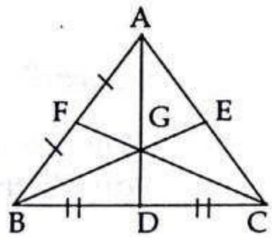
Centre of Triangle

06

1. Centre of a triangle : What do you mean by centroid, Incentre, Circumcentre and orthocentre of a triangle ?

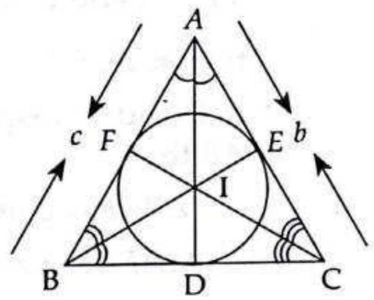
1.1. Centroid : In a triangle line joining the midpoint of a side to the opposite vertex is called a median. The three medians of a triangle meet at a point and the point is called centroid (G) of the triangle. In the adjacent figure points D, E and F are respectively mid point of sides BC, CA and AB. We must learn that



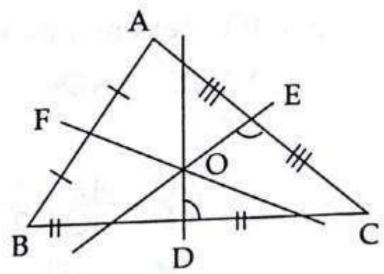
(a) $\frac{AG}{GD} = \frac{BG}{GE} = \frac{CG}{GF} = \frac{2}{1}$ i.e., Centroid divides median in the ratio \2 : 1.

(b) area of ΔAGC = area of ΔBGC = area of ΔAGB i.e., lines joining centroid to the vertices of triangle divide the triangle into three equal areas.

1.2. Incentre : Lines bisecting internal angles (in two equal part) of a triangle are called internal bisector of angles. The internal bisectors of a triangle meet at a point and the point is called incentre of the triangle. In the figure, it is important to note that $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$

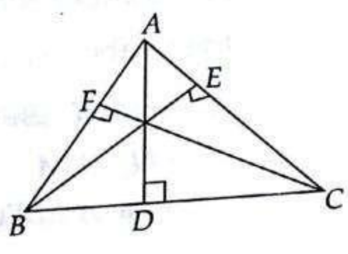


1.3. Circumcentre : Perpendiculars drawn on mid points of sides of a triangle (i.e. perpendicular bisector) meet at a point and the point is called circumcentre of the triangle. Geometrically, circumcentre is equidistant from vertices of a triangle; thus assuming this as centre we can draw a circle passing through all the three vertices of the triangle so,

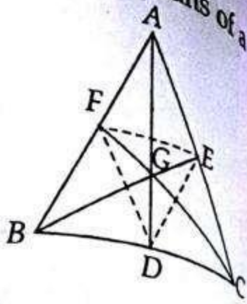


$$AO = BO = CO.$$

1.4. Orthocentre : In a triangle, perpendicular drawn from vertices to the opposite sides (called altitudes) meet at a point and the point is called orthocentre.



2. Important properties of centroid : If AD, BE and CF are medians of a triangle ABC and G be its centroid then.

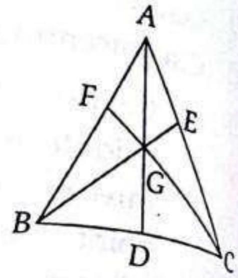


2.1. $\frac{AG}{GD}=\frac{BG}{GE}=\frac{GC}{GF}=\frac{2}{1}$

or, $\frac{AG}{AD}=\frac{2}{3}$,

$\frac{GD}{AD}=\frac{1}{3}$ etc.

2.2. A medians divides triangle into two equal areas



i.e. ar (ΔABD) = ar Δ $(ACD)=\frac{1}{2}$ ar (ΔABC)

ar Δ (BEC) = ar $\Delta(BEA)=\frac{1}{2}$ ar (ΔABC) etc.

2.3. Lines joining centroid to vertices of a triangle divide the triangle into three equal areas.

i.e. ar (ΔAGB) = ar (ΔBGC) = ar $(\Delta CGA)=\frac{1}{3}$ ar ΔABC

2.4. Since GD is the median of triangle BGC.

$\therefore$ ar (ΔBGD) = ar $(\Delta CGD)=\frac{1}{2}$ ar $(\Delta BGC)=\frac{1}{2}\cdot\frac{1}{3}$ ar (ABC)

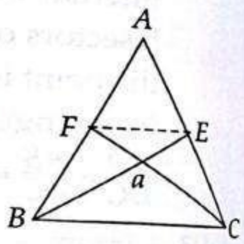
$\therefore$ ar $(\Delta BGD)=\frac{1}{6}\times$ ar (ΔABC) etc.

2.5. G is also the centroid of ΔDEF.

2.6. Since E and F are respectively mid points of AB and AC

therefore, $EF \parallel BC$ and $EF=\frac{1}{2}BC$

2.7. If E and F, respectively mid point of AC and AB then



$\angle AEF = \angle ACB$ $(\because EF \parallel BC)$

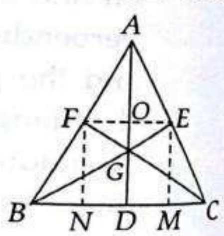
$\angle AFE = \angle ABC$ $(\because EF \parallel BC)$

$\therefore$ $\Delta AFE \sim \Delta ABC$.

2.8. If G be centroid and O is the point of interection of AG and EF then

$\Delta AOE \sim \Delta ADC$ $(\because EF \parallel DC \Rightarrow \angle AEO = \angle ACD$

$\angle AOE = \angle ADC)$



$\therefore$ $\frac{AO}{AD}=\frac{AE}{AC}=\frac{1}{2}$

$\Rightarrow$ $2AO = AD \Rightarrow AO = OD$ $(\because$ E is mid point)

i.e., point O is midpoint of AD. [Also see solved example 16]

2.9. In the above figure if $FN \perp r$ BC and $EM \perp r$ BC then $FN = EM$

$\therefore$ area of $\Delta BFC=\frac{1}{2}\times BC\times FN$; area of $\Delta BEC=\frac{1}{2}\times BC\times EM$

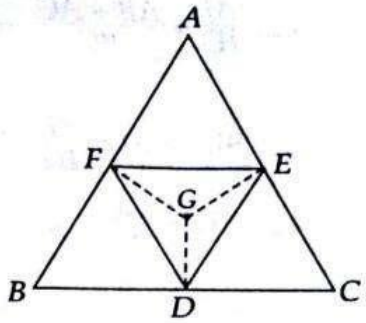
$\because$ $FN = EM$

$\therefore$ area of ΔBFC = area of ΔBEC

2.10. Since D, E, F are mid point of sides then, $\Delta BDF \cong \Delta EFD \cong \Delta FEA \cong \Delta DCE$

and ar (ΔBDF) = ar (ΔCDE) = ar (ΔAEF)

$= \text{ar}(\Delta FDE) = \frac{1}{4}(\text{ar}\ \Delta ABC)$



$\because$ G, is also centroid of ΔDEF

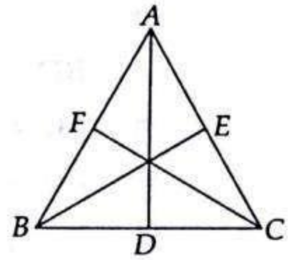
$\therefore$ ar (ΔDGE) = ar (ΔEGF) = ar Δ (DGF)

$= \frac{1}{4} \times \frac{1}{3}$ ar (ΔABC)

$= \frac{1}{12}$ ar (ΔABC)

3. Relation among sides and medians of a triangle : Suppose ABC is a triangle whose medians are AD, BE and CF.

If $AB = c$, $BC = a$ and $AC = b$, then



3.1. $AD = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

3.2. $BE = \frac{1}{2}\sqrt{2a^2 + 2c^2 - b^2}$

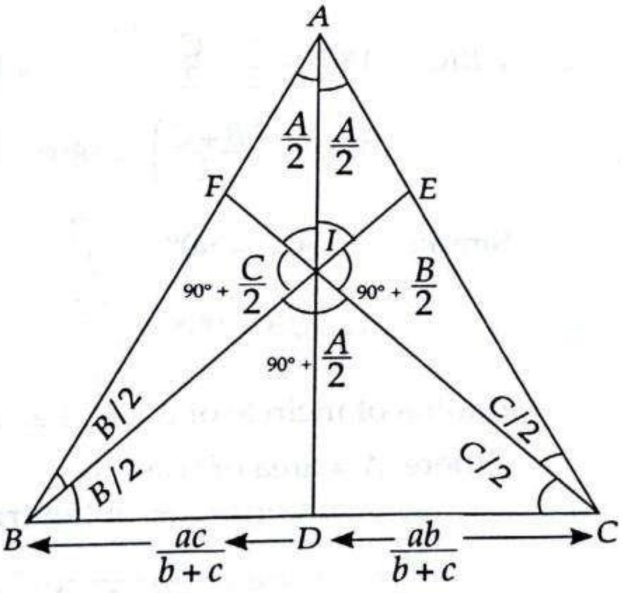
3.3. $CF = \frac{1}{2}\sqrt{2a^2 + 2b^2 - c^2}$

3.4. $3(AB^2 + BC^2 + CA^2) = 4(AD^2 + BE^2 + CF^2)$

or, $AB^2 + BC^2 + CA^2 = \frac{4}{3}(AD^2 + BE^2 + CF^2)$

3.5. area of $ABC = \frac{4}{3}$ × (area formed by taking AD, BE, CF as sides of a triangle)

4. Important properties of Incentre : In the given figure AD, BE and CF are respectively bisectors of $\angle A$, $\angle B$ and $\angle C$. These internal bisectors meet at I which is incentre of the triangle.



Clearly,

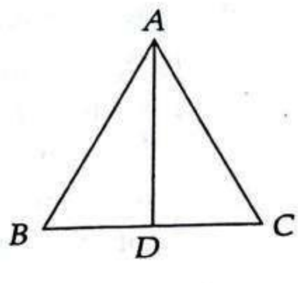
$\angle BAD = \angle CAD = \frac{\angle A}{2}$

$\angle ABE = \angle CBE = \frac{\angle B}{2}$

and $\angle BCF = \angle ACF = \frac{\angle C}{2}$

4.1. If AD is bisector of $\angle A$ then $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$

i.e., angle bisector AD divides side BC in the ratio $AB : AC$.



Similarly, $\frac{CE}{EA} = \frac{BC}{BA}, \frac{AF}{FB} = \frac{CA}{CB}$

4.2. $\frac{AI}{ID} = \frac{AB+AC}{BC} = \frac{c+b}{a}$ (How to recall : AB and AC are connected with AI while ID stand on BC)

Similarly, $\frac{BI}{IE} = \frac{BA+BC}{AC}, \frac{CI}{IF} = \frac{CA+CB}{AB}$

4.3. $BD = \frac{ac}{b+c}, CD = \frac{ab}{b+c}$

$CE = \frac{bc}{c+a}, EA = \frac{ba}{c+a}$

$AF = \frac{cb}{a+b}, BF = \frac{ca}{a+b}$

Explanation : Since $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$

let $BC = ck$ and $DC = bk$

$\therefore$ $BD + DC = ck + bk$

or, $BC = (c+b)k$

or, $a = (b+c)k$

or, $k = \frac{a}{b+c}$

$\therefore$ $BD = ck = \frac{ac}{b+c}, CD = bk = \frac{ba}{b+c}$

(How to recall : Since $BD : CD = c : b$, thus multiplying by $\frac{a}{b+c}$

We get, $BD = \frac{ac}{b+c}, CD = \frac{ab}{b+c}$ etc)

4.4. $\angle BIC = 180° - \frac{B}{2} - \frac{C}{2}$

$= 180° - \left(\frac{B+C}{2}\right) = 180° - \left(\frac{180° - A}{2}\right) = 90° + \frac{\angle A}{2}$

Similarly, $\angle AIC = 90° + \frac{\angle B}{2}$

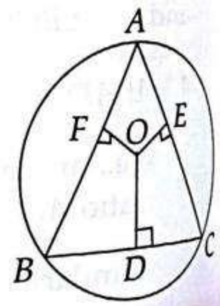
$\angle AIB = 90° + \frac{\angle C}{2}$ [SSC Tier-I 2014]

4.5. Radius of incircle of ΔABC i.e., inradius $\boxed{r = \frac{\Delta}{s}}$

Where, Δ = area of triangle,

and s = semiperimeter of the triangle.

5. Circumcentre : In the given figure O is the circumentre of ΔABC. Hence,

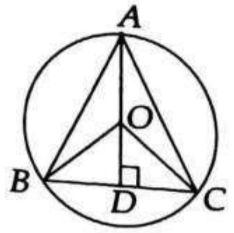


5.1. OD, OE and OF are respectively perpendicular bisector of sides BC, AC and AB i.e., $BD = DC$ and $OD \perp$r BC etc.

5.2. Circumcentre O is equidistant from vertices A, B, C of the triangle i.e., $OA = OB = OC = R$

5.3. R is called circumradius and $\boxed{R = \frac{abc}{4\Delta}}$

Where, Δ is area of triangle.

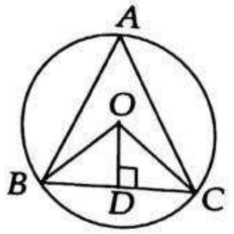


5.4. Angle subtends by arc of a circle at centre is double the angle subtends by it at circumference.

i.e., $\angle BOC = 2\angle A$

$\angle COA = 2\angle B$

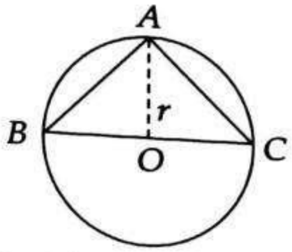
and $\angle AOB = 2\angle C$



5.5. $\Delta OBD \cong \Delta OCD$

$\therefore \quad \angle BOD = \angle COD = \angle A$

and $\angle OBC = \angle OCB = 90° - A$

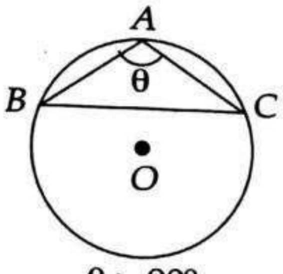


5.6. If ABC is a right angled triangle (with $\angle A = 90°$) then circumcentre O is the mid point of hypotenuse BC.

Since, $OB = OC = OA = r$,

Hence in a right angled triangle, mid point of hypotenuse is equidistant from the vertices of the triangle.

5.7. If ABC is an obtused angle triangle its circumcentre lies out side the triangle ABC.



$\theta > 90°$

6. Important properties of orthocentre

In the given figure $AD \perp r\ BC$, $BE \perp r\ AC$ and $CF \perp r\ AB$. Altitudes AD, BE and CF meet at P which is orthocentre of the ΔABC

6.1. In ΔABD

$$\angle BAD = 180° - 90° - \angle B \qquad (\because \angle ADB = 90°)$$

$$= 90° - \angle B$$

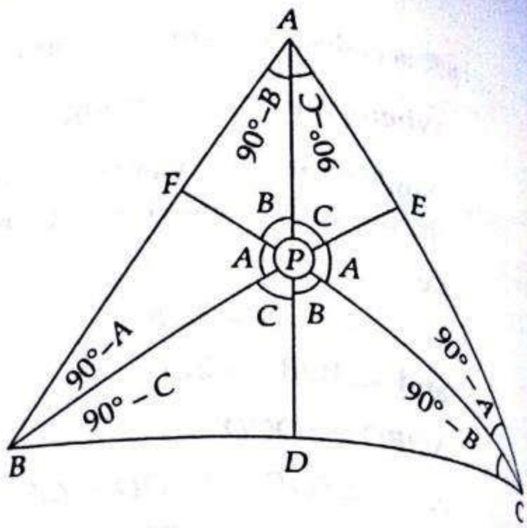
Similarly in ΔADC, $\angle CAD = 90° - \angle C$ etc

See the remaining angles in the figure.

6.2. Angle around orthocentre P :

In ΔBPD,

$$\angle BPD + \angle PBD = 90°$$



or, $\angle BPD + 90° - \angle C = 90°$

or, $\angle BPD = \angle C$

See the remaining angles in the figure,

6.3. $\angle BPC = \angle B + \angle C$

$= 180° - \angle A$

$\angle CPA = \angle C + \angle A = 180° - \angle B$

$\angle APB = \angle A + \angle B = 180° - \angle C$

6.4. $BD = \dfrac{AB^2 + BC^2 - AC^2}{2BC}$

$= \dfrac{c^2 + a^2 - b^2}{2a}$

$CD = \dfrac{AC^2 + BC^2 - AB^2}{2BC}$

$= \dfrac{b^2 + a^2 - c^2}{2a}$

$\therefore\ BD : DC = c^2 + a^2 - b^2 = b^2 + a^2 - c^2$

6.5. Pair of similar triangles are

$\Delta PEC \sim \Delta PFB$

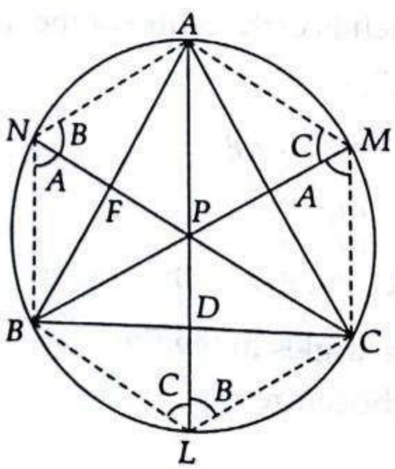
$\Delta PDC \sim \Delta PFA$

$\Delta PEA \sim \Delta PDB$

Write the ratio of sides of triangle yourself. Questions may be asked on these ratio.

6.6. It must be noted that ΔPDB, ΔPDC, ΔPEC, … etc. are right angled triangles.

6.7. P is orthocentre of ΔABC. Draw a circumcircle to the triangle ABC. Since angles in the same segment (or the same base or in the same arc) of a circle are equal,



$\therefore$ On base BL, $\angle BCL = \angle BAL = 90° - \angle B$

On base CL, $\angle CBL = \angle CAL = 90° - \angle C$

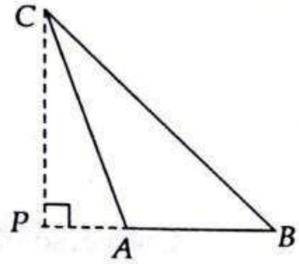
On base BC, $\angle BMC = \angle A$ etc.

See the remaining angles in the figure.

.8. The orthocentre of a right angles triangle is that point where triangle forms the right angle.

.9. The orthocentre of an obtuse angled triangle lies out side the triangle.

In figure, orthocentre P lies outside the triangle.



Mixed properties of centres of a triangle.

7.1. In an equilateral triangle all the four centres are coincident i.e., centroid, incentre, circumcentre and orthocentre of an equilateral triangle lie at the same point.

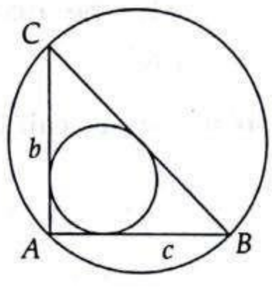
7.2. Centroid (G), orthocentre (P) and circumcentre (O) of a triangle are always collinear (i.e., lie in a straight line) and $PG : GO = 2 : 1$.

7.3. The orthocentre of a right angled triangle lies at the right angled vertex while its circumcentre is mid point of hypotenuse.

7.4. Circumcentre and orthocentre of an obtuse angled triangle always lie outside the triangle.

7.5. The sum of diameters of circumcircle and incircle of a right angled triangle is equal to the sum of its perpendicular sides.

In the given figure ABC is a right angled triangle with $\angle A = 90°$. If radius of circumcircle and incircle of the triangle be respectively R and r then $2(R + r) = b + c$



(See solved example–21)

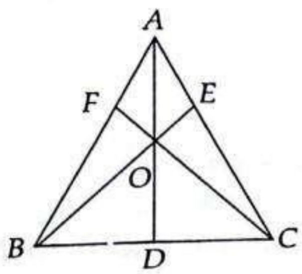
7.6. The distance between incentre and circumcentre of a triangle is $\sqrt{R^2 - 2rR}$ where R is circumradius and r is inradius.

7.7. In an equilateral triangle, length or radius of the circumcircle is equal to twice the radius of its incircle i.e., if ΔABC is equilateral then $R = 2r$.

7.8. Ceva Theorem : If O is any point inside the triangle ABC and AO, BO, CO meet sides BC, CA, AB respectively at point D, E, F then

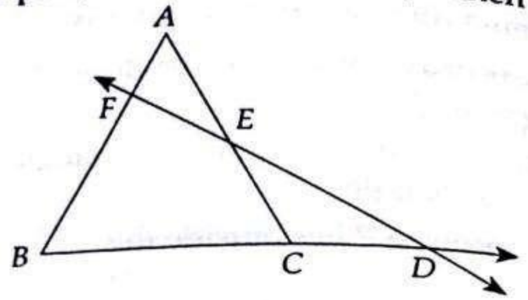
$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$$

Since Ceva Theorem is true for any point inside the triangle, it is therefore also true for centroid, incentre, orthocentre and circumcentre of the triangle.



Converse of Ceva Theorem is also true.

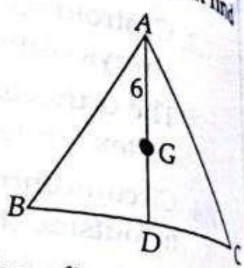
7.9. **Menelaus Theorem :** If a transverse cuts the sides BC, CA and AB (or its produced part) of a triangle at *D, E, F* then $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$



Converse of the theorem is also true.

Solved Example

1. If distance of centroid of triangle *ABC* from vertex *A* is 6 cm them find the length of median through point, *A*.



Solution : Since, $AG : GD = 2 : 1$

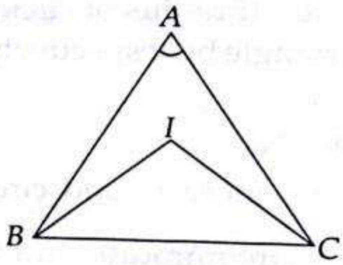
$\therefore \quad \frac{6}{GD} = \frac{2}{1}$

$\Rightarrow \quad GD = 3$

$\therefore \quad AD = AG + GD = 6 + 3 = 9$

2. If *I* be the incentre of triangle *ABC* and $\angle A = 70°$ then find the value of $\angle BIC$.

Solution : Recall that $\angle BIC = 90° + \frac{A}{2}$ (See Article 4 (iv))



Hence, $\angle BIC = 90° + \frac{70°}{2} = 125°$

3. In a triangle *ABC* if $\angle A = \theta$ and perpendiculars drawn from vertices *B* and *C* to respective opposite sides meet in *P* then find the value of $\angle BPC$ in terms of θ.

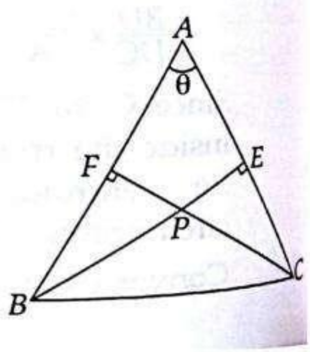
Solution : See the figure, In Quadrilateral *AEPF*

$\theta + 90° + 90° + \angle EPF$

$= 360°$ or, $\angle EPF = 180° - \theta$

$\therefore \quad \angle BPC = \angle EPF = 180° - \theta$

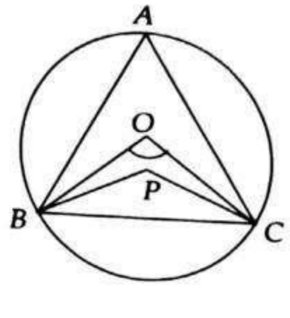
(Vertically opposite angle)



Shortcut : Learn that $\angle BPC = \angle B + \angle C = \pi - A$

4. O is the circumcentre of a triangle ABC whose $\angle A = 50°$. If bisector of $\angle OBC$ and $\angle OCB$ intersect at P then what is the measure of $\angle BPC$?

Solution : Since angle subtened at the centre of the circle is double the angle subtened at circumference



$\therefore \quad \angle BOC = 50° \times 2 = 100°$

$\because \quad OB = OC$

$\therefore \quad \angle OBC = \angle OCB = \dfrac{180° - 100°}{2} = 40°$

In ΔBPC, $\angle BPC + \angle PBC + \angle PCB = 180°$

or, $\angle BPC + \frac{1}{2} \times 40° + \frac{1}{2} \times 40° = 180°$

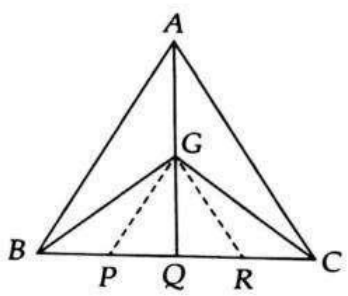
$\therefore \quad \angle BPC = 180° - 20° - 20° = 140°$

Shortcut : $\angle BPC = 90° + \dfrac{\angle BOC}{2} = 90° + A = 90° + \dfrac{100°}{2} = 140°$

5. Three points P, Q, R lie on the side BC of triangle ABC such that $BP = PQ = QR = RC$. If G be centroid of ΔABCthen what is ratio of areas of ΔPGR and ΔABC.

Solution : Clearly Q is mid point of side BC i.e., AQ is median of ΔABC.

We know that



Area of $\Delta BGC = \frac{1}{3} \times$ area of ΔABC

But height of ΔBGC and ΔPGR are equal.

Let this height be h.

$\therefore$ area $\Delta BGC = \frac{1}{2} \times BC \times h$

and area of $\Delta PGR = \frac{1}{2} \times PR \times h$

Now, $\dfrac{\text{area of } \Delta PGR}{\text{area of } \Delta ABC} = \dfrac{\text{area of } \Delta PGR}{3 \times \text{area of } \Delta BGC}$

$$= \frac{\frac{1}{2} \times PR \times h}{3 \times \frac{1}{2} \times BC \times h} = \frac{PR}{3BC}$$

$$= \frac{PR}{3 \cdot 2PR} = \frac{1}{6} \qquad (\because PR = \tfrac{1}{2} BC)$$

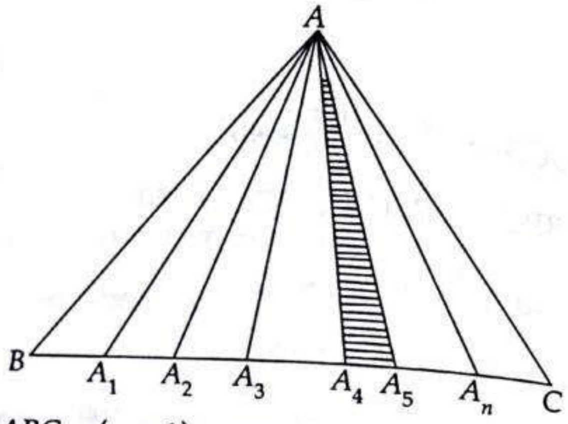
6. A triangle DEF is formed by joining mid points of sides of triangle ABC. Again mid points of sides of triangle DEF are joined together to form a new triangle PQR. If sides of triangle ABC are respectively 4, 5 and 6 cm then what is the distance between centroid of ΔPQR and ΔDEF.

Solution : Centroid of a given triangle and a triangle formed by mid points of the given triangle are coincident (i.e., lie at the same point);

So required distance = 0

7. n equidistant points $A_1, A_2, A_3 \ldots\ldots A_n$ are taken on base BC of ΔABC ... $A_1B = A_1A_2 = A_nC$ and area of $\Delta A\, A_4A_5$ is k cm^2, then what is the area ... ΔABC.

Solution : See the figure, a total of $(n + 1)$ triangles will be formed whose base are same and height are equal.



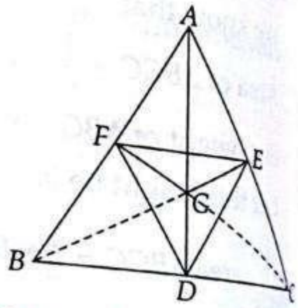
$\therefore$ Area of $\Delta ABC = (n+1)$ area of ΔAA_4A_5
$= (n+1)k$ cm^2

8. Points E and F lie respectively on side AC and AB of a triangle ABC such that $EF \parallel BC$ and $2EF = BC$. If G be the centroid of ΔABC then find the area of triangle formed by joning mid points of sides of triangle EFG with respect to area of ΔABC.

Solution : Since $EF \parallel BC$ and $EF = \frac{1}{2} BC$;

therefore E and F are respectively mid points of sides AC and AB.

We know that line joining the mid points of sides of a triangle divides the triangle in four equal areas.



$\therefore$ Area of $\Delta DEF = \frac{1}{4} \times$ area of ΔABC *(here D is mid point of side BC)*

Now, G is also the centroid of triangle DEF and lines joining centroid and vertices of a triangle divides the triangle into three equal areas.

Hence, area of $\Delta EGF = \frac{1}{3} \times$ area of $\Delta DEF = \frac{1}{3} \times \frac{1}{4} \times$ area of ΔABC

$\therefore$ Area of formed by mid points of sides of $\Delta EGF = \frac{1}{4} \times$ area of ΔEGF

$= \frac{1}{4} \times \frac{1}{3} \times \frac{1}{4} \times$ area of $\Delta ABC = \frac{1}{48} \times$ area of ΔABC

9. The angles of a triangle are in the ratio $3 : 4 : 5$. If I be the incentre of ΔABC then find the measure of $\angle ADC$ and $\angle DIC$ where AD, is the bisector of $\angle A$.

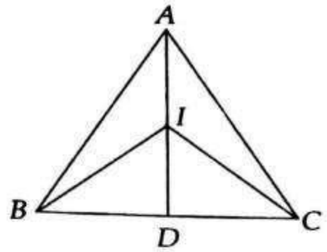
Solution : Let $\angle A = 3k$, $\angle B = 4k$ and $\angle C = 5k$

then, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 3k + 4k + 5k = 180°$

or, $\quad k = \frac{180°}{12} = 15°$

$\therefore \quad \angle A = 45°, \angle B = 60°, \angle C = 75°$



In triangle ACD, $\angle ADC = 180° - \frac{\angle A}{2} - \angle C$

$= 180° - \frac{45°}{2} - 75°$

$= 105° - 22\frac{1°}{2} = 82\frac{1°}{2}$

In triangle CID, $\angle CID = 180° - \angle ADC - \frac{\angle C}{2} = 180° - 82\frac{1°}{2} - \frac{75°}{2}$

$= 180° - 82\frac{1°}{2} - 37\frac{1°}{2} = 180° - 120° = 60°$

10. In triangle ABC, $AB = 6$, $AC = 7$ and $BC = 8$. If AD is bisector of $\angle A$ and I is the incentre of trinagle ABC then find the length of BD and CD. Also find the value of $AI : ID$.

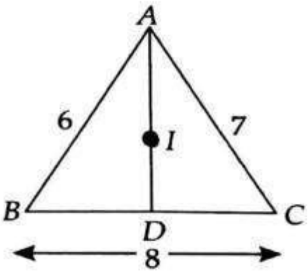
Solution : We know that if AD is bisector of $\angle A$ then $\frac{BD}{DC} = \frac{AB}{AC} = \frac{6}{7}$

Let $\quad BD = 6k, DC = 7k$

then, $BD + DC = BC$

$\Rightarrow \quad 6k + 7k = 8$

$\Rightarrow \quad k = \frac{8}{13}$



$BD = 6k = 6 \times \frac{8}{13} = \frac{48}{13}$

$CD = 8k = \frac{7 \times 8}{13} = \frac{56}{13}$

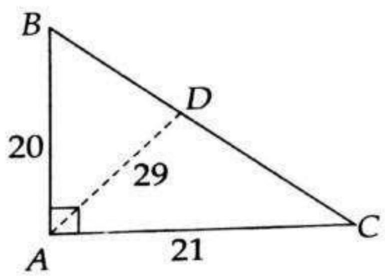
Shortcut : $BD = \frac{ac}{b+c} = \frac{8 \times 7}{6+7} = \frac{56}{13}$

Now, $\frac{AI}{ID} = \frac{AB + AC}{BC} = \frac{6+7}{8} = \frac{13}{8}$

11. In a triangle ABC, if $AB = 20$ cm, $AC = 21$ cm and $BC = 29$ cm, then find the distance between vertex A and mid point of BC.

Solution : Since $20^2 + 21^2 = 400 + 441 = 841 = 29^2$

$\therefore$ ΔABC is a right angled triangle, whose hypotenuse is BC.

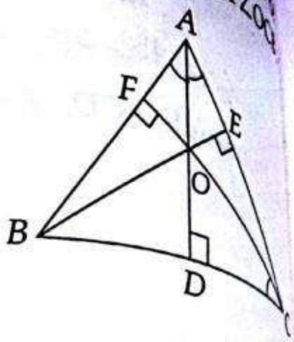


Since mid point of hypotenuse of a right angled triangle is equidistant from each vertex

$\therefore \quad AD = BD = DC = \frac{29}{2}$ cm

or, $AD = 14.5$ cm

12. In triangle ABC, $6\angle A = 4\angle B = 3\angle C$. If AD, BE and CF are altitudes of triangle and O is its point of intersection then find the measure of $\angle COD$, $\angle BOD$ and $\angle BOC$.



Solution : Let $6A = 4B = 3C = K$

then, $\angle A = \frac{K}{6}, \angle B = \frac{K}{4}, \angle C = \frac{K}{3}$

$\therefore \quad \angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad \frac{K}{6} + \frac{K}{4} + \frac{K}{3} = 180°$

$\Rightarrow \quad \frac{2K + 3K + 4K}{12} = 180° \Rightarrow K = \frac{180° \times 12}{9} = 240°$

$\therefore \quad A = 40°, B = 60°$ and $C = 80°$

In right angled ΔBCF, $90° + \angle BCF + \angle B = 180°$

or, $90° + \angle BCF + 60° = 180°$

$\therefore \angle BCF = 180° - 90° - 60° = 30°$

In right angled ΔOCD, $\angle ODC + \angle OCD + \angle COD = 180°$

or, $90° + 30° + \angle COD = 180°$

or, $\angle COD = 60°$

In right angled ΔBEC, $90° + \angle C + \angle EBC = 180°$

or, $90° + 80° + \angle EBC = 180°$

or, $\angle EBC = 10°$

In right angled ΔBOD, $90° + \angle OBD + \angle BOD = 180°$

or, $90° + 10° + \angle BOD = 180°$

or, $\angle BOD = 80°$

$\therefore \quad \angle BOC = \angle COD + \angle BOD = 60° + 80° = 140°$

Shortcut : See the figure of orthcentre in theory part. All the angles can be found directly.

13. If O be the orthocentre of ABC, $OF \perp r\ AB$ and $OE \perp r\ AC$. If $OE = 2$ cm and $BE = 5$ cm then find the value of $OF \times OC$.

Solution : In ΔOBF and ΔOCE,

$\angle OFB = \angle OEC = 90°$ and $\angle BOF = \angle EOC$

$\therefore \quad \Delta OBF \sim \Delta OCE$ *(Vertically opposite angle)*

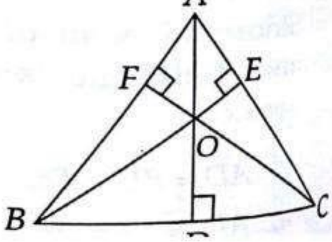
Hence, $\frac{OB}{OC} = \frac{OF}{OE}$ or, $\frac{OB}{OF} = \frac{OC}{OE}$

or, $OB \times OE = OF \times OC$

or, $OF \times OC = OB \times OE$

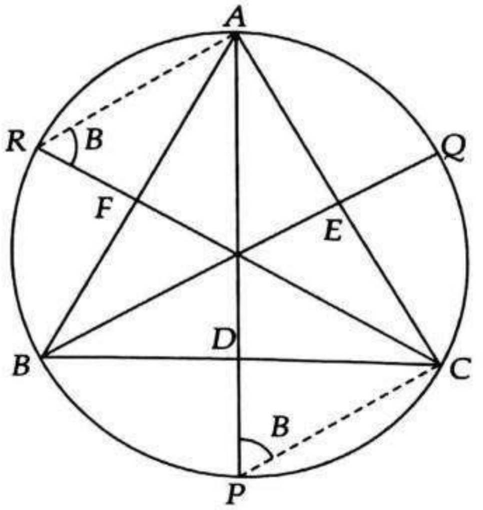
$= (BE - OE)(OE)$

$= (5 - 2) \times 2 = 6 \text{ cm}^2$



14. A circle is drawn circumscribing the ΔABC. If produced part of altitudes AD, BE and CF are bisect on P, Q, R respectively then prove that $\frac{CD}{AF} = \frac{CP}{AR} = \frac{DP}{FR}$

Solution : In ΔCPD, $\angle CDP = 90°$



and $\angle DPC = \angle B$ (angle on the same segment of base AC)

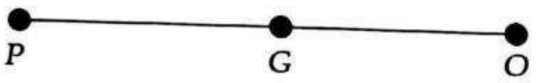
In ΔAFR, $\angle AFR = 90°$, In ΔPDC, $\angle PDC = 90°$

and $\angle ARF = \angle B$ (angle on same segment of base AC)

$\therefore \quad \Delta CDP \sim \Delta AFR \Rightarrow \frac{CD}{AF} = \frac{CP}{AR} = \frac{DP}{FR}$

15. If the distance between centroid and orthocentre of a triangle is 12 cm then find the distance between its orthocentre and circumcentre.

Solution : We know that orthocentre (P), centroid (G) and circumcentre (O) are collinear and $\frac{PG}{GO} = \frac{2}{1}$



According to question $GP = 12.1G$ $\left(\because GO = \frac{1}{2} \cdot PG\right)$

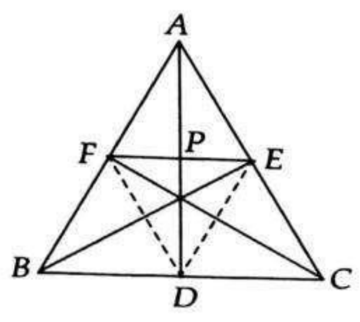
$$= \frac{3}{2} PG = \frac{3}{2} \times 12 = 18 \text{ cm}$$

16. If AD, BE and CF are medians of triangle ABC then prove that median AD divides line segment EF.

Solution : Join $E - D$ and $E - F$

$\therefore$ $AFDE$ will be a parallelogram

$(\because ED \parallel AB \Rightarrow ED \parallel AF$ and $FD \parallel AC \Rightarrow FD \parallel AE)$



Hence AD and EF are diagonals of a parallelogram. Its point of intersection P divides diagonal AD of parallelogram which is median AD of ΔABC.

17. If H be orthocentre of ΔABC and mid point of AH, BH, CH are respectively P, Q, R then prove that H is also the orthocentre of ΔPQR and $\Delta PQR \sim \Delta ABC$ *(Learn the property)*

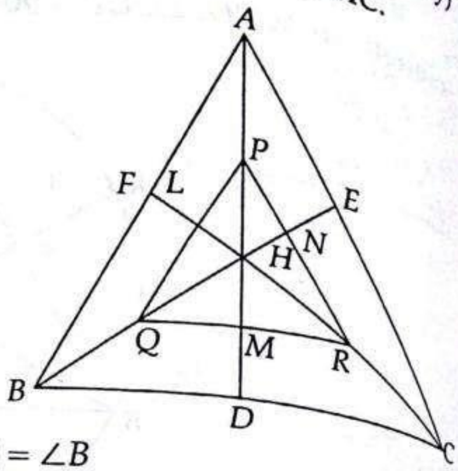
Solution : In ΔHBC, Q is mid point of HB and R is mid point of HC.

Hence $QR \parallel BC$ and $QR = \frac{1}{2} BC$

But $AD \perp r\ BC \Rightarrow PD \perp r\ QR$

Similarly we can prove that $QE \perp r\ PR$ and $RF \perp r\ PQ$

Hence, PD, QE and RF lies on altitudes of ΔPQR. So H, is orthocentre of ΔPQR.



Seccond part : $QR \parallel BC \Rightarrow \angle EQR = \angle EBC$

and $QP \parallel AB \Rightarrow \angle PQE = \angle ABE$

Adding we get $\angle PQR = \angle ABC$ i.e., $\angle Q = \angle B$

Similarly we can prove that $\angle A = \angle P$ and $\angle R = \angle C$

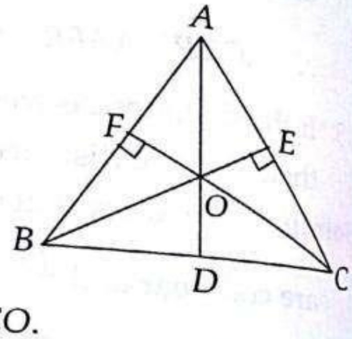
$\therefore$ $\Delta ABC \sim \Delta PQR$

[do your self : $\Delta HQM \sim \Delta HBD$, $\Delta HQR \sim \Delta HBC$ etc.]

18. If O be the orthocentre of ΔABC then orthocentre of ΔOBC is A. Justify the statement.

(Learn the property).

Solution : In figure, AD, BE and CF are respectively altitudes on sides BC, CA and AB. O is orthocentre.



In ΔOBC, CE is perpendicular to produced part of BO.

BF is perpendicular to produced part of CO.

Clearly point of intersection of produced part of BF and CE is A. Thus A is orthocentre of ΔOBC.

19. $ABCD$ is a parallelogram. L and M are respectively mid points of sides AB and AD. Prove that LC and MC divides diagonals BD in three equal parts.

(Learn the property)

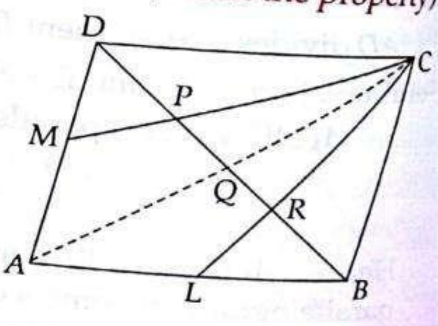
Solution : See the figure; Let CM, CA and CL intersects diagonals DB respectively at the points P, Q and R.

Q is mid point of AC.

CM and DQ are medians of ΔACD which intersects at P.

Hence P is centroid of ΔACD.

$\therefore$ $\frac{DP}{PQ} = \frac{2}{1}$



Similarly in ΔABC, $\frac{BR}{RQ} = \frac{2}{1}$ ... (ii)

[diagonals of parallelogram bisect each other]

But $DQ = QB$

$\Rightarrow DP + PQ = QR + RB$ ... (iii)

$\Rightarrow 2PQ + PQ = QR + 2QR$ (from (i) &(ii))

$\Rightarrow 3PQ = 3QR \Rightarrow PQ = QR$

Now, from (3), $DP + PQ = QR + RB$

$(\because PQ = QR)$

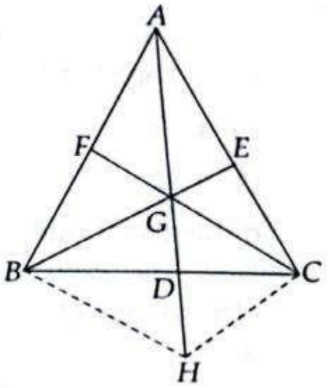
$\Rightarrow DP = RB$ ... (iv)

$\Rightarrow PR = PQ + QR = 2PQ = DP$ ... (v)

From (iv) and (v), $DP = RB = PR$

20. Prove that sum of any two medians of a triangle is greater than the third median. *(Learn the property)*

Solution : In the given figure,



AD, BE and CF are medians of triangle ABC.

G is centroid of ΔABC.

GD is produced to H such that $AG = GH$

Now, see the triangle ΔABH,

Here F is mid point of side AB and G, is the mid point of a side AH.

$\therefore$ $FG \parallel BH \Rightarrow GC \parallel BH$ ... (i)

Now, see the ΔACH.

here E is mid point of side AC and G is mid point of side AH.

$\therefore$ $EG \parallel CH \Rightarrow GB \parallel HC$ ... (ii)

from (i) and (ii), ′$BHCG$ is a parallelogram.

$\therefore$ $BG = CH$ and $GC + BG > GH$ $[CH = BG]$

or, $BG + GC > AG$ $(\because AG = GH$ and $GC = BH)$

or, $\frac{3}{2} BG + \frac{3}{2} GC + \frac{3}{2} AG$

or, $BE + CF > AD$ $(\because \frac{3}{2} BG = BE$ etc.)

21. In a right angled ΔABC, $\angle A = 90°$. If $AC = b$, $BC = a$, $AB = c$ and r and R are respectively radii of incircle and circumcircle of the triangle then prove that $2(r + R) = b + c$ *[Learn the property]*

Solution : Clearly hypotenuse $BC = 2R$

$\therefore$ $a = 2R$

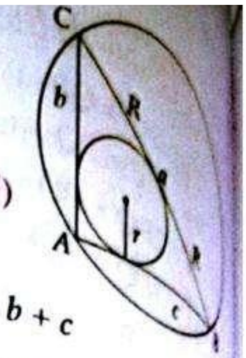
and from $r = \frac{\Delta}{s}$, $r = \frac{\frac{1}{2}bc}{\left(\frac{a+b+c}{2}\right)} = \frac{bc}{a+b+c}$

$$\therefore 2R + 2r = a + \frac{\quad}{a+b+c}$$

$$= \frac{a^2 + ab + ac + 2bc}{a+b+c}$$

$$= \frac{b^2 + c^2 + ab + ac + 2bc}{a+b+c} \quad (\because a^2 = b^2 + c^2)$$

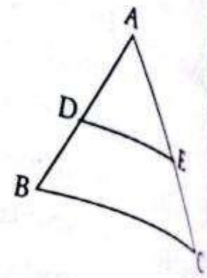
$$= \frac{(b+c)^2 + a(b+c)}{a+b+c} = \frac{(b+c)(b+c+a)}{a+b+c} = b + c$$



Thus sum of diameters of two circles = Sum of mutually perpendicular sides

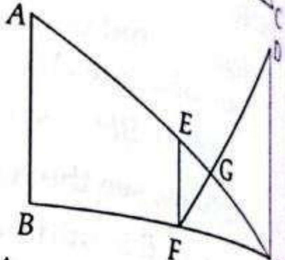
Exercise–6A

1. In the adjacent figure DE is parallel to BC and ratio of areas of ΔADE and trapezium $BDEC$ is 4 : 5. What is the value of $DE : BC$?



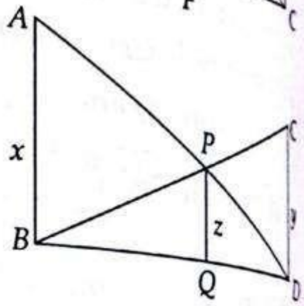
   (a) 1 : 2 (b) 2 : 3
   (c) 4 : 5 (d) None of these

2. In the given figure AB, EF and CD are parallel lines. It is given that $EG = 5$cm, $GC = 10$cm, $AB = 15$ cm and $DC = 18$ cm, What is the value of AC ?



   (a) 20 cm (b) 24 cm
   (c) 25 cm (d) 28 cm

3. In the adjacent figure, $\angle ABD = \angle PQD = \angle CDQ = \frac{\pi}{2}$. If $AB = x$, $PQ = z$ and $CD = y$ then which one of the following is true.



   (a) $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$ (b) $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$
   (c) $\frac{1}{z} + \frac{1}{y} = \frac{1}{x}$ (d) $\frac{1}{x} + \frac{1}{y} = \frac{2}{z}$

4. ΔPQR is right angled at Q; $PR = 5$ cm and $QR = 4$ cm. Another ΔABC is given whose side are respectively 3 cm, 4 cm and 5 cm then which one of the following is true ?
   (a) area of ΔPQR is double the area of ΔABC
   (b) area of ΔABC is double the area of ΔPQR
   (c) $\angle B = \frac{\angle Q}{2}$
   (d) Both triangles are congruent

5. If ratio of length of medians of two equilateral triangles are 3 : 2 then what is the ratio of their sides ?
   (a) 1 : 1 (b) 2 : 3 (c) 3 : 2 (d) $\sqrt{3} : \sqrt{2}$

In which of the following triangle centroid and orthocentre are coincident?

(a) Scalene triangle
(b) Isosceles triangle
(c) Equilateral triangle
(d) Right angled triangle

Consider the triangle ABC. Let D, E are respectively mid points of sides BC, CA while AD and BE intersects at G. Suppose O is a point on AD such that $AO : OD = 2 : 7$.

Assertion (A) : $AO = \frac{(2GD)}{3}$

Reason (R) : $OD = \frac{(2AG)}{3}$

(a) Both Assertion A and Reason R are correct and Reason R is a correct explanation of Assertion A.
(b) Both Assertion A and Reason R are correct but Reason R is not the correct explanation of Assertion A.
(c) Assertion A is correct, Reason R is wrong.
(d) Assertion A is wrong, Reason R is correct.

8. ABC is a given triangle. AD, BE and CF are altitudes of ΔABC.

Assertion (A) : $(AB^2 + BC^2 + CA^2) > (AD^2 + BE^2 + CF^2)$

Reason (R) : $(AE^2 - AF^2) + (BF^2 - BD^2) + (CD^2 - CE^2) = 0$

(a) Both Assertion A and Reason R are correct and Reason R is a correct explanation of Assertion A.
(b) Both Assertion A and Reason R are correct but Reason R is not the correct explanation of Assertion A.
(c) Assertion A is correct, Reason R is wrong.
(d) Assertion A is wrong, Reason R is correct.

9. ABC is a given triangle. An external point X of ΔABC is such that $CD = CX$, where D is the point of intersection of BC and AX. If $\angle BAX = \angle XAC$, then which one of the following is true?

(a) ΔABD and ΔACX are similar
(b) $\angle ABD < \angle ACD$
(c) $AC = CX$
(d) $\angle ADB > \angle DXC$

10. How many point (s) in the plane of ΔABC is equidistant from its vertices?

(a) 0
(b) 1
(c) 2
(d) 3

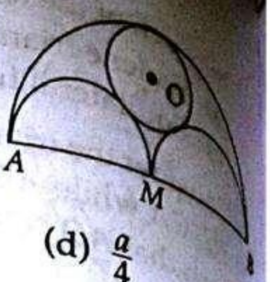
11. In a triangle ABC, interal bisector of $\angle ABC$ and external bisector of $\angle ACB$ meet in D. Which one of the following is true?

(a) $\angle BDC = \angle BAC$
(b) $\angle BDC = \frac{1}{2} \angle ABC$
(c) $\angle BDC = \angle DBC$
(d) None of these

12. The median BD of ΔABC meets side AC at D. If $BD = \frac{1}{2} AC$, then which one of the following is true.

(a) $\angle ACB = 1$ right angle
(b) $\angle BAC = 1$ right angle
(c) $\angle ABC = 1$ right angle
(d) None of the above

13. In the given figure, M is the mid point of line segment AB whose length is $2a$. Semicircles having diameters AM, MB and AB are drawn at the same side of the line. The radius of a circle touching all the three semicircle is



(a) $\frac{2a}{3}$ (b) $\frac{a}{2}$ (c) $\frac{a}{3}$ (d) $\frac{a}{4}$

14. Point of concurrency of altitudes of a triangle is called
(a) Circumcentre (b) Orthocentre
(c) Incentre (d) Centroid

15. Number of circles passing through all the three vertices of a triangle is
(a) one (b) two (c) three (d) infinity

16. Consider the following statements :

Statement–I : Suppose PQR is a triangle with $PQ = 3$ cm, $QR = 4$ cm and $RP = 5$ cm. If D is a point either outside or inside of the plane of triangle then $DP + DQ + DR > 6$ cm.

Statement–II : ΔPQR is a right angled triangle.

Regarding two statements described above which one of the following is true.

(a) Both statement I and II are true and statement II is a correct explanation of statement I.
(b) Both statements I and II are true but statement II is not the correct explanations of statement I.
(c) Statement I is true and statement II is false.
(d) Statement I is false and statement II is correct.

17. ΔABC is a given triangle and AD is perpendicular to BC. It is given that length of three sides AB, BC, CA are rational numbers. Which one of the following is true ?
(a) AD and BD both must be rational.
(b) AD must be rational but BD is not necessarily rational.
(c) BD must be rational but AD is not necessarily rational.
(d) neither AD nor BD is necessarily rational.

18. Centroid of ΔABC is 8 cm away from vertex A. What is the length of median passing through vertex A ?
(a) 20 cm (b) 16 cm (c) 12 cm (d) 10 cm

19. If distance of a vertex of an equilateral triangle from its centroid is 6 cm then area of the triangle is
(a) 24 cm^2 (b) $27\sqrt{3}$ cm^2 (c) 12 cm^2 (d) $12\sqrt{3}$ cm^2

20. In ΔPQR, $PQ = 4$ cm, $QR = 3$ cm and $RP = 3.5$ cm, ΔDEF is similar to ΔPQR. If $EF = 9$ cm then perimeter of ΔDEF is—
(a) 10.5 cm (b) 21 cm (c) 31.5 cm
(d) Cannot be determined as data is insufficient

21. AD is an angle bisector of ΔABC and $BD : DC = 2 : 3$. If $AB = 7$ cm then $AC : BC$ is

(a) $2 : 3$ (b) $3 : 2$
(c) $21 : 10$ (d) data insufficient

22. Assertion (A) : AD is angle bisector of $\angle A$ of the triangle ABC. If $AB = 6$ cm, $BC = 7$ cm, $AC = 8$ cm then $BD = 3$ cm and $CD = 4$ cm.
Reason (R) : The angle bisector AD of the triangle divides base BC in the ratio $AB : AC$.

(a) Both Assertion A and Reason R are correct and Reason R is a correct explanation of Assertion A.
(b) Both Assertion A and Reason R are correct but Reason R is not the correct explanation of Assertion A.
(c) Assertion A is correct, Reason R is wrong.
(d) Assertion A is wrong, Reason R is correct.

23. O is the incentre of ABC and $\angle A = 30^\circ$. Accordingly what is $\angle BOC$?

(a) 100° (b) 105° (c) 110° (d) 90°

24. O is centroid of ΔABC and AD, BE, CF are its three medians. If area of ΔAOE is $15\ cm^2$, then area of quadrilateral $BDOF$ is—

(a) $20\ cm^2$ (b) $30\ cm^2$ (c) $40\ cm^2$ (d) $25\ cm^2$

25. O and C are respectively orthocentre and circumcentre of ΔPQR. Point P and O are joined and produced part meets side QR in S. If $\angle PQS = 60^\circ$ and $\angle QCR = 130^\circ$, then $\angle RPS = ?$

(a) 30° (b) 35° (c) 100° (d) 60°

26. From the circumcentre O of the triangle ABC perpendicular OD is drawn to BC. If $\angle BAC = 60^\circ$ then what is the measure of $\angle BOD$?

(a) 30° (b) 90° (c) 60° (d) 45°

27. O is the circumcentre of a triangle ABC. If $\angle BAC = 85^\circ$ and $\angle BCA = 75^\circ$ then what is the value of $\angle OAC$?

(a) 40° (b) 60° (c) 70° (d) 90°

28. In a triangle ABC medians CD and BE intersect at point O. What is the ratio of area of ΔODE and ΔABC ?

(a) $1 : 6$ (b) $6 : 1$ (c) $1 : 12$ (d) $12 : 1$

29. Suppose O be incentre of ΔABC and D is a point on side BC of ΔABC such that $OD \perp BC$. If $\angle BOD = 15^\circ$ then $\angle ABC = ?$

(a) 75° (b) 45° (c) 150° (d) 90°

30. The radius of incircle of an equilateral triangle is 3 cm. What is the length of each median of the triangle ABC ?

(a) 12 cm (b) $\frac{9}{2}$ cm (c) 4 cm (d) 9 cm

31. I is the incentre of the triangle ABC. If $\angle ABC = 60^\circ$ and $\angle ACB = 50^\circ$ then $\angle BIC$ is

(a) 55° (b) 125° (c) 70° (d) 65°

the value of $\angle BAC$? ... $\angle BOC = 110^\circ$, then wha...

(a) 20° (b) 40° (c) 55° (d) 110°

33. Which groups of centres of a triangle given below always lie in a straight line (i.e., centres are collinear)
   (a) Incentre, circumcentre, centroid
   (b) Incentre, orthocentre, centroid
   (c) Circumcentre, orthocentre, centroid
   (d) None of these

34. If distance between orthocentre and circumcentre of a triangle is 6 cm then what is the distance between its centroid and circumcentre?
   (a) 4 cm (b) 2 cm (c) $\frac{8}{3}$ cm (d) $\frac{4}{3}$ cm

35. Which pair of centres given below lie outside the triangle?
   (a) Circumcentre and centroid (b) Incentre and centroid
   (c) Circumcentre and orthocentre
   (d) None of these

36. What is the distance between circumcentre and orthocentre of a right angled triangle?
   (a) Equal to hypotenuse (b) Half to hypotenuse
   (c) One third to hypotenuse (d) Two third to hypotenuse

37. If hypotenuse of a right angle is 15 cm then what is the distance between its orthocentre and centroid?
   (a) 5 cm (b) 10 cm (c) $\frac{10}{3}$ cm (d) $\frac{20}{3}$ cm

38. A non right angle bisector of a right angled isosceles triangle divides the triangle in those two parts, whose area are in the ratio.
   (a) 1 : 1 (b) $1 : \sqrt{2}$ (c) 1 : 2 (d) $1 : \sqrt{2}-1$

39. In a ΔABC, $BC = 9$ cm, $AC = 40$ cm and $AB = 41$ cm. If bisector of angle A meets side BC at D then ratio of area of ΔABD ad ΔABC is
   (a) 40 : 41 (b) 9 : 40
   (c) 9 : 41 (d) 41 : 81

Directions (40–42) : In a triangle ABC, $AB = 5$ cm, $BC = 6$ cm and $CA = 7$ cm. If bisectors of $\angle A$, $\angle B$, $\angle C$ respectively meet sides BC, CA and AB at D, E, F and I be the incentre of the triangle then

40. What is $BD : DC$?
   (a) 5 : 7 (b) 7 : 5 (c) 5 : 6 (d) 6 : 5

41. What is the length of AE ?
   (a) $\frac{42}{11}$ cm (b) 6 cm (c) $\frac{13}{2}$ cm (d) $\frac{35}{11}$ cm

42. What is $CI : IF$?
   (a) 2 : 1 (b) 11 : 7 (c) 3 : 1 (d) 13 : 5

43. The semiperimeter of a triangle is S and its centroid is G. What is the distance between G and centroid of the triangle formed by mid points of the sides of the given triangle ?

(a) $\frac{5}{3}$ (b) $\frac{5}{6}$ (c) $\frac{5}{18}$ (d) 0

44. If length of three medians of a triangle are respectively 9 cm, 12 cm and 15 cm then what is the area of the triangle in cm^2 ?

(a) 48 (b) 72 (c) 96 (d) 36

45. If incentre of a isosceles right angled triangle is I then ratio of area of triangle formed by joining I to the respective vertices of triangle is

(a) $1:1:\sqrt{2}$ (b) $\frac{2-\sqrt{2}}{2}:\frac{2-\sqrt{2}}{2}:\sqrt{2}-1$

(c) $\sqrt{2}-1:\sqrt{2}-1:\sqrt{2}+1$ (d) None of these

46. In a right angled isosceles triangle $\angle C = 90°$ and I is its incentre their ratio of area of ΔAIB and ΔABC is

(a) $1:\sqrt{2}+1$ (b) $1:\sqrt{2}-1$ (c) $1:\sqrt{2}$ (d) $1:2$

47. A triangle is formed by joining mid points of sides of a triangle and a triangle is formed again by joining mid points of its sides. The ratio of area of this triangle to the area of original triangle is

(a) $1:4$ (b) $1:8$ (c) $1:16$ (d) $1:64$

Directions (48–51) : In a triangle ABC if side $AB = c = 4$ cm, side $AC = b = 6$ cm and $BC = a = 7$, then answer the following questions

48. The length of median AD is

(a) $\frac{53}{2}$ cm (b) $\frac{1}{2}\sqrt{55}$ cm (c) $\sqrt{\frac{53}{2}}$ cm (d) $\sqrt{\frac{63}{2}}$ cm

49. If AD is bisector of angle A then length of BD is

(a) $\frac{16}{5}$ (b) $\frac{21}{5}$ (c) $\frac{12}{5}$ (d) $\frac{14}{5}$

50. If AD be the altitude then what is BD in cm ?

(a) $\frac{29}{12}$ (b) $\frac{39}{12}$ (c) $\frac{29}{14}$ (d) $\frac{69}{14}$

51. If AD be the altitude then $BD : DC$?

(a) $29:69$ (b) $69:29$ (c) $29:39$ (d) $39:29$

52. If area of a triangle is 81 cm^2 and its semiperimeter is 27 cm then area of incircle of the triangle is

(a) 6π cm^2 (b) 3π cm^2 (c) 18π cm^2 (d) 9π cm^2

53. If sides of a triangle are respectively 5 cm, 6 cm and 7 cm then radius of the circumcircle of the triangle is

(a) 9 cm (b) $\frac{35}{\sqrt{6}}$ cm (c) $\frac{35}{4\sqrt{6}}$ cm (d) $\frac{17}{2}$ cm

54. The sides of a triangle are [illegible] radius of the circumcircle is
(a) 5 cm (b) 7.5 cm (c) 7 cm (d) 8.5 cm

55. The sides of a triangle are 8 cm, 15 cm and 17 cm. The sum of radii of circumcircle and incircle of the triangle is
(a) 23 cm (b) 11.5 cm (c) 25 cm (d) 12.5 cm

56. The sides of a triangle are 9 cm, 40 cm and 41 cm. The distance between its orthocentre and circumcentre is
(a) 29 cm (b) 20.5 cm (c) $\sqrt{29}$ cm (d) 15 cm

57. If ratio of sides of a triangle are $4 : 5 : 6$ then what is the ratio of circumradius and inradius ?
(a) $2 : 1$ (b) $16 : 7$ (c) $12 : 7$ (d) $16 : 5$

58. The greatest side of a triangle is two more than double of its smallest side while middle one is one unit less than greatest side. If the smallest side is equal to the least odd prime number then ratio of circumradius and inradius of the triangle is
(a) $12 : 7$ (b) $7 : 2$ (c) $7 : 3$ (d) $4 : 1$

59. In a triangle ABC if $A = 90^o$, $b = 3$ and $c = 4$ then $R : r$ is
(a) $5 : 3$ (b) $7 : 3$ (c) $3 : 2$ (d) $5 : 2$

60. If sides of a triangle are 3 cm, 4 cm and 5 cm then what is the distance between its incentre and circumcentre ?
(a) $\frac{5}{4}$ cm (b) $\sqrt{\frac{5}{2}}$ cm
(c) $\frac{\sqrt{5}}{2}$ cm (d) None of then

61. If triangle formed by medians of a right angled triangle is also a right angled triangle then what is the ratio of sides of the original right angled triangle ?
(a) $1 : \sqrt{2} : \sqrt{3}$ (b) $2 : \sqrt{3} : \sqrt{7}$
(c) $\sqrt{2} : \sqrt{3} : \sqrt{5}$ (d) $3 : 4 : 5$

Answers–6A

1. (b)	2. (c)	3. (a)	4. (d)	5. (c)	6. (c)	7. (c)	8. (b)
9. (a)	10. (b)	11. (d)	12. (c)	13. (c)	14. (b)	15. (a)	16. (a)
17. (c)	18. (c)	19. (b)	20. (c)	21. (c)	22. (a)	23. (b)	24. (b)
25. (b)	26. (c)	27. (a)	28. (c)	29. (c)	30. (d)	31. (b)	32. (b)
33. (c)	34. (b)	35. (c)	36. (b)	37. (a)	38. (b)	39. (d)	40. (a)
41. (d)	42. (d)	43. (d)	44. (b)	45. (b)	46. (a)	47. (c)	48. (b)
49. (d)	50. (c)	51. (a)	52. (d)	53. (c)	54. (a)	55. (b)	56. (b)
57. (b)	58. (b)	59. (d)	60. (c)	61. (a)			

,Exaplanation

1. (b) $\because$ $DE \parallel BC$ and : area (ΔADE) : area (trapezium $BDEC$) = 4 : 5

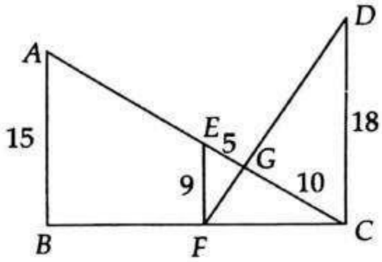
$\therefore$ $\Delta ABC \sim \Delta ADE$

$\Rightarrow \dfrac{\text{area of } \Delta ADE}{\text{area of } \Delta ABC} = \left(\dfrac{DE}{BC}\right)^2$

$\Rightarrow \dfrac{4}{4+5} = \left(\dfrac{DE}{BC}\right)^2$

$\Rightarrow DE : BC = 2 : 3$

2. (c) $\because$ $AB \parallel EF \parallel CD$



$\Rightarrow \dfrac{EG}{GC} = \dfrac{EF}{CD}$

$\Delta EGF \sim CGD$

$\Rightarrow \dfrac{EG}{CG} = \dfrac{EF}{CD} \Rightarrow \dfrac{5}{10} = \dfrac{EF}{18}$

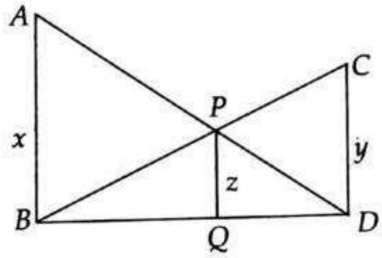
$\Rightarrow EF = 9$ cm

$\because$ $\Delta ABC \sim EFC$

$\therefore$ $\dfrac{EC}{AC} = \dfrac{EF}{AB} \Rightarrow \dfrac{15}{AC} = \dfrac{9}{15}$

$\Rightarrow AC = \dfrac{15 \times 15}{9} = 25$ cm

3. (a) $\because$ $\angle ABD = \angle PQD = 90°$



$\therefore$ $\Delta ABD \sim PQD$

$\Rightarrow \dfrac{x}{z} = \dfrac{BD}{QD}$ ... (i)

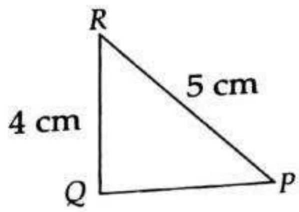
$\because$ $\angle CDB = \angle PQB = 90°$

$\therefore$ $\Delta BCD \sim \Delta BPQ$

$\Rightarrow \dfrac{z}{y} = \dfrac{BQ}{BD} \Rightarrow \dfrac{z}{y} = \dfrac{BD - QD}{BD} \Rightarrow \dfrac{z}{y} = 1 - \dfrac{QD}{BD}$

$\Rightarrow \dfrac{z}{y} = 1 - \dfrac{z}{x} \Rightarrow \dfrac{z}{x} + \dfrac{z}{y} = 1 \Rightarrow \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$

4. (d) In triangle PQR,



$QP^2 = (5)^2 - (4)^2 \Rightarrow QP = 3$

Since sides of ΔABC are also 3 cm, 4 cm, 5 cm therefore the two triangles are congruent.

5. (c) Ratio of medians of two equilateral triangle = ratio of their sides = 3 : 2

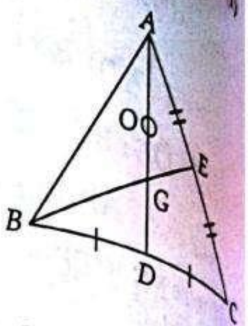
6. (c) All the centres in, an equilateral triangle are coincident, so centroid and orthocentre are also coincident

7. **(c)** Given, $AO : OD = 2 : 7$

$\therefore \quad OA = \frac{2}{9} AD, OD = \frac{7}{9} AD$

We know that centroid divides median in the ratio 2 : 1,

Therefore, $AG = \frac{2}{3} AD, GD = \frac{1}{3} AD$ ...(i)



(A) $OA = \frac{2}{9} AD$

$OA = \left(2 \cdot \frac{1}{3} AD\right)\frac{1}{3}$

$= (2GD)\,\frac{1}{3} = \frac{2GD}{3}$ [from (i)]

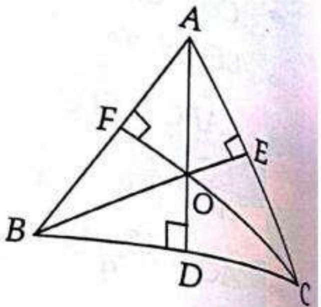
(R) $OD = \frac{7}{9} AD = \left(7 \cdot \frac{2}{3} AD\right) \cdot \frac{1}{3\times 2} = \frac{7AG}{6}$

Hence (A) is true and (R) is false.

8. **(b)** (A) We know that in a right angled triangle hypotenuse is the greatest side.

In ΔABD,

$AB^2 > AD^2$ ... (i)



In ΔBEC,

$BC^2 > BE^2$ ... (ii)

In ΔACF,

$AC^2 > CF^2$ ... (iii)

adding (i), (ii) and (iii)

$(AB^2 + BC^2 + AC^2) > (AD^2 + BE^2 + CF^2)$

Now (R), $(AE^2 - AF^2) + (BF^2 - BD^2) + (CD^2 - CE^2)$

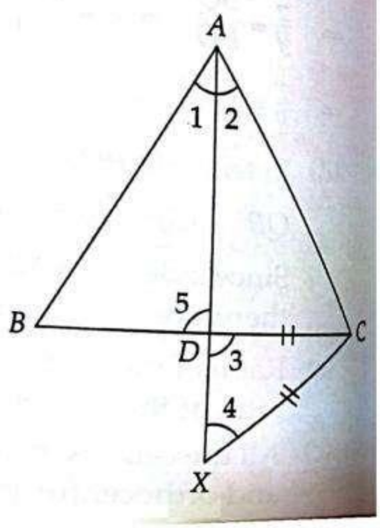
$= (OA^2 - OE^2) - (OA^2 - OF^2) + (OB^2 - OF^2)$

$- (OB^2 - OD^2) + (OC^2 - OD^2) - (OC^2 - OE^2)$

$= 0$

Both (A) and (R) ave correct but (R), (A) is not the correct explanation of (A).

9. **(a)** In ΔDCX,



$CD = CX$ (given)

$\angle 3 = \angle 4$ (opposite angles of equal sides)

but, $\angle 3 = \angle 5$

Hence, $\angle 4 = \angle 5$

In ΔABD and ΔACX,

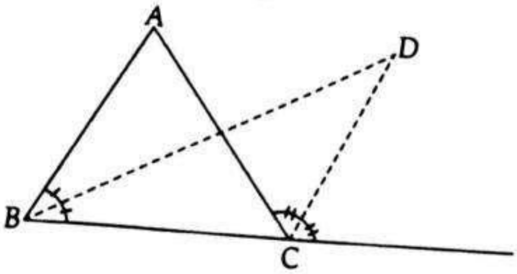
$\angle 1 = \angle 2$ (given)

$\angle 4 = \angle 5$

$\therefore \quad \Delta ABD \sim \Delta ACX$ ($A - A$ condition)

10. (b) In the plane of the triangle circumcentre is the only point which is equidistant from all the three vertices of the triangle.

11. (d) In ΔBCD,

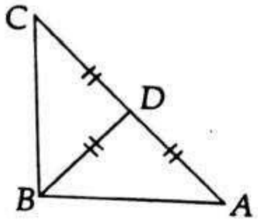


$\angle DBC = \frac{B}{2}, \angle BCD = \frac{A+B}{2}$

$\therefore \quad \angle BDC = \pi - \frac{B}{2} - \frac{A+B}{2}$

$= \pi - \frac{A}{2} - B$

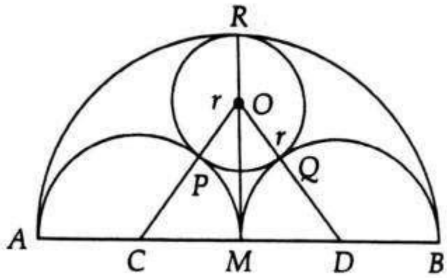
12. (c) Given,



$CD = BD = DA$

It is possible only when ΔABC is a right angled triangle.

13. (c) $AB = 2a \quad \Rightarrow \quad AM = a$



and $AC = CM = BD = MD = \frac{a}{2}$

Now, $OC = OP + PC = OP + CM$

$= r + \frac{a}{2}$

and $OD = OQ + QD = OQ + MD$

$= \left(r + \frac{a}{2}\right)$

$\therefore \quad \Delta OCD$ is an isosceles triangle and M is mid point of CD

$(\because \quad OC = OD)$

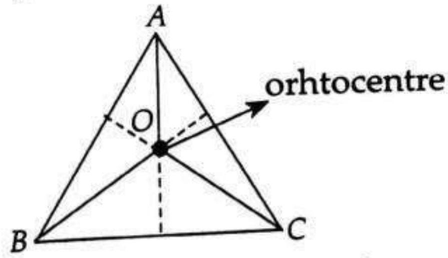
$\Rightarrow \quad \angle OMC = 90^\circ$

In ΔOMC, $OC^2 = OM^2 + CM^2$

$\Rightarrow \quad \left(r + \frac{a}{2}\right)^2 = (a - r)^2 + \left(\frac{a}{2}\right)^2$

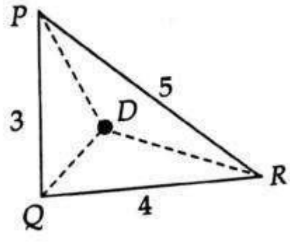
$\Rightarrow \quad r = \frac{a}{3}$

14. (b) Point of concurrency of altitudes of a triangle is called orthocentre



15. (a) One and only one circle passes through three non collinear points.

16. (a) $3^2 + 4^2 = 5^2 \Rightarrow$ triangle is right angled.



In ΔDQR, $DQ + DR > 4$

In ΔDPR, $DP + DR > 5$

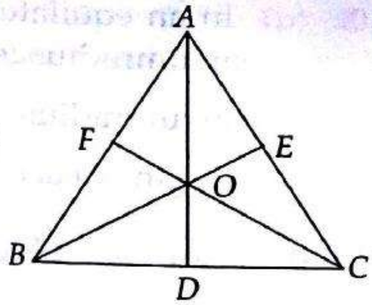
In ΔDQP, $DQ + DP > 3$

24. (b) See the figure,

Δ will be divided into 6 equal parts.

Area of quadrilateral $BDOF = 2 \times$ area of ΔOAE

$= 2 \times 15 = 30 \text{ cm}^2$



25. (b) In ΔPQR,

$\angle QPR = \frac{1}{2} \angle QCR = \frac{1}{2} \times 130° = 65°$

$\angle PQR = \angle PQS = 60°$ (given)

$\therefore \angle PRQ = 180° - 65° - 60° = 55°$

$\because$ O is the orthocentre

$\therefore \angle PSR = 90°$

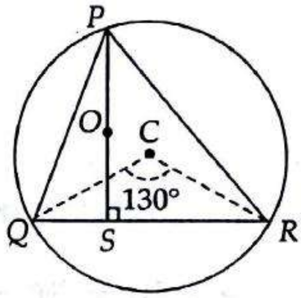
Thus in ΔPSR

$\angle RPS = 180° - 90° - \angle PRS$

$= 180° - 90° - 55°$ $\quad (\because \angle PRS = \angle PRQ = 90°)$

$= 35°$

$\therefore \angle RPS = 35°$



26. (c) $\angle BOD = \frac{1}{2} \times \angle BOC = \frac{1}{2} \times 120° = 60°$

27. (a) $\angle B = 180° - 75° - 85° = 20°$

$\therefore \angle OAC = 2\angle B = 40°$

28. (c) In ΔODE and ΔBOC,

$\angle BOC = \angle DOE$

$\angle DEO = \angle OBC$

$\angle ODE = \angle OCB$

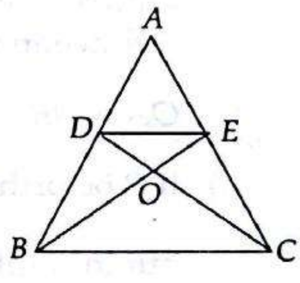
Both triangles are similar.

$\frac{\Delta ODE}{\Delta BOC} = \frac{DE^2}{BC^2}$

$DE \parallel BC$ and $DE = \frac{1}{2} BC$

Area of $\Delta ABC = 3 \times$ Area of ΔOBC

$\therefore \frac{\Delta ODE}{\Delta ABC} = \frac{\Delta ODE}{3 \times \Delta BOC} = \frac{1}{3} \cdot \frac{DE^2}{BC^2} = \frac{1}{3} \cdot \left(\frac{1}{2}\right)^2 = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$



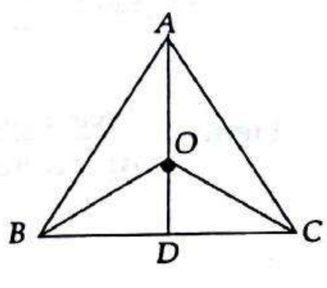
29. (c) BO is bisector of $\angle B$

$\angle ODB = 90°$;

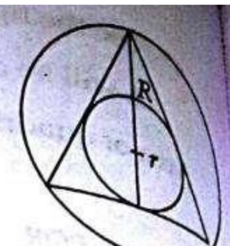
$\angle BOD = 15°$

$\angle OBD = 180° - 90° - 15° = 75°$

$\angle ABC = 2 \times 75° = 150°$



30. (d) In an equilateral triangle ratio of inradius to circumradius is 1 : 2.



$\therefore$ Circumradius = 6 cm

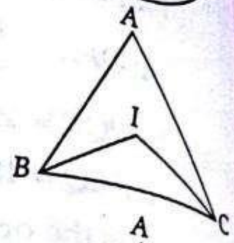
$\therefore$ Length of each median = 3 + 6 = 9 cm

31. (b) Shortcut : $\angle BIC = 90^\circ + \frac{A}{2}$

$$= 90^\circ + \frac{180^\circ - B - C}{2}$$

$$= 90^\circ + \frac{180^\circ - 60^\circ - 50^\circ}{2}$$

$$= 90^\circ + 35^\circ = 125^\circ$$

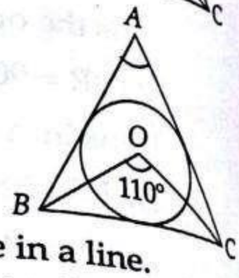


32. (b) Shortcut :

$\angle BOC = 90^\circ + \frac{A}{2}$

$\Rightarrow$ $110^\circ = 90^\circ + \frac{A}{2}$

$\Rightarrow$ $A = 40^\circ$.



33. (c) Note that except incentre all the three centres are in a line.

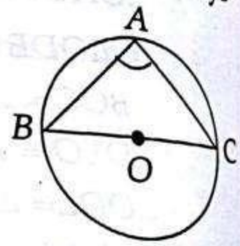
34. (b) The orthocentre (P), centroid (G) and circumcentre (O) always lie on a straight line and $PG : GO = 2 : 1$.

As in question $PO = 6$ cm

$\therefore$ $OG = \frac{1}{3} \times 6 = 2$ cm

35. (c) In an obtused angled triangle circumcentre and orthocentre always lie outside the circle.

36. (b) In the given figure ΔBAC is a right angled triangle. which subtends right angle at A. Clearly A is the orthocentre and O is the circumcentre.



$\therefore$ $OA = OB = OC = \text{radius} = \frac{BC}{2} = \frac{\text{hypotenuse}}{2}$

37. (a) If P be orthocentre, G be centroid, O be circumcentre then $\frac{PG}{OG} = \frac{2}{1}$

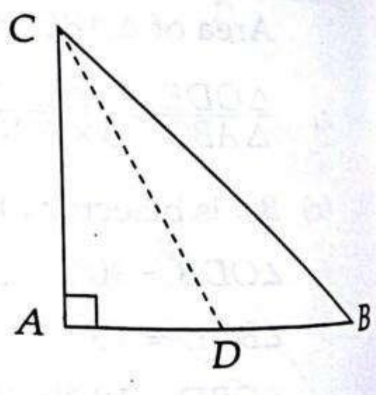
But in right angled triangle $OP = \frac{\text{hypotenuse}}{2} = \frac{15}{2}$ cm

$\therefore$ $PG = \frac{2}{3} OP = \frac{2}{3} \times \frac{15}{2} = 5$ cm

38. (b) In figure, $\angle A = 90^\circ$, $\angle B = \angle C = 45^\circ$

CD, is bisector of non right angle C.

We have $\frac{AD}{DB} = \frac{AC}{BC} = \frac{k}{\sqrt{k^2 + k^2}} = \frac{1}{\sqrt{2}}$



Hence, $\frac{\text{Area } \Delta ACD}{\text{Area } \Delta ABD} = \frac{\frac{1}{2} \times AD \times height}{\frac{1}{2} \times DB \times height}$

$= \frac{AD}{DB}$, (height of both triangles are equal) $= 1 : \sqrt{2}$

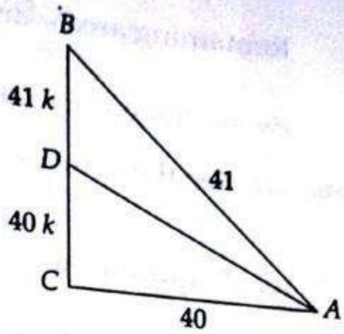
39. (d) $\because 9^2 + 40^2 = 41^2$

$\therefore \Delta ABC$ is a right angled triangle with $\angle C = 90°$

See the figure,

$\because AD$ is bisector of $\angle A$.

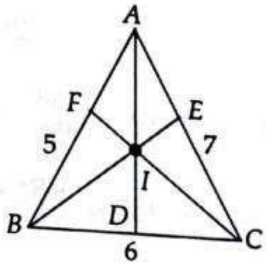
$\therefore \frac{CD}{BD} = \frac{AC}{AB} = \frac{40}{41} \Rightarrow CD = 40k, BD = 41k$



$$\frac{\text{Area } \Delta ABD}{\text{Area } \Delta ABC} = \frac{\frac{1}{2} \times BD \times AC}{\frac{1}{2} \times BC \times AC}$$

$$= \frac{BD}{BC} = \frac{41k}{40k + 41k} = \frac{41}{81}$$

40. (a) $BD : DC = \frac{AB}{AC} = \frac{5}{7}$



41. (d) $\because \frac{AE}{CE} = \frac{AB}{BC} = \frac{5}{6}$

$\therefore$ AE $= \frac{5}{5+6} \times 7 = \frac{35}{11}$ cm

42. (d) Recall that $\frac{CI}{IF} = \frac{CA + CB}{AB} = \frac{7+6}{5} = \frac{13}{5}$

43. (d) The two triangles mentioned in the questions have centroid at the same point.

44. (b) $\because 9^2 + 12^2 = 81 + 144 = 225 = 15^2$

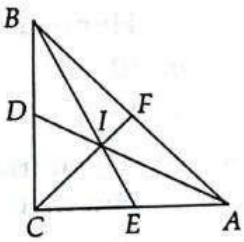
$\therefore$ 9, 12 and 15 are sides of a right angled triangle.

Area of triangle $= \frac{4}{3}$ (Area of triangle formed by taking medians as side of the triangle)

$$= \frac{4}{3} \times \left(\frac{1}{2} \times 9 \times 12\right) = 4 \times 9 \times 2 = 72 \text{ cm}^2$$

45. (b) Let in ΔABC, $\angle C = 90°$, $AC = BC = x$

then $AB = \sqrt{x^2 + x^2} = \sqrt{2}\,x$



$\frac{CI}{IF} = \frac{CA + CB}{AB} = \frac{x + x}{\sqrt{2}\,x} = \frac{2}{\sqrt{2}} = \frac{\sqrt{2}}{1}$ (here, $CF \perp r\ AB$)

$$\therefore \frac{\text{area } \Delta AIB}{\text{area } \Delta ABC} = \frac{\frac{1}{2} \times IF \times AB}{\frac{1}{2} \times CF \times AB}$$

$$= \frac{IF}{CF} = \frac{IF}{CI + IF} = \frac{1}{\sqrt{2} + 1}$$

$$= \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \sqrt{2}-1$$

Remaining area = area of (Δ CIA) + area of (Δ CID) $= 1 - (\sqrt{2} - 1) = 2 - \sqrt{2}$

By symmetry area of $\Delta\, CIA$ = area of $\Delta\, CIB = \frac{2-\sqrt{2}}{2}$

46. **(a)** See the solution of question no. 45.

47. **(c)** Required ratio $= \frac{\frac{1}{4} \times \frac{1}{4}}{1} = \frac{1}{16}$

48. **(b)** $AD = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

$$= \frac{1}{2}\sqrt{2(6)^2 + 2(4)^2 - 7^2} = \frac{1}{2}\sqrt{72 + 32 - 49} = \frac{1}{2}\sqrt{55}$$

49. **(d)** $BD = \frac{ac}{b+c} = \frac{7 \times 4}{6+4} = \frac{28}{10} = \frac{14}{5}$

50. **(c)** $BD = \frac{AB^2 + BC^2 - AC^2}{2BC} = \frac{c^2 + a^2 - b^2}{2a} = \frac{4^2 + 7^2 - 6^2}{2 \times 7} = \frac{29}{14}$

51. **(a)** From above question, $BD = \frac{29}{14}$

and $CD = \frac{b^2 + a^2 - c^2}{2a} = \frac{36 + 49 - 14}{2 \times 7} = \frac{69}{14}$

$\therefore\ BD : DC = 29 : 69$

52. **(d)** $r = \frac{\Delta}{s} = \frac{81}{27} = 3$ cm

$\therefore$ Area $= \pi r^2 = \pi(3)^2 = 9\pi$ cm^2

53. **(c)** Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{9 \times 4 \times 3 \times 2} = 6\sqrt{6}$ cm^2 $\quad \left(\because s = \frac{5+6+7}{2} = 9\right)$

$\therefore\ R = \frac{abc}{4\Delta} = \frac{5 \times 6 \times 7}{4 \times 6\sqrt{6}} = \frac{35}{4\sqrt{6}}$ cm

54. **(a)** $\because\ 6^2 + 8^2 = 10^2$

$\therefore$ Given triangle is right angled.

$\therefore$ Hypotenuse = diameter of circumcircle

or, $10 = 2r$

$\Rightarrow\ r = 5$

55. **(b)** Given triangle is a right angled triangle. If radius of its incircle is r and that of circumcircle is R then

$2(r + R) = a + b,$ (where a and b are perpendicular sides)

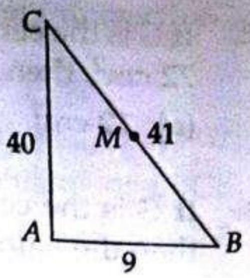
or, $2(r + R) = 8 + 15 = 23$

or, $r + R = \frac{23}{2} = 11.5$ cm

6. (b) $\because$ $9^2 + 40^2 = 81 + 1600 = 1681 = 41^2$

$\therefore$ Given triangle is right angled.

If $AB = 9$, $AC = 40$ and $BC = 41$ then A is orthocentre and mid point of hypotenuse BC is circumcentre of the triangle.



$\therefore$ $AM = BM = CM =$ radius of circumcircle.

or, $AM = \frac{41}{2} = 20.5$ cm

57. (b) Let $a = 4k$, $b = 5k$, $c = 6k$, $s = \frac{4k+5k+6k}{2} = \frac{15k}{2}$

$$\therefore \frac{R}{r} = \frac{\left(\frac{abc}{4\Delta}\right)}{\left(\frac{\Delta}{s}\right)} = \frac{abcs}{4\Delta^2} = \frac{abcs}{4s(s-a)(s-b)(s-c)}$$

$$= \frac{abc}{4(s-a)(s-b)(s-c)} = \frac{(4k)(5k)(6k)}{4\left(\frac{7k}{2}\right)\left(\frac{5k}{2}\right)\left(\frac{3k}{2}\right)}$$

$$= \frac{4\times5\times6\times8}{4\times7\times5\times3} = \frac{6\times8}{7\times3} = \frac{16}{7}$$

58. (b) Least odd prime number = 3 = smallest side

$\therefore$ Greatest side $= 2 \times 3 + 2 = 8$ and middle side $= 8 - 1 = 7$

Now, solve as in above questions.

59. (d) Given triangle is a right angled triangle.

$$\therefore a = \sqrt{3^2+4^2} = 5 = 2R \Rightarrow R = \frac{5}{2}$$

$$r = \frac{\Delta}{s} = \frac{\frac{1}{2}\cdot bc}{\frac{a+b+c}{2}} = \frac{bc}{a+b+c} = \frac{3\times4}{3+4+5} = 1$$

$\therefore$ $R : r = 5 : 2$

60. (c) Distance between incentre and circumcentre $= \sqrt{R^2 - 2Rr}$

From above question $R = \frac{5}{2}$ and $r = 1$

Required distance $= \left(\frac{5}{2}\right)^2 - 2\times\frac{5}{2}\times1 = \sqrt{\frac{25}{4} - 5} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$ cm

61. (a) Required ratio is $1 : \sqrt{2} : \sqrt{3}$. Learn it and try to prove it.

Exercise–6B

1. In ΔABC, AD is the median and $AD = \frac{1}{2}BC$. If $\angle BAD = 30°$, then measure of $\angle ACB$ is
(a) 30° (b) 60° (c) 90° (d) 45°
[SSC Tier-I 2012]

2. If G is the centroid and AD, BE, ... medians of ΔABC with ... 72 cm^2, then the area of ΔBDG is
(a) 12 cm^2 (b) 16 cm^2 (c) 24 cm^2 (d) 8 cm^2
[SSC Tier-I 20...

3. If G is the centroid and AD be a median with length 12 cm of ΔAB..., then the value of AG is
(a) 4 cm (b) 8 cm (c) 10 cm (d) 6 cm
[SSC Tier-I 20...

4. O is the orthocentre of the triangle ABC. If $\angle BOC = 120°$, then $\angle BAC$ is
(a) 150° (b) 60° (c) 135° (d) 90°
[SSC Tier-I 20...

5. Circumcentre of ΔABC is O. If $\angle BAC = 85°$, $\angle BCA = 80°$, then $\angle OAC$ is
(a) 80° (b) 30° (c) 60° (d) 75°
[SSC Tier-I 20...

6. The length of the circum-radius of a triangle having sides of length 12 cm, 16 cm and 20 cm is
(a) 15 cm (b) 10 cm (c) 18 cm (d) 16 cm
[SSC Tier-I 2012]

7. If D is the mid-point of the side BC of ΔABC and the area of ΔABD is 16 cm^2, then the area of ΔABC is
(a) 16 cm^2 (b) 24 cm^2 (c) 32 cm^2 (d) 48 cm^2
[SSC Tier-I 2012]

8. ABC is a triangle. The medians CD and BE intersect each other at O. Then $\Delta ODE : \Delta ABC$ is
(a) 1 : 3 (b) 1 : 4 (c) 1 : 6 (d) 1 : 12
[SSC Tier-I 2012]

9. AB is a diameter of the circumcircle of ΔAPB ; N is the foot of the perpendicular drawn from the point P on AB. If $AP = 8$ cm and $BP = 6$ cm, then the length of BN is
(a) 3·6 cm (b) 3 cm (c) 3·4 cm (d) 3·5 cm
[SSC Tier-I 2012]

10. The bisector of $\angle A$ of ΔABC cuts BC at D and the circumcircle of the triangle at E. Then
(a) $AB : AC = BD : DC$
(b) $AD : AC = AE : AB$
(c) $AB : AD = AC : AE$
(d) $AB : AD = AE : AC$
[SSC Tier-I 2012]

11. O is the centre of the circle passing through the points A, B and C such that $\angle BAO = 30°$, $\angle BCO = 40°$ and $\angle AOC = x°$. What is the value of x?
(a) 70° (b) 140° (c) 210° (d) 280°
[SSC Tier-I 2012]

12. In an obtuse angled triangle ABC, $\angle A$ is the obtuse angle and O is the orthocentre. If $\angle BOC = 54°$, then $\angle BAC$ is
(a) 108° (b) 126° (c) 136° (d) 116°
[SSC Tier-I 2012]

13. Let BE and CF be the two medians of a ΔABC and G be their intersection. Also let EF cut AG at O. Then $AO : OG$ is

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 3 : 1

[SSC Tier-I 2012]

14. If S is the circumcentre of ΔABC and $\angle A = 50^\circ$, then the value of $\angle BCS$ is

(a) 20° (b) 40° (c) 60° (d) 80°

[SSC Tier-I 2012]

15. If I is the in-centre of ΔABC and $\angle A = 60^\circ$, then the value of $\angle BIC$ is

(a) 100° (b) 120° (c) 150° (d) 110°

[SSC Tier-I 2012]

16. O is the circum centre of the triangle ABC with circumradius 13 cm. Let BC = 24 cm and OD is perpendicular to BC. Then the length of OD is

(a) 3 cm (b) 4 cm (c) 5 cm (d) 7 cm

[SSC Tier-I 2012]

17. If G is the centroid of ΔABC and $AG = BC$ then $\angle BGC$ is

(a) 45° (b) 90° (c) 60° (d) 75°

[SSC Tier-I 2012]

18. The three medians AD, BE and CF of ΔABC intersect at point G. If the area of ΔABC is 60 sq. cm then the area of the quadrilateral $BDGF$ is

(a) 15 sq. cm (b) 20 sq. cm (c) 30 sq. cm (d) 10 sq. cm

[SSC Tier-I 2012]

19. In a ΔABC, $\angle B = 90^\circ$, $\angle C = 45^\circ$ and D is mid point of AC. If $AC = 4\sqrt{2}$ unit then BD is

(a) $\frac{5}{2}$ unit (b) 2 unit (c) $2\sqrt{2}$ unit (d) $4\sqrt{2}$ unit

[SSC Tier-I 2012]

Answers–6B

1. (b)	2. (a)	3. (b)	4. (b)	5. (d)	6. (b)	7. (c)	8. (d)
9. (a)	10. (a)	11. (b)	12. (b)	13. (d)	14. (b)	15. (b)	16. (c)
17. (b)	18. (b)	19. (c)					

Exaplanation

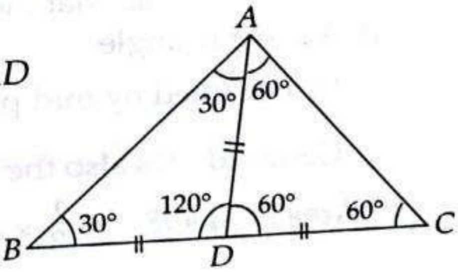
1. (b) $AD = \frac{1}{2}BC \Rightarrow AD = CD = BD$

In ΔABD, $AD = BD \Rightarrow \angle ABD = \angle BAD$

or, $\angle ABD = 30^\circ$

$\therefore \angle ADB = 180^\circ - 30^\circ - 30^\circ = 120^\circ$

And $\angle ADC = 180^\circ - 120^\circ = 60^\circ$



But, $AD = CD \Rightarrow \angle ACD = \angle DAC$

$\Rightarrow \angle ACD + \angle DAC = 180^\circ - 60^\circ = 120^\circ$

$\therefore \angle ACD = \underline{120^\circ} = 60^\circ = \angle ACB$

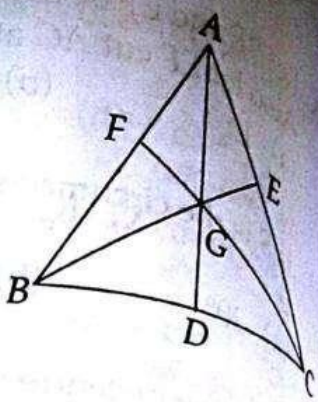
2. (a) If G be the centroid of the triangle then

area of $\Delta BGC = \frac{1}{3}$ (area of ΔABC)

$= \frac{1}{3} \times 72 = 24 \text{ cm}^2$

$\therefore$ area of $\Delta BGD = \frac{1}{2} \times$ area of ΔBGC

$= \frac{1}{2} \times 24 \text{ cm}^2 = 12 \text{ cm}^2$



3. (b) $AG = \frac{2}{3} \times AD = \frac{2}{3} \times 12 = 8$ cm

4. (b) In figure AD, BE and CF are altitudes

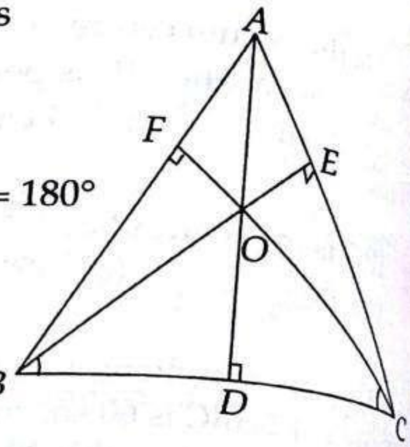
In ΔBFC, $\angle BCF = 90° - B$

In ΔBEC, $\angle CBE = 90° - C$

$\therefore$ In ΔBOC, $120° + (90° - B) + (90° - C) = 180°$

or, $B + C = 120°$

$\therefore$ $\angle A = 180° - 120° = 60°$



5. (d) $\angle ABC = 180° - 80° - 85° = 15°$

$\therefore$ $\angle AOC = 2 \times \angle ABC = 30°$

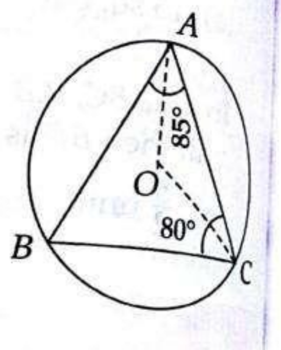
In ΔOAC,

Let $\angle OAC = \theta$

$\therefore$ $\angle OCA = \theta$ ($\because$ $OA = OC$ = radius)

$\therefore$ $\theta + \theta + 30° = 180°$

$\Rightarrow$ $\theta = 75°$



6. (b) $\because 12^2 + 16^2 = 20^2$

$\therefore$ This is a right angled triangle. The diameter of the circumcircle of the triangle is hypotenus of the triangle (Recall that angle of semicircle is right angle)

$\therefore$ Circumradius $= \frac{\text{hypotenuse}}{2} = \frac{20}{2} = 10$ cm

7. (c) Area of $\Delta ABC = 2 \times$ Area of $\Delta ABD = 2 \times 16 = 32 \text{ cm}^2$

(Recall that medians divides triangle into two equal parts)

8. (d) Area of triangle

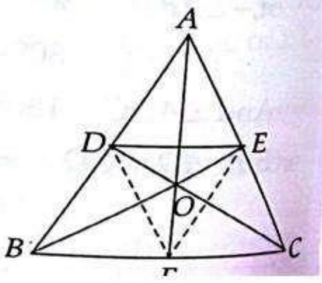
DEF formed by mid points $D, E, F = \frac{1}{4} \times$ (Area of ΔABC)

Centroid O is also the centroid of ΔDEF.

$\therefore$ Area of $\Delta DOE = \frac{1}{3} \times$ (Area of ΔDEF)

$= \frac{1}{3} \times \left(\frac{1}{4} \text{Area of } \Delta ABC\right)$

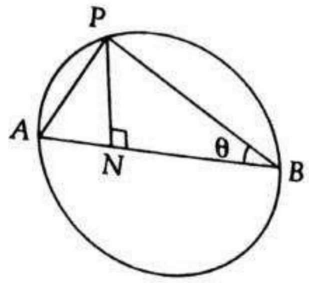
$= \frac{1}{12} \times$ Area of ΔABC



9. (a) Since AB is a diameter of circumcircle of ΔAPB, therefore $\angle APB = 90°$.
Thus triangle is right angled.

$\therefore AB = \sqrt{6^2 + 8^2} = 10$

Area of the triangle $\frac{1}{2} \times AP \times BP = \frac{1}{2} \times PN \times AB$



or, $\frac{1}{2} \times 8 \times 6 = \frac{1}{2} \times PN \times 10$

or, $PN = \frac{48}{10} = \frac{24}{5}$

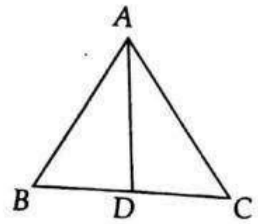
$\therefore BN = \sqrt{PB^2 - (PN)^2}$

$= \sqrt{6^2 - \left(\frac{24}{5}\right)^2} = \frac{\sqrt{30^2 - 24^2}}{5} = \frac{18}{5} = 3{\cdot}6$

10. (a) If AD is bisector of $\angle A$

then $\frac{AB}{AC} = \frac{BD}{DC}$

(It is very important property, learn it)



11. (b) $\because OA = OB =$ radius of circle

$\therefore \angle OBA = \angle OAB = 30°$

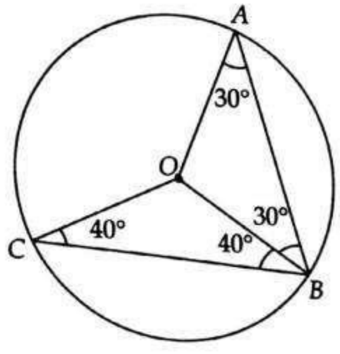
and $\angle AOB = 180° - 30° - 30° = 120°$

Similarly in ΔOBC,

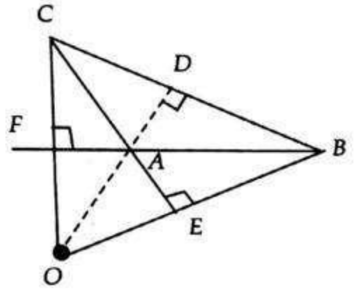
$\angle BOC = 180° - 40° - 40° = 100°$

From figure, $\angle AOC = 360° - \angle AOB - \angle BOC$

$= 360° - 120° - 100° = 140°$



12. (b) In the given figure ABC is an obtused angle triangle. $AD \perp BC$, $CF \perp BA$ (on produced part) and $BE \perp CA$ (on produced part). Altitudes AD, CF and BE, intersect at point O.



Concentrate on Quadrilateral $AFOE$,

Here, $\angle AFO = 90°$, $\angle AEO = 90°$

and $\angle EOF = \angle BOC = 54°$

$\therefore \angle FAE = 360° - 90° - 90° - 54° = 126°$

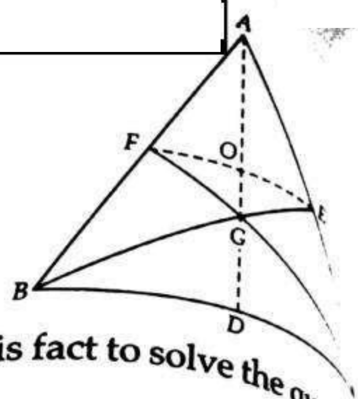
From vertically opposite angle, $\angle BAC = \angle FAE = 126°$

13. (d) $\Delta AOE \sim \Delta ADC$ $(\because FE \,||\, BC \Rightarrow \angle AEO = \angle ACD)$

$\therefore \frac{AO}{AD} = \frac{AE}{AC} = \frac{1}{2}$ $(\because E$ is mid point of $AC)$

But $\frac{AG}{AD} = \frac{2}{3}$

∵ $AD \wedge AG = 2 \wedge 2 = 4$



$\Rightarrow \quad 4AO = 3AG$

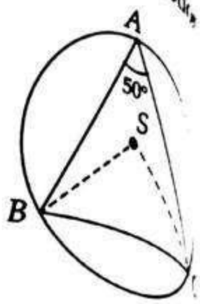
$\Rightarrow \quad 4AO = 3(AO + OG)$

$\Rightarrow \quad AO = 3OG$

$\therefore \quad \dfrac{AO}{OG} = \dfrac{3}{1}$

[Shortcut : O is mid point of AD. Take help of this fact to solve the question.

14. (b) $\angle BSC = 2 \times 50^\circ = 100^\circ$



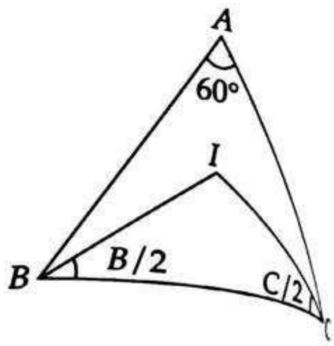
$\because \quad \angle SBC = \angle SCB$

$\therefore$ In ΔBSE

$100^\circ + 2\angle BCS = 180^\circ$

$\Rightarrow \quad \angle BCS = \dfrac{80^\circ}{2} = 40^\circ$

15. (b) $\angle BIC = 180^\circ - \dfrac{B}{2} - \dfrac{C}{2}$

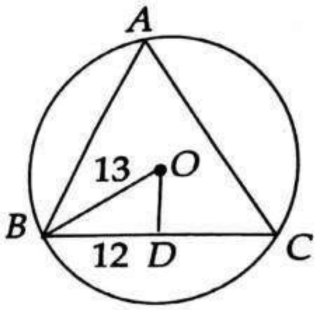


$= 180^\circ - \left(\dfrac{B+C}{2}\right) \quad = 180^\circ - \left(\dfrac{180^\circ - A}{2}\right)$

$= 90^\circ + \dfrac{A}{2}$ (shortcut, learn it direct)

$= 90^\circ + \dfrac{60^\circ}{2} = 120^\circ$

16. (c) See the figure



$$OD = \sqrt{OB^2 - BD^2} = \sqrt{169 - 144} = \sqrt{25} = 5$$

17. (b) $\because \; GD = \dfrac{1}{2} AG$

$\therefore \; GD = \dfrac{1}{2} BC$ $\quad (\because AG = BC)$

or, $GD = CD$ and $GD = BD$

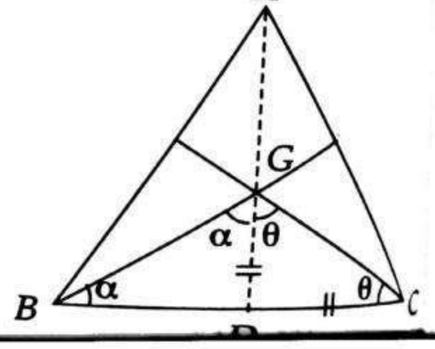
$\therefore \quad \angle DGC = \angle DCG = \theta$ (See the figure)

$\angle DBG = \angle BGD = \alpha$ (See the figure)

In ΔBGC,

$\alpha + \alpha + \theta + \theta = 180$

or, $\alpha + \theta = 90^\circ = \angle BGC$

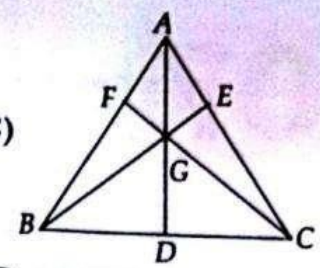


18. (b) area of $BDGF$

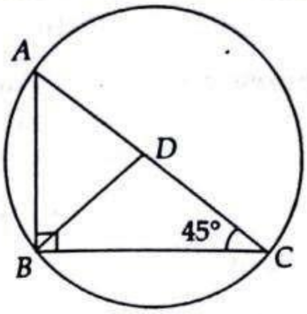
$= \text{Area of } (\Delta BDG) + \text{Area of } (\Delta BGF)$

$= \frac{1}{2} \text{Area of } (\Delta BGC) + \text{Area of } (\Delta ABG)$

$= \frac{1}{2}\left(\frac{1}{3} \times 60 + \frac{1}{3} \times 60\right) = 20 \text{ cm}^2.$



19. (c) ΔABC lies on the semicircle whose centre is D.



$\therefore \quad BD = CD = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$

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