Heights and Distances

INTRODUCTION

Solution of triangles has enormous applications to surveying, navigation, and so on. We will now consider some simple ones from among them. For this purpose, we need to explain certain terms that are generally used in practical problems.

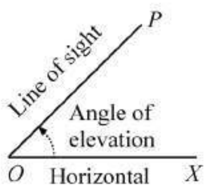


Fig. (a)

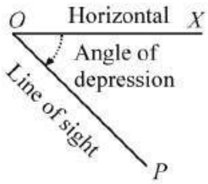


Fig. (b)

1. If OX be a horizontal line through O, the eye of the observer and P be an object in the vertical plane through OX, then $\angle XOP$ is called:
   (i) **the angle of elevation,** if P is *above* OX as in Fig. (a); and
   (ii) **the angle of depression,** if P is *below* OX as in Fig. (b).
   The straight line OP (joining the eye of the observer to the object) is called the *line of sight* of the observer.
2. Values of the trigonometric ratios for some useful angles

angle (θ) / *t*-ratio	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

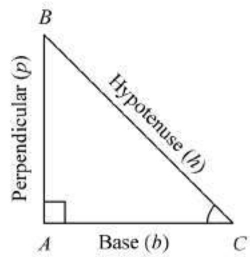
The values of cot θ, sec θ and cosec θ can be found from the above table by using the relations $\cot\theta = \frac{\cos\theta}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$ and $\text{cosec}\,\theta = \frac{1}{\sin\theta}$.

3. **Pythagoras Theorem**

In a right-angled triangle the square of its hypotenuse is equal to the sum of the squares of its legs (i.e., perpendicular and base).

In other words,

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$



or, $$(BC)^2 = (AB)^2 + (AC)^2$$

or, $$h^2 = p^2 + b^2.$$

4. Few important values to memorise:

$\sqrt{2} = 1.414$

$\sqrt{3} = 1.732$

$\sqrt{5} = 2.236.$

Practice Exercises

Difficulty Level-1
(Based on Memory)

1. A 25 m ladder is placed against a vertical wall of a building. The foot of the ladder is 7 m from the base of the building. If the top of the ladder slips 4 m, then the foot of the ladder will slide:

(*a*) 5 m (*b*) 8 m
(*c*) 9 m (*d*) 15 m

[Based on MAT, 2004]

2. The angle of elevation of the top of a tower at a point *G* on the ground is 30°. On walking 20 m towards the tower the angle of elevation becomes 60°. The height of the tower is equal to:

(*a*) $\frac{10}{\sqrt{3}}$ m (*b*) $20\sqrt{3}$ m
(*b*) $\frac{20}{\sqrt{3}}$ m (*d*) $10\sqrt{3}$ m

3. The angles of elevation of the top of a tower from two points at distances *m* and *n* metres are complementary. If the two points and the base of the tower are on the same straight line, then the height of the tower is:

(*a*) $\sqrt{mn}$ (*b*) mn
(*c*) $\frac{m}{n}$ (*d*) None of these

[Based on MAT, 2003]

4. The angles of elevation of an artificial satellite measured from two earth stations are 30° and 40°, respectively. If the distance between the earth stations is 4000 km, then the height of the satellite is:

(*a*) 2000 km (*b*) 6000 km
(*c*) 3464 km (*d*) 2828 km

[Based on MAT, 2002]

5. The angle of elevation of the sun when the length of the shadow of a pole is $\sqrt{3}$ times the height of the pole is:

(*a*) 30° (*b*) 45°
(*c*) 60° (*d*) 75°

[Based on MAT, 2001]

6. The tops of two poles of height 20 m and 14 m are connected by a wire. If the wire makes an angle 30° with the horizontal, then the length of the wire is:

(*a*) 12 m (*b*) 10 m
(*c*) 8 m (*d*) None of these

7. The distance between the tops of two trees 20 m and 28 m high is 17 m. The horizontal distance between the two trees is:

(*a*) 9 m (*b*) 11 m
(*c*) 15 m (*d*) 31 m

[Based on MAT, 2001]

8. A tree breaks due to storm and the broken part bends so that the top of the tree first touches the ground, making an angle of 30 with the horizontal. The distance from the foot of the tree to the point where the top touches the ground is 10 m. The height of the tree is:

(*a*) $10(\sqrt{3}+1)$ m (*b*) $10\sqrt{3}$ m
(*c*) $10(\sqrt{3}-1)$ m (*d*) $\frac{10}{\sqrt{3}}$ m

[Based on MAT, 2001]

9. The angle of elevation of an aeroplane from a point on the ground is 45°. After 15 seconds' flight, the elevation changes to 30°. If the aeroplane is flying at a height of 3000 m, the speed of the plane in Km/h is:

(*a*) 208.34 (*b*) 306.72
(*c*) 402.056 (*d*) 527

[Based on MAT, 2008]

10. A vertical lamp post of height 9 m stands at the corner of a rectangular field. The angle of elevation of its top from the farthest corner is 30°, while from another corner it is 45°. The area of the field is:

(*a*) $9\sqrt{2}$ m^2 (*b*) $81\sqrt{2}$ m^2
(*c*) $8\sqrt{3}$ m^2 (*d*) $9\sqrt{3}$ m^2

[Based on MAT (Feb), 2011]

11. A flagstaff stands vertically on a pillar, the height of the flagstaff being double the height of the pillar. A man on the ground at a distance finds that both the pillar and the flagstaff subtend equal angles at his eyes. The ratio of the height of the pillar and the distance of the man from the pillar is:

(*a*) 1:3 (*b*) $3:\sqrt{1}$
(*c*) $1:\sqrt{3}$ (*d*) $\sqrt{3}:2$

[Based on MAT (Feb), 2011]

12. The angles of elevation of the top of a tower 30 m high, from two points on the level ground on its opposite sides are 45° and 60°. What is the distance between the two points?

(*a*) 47.32 m (*b*) 41.23 m
(*c*) 38.12 m (*d*) 52.10 m

[Based on MAT (Dec), 2010]

13. The angles of depression and elevation of the top of a wall 24 m high from top and bottom of a tree 60° and 30° respectively. The distance of the tree is:

(*a*) 41.56 m (*b*) 32.42 m

(*c*) 56.21 m (*d*) 36.52 m

[Based on MAT (Dec), 2010]

14. A tower stands at the end of a straight road. The angles of elevation of the top of the tower from two points on the road 500 m apart are 45° and 60°, respectively. Find the height of the tower.

(*a*) $\frac{(500\sqrt{3})}{(\sqrt{3}-1)}$ m (*b*) $\frac{(500\sqrt{3})}{(\sqrt{3}+1)}$ m

(*c*) $5000\sqrt{3}$ m (*d*) $450\sqrt{3}$ m

[Based on MAT (Dec), 2010]

15. The distance between two multistoried buildings is 60 m. The angle of depression of the top of the first building as seen from the top of the second building which is 150 m high is 30°. The height of the first building is:

(*a*) $(150 + 20\sqrt{3})$ m (*b*) $(150 - 20\sqrt{3})$ m

(*c*) $(150 + 10\sqrt{3})$ m (*b*) $(15 - 10\sqrt{3})$ m

[Based on MAT (Sept), 2010, 2007]

16. The length of a string between a kite and a point on the ground is 90 m. The string makes an angle of 60° with the level ground. Assuming that there is no slack in the string, the height of the kite is:

(*a*) $45\sqrt{3}$ m (*b*) $45/\sqrt{3}$ m

(*c*) $50\sqrt{3}$ m (*d*) $50/\sqrt{3}$ m

[Based on MAT (Sept), 2010]

17. A Navy captain rowing away from a lighthouse 100 m high observes that it takes 2 minutes to change the angle of elevation of the top of the lighthouse from 60° to 45°. How far has he travelled from the lighthouse for this change to happen?

(*a*) 100 m (*b*) $100\sqrt{3}$ m

(*c*) 90 m (*d*) 75 m

[Based on MAT (May), 2010]

18. From a point *A* on the ground, the angles of elevation of the bottom and top of a transmission tower fixed at the top of a 20 m high building are 45° and 60°, respectively. Find the distance between the point *A* and the building.

(*a*) $\left(20+\frac{1}{\sqrt{3}}\right)$m (*b*) $20/\sqrt{3}$ m

(*c*) $20\sqrt{3}$ m (*d*) 20 m

[Based on MAT (May), 2010]

19. A plane is flying at an altitude of 2 km. The elevation is 30°. After exactly one minute, it is now at an angle of elevation 60° maintaining the same altitude. The speed of the plane is

(*a*) 60 m/s (*b*) 46.76 m/s

(*c*) 25.38 m/s (*d*) 38.49 m/s

[Based on MAT (May), 2010 (Feb), 2008]

20. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. If after 10 second the elevation be 30°, the uniform speed of the aeroplane is:

(*a*) $240\sqrt{3}$ Km/h (*b*) $240/\sqrt{3}$ Km/h

(*c*) $120/\sqrt{3}$ Km/h (*d*) $120\sqrt{3}$ Km/h

[Based on MAT (Feb), 2010]

21. From the top of cliff 25 m high, the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. The height of the tower is:

(*a*) 25 m (*b*) 50 m

(*c*) 75 m (*d*) 100 m

[Based on MAT (Feb), 2010 (Dec), 2008]

22. A portion of 30 m long tree is broken by a tornado and the top strikes the ground making an angle 30° with the ground level. The height of the point where the tree is broken is equal to:

(*a*) $\frac{30}{\sqrt{3}}$ m (*b*) 10 m

(*c*) $30\sqrt{3}$ m (*d*) 60 m

[Based on MAT (Sept), 2009]

23. A balloon leaves the earth at point *A* and rises at a uniform velocity. At the end of $1\frac{1}{2}$ minutes, an observer situated at a distance of 200 m from *A* finds the angular elevation of the balloon to be 60°. The speed of the balloon is:

(*a*) 5.87 m/s (*b*) 4.87 m/s

(*c*) 3.87 m/s (*d*) 6.87 m/s

[Based on MAT (May), 2009]

24. At the foot of a mountain, the elevation of its summit is 45°. After ascending one kilometer towards the mountain upon and incline of 30°, the elevation changes to 60°. The height of the mountain is:

(*a*) 1.366 km (*b*) 1.266 km

(*c*) 1.166 km (*d*) 1.466 km

[Based on MAT (May), 2009]

25. A tree is broken by the wind. The top struck the ground at an angle 30° and at a distance of 30 m from the root. The whole height of the tree is approximately:

(*a*) 52 m (*b*) 17 m

(*c*) 34 m (*d*) 30 m

[Based on MAT (Feb), 2009 (May) 2008]

26. At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is

5/12. On walking 192 m towards the tower, the tangent of the angle of elevation is three-fourths. The height of the tower is:

(*a*) 96 m (*b*) 150 m
(*c*) 180 m (*d*) 226 m

[Based on MAT (Feb), 2009, 2006]

27. The horizontal distance between two towers is 60 m. The angular elevation of the top of the taller tower as seen from the top of the shorter one is 30°. If the height of the taller tower is 150 m, the height of the shorter one, approximately, is:

(*a*) 116 m (*b*) 216 m
(*c*) 200 m (*d*) None of these

[Based on MAT (Dec), 2008]

28. A window on one side of a road is 12 m above ground. A ladder is placed on the road to reach the window. If the ladder is turned on the other side of the road keeping its feet on the same point, it can reach a window which is at a height of 9 m from the ground. Supposing the length of the ladder to be 15 m, what is the width of the road?

(*a*) 9 m (*b*) 21 m
(*c*) 12 m (*d*) None of these

[Based on MAT (Dec), 2008]

29. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 m away from the tree, the angle of elevation becomes 30°. The breadth of the river is:

(*a*) 40 m (*b*) 20 m
(*c*) 30 m (*d*) 60 m

[Based on MAT ((Dec, Sept, May) 2007 (Feb), 2006]

30. A person observes the angle of elevation of a building as 30°. The person proceeds towards the building with a speed of $25(\sqrt{3}-1)$ m/h. After 2 h, he observes the angle of elevation as 45°. The height of the building (in m) is:

(*a*) 100 (*b*) $50(\sqrt{3}+1)$
(*c*) 50 (*d*) $50(\sqrt{3}-1)$

[Based on MAT (Dec), 2007]

31. At a distance *a* from the foot of a tower *AB*, of height *b*, a flagstaff *BC* and the tower subtends equal angles. The length of the flagstaff is:

(*a*) $\frac{a(b^2+a^2)}{a^2-b^2}$ (*b*) $\frac{b(a^2+b^2)}{a^2-b^2}$

(*c*) $\frac{a^2(a^2+b^2)}{a^2-b^2}$ (*d*) $\frac{b^2(a^2+b^2)}{a^2-b^2}$

[Based on MAT (Sept), 2007]

32. The angles of elevation of the top of a tower from the top and the foot of a pole of height 10 m, are 30° and 60°, respectively. The height of the tower is:

(*a*) 20 m (*b*) 15 m
(*c*) 10 m (*d*) None of these

[Based on MAT (May), 2007]

33. A person standing on the bank of a river finds that the angle of elevation of the top of a tower on the opposite bank is 45°. Then, which of the following statements is correct?

(*a*) The breadth of the river is half of the height of the tower.

(*b*) The breadth of the river and the height of the tower are the same.

(*c*) The breadth of the river is twice the height of the tower.

(*d*) None of the above.

[Based on MAT (May), 2009]

34. A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point *A* on the ground is 60° and the angle of depression of the point *A* from the top of the tower is 45°. The height of the tower is:

(*a*) 5.83 m (*b*) 7.83 m
(*c*) 6.83 m (*d*) 4.83 m

[Based on MAT (May), 2006]

35. From a horizontal distance of 50 m, the angles of elevation of the top and the bottom of a vertical cliff face are 45° and 30°, respectively. The height of the cliff face in metres is

(*a*) $50/\sqrt{3}$ (*b*) $50/\sqrt{2}$
(*c*) $50/2\sqrt{3}$ (*d*) $50(1-1/\sqrt{3})$

[Based on MAT, 1997]

36. The angles of elevation of the top of a tower from two points *P* and *Q* at distances of *x* and *y*, respectively, from the base and in the same straight line with it are complimentary. Find the height of the tower.

(*a*) $\sqrt{\frac{y}{x}}$ (*b*) $\sqrt{\frac{x}{y}}$
(*c*) $\sqrt{xy}$ (*d*) None of these

[Based on MAT, 1999]

37. The Qutab Minar casts a shadow 150 m long at the same time when the Vikas Minar casts a shadow of 120 m long on the ground. If the height of the Vikas Minar is 80 m, find the height of the Qutab Minar.

(*a*) 180 m (*b*) 100 m
(*c*) 150 m (*d*) 120 m

[Based on MAT, 1999]

38. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower form the foot of the building is 60°. If height of the tower is 50 m, the height of the building is:

(*a*) $\frac{50\sqrt{3}}{3}m$ (*b*) $16\frac{2}{3}m$

(*c*) $50\sqrt{3}$ (*d*) None of these

[Based on MAT, 2011]

39. As observed from the top of a lighthouse, 100 m high above the sea level, the angle of depression of a ship, sailing directly towards it, changes from 30° and 60°. The distance travelled by the ship during the period of observation is:

(*a*) 173.2 m (*b*) 115.5 m

(*c*) 57.7 m (*d*) None of these

[Based on MAT (Feb), 2012]

40. An electrician has to repair an electric fault on a pole of height 4 m. He needs to reach a point 1.3 m below the top of the pole to undertake the repair work. The length of the ladder that he should use, which when inclined at an angle of 60° to the horizontal would enable him to reach the required position is:

(*a*) $(9\sqrt{3})/10$ m (*b*) $(3\sqrt{3})/10$ m

(*c*) $(3\sqrt{3})/5$ m (*d*) $(9\sqrt{3})/5$ m

[Based on MAT (Feb), 2012]

41. The horizontal distance between two towers is 60 m. The angular elevations of the top of the taller tower as seen from the top of the shorter one is 30 If the height of the taller tower is 150 m, the height of the shorter one is:

(*a*) 100 m (*b*) 106 m

(*c*) 116 m (*d*) None of these

[Based on MAT, 2013]

42. A man observes that when he move up a distance *C* m on a slope, the angle of depression of a point on the horizontal plane from the base of the slope is 30°, and when he moves up a further distance of *C* m, the angle of depression of that point is 45°. The angle of inclination slope with the horizontal is:

(*a*) 45° (*b*) 60°

(*c*) 75° (*d*) 30°

[Based on MAT, 2010]

43. At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$. On walking 192 m towards the tower, the tangent of the angle of elevation is $\frac{3}{4}$. The height of the tower is:

(*a*) 96 m (*b*) 150 m

(*c*) 180 m (*d*) 226 m

[Based on MAT, 2013]

Difficulty Level-2
(Based on Memory)

1. A ladder is inclined to a wall making an angle of 30° with it. A man is ascending the ladder at the rate of 2 m/s. How fast is he approaching the wall?

(*a*) 2 m/s (*b*) 1.5 m/s

(*c*) 1 m/s (4) None of these

[Based on FMS (Delhi), 2004]

2. One side of a parallelogram is 12 cm and its area is 60 cm^2. If the angle between the adjacent sides is 30°, then its other side is:

(*a*) 10 cm (*b*) 8 cm

(*c*) 6 cm (*d*) 4 cm

[Based on IITTM, Gwalior, 2003]

3. A person walking along a straight road towards a hill observes at two points, distance km, the angles of elevation of the hill to be 30° and 60°. The height of the hill is:

(*a*) $\frac{3}{2}$ km (*b*) $\sqrt{\frac{2}{3}}$ km

(*c*) $\frac{\sqrt{3}+1}{2}$ km (*d*) $\sqrt{3}$ km

4. Mr Gidwani's Padyatra Party wanted to go from Gwalior to Bhubaneswar. The walkers travelled 150 km straight and then took a 45° turn towards Varanasi and walked straight for another 120 km. Approximately, how far was the party from the starting point?

(*a*) 250 km (*b*) 90 km

(*c*) 81 km (*d*) 30 km

[Based on FMS (Delhi), 2003]

5. A vertical pole *PO* is standing at the centre *O* of a square *ABCD*. If *AC* subtends an angle of 90° at the point *P* of the pole, then the angle subtended by a side of the square at the point P is:

(*a*) 35° (*b*) 45°

(*c*) 30° (*d*) 60°

[Based on IIFT, 2003]

6. The angle of elevation of the top of an unfinished tower at a point distant 120 m from its base is 45°. If the elevation of the top at the same point is to be 60°, the tower must be raised to a height:

(*a*) $120(\sqrt{3}+1)$ m

(*b*) $120(\sqrt{3}-1)$ m

(*c*) $10(\sqrt{3}+1)$ m

(*d*) None of these

7. What is the height of a tower if the angles of elevation of its top from two points x and y at distances of a and b respectively from the base and on the same straight line with the tower are complementary?

(*a*) $\sqrt{b/a}$ (*b*) $\sqrt{a/b}$

(*c*) $\sqrt{ab}$ (*d*) None of these

[Based on I.P. Univ., 2002]

8. The angle of elevation of the top of a TV tower from three points A, B, C in a straight line through the foot of the tower are α, 2α, 3α, respectively. If $AB = a$, the height of the tower is:

(*a*) $a \tan \alpha$ (*b*) $a \sin \alpha$

(*c*) $a \sin 2\alpha$ (*d*) $a \sin 3\alpha$

9. A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 40 m from the bank, he finds the angle to be 30°. The breadth of the river is:

(*a*) 40 m (*b*) 60 m

(*c*) 20 m (*d*) 30 m

10. A tower subtends an angle of 30° at a point on the same level as the foot of the tower. At a second point h m above the first, the depression of the foot of the tower is 60°. The horizontal distance of the tower from the point is:

(*a*) $h \cot 60°$ (*b*) $h \cot 30°$

(*c*) $\frac{h}{3} \cot 60°$ (*d*) $\frac{h}{3} \cot 30°$

11. An observer standing 72 m away from a building notices that the angles of elevation of the top and the bottom of a flagstaff on the building are respectively 60° and 45°. The height of the flagstaff is:

(*a*) 124.7 m (*b*) 52.7 m

(*c*) 98.3 m (*d*) 73.2 m

[Based on FMS, 2006]

12. The angle of elevation of the top of a hill from each of the vertices A, B, C of a horizontal triangle is α. The height of the hill is:

(*a*) $b \tan \alpha \operatorname{cosec} B$ (*b*) $\frac{a}{2} \tan \alpha \operatorname{cosec} A$

(*c*) $\frac{c}{2} \tan \alpha \operatorname{cosec} C$ (*d*) None of these

13. Angle of depression from the top of a light house of two boats are 45° and 30° due east which are 60 m apart. The height of the light house is:

(*a*) $60\sqrt{3}$ (*b*) $30(\sqrt{3}-1)$

(*c*) $30(\sqrt{3}+1)$ (*d*) None of these

14. Vijay has been invited for dinner in a club. While walking through the garden path towards the club, he observes that there is an electric rod on the top of the building. From the point where he is standing, the angles of elevation of the top of the electric rod and the top of the building are is ϕ and θ respectively. If the heights of the electric rod and the building are p and q respectively, mark all the correct statements.

(*a*) The height of the tower is $\dfrac{p \tan \theta}{\tan \phi - \tan \theta}$

(*b*) The height of the electric rod is $\dfrac{q \tan \theta}{(\tan \theta - \tan \phi)}$

(*c*) The height of the tower is $\dfrac{p \tan \theta}{\tan \theta - \tan \phi}$

(*d*) The height of the electric is $\dfrac{q(\tan \phi - \tan \theta)}{\tan \theta}$

[Based on IIFT, 2006]

15. A ladder 25 m long is placed against a wall with its foot 7 m away from the foot of the wall. How far should the foot be drawn out so that the top of the ladder may come down by half the distance of the total distance if the foot is drawn out?

(*a*) 6 m (*b*) 8 m

(*c*) 8. 75 m (*d*) None of these

[Based on IIFT, 2008]

Directions (Q. 16 and 17): *Based on the following information*

A man standing on a boat South of a light house observes his shadow to be 24 m long, as measured at the sea level. On sailing 300 m Eastwards, he finds his shadow as 30 m long, measured in a similar manner. The height of the man is 6 m above sea level.

16. The height of the light house above the sea level is:

(*a*) 90 m (*b*) 94 m

(*c*) 96 m (*d*) 100 m

[Based on XAT, 2011]

17. What is the horizontal distance of the man from the light house in the second position?

(*a*) 300 m (*b*) 400 m

(*c*) 500 m (*d*) 600 m

[Based on XAT, 2011]

18. The central pole of a conical tent is 3/2 m high. The pole is supported by ropes tied to its top and nails on the ground. If on the ground from the foot of the pole, the distances of the surface of the tent and the nail (s) are in the ratio of 1:3 and if the angles of depression from the top of the pole of the nails and the surface of the tent are in the ratio of 1:2, then the length of one such rope is:

(*a*) 2 m (*b*) 6 m

(*c*) $3\sqrt{2}$ m (*d*) 3 m

[Based on JMET, 2011]

19. A ladder kept in support of a wall makes an angle of 22.5° with the ground. The distance between the bottom of the wall and the foot of the ladder is 2 m. What is the length of the ladder?

(*a*) $\sqrt{8-16\sqrt{2}}$ (*b*) $\sqrt{16-8\sqrt{2}}$

(*c*) $\sqrt{-16+2\sqrt{2}}$ (*d*) $\sqrt{18-8\sqrt{2}}$

[Based on JMET, 2011]

20. Two posts are k m apart and the height of one is double that of the other. If from the middle point of the line joining their feet, an observer finds the angular elevations of their tops to be complementary, then the height (in metres) of the shorter post is:

(*a*) $k\sqrt{2}$ (*b*) $\frac{k}{4}$

(*c*) $\frac{k}{2\sqrt{2}}$ (*d*) $\frac{k}{\sqrt{2}}$

[Based on NMAT, 2005]

21. An aeroplane flying horizontally 1 km above the ground is observed by a person on his right side at an elevation of 60º. If after 10 second the elevation is observed to be, from the same point and in the same direction, 30º, the uniform speed per hour (in km) of the aperoplane is (neglect the height of the person for computations):

(*a*) $360\sqrt{3}$ (*b*) $\frac{720}{\sqrt{3}}$

(*c*) 720 (*d*) $720\sqrt{3}$

[Based on JMET, 2006]

22. From the top of a light house 60 m high with its base at sea level, the angle of depression of a boat is 15°. The distance of the boat from the light house is:

(*a*) $60\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ m (*b*) $60\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ m

(*c*) $30\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$ m (*d*) $30\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$ m

23. From the top of a light house 50 m above the sea, the angle of depression of an incoming boat is 30°. How far is the boat from the light house?

(*a*) $25\sqrt{3}$ m (*b*) $25/\sqrt{3}$ m

(*c*) $50\sqrt{3}$ m (*d*) $50/\sqrt{3}$ m

24. There are two windows on the wall of a building that need repairs. A ladder 30m long is placed against a wall such that it just reaches the first window which is 26 m high. The foot of the ladder is a point A. After the first window is fixed, the foot of the ladder is pushed backwards to point B so that the ladder can reach the second window. The angle made by the ladder with the ground is reduced by half , as a result of pushing the ladder. The distance between points A and B is:

(*a*) <9 m

(*b*) 9≥ m and <9.5 m

(*c*) ≥9.5 m and <10 m

(*d*) ≥10 m and <10.5 m

(*e*) ≥10.5 m

[Based on XAT, 2014]

Answer Keys

Difficulty Level-1

1. (*b*)	**2.** (*d*)	**3.** (*a*)	**4.** (*c*)	**5.** (*a*)	**6.** (*a*)	**7.** (*c*)	**8.** (*b*)	**9.** (*d*)	**10.** (*a*)	**11.** (*c*)	**12.** (*a*)	**13.** (*a*)
14. (*a*)	**15.** (*b*)	**16.** (*a*)	**17.** (*a*)	**18.** (*d*)	**19.** (*d*)	**20.** (*a*)	**21.** (*b*)	**22.** (*b*)	**23.** (*c*)	**24.** (*a*)	**25.** (*a*)	**26.** (*c*)
27. (*a*)	**28.** (*b*)	**29.** (*b*)	**30.** (*c*)	**31.** (*b*)	**32.** (*b*)	**33.** (*b*)	**34.** (*c*)	**35.** (*d*)	**36.** (*c*)	**37.** (*b*)	**38.** (*b*)	**39.** (*b*)
40. (*d*)	**41.** (*c*)	**42.** (*a*)	**43.** (*c*)									

Difficulty Level-2

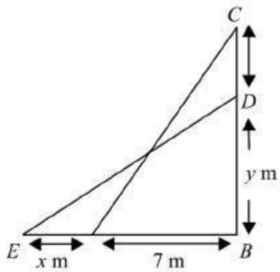
1. (*c*)	**2.** (*a*)	**3.** (*a*)	**4.** (*a*)	**5.** (*d*)	**6.** (*b*)	**7.** (*c*)	**8.** (*c*)	**9.** (*c*)	**10.** (*a*)	**11.** (*b*)	**12.** (*b*)	**13.** (*c*)
14. (*a*, *d*)	**15.** (*d*)	**16.** (*d*)	**17.** (*c*)	**18.** (*d*)	**19.** (*b*)	**20.** (*c*)	**21.** (*b*)	**22.** (*b*)	**23.** (*c*)	**24.** (*e*)		

Explanatory Answers

Difficulty Level-1

1. (*b*) $DE = 25 = AC$ = length of the ladder

$CD = 4$ m



Let, $BD = y$ metres

$\therefore\ (y+4)^2 + 7^2 = 25^2 \Rightarrow y = 20$

Now from ΔDBE,

$(20)^2 + (x+7)^2 = (25)^2$

$\Rightarrow\ (x+7)^2 = (15)^2 \Rightarrow x = 8$

$\Rightarrow$ Foot of the ladder slides 8 m.

2. (*d*) Let $AB = h$ be the height of the tower.

Let $GA = x$.

Then, $\dfrac{h}{x} = \tan 30° = \dfrac{1}{\sqrt{3}}$.

$\therefore\ h = \dfrac{x}{\sqrt{3}}$.

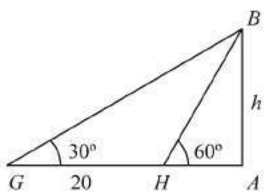
Also, $\dfrac{h}{x-20} = \tan 60° = \sqrt{3}$.

$\therefore\ h = \sqrt{3}\,(x-20)$

$\therefore\ \dfrac{x}{\sqrt{3}} = \sqrt{3}\,(x-20)$

$\Rightarrow\ x = 3(x-20) = 3x - 60$

$\Rightarrow\ 2x = 60 \Rightarrow x = 30$.

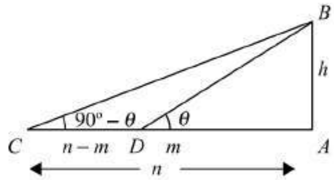


$\therefore\ h = \dfrac{30}{\sqrt{3}} = 10\sqrt{3}$ m.

3. (*a*) From ΔABC,

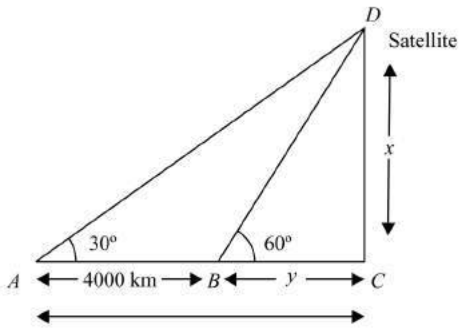
$\dfrac{h}{m} = \tan\theta$ and

$- = \tan(90 - \theta) = \cot\theta$



$\therefore\ \dfrac{h}{m} \times \dfrac{h}{n} = \tan\theta \times \cot\theta = 1 \Rightarrow h = \sqrt{mn}$.

4. (*c*) A and B are two earth stations.



Let, $AB = 4000$ km, $BC = y$ km, $CD = x$ km

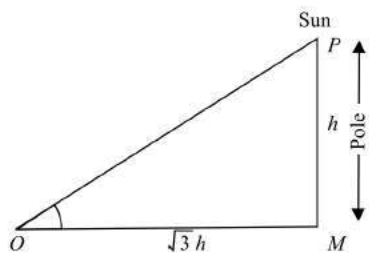
$\therefore\ \dfrac{x}{4000+y} = \tan 30°$

$= \dfrac{1}{\sqrt{3}}$ and $\dfrac{x}{y} = \tan 60° = \sqrt{3}$.

$\Rightarrow\ \sqrt{3}\,x = 4000 + y$ and $y = \dfrac{x}{\sqrt{3}}$

$\Rightarrow\ x = \sqrt{3} \times 2000 = 3464$.

5. (*a*) $\tan \angle MOP = \dfrac{MP}{OM} = \dfrac{1}{\sqrt{3}} = \tan 30°$

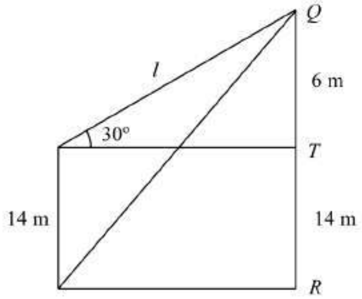


Shadow

$\therefore \quad \angle MOP = 30^\circ.$

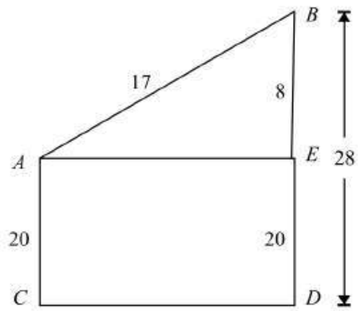
6. (*a*) $\frac{6}{l} = \sin 30^\circ = \frac{1}{2}$

$\therefore \quad l = 12$ m.



7. (*c*) $AE^2 = AB^2 - BE^2 = 289 - 64 = 225$

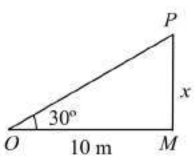
$\Rightarrow \quad AE = 15 \Rightarrow CD = 15.$



8. (*b*) $\frac{MP}{OM} = \tan 30^\circ$

$\Rightarrow \quad \frac{x}{10} = \tan 30^\circ = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad x = \frac{10}{\sqrt{3}}$



$\therefore \quad OP^2 = x^2 + (10)^2$

$= \frac{100}{3} + 100 = \frac{400}{3}$

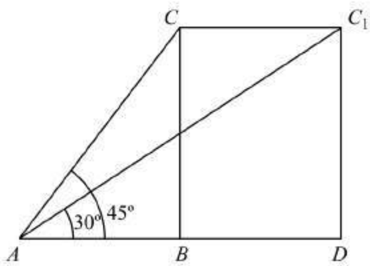
$\Rightarrow \quad OP = \frac{20}{\sqrt{3}}$

$\therefore$ Height of the tree

$= OP + PM = \frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}}$

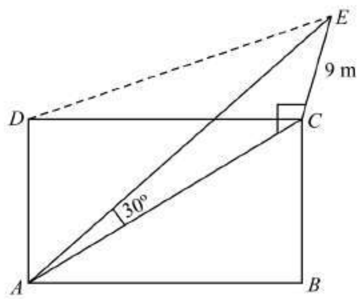
$= \frac{30}{\sqrt{3}} = 10\sqrt{3}.$

9. (*d*) Areroplane moves $BD = 3(\sqrt{3}-1)$ in seconds



$\therefore$ Speed $= \frac{3 \times (\sqrt{3}-1) \times 3600}{15} = 527$ Km/h

10. (*a*) Let *ABCD* be a rectangle and *CE* be a pole of height 9 m.



In right triangle *ACE*,

$\tan 30^\circ = \frac{CE}{AC}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{9}{AC}$

$\Rightarrow \quad AC = 9\sqrt{3}$ m

In right triangle *DCE*,

$\tan 45^\circ = \frac{CE}{DC}$

$\Rightarrow \quad DC = 9$ m

Using Pythagorus theorem in ΔADC,

$DA = \sqrt{AC^2 - DC^2}$

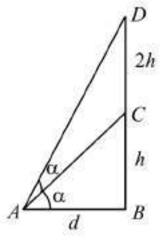
$= \sqrt{(9\sqrt{3})^2 - (9)^2} = 9\sqrt{2}$ m^2

11. (*c*) Let $BC = h$ be the height of the pillar,

Then, $CD = 2h$

$\therefore$ Also, let $\angle BAC = \angle CAD = \alpha$ and $AB = d$

In ΔABC,



$$\tan 2\alpha = \frac{h}{d}$$

and, in $\Delta\, ABD$,

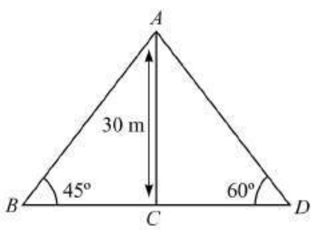
$$\tan 2\alpha = \frac{3h}{d}$$

$$\Rightarrow \quad \frac{2\tan a}{1-\tan^2\alpha} = \frac{3h}{d}$$

$$\Rightarrow \quad \frac{2h/d}{1-\left(\frac{h}{d}\right)^2} = \frac{3h}{d}$$

$$\Rightarrow \quad \frac{2}{3} = 1-\left(\frac{h}{d}\right)^2 \Rightarrow \frac{h}{d} = \frac{1}{\sqrt{3}}$$

12. (*a*) Let AC be the tower.



$$\therefore \quad \frac{AC}{BC} = \tan 45^\circ$$

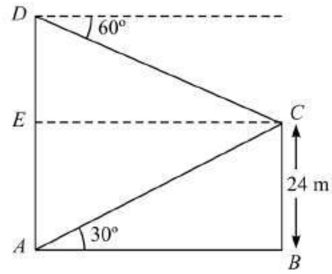
$$\Rightarrow \quad BC = AC = 30 \text{ m}$$

$$\text{and,} \quad \frac{AC}{CD} = \tan 60^\circ$$

$$\Rightarrow \quad CD = \frac{AC}{\sqrt{3}} = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$

So, required distance = $(30+10\sqrt{3})$ m = 47.32 m

13. (*a*)

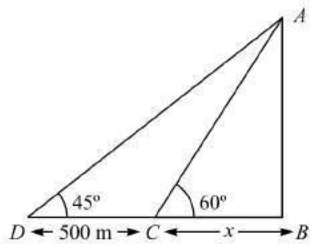


Let AD be the tree and BC be the wall.

$$\frac{BC}{AB} = \tan 30^\circ$$

$$AB = 24 \times \sqrt{3} = 41.5 \text{ m}$$

14. (*a*)



Let AB be the tower and distance BC is x.

$$\text{Then,} \quad \frac{AB}{BC} = \tan 60^\circ$$

$$\Rightarrow \quad AB = x\sqrt{3} \qquad (1)$$

$$\text{and,} \quad \frac{AB}{BD} = \tan 45^\circ$$

$$\Rightarrow \quad AB = x + 500 \qquad (2)$$

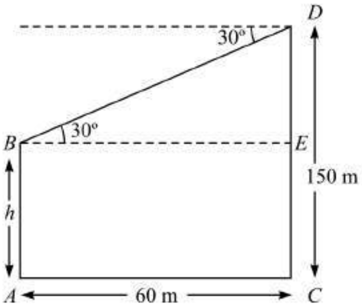
From Eqs. (1) and (2),

$$AB = \frac{AB}{\sqrt{3}} + 500$$

$$\Rightarrow \quad AB\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 500$$

$$\Rightarrow \quad AB = \left(\frac{500\sqrt{3}}{\sqrt{3}-1}\right)\text{m}$$

15. (*b*)



Let height of first building *AB* be *h*.

$\therefore \quad DE = 150 - h$

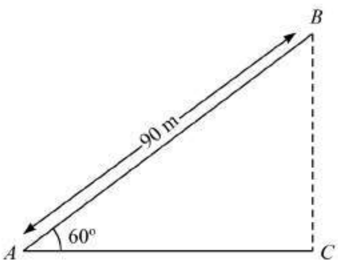
$$\frac{DE}{BE} = \tan 30^\circ$$

$$\Rightarrow \quad 150 - h = 60 \times \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad h = 150 - 60 \times \frac{1}{\sqrt{3}}$$

$$= (150 - 20\sqrt{3}\ )\text{ m}$$

16. (*a*)



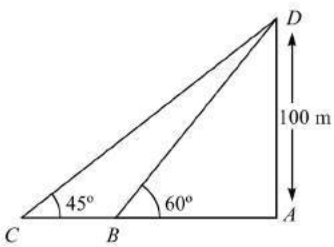
Let *AB* be the string.

$$\frac{BC}{AB} = \sin 60^\circ$$

$$\Rightarrow \quad BC = 90 \times \frac{\sqrt{3}}{2}$$

$$= 45\sqrt{3}\text{ m}$$

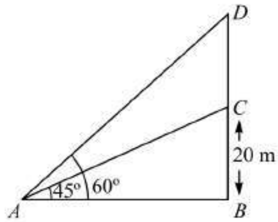
17. (*a*) Let *AD* be the lighthouse.



$$\frac{AD}{AC} = \tan 45^\circ$$

$\therefore \quad AC = 100$ m

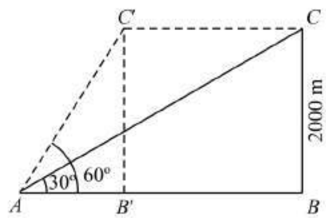
18. (*d*) Let *BC* be the building and *CD* be the tower.



$$\frac{BC}{AB} = \tan 45^\circ$$

$\therefore \quad AB = BC = 20$ m

19. (*d*)



$$\frac{BC}{AB} = \tan 30^\circ$$

$\therefore \quad AB = 2000\sqrt{3}$ m

$$\frac{B'C'}{AB'} = \tan 60^\circ$$

$$\therefore \quad AB' = \frac{2000}{\sqrt{3}}\text{ m}$$

Distance travelled by plan in 60 second = *CC′*

$= AB - AB'$

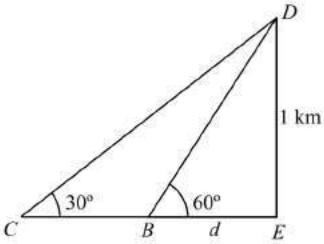
$$= 2000\sqrt{3} - \frac{2000}{\sqrt{3}}$$

$$= \frac{4000}{\sqrt{3}}\text{ m}$$

$$\therefore \text{ Speed of plane} = \frac{4000}{60\sqrt{3}}\text{ m/s}$$

$= 38.49$ m/s

20. (*a*) In ΔDEC, $\tan 30^\circ = \dfrac{1}{EC}$



$$EC = \sqrt{3}$$

So, $EB = d = EC - BC$

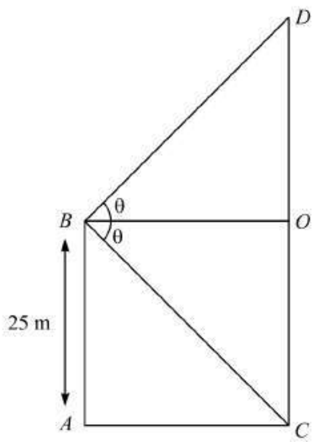
$$d = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$$

Speed of plane

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{10}{3600}} = \frac{2 \times 3600}{\sqrt{3} \times 10}$$

$$= 240\sqrt{3} \text{ km/h}$$

21. (*b*)



Let AB be the cliff and CD be the tower.

In ΔOBC and OBD,

$$\angle OBC = \angle OBD = \theta$$

OB is common.

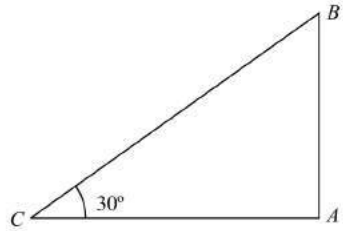
$\therefore \quad BC = BD$

So, triangles are congruent.

$\therefore \quad OC = OD = 25$ m

Height of the tower $= OC + OD = 50$ m

22. (*b*)



Let B is the point from where tree was broken.

$$\frac{AB}{BC} = \sin 30^\circ$$

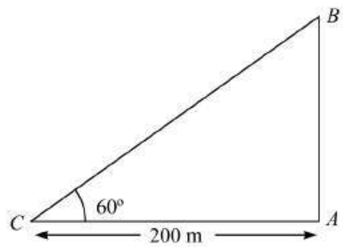
$$AB = \frac{1}{2} \times BC$$

Since, $AB + BC = 30$ m

$AB + 2\,AB = 30$ m

$\therefore \quad AB = 10$ m

23. (*c*)



After $1\frac{1}{2}$ minutes, the balloon will be at point B.

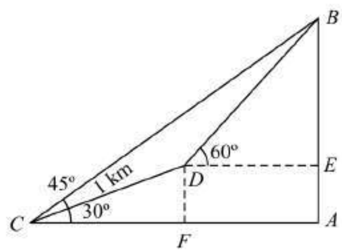
$$\frac{AB}{BC} = \tan 60^\circ$$

$\therefore \quad AB = 200\sqrt{3}$ m

$\therefore$ Speed of balloon $= \dfrac{200\sqrt{3}}{\frac{3}{2} \times 60}$ m/s

$$\approx 3.87 \text{ m/s}$$

24. (*a*)



Let AB be the mountain and its height be h km.

$$\frac{AB}{AC} = \tan 45^\circ$$

$\therefore \quad AC = AB = h$ km

Now, $\quad \dfrac{CF}{CD} = \cos 30^\circ$

$\therefore \quad CF = 1 \times \dfrac{\sqrt{3}}{2}$ km

$\dfrac{DF}{CD} = \sin 30^\circ$

$\therefore \quad DF = \dfrac{1}{2}$ km

Now, $\quad DE = \left(h - \dfrac{\sqrt{3}}{2}\right)$ km

and, $\quad BE = \left(h - \dfrac{1}{2}\right)$ km

$\left(\because DF = AE = \dfrac{1}{2}\right)$

$\dfrac{BE}{DE} = \tan 60^\circ$

$h - \dfrac{1}{2} = \left(h - \dfrac{\sqrt{3}}{2}\right) \times \sqrt{3}$

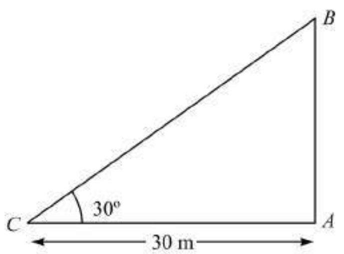
$\Rightarrow \quad h - \dfrac{1}{2} = \sqrt{3}\, h - \dfrac{3}{2}$

$\Rightarrow h(\sqrt{3} - 1) = 1$

$\Rightarrow \quad h = \left(\dfrac{1}{\sqrt{3} - 1}\right)$ km

$= \dfrac{\sqrt{3} + 1}{2}$ km $= 1.366$ km

25. (*a*)



Let the tree was broken at point B, the root be at point A and the top be at point C.

$\dfrac{AB}{AC} = \tan 30^\circ$

$\therefore \quad AB = \dfrac{30}{\sqrt{3}}$ m $= 10\sqrt{3}$ m

Now, $\quad \dfrac{AC}{BC} = \cos 30^\circ$

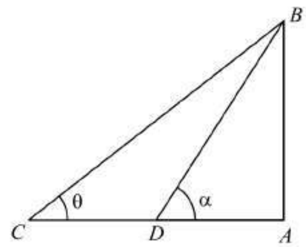
$BC = 30 \times \dfrac{2}{\sqrt{3}} = 20\sqrt{3}$ m

$\therefore$ Whole height of tree

$= (10\sqrt{3} + 20\sqrt{3})$ m

$= 30\sqrt{3}$ m ≈ 52 m

26. (*c*)



Let AB be the tower.

$\tan \alpha = \dfrac{AB}{AD}$

$\Rightarrow \quad \dfrac{3}{4} = \dfrac{AB}{AD}$

$\therefore \quad AD = \dfrac{4}{3} AB \qquad (1)$

Now, $\quad \tan \theta = \dfrac{AB}{AC}$

$\Rightarrow \quad \dfrac{5}{12} = \dfrac{AB}{(192 + AD)}$

$\therefore \quad 192 + AD = \dfrac{12}{5} AB$

$\Rightarrow \quad AD = \dfrac{12}{5} AB - 192 \qquad (2)$

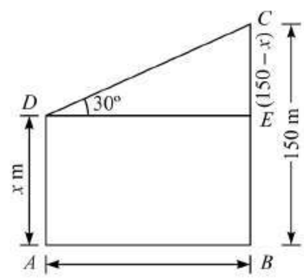
From Eqs. (1) and (2),

$\dfrac{4}{3} AB = \dfrac{12}{5} AB - 192$

$\Rightarrow \quad \dfrac{16}{15} AB = 192$

$\therefore \quad AB = 180$ m

27. (*a*)



Let the height of the shorter tower be x m.

Then, from ΔCDE,

$$\tan 30^\circ = \frac{(150 - x)}{60}$$

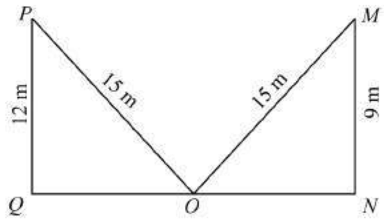
$$\frac{1}{\sqrt{3}} = \frac{150 - x}{60}$$

$$\therefore \quad (150 - x) = \frac{60}{\sqrt{3}} = 20\sqrt{3}$$

$$\therefore \quad x = 150 - 20\sqrt{3}$$

$$= 116 \text{ m (approx)}$$

28. (*b*)



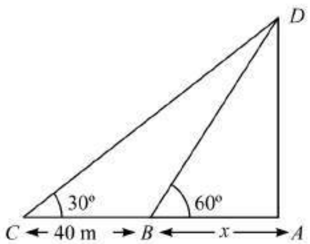
In ΔPOQ, $QO = \sqrt{15^2 - 12^2} = 9$ m

In ΔMON,

$$ON = \sqrt{15^2 - 9^2} = 12 \text{ m}$$

$\therefore$ $QN = QO + ON = 9 + 12 = 21$ m

29. (*b*)



Let AD be tree and AB be the river and its breadth be x m.

Then, $$\frac{AD}{AB} = \tan 60^\circ$$

$$\therefore \quad AD = x\sqrt{3} \text{ m} \qquad (1)$$

and, $$\frac{AD}{x + 40} = \tan 30^\circ$$

$$AD = \frac{(x + 40)}{\sqrt{3}} \qquad (2)$$

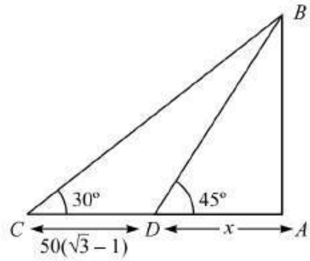
From Eqs. (1) and (2),

$$x\sqrt{3} = \frac{x + 40}{\sqrt{3}}$$

$$\Rightarrow \quad 3x = x + 40$$

$$\therefore \quad x = 20 \text{ m}$$

30. (*c*)



Distance travelled in 2 h,

$$CD = 50(\sqrt{3} - 1) \text{ m}$$

Let AB be the building and its height be h and AD be x.

$$\frac{h}{x} = \tan 45^\circ$$

$$\therefore \quad h = x \qquad (1)$$

$$\frac{h}{x + 50(\sqrt{3} - 1)} = \tan 30^\circ$$

$$\Rightarrow \quad h = \frac{h}{\sqrt{3}} + \frac{50(\sqrt{3} - 1)}{\sqrt{3}} \qquad (2)$$

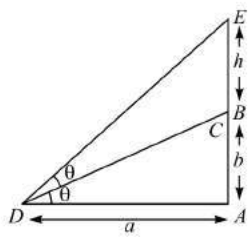
From Eqs. (1) and (2),

$$\Rightarrow \quad h - \frac{x}{\sqrt{3}} = \frac{50\sqrt{3} - 1}{\sqrt{3}}$$

$$\Rightarrow \quad h(\sqrt{3} - 1) = 50(\sqrt{3} - 1)$$

$$\Rightarrow \quad h = 50 \text{ m}$$

31. (*b*)



Let angle made by tower and flagstaff be θ and length of flagstaff be h.

Then, $$\frac{b}{a} = \tan\theta \qquad (1)$$

and, $$\frac{b + h}{a} = \tan 2\theta$$

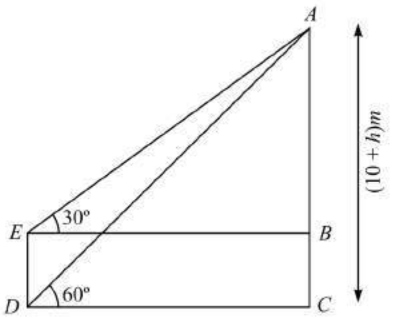
$$\therefore \quad \frac{b + h}{a} = \frac{2\tan}{1 - \tan^2\theta}$$

$$\Rightarrow \quad \frac{b+h}{a} = \frac{\frac{2b}{a}}{1-\left(\frac{b}{a}\right)^2}$$

$$\Rightarrow \quad b+h = \frac{2a^2b}{a^2-b^2}$$

$$\Rightarrow \quad h = \frac{a^2b+b^3}{a^2-b^2} = \frac{b(a^2+b^2)}{a^2-b^2}$$

32. (*b*) Let $AB = h$ m and $DC = EB = x$ m



Here, pole $DE = 10$ m

AC = tower = $10 + h$

In ΔAEB, we have

$$\tan 30^\circ = \frac{AB}{EB} = \frac{1}{\sqrt{3}} = \frac{h}{x} \qquad (1)$$

$$\Rightarrow \quad x = \sqrt{3}\,h$$

In ΔACD, we have

$$\tan 60^\circ = \frac{AC}{CD} = \frac{10+h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{10+h}{x}$$

$$\therefore \quad x = \frac{10+h}{\sqrt{3}} \qquad (2)$$

From Eqs. (1) and (2), we get

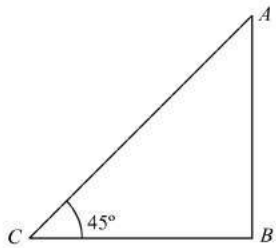
$$\sqrt{3}h = \frac{10+h}{\sqrt{3}}$$

$$\Rightarrow \quad 3h = 10 + h$$

$$\Rightarrow \quad h = 5 \text{ m}$$

$$\therefore \quad AC = 10 + h = 15 \text{ m}$$

33. (*b*)



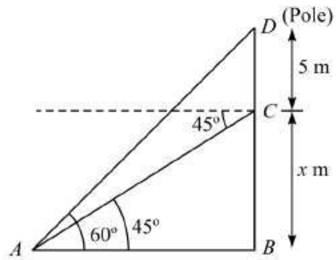
Here, CB = breadth of the river

AB = height of the tower

In ΔABC, we have $\tan 45^\circ = \frac{AB}{BC} = 1$

i.e., $AB = BC$

34. (*c*) Let the height of the tower be x m.



$$\tan 60^\circ = \frac{BD}{AB} = \frac{(5+x)}{AB} \qquad (1)$$

$$\tan 45^\circ = \frac{x}{AB} \qquad (2)$$

From Eq. (1),

$$AB \times \tan 60^\circ = (5 + x)$$

$$x \times \sqrt{3} = (5 + x)$$

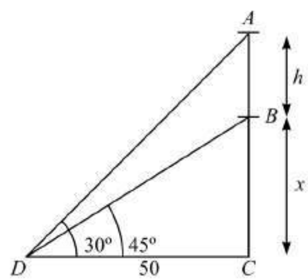
($\because AB = x$ from Eq. (2))

$$x(\sqrt{3}-1) = 5$$

$$x = \frac{5}{(\sqrt{3}-1)} \; \frac{5(\sqrt{3}+1)}{2}$$

$$\approx 6.83 \text{ m} \qquad \text{(approx)}$$

35. (*d*)



Let AB be the cliff of height h.

$BC = x$, $CD = 50$.

$\therefore \quad \dfrac{h+x}{50} = \tan 45^\circ = 1 \quad (1)$

and, $\quad \dfrac{x}{50} = \tan 30^\circ = \dfrac{1}{\sqrt{3}}$

$\therefore \quad x = \dfrac{50}{\sqrt{3}}$

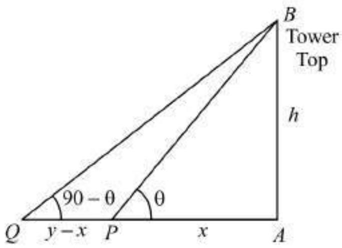
$\therefore$ (1) $\Rightarrow h + x = 50$

i.e., $\quad h = 50 - x$

$h = 50 - \dfrac{50}{\sqrt{3}}$

i.e., $\quad h = 50\left(1 - \dfrac{1}{\sqrt{3}}\right)$.

36. (*c*)



Let the height of the tower AB be h.

$\therefore \quad \dfrac{h}{x} = \tan\theta$ and $\dfrac{h}{y} = \cot\theta$

$\Rightarrow \quad h^2 = xy \quad (\because \tan\theta \times \cot\theta = 1)$

$\Rightarrow \quad h = \sqrt{xy}$.

37. (*b*) Let the height of the Qutab Minar be x m.

Now, according to the question,

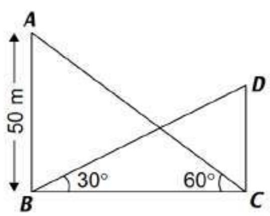
$$\frac{x}{150} = \frac{80}{120}$$

$\therefore \quad x = \dfrac{80 \times 150}{120} = 100$ m.

38. (*a*) Let AB be the tower and CD be the building.

In $\Delta\, ABC$,

$$\tan 60^\circ = \frac{AB}{BC}$$



$\Rightarrow \quad BC = \dfrac{AB}{\tan 60^\circ}$

$\Rightarrow \quad BC = \dfrac{50}{\sqrt{3}}$

Therefore, in $\Delta\, BCD$, we get

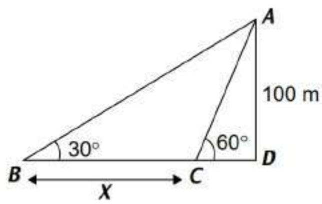
$\tan 30^\circ = \dfrac{CD}{BC}$

$\Rightarrow \quad CD = BC.\tan 30^\circ$

$\Rightarrow \quad CD = \dfrac{50}{\sqrt{3}}\dfrac{1}{\sqrt{3}} = \dfrac{50}{3} = 16\dfrac{2}{3}$ m

39. (*b*) Let the distance travelled by ship be x m.

Now, in ΔACD



$\tan 60^\circ = \dfrac{100}{CD} \quad \Rightarrow \quad CD = \dfrac{100}{\sqrt{3}}$

And in ΔABD

$\tan 30^\circ = \dfrac{100}{BD} \quad \Rightarrow \quad BD = 100\sqrt{3}$

$\Rightarrow \quad BC + CD = 100\sqrt{3}$

$\Rightarrow \quad x + \dfrac{100}{\sqrt{3}} = 100\sqrt{3}$

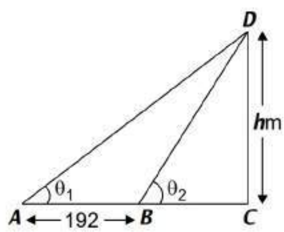
$\Rightarrow \quad x = 100\left(\sqrt{3} - \dfrac{1}{\sqrt{3}}\right)$

$\Rightarrow \quad x = 100 \times \dfrac{2}{\sqrt{3}} = \dfrac{200}{\sqrt{3}} = \dfrac{200}{1.732}$

$= 115.47$ m $= 115.5$ m

40. (*d*) Let height of ladder BD = x m

In ΔBCD,



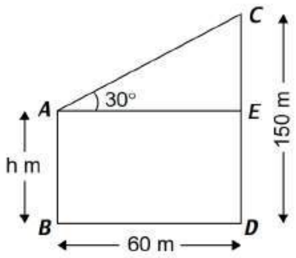
$$\sin 60° = \frac{2.7}{BD}$$

$$\Rightarrow \quad x = \frac{2.7}{\sqrt{3}/2} \quad \Rightarrow \quad x = \frac{5.4}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$x = 1.8\sqrt{3} = \frac{9\sqrt{3}}{5} \text{ m}$$

41. (*c*) Let the height of shorter tower be h m.

Let AB be shorter tower and CD the taller one.



$$\text{In AEC, } \tan 30 = \frac{CE}{AE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{150-h}{60}$$

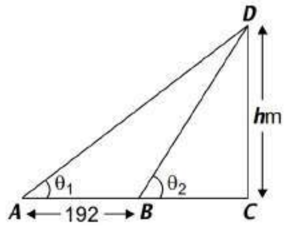
$$\Rightarrow \quad 150 - h = \frac{60}{\sqrt{3}}$$

$$\Rightarrow \quad h = 150 - 20\sqrt{3} \approx 116 \text{ m}$$

42. (*a*) Let A is the point where man can start to move up on a slope. On point D angle of depression is 30° and after going C m. the angle of depression is 45° from the base of slope on point M. After reaching point M, this angle is equal to the angle of inclination from the base of the slope.

Hence, required angle = 45°

43. (*c*) Let the height of tower be h m



We are given,

$$\tan\theta_1 = \frac{h}{192+BC}$$

$$\Rightarrow \quad \frac{5}{12} = \frac{h}{192+BC}$$

$$\Rightarrow \quad \frac{12}{5} = \frac{192+BC}{h} \text{ and } \tan\theta_2 = \frac{h}{BC}$$

$$\Rightarrow \quad \frac{BC}{h} = \frac{4}{3}$$

On subtracting Eq. (ii) from Eq. (i), we get

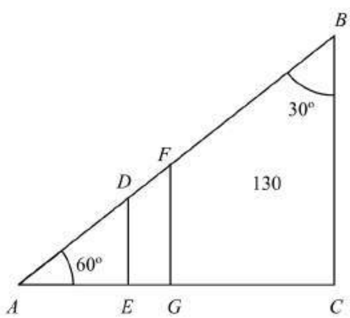
$$\frac{192}{h} = \frac{12}{5} - \frac{4}{3}$$

$$= \frac{36-20}{15} = \frac{16}{5}$$

$$h = \frac{192 \times 15}{16} = 180 \text{ m}$$

Difficulty Level-2 (Based on Memory)

1. (*c*) Let AB be the ladder inclined at an angle of 30° with the wall BC.



After 1 second the man will be at D such that $AD = 2$ m.

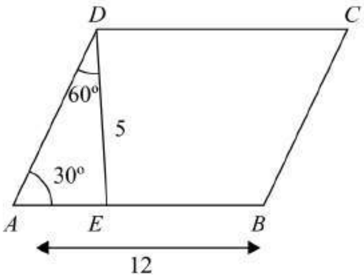
$$\therefore \qquad AE = AD \cos 60° = 2 \times \frac{1}{2} = 1\text{m}$$

Similarly after 2 s, the man will be at F such that AF = 4 m.

$$\therefore \qquad AG = 2 \text{ m}$$

Thus after every second, the man is approaching the wall a distance equal to 1 m, i.e., @ 1 m/s.

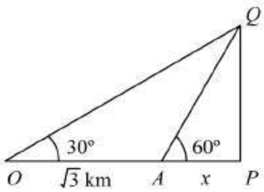
2. (*a*) Area of the parallelogram = $AB \times DE = 60$



$\because$ $AB = 12$, therefore $DE = 5$

Let $AD = x$. Therefore, $\frac{5}{x} = \cos 60° = \frac{1}{2} \Rightarrow x = 10.$

3. (*a*) $\frac{h}{x} = \tan 60 = \sqrt{3}$. $\therefore$ $h = \sqrt{3}\,x.$



Also, $\frac{h}{\sqrt{3}+x} = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore$ $\sqrt{3}\,h = \sqrt{3} + x$

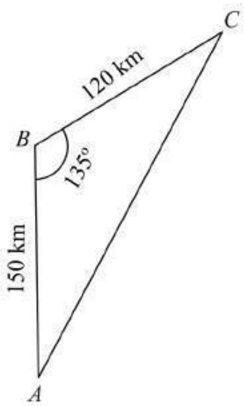
$\therefore$ $\sqrt{3}\,(\sqrt{3}\,x) = \sqrt{3} + x$ or $3x - x = \sqrt{3}$

$\therefore$ $2x = \sqrt{3}$

$\therefore$ $x = \frac{\sqrt{3}}{2}$

$\therefore$ $h = \sqrt{3}\,.\,\frac{\sqrt{3}}{2} = \frac{3}{2}$ km.

4. (*a*) Since $AC^2 + BC^2 - 2AB \times DC \times \cos(135°)$

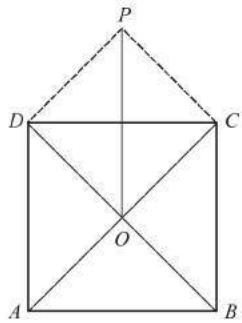


$\Rightarrow AC = \sqrt{(150)^2 + (120)^2 + 150 \times 120 \times 2 \times \sin 45°}$

$\approx$ 250 km.

5. (*d*) Given $\angle APO + \angle CPO = 90°$

$\Rightarrow$ $\angle APO = \angle CPO = 45° = \angle BPO = \angle DPO$



$\cos \angle APB = \cos\theta = \frac{AP^2 + BP^2 - AB^2}{2AP \cdot BP}$

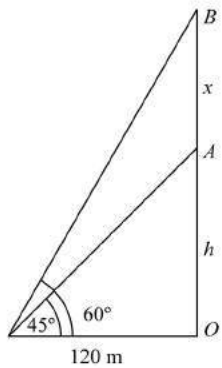
$= \frac{AO^2 + OP^2 + BO^2 + BP^2 - AB^2}{2AP \cdot BP}$

$= \frac{OP^2}{AP \cdot BP} = \frac{OP}{AP} \cdot \frac{OP}{BP}$

$= \sin 45° \times \sin 45° = \frac{1}{2}$.

6. (*b*) $\frac{h+x}{120} = \tan 60° = \sqrt{3}$

$h + x = \sqrt{3}\,(120).$



Also, $\frac{h}{120} = \tan 45° = 1.$

$\therefore$ $h = 120$ m

$\therefore$ $120 + x = 120\sqrt{3}$

$\therefore$ $x = 120\,(\sqrt{3} - 1)$ m.

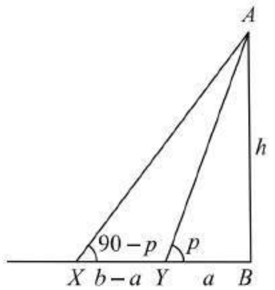
7. (*c*) Let h be the height of the tower AB.

$$\therefore \quad \frac{h}{a} = \tan p$$

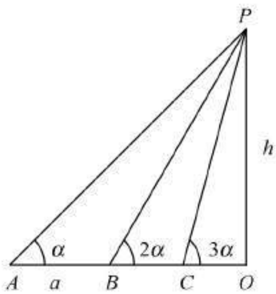
$$\frac{h}{b} = \tan(90 - p) = \cot p$$

$$\therefore \quad \frac{h}{a} \times \frac{h}{b} = \tan p \times \cot p = 1$$

$$\Rightarrow \quad h^2 = ab \Rightarrow h = \sqrt{ab}\,.$$



8. (*c*) Let OP be a vertical tower. The elevation of top P from A, B, C are α, 2α, 3α, respectively. $\angle APB = 2\alpha - \alpha = \angle PAB$.

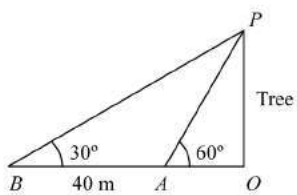


$$\frac{OP}{BP} = \sin 2\alpha$$

$$\therefore \quad OP = BP \sin 2\alpha = a \sin 2\alpha.$$

Thus, height of the tower = $a \sin 2\alpha$.

9. (*c*) Let OA denote the breadth of the river.



$$\frac{OP}{OA} = \tan 60° = \sqrt{3}$$

$$\therefore \quad OP = \sqrt{3}\, OA.$$

$$\text{Also, } \frac{OP}{OA + 40} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\therefore \quad OA + 40 = \sqrt{3}\, OP = \sqrt{3}\,(OA) = 3\, OA.$$

$$\therefore \quad 2OA = 40 \Rightarrow OA = 20 \text{ m}.$$

10. (*a*) Let $PQ = x$ m denote the tower so that $\angle PAQ = 30°$. Let $BA = h$ m

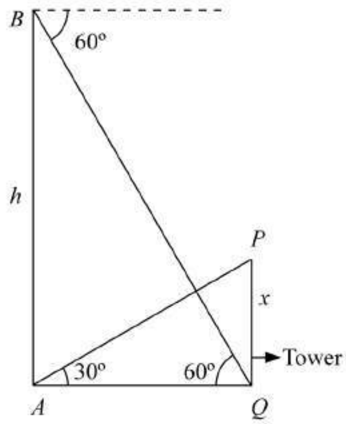
$$\therefore \quad \angle BQA = 60°.$$

$$\text{Now, } \frac{h}{AQ} = \tan 60°$$

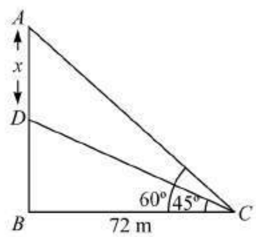
$$= \sqrt{3}\,.$$

$$\therefore \quad AQ = \frac{h}{\sqrt{3}}$$

$$= h \cot 60°.$$



11. (*b*)



$$\text{In } \Delta BCD, \tan 45° = \frac{BD}{BC}$$

$$\Rightarrow \quad BD = 72$$

$$\text{In } \Delta ACB, \tan 60° = \frac{72 + x}{72}$$

$\Rightarrow \quad 72\sqrt{3} = 72 + x$

$\Rightarrow \quad x = 72(\sqrt{3} - 1)$

$= 72 \times 0.732 = 52.7$ m.

12. (*b*) The distance of the foot from each vertex = h cotα.

$\therefore$ The foot is at the circumcentre of the triangle.

$\therefore \quad R = h \cot \alpha$

$\therefore \quad h = R \tan \alpha$

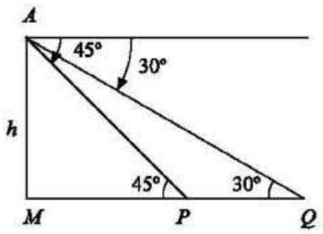
$$= \frac{a}{2\sin\alpha} \tan \alpha$$

$$= \frac{a}{2} \tan\alpha \cdot \text{cosec}\,\alpha.$$

13. (*c*) Let the boats be at P, Q

So that $\quad PQ = 60$ m

Let MA be the light house.



Let, $\quad h = MA$

Then, $\quad \frac{h}{MP} = \tan 45° = 1$

$\therefore \quad h = MP$

Again, $\frac{h}{MP + 60} = \tan 30° = \frac{1}{\sqrt{3}}$

$\therefore \quad MP + 60 = h$

or, $\quad h + 60 = \sqrt{3}\, h$

$\therefore \; (\sqrt{3} - 1)h = 60$

$\therefore \quad h = \frac{60}{\sqrt{3} - 1}$

$$= \frac{60(\sqrt{3} + 1)}{2}$$

$= 30(\sqrt{3} + 1)$ m

14. (*a*, *d*) Option (*a*) is correct as height of tower is

$$q = \frac{p \tan\theta}{\tan\phi - \tan\theta}$$

Option (*b*) is wrong, option (*c*) is wrong, option (*d*) is correct.

15. (*d*) Suppose AB be ladder of 25 m placed against a wall AO at 7 m away from its foot 'O'.

In right angled $\Delta\, AOB$

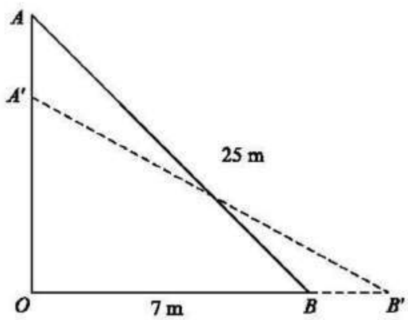
$AO^2 = AB^2 - OB^2$

$= (25)^2 - (7)^2 = 625 - 49$

$AO^2 = 576$

$\therefore \quad AO = 24$

given that, foot of ladder B be drawn out to B's such that $OA' = 12$ m



Now, in right-angled $\Delta\, A'OB'$

$(OB')^2 = (A'B')^2 - (OA')^2$

$\Rightarrow \quad (OB')^2 = (25)^2 - (12)^2$

$= 625 - 144$

$= 481$

$\therefore \quad OB' = \sqrt{481}$

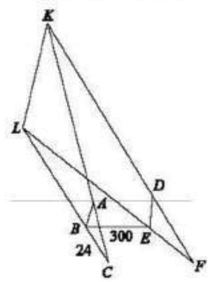
$= 21.8$

$\therefore$ Required distance = (21.8 – 7) m

= 14.8 m.

16. (*d*) $\quad AB = DE = 6$

KL represents the light house. Initially, the man is at BA. His shadow is BC. Then, he walks 300 m to the East. He is at DE. His shadow is EF.



From similar triangles

$$\frac{KL}{AB} = \frac{LC}{BC} \text{ and } \frac{KL}{DE} = \frac{LF}{EF}$$

As $\quad AB = DE, \; \frac{LC}{BC} = \frac{LF}{BF}$

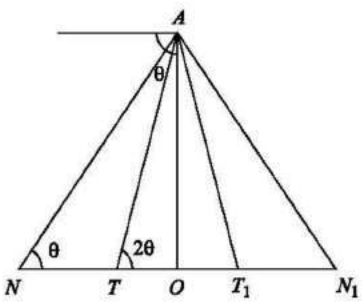
LC is 4 parts $(4x)$, LF is $5x$ or LB is 4 parts $(4y)$ and LE is $5Y$.

As $DLBE$ is right angled at B and E = 300, it follows that LB = 400, LE = 500.

$$KL = \frac{LC}{BC} \times AB = \frac{424}{24} \times 6 = 106$$

17. (*c*) The horizontal distance of the man in the second position from the light house LE = 500 m

18. (*d*)



The above figure gives the front view of the tent where OA is the pole (with O being the bottom), T and T_1 are a pair of diametrically opposite points on the surface of the tent (on the base), N and N_1 are the supporting nails which are collinear with T, O, T_1.

Given that $AO = \frac{3}{2}$ m and $NO = 3\,(TO)$. Let TO be x m and let $\angle ANO = \theta$

$\Rightarrow \quad \angle ATO = 2\theta$

$$\tan\theta = \frac{\frac{3}{2}}{x} = \frac{1}{2x} \text{ and } 2\theta = \frac{3}{2x}$$

$$\Rightarrow \quad \frac{\frac{2}{2x}}{1-\frac{1}{4x^2}} = \frac{3}{2x}$$

$$\Rightarrow \quad \frac{4x}{4x^2-1} = \frac{3}{2x}\left(\tan 2\theta = \frac{2-\tan\theta}{1\tan^2\theta}\right)$$

$$\Rightarrow \quad 8x^2 = 12x^2 - 3$$

$$\Rightarrow \quad x = \frac{\sqrt{3}}{2}$$

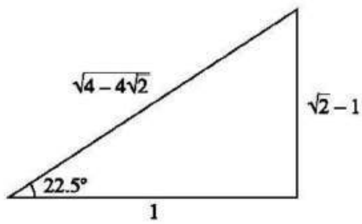
$$\therefore \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta = 30^\circ$$

$$\therefore \quad AN = \frac{AO}{\sin\theta} = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$$

19. (*b*) If the distance is 1 m, the length of the ladder would be $\sqrt{4-2\sqrt{2}}$

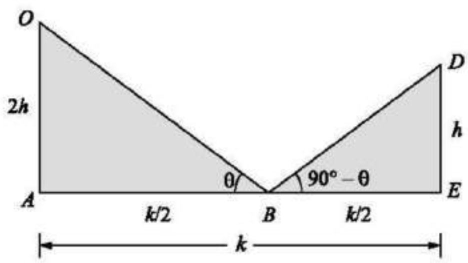
$(\because \tan 22.5^\circ = \sqrt{2}-1)$



$\therefore$ If the distance is 2 m, the length would be

$$2\sqrt{4-4\sqrt{2}} = \sqrt{16-8\sqrt{2}}.$$

20. (*c*)



In ΔABC, $\tan\theta = \frac{2h}{k/2} = \frac{4h}{k}$ (1)

and ΔEBD, $\tan(90^\circ - \theta) = \cot\theta = \frac{h}{k/2}$

$\therefore \quad \tan\theta = \frac{k}{2h}$ (2)

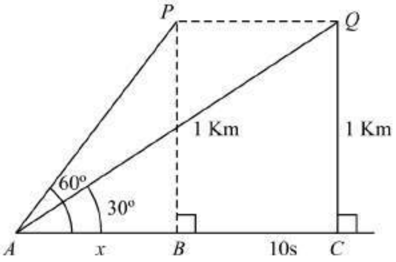
From Eqs. (1) and (2),

$$\frac{4h}{k} = \frac{k}{2h}$$

$$\Rightarrow \quad 8h^2 = k^2$$

$$\Rightarrow \quad h = \sqrt{\frac{k^2}{8}} = \frac{k}{2\sqrt{2}} \text{ m.}$$

21. (*b*) Let the speed of the aeroplane be *s*.



In ΔABP,

$$\tan 60^\circ = \frac{PB}{AB}$$

$$\Rightarrow \quad AB = \frac{1}{\sqrt{3}} = x \qquad (1)$$

In ΔAQC,

$$\tan 30^\circ = \frac{QC}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1}{x + 10\,s} \qquad (2)$$

Using Eqs. (1) and (2),

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1}{\frac{1}{\sqrt{3}} + 10\,s}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} + 10\,s = \sqrt{3}$$

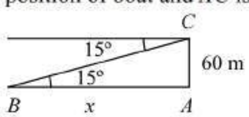
$$\Rightarrow \quad 10\,s = \sqrt{3} - \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad 10\,s = \frac{3-1}{\sqrt{3}}$$

$$\Rightarrow \quad s = \frac{2}{\sqrt{3}} \times \frac{1}{10} = \frac{1}{5\sqrt{3}} \text{ Km/s}$$

$$\Rightarrow \quad s = \frac{60 \times 60}{5\sqrt{3}} = \frac{720}{\sqrt{3}} \text{ Km/h.}$$

22. (*b*) Here, *B* is the position of boat and *AC* is light house.



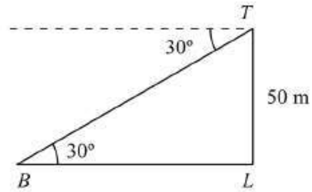
Now, $\dfrac{AC}{x} = \tan 15^\circ = \tan(45^\circ - 30^\circ)$

$$= \frac{1 - \tan 30^\circ}{1 + \tan 30^\circ} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$$

$$\therefore \quad x = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right) 60 \text{ m.}$$

23. (*c*) $\tan 30^\circ = \dfrac{TL}{BL} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{50}{BL}$

$$\therefore \quad BL = 50\sqrt{3} \text{ m.}$$



24. (*e*) $x = \sqrt{30^2 - 26^2} = \sqrt{224} \approx 15\text{m}$

$$\therefore \cos 2\theta = \frac{15}{30} = \frac{1}{2}$$

$$\Rightarrow \quad 2\theta = 60^\circ \Rightarrow \quad \theta = 30^\circ$$

$$\cos 30^\circ = \frac{y}{30} \Rightarrow \frac{\sqrt{3}}{2} = \frac{y}{30} \Rightarrow y = 15\sqrt{3}\text{m}$$

$$AB = y - x = 15\sqrt{3} - 15 = 10.98\text{m}$$