Sample Paper-05
Mathematics
Class – XI

Time allowed: 3 hours Maximum Marks: **100**

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.
c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
d) Use of calculators is not permitted.

Section A

1. Solve for x if $|x| + x = 2 + i$
2. Write the sum of first n odd numbers
3. Write the n[th] tern if the sum of n terms of an AP is $2n^2 + 3n$
4. If $a < b$ write the length of latus rectum of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
5. If $f(x) = 5$ for all real numbers of x find $f(x+5)$
6. What is the maximum number of objects you can weigh if you have four distinct weights.

Section B

7. Prove that $f'(a+b) = f'(a) + f'(b)$ when $f(x) = x^2$ and when $f(x) = x^3$
8. If α, β are the roots of the equation $x^2 + px + q = 0$ Find $\alpha^3 + \beta^3$.
9. A positive 3 digit number has its units digit zero. Find the probability that the number is divisible by 4.
10. Prove that $\tan(45 + x) = \sec 2x + \tan 2x$
11. Prove by mathematical induction that $n(n+1)$ is even
12. Find $n[(A \cup B \cup C)]$ if $n(A) = 4000\ n(B) = 2000\ n(C) = 1000$ and

    $n(A \cap B) = n(B \cap C) = n(A \cap C) = 400,\ n(A \cap B \cap C) = 200$

13. Find the latus rectum, eccentricity and coordinates of the foci of the ellipse $x^2 + 3y^2 = k^2$

14. Find the area of the circle passing through the points $(-8,0),(0,8),12,0)$

15. If S_1, S_2, S_3 are the sums of $n, 2n, 3n$ terms respectively of an AP prove that $S_3 = 3(S_2 - S_1)$

16. Find the least value of $f(x)$ if $f(x) = 3x^2 - 6x - 11$

17. Find $f(x) + f(1-x)$ if $f(x) = \frac{a^x}{a^x + \sqrt{a}}$

18. Prove that $\frac{\tan 2x \tan x}{\tan 2x - \tan x} = \sin 2x$

19. Find the limit $\lim_{n\to\infty} \frac{(n+2)! + (n+1)!)}{(n+2)! - (n+1)!}$

Section C

20. Find $\frac{dy}{dx}$ given that $y = (\sin^n x \cos nx)$

21. If $(5a),(a-b),b$ are in GP prove that $\log\left(\frac{1}{3}(a+b)\right) = \frac{1}{2}(\log a + \log b)$

22. If the nth term of a series is denoted by $\frac{7^{n-1}}{10^n}$. Find the sum to infinity of the series.

23. Calculate the variance and standard deviation of the following data $8,12,13,15,22,14$

24. $f(x) = (1+x)^{\frac{1}{x}}, x \neq 0$. Find $f(1+\frac{a}{y})^{by}$

25. The probability of A hitting a target is $\frac{4}{5}$; the probability of B hitting the target is $\frac{3}{4}$ and the probability of C missing the target is $\frac{1}{3}$.What is the probability of the target being hit at least twice.

26. Find the term independent of x in the expansion $\left(ax^2 - \frac{b}{x}\right)^9$

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ANSWERS

Section A

1. **Solution**

$x = a + ib$

$|x| + x = \sqrt{a^2 + b^2} + a + ib$

$\sqrt{a^2 + b^2} + a = 2$

$a^2 + b^2 = (2 - a)^2$

$b = 1$

$a^2 + 1 = 4 + a^2 - 4a$

$a = \frac{3}{4}$

$x = \frac{3}{4} + i$

2. **Solution**

$S = 1 + 3 + 5 + \cdots\cdots$

$S = \frac{n}{2}[2 + (n-1)(2)]$

$S = n^2$

3. **Solution**

First term = 5

Sum of first and second term = 14

Second term= 9

Common Difference= $9 - 5 = 4$

nth term = $5 + (n-1)4$

= $4n + 1$

4. **Solution**

Length of latus rectum of the ellipse = $\frac{2a^2}{b}$

5. **Solution**

$f(x+5)=5$

6. **Solution**

The number of weights that can be measured = number of subsets can be formed excluding the null set

$2^4-1=15$

Section B

7. **Solution**

When $f(x)=x^2$

$$f(x)=x^2$$
$$f'(x)=2x$$
$$f'(a+b)=2(a+b)$$
$$f'(a)=2a$$
$$f'(b)=2b$$
$$f'(a)+f'(b)=2(a+b)$$
$$=f'(a+b)$$

When $f(x)=x^3$

$$f(x)=x^3$$
$$f'(x)=3x^2$$
$$f'(a+b)=3(a+b)^2$$
$$f'(a)=3a^2$$
$$f'(b)=3b^2$$
$$f'(a)+f'(b)=3(a^2+b^2)$$
$$\neq f'(a+b)$$

8. **Solution**

$$\alpha^3+\beta^3=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]=-p[p^2-3q]$$

9. **Solution**

Total number of 3 digit numbers with 0 in units place = 90

The digits that can go into tens place for the number to be divisible by 4 = $0,2,4,6,8$

100th place can be formed with any of the 9 digits excepting 0

Hence total number of 3 digits number divisible by 4 is $9\times5=45$

Probability= $\frac{45}{90} = \frac{1}{2}$

10. **Solution**

$$\tan(45 + x) = \frac{1 + \tan x}{1 - \tan x}$$
$$= \frac{\cos x + \sin x}{\cos x - \sin x}$$
$$= \frac{\cos^2 x + \sin^2 x + 2\sin x \cos x}{cos^2 x - \sin^2 x}$$
$$= \frac{1 + sin2x}{\cos 2x} = \sec 2x + \tan 2x$$

11. **Solution**

$$P(n) \quad = n(n+1)$$
$$P(1) \quad = 2\ , even$$
$$P(k) \quad = k(k+1)\ let\ this\ be\ true$$
$$P(k+1) \quad = (k+1)(k+2)$$
$$= k^2 + 3k + 2$$
$$= k^2 + k + 2k + 2$$
$$= k(k+1) + 2(k+1)\ True$$

12. **Solution**

$$n[(A \cup B \cup C)] = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$
$$n[(A \cup B \cup C)] = 4000 + 2000 + 1000 - 400 - 400 - 400 + 200$$
$$n[(A \cup B \cup C)] = 6000$$

13. **Solution.**

$$\frac{X^2}{k^2} + \frac{y^2}{\frac{k^2}{3}} = 1$$

Latus rectum is = $\frac{\left(\frac{2k^2}{3}\right)}{k}$

= $\frac{2k}{3}$

$$e = \sqrt{\left(\frac{k^2 - \frac{k^2}{3}}{k^2}\right)}$$
$$= \sqrt{\frac{2}{3}}$$
$$= \frac{\sqrt{6}}{3}$$

Coordinates of foci are $(ae,0)$ and $(-ae,0)$

Coordinates are = $(\frac{\sqrt{6}}{3}k,0)$ and $(\frac{-\sqrt{6}}{3}k,0)$

14. **Solution**

Slope of line AB joining the points $(-8,0)$ and $(12,0)$ = 0

Its midpoint = $(2,0)$

Equation to the line perpendicular to AB and passing through $(2,0)$ is $x=2$

Slope of line AC joining the points $(-8,0)$ and $(0,8)$ = 1

Its midpoint = $(-4,4)$

Equation to the line perpendicular to AC and passing through $(-4,4)$ is $y=-x$

So the center of the circle will be the point of intersection of line AB and line AC. Center of circle at point $(2,-2)$

Radius = $\sqrt{(2-0)^2+(-2-8)^2}$ = $\sqrt{104}$

Area = 104π

15. **Solution**

$$2S_1 = n[2a+(n-1)d]$$
$$2S_2 = 2n[2a+(2n-1)d]$$
$$2S_3 = 3n[2a+(3n-1)d]$$
$$\frac{2S_1}{n} = 2a+(n-1)d$$
$$\frac{2S_3}{3n} = 2a+(n-1)d$$
$$\frac{2S_1}{n}+\frac{2S_3}{3n} = 4a+d(n-1+3n-1)$$
$$\frac{2S_1}{n}+\frac{2S_3}{3n} = 4a+2(2n-1)d$$
$$\frac{2S_1}{n}+\frac{2S_3}{3n} = 2.\frac{2S_2}{2n}$$
$$\frac{2S_3}{3n} = \frac{2S_2}{2n}-\frac{2S_1}{n}$$
$$\frac{2S_3}{3n} = \frac{4S_2}{2n}-\frac{4S_1}{2n}$$
$$S_3 = 3(S_2-S_1)$$

16. **Solution**

$$f(x) = 3x^2 - 6x - 11$$

$$f(x) = 3(x^2 - 2x - \frac{11}{3})$$

$$f(x) = 3(x^2 - 2x + 1 - 1 - \frac{11}{3})$$

$$f(x) = 3[(x-1)^2 - \frac{11}{3} - 1)$$

$$f(x) = 3[(x-1)^2 - \frac{14}{3})$$

$$f(x) = 3(x-1)^2 - 14$$

$f(x)$ will attain minimum when $x = 1$

Minimum value of $f(x) = -14$

17. **Solution**

$$f(x) = \frac{a^x}{a^x + \sqrt{a}}$$

$$f(1-x) = \frac{a^{1-x}}{a^{1-x} + \sqrt{a}}$$

$$f(x) + f(1-x) = \frac{a^x}{a^x + \sqrt{a}} + \frac{a^{1-x}}{a^{1-x} + \sqrt{a}}$$

$$= 1$$

18. **Solution**

$$\frac{\tan 2x \tan x}{\tan 2x - \tan x} = \frac{\frac{2\tan x}{1-\tan^2 x}\tan x}{\frac{2\tan x}{1-\tan^2 x} - \tan x}$$

$$= \frac{2\tan^2 x}{\tan x + \tan^2 x}$$

$$= \frac{2\tan x}{1+\tan^2 x}$$

$$= \sin 2x$$

19. **Solution**

$$\lim_{n\to\infty}\frac{(n+2)!+(n+1)!)}{(n+2)!-(n+1)!} = \lim_{n\to\infty}\frac{(n+1)!(n+2+1)}{(n+1)!(n+2-1)!}$$

$$= \lim_{n\to\infty}\frac{(n+3)}{((n+1)}$$

$$\lim_{n\to\infty}\frac{1+\frac{3}{n}}{1+\frac{1}{n}}$$

$$=1$$

Section C

20. **Solution**

$$\frac{dy}{dx}=\sin^n x.\{-\sin nx).(n)\}+\cos nx.\{n.\sin^{n-1}x.\cos x\}$$

$$\frac{dy}{dx}=n\sin^{n-1}x(\cos nx.\cos x-\sin x.\sin nx)$$

$$\frac{dy}{dx}=n\sin^{n-1}[\cos(n+1)x]$$

21. **Solution**

$$(a-b)^2=5ab$$

$$a^2+b^2-2ab=5ab$$

$$a^2+b^2=7ab$$

$$(a+b)^2=9ab$$

$$a+b=3\sqrt{ab}$$

$$\frac{1}{3}(a+b)=\sqrt{ab}$$

$$\log\left(\frac{1}{3}(a+b)\right)=\frac{1}{2}(\log a+\log b)$$

22. **Solution**

First term = $\frac{1}{10}$

Second term= $\frac{7}{10^2}$

Third term= $\frac{7^2}{10^3}$

r = $\frac{7}{10}$

This is a GP

Sum to infinity= $\dfrac{\frac{1}{10}}{1-\frac{7}{10}} = \dfrac{1}{3}$

23. **Solution**

Mean= 14= $\dfrac{8+12+13+15+22+14}{6}$

x_i	$x_i - Mean$	$(x_i - Mean)^2$
8	-6	36
12	-2	4
13	-1	1
15	1	1
22	8	64
14	0	0
		$\Sigma(x_i - Mean)^2 = 106$

Variance= $\dfrac{1}{n}\Sigma(x_i - Mean)^2 = \dfrac{106}{6} = 17.66$

SD= $\sqrt{Variance} = \sqrt{17.66} = 4.2$

24. **Solution**

$Let\ \dfrac{a}{y} = x$

$by = \dfrac{ab}{x}$

$f(1+\dfrac{a}{y})^{by} = f\left[(1+x)^{\frac{1}{x}}\right]^{ab}$

25. **Solution:**

Probability of all the three hitting the target= $\dfrac{4}{5}.\dfrac{3}{4}.\dfrac{2}{3} = \dfrac{2}{5}$

Probability of A alone missing the target = $\dfrac{1}{5}.\dfrac{3}{4}.\dfrac{2}{3} = \dfrac{1}{10}$

Probability of B alone missing the target = $\dfrac{4}{5}.\dfrac{1}{4}.\dfrac{2}{3} = \dfrac{2}{15}$

Probability of C alone missing the target = $\frac{4}{5}.\frac{3}{4}.\frac{1}{3}=\frac{1}{5}$

The probability that the target being hit at least two= $\frac{2}{5}+\frac{1}{10}+\frac{2}{15}+\frac{1}{5}=\frac{5}{6}$

26. **Solution**

Let T_{r+1} be the term that is independent of x

Then

$$T_{r+1}={}^9C_r(ax^2)^r(-\frac{b}{x})^{9-r}$$
$$2r+(r-9)=0$$
$$r=3$$

4th term is independent of x

$$T_4={}^9C_3(a)^3(-b)^6$$

$$={}^9C_3(a)^3(b)^6$$