

# 11. SIMPLE HARMONIC MOTION

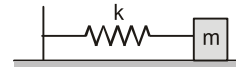
## S.H.M.

$$F = -kx$$

General equation of S.H.M. is  $x = A \sin(\omega t + \phi)$ ;  $(\omega t + \phi)$  is phase of the motion and  $\phi$  is initial phase of the motion.

**Angular Frequency ( $\omega$ ) :** 
$$\omega = \frac{2\pi}{T} = 2\pi f$$

**Time period (T) :** 
$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$$



**Speed :** 
$$v = \omega \sqrt{A^2 - x^2}$$

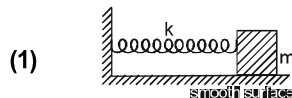
**Acceleration :** 
$$a = -\omega^2 x$$

**Kinetic Energy (KE) :** 
$$\frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 (A^2 - x^2) = \frac{1}{2} k (A^2 - x^2)$$

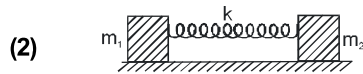
**Potential Energy (PE) :** 
$$\frac{1}{2} Kx^2$$

**Total Mechanical Energy (TME) = K.E. + P.E. =  $\frac{1}{2} k (A^2 - x^2) + \frac{1}{2} Kx^2 = \frac{1}{2} KA^2$  (which is constant)**

## SPRING-MASS SYSTEM



$$T = 2\pi \sqrt{\frac{m}{k}}$$



$$T = 2\pi \sqrt{\frac{\mu}{K}}, \text{ where } \mu = \frac{m_1 m_2}{(m_1 + m_2)} \text{ known as reduced mass}$$

## COMBINATION OF SPRINGS

**Series Combination :** 
$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

**Parallel combination :** 
$$k_{eq} = k_1 + k_2$$

**SIMPLE PENDULUM** 
$$T = 2\pi \sqrt{\frac{\ell}{g}} = 2\pi \sqrt{\frac{\ell}{g_{eff.}}}$$
 (in accelerating Reference Frame);  $g_{eff.}$  is net acceleration due to pseudo force and gravitational force.

## COMPOUND PENDULUM / PHYSICAL PENDULUM

**Time period (T) :**  $T = 2\pi \sqrt{\frac{I}{mg\ell}}$

where,  $I = I_{cm} + m\ell^2$ ;  $\ell$  is distance between point of suspension and centre of mass.

### **TORSIONAL PENDULUM**

**Time period (T) :**  $T = 2\pi \sqrt{\frac{I}{C}}$  where, C = Torsional constant

### **Superposition of SHM s along the same direction**

$x_1 = A_1 \sin \omega t$  &  $x_2 = A_2 \sin (\omega t + \theta)$

If equation of resultant SHM is taken as  $x = A \sin (\omega t + \phi)$

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \theta}$  &  $\tan \phi = \frac{A_2 \sin \theta}{A_1 + A_2 \cos \theta}$

