}{u^2}$$

$$= \frac{1}{2} \log \left| \frac{u-1}{u+1} \right| + \frac{1}{u} + C'$$

$$I_1 = \frac{1}{2} \log \left| \frac{\cos x - 1}{\cos x + 1} \right| + \sec x + C'$$

$$\therefore I = \int \frac{(1-\sin x)dx}{\sin x(1+\sin x)} = \frac{1}{2} \log \left| \frac{\cos x - 1}{\cos x + 1} \right| + 2 \sec x - 2 \tan x + k$$

$\therefore k$ is an arbitrary constant.

$$x = am^2$$

$$ay^2 = (am^2)^3$$

$$ay^2 = a^3 m^6$$

$$y^2 = a^2 m^6$$

$$\therefore y = \pm am^3$$

Considering $(x, y) = (am^2, am^3)$

$$\frac{dy}{dx} = \frac{\frac{dy}{dm}}{\frac{dx}{dm}}$$

$$\frac{dy}{dm} = \frac{d(am^3)}{dm} = 3am^2$$

$$\frac{dx}{dm} = \frac{d(am^2)}{dm} = 2am$$

$$\therefore \left. \frac{dy}{dx} \right|_{(am^2, am^3)} = \frac{3am^2}{2am} = \frac{3m}{2} \quad \text{Slope of tangent at } (am^2, am^3)$$

$$\text{Slope of normal at } (am^2, am^3) = \frac{-1}{\left. \frac{dy}{dx} \right|_{(am^2, am^3)}} = \frac{-1}{\frac{3m}{2}} = \frac{-2}{3m}$$

∴ Equation of normal to the curve at (am^2, am^3) .

$$y - am^3 = -\frac{2}{3m} (x - am^2)$$

$$3my - 3am^4 = -2x + 2am^2$$

$$3my + 2x = 3am^4 + 2am^2 \text{ is the required equation.}$$

12

$$f(x) = \begin{cases} k \sin\left(\frac{\pi}{2}(x+1)\right) & x \leq 0 \\ \frac{\tan x - \sin x}{x^3} & x > 0 \end{cases}$$

$f(x)$ is continuous at $x=0$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$\text{Now } \lim_{x \rightarrow 0^-} f(x) = f(0) = \lim_{x \rightarrow 0^-} k \sin\left(\frac{\pi}{2}(x+1)\right) = k \sin \frac{\pi}{2} = k$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} \frac{\tan x - \sin x}{x^3} = \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \cdot \left(\frac{\sec x - 1}{x^2} \right) \\ &= \lim_{x \rightarrow 0^+} \frac{\sin x}{x} = \lim_{x \rightarrow 0^+} \frac{1 - \cos x}{\cos x \cdot x^2} \end{aligned}$$

$$= \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \times \lim_{x \rightarrow 0^+} \frac{2 \sin^2 \frac{x}{2}}{4 \left(\frac{x}{2} \right)^2}$$

$$\lim_{x \rightarrow 0^+} \cos x$$

$$\text{Now } \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \& \quad \lim_{x \rightarrow 0} \left(\frac{\sin \left(\frac{x}{2} \right)}{\left(\frac{x}{2} \right)} \right)^2 = 1$$

$$\therefore \lim_{x \rightarrow 0^+} f(x) = \frac{1 \times \frac{1}{2}}{1} = \frac{1}{2}$$

$$\therefore \boxed{k = \frac{1}{2}}$$

{ As $f(x)$ is continuous at $x=0$ }

13.

$$\text{Consider } \tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right)$$

$$= \tan^{-1} \left(\frac{1 - \sqrt{\frac{1-x^2}{1+x^2}}}{1 + \sqrt{\frac{1-x^2}{1+x^2}}} \right)$$

$$\text{Put } \text{But } x^2 = \cos 2\theta \quad \therefore \theta = \frac{1}{2} \cos^{-1} x^2$$

$$\therefore \tan^{-1} \left(\frac{1 - \sqrt{\frac{1-x^2}{1+x^2}}}{1 + \sqrt{\frac{1-x^2}{1+x^2}}} \right) = \tan^{-1} \left(\frac{1 - \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}}{1 + \sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}} \right)$$

$$1 - \cos 2\theta = 2\sin^2 \theta$$

$$1 + \cos 2\theta = 2\cos^2 \theta$$

$$\therefore \tan^{-1} \left(\frac{1 - \sqrt{\frac{1-x^2}{1+x^2}}}{1 + \sqrt{\frac{1-x^2}{1+x^2}}} \right) = \tan^{-1} \left(\frac{1 - \tan \theta}{1 + \tan \theta} \right) = \tan^{-1} \left(\frac{\tan \frac{\pi}{4} - \tan \theta}{1 + \tan \frac{\pi}{4} \tan \theta} \right)$$

$$= \tan^{-1} \left(\tan \left(\frac{\pi}{4} - \theta \right) \right)$$

We want

$$\frac{d}{d(\cos^2 x)} \left(\tan^{-1} \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right) \right) = \frac{d}{d\left(\frac{\pi}{4} - \theta\right)} \left(\frac{\pi}{4} - \theta \right) = \frac{\frac{d}{d\theta} \left(\frac{\pi}{4} - \theta \right)}{\frac{d}{d\theta} (2\theta)}$$

$$= \frac{1}{2} \frac{d}{d\theta} \left(\frac{\pi}{4} - \theta \right) = -\frac{1}{2}$$

14. A plane which passes through $A(3, 2, 1)$, $B(4, 2, -2)$ & $C(6, 5, -1)$

is
$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 4-3 & 2-2 & -2-1 \\ 6-3 & 5-2 & -1-1 \end{vmatrix} = 0$$

$$0 = \begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix}$$

$$0 = (x-3)(0 \times (-2) - (-3) \times 3) - (y-2)(1 \times (-2) - (-3) \times 3) + (z-1)(1 \times 3 - 3 \times 0)$$

$$9(x-3) - 7(y-2) + 3(z-1) = 0$$

$$9x - 7y + 3z = 27 - 14 + 3 = 16$$

$\therefore$ Plane passing through points A, B, C is $9x - 7y + 3z = 16$

Now A, B, C & $D(\lambda, 5, 5)$ are coplanar.

$\therefore D$ lies on the plane of A, B, C

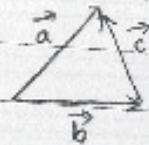
$$9\lambda - 7 \times 5 + 3 \times 5 = 16$$

$$9\lambda = 36$$

$$\lambda = 4$$

15.

$$\vec{a} = \vec{b} + \vec{c}$$



Here $\vec{a} = p\hat{i} + q\hat{j} + r\hat{k}$
 $\vec{b} = s\hat{i} + 3\hat{j} + 4\hat{k}$
 $\vec{c} = 3\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{a} = \vec{b} + \vec{c}$$

$$p\hat{i} + q\hat{j} + r\hat{k} = s\hat{i} + 3\hat{j} + 4\hat{k} + 3\hat{i} + \hat{j} - 2\hat{k}$$

$$p\hat{i} + q\hat{j} + r\hat{k} = (s+3)\hat{i} + 4\hat{j} + 2\hat{k}$$

Equating components,

$$p = s+3$$

$$q = 4$$

$$r = 2$$

Area of triangle = $5\sqrt{6}$

But Area of triangle = $\frac{1}{2} |\vec{a} \times \vec{b}|$

$$5\sqrt{6} = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ p & 4 & 2 \\ p-3 & 3 & 4 \end{vmatrix} = \frac{1}{2} [10\hat{i} - (2p+6)\hat{j} + (12-p)\hat{k}]$$

$$600 = 100 + (2p+6)^2 + (12-p)^2$$

$$500 = 4p^2 + 36 + 24p + 144 + p^2 - 24p \Rightarrow 5p^2 = 320$$

$$\frac{360}{40} = 9$$

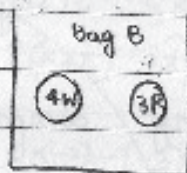
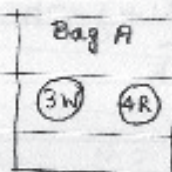
$$p^2 = 64$$

$$p = \pm 8$$

$$\text{If } p = 8, S = p - 3 = 5$$

$$\text{If } p = -8, S = p - 3 = -11$$

16.



Let E_1 : Event that the balls are drawn from bag A

E_2 : Event that the balls are drawn from Bag B

C: Event that 2 white balls & 1 red ball are drawn

$$P(E_2|C) = ?$$

$$\text{Now } P(E_1) = \frac{1}{2} = P(E_2)$$

$$P(C|E_1) = \frac{3}{7} \times \frac{2}{6} \times \frac{4}{5} \times 3 = \frac{4}{35} \times 3$$

$$P(C|E_2) = \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} \times 3 = \frac{6}{35} \times 3$$

(multiplied by 3 because they can be chosen in any order)

By Bayes theorem,

$$P(E_2|C) = \frac{P(E_2)P(C|E_2)}{P(E_1)P(C|E_1) + P(E_2)P(C|E_2)}$$

$$= \frac{\frac{1}{2} \times \frac{6}{35} \times 3}{\frac{1}{2} \times \frac{4}{35} \times 3 + \frac{1}{2} \times \frac{6}{35} \times 3} = \frac{6}{10} = \frac{3}{5}$$

17.

Let the length of the plot be l .

Let the breadth of the plot be b .

Now $l \times b = A$ ($A = \text{Area of the plot}$)

$$(l-50)(b+50) = A$$

$$(l-10)(b-20) = A - 5300$$

$$lb - 50b + 50l - 2500 = A$$

$$-b + l = 50 \quad (1) \quad (A = lb)$$

$$lb - 10b - 20l + 200 = A - 5300$$

$$10b + 20l = 5500$$

$$b + 2l = 550 \quad (2)$$

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} l \\ b \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$$

$\begin{matrix} \text{"} & \text{"} & \text{"} \\ A & X & B \end{matrix}$

$$AX = B$$

$$\therefore X = A^{-1}B$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$(ad-bc = 1 \times 1 - (-2) \times 1 = 3 \neq 0)$$

$\therefore A^{-1}$ exists

Here $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$

$$A^{-1} = \frac{1}{1+2} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{3} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 50 \\ 550 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 600 \\ 450 \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$

$$\begin{bmatrix} l \\ b \end{bmatrix} = \begin{bmatrix} 200 \\ 150 \end{bmatrix}$$

$$l = 200 \text{ m}$$

$$b = 150 \text{ m}$$

He wants to donate the plot to the school because he wants rural places to become developed & he is thereby showing his kind heartedness & his intention to help the society to develop by producing more literates. Children should have an opportunity to learn.

$$2ye^{\frac{x}{y}} dx + (y - 2xe^{\frac{x}{y}}) dy = 0$$

$$\frac{dy}{dx} = \frac{-2ye^{\frac{x}{y}}}{y - 2xe^{\frac{x}{y}}}$$

$$\frac{dx}{dy} = \frac{y - 2xe^{\frac{x}{y}}}{-2ye^{\frac{x}{y}}} = f(x, y) \text{ (say)}$$

$$F(\lambda x, \lambda y) = \frac{\lambda y - 2\lambda x e^{\frac{\lambda x}{\lambda y}}}{-2\lambda y e^{\frac{\lambda x}{\lambda y}}} = \lambda^0 F(x, y)$$

∴ The equation is a homogeneous differential equation

Put $x = vy$

$$\frac{dx}{dy} = v + y \frac{dv}{dy}$$

$$\therefore v + y \frac{dv}{dy} = \frac{y - 2vy e^v}{-2y e^v} = -\frac{1}{2} e^{-v} + v \Rightarrow y \frac{dv}{dy} = -\frac{1}{2} e^{-v}$$

$$\therefore e^v dv = -\frac{1}{2} \frac{dy}{y}$$

Integrating

$$e^v = -\frac{1}{2} \log y + C$$

$\therefore e^{\frac{v}{y}} = -\frac{1}{2} \log y + C$ is the required solution to the differential equation.

19. $(x+1) \frac{dy}{dx} - y = e^{3x} (x+1)^3$

$$\frac{dy}{dx} - \frac{y}{(x+1)} = e^{3x} (x+1)^2$$

This equation is of the form $\frac{dy}{dx} + Py = Q$ { $P, Q =$ functions of x alone }
linear differential equation.

$$\therefore \text{Integrating factor} = e^{\int P dx} = e^{-\int \frac{dx}{x+1}} = e^{-\log(x+1)} = \frac{1}{x+1}$$

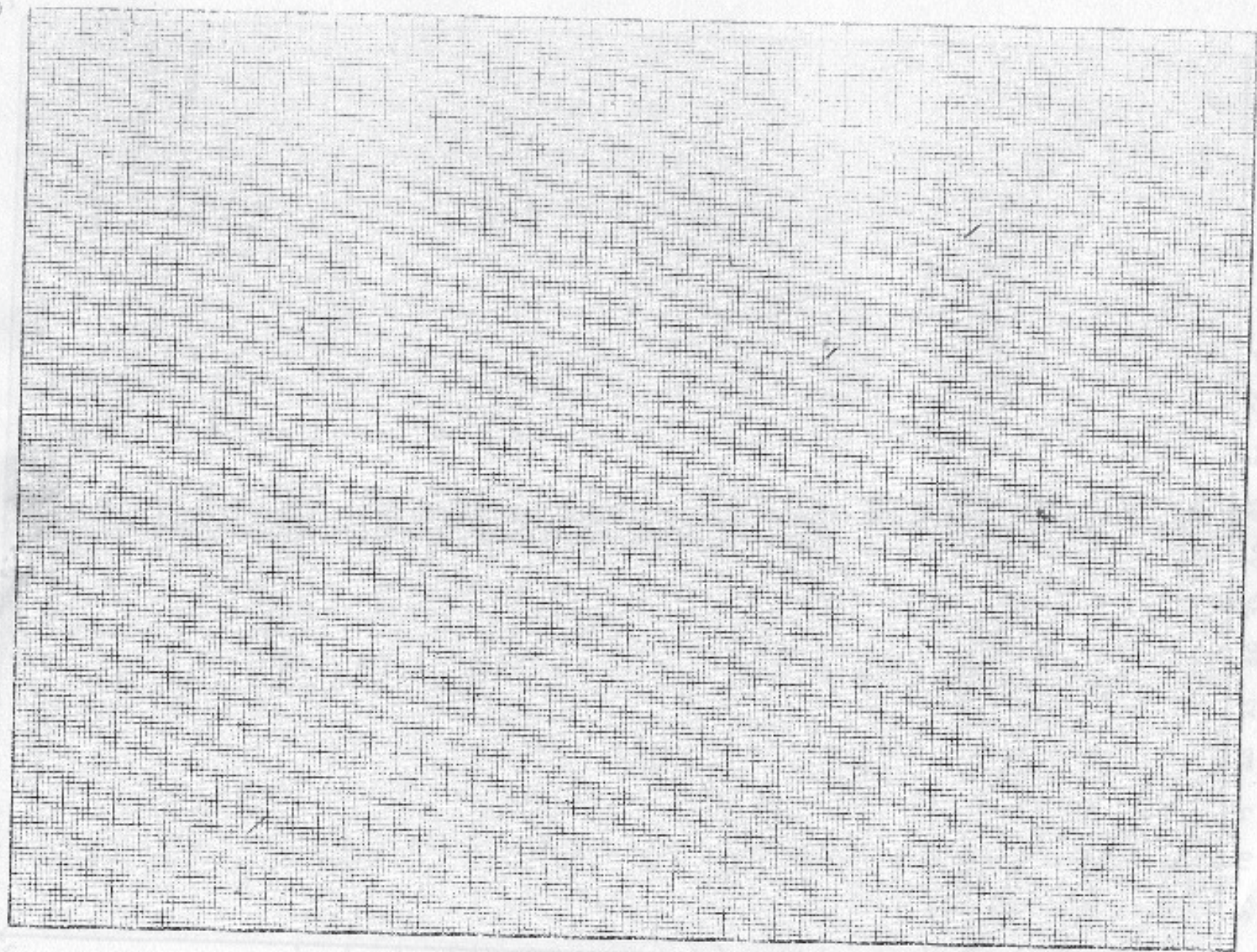
Solution to the differential equation:

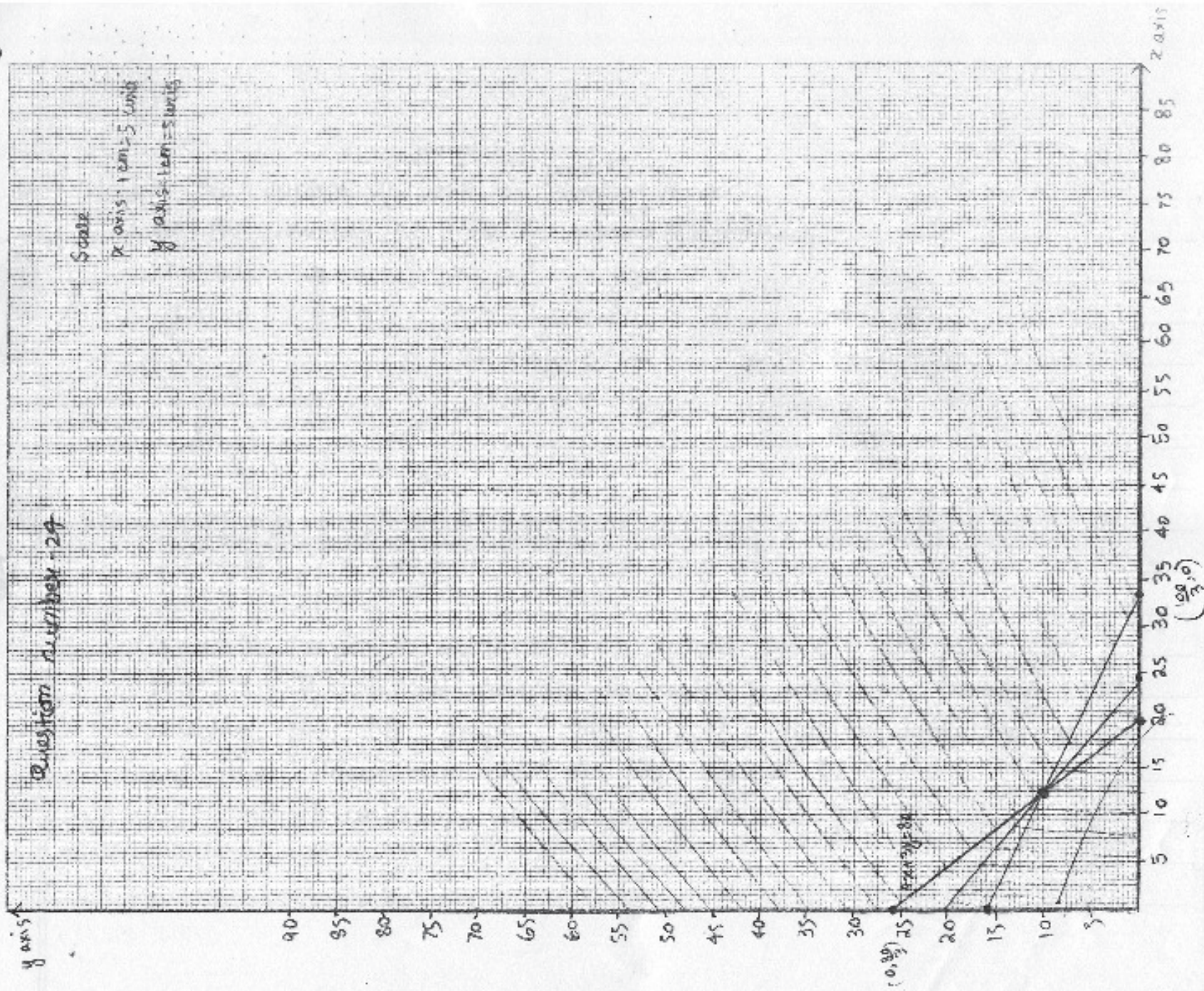
$$y \times \text{I.F.} = \int Q \times \text{I.F.} dx + C$$

$$\therefore \frac{y}{x+1} = \int e^{3x} (x+1)^2 \times \frac{1}{(x+1)} dx + C$$

$$= \int x e^{3x} dx + \int e^{3x} dx + C = \frac{x e^{3x}}{3} - \frac{1}{9} \int e^{3x} dx + \int e^{3x} dx + C$$

$$\frac{y}{x+1} = \frac{x e^{3x}}{3} + \frac{2}{9} e^{3x} + C \text{ is the solution to the required differential equation}$$





Section-C

24

let the number of units of ^{food F₁} Vitamin A = x let the number of units of ^{food F₂} Vitamin Minerals = y

$$x \geq 80 \quad x \geq 0$$

$$x \geq 0$$

$$y \geq 0$$

$$y \geq 100 \quad y \geq 0$$

$$4x + 3y \geq 80$$

$$4x + 3y = 80$$

$$3x + 6y \geq 100$$

$$3x + 6y = 100$$

$$8x + 6y = 160$$

$$5x = 60$$

$$x = 12$$

$$y = \frac{32}{3}$$

$$Z = 5x + 6y$$

Minimize Z (x, y) Z

$$(0, \frac{80}{3})$$

$$Z = 160$$

$$(\frac{100}{3}, 0)$$

$$Z = 166.67$$

$$(12, \frac{32}{3})$$

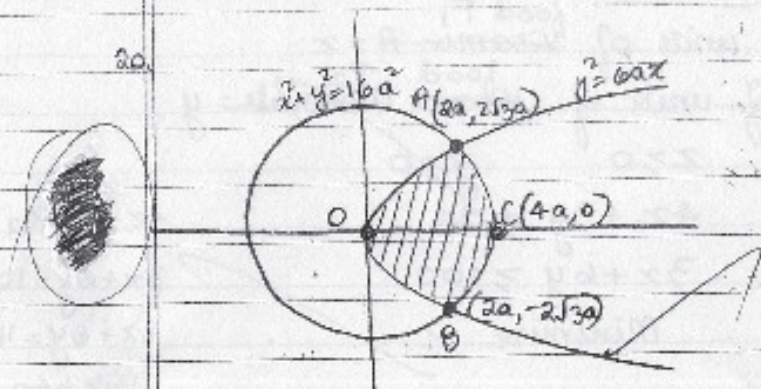
$$Z = 124$$

Since the region is unbounded, we should check if any point other than $(12, \frac{32}{3})$ is in common to region shaded, & $5x + 6y \leq 124$

Clearly, from the graph, only $(12, \frac{32}{3})$ lies on both regions.

$\therefore$ The Minimum cost of the diet = $\text{₹} 124$

with 12 units of food F₁ & $\frac{32}{3}$ units of food F₂



Solving the two curves

$$x^2 + 6ax = 16a^2$$

$$\left(\frac{x}{a}\right)^2 + 6\left(\frac{x}{a}\right) - 16 = 0$$

$$\left(\frac{x}{a} + 8\right)\left(\frac{x}{a} - 2\right) = 0$$

$$x = -8a \text{ or } x = 2a$$

x

$$\therefore x = 2a$$

$$y = \pm 2\sqrt{3}a \quad (y^2 = 12a^2 \quad y = \pm 2\sqrt{3}a)$$

$$\therefore \text{Required Area} = 2 \times \text{Area OACO}$$

$$= 2 \times \int_{2\sqrt{3}a}^{2\sqrt{3}a} (x_1 - x_2) dy$$

$$= 2 \times \int_0^{2\sqrt{3}a} \sqrt{16a^2 - y^2} dy = 2 \times \int_0^{2\sqrt{3}a} \frac{y^2}{6a} dy$$

$$= 2 \times \left[\frac{y}{2} \sqrt{16a^2 - y^2} + \frac{16a^2}{2} \sin^{-1} \frac{y}{4a} \right]_0^{2\sqrt{3}a} = \left[\frac{y^3}{9a} \right]_0^{2\sqrt{3}a}$$

$$= 2\sqrt{3}a \times 2a + 16a^2 \sin^{-1} \frac{\sqrt{3}}{2} - 0 = \frac{8 \times 2\sqrt{3}a^2}{\sqrt{3}}$$

$$= 4\sqrt{3}a^2 = \frac{8\sqrt{3}}{3}a^2 + \frac{16a^2\pi}{3} \quad \left\{ \int \sqrt{A^2 - x^2} dx = \frac{x}{2}\sqrt{A^2 - x^2} + \frac{A^2}{2}\sin^{-1}\frac{x}{A} \right\}$$

$$= \frac{4}{\sqrt{3}}a^2 + \frac{16a^2\pi}{3} \quad \text{sq units}$$

20. $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$

$f(x)$ is increasing strictly in an interval if $f'(x) > 0$ in that interval.
 $f(x)$ is strictly decreasing in an interval if $f'(x) < 0$ in that interval.

$$f'(x) = 4x^3 - 24x^2 + 44x - 24$$

$$= 4(x^3 - 6x^2 + 11x - 8)$$

21.

Maximum & Minimum values of $f(x) = \sec x + \log \cos x$

~~$f(x) = \sec x + \log \cos x$~~

$$f'(x) = 0$$

$$\sec x \tan x + \frac{2}{\cos x} (-\sin x) = 0 \quad x \neq \frac{\pi}{2}$$

$$\therefore \tan x (\sec x - 2) = 0$$

$$\therefore x = \pi \quad \text{or} \quad x = \frac{\pi}{3}, \frac{5\pi}{3}$$

Now $f''(x) < 0$ for maximum $f''(x) > 0$ for minimum

$$f''(x) = \sec x (\sec x - 2) + \tan^2 x \sec x$$

$$f''(\pi) = 1 \times -3 + 0 = -3 < 0$$

$$f''\left(\frac{\pi}{3}\right) = f''\left(\frac{5\pi}{3}\right) = 0 + 3 \times 2 = 6 > 0$$

Function attains maximum value at $x = \pi$ & minimumvalue at $x = \frac{\pi}{3}$ & $\frac{5\pi}{3}$

$$f(\pi) = -1$$

$$f\left(\frac{\pi}{3}\right) = 2 - 2 \log 2$$

But when $x = \frac{\pi}{2}$, function becomes undefined.~~Minimum & Maximum~~ do not exist as Minimum

22 TP: $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ac \\ (a+b)^2 & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)(a^2+b^2+c^2)$

Proof :

$$C_1 \rightarrow C_1 - 2C_3$$

$$\Delta = \begin{vmatrix} b^2+c^2 & a^2 & bc \\ c^2+a^2 & b^2 & ac \\ a^2+b^2 & c^2 & ab \end{vmatrix}$$

$$C_1 \rightarrow C_1 + C_2$$

Now taking $a^2+b^2+c^2$ common from C_1 ,

$$\Delta = (a^2+b^2+c^2) \begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ac \\ 1 & c^2 & ab \end{vmatrix}$$

$$R_2 \rightarrow R_2 - R_1$$

$$R_3 \rightarrow R_3 - R_1$$

$$\Delta = (a^2 + b^2 + c^2) \begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2 - a^2 & ac - bc \\ 0 & c^2 - a^2 & ab - bc \end{vmatrix}$$

Taking $(a-b)$ common from R_2 & $(c-a)$ common from C_3

$$\Delta = (a^2 + b^2 + c^2) (a-b)(c-a) \begin{vmatrix} 1 & a^2 & bc \\ 0 & -(a+b) & c \\ 0 & a+c & -b \end{vmatrix}$$

Expanding along C_1

$$\Delta = (a^2 + b^2 + c^2) (a-b)(c-a) (- (a+b) \times b + (a+c)c)$$

$$= (a^2 + b^2 + c^2) (a-b)(c-a) (b^2 - c^2 + bc - ba)$$

$$= (a^2 + b^2 + c^2) (a-b)(b-c)(c-a)(a+b+c)$$

$$\text{As } (b-c)(a+b+c) = ab + b^2 + bc - ac - bc - c^2$$

$$b^2 - c^2 + ab - ac$$

25.

Equation of plane containing two parallel lines
has DRS of perpendicular

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b}$$

$\vec{b}$ = DRS of the line

$\vec{a}_1$ = Point (Position vector on line 1)

$\vec{a}_2$ = (Position vector) on line 2

$$\vec{a}_1 = \hat{i} - \hat{j}$$

$$\vec{a}_2 = 2\hat{j} - \hat{k}$$

$$\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = -\hat{i} + 3\hat{j} - \hat{k}$$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix} = 8\hat{i} + \hat{j} - 5\hat{k}$$

$$\therefore \text{Equation of plane} = 8x + y - 5z = 8 \times 1 + \hat{j} \times (-1) - 5 \times 0$$

$$8x + y - 5z = 7$$

Now consider the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{2}$

$$8 \times 3 + 1 \times 1 + -5 \times 5 = 0$$

$\therefore$ DRS of line & perpendicular to plane are perpendicular.
& Point on line = (2, 1, 2) $8 \times 2 + 1 - 5 \times 2 = 7$ also satisfies the
equation of plane. $\therefore$ The line is contained in the plane.

86

$$f(x) = 4x^2 + 12x + 15$$

$$\text{Let } f(x) = y$$

$$y = 4x^2 + 12x + 15$$

$$4x^2 + 12x + (15 - y) = 0$$

$$x = \frac{-12 \pm \sqrt{144 - 16(15 - y)}}{8} = \frac{-12 \pm \sqrt{9 - 5 + y}}{8}$$

$$x = \frac{\sqrt{y - 6} - 3}{2}$$

$$\text{Consider } g(x) = \frac{\sqrt{x - 6} - 3}{2}$$

$$\begin{aligned} \text{Now } fog(x) &= f(g(x)) = f\left(\frac{\sqrt{x - 6} - 3}{2}\right) = \frac{4}{4}(\sqrt{x - 6} - 3)^2 + 6(\sqrt{x - 6} - 3) + 15 \\ &= x - 6 + 9 - 6\sqrt{x - 6} + 6\sqrt{x - 6} - 18 + 15 \\ &= x \end{aligned}$$

$$gof(x) = g(f(x)) = g(4x^2 + 12x + 15) = \frac{\sqrt{4x^2 + 12x + 9} - 3}{2} = \frac{2x + 3 - 3}{2} = x$$

$$\text{Hence } fog(x) = gof(x) = x$$

By definition, g is the inverse of f .

As inverse exists, f is invertible &

$$\Rightarrow f^{-1}(x) = \frac{\sqrt{x-6}-3}{2}$$

$$f^{-1}(31) = \frac{\sqrt{31-6}-3}{2} = \frac{5-3}{2} = 1$$

$$f^{-1}(87) = \frac{\sqrt{87-6}-3}{2} = \frac{9-3}{2} = 3$$

23

$X =$	1	2	3	4	5	6
$P(X=)$	$\frac{10}{20} = \frac{1}{2}$	$\frac{6}{20} = \frac{3}{10}$	$\frac{3}{20}$	$\frac{1}{20}$	0	0

(as 5 & 6 can't be minimum values)

$$P(X=1) = \frac{{}^5C_2}{{}^6C_3} = \frac{10}{20}$$

(Numbers from 2-6 can be chosen)
(Any two)

$$P(X=2) = \frac{{}^4C_2}{{}^6C_3} = \frac{6}{20}$$

(Numbers 2-6 can be chosen)
(Any two)

$$P(X=3) = \frac{{}^3C_2}{{}^6C_3} = \frac{3}{20}$$

(Any 2 out of 4, 3, 6 can be chosen)

$$P(X=2) = \frac{{}^2C_2}{{}^6C_3} = \frac{1}{20}$$

only 5, 6 can be chosen

$$\text{Mean} = E(x) = \sum x_i p_i$$

$$= \frac{10}{20} \times 1 + \frac{6}{20} \times 2 + \frac{3}{20} \times 3 + \frac{1}{20} \times 4 + 0 + 0$$

$$= \frac{10}{20} + \frac{12}{20} + \frac{9}{20} + \frac{4}{20} = \frac{35}{20} = 1.75$$

$$\text{Variance} = E(x^2) - (E(x))^2$$

$$= \frac{10}{20} \times 1 + \frac{6}{20} \times 4 + \frac{3}{20} \times 9 + \frac{1}{20} \times 16 - \frac{49}{16}$$

$$= \frac{10 + 24 + 27 + 16}{20} - \frac{49}{16} = \frac{77}{20} - \frac{49}{16}$$

$$= \frac{7}{4} \left(-\frac{11}{5} - \frac{7}{4} \right)$$

$$= \frac{7}{4} \times \frac{9}{20}$$

$$\sigma^2 = \frac{63}{80}$$

$$\frac{44}{9}$$