Quadratic Equation

OLYMPIAD EXCELLENCE BOOK **MATHEMATICS**

QUESTIONS

1. Identify the quadratic equation from the following.

(a) $m+\frac{1}{m}=1, m\neq 0$

(b) $m^2+\frac{1}{m}=1, m\neq 0$

(c) $x^2-\frac{1}{x}=1, x\neq 0$

(d) $x^2+2\sqrt{x}-1=0$

2. If $3.2^{2x+1}-5.2^{x+2}+16=0$ and x is an integer, find the value of x.

(a) 1 (b) 2 (c) 3 (d) 4

3. Which of the following statements is correct?

(a) $x=2$ is a root of $2x^2+5x+1=0$

(b) $x=3$ is not a root of $x^2+3x-5=0$

(c) $x=-1$ is not a root of $5x^2-x-1=0$

(d) $x=-\frac{2}{5}$ is not a root of $x^2-\frac{8x}{5}-\frac{4}{5}=0$

4. Maximum value of $p(x)=-3x^2+5x-12$ is

(a) $\frac{15}{12}$ (b) $-9\frac{11}{12}$ (c) $-\frac{5}{6}$ (d) $\frac{5}{6}$

5. Find the value of 'a' for which $m=\frac{1}{\sqrt{3}}$ is a root of the equation. $am^2+\left(\sqrt{3}-\sqrt{2}\right)m-1=0$.

(a) $\sqrt{2}$ (b) 2 (c) $\sqrt{6}$ (d) 5

6. Which of the following equations has no real roots?

(a) $x^2-4x+3\sqrt{2}=0$

(b) $x^2+2x-6\sqrt{2}=0$

(c) $x^2-4x-3\sqrt{2}=0$

(d) $3x^2-4\sqrt{3x}-4=0$

7. The sides of two square plots are $(2x-1)m$ and $(5x+4)m$. The area of the second square plot is 9 times the area of the first square plot. Find the side of the larger plot.

(a) 50 m (b) 20 m (c) 26 m (d) 39 m

8. Identify the factors of $5x^2=4x-\frac{4}{5}$

(a) $\frac{1}{5},\frac{1}{5}$ (b) $\frac{-3}{2},\frac{3}{2}$ (c) $\frac{2}{5},\frac{2}{5}$ (d) $5,-5$

9. The age of a man is the square of his son's age. One year ago, the man's age was eight times the age of his son. What is the present age of the man?

(a) 60 years (b) 49 years (c) 30 years (d) 40 years

10. Following graphs can be drawn to solve the quadratic equation $4x^2+6x-3=0$. Choose, the correct method?

(a) $y=x^2, 3x-2y-3=0$ (b) $y=4x^2, 6x-2y-3=0$

(c) $y=3x^2, 6x-y-3=0$ (d) $y=2x^2, 6x+2y-3=0$

11. Find two consecutive even numbers whose product is double that of the greater number.

(a) 2, 5 (b) 8, 10 (c) 2, 4 (d) 10, 12

12. Which of the following are the roots of the equation $|x|^2+|x|-12=0$?

(i) 1 (ii) – 1 (iii) 3 (iv) – 3

(a) Both (i) and (ii) (b) Both (iii) and (iv)

(c) (i), (ii), (iii) and (iv) (d) None of these

13. The length and breadth of a rectangle are $(3k+1)$ cm and $(2k-1)$ cm respectively. Find the perimeter of the rectangle if its area is $144\ cm^2$.

(a) 50 cm (b) 20 cm (c) 30 cm (d) 40 cm

14. If the roots of the equation $3ax^2+2bx+c=0$ are in the ratio 2 : 5, then

(a) $8ac=25b^3$ (b) $6ac=19b^2$ (c) $40b^2=75ac$ (d) $8b^2=25ac$

15. The sum of squares of two consecutive positive even integers is 340. Find them.

(a) 12, 14 (b) 4, 6 (c) 6, 8 (d) 10, 12

16. Find the relationship between the coefficients of the equation $rx^2+sx+1=0$, such that $\alpha:\beta=3:4$ where α,β are roots of equation.

(a) $12s^2=49rt$ (b) $22s^2=-49rt$ (c) $59s^2=12rt$ (d) $59t^2=12rs$

17. If $(p^2-q^2)u^2+(q^2-r^2)u+r^2-p^2=0$ and $(p^2-q^2)v^2+(r^2-p^2)v+q^2-r^2=0$ have a common root for p, q, r $\in$ R and u, v being variables in the respective equations, find the common root.

(a) – 3 (b) 1 (c) 3 (d) – 6

18. Which of the following are the roots of $|y|^2 - |y| - 20 = 0$?

(a) 4 (b) –5 (c) 6 (D) ±5

19. Find the value of $\sqrt{20+\sqrt{20+\sqrt{20+....upto\infty}}}$

(a) 5 (b) –4 (c) Either (a) or (b) (d) 6

20. The graphs of $\frac{1}{2}y = x^2$ and $y = rx + t$ intersect at two points (2, 8) and (6, 72). Find the quadratic equation in x whose roots are $r+2$ and $\frac{t}{4}-1$

(a) $2x^2 + 6x - 123 = 0$ (b) $3x^2 + 33x + 378 = 0$

(c) $x^2 - 10x - 121 = 0$ (d) $x^2 - 11x - 126 = 0$

21. The equation $9y^2(\text{m}+3) + 6(\text{m}-3)\text{y} + (\text{m}+3) = 0$, where m is real, has real roots. Which of the following is true?

(a) $m = 0$ (b) $m < 0$ (c) $m \leq 0$ (d) $m \geq 0$

22. Find the maximum or minimum value of the quadratic expression, $x^2 - 5x + 8$, which ever exists.

(a) $y = \frac{1}{2}$ (b) $y = \frac{7}{4}$

(c) $y = 0$ (d) No solution over real numbers

23. Find the roots of the equation $\frac{1}{x} - \frac{1}{x-a} = \frac{1}{b} - \frac{1}{b-a}$ where $a \neq 0$.

(a) $a^3 - b^3$ and $(\text{a}-\text{b})$ (b) b and (a– b)

(c) $a^2 - b$ and $a^2 + b$ (d) a and b

24. If the roots of the equation $2x^2 + 3x + 17 = 0$ are in the ratio p : q, then find the value of $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}}$.

(a) $\frac{\pm 3}{\sqrt{34}}$ (b) $\pm 3\sqrt{34}$ (c) $\pm\frac{3\sqrt{6}}{8}$ (d) $\pm 6\sqrt{17}$

25. If the roots of the quadratic equation $x^2 - kx + k^2 - 3 = 0$ are real, then the range of the values of k is _________.

(a) $[-2, 2]$ (b) $[-\infty, -2] \cup [2, \infty]$

(c) $[-3, 3]$ (d) $[-\infty, -3] \cup [3, \infty]$

26. When is a real number 'a' called the zero of the polynomial f(x)?

(a) $f(0)=a$ (b) $f(a)=f(0)$ (c) $f(a)=0$ (d) $f(\pm a)=\pm a$

27. If a 2nd degree equation $\left(b^2-a^2\right)x^2+\left(c^2-b^2\right)x+\left(a^2-c^2\right)=0$ has equal roots, then which of the following conditions will necessarily be true?

(a) $\sum a^2=a^2b^2c^2$ (b) $b^2+c^2=a^2$

(c) $b^2+c^2=2a^2$ (d) $(a+b+c)^2=ab+bc+ca$

28. What are the values of x which satisfy the equation $\sqrt{3x-2}+\dfrac{1}{\sqrt{3x-2}}=\dfrac{17}{4}$?

(a) $6,\dfrac{11}{16}$ (b) $4,\dfrac{11}{3}$ (c) $\dfrac{11}{9},\dfrac{17}{4}$ (d) $13,\dfrac{11}{9}$

29. If the roots of the quadratic equation $ax^2+bx+c=0$ are α and β, then the equation whose roots are α^2 and β^2 is

(a) $a^2x^2-\left(b^2-2ac\right)x+c^2=0$ (b) $\left(a^2-r\right)x^2+\left(b^2-r\right)x+c^2r=0$

(c) $\left(a^2-ac\right)x^2+\left(b^2+2ac\right)x+c^2=0$ (d) $2a^2x^2-\left(b^2+2ac\right)x+2c^2=0$

30. Find the roots of the equation $l^2\left(m^2-n^2\right)x^2+m^2\left(n^2-l^2\right)x+n^2\left(l^2-m^2\right)=0$

(a) $\dfrac{n^2\left(l^2-m^2\right)}{l^2\left(m^2-n^2\right)},l$ (b) $\dfrac{-m^2\left(l^2-m^2+n^2\right)}{l^2\left(m^2-n^2\right)},\dfrac{1}{2}$

(c) $\dfrac{n^2\left(l^2+m^2+n^2\right)}{m^2\left(m^2-n^2\right)},1$ (d) $\dfrac{-m^2\left(l^2+n^2\right)}{mn\left(m^2-n^2\right)},\dfrac{1}{2}$

31. If $a-b,b-c$ are the roots of $ax^2+bx+c=0$, then find the value of $\dfrac{(a-b)(b-c)}{2(c-a)}$.

(a) $\dfrac{b}{c}$ (b) $\dfrac{c}{2b}$ (c) $\dfrac{ab}{c}$ (d) $\dfrac{bc}{a}$

32. If a and b can take value 1, 2, 5, 6, then the number of the equations of the form $ax^2+bx+1=0$ having real roots is

(a) 11 (b) 9 (c) 15 (d) None of these

33. If α, β are the rots of the equation $ax^2 - 2bx + c = 0$ then $\alpha^3\beta^3 + \alpha^2\beta^3 + \alpha^3\beta^2$

(a) $\frac{-c^2(2b-c)}{a^3}$ (b) $\frac{2bc^3}{a^2}$ (c) $\frac{c^3b^3}{a^6}$ (d) $\frac{b^2(2c+3a)}{a^3}$

34. If the product of the roots of $2x^2 - 7kx + 3e^{3\log k} - 1 = 0$ is 40, then the sum of the roots is

(a) 80 (b) $\frac{7}{2}$ (c) $\frac{21}{2}$ (d) None of these

35. If $sin\theta$, $cos\theta$ are roots of the equation $ax^2 - 2bx + 3c = 0$, then

(a) $4b^2 - a^2 - 6ac = 0$ (b) $a^2 + b^2 + c^2 + abc = 0$

(c) $a^2 + b^2 + 2abc - c^2 = 0$ (d) $a^2 + b^2 - 2ac = 0$

36. tan A, tan B are roots of $3x^2 - 5x + 6 = 0$. then $sin^2(A+B)$

(a) $\frac{14}{15}$ (b) $\frac{25}{34}$ (c) $\frac{13}{25}$ (d) $\frac{11}{46}$

37. If the sum of the roots of $ax^2 + 2bx + 3c = 0$ is equal to the sum of the squares of their reciprocals, then $9bc^2 + 2ab^2$

(a) $18abc$ (b) $3a^2c$ (c) $ab + bc + 2ca$ (d) $18a^2c + 7b^3$

38. Find the value of $\sqrt{3+\sqrt{3+\sqrt{3+--------}}}$

(a) $\frac{1 \pm \sqrt{13}}{2}$ (b) $\frac{1 \pm \sqrt{3i}}{2}$ (c) $\frac{1 \pm \sqrt{6}}{2}$ (d) $\frac{1 \pm \sqrt{3}}{2}$

39. If α and β are the root of the equation $2x^2 - 3x - 7 = 0$ then the value of $\alpha\beta^2 + \alpha^2\beta + \alpha\beta$ are.

(a) 6 (b) $\frac{-35}{4}$ (c) $\frac{3}{4}$ (d) $\frac{-15}{8}$

40. Which of the following quadratic equations have equal roots?

(a) $2x^2 - 3x + 5 = 0$ (b) $3x^2 - 4\sqrt{3x} + 4 = 0$

(c) $2x^2 - 6x + 3 = 0$ (d) $x + \frac{1}{x} = 1$

ANSWER - KEY				
1. A	**2.** A	**3.** B	**4.** B	**5.** C
6. A	**7.** D	**8.** C	**9.** B	**10.** D
11. C	**12.** B	**13.** A	**14.** C	**15.** A
16. A	**17.** B	**18.** D	**19.** A	**20.** D
21. C	**22.** B	**23.** B	**24.** A	**25.** A
26. A	**27.** C	**28.** A	**29.** A	**30.** A
31. B	**32.** B	**33.** A	**34.** C	**35.** A
36. B	**37.** B	**38.** A	**39.** B	**40.** B

SOLUTIONS

1. (A): Upon simplifying $m+\frac{1}{m}=1$

$$\frac{m^2+1}{m}=1 \Rightarrow m^2+1=m \Rightarrow m^2-m+1=0$$

Which is of the form $ax^2+bx+c=0$

2. (A): Not Available

3. (B) Not Available

4. (B): $-3x^2+5x-12$

$$-3\left[x^2\frac{-5}{3}x+4\right]=-3\left[x^2-2.\frac{5}{6}x+\left(\frac{5}{6}\right)^2-\left(\frac{5}{6}\right)^2+4\right]=-3\left[\left\{x-\frac{5}{6}\right\}^2+4-\frac{25}{36}\right]=-3\left[\left\{x-\frac{5}{6}\right\}^2+\frac{119}{36}\right]$$

Maximum value is when $\left\{x-\frac{5}{6}\right\}^2$, square term = 0 $\Rightarrow$ Maximum value $=-3\times\frac{119}{36}$

5. (C): Put $m=\frac{1}{\sqrt{3}} \Rightarrow \frac{a}{3}+\left(\sqrt{3}-\sqrt{2}\right)\frac{1}{\sqrt{3}}-1=0$

$$\Rightarrow a+\left(3-\sqrt{6}\right)-3=0$$

6. (A): **Aliter:** By physical inspection also, one can arrive at the answer:

In (B), (C) and (D) 'c' term is negative which make 4ac as positive & hence b^2-4ac as positive. $\Rightarrow$ (B), (C), (D) cannot be correct options.

7. (D): $(5x+4)^2=9(2x-1)^2 \Rightarrow 5x+4=\pm3(2x-1)$

$\Rightarrow 5x+4+6x-3=0$

Or $5x+4+6x+3=0$

$\Rightarrow 11x=-1$

Or $5x+4-6x+3=0 \Rightarrow -x+7=0 \Rightarrow x=7$

8. (C): Solve the quadratic

9. (B): Let ages be x^2 and $x:(x^2-1)=8(x-1)$

$\Rightarrow x^2-8x+7=0 \Rightarrow (x-1)(x-7)=0$

$\Rightarrow x=1,7$

$x=7$, $x^2=49$

10. (D): Take 6x – 3 on other side: $4x^2+6x-3=0$ which can be written as $2x^2=-\left(3x-\frac{3}{2}\right)$

$\therefore y=2x^2$ and $y=-\left(3x-\frac{3}{2}\right)$ will intersect to give roots.

11. (C): Let even nos. be $n, n+2 \Rightarrow n(n+2)$

$=2(n+2)$ and $n=-2 \Rightarrow n+2=4$ and $n+2=0$

12. (B): (i) Put [x] = y and frame the equation and solve.

13. (A): Form $eq^{ns}.(3k+1)(2k-1=144)$ and find k. Perimeter $=2\{(3k+1)+(2k-1)\}$

14. (C): If the roots of $ax^2+bx+c=0$ are in the ratio m : n, then $(m+n)^2 a.c=mnb^2$. Use this concept to solve the problem.

15. (A): $n^2+(n+2)^2=340 \Rightarrow 2n^2+4n+4$

$=340 \Rightarrow n^2+2n-168=0;$ further solve.

16. (A): Not Available

17. (B): $\left(p^2-q^2\right)u^2+\left(q^2-r^2\right)u+\left(r^2-p^2\right)=0$

The sum of the coefficients

$\left(p^2-q^2\right)u^2+\left(q^2-r^2\right)u+\left(r^2-p^2\right)=0$

$\therefore u=1$ is a root of Eq. (1)

$\left(p^2-q^2\right)v^2+\left(r^2-p^2\right)v+q^2-r^2=0$

$p^2-q^2+r^2-p^2+q^2-r^2=0$

$\therefore v=1$ is a root of Eq. (2)

$\therefore 1$ is the common root of Eq. (1) and (2).

18. (D): $|y|^2-|y|-20=0$

$(|y|-5)(|y|+4)=0$

$|y|=5$ or -4

But $|y|$ must be non - negative.

$\therefore |y|=5$, i.e., $y=\pm 5$

19. (A): $x = \sqrt{20+x}$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

$x = 5$ or -4

But x must be positive.

$\therefore x = 5$

20. (D): At (2, 8) and (6, 72), $y = 2x^2 = rx + t$

8 = 2r + t and 72 = 6r + t.

Solving for r and t, r = 16 and t = – 24.

The required equation is that whose roots are 18 and – 7.

Sum of its roots = 11

Product of its roots = – 126

$\therefore$ The required equation is $x^2 - 11x - 126 = 0$

21. (C): Discriminant

$\left(6(m-3)\right)^2 - 4\left[9(m+3)(m+3)\right]$

$= 36\left[(m-3)^2 - (m+3)^2\right]$

$= 36\left[\left(m^2 - 6m + 9\right) - \left(m^2 + 6m + 9\right)\right]$

$= 36(-12m)$. This must be non - negative for the roots to be real.

$\Rightarrow m < 0$

22. (B): Use the formula and check the coefficient of x^2.

$f(x) = x^2 - 5x + \frac{25}{4} + 8 - \frac{25}{4} = \left(x - \frac{5}{2}\right)^2 + \frac{7}{4}$

$\therefore$ min value $= \frac{7}{4}$

23. (B): Take LCM and form quadratic eq^n and solve.

24. (A): Not Available

25. (A): Discriminant

$\geq 0 \Rightarrow 4k^2 - 4k^2 + 12 \geq 0 \Rightarrow k^2 - 4 \leq 0$

26. (A): Let the roots be α and $\alpha+1$.

$\alpha+(\alpha+1)=$p, i.e., $2\alpha+1=p$

$(\alpha)(\alpha+1)=\frac{q}{1}=q$

27. (C): Trivial Solution is got by observance, put x = 1 and equation is satisfied. Hence x = 1 is a root: According to question, other root is also 1

$\therefore$ sum of roots $=1+1=2=\frac{-\left(c^2-b^2\right)}{\left(b^2-a^2\right)}$

$\Rightarrow 2b^2-2a^2=b^2-c^2 \Rightarrow b^2+c^2=2a^2$

Product of roots $=1\times 1=1=\frac{a^2-c^2}{b^2-a^2}\Rightarrow$ again $b^2+c^2=2a^2$

28. (A): Take square on both sides and simplify

29. (A): Eq^n with roots α^2,β^2 is:

$x^2\left(\alpha^2+\beta^2\right)x+\alpha^2\beta^2=0$

$\Rightarrow x^2-\left\{(\alpha+\beta)^2-2\alpha\beta\right\}x+\alpha^2\beta^2=0$

$\Rightarrow x^2-\left\{\frac{b^2}{a^2}-\frac{2c}{a}\right\}x+\frac{c^2}{a^2}=0$

30. (A): In the equation $ax^2+bx+c=0$ when $a+b+c=0$, then the roots are 1 and $\frac{c}{a}$.

Use the following justification and compare:

here, $a=l^2\left(m^2-n^2\right);b=m^2\left(n^2-l^2\right)$ and $c=n^2\left(l^2-m^2\right)$

31. (B): $(\text{a}-\text{b})(\text{b}-\text{c})=$product of the roots $=\frac{c}{a}$

Also $(\text{c}-\text{a})=-\left[(\text{a}-\text{b})+(\text{b}-\text{c})\right]$

$=-$ sum of roots $=\frac{b}{a}$

$\Rightarrow \frac{(\text{a}-\text{b})(\text{b}-\text{c})}{2(\text{c}-\text{a})}=\frac{\frac{c}{a}}{\frac{2b}{a}}=\frac{c}{2b}$

32. (B): This will happen when $b^2 - 4a \geq 0$

For (a, b) we have pairs

~~(1, 1)~~ (1, 2) ✓ (1, 5) ✓ (1, 6) ✓
~~(2, 1)~~ ~~(2, 2)~~ (2, 5) ✓ (2, 6) ✓
~~(5, 1)~~ ~~(5, 2)~~ (5, 5) ✓ (5, 6) ✓
~~(6, 1)~~ ~~(6, 2)~~ (6, 5) ✓ (6, 6) ✓

$b^2 - 4a < 0$

33. (A): $\alpha^3\beta^3 + \alpha^2\beta^3 + \alpha^3\beta^2 = (\alpha\beta)^3 + \alpha^2\beta^2(\beta + \alpha)$

$$= \frac{c^3}{a^3} + \frac{c^2}{a^2}\left(\frac{-2b}{a}\right) = \frac{c^3 - 2bc^2}{a^3} = \frac{-c^2}{a^3}(2b - c)$$

34. (C): $2x^2 - 7kx + 3 \times k^3 - 1 = 0$

(Since $e^{3\log k} = e^{\log k^3} = k^3$)

$$\therefore \alpha\beta = 40 = \frac{3k^3 - 1}{2}$$

$$\Rightarrow 3k^3 = 81 \Rightarrow k = +3$$

$$\therefore \alpha + \beta = \frac{7k}{2} = \frac{7 \times 3}{2} = \frac{21}{2}$$

35. (A): $\sin\theta + \cos\theta = \dfrac{2b}{a}$ __________(1)

$\sin\theta \times \cos\theta = \dfrac{3c}{a}$ __________(2)

Squaring (1) and subtracting $2\sin\theta \times \cos\theta \Rightarrow$

$$(\sin\theta + \cos\theta)^2 - 2\sin\theta\cos\theta = \sin^2\theta + \cos^2\theta$$

$$= \frac{4b^2}{a^2} = \frac{6c}{a}$$

$$\Rightarrow 1 = \frac{4b^2 - 6ac}{a^2}$$

$$\Rightarrow a^2 = 4b^2 - 6ac$$

$$\Rightarrow 4b^2 - a^2 - 6ac = 0$$

36. (B): $\tan A + \tan B = \dfrac{5}{3}$ and $\tan A \tan B = 2$

$$\tan(A + B) = W = \frac{-5}{3}$$

$$\therefore \sin^2(A+B) = \frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1+\cot^2 B}$$

$$= \frac{1}{1+\frac{9}{25}} = \frac{25}{34}$$

37. (B): Not Available

38. (A)

Let $y = \sqrt{3+\sqrt{3+\sqrt{3+........}}}$

$\Rightarrow y = \sqrt{3+y}$

$\Rightarrow y^2 = 3+y$

$\Rightarrow y^2 - y - 3 = 0$

$\Rightarrow y = \frac{1\pm\sqrt{13}}{2}$

39. (B)

We have, $\alpha\beta^2 + \alpha^2\beta + \alpha\beta = \alpha\beta(\alpha+\beta) + \alpha\beta$

$= \frac{-7}{2} \times \frac{+3}{2} + \frac{-7}{2} = \frac{-21}{4} - \frac{7}{2} = \frac{-35}{4}$

40. (B): It is easy to mentally solve this problem. Just identify a, b, c in each of the equations. In which ever equation, $b^2 - 4ac = 0$, it will have equal roots.

In (B) $a = 3$ $b = -4\sqrt{3}$ and $c = 4$

$\therefore b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4\times3\times4 = 0$