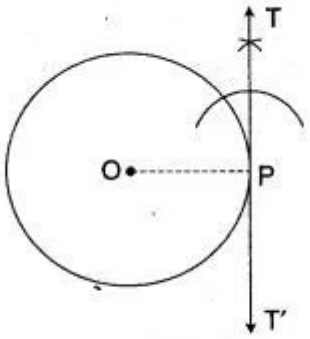
Chapter 16. Circle Constructions

Figure Based Questions

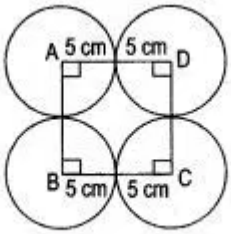
Question 1. Take a point O on the plane at the paper. With O as centre draw a circle of radius 3 cm. Take a point P on this circle and draw a tangent at P.

Solution: Steps of construction:
(i) Take a point O on the plane at the paper and draw a circle at radius 3 cm.
(ii) Take a point P on the circle and join OP.



(iii) Construction $OPT = 90°$
(iv) Produce TP to T′ obtain the required tangent TPT′.

Question 2. Four equal circles, each of radius 5 cm, touch each other as shown in the figure. Find the area included between them. (Take π= 3.14)



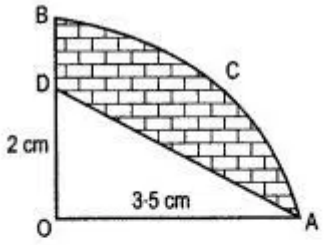
Solution : Required area = Area of square ABCD – Area of 4 quadrants

$$= (10\text{ cm} \quad 10\text{ cm}) - 4 \quad \frac{1}{4} \quad r^2$$

$$= 100 - \frac{22}{7} \quad 5 \quad 5$$

$$= 21{\cdot}5\text{ cm}^2.$$ Ans.

Question 3. In the figure alongside, OAB is a quadrant of a circle. The radius OA = 3.5 cm and OD = 2 cm. Calculate the area of the shaded 22 portion.



Solution : Area of the shaded portion

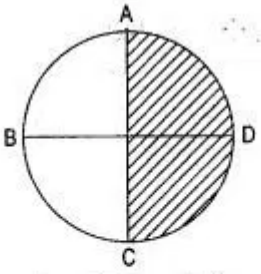
= Area of Quadrant – Area of OAD

$= \frac{}{4}(3{\cdot}5)^2 - \frac{1}{2}$ 3·5 2

$= \frac{22}{7}\cdot\frac{1}{4}$: 3·5 3·5 – 3·5

= 6·125 cm^2 Ans.

Question 4. AC and BD are two perpendicular diameter of a circle ABCD. Given that the area of shaded portion is 308 cm^2 calculate:



(i) The length of AC and

(ii) The circumference of circle.

Solution.(i) Let r be the radius of the circle.

Then

$\frac{1}{2}\ r^2 = 308$

$r^2 = 308 \quad 2 \quad \frac{7}{22} = 14^2$

r = 14 cm

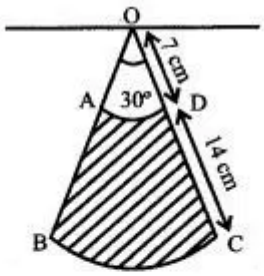
Now, AC = 2r = 28 cm Ans.

(ii) Circumference = 2 r

$= \frac{22}{7}$ 28

= 88 cm. Ans.

Question 5. The diagram represents the area swept by wiper of a car. With the dimension given in figure, calculate the shaded swept by the wiper.



Solution : Area of shaded portion

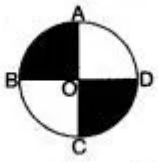
$$= \frac{}{360^\circ}\{(21)^2 - (7)^2\}\ \text{cm}^2$$

$$= \frac{30^\circ}{360^\circ}\ \frac{22}{7}\{441 - 49\}\ \text{cm}^2$$

$$= \frac{22}{12}\ \frac{}{7}\ 392\ \text{cm}^2$$

$$= 102{\cdot}67\ \text{cm}^2.$$

Question 6. AC and BD are two perpendicular diameters of a circle with centre O. If AC = 16 cm, calculate the area and perimeter of the shaded part. (Take π = 3.14).



Solution : Given radius of circle is 8 cm

$$\text{Ar. of shaded part} = \frac{1}{2}\ r^2 = \frac{1}{2}\ 3{\cdot}14\ 8^2$$

$$= 100{\cdot}48\ \text{cm}^2$$

Perimeter of the shaded part

$$= \pi r + 4r$$

$$= 3{\cdot}14 \times 8 + 4 \times 8$$

$$= 25{\cdot}12 + 32$$

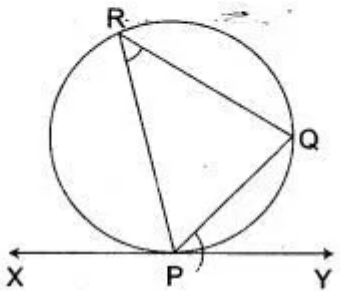
$$= 57{\cdot}12\ \text{cm}$$

Question 7. Draw a circle at radius 4 cm. Take a point on it. Without using the centre at the circle, draw a tangent to the circle at point P.

Solution : Steps of construction :

(i) Draw a chord PQ through the given point on the circle.

(ii) Take a point R on the circle and join P and Q to a point R.



(iii) Construct $\angle QPY = \angle PRQ$ on the opposite sides of the chord PQ.

(iv) Produce YP to X′ to get YPX as the required tangent.

Construct a tangent to the circle from an external point :

In this section we shall study the construction at tangent to a circle from an external point when its center is; (i) Known (ii) Unknown.

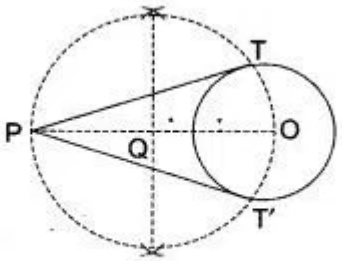
Type (I). Construction at tangent to a circle from an external point when its centre is known.

Steps of construction :

(i) Join the centre O of the circle to the given external point P i.e., join OP.

(ii) Draw right bisector of OP, intersecting OP at Q.

(iii) Taking Q as centre and OQ = PQ as radius, draw a circle to intersect the given circle at T and T′.



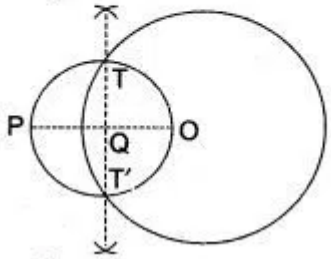
(iv) Join PT and PT′ to get the required tangent as PT and PT′.

Question 8. Draw a circle at radius 3 cm. Take a point at 5.5 cm from the centre at the circle. From point P, draw two tangent to the circle.

Solution : Steps of construction :

(i) Take a point O in the plane paper and draw a circle of radius 3 cm.

(ii) Mark a point P at distance 5·5 cm from the centre O and join OP.



(iii) Draw the right bisector at OP, intersecting OP at Q.

(iv) Taking Q as centre and OQ = PQ as radius, draw a circle to intersect the given circle at T and T′.

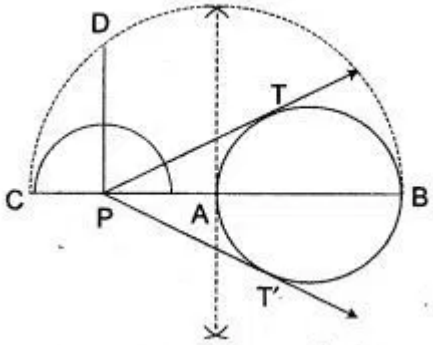
(v) Join PT and PT′ to get the required tangent.

Type (II). Construction of a tangent to a circle from an external point when its centre is known.

Steps of construction :

Let P be the external point form where the tangent are to be drawn to the given circle.

(i) Through P draw a secant PAB to intersect the circle at A and B.



(ii) Join AP to a point C such that AP = DX is equal to the mid-point at AC.

(iii) Draw a semi-circle with BC as diameter.

(iv) Draw PD ⊥ BCX intersecting the semi-circle at D.

(v) With P as centre and PD as radius draw arcs to intersect the given circle at T and T′.

(vi) Join PT and PT′. Then PT and PT′ are the required tangent.

Question 9. Use a ruler and a pair of compasses to construct ΔABC in which BC = 4.2 cm, ∠ ABC = 60° and AB 5 cm. Construct a circle of radius 2 cm to touch both the arms of ∠ ABC of Δ

ABC.

Solution : BC = 4·2 cm, ∠ ABC = 60° and AB = 5 cm.

Steps of construction :

(i) Draw BC of length 4·2 cm.

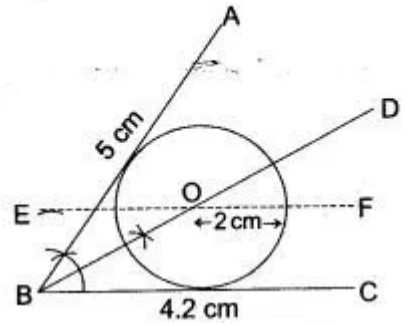
(ii) Draw an angle of 60° at B.

(iii) Cut BA = 5 cm and join A to B.

(iv) Draw angle bisector of ∠ ABC.

(v) Draw BD at 2 cm intersecting EF at O.

(vi) Taking O as centre and 2 cm as radius draw the required circle.



Question 10. Construct an isosceles triangle ABC such that AB = 6 cm, BC = AC = 4 cm. Bisect ∠C internally and mark a point P on this bisector such that CP = 5 cm. Find the points Q and R which are 5 cm from P and also 5 cm from the line AB.

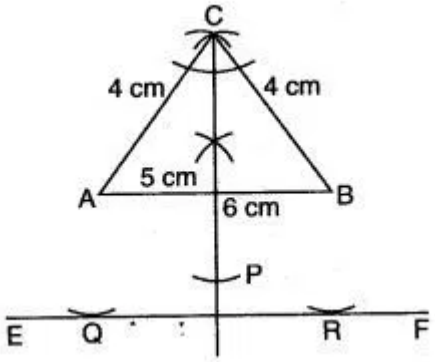
Solution : Steps of construction :

(i) Draw AB = 6 cm and cut arc of 4 cm from A and B these arcs intersect at C join AC and BC.

(ii) Draw the bisector (internal) of ∠ C and mark the point P, taking CP = 5 cm.

(iii) Draw a line EF parallel to AB at a distance of 5 cm.

(iv) Take P as centre cut two points on line EF as PQ and PR are each equal to 5 cm.



Question 11. Draw two lines AB, AC so that ∠ B AC = 40°:
(i) Construct the locus of the centre of a circle that touches AB and has a radius of 3.5 cm.
(ii) Construct a circle of radius 35 cm, that touches both AB and AC, and whose centre lies within the ∠BAC.

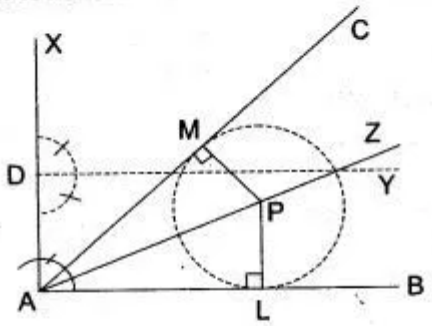
Solution : Steps of construction :

(i) Draw a line AX perpendicular to AB.

(ii) Mark off a point D on AX such that AD = 3·5 cm.

(iii) At D, draw the line DY at right angles to AX.

Then DY is the required locus of the centre of circle that touches AB and has a radius of 3·5 cm.



(iv) Construct the bisector AZ of ∠BAC intersecting DY at P.

(v) Draw PL, PM perpendicular to AB and AC respectively.

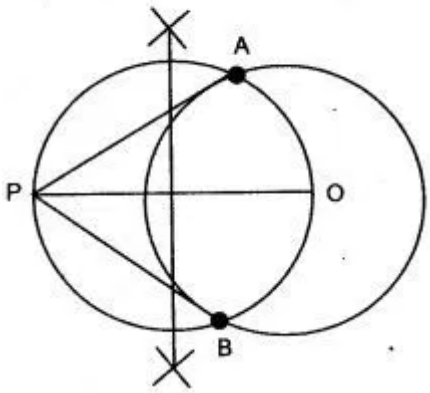
(vi) With P as centre and radius equal to 3·5, draw the circle which will pass through L and M.

Then this is the required circle that touches both AB and AC, and whose centre lies within the ∠ BAC.

Question 12. Draw a circle of radius 3.5 cm. Mark a point P outside the circle at a distance of 6 cm from the centre. Construct two tangents from P to the given circle. Measure and write down the length of one tangent.

Solution : PA & PB are the required tangents

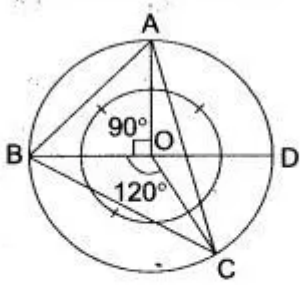
PA = PB = 4·8 cm



Question 13. Construct a triangle ABC, given that the radius of the circumcircle of triangle ABC is 3.5 cm, ∠ BCA = 45° and ∠ BAC = 60°.

Solution: Steps of construction:

(i) Draw a circle with radius = 3·5 cm.



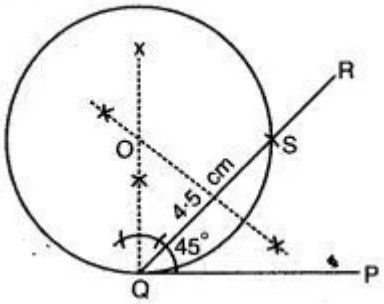
(iii) Again make BOC = 120°.

(ii) Draw diameter BOD and construct BOA = 90°.

(iv) Join AB, AC and BC. Then ABC is the required triangle.

Question 14. Construct an angle PQR = 45°. Mark a point S on QR such that QS = 4.5 cm. Construct a circle to touch PQ at Q and also to pass through S.

Solution :



Steps of construction

(i) Draw PQX = 90°

(ii) Bisect PQX and draw PQR = 45°

(iii) Cut off QS = 4·5 cm from QR.

(iv) r bisector of QS and it is intersect of QX at O.

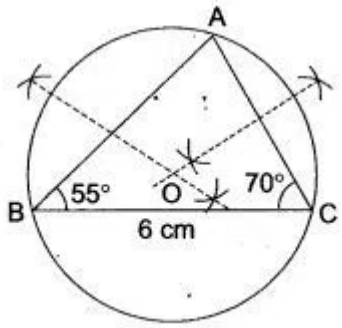
(v) With O as centre and radius = OQ (or OS draw the required circle).

Question 15. Construct the circumcircle of the ABC when BC = 6 cm, B = 55° and C = 70°.

Solution : Steps of construction :

(i) Draw a line segment BC = 6 cm.

(ii) At point B, make CBA = 55° and at point C, make an angle BCA = 70° with the help of a protractor. Join AB and AC.



(iii) Draw the perpendicular bisectors of sides AB and AC. Let them intersect at O.

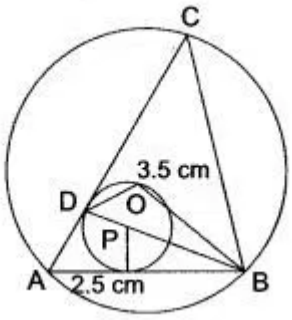
(iv) With O as centre and radius OA, draw a circle which passes through the point A, B and C.

This is the required circumcircle of ABC.

Question 16. Using ruler and compass only, construct a triangle ABC such that AB = 5 cm, ABC = 75° and the radius of the circumcircle of triangle ABC is 3.5 cm.
On the same diagram, construct a circle, touching AB at its middle point and also touching the side AC.

Solution : Steps : Draw a line segment AB = 5 cm long. Make an angle of 75° at 'B' draw perpendicular bisector of AB and angular bisector of B.

Mark 3·5 cm on the perpendicular bisector with O as centre and radius equal to OA or OB draw circum-circle. Mark 2·5 cm on AC from A. Join BD, it will intersect at P, with P as centre and PD as radius draw another circle.

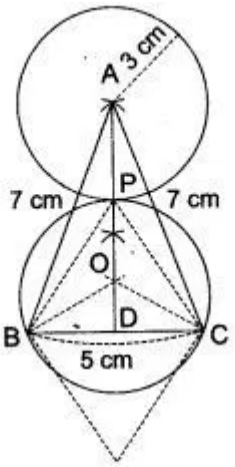


Question 17. (a) Only ruler and compass may be used in this question. All contraction lines and arcs must be clearly shown and be of sufficient length and clarity to permit assessment.
(i) Construct a ABC, such that AB = AC = 7 cm and BC = 5 cm.
(ii) Construct AD, the perpendicular bisector of BC.
(iii) Draw a circle with centre A and radius 3 cm. Let this drcle cut AD at P.
(iv) Construct another circle, to touch the circle with centre A, externally at P, and pass through B and C.

Solution : Steps of Construction :

(i) (1) Draw BC = 5 cm.

(2) With B and C as centres draw two arcs to length 7 cm cutting each other at A. (3) Join AB and AC. (4) Then ABC is required triangle.



(ii) Draw AD, the right bisector of BC.

(iii) With A as centre and radius 3 cm draw a circle meeting AD in P.

(iv) (1) Join BP and CP.

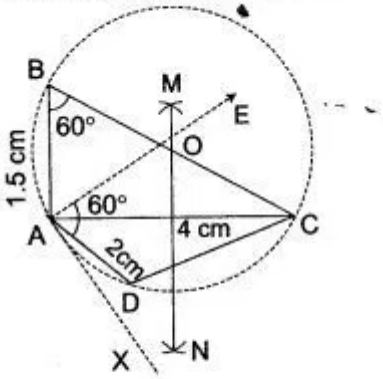
(2) Draw the right bisector of CP meeting AD in O.

(3) With O as centre and radius equal to OP draw the required circle, passing through B and C.

Question 18. Using ruler and compass construct a cyclic quadrilateral ABCD in which AC = 4 cm, ∠ ABC = 60°, AB 1.5 cm and AD = 2 cm. Also write the steps of construction.

Solution : Steps of construction :

(i) Draw a line segment AC = 4 cm.



(ii) Draw ∠CAX = 60°.

(iii) Draw the perpendicular bisector MN of AC.

(iv) Draw EA ⊥ AX at the point A which intersects MN at O.

(v) With centre O and radius OA draw a circle.

(vi) Mark a point B on the circumference of the circle such that AB = 1·5 cm and mark a point D on the circumference so that AD = 2·0 cm.

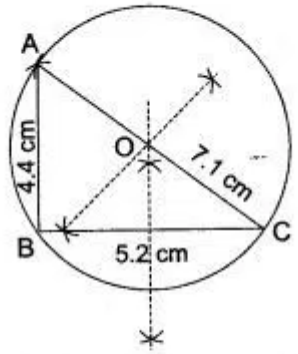
(vii) Join BC and measure ∠ ABC = 60°.

Then ABCD is the required cyclic quadrilateral.

Question 19. Construct a triangle whose sides are 4.4 cm, 5.2 cm and 7.1 cm. Construct its circumcircle. Write also the steps of construction.

Solution: Steps of construction:

1. Draw an line segment BC = 5·2.
2. With centre B and radius BA = 4·4 cm, draw an arc.

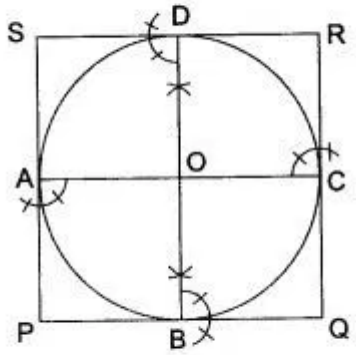


3. With C centre and radius CA = 7·1 cm, draw an arc intersecting the previous arc at A. Then ABC is the given triangle.
4. Draw the perpendicular bisectors of any two sides, say BC and AC, intersecting at O. Then O is the circumcentre of Δ ABC.

Question 20. Draw a circle of radius 3 cm. Construct a square about the circle.

Solution : Steps of construction :

1. Draw a circle with centre O and radius equal to 3 cm.
2. Draw a diameter AC.
3. Draw another diameter BD which bisects AC at rt. ∠s.

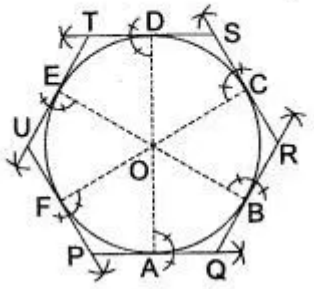


4. Join AB, BC, CD and DA.
5. Now draw tangents to the given circle at the points A, B, C, D and let them meet at P, Q, R, S. Then PQRS is the required square about the given circle.

Question 21. Draw a circle of radius 2.5 cm and circumscribe a regular hexagon about it.

Solution : Steps of construction :

1. Draw a circle with centre O and radius = 2·5 cm.



2. With radius (= 2·5 cm.) cut off six equal arcs along circumference and take these points as A, B, C, D, E and F respectively.

3. Draw tangents at A, B, C, D, E and F meeting to form circumscribed hexagon PQRSTU.

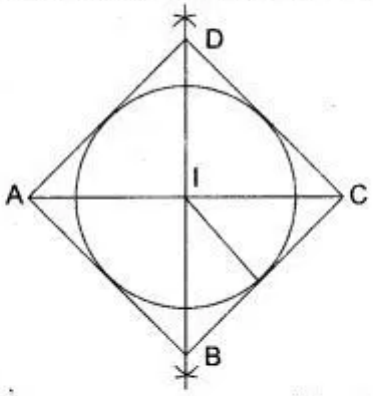
Question 22. Construct the rhombus ABCD whose diagonals AC and BD are of lengths 8 cm and 6 cm respectively. Construct the inscribed circle of the rhombus. Measure its radius.

Solution : Steps of construction :

1. Draw AC = 8 cm.

2. Draw perpendicular bisector of AC = cut it at I.

3. cut IB $= \frac{6}{2} = 3$ cm and ID = 3 cm and join ABCD. Then ABCD is the required rhombus.



4. Now form I draw a perpendicular to BC to meet it at L.

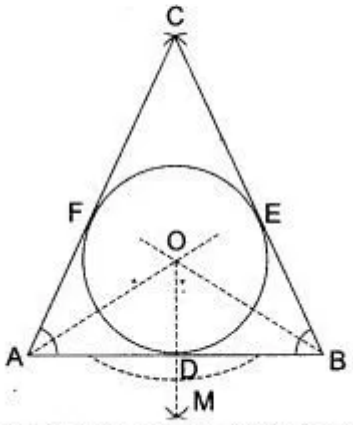
5. With I as centre and IL as radius, draw a circle which is the required circle. Its radius = IL = 2·4 cm.

Question 23. Draw an isosceles triangle with sides 6 cm, 4 cm and 6 cm. Construct the in circle of the triangle. Also write the steps of construction.

Solution : Steps of construction :

1. Construct a ΔABC such that AB = 4 cm, AC = 6 cm, BC = 6 cm.

2. Draw the bisectors of $\angle A$ and $\angle B$ and let them meet at O.



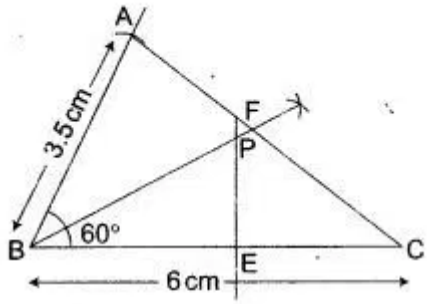
3. With O as centre draw OM the right bisector of AB meeting AB at D.

4. With O as centre and OD as radius draw a circle. The circle touches the sides of the ΔABC at D, E and F.

Then this is the required in-circle of Δ ABC.

Question 24. Use ruler and compasses only for this question:
(i) Construct A ABC, where AB = 3.5 cm, BC = 6 cm and $\angle$ ABC = 60°.
(ii) Construct the locus of points inside the triangle which are equidistant from BA and BC.
(iii) Construct the locus of points inside the triangle which are equidistant from B and C.
(iv) Mark the point P which is equidistant from AB, BC and also equidistant from B and C. Measure and record the length of PB.

Solution :



(i) See figure.

(ii) Inside the triangle point P is equidistant from BA and BC.

(iii) Line EF is the locus of points inside the triangle.

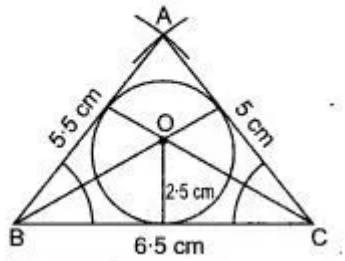
which are equidistant from B and C.

(iv) PB = 3·5 cm Ans.

Question 25. Construct a Δ ABC with BC = 6.5 cm, AB = 5.5 cm, AC = 5 cm. Construct the incircle of the triangle. Measure and record the radius of the incircle.

Solution : Given that

BC = 6·5 cm, AB = 5·5 cm, AC = 5 cm



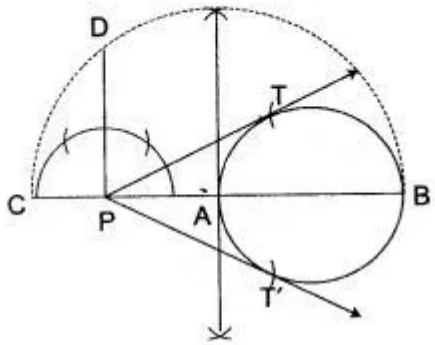
Radius of in circle is 2·5 cm.

Question 26. Draw a circle of radius 4 cm. Take a point P out side the circle without using the centre at the circle. Draw two tangent to the circle from point P.

Solution: Steps of construction:

(i) Draw a circle of radius 4 cm.

(ii) Take a point P out side the circle and draw a secant PAB, intersecting the circle at A and B.



(iii) Produce AP to C such that AP = CP.

(iv) Draw a semi-circle with CB as diameter.

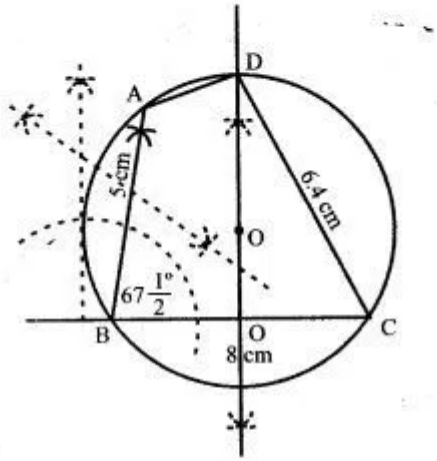
(v) Draw PD ⊥ CB intersecting the semi -circle at D.

(vi) With P as centre and PD as radius draw arcs to intersect the given circle at T and T′.

(vii) Join PT and PT′. Then PT and PT′ are the required tangents.

Question 27. Use ruler and compass only for this question construct the cyclic quadrilateral ABCD in which AB = 5 cm, BC = 8 cm, $\angle ABC = 67\frac{1}{2}^\circ$ and D is equidistant from B and C.

Solution :



Steps of Constuction :

(1) Draw BC = 8 cm, construct $\angle B = 67\frac{1}{2}^\circ$ and cut off AB = 5 cm.

(2) Draw bisector of BC and AB and produce these to meet at O.

(3) With O as centre, radius OA or OB or OC draw a circle to pass through ABC.

(4) Produce the bisector of BC to intersect the circumference of the circle. This intersection point is D.

(5) Join CD and AD.

(6) ABCD is the required cyclic quadrilateral.

Length of CD = 6.4 cm. (By measurement)

Question 28. Ruler and compasses only may be used in this question. All constructions lines and arcs must be clearly shown, and the be sufficient length and clarity to permit assessment:
(i) Construct a triangle ABC, in which AB = 9 cm, BC = 10 cm and angle ABC = 45°.
(ii) Draw a circle, with centre A and radius 2.5 cm. Let it meet AB at D.
(iii) Construct a circle to touch the circle with center A externally at D and also to touch the line

BC.

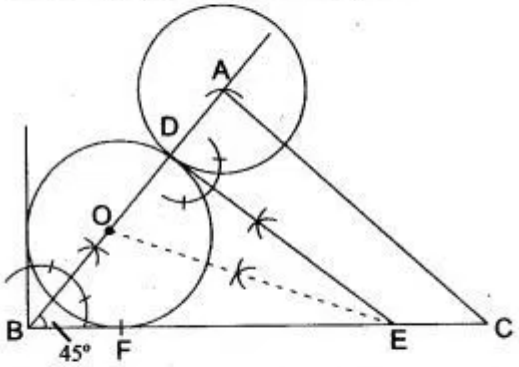
Solution : Steps of construction :

(i) Take BC = 10 cm.

(ii) Make ABC = 45° and with centre B, cut the arc BA = 9 cm.

(iii) Join AC, so ABC is the required triangle.

(iv) With A as centre and radius = 2·5 cm, draw a circle. It will pass through D.



(v) Draw DE AB, which cuts BC at E.

(vi) Draw the angle bisector of BED which cut BD at O.

(vii) Taking radius = OD draw a circle which touches the first circle at D and also touches the line BC at F.

(viii) This is the required circle. The radius OD = 2·7 cm.

Question 29. (i) Construct a triangle ABC, in which AB = 5·0 cm, BC = 3·5 cm and $ABC = 67\frac{1}{2}^\circ$ (Use a pair of compasses and ruler only).

(ii) Construct a circle to touch AB at B and it pass though C.

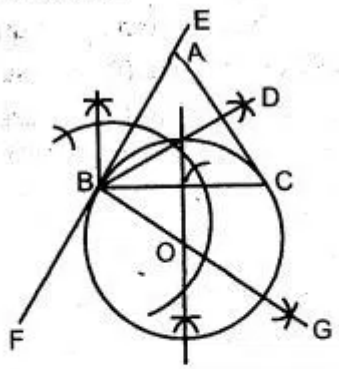
Solution : Steps of construction :

1. Draw BC = 3·5 cm.

2. At B draw BE such that $EBC = 67\frac{1}{2}^\circ$. From BE cut off BA = 5 cm.

3. Join AC. Then ABC is the required triangle.

4. Produce EB to F.



5. At B draw BG such that $\angle EBG = 90°$.

6. Draw perpendicular bisector of BC to cut BG at O.

7. With O as centre and OB as radius draw a circle. This is the required circle to touch AB at B and pass through C.

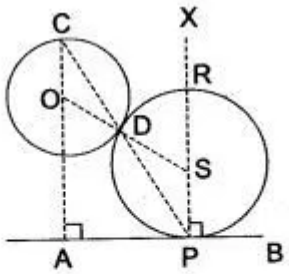
Question 30. The centre O of a circle of a radius 1.3 cm is at a distance of 3.8 cm from a given straight line AB. Draw a circle to touch the given straight line AB at a point P so that OP = 4.7 cm and to touch the given circle externally.

Solution : Steps of construction :

1. Take a point O at a distance of 3·8 cm from AB and with O as centre draw a circle of radius 1·3 cm.

2. With O as centre and radius equal to 4·7 cm, draw an arc cut BA at P. Draw PX perpendicular to AB.

3. Produce AO to cut the circle at C and join CP cutting the circle at D.

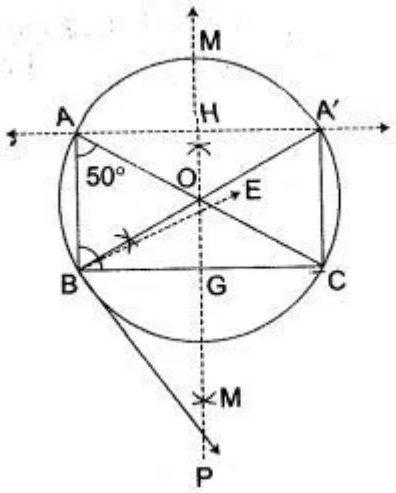


4. Join OD and produce it to cut PX at S. With S as centre and radius = SD, draw the circle PDR. This is the required circle.

Question 31. Construct a triangle having base 6 cm, vertical angle 60° and median through the vertex is 4 cm.

Solution : Steps of construction :

1. Draw a line segment AB = 6 cm.
2. Make an angle PAB = 60°
3. Draw the perpendicular bisectors LM of AB.
4. Draw TA ⊥ AP at A.



5. Let LM and TA intersect at O.
6. Draw the circle with centre O and radius OA. Then any angle in the major segment, ∠PAB = 60° (Angle in alternate segment).
7. Let LM intersect AB at N, then N is the mid-point of AB. Taking N as centre and 4 cm radius, draw arcs intersecting the circle C, C′.
8. Join CA, CB and C′A and C′B.

Then the required triangle is ABC or ABC′.

Question 32. Using a ruler and compasses only:
(i) Construct a triangle ABC with the following data:
AB = 3.5 cm, BC = 6 cm and ∠ ABC = 120°.
(ii) In the same diagram, draw a circle with BC as diameter. Find a point P on the circumference of the circle which is equidistant from AB and BC.
(iii) Measure ∠ BCP.

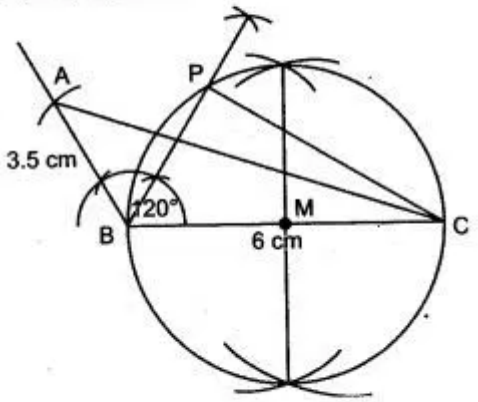
Solution :

(i) 1. Draw a line segment BC = 6 cm.

2. Draw $\angle ABC = 120°$.

3. With centre B and radius BA = 3·5 cm, draw an arc.

4. Join AB and AC.



(ii) 1. Draw the right bisector on line BC cut the BC line at M.

2. Draw another bisector of line AC.

3. With M as centre and radius MB draw a circle cut AC bisector at P.

(iii) $\angle BCP = 30°$.

Question 33. Draw a circle of radius 3 cm and construct a tangent to it from an external point without using the centre.

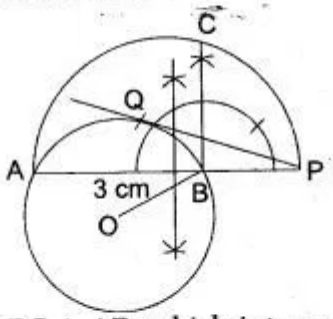
Solution : Steps of construction :

1. With centre O and radius = 3 cm, draw a circle.

2. Take any point P outside the circle.

3. Through the external point P draw a straight line PBA meeting the circle at A and B.

4. Draw a semi-circle on AP as diameter.



5. Draw BC ⊥ AP, which intersects the semi-circle at C.

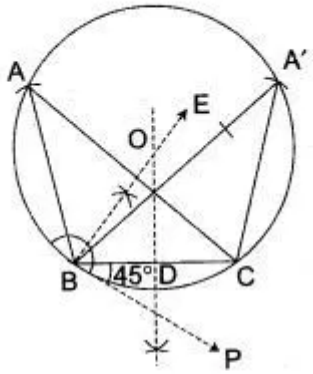
6. With centre P and radius PC draw an arc cutting the circle at Q.

7. Join PQ. Then PQ is the required tangent.

Question 34. Construct a ΔABC with base BC = 3.5 cm, vertical angle ∠BAC = 45° and median through the vertex A is 3.5 cm. Write also the steps of construction.

Solution : Steps of construction :

1. Draw a line segment BC = 3 cm and make ∠CBP = 45°.



2. Construct EB ⊥ BP.

3. Draw the perpendicular bisector of BC it's intersecting BE in O and BC in D.

4. Draw a circle taking O as centre and OB as radius.

5. Now with D as centre and radius = 3·5 cm draw arcs of the circle intersecting the above drawn circle in A and A′.

6. Join AB, AC and A′B, A′C.

Then the Δs, ABC and A′BC are the required triangles.