Note

(ਬਹੁਪਦ)
Polynomials

Monomials	Binomials	Trinomials
ਇੱਕ ਪਦੀ	ਦੋ ਪਦੀ	ਤਿੰਨ ਪਦੀ
$4x$	$4x + 7$	$x^8 + 3x^4 + 5$

(ਬਹੁਪਦ)
Polynomials   (ਘਾਤ ਦੇ ਆਧਾਰ ਤੇ)

Linear	Quadratic	Cubic
ਰੇਖੀ	ਦੋ ਘਾਤੀ	ਤਿੰਨ ਘਾਤੀ
$4x + 7$	$4x^2 + 7$	$4x^3 + 7$

Exercise : 2.1

Q: ① ਕਿਹੜਾ ਬਹੁਪਦ ਇੱਕ ਚਲ ਵਿੱਚ ਹੈ ਤੇ ਕਿਹੜਾ ਨਹੀਂ ਹੈ?

Which of following expressions are polynomial in one variable and which are not?

(i) $4x^2 - 3x + 7$

ਇੱਕ ਚਲ ਵਾਲਾ ਬਹੁਪਦ ਹੈ ਕਿਉਂਕਿ x ਦੀਆਂ ਸਾਰੀਆਂ ਘਾਤਾਂ ਪੂਰਨ ਸੰਖਿਆਵਾਂ ਹਨ।

It is a polynomial in one variable x because all the exponents of x are whole numbers

(ii) $y^2 + \sqrt{2}$

ਇੱਕ ਚਲ ਵਾਲਾ ਘਾਤਅੰਕ ਹੈ ਕਿਉਂਕਿ y ਦੀ ਘਾਤ ਇੱਕ ਪੂਰਨ ਸੰਖਿਆ ਹੈ।

It is a polynomial in one variable y because exponent of y is a whole number $\times 1$

(iii) $3\sqrt{t} + t\sqrt{2} = 3t^{1/2} + t\sqrt{2}$

ਇਹ ਘਾਤਅੰਕ ਨਹੀਂ ਹੈ ਕਿਉਂਕਿ ਇੱਥੇ t ਦੀ ਘਾਤ $\frac{1}{2}$ ਹੈ ਜੋ ਕਿ ਪੂਰਨ ਸੰਖਿਆ ਨਹੀਂ ਹੈ।

It is not a polynomial because here exponent of t is $\frac{1}{2}$ which is not a whole number.

(iv) $y + \dfrac{2}{y} = y + 2y^{-1}$

ਇਹ ਘਾਤਅੰਕ ਨਹੀਂ ਹੈ ਕਿਉਂਕਿ ਇੱਥੇ y ਦੀ ਘਾਤ -1 ਹੈ ਜੋ ਕਿ ਇੱਕ ਪੂਰਨ ਸੰਖਿਆ ਨਹੀਂ ਹੈ।

It is not a polynomial because here exponent of y is -1 which is not a whole number.

(v) $x^{10} + y^3 + t^{50}$

ਇਹ ਇੱਕ ਚਲ ਵਾਲਾ ਘਾਤਅੰਕ ਨਹੀਂ ਹੈ ਕਿਉਂਕਿ ਇਸ ਵਿੱਚ ਤਿੰਨ ਚਲ ਹਨ x, y, t.

It is not a polynomial in one variable because there are three variables x, y, t.

Q (2): x^2 ਦਾ ਗੁਣਾਂਕ ਲਿਖੋ। Write coefficient of x^2

(i) $2 + x^2 + x$

x^2 ਦਾ ਗੁਣਾਂਕ $= 1$

Coefficient of $x^2 = 1$

(iii) $\dfrac{\pi}{2} x^2 + x$

x^2 ਦਾ ਗੁਣਾਂਕ $= \dfrac{\pi}{2}$

Coefficient of $x^2 = \dfrac{\pi}{2}$

(ii) $2 - x^2 + x^3$

x^2 ਦਾ ਗੁਣਾਂਕ $= -1$

Coefficient of $x^2 = -1$

(iv) $\sqrt{2}\, x - 1$

x^2 ਦਾ ਗੁਣਾਂਕ $= 0$

Coefficient of $x^2 = 0$

Q (3): 35 ਘਾਤ ਦੇ ਦੋ ਪਦ ਦਾ ਅਤੇ ਘਾਤ 100 ਦੇ ਇੱਕ ਪਦੀ ਦਾ ਇੱਕ -2 ਉਦਾਹਰਣ ਲਿਖੋ।

Give one example each of a binomial of degree 35 and of a monomial of degree 100,

Sol: 35 ਘਾਤ ਦਾ 2 ਪਦੀ $= x^{35} - 7$

100 ਘਾਤ ਦਾ ਇੱਕ ਪਦੀ $= x^{100}$

Binomial of degree 35 is $x^{35} - 7$

Monomial of degree 100 is x^{100}

Q. (4): ਹਰੇਕ ਬਹੁਪਦ ਦੀ ਘਾਤ ਲਿਖੋ।

Write degree of each polynomial

(i) $5x^3 + 4x^2 + 7x$.

ਘਾਤ $= 3$

Degree $= 3$

(ii) $4-y^2$

ਘਾਤ $3 = 2$ ⟶ Degree $= 2$

(iii) $5t-\sqrt{7}$

ਘਾਤ $3 = 1$ ⟶ Degree $= 1$

(iv) $3 = 3x^0$

ਘਾਤ $3 = 0$ ⟶ Degree $= 0$.

Q(5) Classify the following as linear, quadratic, cubic:

ਕਿੜਾ ਬਹੁਪਦ ਰੇਖੀ ਹੈ ਕਿੜਾ ਦੋ-ਘਾਤੀ ਤੇ ਕਿੜਾ ਤਿੰਨ ਘਾਤੀ

(i) x^2+x ਦੋਘਾਤੀ ਬਹੁਪਦ   Quadratic polynomial

(ii) $x-x^3$ ਤਿੰਨ ਘਾਤੀ ਬਹੁਪਦ   Cubic polynomial

(iii) $y+y^2+4$ ਦੋ ਘਾਤੀ ਬਹੁਪਦ   Quadratic polynomial

(iv) $1+x$ ਰੇਖੀ ਬਹੁਪਦ   Linear polynomial

(v) $3t$ ਰੇਖੀ ਬਹੁਪਦ   Linear polynomial

(vi) r^2 ਦੋਘਾਤੀ ਬਹੁਪਦ   Quadratic polynomial

(vii) $7x^3$ ਤਿੰਨ ਘਾਤੀ ਬਹੁਪਦ   Cubic polynomial

ਘੁਪਦ $5x - 4x^2 + 3$ ਦਾ ਅੰਸ ਲੇਵੋ।

Q ①

Find the value of polynomial $5x - 4x^2 + 3$

(i) $x = 0$

$$p(x) = 5x - 4x^2 + 3$$
$$p(0) = 5(0) - 4(0)^2 + 3$$
$$= 0 - 0 + 3$$
$$\therefore p(0) = 3 \quad \text{Ans.}$$

(ii) $x = -1$

$$p(x) = 5x - 4x^2 + 3$$
$$p(-1) = 5(-1) - 4(-1)^2 + 3$$
$$= -5 - 4(1) + 3$$
$$= -5 - 4 + 3$$
$$= -9 + 3$$
$$\therefore p(-1) = -6 \quad \text{Ans.}$$

(iii) $x = 2$

$$p(x) = 5x - 4x^2 + 3$$
$$p(2) = 5(2) - 4(2)^2 + 3$$
$$= 10 - 4(4) + 3$$
$$= 10 - 16 + 3$$
$$= 10 + 3 - 16$$
$$= 13 - 16$$
$$\therefore p(2) = -3 \quad \text{Ans.}$$

Q-②: $p(0), p(1)$ ਅਤੇ $p(2)$ ਲੇਵੋ :- Find $p(0), p(1)$ and $p(2)$

(i) $p(y) = y^2 - y + 1$
$$p(0) = 0^2 - 0 + 1$$
$$\therefore p(0) = 1 \quad \text{Ans.}$$
$$p(1) = (1)^2 - 1 + 1$$
$$\therefore p(1) = 1 - 1 + 1 = 1 \quad \text{Ans.}$$
$$p(2) = (2)^2 - 2 + 1$$
$$= 4 - 2 + 1 = 2 + 1$$
$$\therefore p(2) = 3 \quad \text{Ans.}$$

(iii) $p(x) = x^3$
$$p(0) = 0^3 = 0$$
$$p(1) = (1)^3 = 1$$
$$p(2) = (2)^3 = 8$$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

$p(0) = 2 + 0 + 2(0)^2 - 0^3$

$\quad = 2 + 0$

$\Rightarrow p(0) = 2$

$p(1) = 2 + 1 + 2(1)^2 - (1)^3$

$\quad = 3 + 2 - 1$

$\quad = 5 - 1 = 4$

$\Rightarrow p(1) = 4$

$p(2) = 2 + 2 + 2(2)^2 - (2)^3$

$\quad = 2 + 2 + 2(4) - 8$

$\quad = 4 + 8 - 8$

$\Rightarrow p(2) = 4$

(iv) $p(x) = (x-1)(x+1)$

$p(0) = (0-1)(0+1)$

$\quad = (-1)(1) = -1$

$\Rightarrow p(0) = -1$

$p(1) = (1-1)(1+1)$

$\quad = 0(2)$

$\Rightarrow p(1) = 0$

$p(2) = (2-1)(2+1)$

$\quad = 1(3)$

$\Rightarrow p(2) = 3$

Q ③ : Verify whether following are zeroes of polynomial

ਜਾਂਚ ਕਰੋ ਕਿ ਦਿੱਤੇ ਗਏ ਮੁੱਲ ਸੰਗਤ ਬਹੁਪਦ ਦੀਆਂ ਸਿਫ਼ਰੇ ਹਨ।

(i) $p(x) = 3x + 1$, $x = -\dfrac{1}{3}$

$p\left(\dfrac{-1}{3}\right) = 3\left(\dfrac{-1}{3}\right) + 1$

$\quad = -1 + 1 = 0$

$\Rightarrow x = \dfrac{-1}{3}$, $p(x)$ ਦੀ ਸਿਫ਼ਰ ਹੈ।

$\Rightarrow x = \dfrac{-1}{3}$ is zero of $p(x)$.

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(ii) $p(x) = 5x - \pi$, $x = \frac{4}{5}$

$p\left(\frac{4}{5}\right) = 5\left(\frac{4}{5}\right) - \pi = 4 - \pi \neq 0$

$\Rightarrow$ $x = \frac{4}{5}$ is not zero of $p(x)$.

$\Rightarrow$ $x = \frac{4}{5}$, $p(x)$ ਦੀ ਸਿਫਰ ਨਹੀਂ ਹੈ।

(iii) $p(x) = x^2 - 1$, $x = 1, -1$

$p(1) = (1)^2 - 1 = 1 - 1 = 0$

$p(-1) = (-1)^2 - 1 = 1 - 1 = 0$

$\Rightarrow$ $x = 1, -1$ both are zeroes of $p(x)$

$\Rightarrow$ $x = 1, -1$ ਦੋਨੋ ਜੀ $p(x)$ ਦੇ ਸਿਫਰ ਹਨ।

(iv) $p(x) = (x+1)(x-2)$ $x = -1, 2$

$p(-1) = (-1+1)(-1-2) = 0(-3) = 0$

$p(2) = (2+1)(2-2) = 3(0) = 0$

$\Rightarrow$ $x = -1, 2$ both are zeroes of $p(x)$.

$\Rightarrow$ $x = -1, 2$ ਦੋਨੋ ਜੀ $p(x)$ ਦੇ ਸਿਫਰ ਹਨ।

(v) $p(x) = x^2$, $x = 0$

$p(0) = 0^2 = 0$

$\Rightarrow$ $x = 0$ is zero of $p(x)$

$\Rightarrow$ $x = 0$, $p(x)$ ਦਾ ਸਿਫਰ ਹੈ।

(vi) $p(x) = \ell x + m$, $x = -\dfrac{m}{\ell}$

$$p\left(\dfrac{-m}{\ell}\right) = \ell\left(\dfrac{-m}{\ell}\right) + m$$

$$= -m + m = 0$$

$\Rightarrow x = \dfrac{-m}{\ell}$ is zero of $p(x)$.

$\Rightarrow x = \dfrac{-m}{\ell}$, $p(x)$ दी निळक है।

(vii) $p(x) = 3x^2 - 1$, $x = \dfrac{-1}{\sqrt{3}}, \dfrac{2}{\sqrt{3}}$

$$p\left(\dfrac{-1}{\sqrt{3}}\right) = 3\left(\dfrac{-1}{\sqrt{3}}\right)^2 - 1 = 3\left(\dfrac{1}{3}\right) - 1 = 1 - 1 = 0$$

$$p\left(\dfrac{2}{\sqrt{3}}\right) = 3\left(\dfrac{2}{\sqrt{3}}\right)^2 - 1 = 3\left(\dfrac{4}{3}\right) - 1 = 4 - 1 = 3 \neq 0$$

$\Rightarrow x = \dfrac{-1}{\sqrt{3}}$ is zero but $\dfrac{2}{\sqrt{3}}$ is not zero of $p(x)$

$\Rightarrow x = \dfrac{-1}{\sqrt{3}}$, $p(x)$ दी निळक है पर $x = \dfrac{2}{\sqrt{3}}$ नणी।

(viii) $p(x) = 2x + 1$, $x = \dfrac{1}{2}$

$$p\left(\dfrac{1}{2}\right) = 2\left(\dfrac{1}{2}\right) + 1 = 1 + 1 = 2 \neq 0$$

$\Rightarrow x = \dfrac{1}{2}$ is not zero of $p(x)$

$\Rightarrow x = \dfrac{1}{2}$, $p(x)$ दी निळक नणी है।

ਵਾਜ਼ਪਦ ਦੀ ਜ਼ਿਰੋ ਲੱਭੋ।

Find zero of polynom

Q(4):

(i) $p(x) = x + 5$

$$p(x) = 0$$

$$x + 5 = 0$$

$$x = -5 \text{ Ans}$$

(ii) $p(x) = x - 5$

$$p(x) = 0$$

$$x - 5 = 0$$

$$x = 5 \text{ Ans.}$$

(iii) $p(x) = 2x + 5$

$$p(x) = 0$$

$$2x + 5 = 0$$

$$2x = -5$$

$$x = \frac{-5}{2} \text{ Ans.}$$

(iv) $p(x) = 3x - 2$

$$p(x) = 0$$

$$3x - 2 = 0$$

$$3x = 2$$

$$x = \frac{2}{3} \text{ Ans}$$

(v) $p(x) = 3x$

$$p(x) = 0$$

$$3x = 0$$

$$x = \frac{0}{3}$$

$$x = 0 \text{ Ans}$$

(vi) $p(x) = ax, \; a \neq 0$

$$p(x) = 0$$

$$ax = 0$$

$$x = \frac{0}{a}$$

$$x = 0 \text{ Ans}$$

(vii) $p(x) = cx + d, \; c \neq 0.$

$$p(x) = 0$$

$$cx + d = 0$$

$$cx = -d$$

$$x = \frac{-d}{c} \text{ Ans.}$$

Q ① $x^3 + 3x^2 + 3x + 1$ ਨੂੰ ਹੇਠ ਲਿਖਿਆਂ ਨਾਲ ਭਾਗ ਦੇਣ ਤੇ ਬਾਕੀ ਲੱਭੋ।

Find remainder when $x^3 + 3x^2 + 3x + 1$ is divided by

(i) $x + 1$

$x + 1 = 0$

$x = -1$

⇒ Remainder / ਬਾਕੀ = $p(-1) = (-1)^3 + 3(-1)^2 + 3(-1) + 1$

$= -1 + 3(1) - 3 + 1$

$= -\cancel{1} + \cancel{3} - \cancel{3} + \cancel{1} = 0$ Ans.

(ii) $x - \dfrac{1}{2}$

$x - \dfrac{1}{2} = 0$

$x = \dfrac{1}{2}$

⇒ Remainder / ਬਾਕੀ = $p\left(\dfrac{1}{2}\right) = \left(\dfrac{1}{2}\right)^3 + 3\left(\dfrac{1}{2}\right)^2 + 3\left(\dfrac{1}{2}\right) + 1$

$= \dfrac{1}{8} + \dfrac{3}{4} + \dfrac{3}{2} + \dfrac{1}{1}$

$= \dfrac{1 + 6 + 12 + 8}{8} = \dfrac{27}{8}$ Ans.

(iii) x

$x = 0$

⇒ Remainder / ਬਾਕੀ = $p(0) = (0)^3 + 3(0)^2 + 3(0) + 1$

$= 0 + 0 + 0 + 1$

$= 1$ Ans.

(iv) $x + \pi$

$x + \pi = 0 \implies x = -\pi$

$\implies$ Remainder = $p(-\pi) = (-\pi)^3 + 3(-\pi)^2 + 3(-\pi) + 1$
चाहो
$= -\pi^3 + 3\pi^2 - 3\pi + 1$ Au.

(v) $5 + 2x$

$5 + 2x = 0 \implies 2x = -5$
$\implies x = \dfrac{-5}{2}$

$\implies$ Remainder = $p\left(\dfrac{-5}{2}\right) = \left(\dfrac{-5}{2}\right)^3 + 3\left(\dfrac{-5}{2}\right)^2 + 3\left(\dfrac{-5}{2}\right) + 1$
चाहो

$= \dfrac{-125}{8} + 3\left(\dfrac{25}{4}\right) - \dfrac{15}{2} + 1$

$= \dfrac{-125}{8} + \dfrac{75}{4} - \dfrac{15}{2} + 1$

$= \dfrac{-125 + 150 - 60 + 8}{8}$

$= \dfrac{-125 - 60 + 150 + 8}{8}$

$= \dfrac{-185 + 158}{8} = \dfrac{-27}{8}$ Ans.

Q.② Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $x - a$.

$x^3 - ax^2 + 6x - a$ को $x - a$ रोड अग हैर ड़ चाहो मेत

$$x - a = 0 \implies x = a$$

$\implies$ Remainder = $p(a) = (a)^3 - a(a)^2 + 6(a) - a$
घारी
$$= a^3 - a^3 + 6a - a$$
$$= 5a \quad \text{Ans.}$$

Q.(3) ਜਾਂਚ ਕਰੋ ਕਿ $7 + 3x$, $3x^3 + 7x$ ਦਾ ਇੱਕ ਗੁਣਨਖੰਡ ਹੈ
Check whether $7 + 3x$ is a factor of $3x^3 + 7x$ ਜਾਂ ਨਹੀਂ।

$$7 + 3x = 0 \implies 3x = -7$$
$$x = \frac{-7}{3}$$

$\implies$ Remainder = $p\left(\frac{-7}{3}\right) = 3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right)$
घारी
$$= 3\left(\frac{-343}{27}\right) - \frac{49}{3}$$
$$= \frac{-343}{9} - \frac{49}{3}$$
$$= \frac{-343 - 147}{9} = \frac{-490}{9} \neq 0$$

$\implies$ Remainder $\neq 0$
घारी $\neq 0$

$\implies 7 + 3x$, $p(x)$ ਦਾ ਗੁਣਨਖੰਡ ਨਹੀਂ ਹੈ।
$\implies 7 + 3x$ is not factor of $p(x)$.

Q ① ਹੋਏ ਕਿ ਤੋਂ ਦਿੱਤੇ ਜੁਏ ਬਹੁਪਦਾਂ ਵਿੱਚੋਂ ਕਿਹੜੇ ਬਹੁਪਦ ਦਾ
ਇੱਕ ਗੁਣਨਖੰਡ x+1 ਹੈ?

Determine which of following polynomials has
(x+1) as factor.

(i) $x^3 + x^2 + x + 1$

$x + 1 = 0 \implies x = -1$

$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -\cancel{1} + \cancel{1} - 1 + \cancel{1} = 0$

$\implies$ By Factor theorem, $x+1$ is a factor of $p(x)$

$\implies$ ਗੁਣਨਖੰਡ ਵਿਧੀ ਰਾਹੀਂ, $x+1$, $p(x)$ ਦਾ ਗੁਣਨਖੰਡ ਹੈ।

(ii) $p(x) = x^4 + x^3 + x^2 + x + 1$

$x + 1 = 0 \implies x = -1$

$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + 1 = 1 \neq 0$

$\implies$ By Factor theorem, $x+1$ is not a factor of $p(x)$

$\implies$ ਗੁਣਨਖੰਡ ਵਿਧੀ ਰਾਹੀਂ, $x+1$, $p(x)$ ਦਾ ਗੁਣਨਖੰਡ ਨਹੀਂ ਹੈ।

(iii) $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$x + 1 = 0 \implies x = -1$

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= \cancel{1} + 3(-1) + 3(1) - \cancel{1} + \cancel{1}$

$= 1 - 3 + 3 = 1 \neq 0$

$\implies$ By Factor theorem, $x+1$ is not a factor of $p(x)$

$\implies$ ਗੁਣਨਖੰਡ ਵਿਧੀ ਰਾਹੀਂ, $x+1$, $p(x)$ ਦਾ ਗੁਣਨਖੰਡ ਨਹੀਂ ਹੈ।

(iv) $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

$x + 1 = 0 \Rightarrow x = -1$

$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$

$= -1 - (1) + 2 + \sqrt{2} + \sqrt{2}$

$= -\cancel{2} + \cancel{2} + 2\sqrt{2} = 2\sqrt{2} \neq 0$

$\Rightarrow$ By Factor theorem, $x+1$ is not a factor of p(x)

$\Rightarrow$ ਗੁਣਨਖੰਡ ਵਿਭਾਜਕ ਨਹੀਂ, $x+1$ $p(x)$ ਦਾ ਗੁਣਨਖੰਡ ਨਹੀਂ ਹੈ।

Q. ② $g(x)$, $p(x)$ ਦਾ ਗੁਣਨਖੰਡ ਹੈ ਜਾਂ ਨਹੀਂ?

Whether $g(x)$ is a factor of $p(x)$ or not.

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

$x + 1 = 0 \Rightarrow x = -1$

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= 2(-1) + 1 + 2 - 1$

$= -\cancel{2} + \cancel{1} + \cancel{2} - \cancel{1} = 0$

$\Rightarrow$ By Factor theorem, $g(x)$ is a factor of $p(x)$.

$\Rightarrow$ ਗੁਣਨਖੰਡ ਵਿਭਾਜਕ ਨਹੀਂ, $g(x)$, $p(x)$ ਦਾ ਗੁਣਨਖੰਡ ਹੈ।

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

$x + 2 = 0 \Rightarrow x = -2$

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 3(4) - 6 + 1$

$= -8 + 12 - 6 + 1 = -8 - 6 + 12 + 1$

$= -14 + 13 = -1 \neq 0$

$\Rightarrow$ By Factor theorem, $g(x)$ is not factor of p(x)

$\Rightarrow$ ਗੁਣਨਖੰਡ ਵਿਭਾਜਕ ਨਹੀਂ, $g(x)$, $p(x)$ ਦਾ ਗੁਣਨਖੰਡ ਨਹੀਂ ਹੈ।

(iii) $p(x) = x^3 - 4x^2 + x + 6$ $\qquad$ $g(x) = x - 3$

$x - 3 = 0 \Rightarrow x = 3$

$p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 4(9) + 3 + 6$

$= 27 - 36 + 9 \qquad = 27 + 9 - 36$

$= 36 - 36 = 0$

⇒ By Factor theorem, $g(x)$ is factor of $p(x)$

⇒ ਗੁਣਕਬੱਧ ਸਿਧੱਤਰ ਹੁੰ , $g(x)$, $p(x)$ ਦਾ ਗੁਣਕੇਬ

Q.③ Find the value of k, if $x - 1$ is a factor

of $p(x)$. $\quad$ K ਦਾ ਮੁੱਲ ਲੱਭੋ ਜੇਕਰ $x - 1$, $p(x)$ ਦਾ ਗੁਣਕ

(i) $p(x) = x^2 + x + k$

∵ $x - 1$ is a factor of $p(x)$

∴ $x - 1$, $p(x)$ ਦਾ ਗੁਣਕ ਹੈ।

$\Rightarrow p(1) = 0$

$(1)^2 + 1 + k = 0$

$1 + 1 + k = 0$

$2 + k = 0$

$k = -2 \quad$ Ans.

(ii) $\quad p(x) = 2x^2 + kx + \sqrt{2}$

∵ $x - 1$ is a factor of $p(x)$

∵ $x - 1$, $p(x)$ ਦਾ ਵਿੱਚ ਗੁਣਕਬੱਧ ਹੈ।

$\Rightarrow p(1) = 0$

$2(1)^2 + k(1) + \sqrt{2} = 0$

$2 + k + \sqrt{2} = 0$

$k = -2 - \sqrt{2} \quad$ Ans.

(iii) $p(x) = kx^2 - \sqrt{2}\, x + 1$

$\because$ $x-1$ is a factor of $p(x)$.

$\therefore$ $x-1$, $p(x)$ ਦਾ ਇੱਕ ਗੁਣਨਖੰਡ ਹੈ।

$\Rightarrow$ $p(1) = 0$

$k(1)^2 - \sqrt{2}(1) + 1 = 0$

$k - \sqrt{2} + 1 = 0$

$k = \sqrt{2} - 1$ Ans.

(iv) $p(x) = kx^2 - 3x + k$

$\because$ $x-1$ is a factor of $p(x)$.

$\therefore$ $x-1$, $p(x)$ ਦਾ ਇੱਕ ਗੁਣਨਖੰਡ ਹੈ।

$\Rightarrow$ $p(1) = 0$

$k(1)^2 - 3(1) + k = 0$

$k - 3 + k = 0$

$2k - 3 = 0$ $\Rightarrow$ $2k = 3$

$k = \dfrac{3}{2}$ Ans.

Q (4)   ਗੁਣਨਖੰਡ ਪਤਾ ਕਰੋ।   Factorise :

(i) $12x^2 - 7x + 1$

$\underbrace{\qquad\qquad}$

$12 \times 1 = 12$

$12x^2 - 4x - 3x + 1$

$4x(3x-1) - 1(3x-1)$

$(4x - 1)(3x - 1)$ Ans.

$12 \times 1 = 12$
$6 \times 2 = 12$
$\boxed{4 \times 3 = 12}$

$-4 - 3 = -7$

(ii) $\dfrac{2x^2+7x+3}{2\times 3=6}$

$2x^2+x+6x+3$

$x(2x+1)+3(2x+1)$

$(2x+1)(x+3)$ Ans.

(iii) $\dfrac{6x^2+5x-6}{6\times 6=36}$

$6x^2+9x-4x-6$

$3x(2x+3)-2(2x+3)$

$(2x+3)(3x-2)$ Ans

$1\times 36=3$
$2\times 18=3$
$3\times 12=3$
$4\times 9=3$
$6\times 6=3$

$9-4=5$

(iv) $\dfrac{3x^2-x-4}{3\times 4=12}$

$3x^2+3x-4x-4$

$3x(x+1)-4(x+1)$

$(x+1)(3x-4)$ Ans.

$1\times 12=12$
$2\times 6=12$
$3\times 4=12$

$3-4=$

Q(5) ਹੇਠਲਿਖੇ ਪਤ ਕੋਂ:   Factorise

(i) x^3-2x^2-x+2

$x^2(x-2)-1(x-2)$

$(x-2)(x^2-1)$

$(x-2)(x^2-1^2)=(x-2)(x+1)(x-1)$ Ans.

(ii) $p(x) = x^3 - 3x^2 - 9x - 5$

$p(1) = 1^3 - 3(1)^2 - 9(1) - 5 = 1 - 3 - 9 - 5 = 1 - 17 = -16 \neq 0$

$p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5 = -1 - 3 + 9 - 5 = -9 + 9 = 0$

$\Rightarrow$ $x + 1$ is a factor of $p(x)$

$\Rightarrow$ $x + 1$, $p(x)$ ਦਾ ਇੱਕ ਗੁਣਨਖੰਡ ਹੈ।

$$
\begin{array}{r}
x^2 - 4x - 5 \\
x+1 \overline{\smash{)}\ x^3 - 3x^2 - 9x - 5} \\
\underline{x^3 + x^2} \\
-4x^2 - 9x - 5 \\
\underline{-4x^2 - 4x} \\
-5x - 5 \\
\underline{-5x - 5} \\
x
\end{array}
$$

$\underline{x^2 - 4x - 5}$

$1 \times 5 = 5$

$x^2 + x - 5x - 5$

$x(x+1) - 5(x+1)$

$(x+1)(x-5)$

$\Rightarrow$ $p(x) = (x+1)(x^2 - 4x - 5)$

$= (x+1)(x+1)(x-5)$ **Ans**

(iii) $p(x) = x^3 + 13x^2 + 32x + 20$

$p(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20 = -1 + 13 - 32 + 20$

$\qquad = -33 + 33 = 0$

$\Rightarrow$ $x+1$ is a factor of $p(x)$

$\Rightarrow$ $x+1$, $p(x)$ ਦਾ ਇੱਕ ਗੁਣਨਖੰਡ ਹੈ।

$$
\begin{array}{r}
x^2 + 12x + 20 \\
x+1\overline{\smash{\big)}\ x^3 + 13x^2 + 32x + 20} \\
\underline{x^3 + x^2} \\
12x^2 + 32x + 20 \\
\underline{12x^2 + 12x} \\
20x + 20 \\
\underline{20x + 20} \\
0
\end{array}
$$

$\underbrace{x^2 + 12x + 20}$

$1 \times 20 = 20$

$x^2 + 2x + 10x + 20$

$x(x+2) + 10(x+2)$

$(x+2)(x+10)$

$\Rightarrow$ $p(x) = (x+1)(x+2)(x+10)$ Ans

$1 \times 20 = 20$

$2 \times 10 = 20$

$4 \times 5 = 20$

$\begin{array}{r|l} 2 & 20 \\ 2 & 10 \\ 5 & 5 \\ \hline & 1 \end{array}$

(iv) $2y^3 + y^2 - 2y - 1$

$y^2(2y+1) - 1(2y+1)$

$(2y+1)(y^2 - 1)$

$\Rightarrow (2y+1)(y+1)(y-1)$ Ans

Note:

Algebraic Identities ਧੀਜਗਣਿਤੀਤ੨ ਸ਼ਰਘਸਸਤੂੰ

* $(x+y)^2 = x^2 + 2xy + y^2$
* $(x-y)^2 = x^2 - 2xy + y^2$

* $x^2 - y^2 = (x+y)(x-y)$
* $(x+a)(x+b) = x^2 + (a+b)x + ab$

* $(x+y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$
* $(x-y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$

* $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$
* $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

* $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
* $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

EXERCISE : 2.5

① ਖ਼ੁਰਦੀ ਸਮੀਕਰਨਾਂ ਦਾ ਪ੍ਰਯੋਗ ਕਰਕੇ ਹੇਠ ਲਿਖੇ ਗੁਣਨਫਲਾਂ ਦਾ ਮੁੱਲ ਪਤਾ ਕਰੋ:

Use suitable identities to find following products

Sol:

(i) $(x+4)(x+10)$

$(x+a)(x+b) = x^2 + (a+b)x + ab$ $\boxed{\begin{array}{l} a=4 \\ b=10 \end{array}}$

$(x+4)(x+10) = x^2 + (4+10)x + 4\times 10$

$= x^2 + 14x + 40$ Ans.

(ii) $(x+8)(x-10)$

$(x+a)(x+b) = x^2 + (a+b)x + ab$ $\boxed{\begin{array}{l} a=8 \\ b=-10 \end{array}}$

$(x+8)(x-10) = x^2 + (8-10)x + 8(-10)$

$= x^2 - 2x - 80$ Ans.

(iii) $(3x+4)(3x-5)$

$(x+a)(x+b) = x^2 + (a+b)x + ab$ $\boxed{\begin{array}{l} X=3x \\ a=4 \\ b=-5 \end{array}}$

$(3x+4)(3x-5) = (3x)^2 + (4-5)3x + 4(-5)$

$= 9x^2 - 3x - 20$ Ans.

(iv) $\left(y^2 + \dfrac{3}{2}\right)\left(y^2 - \dfrac{3}{2}\right)$

$(a+b)(a-b) = a^2 - b^2$ $\boxed{\begin{array}{l} a=y^2 \\ b=\dfrac{3}{2} \end{array}}$

$\left(y^2 + \dfrac{3}{2}\right)\left(y^2 - \dfrac{3}{2}\right) = (y^2)^2 - \left(\dfrac{3}{2}\right)^2$

$= y^4 - \dfrac{9}{4}$ Ans.

(v) $(3-2x)(3+2x)$

$(a+b)(a-b) = a^2 - b^2$ $\boxed{\begin{array}{l} a=3 \\ b=2x \end{array}}$

$(3+2x)(3-2x) = (3)^2 - (2x)^2$

$= 9 - 4x^2$ Ans.

② ਮਿਥੀ ਗੁਣਾ ਕੀਤੇ ਬਿਨਾਂ ਹੇਠ ਲਿਖਿਆਂ ਗੁਣਨਫਲਾ ਦੇ ਮੁੱਲ
ਪਤਾ ਕਰੋ

Evaluate the following products without multiply directly.

(i) 103×107

$= (100 + 3)(100 + 7) \quad \rightarrow (x+a)(x+b) = x^2 + (a+b)x + ab$

$= (100)^2 + (3+7)100 + 3 \times 7$

$\boxed{\begin{array}{l} x = 100 \\ a = 3 \\ b = 7 \end{array}}$

$= 10000 + (10)100 + 21$

$= 10000 + 1000 + 21 = 11021 \quad$ Ans

(ii) 95×96

$= (100 - 5) \times (100 - 4) \quad \rightarrow (x+a)(x+b) = x^2 + (a+b)x + ab$

$= (100)^2 + (-5-4)100 + (-5)(-4)$

$\boxed{\begin{array}{l} x = 100 \\ a = -5 \\ b = -4 \end{array}}$

$= 10000 - 900 + 20$

$= 9100 + 20 = 9120 \quad$ Ans.

(iii) 104×96

$= (100 + 4)(100 - 4) \qquad (a+b)(a-b) = a^2 - b^2$

$= (100)^2 - (4)^2$

$\boxed{\begin{array}{l} a = 100 \\ b = 4 \end{array}}$

$= 10000 - 16 = 9984 \quad$ Ans.

③ ਢੁੱਕਵੀਂ ਸਮੀਕਰਣ ਦਾ ਪ੍ਰਯੋਗ ਕਰਕੇ ਗੁਣਨਖੰਡੀਕਰਣ ਕਰੋ।

Factorise the following using appropriate identity

(i) $9x^2 + 6xy + y^2$

$= (3x)^2 + 2(3x)(y) + (y)^2$

$= (3x + y)^2 \qquad \left[\because (a+b)^2 = a^2 + 2ab + b^2 \right]$

$= (3x+y)(3x+y) \quad$ Ans.

$\boxed{\begin{array}{l} a = 3x \\ b = y \end{array}}$

(ii) $4y^2 - 4y + 1$

$= (2y)^2 - 2(2y)(1) + (1)^2$ $[\because (a-b)^2 = a^2 - 2ab + b^2]$

$= (2y-1)^2$ $\boxed{\begin{array}{l} a = 2y \\ b = 1 \end{array}}$

$= (2y-1)(2y-1)$ Ans.

(iii) $x^2 - \dfrac{y^2}{100}$

       $[\because a^2 - b^2 = (a+b)(a-b)]$

$= x^2 - \left(\dfrac{y}{10}\right)^2$ $\boxed{\begin{array}{l} a = x \\ b = \dfrac{y}{10} \end{array}}$

$= \left(x + \dfrac{y}{10}\right)\left(x - \dfrac{y}{10}\right)$ Ans.

④   ਢੁਰਦੀ ਸਰਬੱਸਤਾ ਦਾ ਯੂਜੋਗ ਕਰਕੇ ਵਿਸਤਾਰ ਕਰੋ :-

Expand using suitable identities :—

(i) $(x + 2y + 4z)^2$ $\boxed{a=x,\ b=2y,\ c=4z}$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(x+2y+4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2(x)(2y) + 2(2y)(4z) + 2(4z)(x)$

$\qquad = x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$ Ans.

(ii) $(2x - y + z)^2$ $\boxed{a=2x,\ b=-y,\ c=z}$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(2x-y+z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2(2x)(-y) + 2(-y)z + 2(z)2x$

$\qquad = 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$ Ans.

(iii) $(-2x + 3y + 2z)^2$ $\qquad$ $\boxed{a = -2x, \; b = 3y, \; c = 2z}$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(-2x+3y+2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2(-2x)(3y) +$
$$2(3y)(2z) + 2(2z)(-2x)$$

$\qquad = 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$

(iv) $(3a - 7b - c)^2$ $\qquad$ $\boxed{x = 3a, \; y = -7b, \; z = -c}$

$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2(3a)(-7b) + 2(-7b)(-c)$
$$+ 2(-c)(3a)$$

$\qquad = 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$ Ans.

(v) $(-2x + 5y - 3z)^2$ $\qquad$ $\boxed{a = -2x, \; b = 5y, \; c = -3z}$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(-2x+5y-3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2(-2x)(5y)$
$$+ 2(5y)(-3z) + 2(-3z)(-2x)$$

$\qquad = 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$ Ans.

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$ $\qquad$ $\boxed{x = \frac{1}{4}a, \; y = \frac{-1}{2}b, \; z = 1}$

$(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2 = \left(\frac{1}{4}a\right)^2 + \left(\frac{-1}{2}b\right)^2 + (1)^2 + 2\left(\frac{1}{4}a\right)\left(\frac{-1}{2}b\right) + 2\left(\frac{-1}{2}b\right)$
$$+ 2(1)\left(\frac{1}{4}a\right)$$

$\qquad = \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$ Ans

Q ⑤: ਗੁਣਨਖੰਡ/ਵੰਡ ਕਰੋ: Factorise

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2(2x)(3y) + 2(3y)(-4z) + 2(-4z)(2x)$

$= (2x + 3y - 4z)^2$

$= (2x + 3y - 4z)(2x + 3y - 4z)$ Ans.

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

$= (-\sqrt{2}x)^2 + y^2 + (2\sqrt{2}z)^2 + 2(-\sqrt{2}x)y + 2(y)(2\sqrt{2}z) + 2(2\sqrt{2}z)(-\sqrt{2}x)$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$ Ans

Q ⑥ Write the following cubes in expanded form

ਹੇਠ ਲਿਖੇ ਘਣਾਂ ਨੂੰ ਵਿਸਤ੍ਰਿਤ ਰੂਪ ਵਿੱਚ ਲਿਖੋ:

(i) $(2x+1)^3$ $\boxed{\begin{array}{l} a = 2x \\ b = 1 \end{array}}$

$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$(2x+1)^3 = (2x)^3 + (1)^3 + 3(2x)^2(1) + 3(2x)(1)^2$

$= 8x^3 + 1 + 3(4x^2) + 3(2x)$

$= 8x^3 + 1 + 12x^2 + 6x$

$= 8x^3 + 12x^2 + 6x + 1$ Ans

(ii) $(2a - 3b)^3$ $\boxed{\begin{array}{l} x = 2a \\ y = 3b \end{array}}$

$(x-y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$

$(2a-3b)^3 = (2a)^3 - (3b)^3 - 3(2a)^2(3b) + 3(2a)(3b)^2$

$= 8a^3 - 27b^3 - 3(4a^2)(3b) + 3(2a)(9b^2)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$ Ans

(iii) $\left[\dfrac{3}{2}x+1\right]^3$ $\boxed{a=\dfrac{3}{2}x,\ b=1}$

$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

$\left(\dfrac{3}{2}x+1\right)^3 = \left(\dfrac{3}{2}x\right)^3 + (1)^3 + 3\left(\dfrac{3}{2}x\right)^2(1) + 3\left(\dfrac{3}{2}x\right)(1)^2$

$= \dfrac{27}{8}x^3 + 1 + 3\left(\dfrac{9}{4}x^2\right) + \dfrac{9}{2}x$

$= \dfrac{27}{8}x^3 + 1 + \dfrac{27}{4}x^2 + \dfrac{9}{2}x \quad$ Ans

(iv) $\left(x-\dfrac{2}{3}y\right)^3$ $\boxed{\begin{array}{l}a=x\\ b=+\dfrac{2}{3}y\end{array}}$

$(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$\left(x-\dfrac{2}{3}y\right)^3 = (x)^3 - \left(\dfrac{2}{3}y\right)^3 - 3(x)^2\left(\dfrac{2}{3}y\right) + 3(x)\left(\dfrac{2}{3}y\right)^2$

$= x^3 - \dfrac{8}{27}y^3 - \cancel{3}x^2 \times \dfrac{2}{\cancel{3}}y + \cancel{3}x\left(\dfrac{4}{\cancel{9}_3}y^2\right)$

$= x^3 - \dfrac{8}{27}y^3 - 2x^2y + \dfrac{4}{3}xy^2 \quad$ Ans

Q(7) ਛੇਵੀਂ ਮਾਧਮਭੂਤਾ ਦਾ ਪ੍ਰਯੋਗ ਕਰਦੇ ਮੁੱਲ ਪਤਾ।

Evaluate using suitable identities:

(i) $(99)^3$

$= (100-1)^3$ $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$\boxed{a=100,\ b=1}$

$= (100)^3 - (1)^3 - 3(100)^2(1) + 3(100)(1)^2$

$= 1000000 - 1 - 30000 + 300$

$= 1000000 + 300 - 1 - 30000 = 1000300 - 30001$

$= 970299$

$$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$$

(ii) $(102)^3$ $\boxed{a=100, b=2}$

$= (100+2)^3$

$= (100)^3 + (2)^3 + 3(100)^2(2) + 3(100)(2)^2$

$= 1000000 + 8 + 60000 + 1200$

$=\quad 1061208$ Ans

(iii) $(998)^3$ $(a-b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$

$= (1000-2)^3$ $\boxed{a=1000, b=2}$

$= (1000)^3 - (2)^3 - 3(1000)^2(2) + 3(1000)(2)^2$

$= 1000000000 - 8 - 6000000 + 12000$

$= 1000000000 + 12000 - 8 - 6000000$

$= 1000012000 - 6000008$

$=\quad 994011992$ Ans

Q.8   Factorise   गुरुनवंड/20र 23 :

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 3(2a)^2(b) + 3(2a)(b)^2$

$=\quad (2a+b)^3$ $\rightarrow \left[\because (x+y)^3 = x^3 + y^3 + 3x^2y + 3xy^2 \right]$

$=\quad (2a+b)(2a+b)(2a+b)$ Ans

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

$= (2a)^3 - (b)^3 - 3(2a)^2(b) + 3(2a)(b)^2$

$=\quad (2a-b)^3$ $\rightarrow \left[\because (x-y)^3 = x^3 - y^3 - 3x^2y + 3xy^2 \right]$

$=\quad (2a-b)(2a-b)(2a-b)$ Ans

(iii) $27 - 125a^3 - 135a + 225a^2$

$= (3)^3 - (5a)^3 - 3(3)^2(5a) + 3(3)(5a)^2$

$= (3 - 5a)^3$

$= (3 - 5a)(3 - 5a)(3 - 5a)$ Ans.

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 3(4a)^2(3b) + 3(4a)(3b)^2$

$= (4a - 3b)^3$

$= (4a - 3b)(4a - 3b)(4a - 3b)$ Ans.

(v) $27p^3 - \dfrac{1}{216} - \dfrac{9}{2}p^2 + \dfrac{1}{4}p$

$= (3p)^3 - \left(\dfrac{1}{6}\right)^3 - 3(3p)^2\left(\dfrac{1}{6}\right) + 3(3p)\left(\dfrac{1}{6}\right)^2$

$= \left(3p - \dfrac{1}{6}\right)^3 = \left(3p - \dfrac{1}{6}\right)\left(3p - \dfrac{1}{6}\right)\left(3p - \dfrac{1}{6}\right)$ Ans

Q.(9) सिद्ध करें : Verify :

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

R.H.S $= (x + y)(x^2 - xy + y^2)$

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 + y^3 = $ L.H.S.

(ii) $\quad x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

$\text{R.H.S.} = (x-y)(x^2 + xy + y^2)$

$\qquad = x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$\qquad = x^3 + x^2 y + xy^2 - x^2 y - xy^2 + y^3$

$\qquad = x^3 - y^3 = \text{L.H.S.}$

Q. ⑩ गुरुकुंडीड्ठाटुुं 2ठ: Factorise :

(i) $\quad 27y^3 + 125z^3$

$= (3y)^3 + (5z)^3 \qquad \left[\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)\right]$

$= (3y + 5z)\left[(3y)^2 - (3y)(5z) + (5z)^2\right]$

$= (3y + 5z)(9y^2 - 15yz + 25z^2) \quad \text{Ans}$

(ii) $\quad 64m^3 - 343n^3$

$= (4m)^3 - (7n)^3 \qquad \left[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)\right]$

$= (4m - 7n)\left[(4m)^2 + (4m)(7n) + (7n)^2\right]$

$= (4m - 7n)(16m^2 + 28mn + 49n^2) \quad \text{Ans}$

Q. ⑪ $\quad 27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)\left[(3x)^2 + y^2 + z^2 - (3x)(y) - (y)(z) - (z)(3x)\right]$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3zx)$

$\rightarrow \left[\because a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 \right.$

$\left. - ab - bc - ca)\right]$

Q 12   ਜਾਂਚ ਕਰੋ: Verify:

$$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z)\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right.$$

Sol: R.H.S $= \frac{1}{2}(x+y+z)\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right]$

$$= \frac{1}{2}(x+y+z)\left[x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + z^2 + x^2 - 2zx\right]$$

$$= \frac{1}{2}(x+y+z)\left(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx\right)$$

$$= \frac{1}{2}(x+y+z)\, 2\,(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

$$= x^3 + y^3 + z^3 - 3xyz = L.H.S$$

Q 13   ਜੇ $x+y+z = 0$ ਹੈ ਤਾਂ ਦਿਖਾਓ ਕਿ $x^3 + y^3 + z^3 = 3xyz$.

If $x+y+z=0$, show that $x^3 + y^3 + z^3 = 3xyz$

Sol:
$$x+y+z = 0$$
$$x+y = -z \longrightarrow ①$$
$$(x+y)^3 = (-z)^3$$
$$x^3 + y^3 + 3xy(x+y) = -z^3 \qquad\qquad ① \tfrac{y}{3}$$
$$x^3 + y^3 + 3xy(-z) = -z^3 \longrightarrow [\text{By } ①]$$
$$x^3 + y^3 + z^3 = 3xyz$$

<div align="center">Hence proved.</div>

<div align="center">ਸਿੰਘ ਜਿਲਾ</div>

Q.(14)   without actually calculating cubes, find value

ਘਣਾਂ ਦੀ ਗਣਨਾ ਕੀਤੇ ਬਿਨਾ ਮੁੱਲ ਪਤਾ ਕਰੋ:

(i) $(-12)^3 + (7)^3 + (5)^3$

ਸੰਨ ਕਰੋ Let $a = -12$, $b = 7$, $c = 5$

$$a + b + c = -12 + 7 + 5$$
$$= -12 + 12 = 0$$
$$\Rightarrow a + b + c = 0$$
$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$
$$\therefore (-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5) = -1260$$

An

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Let $a = 28$, $b = -15$, $c = -13$

$$a + b + c = 28 - 15 - 13 = 28 - 28 = 0$$
$$\Rightarrow a + b + c = 0$$
$$\Rightarrow a^3 + b^3 + c^3 = 3abc$$
$$\therefore (28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$$
$$= 16380 \text{ An}$$

Q.(15)   ਸੰਭਾਵੀਆਂ ਦਿੱਤੇ ਗਏ ਹਨ। ਲੰਬਾਈ ਅਤੇ ਚੌੜਾਈ ਪਤਾ ਕਰੋ।

Give possible expressions for length & breadth

(i)   Area = ਖੇਤਰਫਲ = $25a^2 - 35a + 12$

$$= 25a^2 - 20a - 15a + 12$$
$$= 5a(5a - 4) - 3(5a - 4)$$
$$= (5a - 4)(5a - 3)$$

$\Rightarrow$ Length = 5a-4   ,   Breadth = 5a-3
ਲੰਬਾਈ   ਚੌੜਾਈ

(ii) Area = क्षेत्रफल = $35y^2 + 13y - 12$

$= 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

$= (5y + 4)(7y - 3)$

⟹ Length = $5y + 4$ Breadth = $7y - 3$

ਲੰਬਾਈ   ਚੌੜਾਈ

Q16. ਘਣਾਂ ਦੇ ਮਾਪਾਂ ਵਿੱਤ ਜਾ। ਮਾਪਾਂ ਲਈ ਸੰਭਵ ਵਿਆਪਕ ਦੱਸੋ

What are the possible expressions for dimensions of cuboids whose volume are given :-

(i) Volume = ਮਾਪਾਂ = $3x^2 - 12x$

$= 3x(x - 4)$

⟹ Length = 3 , Breadth = x , Height = $x - 4$

ਲੰਬਾਈ   ਚੌੜਾਈ   ਉਚਾਈ

(ii) Volume = ਮਾਪਾਂ = $12ky^2 + 8ky - 20k$

$= 4k(3y^2 + 2y - 5)$

$= 4k(3y^2 + 5y - 3y - 5)$

$= 4k[y(3y + 5) - 1(3y + 5)]$

$= 4k(3y + 5)(y - 1)$

⟹ Length = $4k$ Breadth = $3y + 5$ Height = $y - 1$

ਲੰਬਾਈ   ਚੌੜਾਈ   ਉਚਾਈ