JEE Advanced 2024
Sample Paper - 2

Time Allowed: 3 hours **Maximum Marks: 180**

General Instructions:

This question paper has THREE main sections and four sub-sections as below.

MRQ

- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) the correct answer(s).
- You will get +4 marks for the correct response and -2 for the incorrect response.
- You will also get 1-3 marks for a partially correct response.

MCQ

- Each question has FOUR options (A), (B), (C) and (D). ONLY ONE of these four options is the correct answer.
- You will get +3 marks for the correct response and -1 for the incorrect response.

NUM

- The answer to each question is a NON-NEGATIVE INTEGER.
- You will get +4 marks for the correct response and 0 marks for the incorrect response.

MATCH

- FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
- You will get +3 marks for the correct response and -1 for the incorrect response.

Mathematics (MRQ)

1. Let $\alpha, \beta, \gamma, \delta$ be real numbers such that $\alpha^2 + \beta^2 + \gamma^2 \neq 0$ and $\alpha + \gamma = 1$. Suppose the point (3, 2, -1) is the mirror image of the point (1, 0, -1) with respect to the plane $\alpha x + \beta y + \gamma z = \delta$. Then which of the following statements is/are TRUE? **[4]**

   a) $\delta - \gamma = 3$ b) $\delta + \beta = 4$

   c) $\delta + \beta + \gamma = \delta$ d) $\alpha + \beta = 2$

2. If M and A are any two events, the probability that exactly one of them occurs is **[4]**

   a) $P(M^c) + P(N^c) - 2P(M^c \cap N^c)$ b) $P(M) + P(N) - P(M \cap N)$

   c) $P(M) + P(N) - 2P(M \cap N)$ d) $P(M \cap N^c) + P(M^c \cap N)$

3. Let $M = (a_{ij})$, $i, j \in \{1, 2, 3\}$, be the 3×3 matrix such that $a_{ij} = 1$, if $j + 1$ is divisible by i, otherwise $a_{ij} = 0$. Then which of the following statements is(are) true? **[4]**

   a) The matrix (M - 2I) is invertible, where I is the 3×3 identity matrix. b) M is invertible.

c) The set $\{X \in \mathbb{R}^3 : MX = 0\} \neq \{0\}$ where $0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

d) There exists a nonzero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that M $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ = $\begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$.

Mathematics (MCQ)

4. Let f be a non-negative function defined on the interval [0, 1]. If $\int_0^x \sqrt{1-(f'(t))^2}dt = \int_0^x$ f(t) dt, $0 \leq$ x $\leq$ 1, and f(0) = 0, then **[3]**

a) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

b) $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

c) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

d) $f\left(\frac{1}{2}\right) > \frac{1}{2}$ and $f\left(\frac{1}{3}\right) > \frac{1}{3}$

5. Let a > 0, b > 0 and c > 0. Then the roots of the equation $ax^2 + bx + c = 0$ **[3]**

a) both are real and negative and have negative real parts

b) have negative real parts

c) have positive real parts

d) are real and negative

6. Let ω be a complex cube root of unity with $\omega \neq 1$. A fair die is thrown three times. If r_1, r_2 and r_3 are the numbers obtained on the die, then the probability that $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$, is **[3]**

a) 1/36

b) 1/9

c) 1/18

d) 2/9

7. A tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ meets x-axis at P and y-axis at Q. Lines PR and QR are drawn such that OPRQ is a rectangle (where O is the origin). Then R lies on: **[3]**

a) $\frac{2}{x^2} + \frac{4}{y^2} = 1$

b) $\frac{4}{x^2} - \frac{2}{y^2} = 1$

c) $\frac{4}{x^2} + \frac{2}{y^2} = 1$

d) $\frac{2}{x^2} - \frac{4}{y^2} = 1$

Mathematics (NUM)

8. Let y'(x) + y(x) g'(x) = g(x) g'(x), y(0) = 0, x $\in$ R, where f'(x) denotes $\frac{df(x)}{dx}$ and g(x) is a given non-constant differentiable function on R with g(0) = g(2) = 0. Then, the value of y(2) is **[4]**

9. Let $z = \frac{-1+\sqrt{3}i}{2}$, where i = $\sqrt{-1}$ and r, $s \in \{1, 2, 3\}$. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then, the total number of ordered pairs (r, s) for which $P^2 = -I$ is **[4]**

10. Let X be the set of all five digit numbers formed using 1, 2, 2, 2, 4, 4, 0. For example, 22240 is in X while 02244 and 44422 are not in X . Suppose that each element of X has an equal chance of being chosen. Let p be the conditional probability that an element chosen at random is a multiple of 20 given that it is a multiple of 5. Then the value of 38p is equal to **[4]**

11. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 and p(3) = 2, then p'(0) is equal to **[4]**

12. Let $\omega = e^{\frac{i\pi}{3}}$ and a, b, c, x, y, z be non-zero complex numbers such that a + b + c = x, $a + b\omega + c\omega^2 = y$, $a + b\omega^2 + c\omega$ = z. Then, the value of $\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is **[4]**

13. Let k be a positive real number and let **[4]**

$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix}$ and

$B = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}$

If det (adj A) + det (adjB) = 10^6, then [k] is equal to

Mathematics (MATCH)

14. Let $f_1 : R \to R, f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R, f_3 : \left(-1, e^{\frac{\pi}{2}} - 2\right)$ → R and f_4 : R → R be functions defined by **[3]**

i. $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right)$,

ii. $f_2(x) = \begin{cases} \frac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$, where the inverse trigonometric functions $\tan^{-1}$ x assumes values in $\text{in } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

iii. f_3(x) = [sin($\log_e$(x + 2))], where, for t ∈ R, [t] denotes the greatest integer less than or equal to t,

iv. $f_4(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$.

LIST - I	LIST - II
(P) The function f_1 is	(1) **NOT** continuous at x = 0
(Q) The function f_2 is	(2) continuous at x = 0 and **NOT** differentiable at x = 0
(R) The function f_3 is	(3) differentiable at x = 0 and its derivative is **NOT** continuous at x = 0
(S) The function f_4 is	(4) differentiable at x = 0 and its derivative is continuous at x = 0

a) P → 2; Q → 1; R → 4; S → 3

b) P → 4; Q → 2; R → 1; S → 3

c) P $\rightarrow$ 4; Q $\rightarrow$ 1; R $\rightarrow$ 2; S $\rightarrow$ 3 d) P $\rightarrow$ 2; Q $\rightarrow$ 3; R $\rightarrow$ 1; S $\rightarrow$ 4

15. Match List I with List II and select the correct answer using the code given below the lists: **[3]**

List-I	**List-II**
(P) Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a} \times \vec{b}), 3(\vec{b} \times \vec{c})$ and $2(\vec{c} \times \vec{a})$ is	(1) 100
(Q) Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a} + \vec{b}), 3(\vec{b} + \vec{c})$ and $2(\vec{c} + \vec{a})$ is	(2) 30
(R) Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a} + 3\vec{b})$ and $(\vec{a} - \vec{b})$	(3) 24
(S) Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a} + \vec{b})$ and $\vec{a}$ is	(4) 60

a) P $\rightarrow$ 1, Q $\rightarrow$ 4, R $\rightarrow$ 3, S $\rightarrow$ 2 b) P $\rightarrow$ 2, Q $\rightarrow$ 3, R $\rightarrow$ 1, S $\rightarrow$ 4

c) P $\rightarrow$ 3, Q $\rightarrow$ 4, R $\rightarrow$ 1, S $\rightarrow$ 2 d) P $\rightarrow$ 4, Q $\rightarrow$ 2, R $\rightarrow$ 3, S $\rightarrow$ 1

16. Match List I with List II and select the correct answer using the code given below the lists: **[3]**

List I	**List II**
(P) $\left(\frac{1}{y^2}\left(\frac{\cos(\tan^{-1} y)+y\sin(\tan^{-1} y)}{\cot(sin^{-1}y)+\tan(\sin^{-1} y)}\right)^2+y^4\right)^{\frac{1}{2}}$ take value	(1) $\frac{1}{2}\sqrt{\frac{5}{3}}$
(Q) If cosx + cosy + cosz = 0 = sinx + siny + sinz then possible value of cos $\frac{x-y}{2}$ is	(2) $\sqrt{2}$
(R) If $\cos\left(\frac{\pi}{4}-x\right)$cos 2x + sinx sin 2 secx = cosx sin2x secx + $\cos\left(\frac{\pi}{4}+x\right)$cos 2x then possible value of sec x is	(3) $\frac{1}{2}$
(S) If cot$\left(\sin^{-1}\sqrt{1-x^2}\right)=\sin\left(\tan^{-1}(x\sqrt{6})\right)$, x $\neq$ 0, then possible value of x is	(4) 1

a) (P) - (4), (Q) - (3), (R) - (1), (S) - (2) b) (P) - (4), (Q) - (3), (R) - (2), (S) - (1)

c) (P) - (3), (Q) - (4), (R) - (1), (S) - (2) d) (P) - (3), (Q) - (4), (R) - (2), (S) - (1)

17. In a high school, a committee has to be formed from a group of 6 boys M_1, M_2, M_3, M_4, M_5, M_6 and 5 girls G_1, G_2, G_3, G_4, G_5. **[3]**

i. Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 boys and 2 girls.

ii. Let α_2 be the total number of ways in which the committee can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

iii. Let α_3 be the total number of ways in which the committee can be formed such that the committee has 5 members, at least 2 of them being girls.

iv. Let α_4 be the total number of ways in which the committee can be formed such that the committee has 4 members, having at least 2 girls such that both M_1 and G_1 are **NOT** in the committee together.

LIST-I	**LIST-II**
(P) The value of α_1 is	(1) 136
(Q) The value of α_2 is	(2) 189
(R) The value of α_3 is	(3) 192
(S) The value of α_4 is	(4) 200
	(5) 381
	(6) 461

a) P $\to$ 4; Q $\to$ 6; R $\to$ 5; S $\to$ 2

b) P $\to$ 4; Q $\to$ 6; R $\to$ 2; S $\to$ 1

c) P $\to$ 4; Q $\to$ 2; R $\to$ 3; S $\to$ 1

d) P $\to$ 1; Q $\to$ 4; R $\to$ 2; S $\to$ 3

Physics (MRQ)

18. The filament of a light bulb has surface area 64 mm^2. The filament can be considered as a black body at temperature 2500 K emitting radiation like a point source when viewed from far. At night the light bulb is observed from a distance of 100 m. Assume the pupil of the eyes of the observer to be circular with radius 3 mm. Then **[4]**

(Take Stefan-Boltzmann constant = 5.67×10^{-8} $Wm^{-2}K^{-4}$, Wien's displacement constant = 2.90×10^{-3} m-K, Planck's constant = 6.63×10^{-34} Js, speed of light in vacuum = 3.00×10^8 ms^{-1})

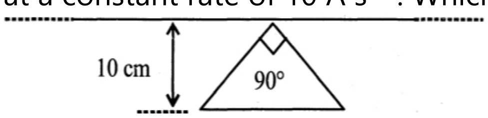
a) radiated power entering into one eye of the observer is in the range 3.15×10^{-8} W to3.25×10^{-8} W

b) the wavelength corresponding to the maximum intensity of light is 1160 nm

c) power radiated by the filament is in the range 642 W to 645 W

d) taking the average wavelength of emitted radiation to be 1740 nm, the total number of photons entering per second into one eye of the observer is in the range 2.75×10^{11} to 2.85×10^{11}

19. A conducting loop in the shape of a right-angled isosceles triangle of height 10 cm is kept such that the 90^{o} vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counter-clockwise direction and increased at a constant rate of 10 A s^{-1}. Which of the following statement(s) is(are) true? **[4]**



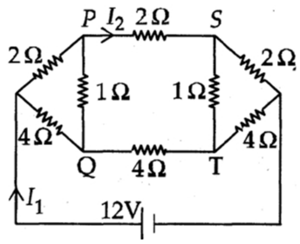
a) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

b) There is a repulsive force between the wire and the loop

c) The induced current in the wire is in opposite direction to the current along the hypotenuse

d) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$volt

20. For the resistance network shown in the figure, choose the correct option(s) **[4]**



a) I_2 = 2 A

b) The current through PQ is zero

c) The potential at S is less than that at Q.

d) I_1 = 3 A

Physics (MCQ)

21. X-rays can not be diffracted by means of an ordinary grating due to: **[3]**

a) Low speed

b) large wavelength

c) high speed

d) short wavelength

22. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by: **[3]**

a) $\frac{\mu_0}{2\pi d}\left(I_1^2+I_2^2\right)$

b) $\frac{\mu_0}{2\pi d}\left(I_1+I_2\right)$

c) $\frac{\mu_0}{2\pi}\left(\frac{I_1+I_2}{d}\right)^{\frac{1}{2}}$

d) $\frac{\mu_0}{2\pi d}\left(I_1^2+I_2^2\right)^{\frac{1}{2}}$

23. A tank is filled with water of density 1 g per cm^3 and oil of density 0.9 g per cm^3. The height of water layer is 100 cm and of the oil layer is 400 cm. If g = 980 cm/sec^2, then the velocity of efflux from an opening in the bottom of the tank is: **[3]**

a) $\sqrt{950\times 980}\mathrm{cm/sec}$

b) $\sqrt{900\times 980}\mathrm{cm/sec}$

c) $\sqrt{920\times 980}\mathrm{cm/sec}$

d) $\sqrt{1000\times 980}\mathrm{cm/sec}$

24. An open pipe is in resonance in 2nd harmonic with frequency f_1. Now one end of the tube is closed and frequency is increased to f_2 such that the resonance again occurs in nth harmonic. Choose the correct option. **[3]**

a) $n = 3, f_2 = \left(\frac{5}{4}\right) f_1$ b) $n = 3, f_2 = \left(\frac{3}{4}\right) f_1$

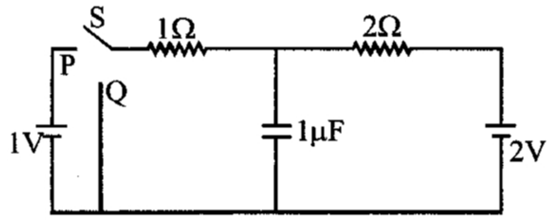
c) $n = 5, f_2 = \left(\frac{5}{4}\right) f_1$ d) $n = 5, f_2 = \left(\frac{3}{4}\right) f_1$

Physics (NUM)

25. A string of length 1m and mass 2×10^{-5}kg is under tension T. When the string vibrates, two successive harmonics are found to occur at frequencies 750 Hz and 1000 Hz. The value of tension T is ________ Newton. **[4]**

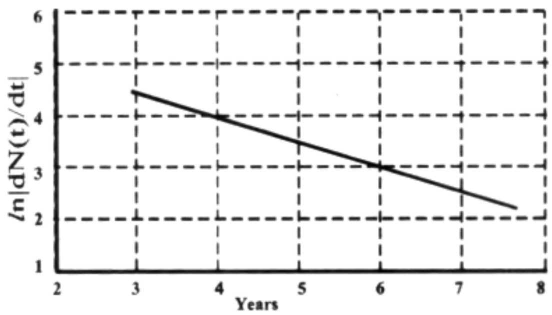
26. A projectile is fired from horizontal ground with speed v and projection angle θ. When the acceleration due to gravity is g, the range of the projectile is d. If at the highest point in its trajectory, the projectile enters a different region where the effective acceleration due to gravity is $g' = \frac{g}{0.81}$, then the new range is d' = nd. The value of n is ________. **[4]**

27. In the circuit shown below, the switch S is connected to position P for a long time so that the charge on the capacitor becomes q_1 μC. Then S is switched to position Q. After a long time, the charge on the capacitor is q_2 μC. **[4]**

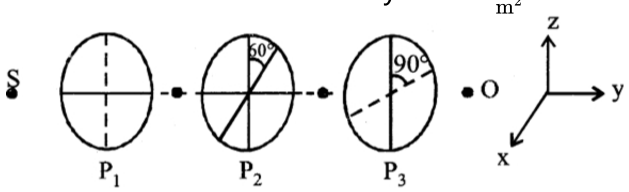


The magnitude of q_1 is ________.

28. To determine the half life of a radioactive element, a student plots a graph of In $\left|\frac{dN(t)}{dt}\right|$ versus t. Here $\left|\frac{dN(t)}{dt}\right|$ is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is **[4]**



29. As shown in figures, three identical polaroids P_1, P_2 and P_3 are placed one after another. The pass axis of P_2 and P_3 are inclined at angle of 60° and 90° with respect to axis of P_1. The source S has an intensity of $256 \frac{\mathrm{W}}{\mathrm{m}^2}$. The intensity of light at O is ________ $\frac{\mathrm{W}}{\mathrm{m}^2}$. **[4]**



30. Consider one mole of helium gas enclosed in a container at initial pressure P_1 and volume V_1. It expands isothermally to volume $4V_1$. After this, the gas expands adiabatically and its **[4]**

volume becomes $32V_1$. The work done by the gas during isothermal and adiabatic expansion processes are W_{iso} and W_{adia}, respectively. If the ratio $\frac{W_{iso}}{W_{adia}} = f \ln 2$, then f is ________.

Physics (MATCH)

31. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively, ignore the gravitational force between the satellites. Define V_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and v_2, L_2, K_2, and T_2 to be the corresponding quantities of satellite 2. Given $\frac{m_1}{m_2}$ = 2 and $\frac{R_1}{R_2}$ = $\frac{1}{4}$, match the ratios in List - I to the numbers in List - II. **[3]**

LIST - I	LIST - II
P. $\frac{v_1}{v_2}$	1. $\frac{1}{8}$
Q. $\frac{L_1}{L_2}$	2. 1
R. $\frac{K_1}{K_2}$	3. 2
S. $\frac{T_1}{T_2}$	4. 8

a) P → 2; Q → 3; R → 1; S → 4
b) P → 4; Q → 2; R → 1; S → 3
c) P → 3; Q → 2; R → 4; S → 1
d) P → 2; Q → 3; R → 4; S → 1

32. A series LCR circuit is connected to a 45 sin(ωt) Volt source. The resonant angular frequency of the circuit is 10^5 rad s^{-1} and current amplitude at resonance is I_0. When the angular frequency of the source is $\omega = 8 \times 10^4$ rad s^{-1}, the current amplitude in the circuit is 0.05 I_0. If L = 50 mH, match each entry in List-I with an appropriate value from List-II and choose the correct option. **[3]**

List-I	List-II
(P) I_0 in mA	(1) 44.4
(Q) The quality factor of the circuit	(2) 18
(R) The bandwidth of the circuit in rad s^{-1}	(3) 400
(S) The peak power dissipated at resonance in Watt	(4) 2250
	(5) 500

a) P → 4, Q → 5, R → 3, S → 1
b) P → 2, Q → 3, R → 5, S → 1
c) P → 3, Q → 1, R → 4, S → 2
d) P → 4, Q → 2, R → 1, S → 5

33. Match the temperature of a black body given in List I with an appropriate statement in List II, and choose the correct option. **[3]**
[Given: Wien's constant as 2.9×10^{-3} m-K and $\frac{hc}{e}$ = 1.24×10^{-6} V-m]

List - I	List - II
(P) 2000 K	(1) The radiation at peak wavelength can lead to emission of photoelectrons from a metal of work function 4eV.
(Q) 3000 K	(2) The radiation at peak wavelength is visible to human eye.
(R) 5000 K	(3) The radiation at peak emission wavelength will result in the widest central maximum of a single slit diffraction.
(S) 10000 K	(4) The power emitted per unit area is $\frac{1}{16}$ of that emitted by a blackbody at temperature 6000 K.
	(5) The radiation at peak emission wavelength can be used to image human bones.

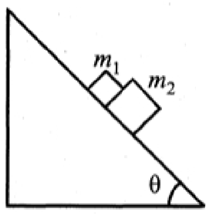
a) P $\rightarrow$ 3, Q $\rightarrow$ 4, R $\rightarrow$ 2, S $\rightarrow$ 1

b) P $\rightarrow$ 1, Q $\rightarrow$ 2, R $\rightarrow$ 5, S $\rightarrow$ 3

c) P $\rightarrow$ 3, Q $\rightarrow$ 2, R $\rightarrow$ 4, S $\rightarrow$ 1

d) P $\rightarrow$ 3, Q $\rightarrow$ 5, R $\rightarrow$ 2, S $\rightarrow$ 3

34. A block of mass m_1 = 1 kg another mass m_2 = 2 kg, are placed together (see figure) on an inclined plane with angle of inclination θ. Various values of 0 are given in List-I. The coefficient of friction between the block m_1 and plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to μ = 0.3. In List-II expressions for the friction on block m_2 are given. Match the correct expression of the friction in List-II with the angles given in List-I, and choose the correct option. The acceleration due to gravity is denoted by g. [Useful information: $\tan(5.5^\circ) \approx 0.1; \tan(11.5^\circ) \approx 0.2; \tan\,(16.5^\circ) \approx 0.3$ **[3]**



List - I	List - II
(P) θ = 5°	(i) $m_2 g\sin\theta$
(Q) θ = 10°	(ii) $(m_1 + m_2)g\ \sin\theta$
(R) θ = 15°	(iii) $\mu m_2 g\cos\theta$
(S) θ = 20°	(iv) $\mu(m_1 + m_2)g\ \cos\theta$

a) (P) - (ii), (Q) - (ii), (R) - (ii), (S) -(iii)

b) (P) - (i), (Q) - (i), (R) - (i), (S) - (iii)

c) (P) - (i), (Q) - (ii), (R) - (ii), (S) - (iii)

d) (P) - (ii), (Q) - (ii), (R) - (ii), (S) - (iv)

Chemistry (MRQ)

35. A positive carbylamine test is given by **[4]**

a) N-methyl-o-methylaniline

b) 2, 4-dimethylaniline

c) p-methylbenzylamine

d) N, N-dimethylaniline

36. For the given aqueous reactions, which of the statement (s) is (are) true? **[4]**

excess KI + $K_3[Fe(CN)_6] \xrightarrow{dilute\ H_2SO_4}$ brownish-yellow solution $\xrightarrow{ZnSO_4}$ white precipitate + $\underbrace{brownish - yellow\ filtrate}$ $\xrightarrow{Na_2S_2O_3}$ colourless solution

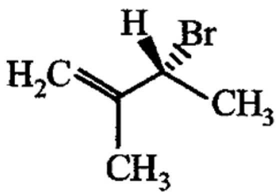
a) The first reaction is a redox reaction.

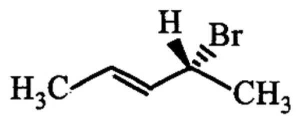
b) White precipitate is $Zn_3[Fe(CN)_6]_2$.

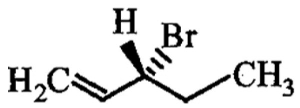
c) Addition of filtrate to starch solution gives blue colour.

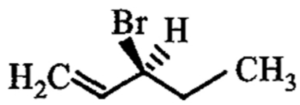
d) White precipitate is soluble in NaOH solution.

37. Compound(s) that on hydrogenation produce(s) optically inactive compound(s) is (are) **[4]**

a) 

b) 

c) 

d) 

Chemistry (MCQ)

38. Which of the following compound is added to the sodium extract before addition of silver nitrate for testing of halogens? **[3]**

a) Nitric acid

b) Hydrochloric acid

c) Sodium hydroxide

d) Ammonia

39. The standard reduction potential values of three metallic cations, X, Y, Z are 0.52, - 3.03 and - 1.18 V respectively. The order of reducing power of the corresponding metals is **[3]**

a) Z > Y > X

b) Z > X > Y

c) Y > Z > X

d) X > Y > Z

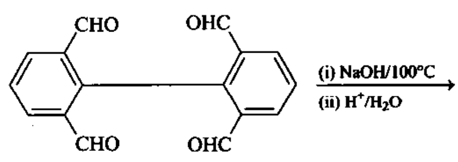
40. Which one of the following is the strongest base? **[3]**

a) AsH_3

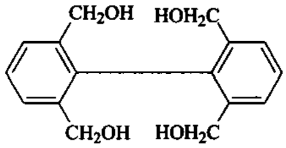
b) NH_3

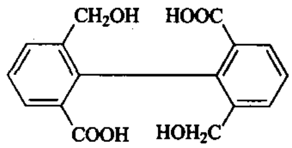
c) PH_3

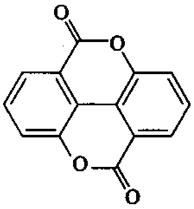
d) SbH_3

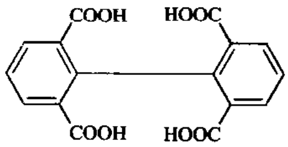
41.  **[3]**

Major product is:

a) 

b) 

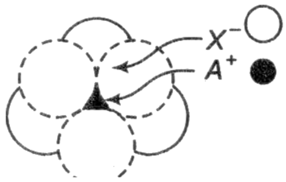
c) 

d) 

Chemistry (NUM)

42. The conductance of a 0.0015 M aqueous solution of a weak monobasic acid was determined by using a conductivity cell consisting of platinized Pt electrodes. The distance between the electrodes is 120 cm with an area of cross section of 1 cm^2. The conductance of this solution was found to be 5×10^{-7} S. The pH of the solution is 4. The value of limiting molar conductivity (Λ_m°) of this weak monobasic acid in an aqueous solution is Z $\times$ 10^2 S cm^2 mol^{-1}. The value of Z is **[4]**

43. The arrangement of X^- ions around A^+ ion in solid AX is given in the figure (not drawn to scale). If the radius of X^- is 250 pm, the radius of A^+ is **[4]**



44. Amongst the following, the total number of compounds whose aqueous solution turns red litmus paper blue is KCN, K_2SO_4, $(NH_4)_2C_2O_4$, NaCl, $Zn(NO_3)_2$, $FeCl_3$, K_2CO_3, NH_4NO_3 and LiCN **[4]**

45. A closed vessel with rigid walls contains 1 mole of $^{238}_{92}U$ and 1 mole of air at 298 K. Considering complete decay of $^{238}_{92}U$ to $^{206}_{82}Pb$, the ratio of the final pressure to the initial pressure of the system at 298 K is **[4]**

46. Dissolving 1.24 g of white phosphorous in boiling NaOH solution in an inert atmosphere gives a gas Q . The amount of $CuSO_4$ (in g) required to completely consume the gas Q is ________. **[4]**

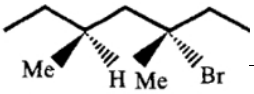
[**Given:** Atomic mass ofH = 1, 0 = 16, Na = 23, P = 31, S = 32, Cu = 63]

47. Among the complex ions, $[Co(NH_2CH_2CH_2\text{-}NH_2)_2\ Cl_2]^+$, $[CrCl_2\ (C_2O_4)_2]^{3-}$, $[Fe(H_2O)_4(OH)_2]^+$, $[Fe(NH_3)_2(CN)_4]^-$, $Co(NH_2\text{-}CH_2\text{-}CH_2\text{-}NH_2)_2(NH_3)Cl]^{2+}$ and $[Co(NH_3)_4(H_2O)Cl]^{2+}$ the number of complex ion(s) that show(s) cis-trans isomerism is **[4]**

Chemistry (MATCH)

48. Match the reactions in List-I with the features of their products in List-II and choose the correct option. **[3]**

List-I	List-II

(P) $(-)-1-\text{Bromo}-2-\text{ethylpentane} \xrightarrow[\text{S}_\text{N}2\text{ reaction}]{\text{aq.NaOH}}$ (single enantiomer)	(1) Inversion of configuration
(Q) $(-)-2-\text{Bromopentane} \xrightarrow[\text{S}_\text{N}2\text{ reaction}]{\text{aq.NaOH}}$ (single enantiomer)	(2) Retention of configuration
(R) $(-)-3-\text{Bromo}-3-\text{methylhexane} \xrightarrow[\text{S}_\text{N}1\text{ reaction}]{\text{aq.NaOH}}$ (single enantiomer)	(3) Mixture of enantiomers
(S)  (single enantiomer) $\xrightarrow[\text{S}_\text{N}1\text{ reaction}]{\text{aq.NaOH}}$	(4) Mixture of structural isomers
	(5) Mixture of diastereomers

a) P → 1; Q → 2; R → 5; S → 4
b) P → 2; Q → 1; R → 3; S → 5
c) P → 2; Q → 4; R → 3; S → 5
d) P → 1; Q → 2; R → 5; S → 3

49. The major products obtained from the reactions in List-II are the reactants for the named reactions mentioned in List-I. Match List-I with List-II and choose the correct option. **[3]**

List-I	**List-II**
(P) Etard reaction	(1) Acetophenone $\xrightarrow{\text{Zn-Hg,HCl}}$
(Q) Gattermann reaction	(2) Toluene $\xrightarrow[\text{(ii)SOCl}_2]{\text{(i)KMnO}_4\text{,KOH,}\Delta}$
(R) Gattermann-Koch reaction	(3) Benzene $\xrightarrow[\text{anhyd.AlCl}_3]{\text{CH}_3\text{Cl}}$
(S) Rosenmund reduction	(4) Aniline $\xrightarrow[\text{273-278K}]{\text{NaNO}_2\text{/HCl}}$
	(5) Phenol $\xrightarrow{\text{Zn, }\Delta}$

a) P → 3; Q → 4; R → 5; S → 2
b) P → 3; Q → 2; R → 1; S → 4
c) P → 1; Q → 3; R → 5; S → 2
d) P → 2; Q → 4; R → 1; S → 3

50. Dilution processes of different aqueous solutions, with water, are given in LIST-I. The effects of dilution of the solutions on $[H^+]$ are given in LIST-II. **[3]**
(Note: Degree of dissociation (α) of weak acid and weak base is << 1; degree of hydrolysis of salt << 1; $[H^+]$ represents the concentration of H^+ ions)

LIST-I	**LIST-II**
(P) (10 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 60 mL	(1) the value of $[H^+]$does not change on dilution
(Q) (20 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 80 mL	(2) the value of $[H^+]$ changes to half of its initial value on dilution

(R) (20 mL of 0.1 M HCl + 20 mL of 0.1 M ammonia solution) diluted to 80 mL	(3) the value of $[H^+]$ changes to two times of its initial value on dilution
(S) 10 mL saturated solution of $Ni(OH)_2$ in equilibrium with excess solid $Ni(OH)_2$ is diluted to 20 mL (solid $Ni(OH)_2$ is still present after dilution).	(4) the value of $[H^+]$ changes to $\frac{1}{\sqrt{2}}$ times of its initial value on dilution
	(5) the value of $[H^+]$ changes to $\sqrt{2}$ times of its initial value on dilution

a) P- 4; Q - 2; R - 3; S - 1

b) P - 1; Q - 5; R - 4; S - 1

c) P - 4; Q - 3; R - 2; S - 3

d) P - 1; Q - 4; R - 5; S - 3

51. Consider the Bohr's model of a one - electron atom where the electron moves around the nucleus. In the following List-I contains some quantities for the n^{th} orbit of the atom and List-II contains options showing how they depend on n **[3]**

List-I	**List-II**
(I) Radius of the n^{th} orbit	(P) $\propto n^{-2}$
(II) Angular momentum of the electron in the n^{th} orbit of the atom	(Q) $\propto n^{-1}$
(III) Kinetic energy of the electron in the n^{th} orbit	(R) $\propto n^0$
(IV) Potential energy of the electron in the n^{th} orbit	(S) $\propto n^1$
	(T) $\propto n^2$
	(U) $\propto n^{\frac{1}{2}}$

Which of the following options has the correct Combination considering List-I and List-II?

a) (I), (P)

b) (I), (T)

c) (II), (Q)

d) (II), (R)

JEE Advanced 2024

Sample Paper - 2

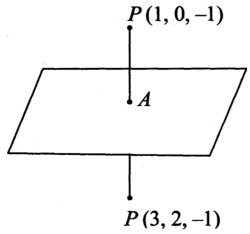
Solution

Mathematics (MRQ)

1. **(a)** $\delta - \gamma = 3$

   (b) $\delta + \beta = 4$

   (d) $\alpha + \beta = 2$

   Explanation: 

   Mid-point of PQ = A (2, 1, -1)

   D.r's of PQ = 2, 2, 0

   Since PQ perpendicular to plane and mid-point lies on plane

   $\therefore$ Equation of plane : 2(x - 2) + 2(y - 1) + 0 (z + 1) = 0 $\Rightarrow$ x - 2 + y - 1 = 0

   $\Rightarrow$ x + y = 3 comparing with ax + βy + γz = δ,

   we get α = 1, β = 1, γ = 0 and δ = 3.

   $\therefore$ option (a), (b), (c) are true,

2. **(a)** $P(M^C) + P(N^C) - 2P(M^C \cap N^C)$

   (c) $P(M) + P(N) - 2P(M \cap N)$

   (d) $P(M \cap N^C) + P(M^C \cap N)$

   Explanation:

   - $P(M) + P(N) - 2P(M \cap N)$
     $= P(M) - P(M \cap N) + P(N) - P(M \cap N)$
     $= P(M \cap N^C) + P(M^C \cap N)$
     $\Rightarrow$ Prob. that exactly one of M and N occurs.
   - $P(M) + P(N) - P(M \cap N) = P(M \cup N)$
     $\Rightarrow$ Prob. that at least one of M and N occurs.
   - $P(M^C) + P(N^C) - 2P(M^C \cap N^C)$
     $= 1 - P(M) + 1 - P(N) - 2[1 - P(M \cup N)]$
     $= P(M) + P(N) - 2P(M \cup N)$
     $= P(M) - P(M \cap N) + P(N) - P(M \cap N)$
     $= P(M \cap N^C) + P(M^C \cap N)$
     $\Rightarrow$ Prob. that exactly one of M and N occurs.
   - $P(M \cap N^C) + P(M^C \cap N)$
     $\Rightarrow$ Prob that M occurs but not N or prob that M does not occur but N occurs.
     $\Rightarrow$ Prob. that exactly one of M and N occurs.

   Thus we can conclude that $P(M) + P(N) - 2P(M \cap N)$, $P(M^C) + P(N^C) - 2P(M^C \cap N^C)$ and $P(M \cap N^C) + P(M^C \cap N)$ are the correct options.

3. **(c)** The set $\{X \in \mathbb{R}^3 : MX = 0\} \neq \{0\}$ where 0 = $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

(d) There exists a nonzero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that M $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ = $\begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$.

Explanation: Given that a_{ij} = -1 if j +1 is divisible by i, otherwise a_{ij} = 0

M = $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

i. = |M| = -1 + 1 = 0 $\Rightarrow$ M is singular so non-invertible.
So, M is invertible is incorrect.

ii. M $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ = $\begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix}$ $\Rightarrow$ $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ = $\begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix}$

$\left.\begin{array}{l} a_1 + a_2 + a_3 = -a_1 \\ a_1 + a_3 = -a_2 \\ a_2 = -a_3 \end{array}\right\}$ $\Rightarrow$ a_1 = 0 and a_2 + a_3 = 0 infinite solutions.

So, There exists a nonzero column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ such that M $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ = $\begin{pmatrix} -a_1 \\ -a_2 \\ -a_3 \end{pmatrix}$ is correct.

iii. MX = 0 $\Rightarrow$ $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ = $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

x + y + z = 0
x + z = 0
y = 0
$\therefore$ Infinite solution.

So, The set $\{X \in \mathbb{R}^3 : MX = 0\} \neq \{0\}$ where 0 = $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ is correct.

iv. M - 21 = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ -2 $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = $\begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$

|M - 2I| = 0
So, The matrix (M - 2I) is invertible, where I is the 3 $\times$ 3 identity matrix is incorrect.

Mathematics (MCQ)

4. **(a)** $f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

Explanation: Given that f is a non negative function defined on [0, 1] and

$\int\limits_0^x \sqrt{1-(f'(t))^2}dt = \int\limits_0^x$ f(t) dt, $0 \leq x \leq 1$

Differentiating both sides with respect to x, we get $\sqrt{1-[f'(x)]^2}$ = f(x)

$\Rightarrow$ 1 - $[f'(x)]^2$ = $[f(x)]^2$ $\Rightarrow$ $[f'(x)]^2$ = 1 - $[f(x)]^2$

$\Rightarrow \frac{d}{dx} f(x) = \pm\sqrt{1-[f(x)]^2} \Rightarrow \pm\frac{df(x)}{\sqrt{1-[f(x)]^2}} = dx$

Integrating both sides with respect to x, we get

$\pm\int \frac{df(x)}{\sqrt{1-[f(x)]^2}} = \int dx \Rightarrow \pm\sin^{-1} f(x) = x + C$

∵ Given that f(0) = 0 ⇒ C = 0

Hence f(x) = ± sin x

But as f(x) is a non negative function on [0, 1]

∴ (x) = sin x

Now sin x < x, ∀ x > 0

$\therefore f\left(\frac{1}{2}\right) < \frac{1}{2}$ and $f\left(\frac{1}{3}\right) < \frac{1}{3}$

5. **(a)** both are real and negative and have negative real parts

Explanation: Since, a, b, c > 0; therefore a, b, c should be real because order relation is not defined in the set of complex numbers.

∴ Roots of equation are either real or complex conjugate.

Let α, β be the roots of $ax^2 + bx + c = 0$, then

$\alpha + \beta = -\frac{b}{a} = -ve, \alpha\beta = \frac{c}{a} = +ve$

⇒ Either both α, β are -ve, if roots are real or both α, β have -ve real parts, if roots are complex conjugate.

6.

(d) 2/9

Explanation: <u>Sample space:</u> A dice is thrown thrice, n (s) = 6 × 6 × 6

<u>Favorable events:</u> $\omega^{r_1} + \omega^{r_2} + \omega^{r_3} = 0$

i.e. (r_1, r_2, r_3) are ordered 3 triples which can take values,

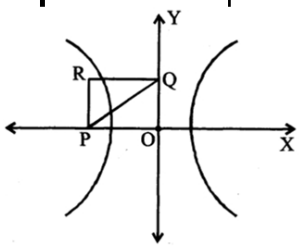
$\left.\begin{array}{llll}(1,2,3), & (1,5,3), & (4,2,3), & (4,5,3)\\(1,2,6), & (1,5,6), & (4,2,6), & (4,5,6)\end{array}\right\}$ i.e. 8 ordered pairs and each can be arranged in 3 ! ways = 6

$\therefore \quad n(E) = 8 \times 6 \Rightarrow P(E) = \frac{8\times6}{6\times6\times6} = \frac{2}{9}$

7.

(b) $\frac{4}{x^2} - \frac{2}{y^2} = 1$

Explanation: Equation of the tangent at the point θ is



$\frac{x\sec\theta}{a} - \frac{y\tan\theta}{b} = 1$

⇒ P = (a cosθ, 0) and Q = (0, -b cotθ)

Let R be (h, k)

⇒ h = a cos θ, k = -b cotθ

$\Rightarrow \frac{k}{h} = \frac{-b}{a\sin\theta} \Rightarrow \sin\theta = \frac{-bh}{ak}$ and $\cos\theta = \frac{h}{a}$

By squaring and adding, $\frac{b^2h^2}{a^2k^2} + \frac{h^2}{a^2} = 1$

$\Rightarrow \frac{b^2}{k^2} + 1 = \frac{a^2}{h^2} \Rightarrow \frac{a^2}{h^2} - \frac{b^2}{k^2} = 1$

Now, given eq^n of hyperbola is $\frac{x^2}{4} - \frac{y^2}{2} = 1$

$\Rightarrow$ $a^2 = 4, b^2 = 2$

$\therefore$ R lies on $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$ i.e. $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$

Mathematics (NUM)

8. 0

Explanation:

$\frac{dy}{dx} + y \cdot g'(x)$ = g(x) g' (x)

IF $= e^{\int g'(x)dx} = e^{g(x)}$

$\therefore$ Solution is $y(e^{g(x)}) = \int g(x) \cdot g'(x) \cdot e^{g(x)} dx + C$

Put g(x) = t, g'(x) dx = dt

$y(e^{g(x)}) = \int t \cdot e^t dt + C$

$= t \cdot e^t - \int 1 \cdot e^t dt + C = t \cdot e^t - e^t + C$

$y\ e^{g(x)} = (g(x) - 1)\ e^{g(x)} + C$ ...(i)

Given, y(0) = 0, g(0) = g(2) = 0

$\therefore$ Eq. (i) becomes,

$y(0) \cdot e^{g(0)} = (g(0) - 1) \cdot e^{g(0)} + C$

$\Rightarrow 0 = (-1) \cdot 1 + C \Rightarrow C = 1$

$\therefore y(x) \cdot e^{g(x)} = (g(x) - 1)\ e^{g(x)} + 1$

$\Rightarrow y(2) \cdot e^{g(2)} = (g(2) - 1)\ e^{g(2)} + 1$, where g(2) = 0

$\Rightarrow y(2) \cdot 1 = (-1) \cdot 1 + 1$

y(2) = 0

9. 1

Explanation:

Here, $z = \frac{-1+i\sqrt{3}}{2} = \omega$

$\because P = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$

$P^2 = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix} \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$

$= \begin{bmatrix} \omega^{2r} + \omega^{4s} & \omega^{r+2s}[(-1)^r + 1] \\ \omega^{r+2s}[(-1)^r + 1] & \omega^{4s} + \omega^{2r} \end{bmatrix}$

Given, $P^2 = -I$

$\omega^{2r} + \omega^{4s} = -1$ and $\omega^{r+2s}[(-1)^r + 1] = 0$

Since, $r \in \{1, 2, 3\}$ and $(-1)^r + 1 = 0$

$\Rightarrow$ r = {1, 3}

Also, $\omega^{2r} + \omega^{4s} = -1$

If r = 1, then $\omega^2 + \omega^{4s} = -1$

which is only possible, when s = 1.

As, $\omega^2 + \omega^4 = -1$

$\therefore$ r = 1, s = 1

Again, if r = 3, then

$\omega^6 + \omega^{4s} = -1$

$\Rightarrow \omega^{4s} = -2$ [never possible]

$\therefore r \neq 3$

⇒ (r, s) = (1, 1) is the only solution.
Hence, the total number of ordered pairs is 1.

10. 31.0
Explanation:
Number of five digit numbers divisible by 5

$\underbrace{----}_{1,2,2,2}$ $\underset{\text{fixed}}{0}$ → $\frac{|\underline{4}}{|\underline{3}} = 4$

$\underbrace{----}_{1,4,2,2}$ $\underset{\text{fixed}}{0}$ → $\frac{|\underline{4}}{|\underline{2}} = 12$

$\underbrace{----}_{4,2,2,2}$ $\underset{\text{fixed}}{0}$ → $\frac{|\underline{4}}{|\underline{3}} = 4$

$\underbrace{----}_{2,2,4,4}$ $\underset{\text{fixed}}{0}$ → $\frac{|\underline{4}}{|\underline{2}\,|\underline{2}} = 6$

$\underbrace{----}_{1,2,4,4}$ $\underset{\text{fixed}}{0}$ → $\frac{|\underline{4}}{|\underline{2}} = 12$

} Total = 38

Number of five digit numbers divisible by 5 but not by 20

$\underset{2,2,2}{---}$ $\underset{\text{fixed}}{1\ 0}$ → $\frac{|\underline{3}}{|\underline{3}} = 1$

$\underset{2,2,4}{---}$ $\underset{\text{fixed}}{1\ 0}$ → $\frac{|\underline{3}}{|\underline{2}} = 3$

$\underset{2,4,4}{---}$ $\underset{\text{fixed}}{1\ 0}$ → $\frac{|\underline{3}}{|\underline{2}} = 3$

} Total = 7

$\therefore p = \frac{38-7}{38}$ ∴ 38p = 31

11. 9
Explanation:
Then, $f'(x) = \lambda(x-\alpha)(x-\beta)$
Here, $p'(x) = \lambda(x-1)(x-3) = \lambda\left(x^2-4x+3\right)$
On integrating both sides between 1 to 3, we get
$\int_1^3 p'(x)dx = \int_1^3 \lambda\left(x^2-4x+3\right)dx$
⇒ $(p(x))_1^3 = \lambda\left(\frac{x^3}{3}-2x^2+3x\right)_1^3$
⇒ p(3) - p(1) =$\lambda\left((9-18+9)-\left(\frac{1}{3}-2+3\right)\right)$
⇒ 2 - 6 = $\lambda\left\{\frac{-4}{3}\right\}$
⇒ λ = 3
⇒ p' (x) = 3 (x - 1) (x - 3)
∴ p'(0) = 9

12. 3
Explanation:
$\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2} = \frac{x\bar{x}+y\bar{y}+z\bar{z}}{|a|^2+|b|^2+|c|^2}$
(a + b + c) $(\bar{a}+\bar{b}+\bar{c})$ + (a + bω + cω^2)
= $\frac{\left(\bar{a}+\bar{b}\omega^2+\bar{c}\omega\right)+\left(a+b\omega^2+c\omega\right)\left(\bar{a}+\bar{b}\omega+\bar{c}\omega^2\right)}{|a|^2+|b|^2+|c|^2}$
= $\frac{3\left(|a|^2+|b|^2+|c|^2\right)}{|a|^2+|b|^2+|c|^2}$ = 3

13. 4

Explanation:

|A| = $\begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$

= $\begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 2\sqrt{k} & 1+2k & -2k \\ -2\sqrt{k} & 1+2k & -1 \end{vmatrix}$ [$C_2 \to C_2 - C_3$]

= $\begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 4\sqrt{k} & 0 & 1-2k \\ -2\sqrt{k} & 1+2k & -1 \end{vmatrix}$ [$R_2 \to R_2 - R_3$]

= (1 + 2k) (8k - 4k + 4k^2 +1) = (2k + 1)^3

Since B is skew symmetric of odd order,

∴ |B| = 0

Hence, |Adj A| + |Adj B| = |A|^2 + |B|^2 = 10^6

⇒ (2k + 1)^6 = 10^6 ⇒ k = 4.5, ∴ [k] = 4

Mathematics (MATCH)

14. **(a)** P → 2; Q → 1; R → 4; S → 3

Explanation:

i. $f_1'(0) = \lim\limits_{h \to 0}\left[\frac{\sin\sqrt{1-e^{-h^2}} - 0}{h}\right]$

$= \lim\limits_{h \to 0}\left[\frac{\sin\sqrt{1-e^{-h^2}}}{\sqrt{1-e^{-h^2}}} \times \frac{\sin\sqrt{1-e^{-h^2}}}{h^2} \times \frac{|h|}{h}\right]$

$= \lim\limits_{h \to 0}\left[1 \times 1 \times \frac{|h|}{h}\right] = \lim\limits_{h \to 0}\frac{|h|}{h}\left[\because \lim\limits_{x \to 0}\frac{\sin x}{x} = 1\right]$

which does not exit.

∴ for (P),(2) is correct.

ii. $\lim\limits_{x \to 0} f_2(x) = \lim\limits_{x \to 0}\left[\frac{|\sin x|}{\tan^{-1} x}\right]$

$\lim\limits_{x \to 0}\left[\frac{|\sin x|}{|x|} \times \frac{x}{\tan^{-1} x} \times \frac{|x|}{x}\right]$

$\lim\limits_{x \to 0}\left[1 \times 1 \times \frac{|x|}{x}\right] = \lim\limits_{x \to 0}\frac{|x|}{x}\left[\because \lim\limits_{x \to \infty}\frac{x}{\tan^{-1} x} = 1\right]$

which does not exist, so for Q,(1) is correct.

iii. $\lim\limits_{x \to 0} f_3(x) = \lim\limits_{x \to 0}\left[\sin(\log_e(x+2))\right]$

if x → 0 ⇒ (x + 2) → 2 ⇒ $\log_e$(x + 2) → $\log_e$ 2 < 1

⇒ 0 < $\lim\limits_{x \to 0}\sin(\log_e(x+2) < \sin 1$

⇒ $\lim\limits_{x \to 0}\left[\sin(\log_e(x+2))\right] = 0$

f_3(x) = 0 $\forall x \in \left[-1, e^{\frac{\pi}{2}} - 2\right)$

⇒ f'_3(x) = 0 $\forall x \in \left[-1, e^{\frac{\pi}{2}} - 2\right)$

⇒ f''_3(x) = 0 $\forall x \in \left[-1, e^{\frac{\pi}{2}} - 2\right)$

∴ for (R), (4) is correct.

iv. $\lim_{x\to 0} f_4(x) = \lim_{x\to 0}\left(x^2 \sin\frac{1}{x}\right) = \lim_{x\to 0} x^2\left(\sin\frac{1}{x}\right) = 0$

$f'_4(0) = \lim_{x\to 0}\frac{h^2\sin\left(\frac{1}{h}\right)-0}{h} = \lim_{x\to 0} h\sin\left(\frac{1}{4}\right) = 0$

$f'_4(x) = -\cos\frac{1}{x} + 2x\sin\frac{1}{x}, x \neq 0$

$\lim_{x\to 0} f'_4(x) = \lim_{x\to 0}\left[-\cos\frac{1}{x} + 2x\sin\frac{1}{x}\right] = -\lim_{x\to 0}\cos\frac{1}{x}$

which does not exist

So for (S), (3) is correct.

15.

(c) P → 3, Q → 4, R → 1, S → 2

Explanation:

P. Given that $\left[\vec{a} \quad \vec{b} \quad \vec{c}\right] = 2$

$\therefore [2(\vec{a}\times\vec{b})3(\vec{b}\times\vec{c})\vec{c}\times\vec{a}]$

$= 6\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right]$

$= 6\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]^2 = 6\times 4 = 24$

$\therefore (P)\to(3)$

Q. Given that $[\vec{a}\vec{b}\vec{c}] = 5$

$\therefore [3(\vec{a}+\vec{b})\vec{b}+\vec{c} \quad 2(\vec{c}+\vec{a})]$

$= 6\left[\vec{a}+\vec{b} \quad \vec{b}+\vec{c} \quad \vec{c}+\vec{a}\right]$

$= 6\times 2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right] = 6\times 2\times 5 = 60$

$\therefore (Q)\to(4)$

R. Given that $\frac{1}{2}|\vec{a}\times\vec{b}| = 20 \Rightarrow |\vec{a}\times\vec{b}| = 40$

$\therefore \frac{1}{2}|(2\vec{a}+3\vec{b})\times(\vec{a}-\vec{b})| = \frac{1}{2}|-2\vec{a}\times\vec{b}+3\vec{b}\times\vec{a}|$

$= \frac{1}{2}\times 5|\vec{a}\times\vec{b}| = \frac{5}{2}\times 40 = 100$

$\therefore (R)\to(1)$

S. Given that $|\vec{a}\times\vec{b}| = 30$

$\therefore |(\vec{a}+\vec{b})\times\vec{a}| = |(\vec{b}\times\vec{a})| = 30$

$\therefore (s)\to(2)$

16.

(b) (P) - (4), (Q) - (3), (R) - (2), (S) - (1)

Explanation:

P. $\left[\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)}{\cot(\sin^{-1}y)+\tan(\sin^{-1}y)}\right)^2 + y^4\right]^{\frac{1}{2}}$

$$= \left[\frac{1}{y^2}\left(\frac{\cos\left(\cos^{-1}\frac{1}{\sqrt{1+y^2}}\right)+y\sin\left(\sin^{-1}\frac{y}{\sqrt{1+y^2}}\right)}{\cot\left(\cot^{-1}\frac{\sqrt{1-y^2}}{y}\right)+\tan\left(\tan^{-1}\frac{y}{\sqrt{1-y^2}}\right)}\right)^2 + y^4\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{y^2}\left(\frac{\sqrt{1+y^2}}{\frac{1}{y(\sqrt{1-y^2})}}\right)^2 + y^4\right]^{\frac{1}{2}}$$

$= \left(1-y^4+y^4\right)^{\frac{1}{2}} = 1 \therefore (P)\to(4)$

Q. cos x + cos y = - cos z ...(i)
and sin x + sin y = -sin z ...(ii)
On squaring (i) and (ii) and then adding, we get
$(\cos x + \cos y)^2 + (\sin x + \sin y)^2 = \cos^2 z + \sin^2 z$
$\Rightarrow 2 + 2\cos(x - y) = 1$
$\Rightarrow 4\cos^2 \frac{x-y}{2} = 1 \Rightarrow \cos\frac{x-y}{2} = \pm\frac{1}{2}$
$\therefore$ Q $\rightarrow$ (3)

R. $\cos\left(\frac{\pi}{4} - x\right)$ cos2x + sinx sin2x secx
= cosx sin2x secx + $\cos\left(\frac{\pi}{4} + x\right)\cos 2x$
$\Rightarrow \cos 2x\left[\cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{\pi}{4} + x\right)\right]$
= sin2x secx (cosx - sinx)
$\Rightarrow 2\sin\frac{\pi}{4}$sin x cos2x = 2sin x (cos x - sin x)
$\Rightarrow 2\sin x\left[\frac{1}{\sqrt{2}}\left(\cos^2 x - \sin^2 x\right) - (\cos x - \sin x)\right] = 0$
$\Rightarrow 2\sin x(\cos x - \sin x)\left(\frac{\cos x + \sin x}{\sqrt{2}} - 1\right) = 0$
$\Rightarrow$ sinx = 0 or tanx = 1 or $\cos\left(x - \frac{\pi}{4}\right) = 1$
$\Rightarrow$ x = 0 or $\frac{\pi}{4} \Rightarrow$ secx = 1 or $\sqrt{2}$
$\therefore$ (R) $\rightarrow$ (2, 4)

S. $\cot\left(\sin^{-1}\sqrt{1-x^2}\right) = \sin\left(\tan^{-1} x\sqrt{6}\right)$
$\Rightarrow \frac{x}{\sqrt{1-x^2}} = \frac{x\sqrt{6}}{\sqrt{1+6x^2}} \Rightarrow x = \pm\frac{1}{2}\sqrt{\frac{5}{3}}$
$\therefore$ (S) $\rightarrow$ (1)
Hence (P) $\rightarrow$ (4), (Q) $\rightarrow$ (3), (R) $\rightarrow$ (2, 4), (S) $\rightarrow$ (1)

17. **(a)** P $\rightarrow$ 4; Q $\rightarrow$ 6; R $\rightarrow$ 5; S $\rightarrow$ 2

Explanation: Given 6 boys M_1, M_2, M_3, M_4, M_5, M_6 and 5 girls G_1, G_2, G_3, G_4, G_5.

a. $\alpha_1 \rightarrow$ Total number of ways of selecting 3 boys and 2 girls from 6 boys and 5 girls
i.e., ${}^6C_3 \times {}^5C_2$ = 20 × 10 = 200 $\therefore \alpha_1$ = 200

b. $\alpha_2 \rightarrow$ Total number of ways selecting at least 2 member and having equal number of boys and girls
i.e., ${}^6C_1{}^5C_1 + {}^6C_2{}^5C_2 + {}^6C_3{}^5C_3 + {}^6C_4{}^5C_4 + {}^6C_5{}^5C_5$
= 30 + 150 + 200 + 75 + 6 = 461 $\Rightarrow \alpha_2$ = 461

c. $\alpha_3 \rightarrow$ Total number of ways of selecting 5 members in which at least 2 of then girls
i.e., ${}^5C_2{}^6C_3 + {}^5C_3{}^6C_2 + {}^5C_4{}^6C_1 + {}^5C_5{}^6C_0$
= 200 + 150 + 30 + 1 = 381 $\Rightarrow \alpha_3$ = 381

d. $\alpha_4 \rightarrow$ Total number of ways for selecting 4 members in which at least two girls such that M_1 and G_1 are not included together.
G_1 is included $\rightarrow {}^4C_1 \cdot {}^5C_2 + {}^4C_2 \cdot {}^5C_1 + {}^4C_3$
= 40 + 30 + 4 = 74
M_1 is included $\rightarrow {}^4C_2 \cdot {}^5C_1 + {}^4C_3$ = 30 + 4 = 34
G_1 and M_1 both are not included
${}^4C_4 + {}^4C_3 \cdot {}^5C_1 + {}^4C_2 \cdot {}^5C_2$
1 + 20 + 60 = 81
$\therefore$ Total number = 74 + 34 + 81 = 189
α_4 = 189
Now, P $\rightarrow$ 4; Q $\rightarrow$ 6; R $\rightarrow$ 5; S $\rightarrow$ 2

Physics (MRQ)

18. **(a)** radiated power entering into one eye of the observer is in the range 3.15 $\times$ 10^{-8} W to3.25 $\times$ 10^{-8} W

(b) the wavelength corresponding to the maximum intensity of light is 1160 nm

(d) taking the average wavelength of emitted radiation to be 1740 nm, the total number of photons entering per second into one eye of the observer is in the range 2.75 $\times$ 10^{11} to 2.85 $\times 10^{11}$

Explanation: According to question,

surface area of filament of light bulb, A = 64 mm^2

Temperature of filament, T = 2500 K

distance of bulb or source from observer, d = 100 m

Radius of the pupil of the eyes of the observer, Re = 3 mm = 3 $\times$ 10^{-3} m

a. Power radiated by the filament P = σAeT^4

= 5.67 $\times 10^{-8}$ $\times$ 64 $\times$ 10^{-6} $\times$ 1 $\times$ $(2500)^4$ = 141.75 w ($\because$ e = 1 for black body)

Hence, option (power radiated by the filament is in the range 642 W to 645 W) is incorrect.

b. Radiated power entering into one eye of the observer,

I $= \frac{P}{4\pi d^2} \times (\pi R_1^2)$

$= \frac{141.75}{4\pi \times (100)^2} \times \pi \times (3 \times 10^{-3})^2$= 3.189375 $\times$ 10^{-8}W

Hence, option (radiated power entering into one eye of the observer is in the range 3.15 $\times$ 10^{-8} W to 3.25 $\times$ 10^{-8} W) is correct

c. From wein's displacement law, λ_m T = b

or, λ_m $\times$ 2500 = 2.9 $\times$ 10^{-3}

or, λ_m = 1.16 $\times$ 10^{-6} =1160 nm

Hence, option (The wavelength corresponding to the maximum intensity of light is 1160 nm) is correct

d. Total no. of photons entering per second into one eye of the observer $= (\frac{hc}{\lambda}) \times N_{photons}$ = I

3.189375 $\times$ 10^{-8} $= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1740 \times 10^{-9}} \times N_{phtotons}$

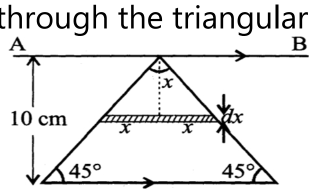
$\therefore$ $N_{photons}$ = 2.79 $\times$ 10^{11}

Hence, option (taking the average wavelength of emitted radiation to be 1740 nm, the total number of photons entering per second into one eye of the observer is in the range 2.75 $\times$ 10^{11} to 2.85 $\times 10^{11}$) is correct.

19. **(b)** There is a repulsive force between the wire and the loop

(d) The magnitude of induced emf in the wire is $(\frac{\mu_0}{\pi})$volt

Explanation: The flux passing through the triangular wire if i current flows through the inifinitely long conducting wire



$$d\phi = \int_0^{0.1} \frac{\mu_0 i}{2\pi x} \times 2\pi dx$$

$$\phi = \frac{\mu_0 i}{10\pi} = Mi$$

$\therefore M = \frac{\mu_0}{10\pi}$ ($\frac{dI}{dt} = 10AS^{-1}$ given)

Induced emf in the wire, e $= M\frac{di}{dt} = \frac{\mu_0}{10\pi} \times 10 = \frac{\mu_0}{\pi}V$

There will be no extra induced emf in the wire because there is no change in the magnetic. Flux due to rotation of loop.

As the current in the triangular wire is decreasing the induced current in AB is in the same direction as the current in the hypotenuse of the triangular wire. Therefore force will be repulsive.

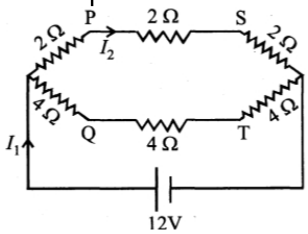
20. **(a)** I_2 = 2 A

(b) The current through PQ is zero

(c) The potential at S is less than that at Q.

(d) I_1 = 3 A

Explanation: Resistance of arm PQ and ST becomes ineffective as P & Q and 5 & Tare at the same potential. The equivalent circuit is as shown in the figure.



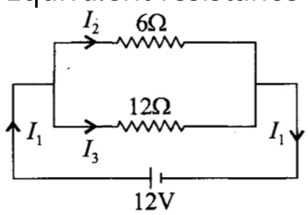
The resistance of the upper arm

R_1 = 2 Ω + 2 Ω + 2 Ω = 6 Ω

The resistance of the lower arm

R_2 = 4 Ω + 4 Ω + 4 Ω = 12 Ω

Equivalent resistance of the circuit,



$R_{eq} = \frac{R_1R_2}{R_1+R_2} = \frac{(6\Omega)(12\Omega)}{6\Omega+12\Omega} = 4\Omega$

$\therefore I_1 = \frac{12V}{4\Omega} = 3$ A $I_2 = \left(\frac{12}{6+12}\right) \times 3$ = 2A

$I_3 = I_1 - I_2$ = 1A

Potential difference across A and P,

$V_A - V_P = I_2 \times 2$ Ω = (2A) (2 Ω)

12V - V_P = 4V or V_P = 8V

Potential difference across A and Q

$V_A - V_Q = I_3 \times 2$ Ω = (1A)(4 Ω)

12V - V_Q = 4V

V_Q = 12V - 4V = 8V

Potential difference across P and S,

$V_P - V_S$ = (2A)(2W) = 4V

8V - V_S = 4V ⇒ V_S = 4V

$V_S < V_Q$

Physics (MCQ)

21.

(d) short wavelength

Explanation: short wavelength

22.

(d) $\frac{\mu_0}{2\pi d}\left(I_1^2+I_2^2\right)^{\frac{1}{2}}$

Explanation: The field at the same point at the same distance from the mutually perpendicular wires carrying current will be having the same magnitude but in perpendicular directions.

$\therefore \quad B=\sqrt{B_1^2+B_2^2}$

$\therefore \quad B=\frac{\mu_0}{2\pi d}\left(I_1^2+I_2^2\right)^{1/2}$

23.

(c) $\sqrt{920\times 980}\mathrm{cm/sec}$

Explanation: Let d_W and d_0 be the densities of water and oil; then the pressure at the bottom of the tank

$=h_W d_W$ g + $h_0 d_0$ g

Let this pressure be equivalent to pressure due to water of height h.

Then, h d_W g = $h_W d_W$ g + h_O d_Og

$\therefore h=h_w+\frac{h_0 d_o}{d_w}=100+\frac{400\times 0.9}{1}$

= 100 + 360 = 460

According to Toricelli's theorem,

$v=\sqrt{2gh}=\sqrt{2\times 980\times 460}$ cm/sec

$=\sqrt{920\times 980}$ cm/sec

24.

(c) n = 5, f_2 = $(\frac{5}{4})$ f_1

Explanation: $f_1=\frac{v}{l}$ (2nd harmonic of open pipe)

f_2 = n$\left(\frac{v}{4l}\right)$ (nth harmonic of closed pipe)

Here, n is odd and f_2 > f_1. It is possible when n = 5

because with n = 5

$f_2=\frac{5}{4}\left(\frac{v}{l}\right)=\frac{5}{4}f_1$

Physics (NUM)

25. 5

Explanation:

As two successive harmonics are found to occur at frequency 750 Hz and 1000 Hz and frequency

$\mathrm{f}=\frac{\mathrm{P}}{2\ell}\sqrt{\frac{\mathrm{T}}{\mu}}$ So,

$750=\frac{P}{2}\sqrt{\frac{T}{\mu}}$ ...(i)

and, $1000=\frac{P+1}{2}\sqrt{\frac{T}{\mu}}$ ...(ii)

Dividing eq (ii) by (i)

$\frac{4}{3}=\frac{P+1}{P}\therefore P=3$

Putting this value of P = 3 in eq. (ii) and solving we get tension T

$1000=\frac{4}{2}\sqrt{\frac{\mathrm{T}}{2\times 10^{-5}}}\therefore \mathrm{T}=5\ \mathrm{N}$

26. 0.95

Explanation:

After entering in new region, time taken by projectile to reach ground is given as

$t=\sqrt{\frac{2h}{g_{eff}}}=\sqrt{\frac{2\times 0.81\times u^2\sin^2\theta}{g\times 2g}}=0.9\frac{\mathrm{u}\sin\theta}{\mathrm{g}}$

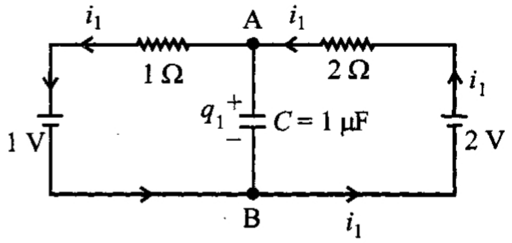
So, horizontal displacement done by projectile in new region is given as

$x = 0.9 \times \frac{u \sin \theta}{g} \times u \cos \theta = 0.9 \left(\frac{d}{2}\right)$

Now, new range = $\frac{d}{2}$ + 0.9$\frac{d}{2}$ = 0.95d

27. 1.33

Explanation:



When switch is connected to position P From KVL,

$V_A - 1 \cdot i_1 - 1 + 2 - 2i_1 = V_A \Rightarrow 3i_1 = 1 \therefore i_1 = \frac{1}{3}$ A

Again $V_A - 1 \cdot i_1 - 1 = V_B$ or, $V_A - V_B = 1 + i_1 = \frac{4}{3}$V

Potential drop across capacitor $\Delta V = \frac{4}{3}$V

∴ Charge on capacitor, $q_1 = C\Delta V = 1 \times \frac{4}{3} \mu C$

$q_1 = 1.33\ \mu C$

28. 8

Explanation:

We know that $N = N_0 e^{-\lambda t}$

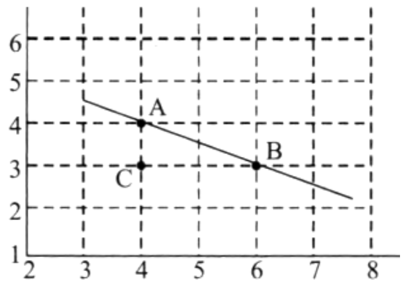
$\therefore \frac{dN}{dt} = N_0 e^{-\lambda t}(-\lambda) = N_0 \lambda e^{-\lambda t}$

Taking log on both sides

$\log_e \frac{dN}{dt} = \log_e (\lambda N_0) - \lambda t$

Comparing it with the graph line,

Decay constant, $\lambda = \frac{1}{2}$yr^{-1} $\left[\frac{AC}{BC} = \frac{1}{2}\right]$



$\therefore T_{1/2} = \frac{0.693}{\lambda}$ = 0.693 × 2 = 1.386 years

$n(t_{1/2})$ = 4.16 ∴ n = $\frac{4.16}{1.386} \approx 3$

$\therefore N = N_0\left(\frac{1}{2}\right)^3$

$\therefore \frac{1}{P} = \frac{1}{8}$ [∴ P = 8]

29. 24.0

Explanation:

Given, intensity of light, $I = 256 \frac{W}{m^2}$

By first polaroid P_1 intensity will be halved.

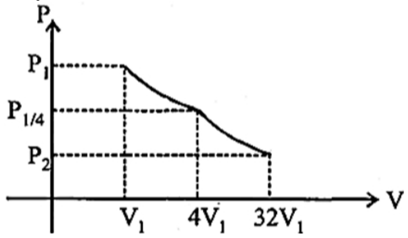
After P_2, $I_2 = \frac{I}{2} \cos^2 60^\circ = \frac{I}{8}$

After P_3, $I_3 = \frac{I}{8} \cos^2 30^\circ = \frac{I}{8}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3I}{32}$

Intensity $= \frac{256 \times 3}{32} = 24 \frac{W}{m^2}$

30. 1.78

Explanation:



For monatomic gas, $\gamma = \frac{5}{3}$

In adiabatic process

$\mathrm{P_1\ V_1^{\gamma} = P_2\ V_2^{\gamma}}$

$\Rightarrow \frac{P_1}{4}(4V_1)^{5/3} = P_2(32V_1)^{5/3}$

$\Rightarrow P_2 = \frac{P_1}{4}\left(\frac{1}{8}\right)^{5/3} = \frac{P_1}{128}$

And $\mathrm{W_{adi}} = \frac{\mathrm{P_1\ V_1 - P_2\ V_2}}{\gamma - 1} = \frac{\mathrm{P_1\ V_1} - \frac{\mathrm{P_1}}{128}(32\ \mathrm{V_1})}{\frac{5}{2} - 1}$

$= \frac{P_1 V_1(3/4)}{2/3} = \frac{9}{8} P_1 V_1$

In isothermal process,

$\mathrm{W_{iso}} = 2.303\mu\mathrm{RT}\log_{10}\left(\frac{\mathrm{V_2}}{\mathrm{V_1}}\right)$

$\Rightarrow \mathrm{W_{iso}} = \mathrm{P_1\ V_1} \ln\left(\frac{4\ \mathrm{V_1}}{\mathrm{V_1}}\right) = 2\mathrm{P_1\ V_1 \ In}\, 2$

$\therefore \frac{\mathrm{W_{iso}}}{\mathrm{W_{adia}}} = \frac{2\mathrm{P_1\ V_1\ In\ 2}}{\frac{9}{8}\mathrm{P_1\ V_1}} = \frac{16}{9}\ln 2 = \mathrm{f}\ln 2$

So, f = $\frac{16}{9} = 1.7778 \approx 1.78$

Physics (MATCH)

31.

(c) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 1

Explanation: $\because$ Orbital velocity,

$V = \sqrt{\frac{GM}{R}}$, or, $V \propto \frac{1}{\sqrt{R}} \therefore \frac{V_1}{V_2} = \sqrt{\frac{R_2}{R_1}} = \frac{2}{1}$

$\frac{L_1}{L_2} = \frac{m_1 v_1 R_1}{m_1 v_2 R_2} = \frac{2\times2\times1}{1\times1\times4} = \frac{1}{1}$

Kinetic energy, $K = \frac{GMm}{2R}$

$\therefore \frac{k_1}{k_2} = \frac{m_1}{m_2} \times \frac{R_1}{R_2} = \frac{2\times4}{1\times1} = \frac{8}{1}$

From Kepler's law of planetary motion.

$\mathrm{T^2 \propto R^3} \therefore \frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{\frac{3}{2}} = \frac{1}{8}$

32.

(c) P $\rightarrow$ 3, Q $\rightarrow$ 1, R $\rightarrow$ 4, S $\rightarrow$ 2

Explanation: As, V = V_0 sin ωt and resonant angular frequency

ω_0 = $\frac{1}{\sqrt{LC}} \Rightarrow \omega_0$ = 10^5 rad/s = $\frac{1}{\sqrt{LC}} \Rightarrow$ C = $\frac{1}{L\omega_0^2}$

= $\frac{1}{5\times10^{-2}\times10^{10}}$ = 2 $\times 10^{-9}$ F

[$\because$ L = 50 mH = 5 $\times$ 10^{-2} henry]

I_0 = $\frac{V_0}{R}$ = $\frac{45}{R}$ [$\because$ V = 45 sin ωt given]

ω = 8 $\times 10^4$ rad/s = 0.8 ω_0

I = 0.05 I_0 = $\frac{I_0}{20} \Rightarrow$ Z = 20R

$X_L = L\omega = 8 \times 10^4 \times 5 \times 10^{-2}\ \Omega = 4k\Omega$

$X_C = \frac{1}{C\omega} = \frac{1}{8\times10^4\times2\times10^{-9}} = \frac{1}{16} \times 10^5\Omega = \frac{25}{4}k\Omega$

$Z^2 = R^2 + (X_C - X_L)^2$ or, $400\ R^2 = R^2 + \left(\frac{9}{4}k\Omega\right)^2$

$\therefore R = \frac{\frac{9}{4}\text{k}\Omega}{\sqrt{399}} = \frac{9}{80}\text{k}\Omega = \frac{900}{8}\Omega$

$I_0 = \frac{V_0}{R} = \frac{45\times8}{900} = \frac{8}{20}\text{ A} \approx 0.4\text{ A} = 400\text{ mA}$

Quality factor,

$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{8}{900}\sqrt{\frac{5\times10^{-2}}{2\times10^{-9}}} = \frac{8}{900}\sqrt{25\times10^6}$

$= \frac{8}{900} \times 5000 = 44.4$

$Q = \frac{\omega_0}{\Delta_\omega}$ $\therefore$ Bandwidth, $\Delta\omega = \frac{\omega_0}{Q} = \frac{10^5}{44.4} = 2250.0$

Peak Power dissipated, $P_{max} = I_0^2R = \frac{45^2}{R^2} \times R = \frac{45^2}{R}$

$= \frac{45^2}{900} \times 8 = 18.4\text{ W} \simeq 18\text{W}$

33. **(a)** P $\to$ 3, Q $\to$ 4, R $\to$ 2, S $\to$ 1

Explanation: From Wien's displacement lens, λ_m T = b (constant) and b = 2.9 $\times$ 10^{-3} mK

When T = 200 K(P), $\lambda_m = \frac{b}{T} = \frac{2.9\times10^{-3}}{2000}$

= 1450 nm (max)

When T = 3000 K(Q)

$\lambda m = \frac{2.9\times10^{-3}}{3000} = 966.66\text{ nm}$

When T = 5000 K (R)

$\lambda_m = \frac{2.9\times10^{-3}}{5000} = 580\text{ nm}$

When T = 10000 K (s)

$\lambda_m = \frac{2.9\times10^{-3}}{10000} = 290\text{ nm min}$

For option (P) λ maximum and $\beta = \frac{\lambda D}{d}$

$\therefore$ widest central maximum

For option (Q) Power

$P_{3000} = \sigma A\ (3000)^4$

$P_{6000} = \sigma A\ (6000)^4$

$\frac{P_{3000}}{P_{6000}}\left(\frac{1}{2}\right)^4 = \frac{1}{16} \therefore P_{3000} = \frac{1}{16}P_{6000}$

For (R) Wavelength λ = 580 nm

Visible to human eyes

For (S)$\lambda = \frac{hc}{\phi} = \frac{1.24\times10^{-6}}{4}$ = 30nm

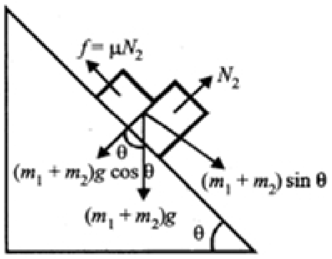
290 nm < 310 nm so this radiation lead to emission of photo electrons from a metal of work function ϕ = 4 ev.

For imaging human bones x-rays wavelength range 1 - 10 nm is used.

34.

(c) (P) - (i), (Q) - (ii), (R) - (ii), (S) - (iii)

Explanation: Block will not be slip or will be at rest if



$(m_1 + m_2)\, g \sin\theta \leq \mu m_2 g \cos\theta$

$\tan\theta \leq \frac{\mu m_2 g}{(m_1+m_2)g}$

$\Rightarrow \tan\theta \leq \frac{\mu m_2}{m_1+m_2}$

$\Rightarrow \tan\theta \leq \frac{0.3\times 2}{1+2} \leq \frac{1}{5}$

$\Rightarrow \tan\theta \leq 0.2 \text{ i.e., } \theta \leq 11.5^\circ$

i.e., If the angle 0 < 11.5° the frictional force is less than

$\mu N_2 = \mu m_2 \text{ g} = 0.3 \times 2 \times g = 0.6 \text{ g}$

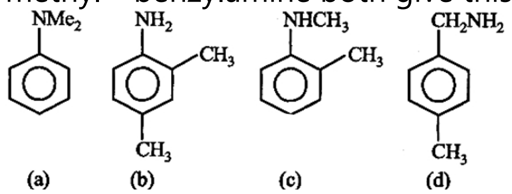
and is equal to $(m_1 + m_2)\, g \sin\theta$

Blocks will not slip on the inclined plane and friction is static.

At 0 > 11.5° the bodies start moving on the inclined plane and friction is kinetic and equal to $\mu m_2 g \cos\theta$

Chemistry (MRQ)

35. **(b)** 2, 4-dimethylaniline
 (c) p-methylbenzylamine
 Explanation: Only primary amines give carbylamine test. Hence 2, 4-dimethylaniline and P-methyl—benzylamine both give this test.



36. **(a)** The first reaction is a redox reaction.
 (c) Addition of filtrate to starch solution gives blue colour.
 (d) White precipitate is soluble in NaOH solution.

(a) $\overset{-1}{KI}(aq) + K_3[\overset{+3}{Fe}(CN)_6](aq) \longrightarrow$

Explanation:

$\overset{-3}{KI_3}(aq) + K_4[\overset{+2}{Fe}(CN)_6](aq)$
Brownish-yellow

$\downarrow ZnSO_4(aq)$

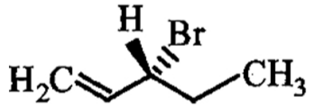
(b) $K_2Zn_3[Fe(CN)_6]_2$ or $\{K_2Zn[Fe(CN)_6]\}\downarrow + KI_3(aq)$
white ppt white ppt

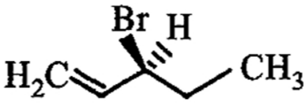
$\downarrow Na_2S_2O_3$

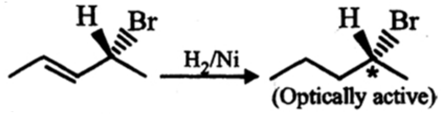
$I^-(aq) + S_4O_6^{2-}(aq)$
Colourless

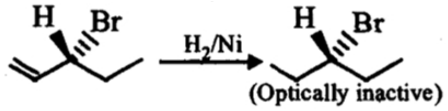
(c) When the filterate containing KI_3 add to start solution, the dissolved I_2 will produce a blue colour solution.

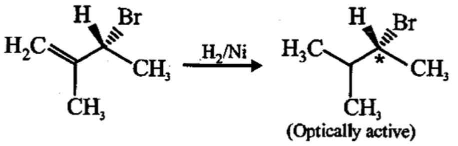
(d) $\underset{\text{white ppt.}}{K_2Zn[Fe(CN)_6]} + NaOH \longrightarrow [Zn(OH)_4]^{2-}(aq) + [Fe(CN)_6]^{4-}(aq)$

37. **(c)** 

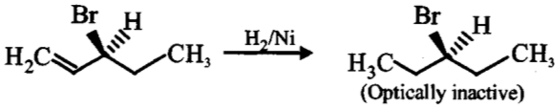
(d) 

Explanation: (a) 

(b) 

(c) 

(d)



Chemistry (MCQ)

38. **(a)** Nitric acid

Explanation: For the test of halogens, it is necessary to remove NaCN and Na_2S from the sodium extract if nitrogen and sulphur are present. This is done by boiling the sodium extract with conc. HNO_3. If NaCN and Na_2S are not decomposed, white or black ppt of AgCN and Ag_2S respectively are formed with $AgNO_3$ solution.

39.

(c) Y > Z > X

Explanation: Lower the value of E°, stronger the reducing agent. Reducing power:

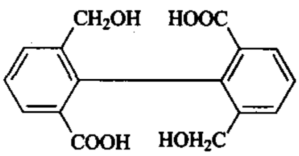
Y (E^O = -3.03 V) > Z (E^O = -1.18 V) > X (E^O = 0.52 V)

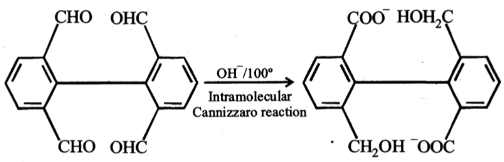
40.

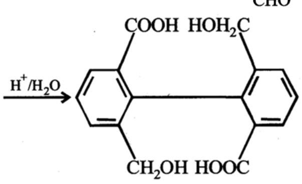
(b) NH_3

Explanation: Amongst XH_3 where 'X ' is group-15 elements, basic strength decreases from top to bottom. Hence, NH_3 is the strongest base.

41.

(b) 

Explanation: 



The above reaction is an example of intramolecular Cannizzaro reaction.

Chemistry (NUM)

42. 6

Explanation:

The formula for conductance is G = $\kappa \times \frac{a}{l}$

$5 \times 10^{-7} = \kappa \times \frac{1}{120}$

$\kappa = 6 \times 10^{-5}$ S cm^{-1}

$\Lambda_m^c = \frac{\kappa \times 1000}{M} = \frac{6 \times 10^{-5} \times 1000}{0.0015}$ = 40

$\because$ pH = 4

$\therefore$ [H$^+$] = $10^{-4} = c\alpha = 0.0015\ \alpha$

$\alpha = \frac{10^{-4}}{0.0015}$

Also, $\alpha = \frac{\Lambda_m^c}{\Lambda_m^o} \Rightarrow \frac{10^{-4}}{0.0015} = \frac{40}{\Lambda_m^o}$

$\Lambda_m^\circ = \frac{40 \times 0.0015}{10^{-4}}$ = 6×10^2 S cm^2 mol^{-1}

Hence, Z $\approx$ 6

43. 104

Explanation:

Given arrangement represents octahedral void and for this

$\frac{r_+(\text{cation})}{r_-(\text{anion})} = 0.414$

$\frac{r(A^+)}{r(X^-)} = 0.414$

= r(A$^+$) = 0.414 $\times$ r(X$^-$) = 0414 $\times$ 250pm

= 103.5pm $\approx$ 104 pm

44. 3

Explanation:

KCN, K_2CO_3 and LiCN are the salts of weak acid and strong base. So, their aqueous solutions turns red litmus paper blue.

45. 9

Explanation:

$_{92}U^{238} \rightarrow {}_{82}Pb^{206} + 8\ {}_2He^4(g) + 6\ {}_{-1}\beta^0$

n(gas)[Initial] = 1 (air)

n(gas) [Final] = 8 (He) + 1 (air) = 9

$\Rightarrow$ At constant temperature and volume;

p $\propto$ n

So, $\frac{p_f}{p_i} = \frac{n_f}{n_i} = \frac{9}{1}$ = 9

46. 2.38

Explanation:

$$\underset{\text{(White phosphorous)}}{P_4} + 3NaOH + 3H_2O \xrightarrow[\text{atmosphere}]{\text{Inert}} \underset{\text{Phosphine}}{PH_3 \uparrow} + 3NaH_2PO_2$$

PH_3 : a non-inflammable gas in its pure form; slightly soluble in water.

When PH_3 is absorbed in $CuSO_4$ solution cupric phosphide forms:

$2PH_3 + 3CuSO_4 \longrightarrow CuP_2 + 3H_2SO_4$

1 mol of P_4 = 31 $\times$ 4 = 124g

$\therefore$ 1.24 g of white phosphorous = 0.01 mol

$\therefore$ 0.01 mol of P_4 forms 0.01 mol of P_3

No. of moles of $CuSO_4$ is required for complete consumption of 0.01 mol = 0.01 $\times \frac{3}{2}$ = 15 $\times$ 10^{-3}

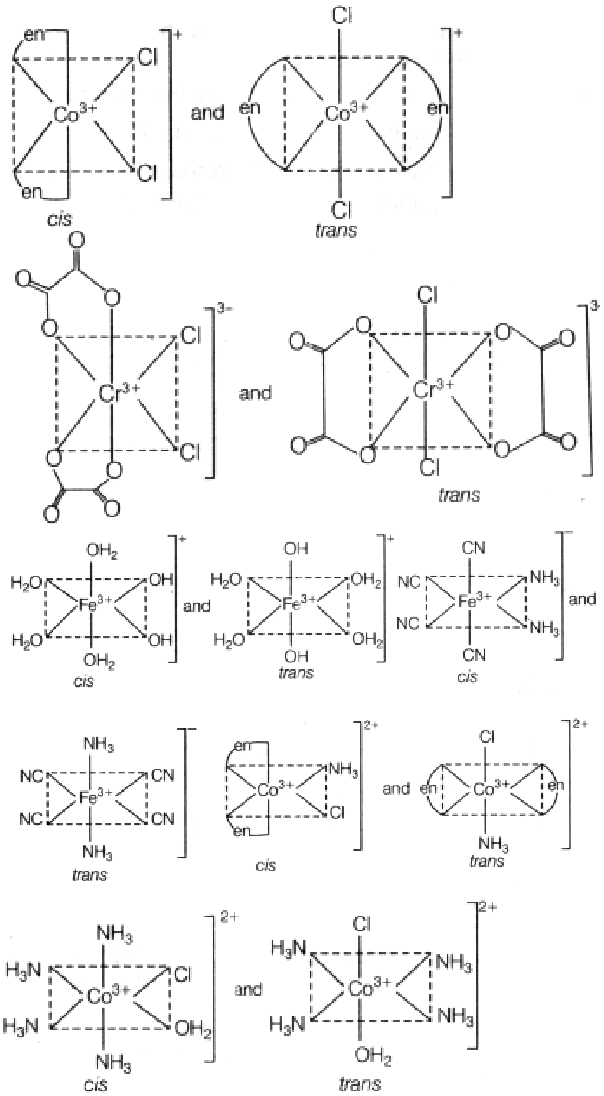
M. W. of $CuSO_4$ = 159 g/mol

$\therefore$ Amount of $CuSO_4$ required = $15 \times 10^{-3} \times 159 = 2.38$ g

47. 6

Explanation:

All six complex will show cis-trans isomerism.

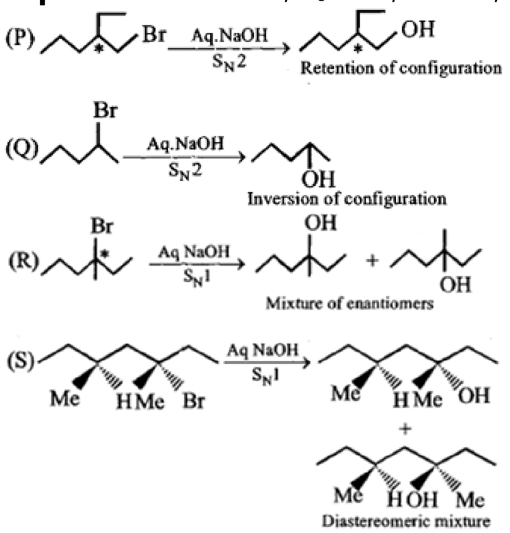


Chemistry (MATCH)

48.

(b) P $\rightarrow$ 2; Q $\rightarrow$ 1; R $\rightarrow$ 3; S $\rightarrow$ 5

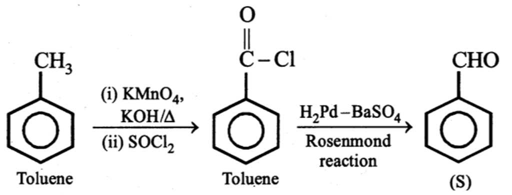
Explanation: P $\rightarrow$ 2, Q $\rightarrow$ 1, R $\rightarrow$ 3, S $\rightarrow$ 5

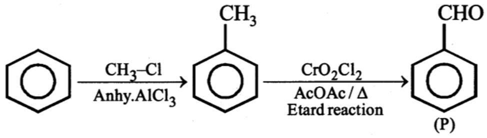


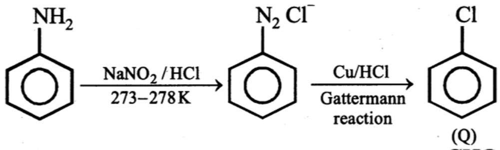
49. **(a)** P $\rightarrow$ 3; Q $\rightarrow$ 4; R $\rightarrow$ 5; S $\rightarrow$ 2

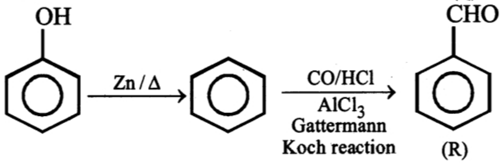
Explanation: P $\rightarrow$ 3, Q $\rightarrow$ 4, R $\rightarrow$ 5, S $\rightarrow$ 2

i. $\underset{\text{Acetophenone}}{Ph - \overset{\overset{O}{||}}{C} - CH_3} \xrightarrow{Zn-Hg/HCl} Ph - CH_2 - CH_3$

ii.


iii.


iv.


v.


50.

(b) P - 1; Q - 5; R - 4; S - 1

Explanation: P - 1; Q - 5; R - 4; S - 1

51.

(b) (I), (T)

Explanation: r $\propto \frac{n^2}{Z}$ or r = 0.529 $\times \frac{n^2}{Z}$;(I), (T)

|L| $\propto$ n or mvr = $\frac{nh}{2\pi}$; (II), (S)