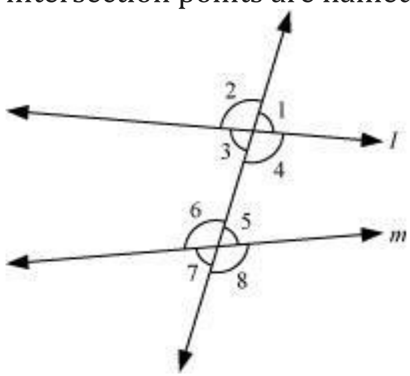
Lines and Angles

- **Corresponding angles**

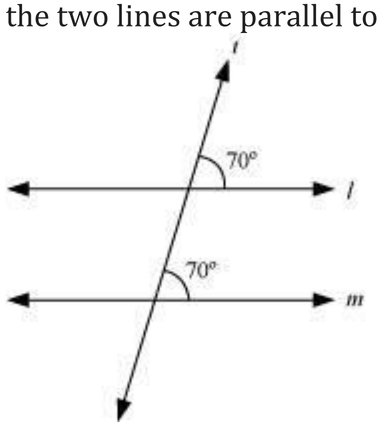
  When a transversal intersects two lines *l* and *m*, the **c**orresponding angles so formed at the intersection points are named as follows:



  ∠1 and ∠5, ∠2 and ∠6
  ∠3 and ∠7, ∠4 and ∠8

- **Corresponding angles axiom**
  If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.
  In the above figure, if lines *l* and *m* become parallel then we will have following pair of equal angles:
  ∠1 =∠5, ∠2 =∠6, ∠3 =∠7 and ∠4 = ∠8

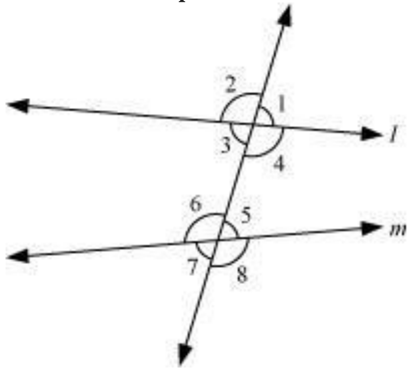
- **Converse of corresponding angles axiom**
  If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.



  In the figure, the corresponding angles are equal. Therefore, the lines *l* and *m* are parallel to each other.

- **Alternate angles**

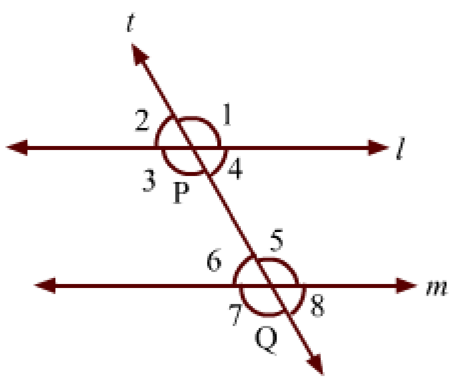
When a transversal intersects two lines *l* and *m*, the alternate angles so formed at the intersection points are named as follows:



Alternate interior angles
∠3 and ∠5, ∠4 and ∠6

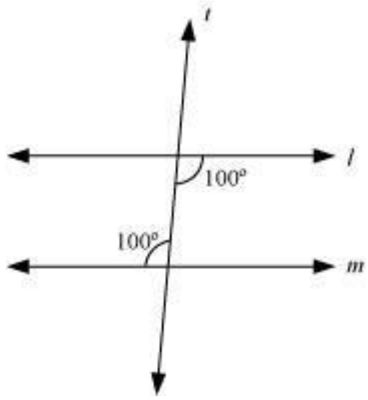
Alternate exterior angles
∠1 and ∠7, ∠2 and ∠8

- **Alternate angles axiom**
  If a transversal intersects two parallel lines, then the angles in each pair of alternate angles are equal.



In the above figure, lines *l* and *m* are parallel. So, by using the alternate angles axiom, we can say that:
∠1 = ∠7, ∠2 = ∠8, ∠3 = ∠5 and ∠4 = ∠6

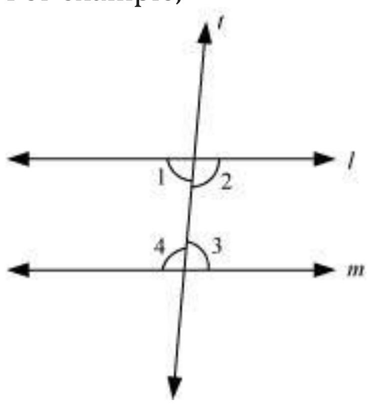
- **Converse of alternate angles axiom**
  If a transversal intersects two lines such that the angles in a pair of alternate angles are equal, then the two lines are parallel.



In the above figure, alternate interior angles are equal (100°) and thus, lines *l* and *m* are parallel.

- **Property of interior angles on the same side of a transversal:**

  If a transversal intersects two parallel lines, then the angles in a pair of interior angles on the same side of the transversal are supplementary.
  For example,

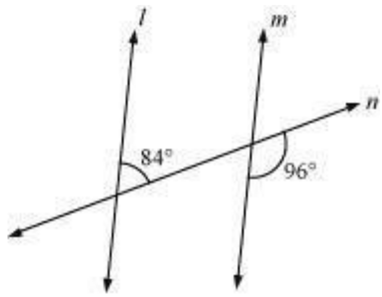


  In the given figure, if lines *l* and *m* are parallel to each other then $\angle 1 + \angle 4 = 180°$ and $\angle 2 + \angle 3 = 180°$.

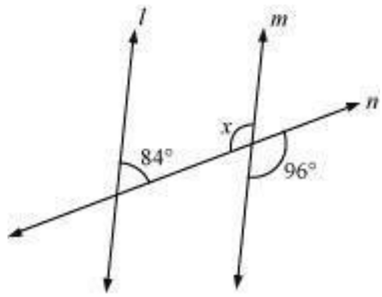
- **Converse of the property of interior angles on the same side of a transversal:**

  If a transversal intersects two lines such that the interior angles on the same side of the transversal are supplementary, then the lines intersected by the transversal are parallel.

  Example: In the given figure, decide whether *l* is parallel to *m* or not.



Solution:



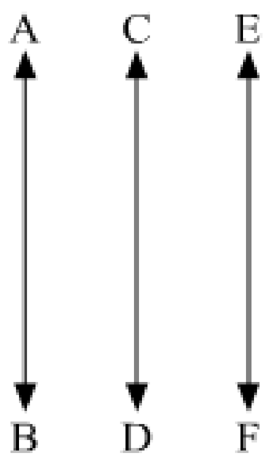
$\angle x = 96°$ (Vertically opposite angles)

$\angle x + 84 = 96° + 84° = 180°$

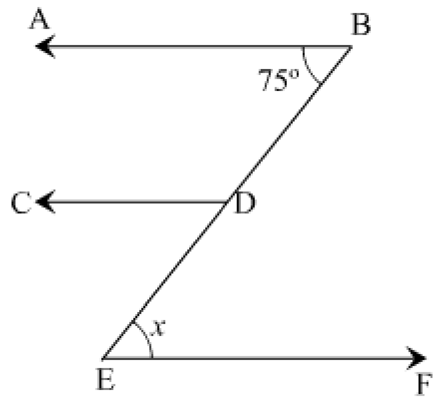
i.e., Sum of the interior angles on the same side of the transversal is supplementary.

Therefore, $l || m$.

- Lines which are parallel to the same line are parallel to each other.
  In the given figure, AB ∥ CD and CD ∥ EF, therefore AB ∥ EF.

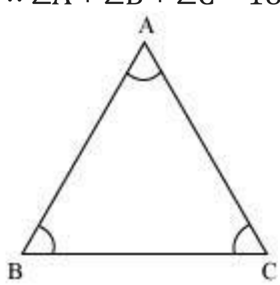


Example: In the given figure, line AB is parallel to CD and CD is parallel to EF. Find the value of x



Solution: It is given that AB || CD and CD || EF.
We know that the lines which are parallel to the same line are parallel to each other.
Therefore, AB || EF.
⇒ $x = 75°$ (Alternate interior angles)

- **Angle sum property of triangles:**
  The sum of all the three interior angles of a triangle is 180°.
  $\therefore \angle A + \angle B + \angle C = 180°$



Example: If the measures of the angles of a triangle are in the ratio 2: 4: 6, then find all the angles of the triangle.

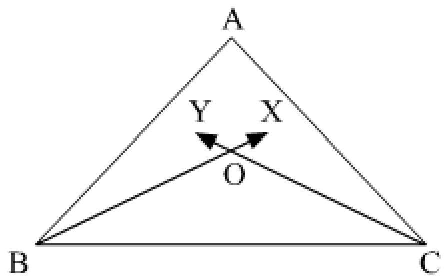
Solution: Ratio of the measures of angles = 2: 4: 6
Therefore, let the angles of the triangle measure $2x$, $4x$, and $6x$.
Now, $2x + 4x + 6x = 180°$ {By angle sum property of triangles}
⇒ $12x = 180°$
⇒ $x = 15°$
Thus, the angles of the triangle are
$2x = 2 \times 15° = 30°$,
$4x = 3 \times 15° = 60°$
$6x = 6 \times 15° = 90°$.

The measure of one of angle is 90°.

- **Facts deduced from angle sum property of triangles:**

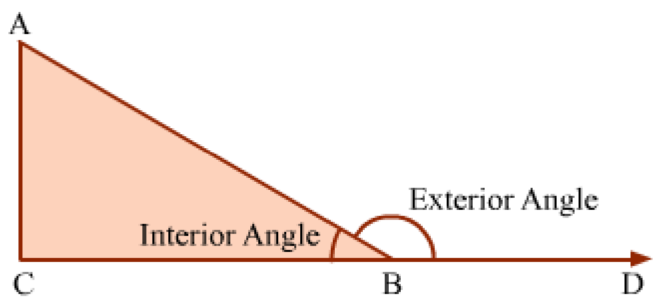
There can be no triangle with two right angles or two obtuse angles.
There can be no triangle with all angles less than or greater than 60°.

- **Relation between the vertex angle and the angles made by the bisectors of the remaining angles:**



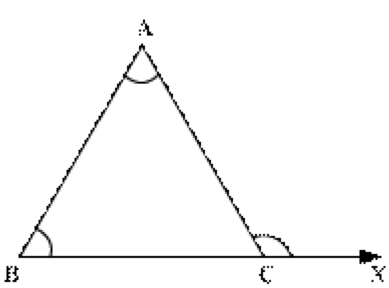
In ΔABC, BX and CY are bisectors of ∠B and ∠C respectively. Also, O is the point of intersection of BX and CY.
Therefore, $\angle BOC = 90° + \frac{1}{2}\angle A$.

- The angle formed by a side of a triangle with an extended adjacent side is called an **exterior angle of the triangle**.



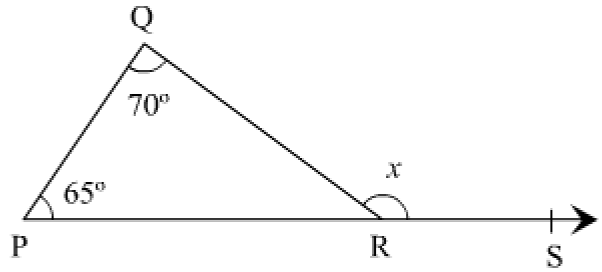
It can be seen that in **Δ**ABC, side CB is extended up to point D. This extended side forms an angle with side AB, i.e., ∠ABD. This angle lies exterior to the triangle. Hence, ∠**ABD is an exterior angle of ΔABC.**

- If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.



∠ACX = ∠BAC + ∠ABC
This property is known as exterior angle property of a triangle.

Example: Find the value of x in the following figure.



Solution: $\angle QRS$ is an exterior angle of ΔPQR. It is thus equal to the sum of its interior opposite angles.
$\therefore \angle QRS = \angle QPR + \angle PQR$
$\Rightarrow x = 65° + 70° = 135°$
Thus, the value of x is 135°.

- Two exterior angles can be drawn at each vertex of triangle. The two angles thus drawn have an equal measure and are equal to the sum of the two opposite interior angles.