CBSE Class 11 Mathematics

Important Questions

Chapter 13

Limits and Derivatives

1 Marks Questions

1. Evaluate $\lim_{x \to 3}\left[\frac{x^2-9}{x-3}\right]$

Ans. $\lim_{x \to 3}\frac{x^2-9}{x-3}\ \frac{0}{0}\ form$

$$\lim_{x \to 3}\frac{(x+3)(x-3)}{(x-3)} = 3+3 = 6$$

2. Evaluate $\lim_{x \to 0}\frac{\sin 3x}{5x}$

Ans. $\lim_{x \to 0}\frac{\sin 3x}{5x}$

$= \lim_{3x \to 0}\ \frac{\sin 3x}{3x} \times \frac{3}{5}$

$= 1 \times \frac{3}{5} = \frac{3}{5}\left[\because \lim_{x \to 0}\frac{\sin x}{x} = 1\right]$

3. Find derivative of 2^x

Ans. Let $y = 2^x$

$$\frac{dy}{dx} = \frac{d}{dx}2 = 2^x 10g2$$

4. Find derivative of $\sqrt{\sin 2x}$

Ans. $\frac{d}{dx}\sqrt{\sin 2x} = \frac{1}{2\sqrt{\sin 2x}}\frac{d}{dx}\sin 2x$

$= \frac{1}{\cancel{2}\sqrt{\sin 2x}} \times \cancel{2}\cos 2x$

$= \frac{\cos 2x}{\sqrt{\cos 2x}}$

5. Evaluate $\lim_{x\to 0}\frac{\sin^2 4x}{x^2}$

Ans.

$\lim_{x\to 0}\frac{\sin^2 4x}{x^2 4^2}\times 4^2 = \lim_{4x\to 0}\left(\frac{\sin 4x}{4x}\right)^2 \times 16$

$= 1\times 16 = 16$

6. What is the value of $\lim_{x\to a}\left(\frac{x^2 - a^n}{x-a}\right)$

Ans. $\lim_{x\to a}\frac{x^n - a^n}{x-a} = 1$

7. Differentiate $\frac{2^x}{x}$

Ans. $\frac{d}{dx}\frac{2^x}{x} = \frac{x\frac{d}{dx}2^x - 2^x\frac{d}{dx}x}{x^2}$

$= \frac{x\times 2^x 10g2 - 2^x \times 1}{x^2}$

$$= 2x\frac{[x+10g2-1]}{x^2}$$

8. If $y = e^{\sin x}$ **find** $\frac{dy}{dx}$

Ans. $y = e^{\sin x}$

$$\frac{dy}{dx} = \frac{d}{dx}e^{\sin x}$$

$$= e^{\sin x} \times \cos x = \cos x e^{\sin x}$$

9. Evaluate $\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$

Ans. $\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$

$$= \frac{\lim_{x\to 1}\frac{x^{15}-1^{15}}{x-1}}{\lim_{x\to 1}\frac{x^{10}-1^{10}}{x-1}} = \frac{15\times 1^{14}}{10\times 1^{9}}$$

$$= \frac{15}{10} = \frac{3}{2}$$

10. Differentiate $x\sin x$ **with respect to** x

Ans. $\frac{d}{dx}x\sin x = x\cos x + \sin x\,1$

$= x\cos x + \sin x$

11. Evaluate $\lim_{x \to 1} \frac{x^2+1}{x+100}$

Ans. $\lim_{x \to 1} \frac{x^2+1}{x+100} = \frac{2}{101}$

12. Evaluate $\lim_{x \to 0} \left[\cos ecx - \cot x\right]$

Ans. $\lim_{x \to 0} \left[\cos ec - \cot x\right]$

$= \lim_{x \to 0} \left[\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right]$

$= \lim_{x \to 0} \frac{1-\cos x}{\sin x} = \lim_{x \to 0} \frac{2\sin^2 \frac{x}{2}}{2\sin \frac{x}{2} \cos \frac{x}{2}}$

$= \lim_{x \to 0} \tan \frac{x}{2} = 0$

13. Find $f^1(x)$ $at\ x = 100$

$if\ f(x) = 99x$

Ans. $f(x) = 99x$

$f^1(x) = 99$ at x=100

$f^1(x) = 99$

14. Evaluate $\lim_{x \to -2} \frac{\tan \pi x}{x+2}$

Ans. $\lim_{x\to -2}\frac{\tan \pi x}{x+2}$ $\quad \frac{1}{0}\,form$

$let\ x+2=y$

$x=y-2$

$$\lim_{y\to 0}\frac{\tan \pi(y-2)}{y}$$

$$\lim_{y\to 0}\frac{-\tan \pi(2-y)}{y}=\lim_{y\to 0}\frac{\tan[2\pi-2y]}{y}$$

$$=\lim_{2y\to 0}\frac{+\tan 2y}{2y}\times 2$$

$$=1\times 2=2$$

15. Find derivative of $\sin^n x$

Ans. $\frac{d}{dx}\sin^n x$

$$=n\sin^{n-1}x\ \ \frac{d}{dx}\sin x$$

$$=n\sin^{n-1}x\ \cos x$$

16. Find derivative of $1+x+x^2+x^3+---+x^{50}\,at\ x=1$

Ans. $f(x)=1+x+x^2+x^3+---+x^{50}$

$f^1(x)=1+2x+3x^2+---+50x^{49}$

$at\ x=1$

$$f^1(1)=1+2+3+---+50=\frac{50(50+1)}{2}$$

$= 25 \times 51 = 1275$

17. The value of $\lim_{h \to 0} \frac{e^{2h} - 1}{h}$

Ans. $\lim_{2h \to 0} \frac{e^{2h} - 1}{2h} \times 2$

$= 1 \times 2 = 2$

18. Evaluate $\lim_{x \to 0} \frac{(1+x)^6 - 1}{(1+x)^2 - 1}$

Ans. $\lim_{x \to 0} \left[\frac{(1+x)^6 - 1}{(1+x)^2 - 1} \right]$

$let\ 1 + x = y$

$x \to 0, y \to 1$

$$\lim_{y \to 1} \frac{y^6 - 1}{y^2 - 1} = \lim_{y \to 1} \frac{\frac{y^6 - 1^6}{y - 1}}{\frac{y^2 - 1^2}{y - 1}}$$

$$= \frac{6 \times 1^5}{2 \times 1} = \frac{6}{2} = 3$$

19. $\lim_{x \to a} \frac{x^7 + a^7}{x + a} = 7$ **find the value of 'a**

Ans. $\lim_{x \to a} \frac{x^7 + a^7}{x + a} = 7$

$$= \frac{a^7 + a^7}{a + a} = 7$$

$$= \frac{\cancel{2}a^7}{\cancel{2}a} = 7$$

$$= a^6 = 7$$

$$= a = \sqrt[6]{7}$$

20. Differentiate $x^{-3}(5+3x)$

Ans. $\frac{d}{dx} x^{-3}(5+3x)$

$$= \frac{d}{dx}\left[5x^{-3} + 3x^{-2}\right]$$

$$= 5 \times -3x^{-4} + 3 \times -2x^{-3}$$

$$= \frac{-15}{x^4} - \frac{6}{x^3}$$

CBSE Class 12 Mathematics

Important Questions

Chapter 13

Limits and Derivatives

4 Marks Questions

1. Prove that $\lim_{x \to 0}\left(\frac{e^x - 1}{x}\right) = 1$

Ans. We have

$$\lim_{x \to 0} \frac{e^x - 1}{x}$$

$$\lim_{x \to 0} \left\{ \frac{\left[\cancel{1} + x + \frac{x^2}{2!} + \frac{x^3}{3!} + - - -\right] - \cancel{1}}{x} \right\} \left[\because e^x = 1 + x + \frac{x^2}{2!} + - - -\right]$$

$$\lim_{x \to 0} \left\{ \frac{x + \frac{x^2}{2!} + \frac{x^3}{3!} + - - -}{x} \right\}$$

$$\lim_{x \to 0} \cancel{x} \left\{ \frac{1 + \frac{x}{2!} + \frac{x^2}{3!} + - - -}{\cancel{x}} \right\}$$

$$= 1 + 0 = 1$$

2. Evaluate $\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{(2x^2+x-3)}$

Ans. $\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{(2x^2+x-3)}$

$=\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)}$

$\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(x-1)}\times\frac{(\sqrt{x}+1)}{(\sqrt{x}+1)}$

$\lim_{x\to 1}\frac{(2x-3)(x-1)}{(2x+3)(x-1)(\sqrt{x}+1)}$

$\lim_{x\to 1}\frac{(2x-3)}{(2x+3)(\sqrt{x}+1)}=\frac{2\times 1-3}{(2\times 1+3)(\sqrt{1}+1)}$

$=\frac{-1}{10}$

3. Evaluate $\lim_{x\to 0}\frac{x\tan 4x}{1-cas4x}$

Ans. $\lim_{x\to 0}\frac{x\tan 4x}{1-cas4x}$

$=\lim_{x\to 0}\frac{x\sin 4x}{\cos 4x\left[2\sin^2 2x\right]}$

$$= \lim_{x \to 0} \frac{2x \sin 2x \cos 2x}{\cos 4x \left(2 \sin^2 2x\right)}$$

$$= \lim_{x \to 0} \left(\frac{\cos 2x}{\cos 4x} \cdot \frac{2x}{\sin 2x} \times \frac{1}{2} \right)$$

$$= \frac{1}{2} \frac{\lim_{2x \to 0} \cos 2x}{\lim_{4x \to 0} \cos 4x} \times \lim_{2x \to 0} \left(\frac{2x}{\sin 2x} \right) = \frac{1}{2} \times 1 = \frac{1}{2}$$

4. It $y = \frac{(1 - \tan x)}{(1 + \tan x)}$ **. Show that** $\frac{dy}{dx} = \frac{-2}{(1 + \sin 2x)}$

Ans. $y = \frac{(1 - \tan x)}{(1 + \tan x)}$

$$\frac{dy}{dx} = \frac{(1 + \tan x)\frac{d}{dx}(1 - \tan x) - (1 - \tan x)\frac{d}{dx}(1 + \tan x)}{(1 + \tan x)^2}$$

$$= \frac{(1 + \tan x)(-\sec^2 x) - (1 - \tan x)\sec^2 x}{(1 + \tan x)^2}$$

$$= \frac{-\sec^2 x - \tan x \sec^2 x - \sec^2 + \tan x \sec^2 x}{(1 + \tan x)^2}$$

$$= \frac{-2\sec^2 x}{(1 + \tan x)^2} = \frac{-2}{\cos^2 x \left[1 + \frac{sicx}{\cos x}\right]^2}$$

$$= \frac{-2}{\cos^2 x \left[\frac{\cos x + \sin x}{\cos^2 x}\right]^2}$$

$$= \frac{-2}{\cos^2 x + \sin^2 x + 2\sin x \cos x} = \frac{-2}{1 + \sin^2 x}$$

$\therefore \frac{dy}{dx} = \frac{-2}{1 + \sin 2x}$ Hence proved

5. Differentiate $e^{\sqrt{\cot x}}$

Ans. Let $y = e^{\sqrt{\cot x}}$

$$\frac{dy}{dx} = \frac{d}{dx} e^{\sqrt{\cot x}} = e^{\sqrt{\cot x}} \frac{d}{dx} \sqrt{\cot x}$$

$$= e^{\sqrt{\cot x}} \times \frac{1}{2\sqrt{\cot x}} . \frac{d}{dx} \cot x$$

$$= \frac{e^{\sqrt{\cot x}}}{2\sqrt{\cot x}} - \cos ec^2 x$$

$$= \frac{-\cos cec^2 e^{\sqrt{\cot x}}}{2\sqrt{\cot x}}$$

6. Let $f(x) \begin{cases} a + bx, x < 1 \\ 4, x = 1 \\ b - ax, x > 1 \end{cases}$ **and if** $\lim_{x \to 1} f(x) = f(1)$ **What are the possible value of a and b ?**

Ans. Given f (1) =4

$$\because \lim_{x \to 1} f(x) = f(1) = 4$$

$$\therefore \lim_{x \to 1} f(x) exist$$

$$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = 4 ----(1)$$

$$\lim_{x\to1^-} f(x) = \lim_{x\to1}(a+bx) \quad \begin{bmatrix} \because for x<1 \\ f(x)=a+bx \end{bmatrix}$$

$= a+b$

$$\lim_{x\to1^+} f(x) = \lim_{x\to1}(b-ax) \begin{bmatrix} \because for x>1 \\ f(x)=b-ax \end{bmatrix}$$

$= b-a$

By eq. (1)

$a+b=b-a=4$

$a+b=4$

$b-a=4$

$\therefore a=0 \;\; and \;\; b=4$

7. If $y=\dfrac{1}{\sqrt{a^2-x^2}}$, **find** $\dfrac{dy}{dx}$

Ans. $y=\dfrac{1}{\sqrt{a^2-x^2}}$

$put \;\; (a^2-x^2)=t$

$y=\dfrac{1}{\sqrt{t}}$ and $t=a^2-x^2$

$\dfrac{dy}{dt}=\dfrac{d}{dt}t^{\frac{-1}{2}}$

$=\dfrac{-1}{2}t^{\frac{-1}{2}-1}$

$$= \frac{-1}{2} t \frac{-3}{2}$$

$$\frac{dt}{dx} = -2x$$

so.

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{-1}{\not{2}} t^{\frac{-3}{2}} \times \left(-\not{2}x\right) = x\, t^{\frac{-3}{2}}$$

$$= x\left(a^2 - x^2\right)^{\frac{-3}{2}}$$

8. Differentiate $\sqrt{\frac{1-\tan x}{1+\tan x}}$

Ans. let $y = \sqrt{\frac{1-\tan x}{1+\tan x}}$

put $\frac{1-\tan x}{1+\tan x} = t$

$y = \sqrt{t}$ and $t = \frac{1-\tan x}{1+\tan x}$

$$\frac{dy}{dt} = \frac{d}{dt} t^{\frac{1}{2}}$$

$$= \frac{1}{2} t^{\frac{1}{2}-1} = \frac{1}{2} t^{\frac{-1}{2}}$$

$$= \frac{1}{2\sqrt{t}} = \frac{1}{2\sqrt{\frac{1-\tan x}{1+\tan x}}} = \frac{1}{2}\sqrt{\frac{1+\tan x}{1-\tan x}}$$

$$\frac{dt}{dx}=\frac{(1+\tan x)\frac{d}{dx}(1-\tan x)-(1-\tan x)\frac{d}{dx}(1+\tan x)}{(1+\tan x)^2}$$

$$=\frac{(1+\tan x)(0-\sec^2 x)-(1-\tan x)(0+\sec^2 x)}{(1+\tan x)^2}$$

$$=\frac{\sec^2 x[-1-\tan x-1+\tan x]}{(1+\tan x)^2}$$

$$=\frac{-2\sec^2 x}{(1+\tan x)^2}$$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dt}{dx}}=\frac{1}{2}\sqrt{\frac{1+\tan x}{1-\tan x}}$$

$$\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$$

$$=\frac{-2\sec^2 x}{(1+\tan x)^2}\times\frac{1}{2}\sqrt{\frac{1+\tan x}{1-\tan x}}$$

$$=\frac{-\sec^2 x}{(1+\tan x)^{\frac{3}{2}}(1-\tan x)^{\frac{1}{2}}}$$

9. Differentiate $(i)\ \left(\frac{\sin x+\cos x}{\sin x-\cos x}\right)$ $(ii)\ \left(\frac{\sin x-1}{\sec x+1}\right)$

Ans. (i) $\frac{d}{dx}\left(\frac{\sin x+\cos x}{\sin x-\cos x}\right)$

$$= \frac{(\sin x - \cos x).\frac{d}{dx}(\sin x + \cos x) - (\sin x + \cos x).\frac{d}{dx}(\sin x - \cos x)}{(\sin x - \cos x)^2}$$

$$= \frac{(\sin x - \cos x)(\cos x - \sin x) - (\sin x + \cos x)(\cos x + \sin x)}{(\sin x - \cos x)^2}$$

$$= \frac{-\left[(\sin x - \cos x)^2 + (\sin x + \cos x)^2\right]}{(\sin x - \cos x)^2}$$

$$= \frac{-2(\sin^2 x + \cos^2 x)}{(\sin^2 x + \cos^2 x - 2\sin x \cos x)}$$

$$= \frac{-2}{(1 - \sin 2x)}$$

(ii) $\frac{d}{dx}\left[\frac{\sec x - 1}{\sec x + 1}\right]$

$$= \frac{(\sec x + 1).\frac{d}{dx}(\sec x - 1) - (\sec x - 1).\frac{d}{dx}(\sec x + 1)}{(\sec x + 1)^2}$$

$$= \frac{(\sec x + 1)\sec x \tan x - (\sec x - 1)\sec x \tan x}{(\sec x + 1)^2}$$

$$= \frac{2\sec x \tan x}{(\sec x + 1)^2}$$

10. Evaluate $\lim_{x \to \frac{\pi}{4}} \frac{\sin x - cox}{\left(x - \frac{\pi}{4}\right)}$

Ans. put $\left(x-\frac{\pi}{4}\right)=y$, so that when $x \to \frac{\pi}{4}$ then $y \to 0$

$$\therefore \lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{\left(x-\frac{\pi}{4}\right)}$$

$$= \lim_{y \to 0} \frac{\left[\sin\left(\frac{\pi}{4}+y\right)-\cos\left(\frac{\pi}{4}+y\right)\right]}{y} \left[\text{puthing } \left(x-\frac{\pi}{4}=y\right)\right]$$

$$= \lim_{y \to 0} \frac{\left[\left(\sin\frac{\pi}{4}\cos y+\cos\frac{\pi}{4}\sin y-\sin\frac{\pi}{4}\sin y\right)\right]}{y}$$

$$= \frac{2}{\sqrt{2}} \times \lim_{y \to 0}\left(\frac{\sin y}{y}\right) = \left(\sqrt{2}\times 1=\sqrt{2}.\right)$$

11. Evaluate $\lim_{x \to 0} \frac{(1+x)^6-1}{(1+x)^5-1}$

Ans. put $(1+x)=y$, so that when $x \to 0$ then $y \to 1$

$$\therefore \lim_{x \to 0} \frac{(1+x)^6-1}{(1+x)^5-1}$$

$$= \lim_{y \to 1}\left[\frac{y^6-1}{y^5-1}\right] = \frac{\lim_{y \to 1}\left(\frac{y^6-1}{y-1}\right)}{\lim_{y \to 1}\left(\frac{y^5-1}{y-1}\right)}$$

$$=\frac{\lim\limits_{y\to 1}\left(\dfrac{y^6-1}{y-1}\right)}{\lim\limits_{y\to 1}\left(\dfrac{y^5-1^5}{y-1}\right)}=\frac{6\times 1^{(6-1)}}{5\times 1^{(5-1)}}$$

$$=\frac{6\times 1^5}{5\times 1^4}=\frac{6}{5}\left[\because \lim_{x\to a}\left(\frac{x^n-a^n}{x-a}\right)=na^{n-1}\right]$$

12. Evaluate $\lim\limits_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$

Ans. $\lim\limits_{x\to a}\dfrac{\left(\sqrt{a+2x}-\sqrt{3x}\right)}{\left(\sqrt{3a+x}-2\sqrt{x}\right)}$

$$=\lim_{x\to a}\frac{\left(\sqrt{a+2x}-\sqrt{3x}\right)}{\left(\sqrt{3a+x}-2\sqrt{x}\right)}\times\frac{\left(\sqrt{3a+x}+2\sqrt{x}\right)}{\left(\sqrt{3a+x}+2\sqrt{x}\right)}\times\frac{\left(\sqrt{a+2x}+\sqrt{3x}\right)}{\left(\sqrt{a+2x}+\sqrt{3x}\right)}$$

$$=\lim_{x\to a}\frac{\left[(a+2x)-3x\right]\times\left(\sqrt{3a+x}+2\sqrt{x}\right)}{\left[(3a+x)-4x\right]\times\left(\sqrt{a+2x}+\sqrt{3x}\right)}$$

$$=\lim_{x\to a}\frac{(a-x)\times\left(\sqrt{3a}+x+2\sqrt{x}\right)}{3(a-x)\times\left(\sqrt{a+2x}+\sqrt{3x}\right)}$$

$$=\lim_{x\to a}\frac{\left(\sqrt{3a+x}+2\sqrt{x}\right)}{3\left[\sqrt{a+2x}+\sqrt{3x}\right]}$$

$$=\frac{\left(\sqrt{4a}+2\sqrt{a}\right)}{3\left(\sqrt{3a}+\sqrt{3a}\right)}=\frac{4\sqrt{a}}{6\sqrt{3}\sqrt{a}}=\frac{\sqrt{2}}{3\sqrt{3}}$$

13. Find the derivative of $f(x)=1+x+x^2+--+x^{50}$ at $x=1$

Ans. $f(x)=1+x+x^2+---+x^{50}$

$$f1(x)=\frac{d}{dx}\left(1+x+x^2+---+x^{50}\right)$$

$$=0+1+2x+3x^2+--+50x^{49}$$

At $x=1$

$$f^1(1)=1+2+3+--50$$

$$=\frac{50^{25}(50+1)}{2}=25\times51\left[\begin{array}{l}1+2+3--+n\\=\frac{n(n+1)}{2}\end{array}\right]$$

$$=1305$$

14. Find the derivative of $\sin^2 x$ with respect to x using product rule

Ans. let

$$y=\sin^2 x$$
$$y=\sin x\times\sin x$$

$$\frac{dy}{dx}=\frac{d}{dx}.\sin x\times\sin x$$

$$=\sin x\frac{d}{dx}\sin x+\sin x\frac{d}{dx}\sin x$$

$$=\sin x.\cos x+\sin x.\cos x$$

$$=2\sin x.\cos x=\sin 2x$$

15. Find the derivative of $\frac{x^5 - \cos x}{\sin x}$ **with respect to** x

Ans. let

$$y = \frac{x^5 - \cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{x5 - \cos x}{\sin x}\right)$$

$$= \frac{\sin x \frac{d}{dx}\left(x^5 - \cos x\right) - \left(x^5 - \cos x\right)\frac{d}{dx}\sin x}{\sin^2 x}$$

$$= \frac{\sin x\left[5x^4 + \sin x\right] - \left(x^5 - \cos x\right)\cos x}{\sin^2 x}$$

$$= \frac{5x^4 \sin x + \sin^2 x - x^5 \cos x + \cos^2 x}{\sin^2 x}$$

$$= \frac{5x^4 \sin x - x^5 \cos x + 1}{\sin^2 x}$$

16. Find $\lim_{x \to 0} f(x)$.

when $f(x) = \begin{cases} \frac{|x|}{x}; x \neq 0 \\ 0; x = 0 \end{cases}$

Ans. $f(x) = \begin{cases} \frac{|x|}{x}; x \neq 0 \\ 0; x = 0 \end{cases}$

We know that $|x| = \begin{cases} x; x \geq 0 \\ -x; x < 0 \end{cases}$

$$\therefore f(x) = \begin{cases} \frac{x}{x} = 1. x > 0 \\ \frac{-x}{x} = -1. x < 0 \\ 0. x = 0 \end{cases}$$

L. H. L. $\lim_{x \to 0^-} f(x) = \lim_{x \to 0} -1 = -1$

R. H. L. $\lim_{x \to 0^+} d(x) = \lim_{x \to 0} 1 = 1$

L. H. L. $\neq$ R. H. L $\therefore \lim_{x \to 0} f(x)$ does not exist

17. Find the derivative of the function $f(x) = 2x^2 + 3x - 5$ at $x = -1$. Also show that $f'(0) + 3f'(-1) = 0$

Ans. $f(x) = 2x^2 + 3x - 5$

$f^1(x) = \frac{d}{dx}(2x^2 + 3x - 5)$

$= 4x + 3$

At $x = -1$

$f^1(-1) 4 \times -1 + 3 = -4 + 3 = -1$

$f^1(0) = 4 \times 0 + 3 = 3$

$f^1(0) + 3f^1(-1) = 3 + 3 \times -1$

$= 3 - 3 = 0$ Hence proved

18. Evaluate $\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}$

Ans. $\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}$

$=\lim_{x\to 0}\frac{x[a+\cos x]}{xb\frac{\sin x}{x}}$

$\frac{\lim_{x\to 0}(a+\cos x)}{\lim_{x\to 0} b\frac{\sin x}{x}}$

$\frac{a+1}{b\times 1}=\frac{a+1}{b}\begin{bmatrix}\because \lim_{x\to 0}\cos x=1\\ \lim_{x\to 0}\frac{sicx}{x}=1\end{bmatrix}$

19. Find derivative of $\tan x$ **by first principle**

Ans. $let\, f(x)=\tan x$

$f(x+h)=\tan(x+h)$

$f^1(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$=\lim_{h\to 0}\frac{\tan(x+h-\tan x)}{h}$

$=\lim_{h\to 0}\frac{\frac{\sin(x+h)}{\cos(x+h)}-\frac{\sin x}{\cos x}}{h}$

$$= \lim_{h \to 0} \frac{\sin(x+h)\cos x - \cos(x+h)\sin x}{\cos(x+h)\cos x\, h}$$

$$= \lim_{h \to 0} \frac{\sin[x+h-x]}{\cos(x+h)\cos x\, h}$$

$$= \frac{\lim_{h \to 0} \frac{\sinh}{h}}{\lim_{h \to 0} \cos(x+h)\cos x} = \frac{1}{\cos(x+0)\cos x}$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

20. Evaluate $\lim_{x \to 1} \frac{x + x^2 + x^3 + --- + x^n - n}{(x-1)}$

Ans. $\lim_{x \to 1} \frac{x + x^2 + x^3 + --- + x^n - 1}{(x-1)}$

$$= \lim_{x \to 1} \frac{(x-1) + (x^2 - 1) + (x^3 - 1 + -- + (x^n - 1))}{(x-1)}$$

$$= \lim_{x \to 1} \frac{(x-1)[1 + (x+1) + (x^2 + x + 1) + ---- + x^{n-1} + x^{n-2} + ---- + 1]}{(x-1)}$$

$$= 1 + 2 + 3 + --- + n = \frac{n(n+1)}{2}$$

21. Evaluate $\underset{x \to 4}{Lt} \frac{|4-x|}{x-4}$ **(if it exist)**

Ans. $\lim_{x \to 4} \frac{|4-x|}{x-4}$

$L.H.L. \quad \lim_{x\to 4^-} \frac{-(4-x)}{x-4} = \lim_{x\to 4} \frac{-(4-x)}{-(4-x)} = 1$

$R.H.L. \quad \lim_{x\to 4^+} \frac{4-x}{x-4} = \lim_{x\to 4} \frac{-(x-4)}{(x-4)} = -1$

$L.H.L. \neq R.H.L.$

$\therefore \lim_{x\to 4} \frac{|4-x|}{x-4}$ does not exist

22. For what integers m and n does both

$\underset{x\to 4}{Lt} f(x)$ *and* $\underset{x\to 1}{Lt} f(x)$ exist it

$$f(x) = \begin{cases} mx^2 + n; x < 0 \\ nx + m; 0 \le x \le 1 \\ nx^3 + m; x > 1 \end{cases}$$

Ans. *for* $x = 0$

$L.H.L. \quad \lim_{x\to 0^-} f(x) = \lim_{x\to 0} mx^2 + n$

$= n$

$R.H.L. \quad \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} nx + m$

$= m$

$\because \lim_{x\to 0} f(x)$ exist

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$n = m$

For all real number $m = n \lim_{x\to 0} f(x)$ exist

For $x = 1$

$L.H.L.\ \lim_{x \to 1^-} f(x) = \lim_{x \to 1} nx + m$

$= n + m$

$R.H.L.\ \lim_{x \to 1} f(x) = \lim_{x \to 1} nx^3 + m$

$= n + m$

$\therefore \lim_{x \to 1} f(x) = \lim_{x \to 1^-} f(x)$

$m + n = m + n$

$\therefore$ all integral values of $m + n \lim_{x \to 1} f(x)$ exist

23. If $y = \sqrt{x + } \frac{1}{\sqrt{x}}$. prove that $2x \frac{dy}{dx + y} + y = 2\sqrt{x}$

Ans. $y = \sqrt{x} + \frac{1}{\sqrt{x}} = x^{\frac{1}{2}} + x^{\frac{-1}{2}}$

Differentiating w. r. t. x we gill

$\frac{dy}{dx} = \frac{1}{2} x^{\frac{-1}{2}} + \left(\frac{-1}{2}\right) x^{\frac{-3}{2}}$

$= \frac{1}{2\sqrt{x}} - \frac{1}{2x^{\frac{3}{2}}}$

$2x \frac{dy}{dx} = \sqrt{x} - \frac{1}{\sqrt{x}}$

$2x \frac{dy}{dx} + y = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) + \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)$

$2x\frac{dy}{dx}+y=2\sqrt{x}$ Hence proved

24. Evaluate $\lim_{x\to\frac{\pi}{2}}\frac{1+\cos 2x}{(\pi-2x)^2}$

Ans. $\lim_{x\to\frac{\pi}{2}}\frac{1+\cos 2x}{(\pi-2x)^2}$

$let\ \pi-2x=y$

$2x=\pi-y$

$x\to\frac{\pi}{2}, y\to 0$

$\lim_{y\to 0}\frac{1+\cos(\pi-y)}{y^2}=\lim_{y\to 0}\frac{1-\cos y}{y^2}\frac{1}{2}$

$=\lim_{y\to 0}\frac{2\sin^2\frac{y}{2}}{4\times\frac{y^2}{4}}$

$=\lim_{y\to 0}\frac{1}{2}\times\frac{\sin^2\frac{y}{2}^2}{\left(\frac{y}{2}\right)^2}\quad =$

$=\frac{1}{2}\lim_{\frac{y}{2}\to 0}\left[\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right]^2$

$=\frac{1}{2}\times 1$

25. Differentiate the function $y = \dfrac{(x+2)(3x-1)}{(2x+5)}$ **with respect to** x

Ans. $y = \dfrac{(x+2)(3x-1)}{(2x+5)}$

$$\frac{dy}{dx} = \frac{d}{dx}\frac{(x+2)(3x-1)}{(2x+5)}$$

$$= \frac{(2x+5)\frac{d}{dx}(x+2)(3x-1) - (x+2)(3x-1)\frac{d}{dx}(2x+5)}{(2x+5)^2}$$

$$= \frac{(2x+5)\left[(x+2)\frac{d}{dx}(3x-1) + (3x-1)\frac{d}{dx}(x+2)\right] - (x+2)(3x-1)[2+0]}{(2x+5)^2}$$

$$= \frac{(2x+5)[(x+2)\times 3 + (3x-1)\times 1] - 2\left[3x^2 + 6x - x - 2\right]}{(2x+5)^2}$$

$$= \frac{(2x+5)[3x+6+3x-1] - 6x^2 - 12x + 2x + 4}{(2x+5)^2}$$

$$= \frac{12x^2 + 30x + \cancel{10x} + 25 - 6x^2 - \cancel{10x} + 4}{(2x+5)^2}$$

$$= \frac{6x^2 + 30x + 29}{(2x+5)^2}$$

26. Find $\lim\limits_{x \to 5} |x| - 5$

Ans. $L.H.S. \lim\limits_{x \to 5^-} f(x)$

$x = 5 - h$

$x \to 5, h \to 0$

$\lim_{h \to 0} f(5-h)$

$\lim_{h \to 0} |5-h| - 5$

$= 0$

$R.H.S. \lim_{x \to 5^+} f(x)$

$put\ x = 5 + h$

$x \to 5,\ h \to 0$

$\lim_{h \to 0} f(5+h) = \lim_{h \to 0} |5+h| - 5$

$= 0$

$R.H.S. = R.H.S.$

$\therefore \lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x)$

$\therefore \lim_{x \to 5} f(x)$ exist

$\therefore \lim_{x \to 5} f(x) = 0$

27. Find $\lim_{x \to 0} f(x)$ *and* $\lim_{x \to 1} f(x)$ *where* $f(x) = \begin{cases} 2x+3; x \le 0 \\ 3(x+1); (x > 0) \end{cases}$

Ans. $given\ f(x) = \begin{cases} 2x+3, x \le 0 \\ 3(x+1), x > 0 \end{cases}$

$for\ x = 0$

$L.H.S. \lim_{x \to 0^-} f(x) = \lim_{x \to 0} 2x + 3 = 3$

$R.H.S. \lim_{x\to 0^+} f(x) = \lim_{x\to 0} 3(x+1) = 3$

$L.H.S. = R.H.S.$

$\therefore \lim_{x\to 0} f(x)$ $exist$

$\therefore \lim_{x\to 0} f(x) = 3$

$for\ x = 1$

$L.H.S.$

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1} 3(x+1)$

$= 3(1+1) = 6$

$R.H.S.$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1} 3(x+1)$

$= 3(1+1) = 6$

$L.H.S. = R.H.S.$

$\therefore \lim_{x\to 1} f(x)$ $exist$

$\lim_{x\to 1} f(x) = 6$

28. Find derivative of $\sec x$ by first principle

Ans. $let\ f(x) = \sec x$

$f(x+h) = \sec(x+h)$

$$f^1(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sec(x+h) - \sec x}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{\cos(s+h)} - \frac{1}{\cos x}}{h}$$

$$= \lim_{h \to 0} \frac{\cos x - \cos(x+h)}{\cos(x+h)\cos xh}$$

$$= \lim_{h \to 0} \frac{-2\sin\left[\frac{2x+h}{2}\right]\sin\left[\frac{-h}{2}\right]}{\cos(x+h)\cos xh}$$

$$= \lim_{h \to 0} \frac{2\sin\left[\frac{2x+h}{2}\right]\sin\frac{h}{2}}{\cos(x+h)\cos xh} \qquad [\sin(-\theta = -\sin\theta)]$$

$$= \lim_{h \to 0} \frac{2\sin\frac{h}{2}}{2\frac{h}{2}} \times \frac{\lim_{h \to 0} \sin\frac{(2x+h)}{2}}{\lim_{h \to 0} \cos(x+h)\cos x}$$

$$= 1 \times \frac{\sin\left(\frac{2x+0}{2}\right)}{\cos(x+0)\cos x} = \frac{\sin x}{\cos x \ \cos x}$$

$$= \tan x \ \sec x$$

29. Find derivative of $f(x) = \frac{4x + 5\sin x}{3x + 7\cos x}$

Ans. $f(x) = \frac{4x + 5\sin x}{3x + 7\cos x}$

$$f^1(x)=\frac{(3x+7\cos x)\frac{d}{dx}(4x+5\sin x)-(4x+5\sin x)\times\frac{d}{dx}(3x+7\cos x)}{(3x+7\cos x)^2}$$

$$=\frac{(3x+7\cos x)(4+5\cos x)-(4x+5\sin x)(3-7\sin x)}{(3x+7\cos x)^2}$$

$$=\frac{\cancel{12x}+15x\cos x+28\cos x+35\cos^2 x-\cancel{12x}+28\sin x+15\sin x+35\sin^2 x}{(3x+7\cos x)^2}$$

$$=\frac{15x\cos x+35\left[\sin^2 x+\cos^2 x\right]+28+\cos x+43\sin x}{(3x+7\cos x)^2}$$

$$=\frac{15x\cos x+35+28\cos x+45\sin x}{(3x+7\cos x)^2}$$

30. Find derivative of $\frac{x^n-a^n}{x-a}$

Ans. $\frac{d}{dx}\;\frac{x^n-a^n}{x-a}$

$$=\frac{(x-a)\frac{d}{dx}(x^n-a^n)-(x^n-a^n)\frac{d}{dx}(x-a)}{(x-a)^2}$$

$$=\frac{(x-a)\left[nx^{n-1}-0\right]-(x^n-a^n)[1-0]}{(x-a)^2}$$

$$=\frac{nx^{n-1}(x-a)-x^n+a^n}{(x-a)^2}$$

$$=\frac{nx^n-nax^{n-1}-x^n+a^n}{(x-a)^2}=\frac{x^n(n-1)-nax^{n-1}+a^n}{(x-a)^2}$$

CBSE Class 12 Mathematics

Important Questions

Chapter 13

Limits and Derivatives

6 Marks Questions

1. Differentiate tan x from first principle.

Ans. $f\left(x\right)=\tan x$

$f\left(x+h\right)=\tan\left(x+h\right)$

$f^{1}\left(x\right)=\lim_{h\to 0}\frac{f\left(x+h\right)-f\left(x\right)}{h}$

$=\lim_{h\to 0}\frac{\tan\left(x+h\right)-\tan x}{h}$

$=\lim_{h\to 0}\frac{\frac{\sin\left(x+h\right)}{\cos\left(x+h\right)}-\frac{\sin x}{\cos x}}{h}$

$=\lim_{h\to 0}\frac{\sin\left(x+h\right)\cos x-\cos\left(x+h\right)\sin x}{h\cos\left(x+h\right)\cos x}$

$=\lim_{h\to 0}\frac{\sin\left[x+h-x\right]}{h\cos\left(x+h\right)\cos x}\left[\begin{array}{l}\because \sin\left(A-B\right)=\\ \sin A\cos B-\cos A\sin B\end{array}\right]$

$=\lim_{h\to 0}\frac{\text{sinh}}{h\cos\left(x+h\right)\cos x}$

$$= \frac{\lim_{h\to 0} \frac{\sin h}{h}}{\lim_{h\to 0} \cos(x+h)\cos x} = \frac{1}{\cos x.\cos x} \left[\because \lim_{h\to 0} \frac{\sin h}{h} = 1\right]$$

$$= \frac{1}{\cos^2 x} = \sec^2 x$$

2. Differentiate $(x+4)^6$ From first principle.

Ans. let $f(x) = (x+4)^6$

$$f(x+h) = (x+h+4)^6$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h+4)^6 - (x+4)^6}{h}$$

$$= \lim_{(x+h+4)\to(x+4)} \frac{(x+h+4)^6 - (x+4)^6}{(x+h+4) - (x-4)}$$

$$= 6(x+4)^{(6-1)} \left[\because \lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}\right]$$

$$= 6(x+4)^5$$

3. Find derivative of cosec x by first principle

Ans. proof let $f(x) = \text{cosec}\, x$

$$\text{By def.} f(x) = \underset{h\to 0}{Lt} \frac{f(x+h) - f(h)}{h}$$

$$= \underset{h\to 0}{Lt} \frac{\text{cosec}(x+h) - \text{cosec}\, x}{h}$$

$$= \underset{h\to 0}{Lt} \frac{\frac{1}{\sin(x+h)} - \frac{1}{\sin x}}{h} = \underset{h\to 0}{Lt} \frac{\sin x - \sin(x+h)}{h\sin(x+h)\sin x}$$

$$= \underset{h\to 0}{Lt} \frac{2\cos\frac{x+x+h}{2}\sin\frac{x-x+h}{2}}{h\sin(x+h)\sin x}$$

$$= \underset{h\to 0}{Lt} \frac{2\cos\left(x+\frac{h}{2}\right)\sin\left(-\frac{h}{2}\right)}{h\sin(x+h)\sin x}$$

$$= \frac{\underset{\frac{h}{2}\to 0}{Lt}\cos\left(x+\frac{h}{2}\right)}{\cos x.\underset{h\to 0}{L}+\sin(x+h)} . \underset{\frac{h}{2}\to 0}{Lt} \frac{\sin\frac{h}{2}}{\frac{h}{2}}$$

$$= -\frac{\cos x}{\sin x.\sin x}.1 = -\cos ecx\cot x$$

4. Find the derivatives of the following fuchsias:

$$(i)\ \left(x-\frac{1}{x}\right)^3 \quad (ii)\ \frac{(3x+1)\left(2\sqrt{x-1}\right)}{\sqrt{x}}$$

Ans. (i) let $f(x) = \left(x-\frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x + \frac{1}{x}\left(x-\frac{1}{x}\right)$

$= x^3 - x^{-3} - 3x + 3x^{-1}.d.ff\ wr.t4, we$ get

$f(x) = 3\times x^2 - (-3)x^{-4} - 3\times 1 + 3\times(-1)x^{-2}$

$= 3x^2 + \frac{3}{x^4} - 3 - \frac{3}{x^2}$.

(ii) let $f(x) = \frac{(3x+1)(2\sqrt{x}-1)}{\sqrt{x}} = \frac{6x^{\frac{3}{2}} - 3x + 2\sqrt{x} - 1}{\sqrt{x}}$

$= 6x - 3x^{\frac{1}{2}} + 2 - x^{-\frac{1}{2}}$, $d:ff$ w.r.t. x.weget

$f(x) = 6\times 1 - 3\times\frac{1}{2}\times x^{-\frac{1}{2}} + 0 - \left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$

$= 6 - \frac{3}{2\sqrt{x}} + \frac{1}{2.x^{\frac{3}{2}}}$.

5. If $f(x) = \begin{cases} |x| + a; x < 0 \\ 0; x = 0 \\ |x| - a; x > 0 \end{cases}$ **for what Values of 'a' does** $\lim_{x\to 0} f(x)$ **exist**

Ans.given $f(x) = \begin{cases} |x| + a; x < 0 \\ 0; x = 0 \\ |x| - a; x > 0 \end{cases}$

a=0

L.H.L. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} |x| + a$

$= \lim_{x\to 0} -x + a = a$

R.H.L. $\lim_{x\to 0^+} f(x) = \lim_{x\to 0} |x| - a$

$\because \lim_{x\to 0} f(x)$ exist

$\therefore \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$a = -a$

$2a = 0$

$a = 0$

At $a = 0 \lim_{x\to 0} f(x)$ exist

6. Find the derivative of sin $(x+1)$. with respect to x, from first principle.

Ans. let $f(x) = \sin(x+1)$

$f(x+h) = \sin(x+h+1)$

$$f^1(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h+1) - \sin(x+1)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left[\frac{x+h+1+x+1}{2}\right]\sin\left[\frac{x+h+1-x-1}{2}\right]}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left[x+1+\frac{h}{2}\right]\sin\frac{h}{2}}{h}$$

$$= \lim_{h\to 0} \cancel{2}\cos\left(x+1+\frac{h}{2}\right) \times \lim_{h\to 0} \frac{\sin\frac{h}{2}}{\cancel{2}\frac{h}{2}}$$

$$= \cos(x+1) \times 1 = \cos(x+1)$$

7. Find the derivative of $\sin x+\cos x$ **from first principle**

Ans. $let\ f(x)=\sin x+\cos x$

$$f(x+h)=\sin(x+h)+\cos(x+h)$$

$$f^{1}(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\left[\sin(x+h)\cos(x+h)\right]-\left[\sin x+\cos x\right]}{h}$$

$$=\lim_{h\to 0}\frac{\left[\sin(x+h)-\sin x\right]+\left[\cos(x+h)-\cos x\right]}{h}$$

$$=\lim_{h\to 0}\frac{2\cos\left[\frac{x+h+x}{2}\right]\sin\frac{(x+h-x)}{2}-2\sin\frac{(x+h+x)}{2}\times\sin\left[\frac{x+h-x}{2}\right]}{h}$$

$$=\lim_{h\to 0}\frac{2\cos\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{h}+\lim_{h\to 0}\frac{-2\sin\left(x+\frac{h}{2}\right)\sin\frac{h}{2}}{h}$$

$$=\lim_{h\to 0}\not{2}\cos\left(x+\frac{h}{2}\right)\frac{\sin\frac{h}{2}}{\not{2}\frac{h}{2}}+\lim_{h\to 0}-\not{2}\sin\left(x+\frac{h}{2}\right)\frac{\sin\frac{h}{2}}{\not{2}\frac{h}{2}}$$

$$=\cos(x+0)\times 1-\sin(0+x)\times 1$$

$$=\cos x-\sin x$$

8. Find derivative of

$$(i)\frac{x\sin x}{1+\cos x}(ii)(ax+b)(x+d)^{2}$$

Ans. (i) $\frac{d}{dx}\frac{x\sin x}{1+\cos x}$

$$=\frac{(1+\cos x)\frac{d}{dx}(x\sin x)-x\sin x\frac{d}{dx}(1+\cos x)}{(1+\cos x)^2}$$

$$=\frac{(1+\cos x)\left[x\frac{d}{dx}(\sin x)+\sin x\frac{d}{dx}(x)\right]-x\sin x[0-\sin x]}{(1+\cos x)^2}$$

$$=\frac{(1+\cos x)[x\cos x+\sin x\times 1]+x\sin^2 x}{(1+\cos x)^2}$$

$$=\frac{x\cos x+x\cos^2 x+\sin x+\sin x\cos x+x\sin^2 x}{(1+\cos x)^2}$$

$$=\frac{x(\cos^2 x+\sin^2 x)+x\cos x+\sin x+\sin x\cos x}{(1+\cos x)^2}$$

$$=\frac{x+x\cos x+\sin x+\sin x\cos x}{(1+\cos x)^2}$$

(ii) $\frac{d}{dx}(ax+b)(cx+d)^2$

$$=(ax+b)\frac{d}{dx}(cx+d)^2+(cx+d)^2\frac{d}{dx}(ax+b)$$

$$=(ax+b)2(cx+d)\frac{d}{dx}(cx+d)+(cx+d)^2\times a$$

$$=2(ax+b)(cx+d)\times c+a(cx+d)^2$$

$$=(cx+d)[2c(ax+b)+a(cx+d)]$$

$$=(cx+d)[2acx+2bc+acx+ad]$$

$$=(cx+d)[3acx+2abc+ad]$$

9. Evaluate $\lim_{h\to 0}\frac{(a+h)^2\sin(a+h)-a^2\sin a}{h}$

Ans. $\lim_{h\to 0}\frac{(a+h)^2\sin(a+x)-a^2\sin a}{h}$

$$=\lim_{h\to 0}\frac{(a^2+2ah+h^2)\sin(a+h)-a^2\sin a}{h}$$

$$=\lim_{h\to 0}\frac{a^2\sin(a+h)+2ah\sin(a+h)+h^2\sin(a+h)-a^2\sin a}{h}$$

$$=\lim_{h\to 0}\frac{a^2[\sin(a+h)-\sin a]+2ah\sin(a+h)+h^2\sin(a+h)}{h}$$

$$=\lim_{h\to 0}\frac{a^2 2\cos\left[\frac{2a+h}{2}\right]\sin\frac{h}{2}}{2\frac{h}{2}}+\lim_{h\to 0}2a\sin(a+h)+\lim_{h\to 0}h\sin(a+h)$$

$$=a^2\cos\left[\frac{2a+0}{2}\right]\times 1+2a\sin[a+0]+0\times\sin a$$

$$=a^2\cos a+2a\sin a$$

10. Differentiate

$(i)\left(\frac{a}{x^4}\right)-\frac{b}{x^2}+\cos x\quad (ii)(x+\cos x)(x-\tan x)$

Ans. (i) $\frac{d}{dx}\left[\frac{a}{x^4}-\frac{b}{x^2}+\cos x\right]$

$$=\frac{d}{dx}ax^{-4}-\frac{d}{dx}bx^{-2}+\frac{d}{dx}\cos x$$

$$=a\left(-4x^{-5}\right)-b\left(-2x^{-3}\right)-\sin x$$

$$=\frac{-4a}{x^5}+\frac{2b}{x^3}-\sin x$$

(ii) $\frac{d}{dx}(x+\cos x)(x-\tan x)$

$$=(x+\cos x)\frac{d}{dx}(x-\tan x)+(x-\tan x)\frac{d}{dx}(x+\cos x)$$

$$=(x+\cos x)\left(1-\sec^2 x\right)+(x-\tan x)(1-\sin x)$$

$$=x-x\sec^2 x+\cos x-\cos x\sec^2 x+x-x\sin x-tacx+\tan x\sin x in x$$

$$=2x-x\sec^2 x+\cancel{\cos x}-\cancel{\cos x}-x\sin x-\tan x+\tan x\sin x$$

$$=2x-x\sec^2 x-x\sin x-\tan x+\tan x s$$