Ch- - Laplace Transform ::

Purpose: All differential and integrals are converted to simple algebric eqn.

⇒ L.T. expresses signals as linear combination of complex exponentials, which are eigen functions of DE which describe continuous - time LTI systems.

⇒ The primary role of the L.T in engineering is the transient & stability analysis of causal LTI systems.

⇒ In addition to its simplicity, many design techniques in circuits, filters & control systems have been developed in L.T. domain.

⇒ Generalization of F.T. is laplace Trans.

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

⇒ Let, Input $x(t) = e^{st}$ $(s = \sigma + j\omega)$

$\Downarrow$

Complex Variable

$\Rightarrow$ O/p is $y(t) = e^{st} \cdot H(s)$.

$$\rightarrow y(t) = x(t) * h(t).$$

$$= \int_{-\infty}^{+\infty} x(t-\tau) \cdot h(\tau) d\tau.$$

$$= \int_{-\infty}^{+\infty} e^{s(t-\tau)} \cdot h(\tau) \cdot d\tau.$$

$$y(t) = e^{st} \cdot \int_{-\infty}^{+\infty} e^{-s\tau} \cdot h(\tau) \cdot d\tau.$$

$$\therefore \boxed{y(t) = e^{st} \cdot H(s)}$$

$\Rightarrow$ L.T. of general signal $x(t)$

$$\boxed{L[x(t)] = \int_{-\infty}^{+\infty} x(t) \cdot e^{-st} \, dt} = X(s).$$

$s = \sigma + j\omega$.

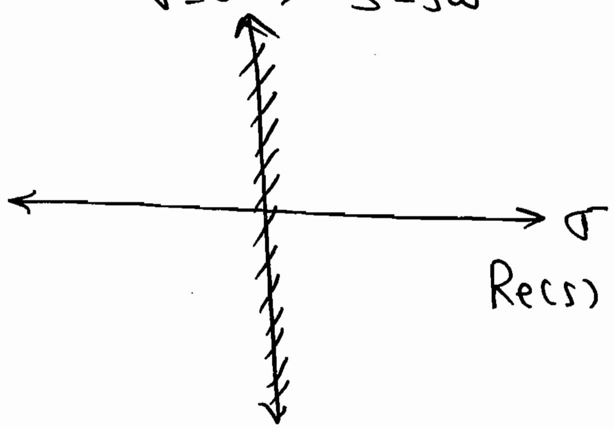
$$\Rightarrow X(\sigma + j\omega) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-(\sigma + j\omega)t} \cdot dt.$$

$$= \int_{-\infty}^{+\infty} \left[x(t) \cdot e^{-\sigma t}\right] \cdot e^{-j\omega t} \cdot dt$$

$$\therefore \boxed{L[x(t)] = F\{x(t) \cdot e^{-\sigma t}\}}.$$

$\Rightarrow$ $e^{-\sigma t}$ may be decaying (or) growing depending on whether 'σ' is +ve (or) -ve.

$\Rightarrow$ $\sigma = 0 \Rightarrow s = j\omega$



L.T.

BLT $\Downarrow$ causality, stability, freq. response

U.L.T $\Downarrow$ used to solve D.E. with I.C.S.

* Region of Convergence of L.T. : (ROC)

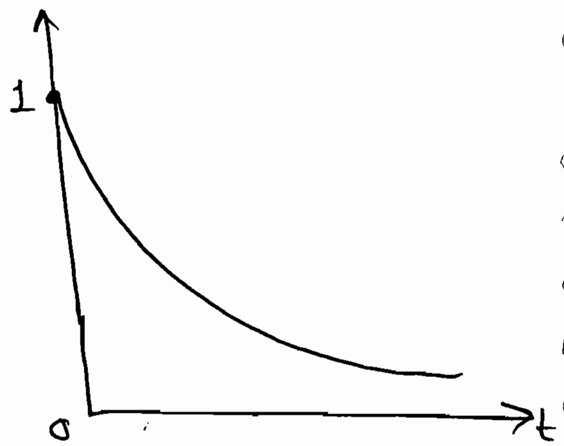
$$X(s) < \infty.$$

$$\Rightarrow \int_{-\infty}^{+\infty} \left| x(t) \cdot e^{-\sigma t} \right| dt < \infty.$$

Necessary.

* L.T. of Standard Signals:

① $x_1(t) = e^{-at} u(t)$; $\text{Rea}\{a\} > 0.$



$$\Rightarrow X(s) = \int_{-\infty}^{\infty} x(t) \cdot e^{-st} \cdot dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-st} \cdot dt.$$

$$= \int_{0}^{\infty} e^{-(s+a)t} \cdot dt.$$

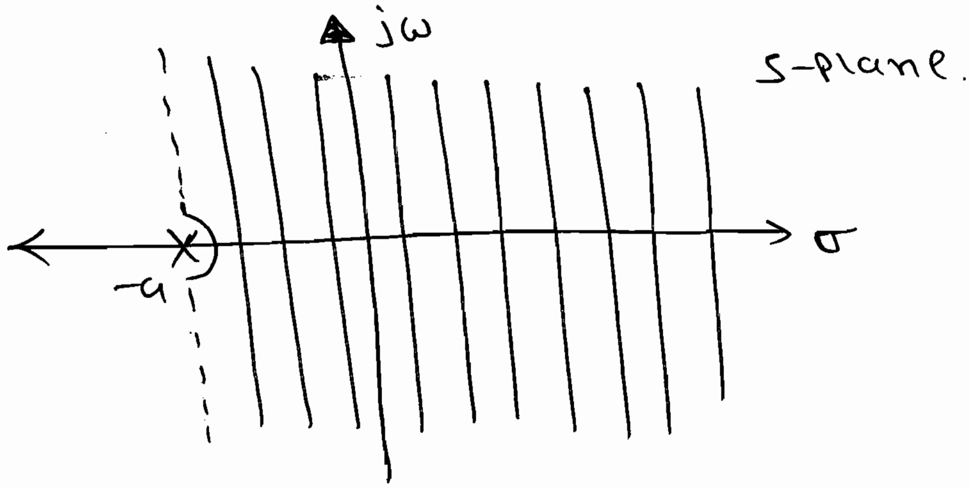
$$= \left[\frac{e^{-(s+a)t}}{-(s+a)} \right]_0^{\infty}.$$

$$= \frac{1}{\infty} + \frac{1}{s+a}$$

$$= \frac{1}{s+a} \quad ; \quad \sigma + a > 0$$

$$\underbrace{\sigma > Re\{-a\}.}_{R.o.C.}$$

$$\rightarrow \quad e^{-at} \cdot u(t) \longleftrightarrow \frac{1}{s+a} \; ; \quad Re(s) > -a.$$



~~2) $x(t) = -e^{-at} u(-t)$.~~

$\Rightarrow$ R.o.C. of LT consist of lines ||el to jω- axis.

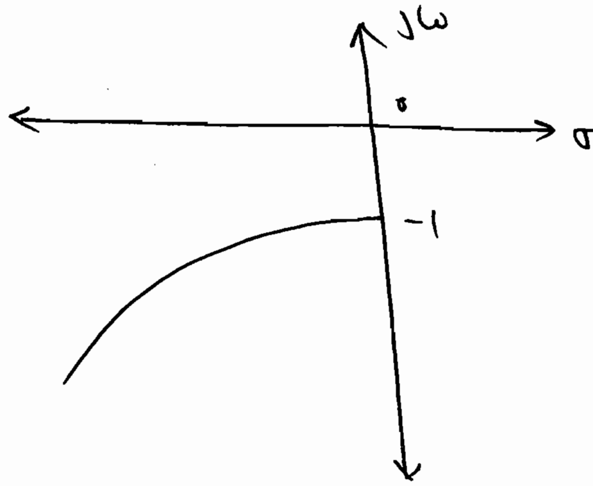
$\Rightarrow$ ROC do not contain any Pole.

$\Rightarrow$ For Stability in laplace domain ROC must include Imaginary axis.

(2) $x_2(t) = -e^{-at} u(-t)$; $Re\{a\} > 0$.

$$\Rightarrow \quad x_2(t) = 0 \; ; \quad t > 0$$
$$= -e^{-at} \; ; \quad t < 0.$$

$$\Rightarrow X_2(s) = \int_{-\infty}^{0} -e^{-at} \cdot e^{-st} \, dt$$



$$= \int_{-\infty}^{0} -e^{-(s+a)t} \, dt.$$

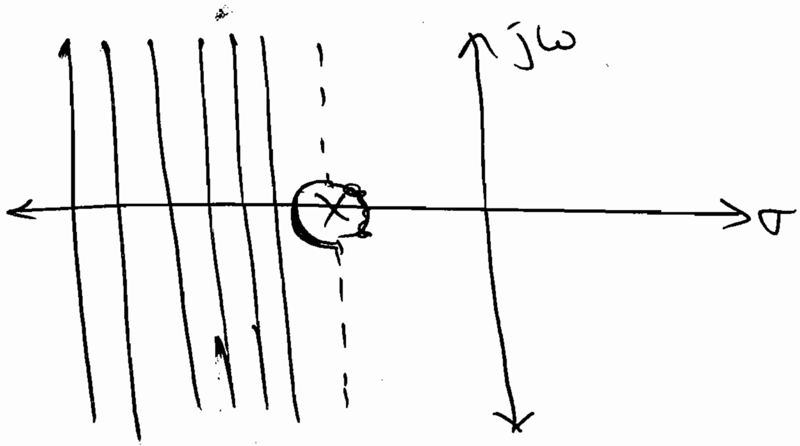
$$= \left[\frac{e^{-(s+a)t}}{(s+a)} \right]_{-\infty}^{0} = \left[\frac{e^{-(\underline{\sigma+a}+j\omega)t}}{s+a} \right]_{-\infty}^{0}$$

$$X_2(s) = \frac{1}{s+a} \; ; \quad \sigma + a < 0. \; , \quad Re(s) < -a$$

$$Re(\sigma + j\omega) < -a$$

$$\Rightarrow \boxed{\sigma < -a.}$$

$\Rightarrow$

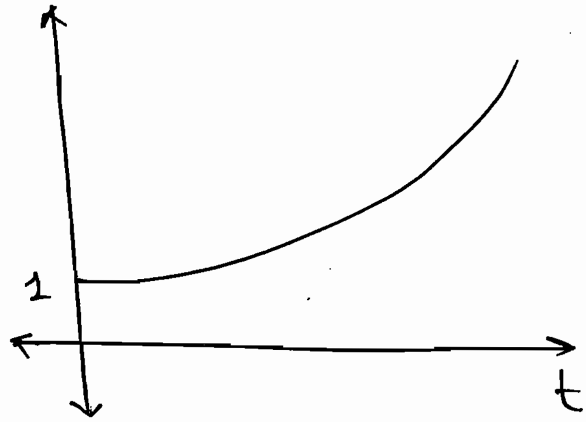


$$\Rightarrow \quad e^{-at} u(t) \longleftrightarrow \frac{1}{s+a} \; ; \quad \sigma > Re\{-a\}.$$

$$-e^{-at} \cdot u(-t) \longleftrightarrow \frac{1}{s+a} \; ; \quad \sigma < Re\{-a\}.$$

$\Rightarrow$ Note: Solution of Laplace transform is unique only when the RoC is given.

③ $x_3(t) = e^{at}u(t)$; $Re\{a\} > 0$.



$$\Rightarrow X_3(s) = \int_0^\infty e^{at} \cdot e^{-st} \cdot dt.$$

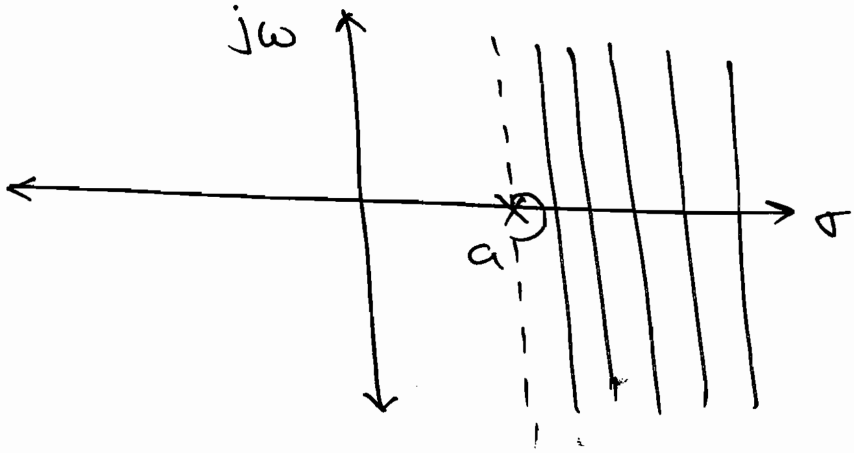
$$= \int_0^\infty e^{-(s-a)t} \cdot dt.$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty$$

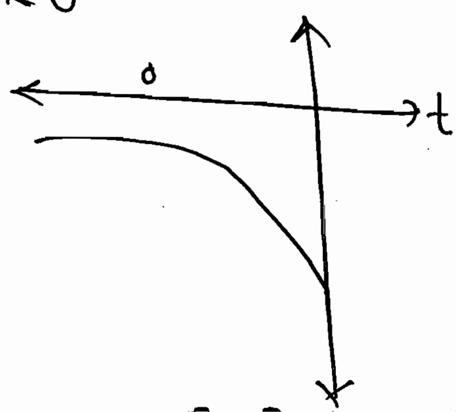
$$= \left[\frac{e^{-(\sigma-a+j\omega)t}}{-(s-a)}\right]_0^\infty.$$

$$\therefore X_3(s) = \frac{1}{(s-a)}; \quad \sigma - a > 0$$

$$\sigma > Re\{a\}.$$



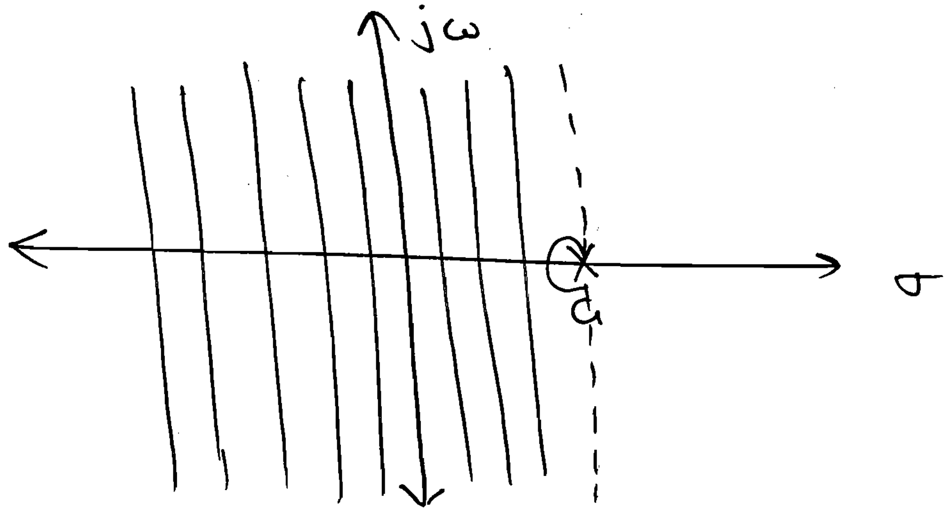
④ $x_4(t) = -e^{at}u(-t)$. , $Re\{a\} \geq 0$.



$$\Rightarrow X_4(s) = \frac{1}{s-a}; \quad Re\{s\} < a.$$

$$\sigma < Re\{a\}$$

$$-e^{at}u(-t) \longleftrightarrow \frac{1}{s-a} \; ; \; \sigma < a.$$



$\Rightarrow$

$$\boxed{\begin{array}{llll} e^{-at}u(t) & \longleftrightarrow & \dfrac{1}{s+a} & ; \; \sigma > Re\{-a\} \\ -e^{-at}u(-t) & \longleftrightarrow & \dfrac{1}{s+a} & ; \; \sigma \not> Re\{-a\}. \\ e^{at}u(t) & \longleftrightarrow & \dfrac{1}{s-a} & ; \; \sigma > Re\{a\}. \\ -e^{at}u(-t) & \longleftrightarrow & \dfrac{1}{s-a} & ; \; \sigma < Re\{a\}. \end{array}}$$

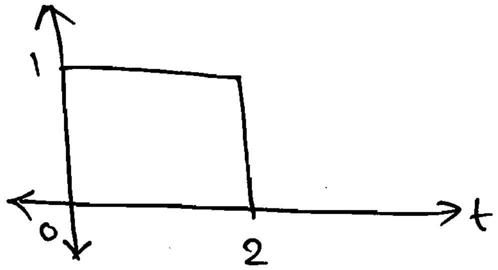
$\Rightarrow$ If $x(t)$ is a finite duration then the RoC is complete s-plane.

e.g. $x(t) = \delta(t)$.

$\downarrow$ L.T.

$X(s) = 1$, RoC entire s-plane.

e.g.



$x(t) = u(t) - u(t-2).$

$X(s). = \frac{1}{s} - \frac{e^{-2s}}{s}.$

$$\lim_{s \to 0} \frac{1 - e^{-2s}}{s}.$$

$$= \lim_{s \to 0} \frac{0 + 2e^{-2s}}{(1)} \quad (\because \text{L' hospital rule.}$$

$$= 2 //$$

Roc is entire s-plane.

* Properties of L.T.

(1.) Linearity :

$\Rightarrow$ If $x_1(t) \longleftrightarrow X_1(s)$ with $Roc = R_1$

$x_2(t) \longleftrightarrow X_2(s)$ with $Roc = R_2.$

then $a x_1(t) + b x_2(t) \longleftrightarrow a X_1(s) + b X_2(s)$

with $Roc = R_1 \cap R_2.$

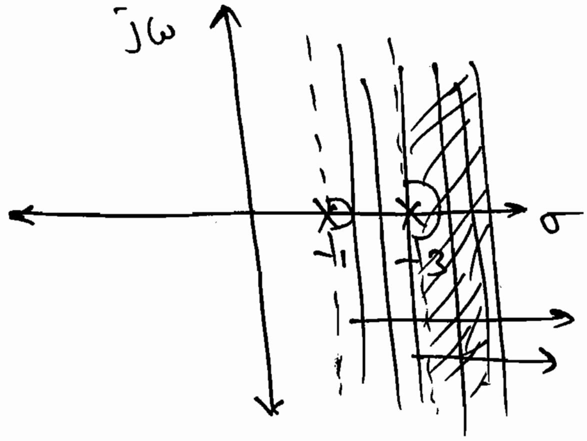
$\boxed{\text{P 5.1.2}}$ Find the L.T. of the following signals with R.O.C.?

1) $x_1(t) = e^{-t} u(t) + e^{-3t} u(t).$

Soln: $X_1(s) = \frac{1}{(s+1)} + \frac{1}{(s+3)}$

$\sigma > -1$ $\quad$ $\sigma > -3.$ $\quad$ common ROC. $\boxed{\sigma > -1}$

$\Rightarrow$



<u>Note</u> :- If the L.T. $X(s)$ of $x(t)$ is rational, then if $x(t)$ is right sided the ROC is the region in the s-plane to the right of the right most pole and if $x(t)$ is left sided, ROC is left of the left most pole.

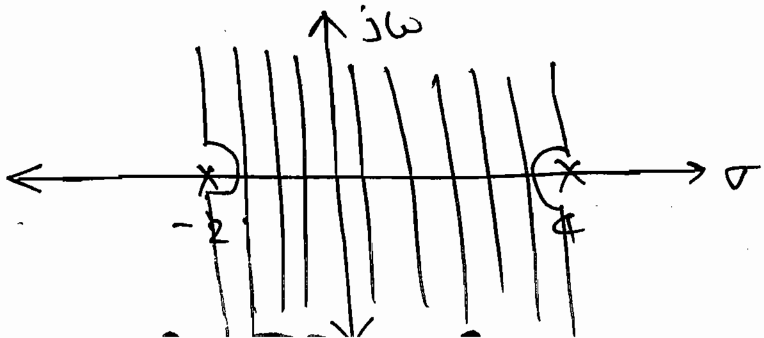
$\rightarrow$ In above case right of the right most pole is -3 so, ROC: $\sigma > -3$.

(2) $x_2(t) = e^{-2t} u(t) + e^{+4t} u(-t)$.

Sol^n: $X_2(s) = \dfrac{1}{(s+2)} - \dfrac{1}{(s-4)}$

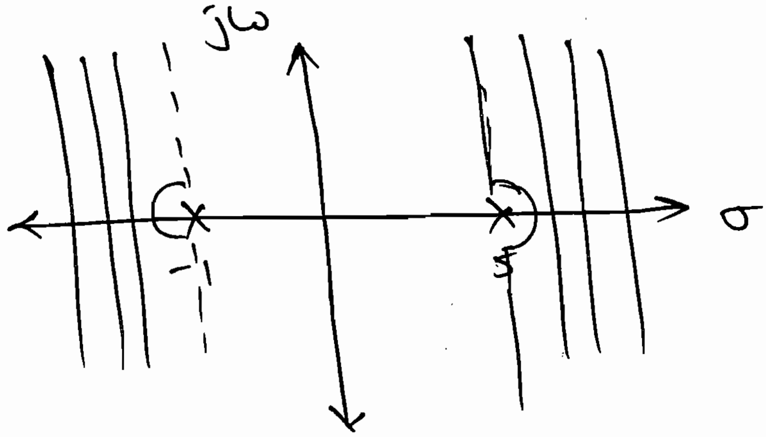
$\uparrow$ $\sigma > -2$ $\uparrow$ $\sigma < 4$.

So, ROC: $-2 < \sigma < 4$.



(3) $x_3(t) = e^{-t}u(-t) + e^{5t}u(t)$.

$\underline{\underline{\text{Sol}^n}}$: $X_3(s) = \underset{\substack{\uparrow \\ \sigma < -1}}{-\dfrac{1}{(s+1)}} + \underset{\sigma > 5.}{\dfrac{1}{(s-5)}}$



No Common ROC so L.T. can not exist.

(4) $X_4(t) = 1 \quad \forall t$.

$\underline{\underline{\text{Sol}^n}}$: $X_4(t) = \underset{\substack{\uparrow \\ \sigma > 0}}{u(t)} + \underset{\substack{\uparrow \\ \sigma < 0}}{u(-t)}$.

No Common ROC. So L.T do not exist.

(5) $x_5(t) = \text{sgn}(t)$.

$\underline{\underline{\text{Sol}^n}}$: $x_5(t) = \underset{\substack{\uparrow \\ \sigma > 0}}{u(t)} - \underset{\substack{\uparrow \\ \sigma < 0}}{u(-t)}$

No Common ROC. So L.T. do not exist.

$\boxed{\text{PS-1.3}}$ Consider the signal $x(t) = e^{-5t}u(t) + e^{-\beta t}u(t)$ & its L.T. is $X(s)$. What are

Constraints placed on the real & imaginary parts of β if the ROC of $X(s)$ is $Re\{s\} > -3$?

Soln:

$$x(t) = e^{-5t}u(t) + e^{-\beta t}u(t).$$

$$X(s) = \frac{1}{(s+5)} + \frac{1}{(s+\beta)}.$$

$\sigma > -5$ $\qquad$ $\sigma > -\beta$.

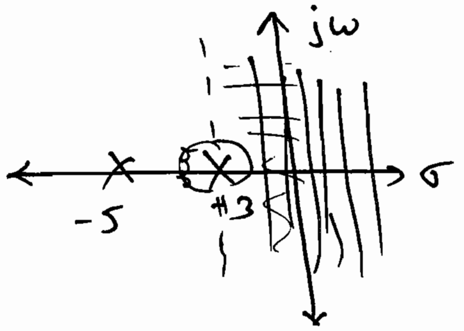
$$\boxed{\begin{array}{l}\sigma + Re\{\beta\} > 0 \\ \sigma + 3 > 0.\end{array}}$$

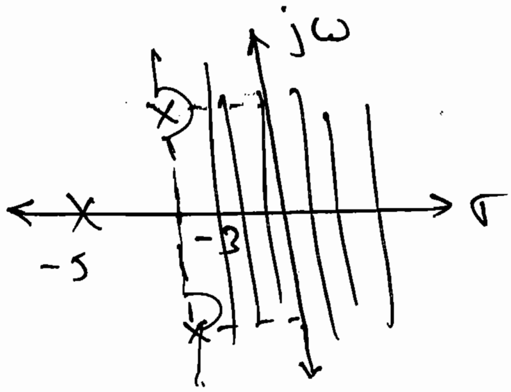
Now, ROC of $X(s)$ is $Re\{s\} > -3$

so, $\sigma > Re\{\beta\}$.

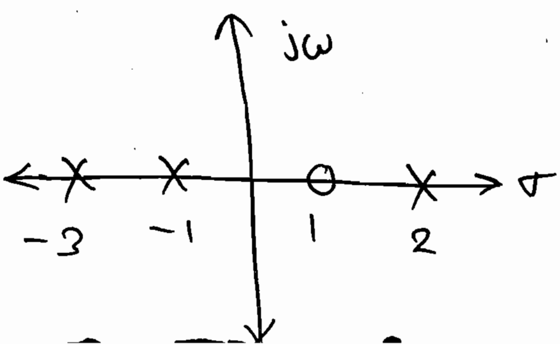
So, $\boxed{Re\{\beta\} = 3.}$

$Im\{\beta\} \Rightarrow$ any value.





$\boxed{P5.1.4}$ How many possible ROC are there for the pole-zero plot shown in fig (1)?



Soln: Possible RoC.

① $\sigma < -3$ ③ $-3 < \sigma < -1$.

② $\sigma > 2$ ④ $-1 < \sigma < 2$.

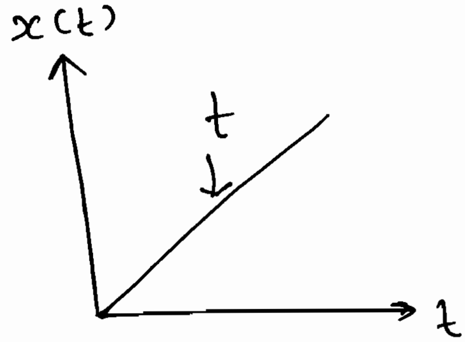
P 5.1.5 In what range should $Re\{s\}$ remain so that the L.T. of the fn $e^{(a+2)t+5} \cdot u(t)$ exists?

Soln:
$$x(t) = e^{(a+2)t} \cdot e^{5} \cdot u(t).$$

so, ~~Re (a+2)~~ $\boxed{Re\{s\} > (a+2)}$

a) $x(t) = t \cdot u(t)$.

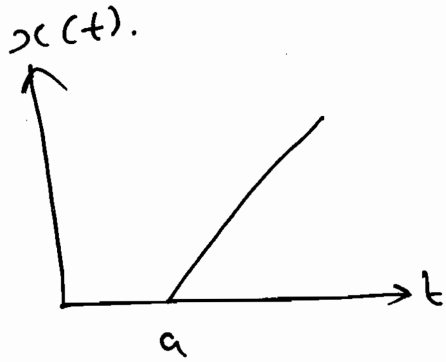
Soln:
$$X(s) = \frac{1}{s^2}$$



a) $(t-a)\, u(t-a)$.

Soln: shifting Property.

$$X(s) = \frac{e^{-as}}{s^2}.$$



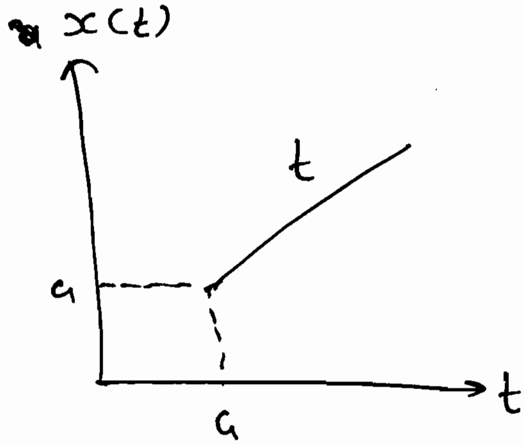
a) $(t-a)\, u(t)$.

Soln: $x(t) = (t-a)\, u(t)$

$= t \cdot u(t) - a u(t)$.

$$\Rightarrow \quad X(s) = \frac{1}{s^2} - \frac{a}{s}.$$

$\boxed{e}$ $\quad t\,u(t-a)$.



Sol^n:

$$x(t) = t\,u(t-a).$$

$$\Rightarrow x(t) = [(t-a)+a]\,u(t-a).$$

$$= (t-a).\,u(t-a) + a\,u(t-a).$$

$\downarrow$ L.T.

$$X(s) = \frac{e^{-as}}{s^2} + \frac{a.e^{-as}}{s^2}.$$

$$\boxed{X(s) = \frac{e^{-as}}{s^2}[1+sa].}$$

(or)

$$\Rightarrow \quad x(t) = t\,u(t-a).$$

$$L[x(t)] = L[t\,u(t-a)].$$

$$= e^{-as}\,L[t+a].$$

$$= e^{-as}\left[\frac{1}{s^2} + \frac{a}{s}\right].$$

$$\boxed{X(s) = \frac{e^{-as}}{s^2}[1+as]}$$

(*) $$X(s) = \int_{-\infty}^{+\infty} x(t).\,e^{-st}\,dt.$$

$$X(s) = F.T.\left\{x(t).\,e^{-\sigma t}\right\}.$$

when $\sigma = 0 \Rightarrow s = j\omega$

$$X(s) = \text{F.T.}\{x(t)\}.$$

So, $\boxed{\text{L.T. of } x(t) \text{ calculated on } j\omega \text{ axis (i.e. } s=j\omega) \text{ is nothing but the F.T. of } x(t).}$

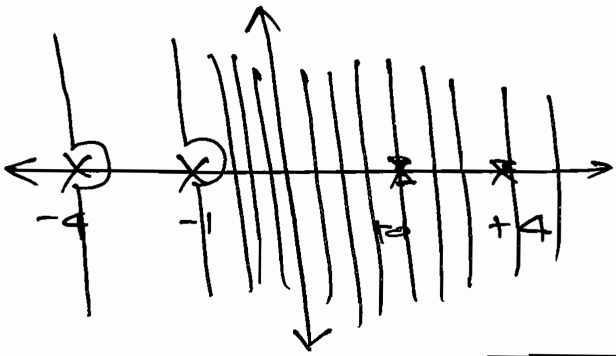
$\Rightarrow$ $\boxed{\begin{array}{l}\text{Right-sided} \Rightarrow u(t). \\ \text{Left-sided} \Rightarrow -u(-t).\end{array}}$ ** *

e.g. $X(s) = \dfrac{1}{(s+1)(s+4)}$.

Soln: $X(s) = \dfrac{1}{(s+1)(s+4)}$.

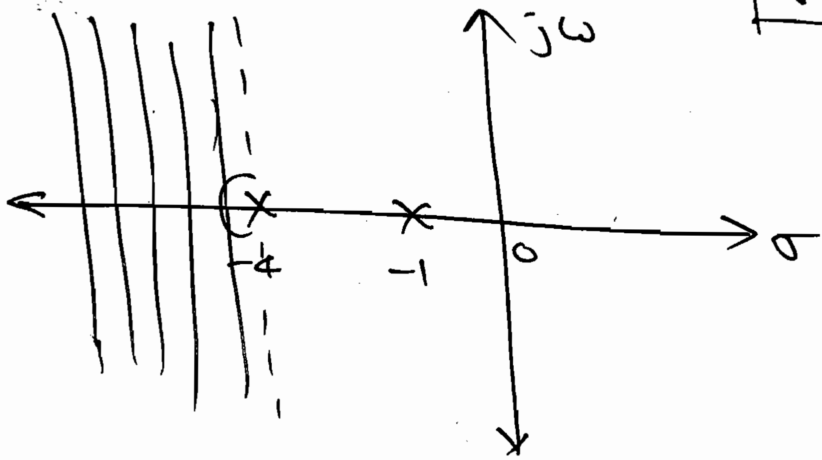
$$= \frac{\frac{1}{3}}{(s+1)} - \frac{1/3}{(s+4)}.$$

① $x(t)$ is right sided, $\boxed{\sigma > -1}$



$\Rightarrow$ $\boxed{x(t) = \frac{1}{3} e^{-t} u(t) - \frac{1}{3} e^{-4t} \cdot u(t).}$

② $x(t)$ is left-sided. $\boxed{\sigma < -4}$



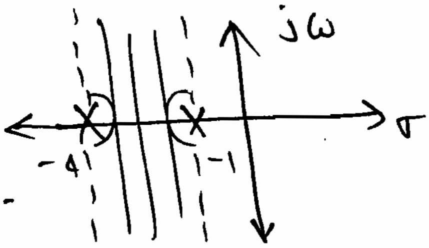
$\Rightarrow$ $\boxed{x(t) = -\frac{1}{3}e^{-t}u(-t) + \frac{1}{3}e^{-4t}u(-t).}$

③ $x(t)$ is two sided. $\boxed{-4 < \sigma < -1}$

① $-4 < \sigma$ ② $\sigma < -1$

$\Rightarrow$ $\sigma > -4$ $\Downarrow$

$u(t)$ $-u(-t)$.



So, $\boxed{x(t) = -\frac{1}{3}e^{-t}\cdot u(-t) + \frac{1}{3}e^{-4t}\cdot u(t).}$

$\boxed{P5.1.1}$ Given $X(s) = \frac{2s+5}{s^2+5s+6}$, find all the time-domain signals?

Sol^n:

$$X(s) = \frac{2s+5}{s^2+5s+6}.$$

$$\Rightarrow X(s) = \frac{2s+5}{(s+2)(s+3)} = \frac{1}{(s+2)} + \frac{1}{(s+3)}.$$

① $x(t)$ is left sided.

R.O.C.:- $\boxed{\sigma < -3}$

$$\Rightarrow \boxed{x(t) = -e^{-2t}u(-t) - e^{-3t}u(-t).}$$

② $x(t)$ is Right sided.

R.o.c. $\boxed{\sigma > -2}$

$$\boxed{x(t) = e^{-2t}\cdot u(t) + e^{-3t}\cdot u(t)}$$

③ $x(t)$ is Two sided.

R.o.c. $-3 < \sigma < -2.$

$$\therefore \boxed{x(t) = -e^{-2t}\cdot u(-t) + e^{-3t}\cdot u(t).}$$

2. Time- Shifting:-

$\Rightarrow$ let, $x(t) \longleftrightarrow X(s).$, $Roc = R$

then $\boxed{x(t-t_0) \longleftrightarrow e^{-st_0}\cdot X(s)}$, with $Roc = R.$

$\boxed{P\,5.1.6.}$

(b) $x(t) = u(t-5).$

Soln:

$$X(s) = e^{-5s}\cdot \mathcal{L}[u(t)].$$

$$= e^{-5s}/s.$$

(c) $y(t) = e^{5t} u(-t+3)$.

Soln:

$$y(t) = e^{5t} \cdot u(-(t-3))$$

$$\neq \cancel{e^{5t}} \cdot \cancel{e^{-3s}} \; L[\cancel{u(-t)}]$$

$$= e^{-3s} \cdot L[e^{5t} \cdot u(-t)]$$

$$= e^{+3(s-5)} \cdot \frac{1}{-(s-5)}$$

$\boxed{P\,5.1.8}$ Consider the signal $x(t) = e^{-5t} u(t-1)$

with L.T. $X(s)$.

a) Find $X(s)$ with R.O.C.?

Soln:

$$x(t) = e^{-5t} \cdot u(t-1)$$

$$X(s) = e^{-5t+5-5} \cdot u(t-1)$$

$$= e^{-5(t-1)} \cdot u(t-1) \cdot e^{-5}$$

$$X(s) = \frac{e^{-5} \cdot e^{-s}}{s+5} \quad ; \quad \text{ROC } \boxed{\sigma > -5}$$

(or)

$$X(s) = e^{-s}\left[L\{e^{-5(t+1)}\}\right]$$

$$= e^{-5}\left[\frac{e^{-5}}{s+5}\right]$$

$$\Rightarrow \quad X(s) = \frac{e^{-5} \cdot e^{-s}}{s+5} \; ; \quad \text{R.o.C.} \; \boxed{\sigma > -5}$$

(b) Find the values of 'A' & 't_0' such that the L.T. $G(s)$ of $g(t) = A e^{-5t} \cdot u(-t-t_0)$.

has same algebraic form as $X(s)$.

What is the R.o.C. of corresponding to $G(s)$?

Sol^n:

$$g(t) = A \cdot e^{-5t} \cdot u(-t-t_0).$$

$$= A \cdot e^{-5t} \cdot u(-(t+t_0)).$$

s become $-s$

$$= A \cdot e^{-t_0 s} \cdot L\left[\frac{e^{-5(t-t_0)}}{} \right].$$

$$= A \cdot e^{5t_0} \cdot e^{-t_0(-s)} \times \frac{1}{-s+5}$$

$$G(s) = \frac{-A \cdot e^{5t_0} \cdot e^{t_0 s}}{s-5} \; ; \quad \text{R.o.C.}$$

$s-5 > 0$

$\sigma > 5$

but $\boxed{\sigma < -5}$

($\because$ s becomes $-s$).

$\therefore$ So, $\boxed{A = -1}$, $\boxed{t_0 = -1}$,

& R.o.C. $\boxed{\sigma < -5}$.

(3) Shift in S-domain:-

$\Rightarrow$ If $x(t) \longleftrightarrow X(s)$ with Roc $= R$ then

$$\boxed{x(t) \cdot e^{s_0 t} \longleftrightarrow X(s - s_0) \quad \text{with Roc} = R + Re(s_0)}$$

(4) Time - reversal :-

$\Rightarrow \quad x(t) \longleftrightarrow X(s)$ then $x(-t) \leftrightarrow X(-s)$,

ROC $= -R$.

P 5.1.7. Find the I.L.T. of

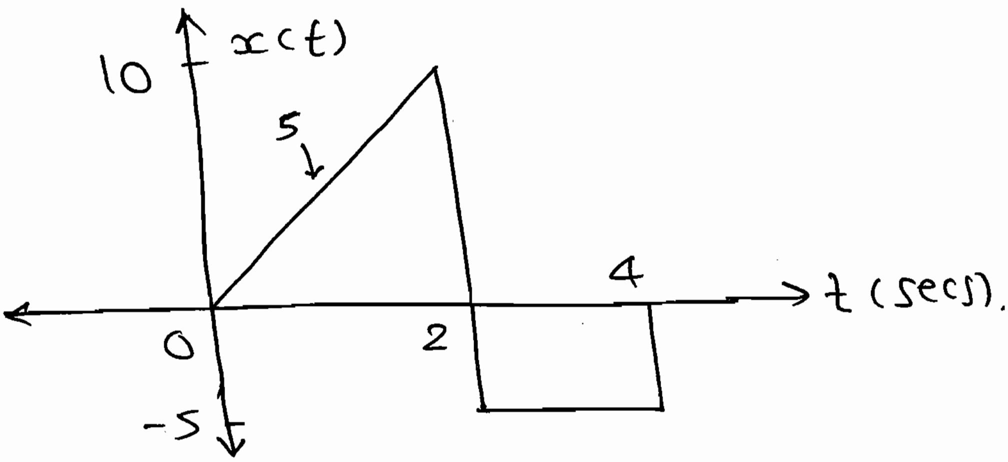
$$Y(s) = \frac{e^{-3s}}{(s+1)(s+2)}, \quad \sigma > -1.$$

Sol^n:

$$Y(s) = \underset{t_0 = 3}{e^{-3s}} \left[\frac{1}{(s+1)} - \frac{1}{(s+2)} \right].$$

$$\boxed{y(t) = e^{-(t\mp 3)} \cdot u(t \mp 3) - e^{-2(t\mp 3)} \cdot u(t \mp 3).}$$

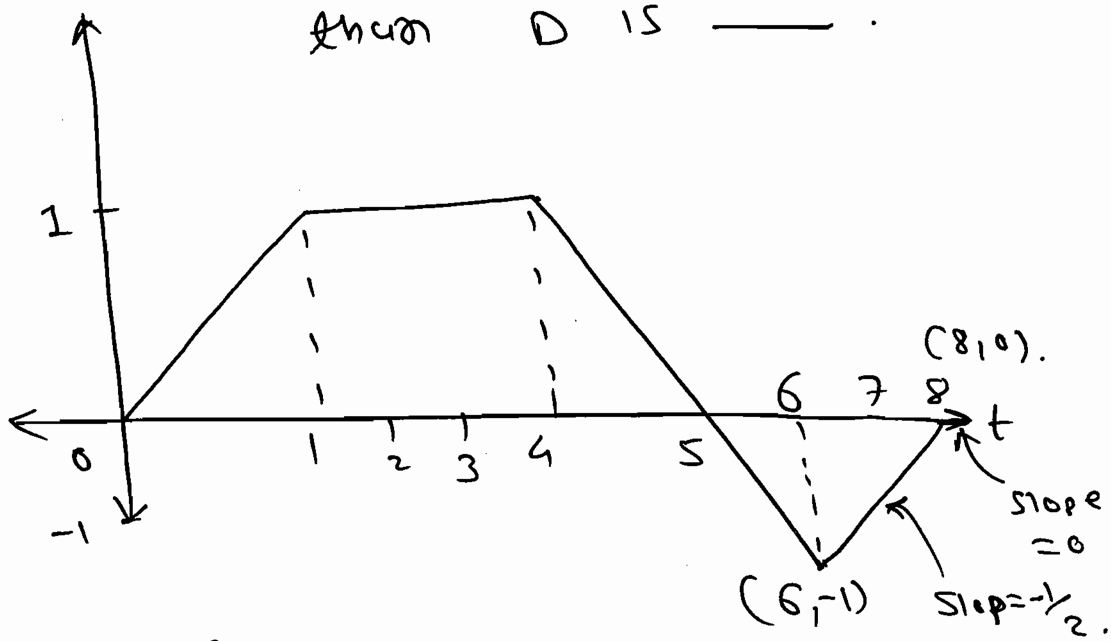
P 5.1.9. Find the L.T. of the waveform shown in figure?



Sol^n: $x(t) = 5r(t) - 5r(t-2) - 15u(t-2) + 5u(t-4)$.

$$\Rightarrow X(s) = \frac{5}{s^2} - \frac{5e^{-2s}}{s^2} - \frac{15e^{-2s}}{s} + \frac{5e^{-4s}}{s}.$$

P 5.1.10 L.T. of the waveform shown in is $\frac{1}{s^2}(1 + Ae^{-s} + Be^{-4s} + Ce^{-6s} + De^{-8s})$. then D is ____ .



$\underline{\underline{\text{sol}^n}}$:

$$\text{slop} = \frac{-1-0}{6-8} = \frac{1}{2}.$$

D (8 to 6).

so, $\boxed{D = \frac{1}{2}}$

A (1 to 0).

so, $\boxed{A = \text{slop} = 1.}$

P 5.1.11 Find the L.T. of

1) $x_1(t) = \cos\omega_0 t\, u(t)$

$\underline{\underline{\text{sol}^n}}$:

$$x_1(t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \cdot u(t).$$

$$x_1(t) = \frac{1}{2} \cdot u(t) \cdot e^{j\omega_0 t} + \frac{1}{2} \cdot u(t) \cdot e^{-j\omega_0 t}$$

let, $x(t) = u(t)$

L.T. $\rightarrow$ $X(s) = 1/s$.

$$X_1(s) = \frac{1}{2} X(s - j\omega_0) + \frac{1}{2} X(s + j\omega_0).$$

$$\Rightarrow \quad X_1(s) = \frac{1}{2}\left[\frac{1}{s-j\omega_0} + \frac{1}{s+j\omega_0}\right].$$

$$\therefore \quad \boxed{X_1(s) = \frac{s}{s^2+\omega_0^2}} \quad ; \quad \sigma > 0+0$$

$$\boxed{\sigma > 0}.$$

(2) $x_2(t) = t \cdot e^{-3t} u(t).$

Soln: $x_2(t) = t \cdot e^{-3t} \cdot u(t).$

let, $x(t) = t \Rightarrow X(s) = 1/s^2.$

$$\therefore \quad X_2(s) = X(s+3).$$

$$\therefore \quad \boxed{X_2(s) = \frac{1}{(s+3)^2}} \quad ; \quad \boxed{\sigma > 0}$$

(3) $x_3(t) = e^{-at} \cdot \sin\omega_0 t \, u(t).$

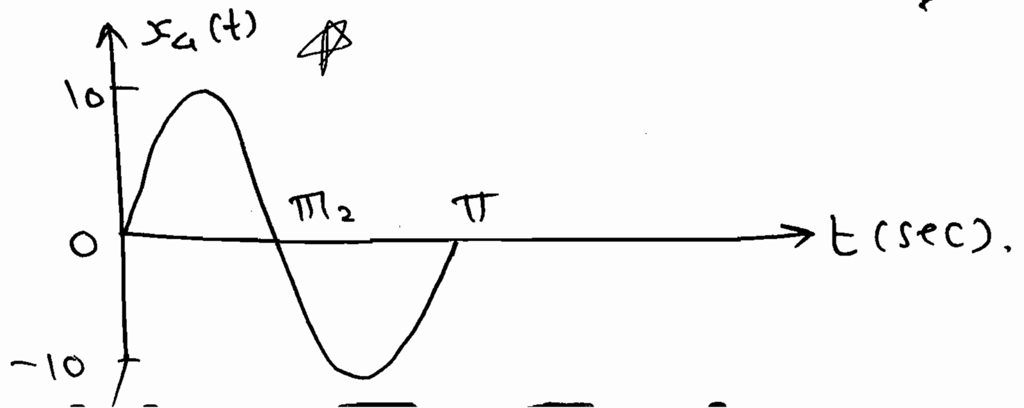
Soln: $L[\sin\omega_0 t] = \frac{\omega_0}{s^2+\omega_0^2}.$

$$\therefore \quad X_3(s) = X(s+a).$$

$$\therefore \quad \boxed{X_3(s) = \frac{\omega_0}{(s+a)^2+\omega_0^2}} \quad ; \quad \sigma > 0 + Re\{-a\}$$

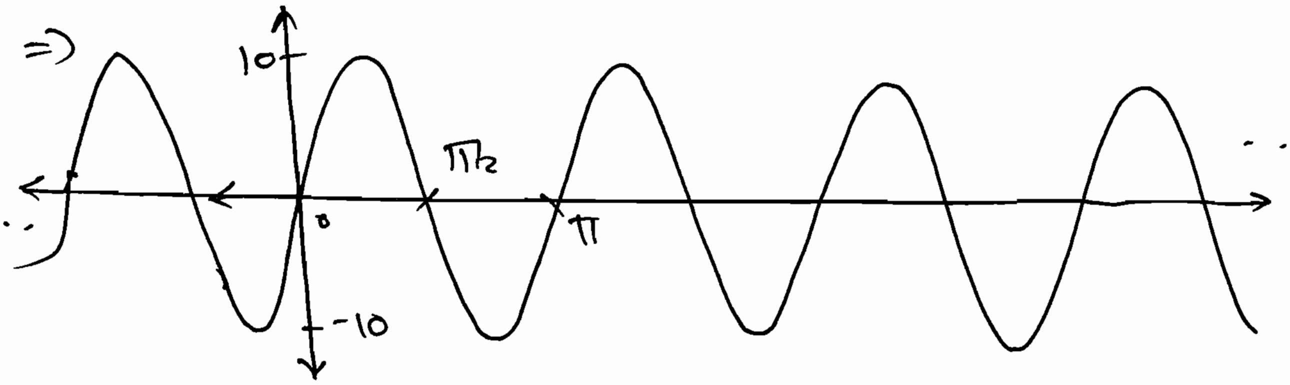
$$; \boxed{\sigma > Re\{-a\}}.$$

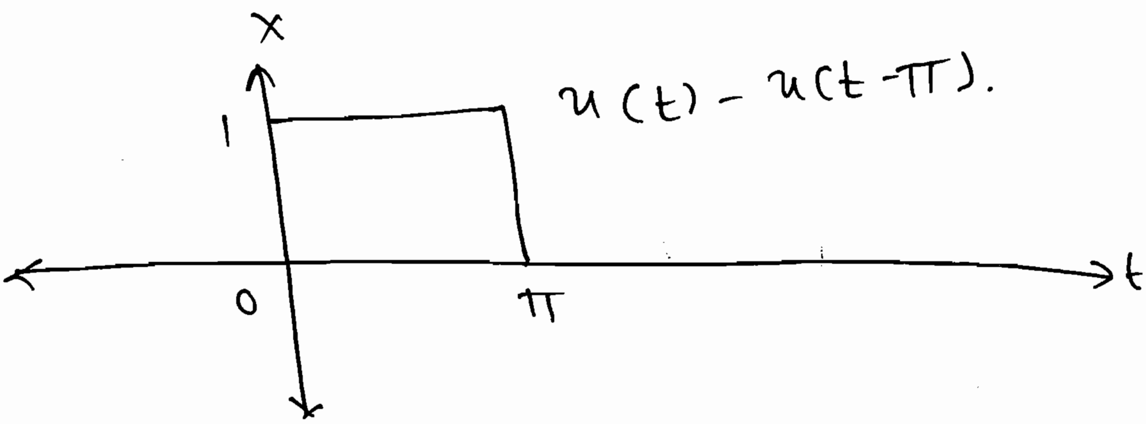
(4)



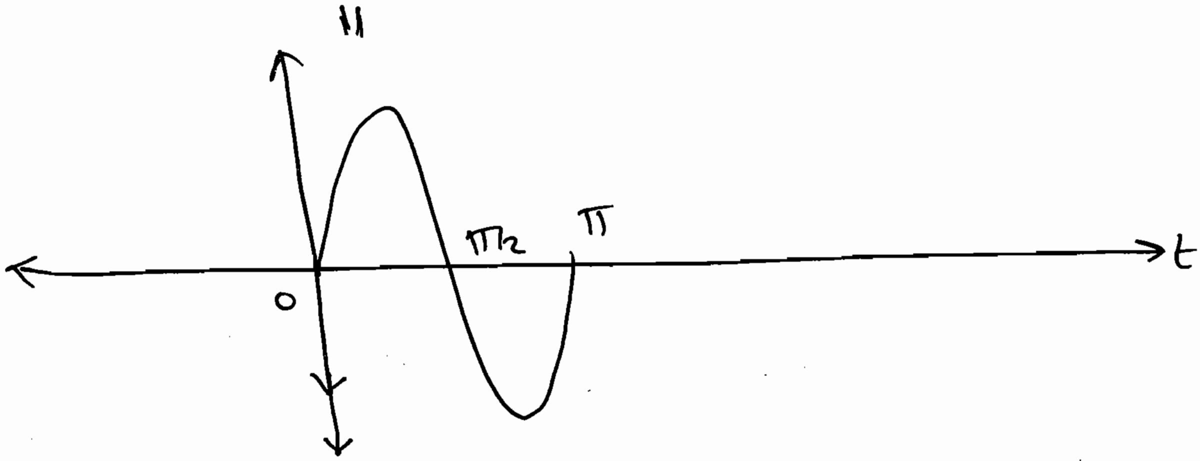
$\Rightarrow \quad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{\pi} = 2.$

$\Rightarrow$





$u(t) - u(t-\pi).$



$\Rightarrow \quad x_a(t) = 10 \sin \omega_0 t \cdot [u(t) - u(t-\pi)].$

$x_a(t) = \sin 2t \cdot [u(t) - u(t-\pi].$

$\downarrow^{L.T.} = \frac{20}{s^2+4} - \frac{20e^{-\pi s}}{s^2+4}.$

$$\boxed{X_a(s) = \frac{20}{s^2+4}\left[1 - e^{-\pi s}\right].}$$

P5.1.12 Let $x(t)$ be a signal that has a rational L.T. with exactly 2 poles located at $s=-1$ and $s=-3$. If $g(t) = e^{2t}x(t)$ & $G(\omega)$ converges, determine whether $g(t)$ is

(a) left sided (b) right-sided

(c) two-sided (d) finite-duration.

Soln:

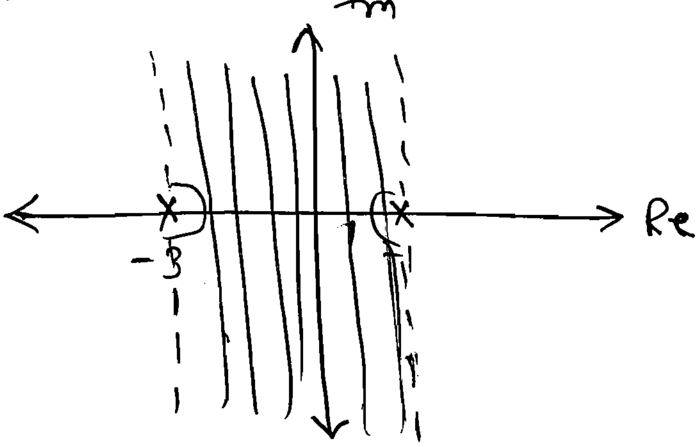
$$X(s) = \frac{1}{(s+1)(s+3)}.$$

$\Rightarrow$ Now, $g(t) = e^{2t} \cdot x(t).$

$$\Rightarrow \quad G(s) = X(s-2).$$

$$\Rightarrow \quad G(s) = \frac{1}{(s-1)(s+3)}$$

Now, $G(\omega)$ converges means F.T. is define and L.T.'s ROC includes $j\omega$ (or) "Im" axis.



R.O.C. must be

$\Rightarrow$ $\boxed{-3 < \sigma < -1}$ So, Ans (c) two sided

$\boxed{P\ 5.1.13}$ Let $g(t) = x(t) + \alpha x(-t)$ where

$x(t) = \beta e^{-t} u(t)$. & $G(s) = \frac{s}{s^2-1}$, $-1 < Re\{s\} < 1$,

find α & β?

Soln:

$$g(t) = x(t) + \alpha x(-t).$$

$$\therefore G(s) = X(s) + \alpha X(-s).$$

$$\Rightarrow x(t) = \beta \cdot e^{-t} u(t).$$

$$\Rightarrow X(s) = \frac{\beta}{(s+1)}, \quad Re\{s\} > -1.$$

$$X(-s) = \frac{\beta}{-s+1}, \quad R\{s\} < 1.$$

$$\therefore G(s) = \frac{\beta}{(s+1)} + \frac{\alpha\beta}{(-s+1)}.$$

$$G(s) = \frac{\beta}{(s+1)} - \frac{\alpha\beta}{s-1}, \quad \text{--- } ①$$

given $G(s) = \frac{s}{s^2-1} = \frac{s}{(s+1)(s-1)}$.

$$G(s) = \frac{1/2}{(s-1)} - \frac{-1/2}{(s-1)}. \quad \text{--- } ②$$

Compare eqn ① & ②.

$\therefore \boxed{\beta = \frac{1}{2}}$

$$\alpha \cdot \beta = -\tfrac{1}{2}.$$

$$\alpha \cdot \cancel{\left(\tfrac{1}{2}\right)} = \left(-\cancel{\tfrac{1}{2}}\right).$$

$$\boxed{\alpha = -1}$$

$\boxed{Q}$ $Y(s) = \dfrac{s^2 - s + 1}{(s+1)^2}$; $\sigma > -1$.

Sol^n:

$$Y(s) = \frac{s^2 - s + 1}{(s+1)^2}.$$

$$= \frac{s^2 + 2s + 1 - 3s}{(s+1)^2}.$$

$$= \frac{(s+1)^2 - 3(s+1-1)}{(s+1)^2}.$$

$$Y(s) = 1 - \frac{3}{(s+1)} + \frac{3}{(s+1)^2}.$$

$\downarrow$ I.L.T.

$$\boxed{y(t) = \delta(t) - 3e^{-t} u(t) + 3t \cdot e^{-t} \cdot u(t)}$$

5) Differentiation in time :-

If $x(t) \longleftrightarrow X(s)$ with ROC = R

then $\dfrac{dx(t)}{dt} \longleftrightarrow sX(s)$. with ROC = R.

$\boxed{P5.1.14}$ Consider 2 right-sided signals $x(t)$ & $y(t)$ related through the equation

$$\frac{dx(t)}{dt} = -2y(t) + \delta(t) \;\&$$

$$\frac{dy(t)}{dt} = 2x(t).$$

find $X(s)$ & $Y(s)$ with ROCs?

Soln:

$$sX(s) = -2Y(s) + 1$$

& $$sY(s) = 2X(s).$$

$$\Rightarrow Y(s) = \frac{2X(s)}{s}.$$

$$\therefore \quad sX(s) = -2\left[\frac{2X(s)}{s}\right] + 1.$$

$$\therefore \quad sX(s) = -\frac{4X(s)}{s} + 1.$$

$$X(s)\left[s + \frac{4}{s}\right] = 1.$$

$$\therefore \quad \boxed{X(s) = \frac{s}{s^2+4}}. \quad \text{ROC } \boxed{\sigma > 0}$$

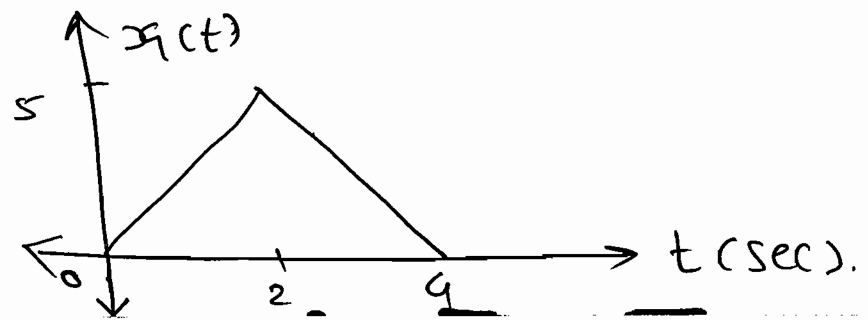
$$\therefore \quad x(t) = \cos 2t$$

$$\therefore \quad Y(s) = \frac{2}{\cancel{s}} \times \frac{\cancel{s}}{s^2+4} = \frac{2}{s^2+4}.$$

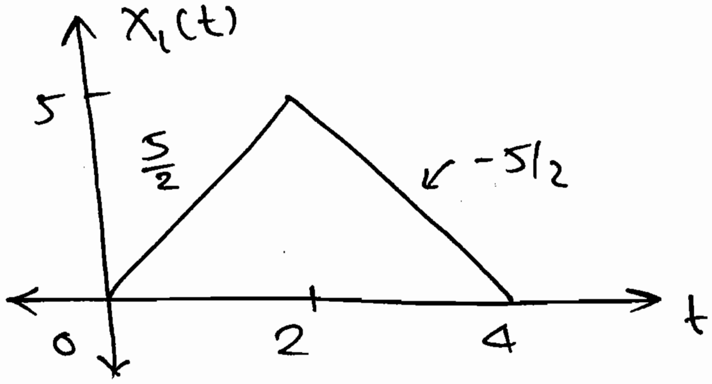
$$\Rightarrow \boxed{Y(s) = \frac{2}{s^2+4}} \Rightarrow y(t) = \sin 2t.$$

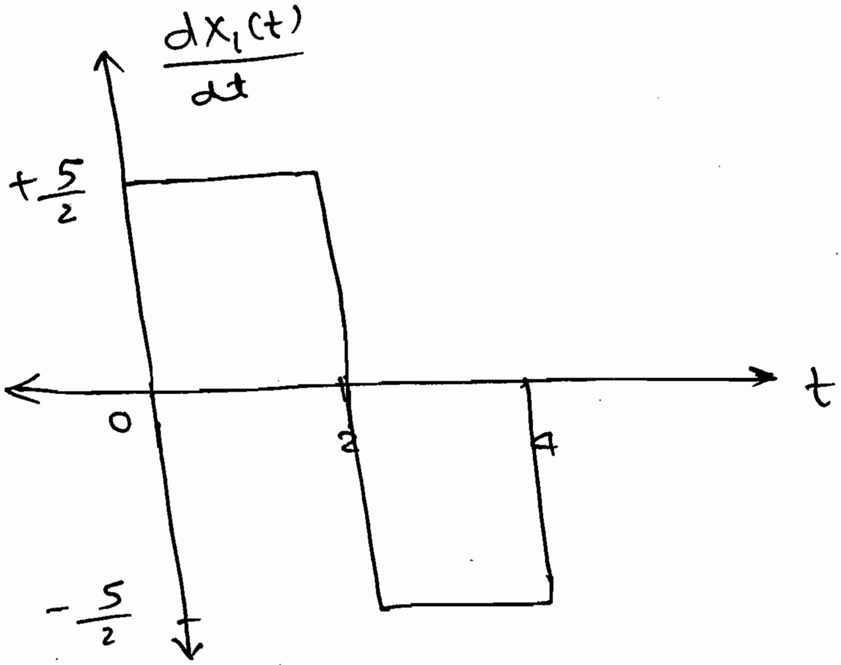
$$\sigma > 0$$

$\boxed{\text{P 5.1.15}}$ By using derivative method find the L.T. of the following signal?



Sol^n:



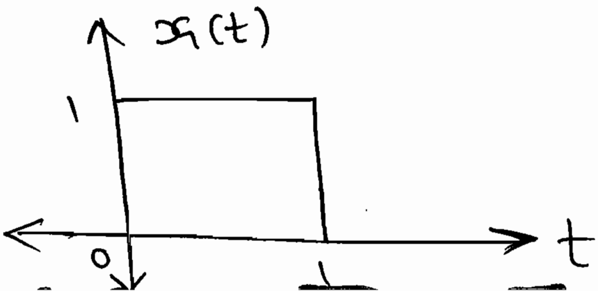


$$\therefore \frac{dx_1(t)}{dt} = \frac{5}{2}u(t) - \frac{10}{2}u(t-2) + \frac{5}{2}u(t-4).$$

$$\therefore sX_1(s) = \frac{5}{2s} - 5 \cdot \frac{e^{-2s}}{s} + \frac{5}{2} \cdot \frac{e^{-4s}}{s}.$$

$$\therefore \boxed{X_1(s) = \frac{5}{2s^2} - \frac{5 \cdot e^{-2s}}{s^2} + \frac{5 \cdot e^{-4s}}{2s^2}.}$$

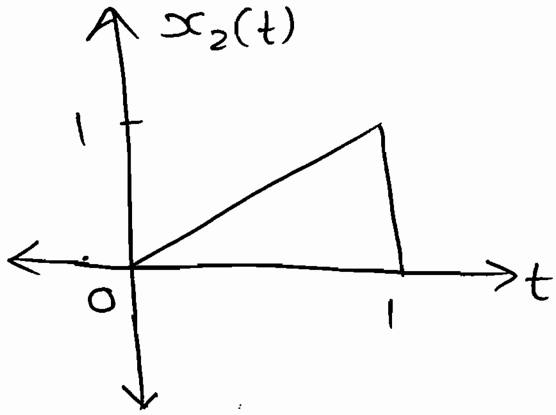
P 5.1.16 Find the Laplace transform of the following signals?

① 

Sol^n: $x_1(t) = u(t) - u(t-1)$.

$$\xrightarrow{L.T.}\quad X_1(S) = \frac{1}{s} - \frac{e^{-s}}{s} = \frac{1-e^{-s}}{s}.$$

(2)



Sol^n: $x_2(t) = t \cdot [u(t) - u(t-1)]$.

$x_2(t) = t \cdot x_1(t)$

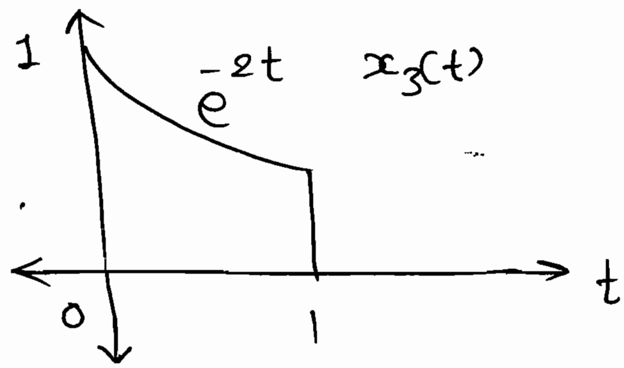
$\downarrow$ L.T.

$$\therefore\ X_2(S) = (-1)^1 \cdot \frac{d}{ds}(X_1(S)).$$

$$= -\left[\frac{-1}{s^2} - \frac{s(-1)\cdot e^{-s} - e^{-s}}{s^2} \right].$$

$$X_2(S) = \frac{1}{s^2} + \frac{-s\cdot e^{-s} - e^{-s}}{s^2}.$$

P 5.1.16



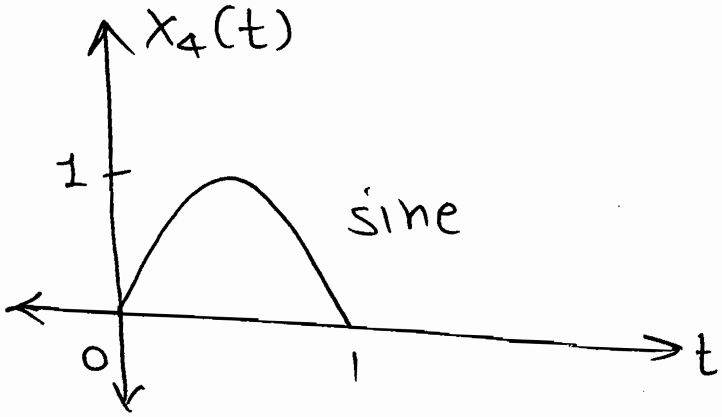
Sol^n: $x_3(t) = x_1(t)\,[u(t) - u(t-1)]$.

$$\therefore \quad x_3(t) = e^{-2t} \cdot x_1(t).$$

$$\Rightarrow \quad X_3(s) = X_1(s+2).$$

$$\boxed{X_3(s) = \frac{1 - e^{-(s+2)}}{(s+2)}}$$

(4)



$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2}$$

$$\omega_0 = \pi$$

Sol^n:

$$\therefore \quad x_4(t) = \sin \omega_0 t \cdot [x_1(t)].$$

$$= \sin \pi t \cdot [x_1(t)].$$

$$= \frac{e^{+j\pi t} - e^{-j\pi t}}{2j} \cdot x_1(t).$$

$$x_4(t) = \frac{e^{+j\pi t} \cdot x_1(t) - e^{-j\pi t} \cdot x_1(t)}{2j}$$

$$\big\downarrow \text{L.T.}$$

$$\therefore \quad X_4(s) = \frac{X_1(s+j\pi) - X_1(s+j\pi)}{2j}.$$

$$\therefore \quad X_4(s) = \frac{1 - e^{-(s-j\pi)}}{(s+2-j\pi)2j} - \frac{1 - e^{-j(s+j\pi+2)}}{(s+j\pi+2)}$$

6) Differentiation in S-domain :-

$\Rightarrow$ If $x(t) \longleftrightarrow X(S)$ with $ROC = R$,

then $t\,x(t) \longleftrightarrow -\frac{d}{dS} X(S)$ $ROC = R$.

$$\Rightarrow \boxed{t^n . x(t) \longleftrightarrow (-1)^n . \frac{d^n}{dS^n} X(S).}$$

P 5.1.17 Find the I.L.T. of $X(S) = \log\left[\frac{S+5}{S+6}\right]$?

Soln:

$$X(S) = \log\left[\frac{S+5}{S+6}\right].$$

$$\therefore \frac{dX(S)}{dS} = \frac{1.\cancel{(S+6)}}{(S+5)} \times \left[\frac{(S+6)(1) - (S+5)}{(S+6)^{\cancel{2}}}\right]$$

$$= \frac{1}{(S+5)(S+6)}.$$

$$\therefore \frac{dX(S)}{dS} = \frac{1}{(S+5)} - \frac{1}{(S+6)}.$$

$$\therefore -t . x(t) = \left(e^{-5t} - e^{-6t}\right) u(t).$$

$$\therefore \boxed{x(t) = \left[\frac{e^{-6t} - e^{-5t}}{t}\right] u(t).}$$

So, in general,

$$\boxed{x(t) = \left[\frac{e^{-\alpha t} - e^{-\beta t}}{t}\right] u(t) \xleftrightarrow{L.T.} \text{Log}\left[\frac{S+\beta}{S+\alpha}\right]}$$

$\boxed{P\ 5.1.18}$ Find the ILT of

(a) $$X(s) = \frac{4}{(s+2)(s+1)^3}.$$

Sol^n:

$$X(s) = \frac{4}{(s+2)(s+1)^3}.$$

$$X(s) = 4\left[\frac{A=-4}{(s+2)} + \frac{B=4}{(s+1)^3} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)}\right]$$

$$X(s) = \cancel{4}\left[\frac{-4}{(s+2)} + \frac{4}{(s+1)^3} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)}\right]$$

$$= \frac{4}{(s+2)(s+1)^3}.$$

Put $s=0$.

$$\therefore \frac{4}{2} = -\frac{4}{2} + 4 + C + D.$$

$$\boxed{C=-D}$$

Put $s=1$.

$$\therefore \frac{1}{6} = -\frac{4}{3} + \frac{1}{2} + \frac{C}{4} + \frac{D}{2}.$$

$\Rightarrow$ $C + 2D = 4$.

$\Rightarrow$ $\boxed{D=4}$ & $\boxed{C=-4}$

$$\therefore X(s) = \frac{-4}{(s+2)} + \frac{4}{(s+1)^3} - \frac{4}{(s+1)^2} + \frac{4}{(s+1)}.$$

$\downarrow$ I.L.T.

$$\boxed{x(t) = -4e^{-2t} + \frac{4}{2!}e^{-t}\cdot t^2 - 4te^{-t} + 4e^{-t}}$$

(b) $$X(s) = e^{-2s} \frac{d}{ds}\left[\frac{1}{(s+1)^2}\right].$$

Solⁿ:

$$X(s) = e^{-2s}\left[\frac{-2}{(s+1)^3}\right].$$

$$\downarrow_{\text{I.L.T.}} \quad = \frac{-2e^{-2s}}{(s+1)^3}.$$

$$x(t) = -2\frac{(t+2)^2}{2!} \cdot e^{-(t+2)} u(t+2).$$

$$\therefore \boxed{x(t) = -(t+2)^2 \cdot e^{-(t+2)} \cdot u(t+2)}$$

7) Convolution in time :-

⇒ If $x(t) \longleftrightarrow X(s)$ with $ROC = R_1$ &

$h(t) \longleftrightarrow H(s)$ with $ROC = R_2$.

then $$\boxed{x(t) * h(t) \longleftrightarrow X(s) \cdot H(s) \quad ROC = R_1 \cap R_2}$$

⇒ L.T. of impulse response is known as system (or) transfer function.

$\boxed{\text{P 5.1.19}}$ Solve the following equation.

$$y(t) + \int_0^\infty y(\tau) \cdot x(t-\tau)\, d\tau = x(t) + \delta(t)?$$

Solⁿ: $$y(t) + y(t) * x(t) = x(t) + \delta(t).$$

$\xrightarrow{L.T.}$ $Y(s) + \cancel{X(s)} + X(s) \cdot Y(s) = X(s) + 1.$

$\therefore \quad Y(s)\,[\cancel{1 + X(s)}] = \cancel{(1 + X(s))}$

$\therefore \quad Y(s) = 1$

$\therefore \quad \boxed{y(t) = \delta(t).}$

P 5.1.20 ~~Solve the following~~

Consider a signal $y(t) = x_1(t-2) * x_2(-t+3)$ where $x_1(t) = e^{-2t} \cdot u(t)$ & $x_2(t) = e^{-3t} \cdot u(t)$. Find $Y(s)$ with ROC?

Soln:

$$X_1(s) = \frac{1}{(s+2)} \quad \sigma > -2, \quad X_2(s) = \frac{1}{(s+3)} \quad \sigma > -3.$$

$\therefore \quad y(t) = x_1(t-2) * x_2(-t+3).$

$$Y(s) = e^{-2s} X_1(s) \cdot e^{-3s} \cdot X_2(-s).$$

$$\therefore \quad \boxed{Y(s) = \frac{e^{-2s}}{\underbrace{(s+2)}_{\sigma > -2}} \cdot \frac{e^{-3s}}{\underbrace{(-s+3)}_{\sigma < 3}}}$$

So, ROC $\boxed{-2 < \sigma < 3.}$

P 5.2.21 Find the transfer function & impulse response of a filter whose input-output relation is described by

$$y(t) = x(t) + \int_{-\infty}^{t} y(\lambda) \cdot e^{-3(t-\lambda)} \cdot u(t-\lambda)\, d\lambda.$$

$\underline{\underline{\text{Sol}^n}}$:

~~Y/X(S)/7~~

$$y(t) = \left[x(t) + y(t) * e^{-3t}\right].$$

$$\therefore \quad Y(s) = X(s) + Y(s) \cdot \frac{1}{s+3}.$$

$$\therefore \quad Y(s)\left[1 - \frac{1}{s+3}\right] = X(s).$$

$$\therefore \quad \frac{Y(s)}{X(s)} = H(s) = \frac{s+3}{s+2} = \frac{s+2+1}{(s+2)}.$$

$$\therefore \quad H(s) = 1 + \frac{1}{(s+2)}.$$

$$\therefore \quad \boxed{h(t) = \delta(t) + e^{-2t}.u(t).}$$

P 5.1.23 An Input $x(t) = \exp(-2t).u(t) + \delta(t-6)$ is applied to an L.T.I. system with impulse response $h(t) = u(t)$. The output is

$\underline{\underline{\text{Sol}^n}}$:

$$X(s) = \frac{1}{(s+2)} + e^{-6s}.$$

$$H(s) = \frac{1}{s}.$$

$$\therefore \quad H(s) = \frac{Y(s)}{X(s)}.$$

$$\therefore \quad Y(s) = H(s) \cdot X(s).$$

$$= \frac{1}{s} \cdot \left[\frac{1}{s+2} + e^{-6s}\right].$$

$$\therefore \quad Y(s) = \frac{1}{s(s+2)} + \frac{e^{-6s}}{s}.$$

$$= \frac{1}{2s} - \frac{1}{2(s+2)} + \frac{e^{-6s}}{s}.$$

$\therefore \quad y(t) = \frac{1}{2}u(t) \quad -\frac{1}{2}e^{-2t}\cdot u(t) \quad + u(t+6).$

$\therefore \quad \boxed{y(t) = 0.5\,[1 - \exp(-2)]\,u(t) + u(t+6).}$

$\boxed{P5.1.14}$ Let the L.T. of a function $f(t)$ which exists for $t>0$ be $F_1(s)$ and the L.T. of its delayed version $f(t-\gamma)$ be $F_2(s)$. ~~the~~ let $F_1^*(s)$ be the complex conjugate of $F_1(s)$ with $s = \sigma + j\omega$ of

$$G(s) = \frac{F_2(s).\ F_1^*(s)}{|F_1(s)|^2},$$ then the L.T.I. of

$G(s)$ is

(a) $\delta(t)$ (b) $\delta(t-\gamma)$ (c) $u(t)$ (d) $u(t-\gamma)$.

Soln:

$$F_2(s) = e^{-\gamma s}.\ F_1(s).$$

$$\Rightarrow \quad G(s) = \frac{F_2(s).\ F_1^*(s)}{|F_1(s)|^2}.$$

$$\Rightarrow \quad G(s) = \frac{e^{-Ts}.\ \cancel{F_1(s).\ F_1^*(s)}}{\cancel{|F_1(s)|^2}}$$

$$G(s) = e^{-Ts}.\ 1$$

$\Rightarrow \quad \boxed{g(t) = \delta(t-T).}$

Ans: (b) $\delta(t-\gamma)$.

* **Frequency Integration :-**

$$\Rightarrow \boxed{\frac{x(t)}{t} \longleftrightarrow \int_{s}^{\infty} X(s)\cdot ds.}$$

$\boxed{P\ 5.1.25}$ Find the L.T. of $\frac{\sin\omega_0 t}{t}\cdot u(t)$?

Soln: let, $x(t) = \sin\omega_0 t$

$$X(s) = \frac{\omega_0}{s^2+\omega_0^2}.$$

$$\Rightarrow L\left[\frac{\sin\omega_0 t}{t}\cdot u(t)\right] = \int_{s}^{\infty} X(s)\cdot ds.$$

$$= \int_{s}^{\infty} \frac{\omega_0}{s^2+\omega_0^2}\cdot ds.$$

$$= \frac{\cancel{\omega_0}}{\cancel{\omega_0}} \tan^{-1}\left(\frac{s}{\omega_0}\right)_s^{\infty}$$

$$= \frac{\pi}{2} - \tan^{-1}(s/\omega_0).$$

(*) **Integration in time :-**

$$\Rightarrow \boxed{\int_{0}^{t} x(\tau)\,d\tau \longleftrightarrow \frac{X(s)}{s}.}$$

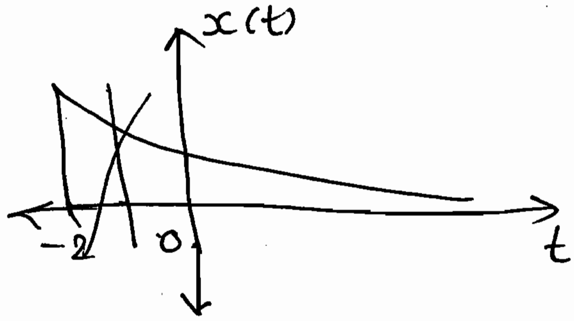
5.2 Unilateral L.T.

$$\boxed{X(s) = \int_{0}^{\infty} x(t)\cdot e^{-st}\cdot dt.}$$

$\boxed{P\ 5.2.1}$ Find the U.L.T. of the following signals & find the ROC ?

(a) $x(t) = e^{-3t} \cdot u(t+2)$.

Soln:



$$\Rightarrow X(s) = \int_{0}^{\infty} e^{-3t} \cdot e^{-st} \cdot dt$$

$$\boxed{X(s) = \frac{1}{s+3} \quad ; \quad \text{R.O.C } \sigma > -3.}$$

(b) $x(t) = \delta(t+2) + \delta(t-4)$.

Soln:

$$\boxed{X(s) = e^{-4s}}$$

* Differentiation in time :-

$$\Rightarrow \frac{d}{dt} x(t) \longleftrightarrow sX(s) - x(0).$$

$$\Rightarrow \frac{d^2}{dt^2} x(t) \longleftrightarrow s^2 X(s) - sx(0) - x'(0).$$

$\boxed{P\ 5.2.2}$ A system described by a linear, constant coefficient, ordinary, first order differential equation has an exact solution given by $y(t)$ for $t>0$, when the forcing function is $x(t)$ and the initial condition is $y(0)$. If one whishes to modify the system so that solution becomes $-2y(t)$ for $t>0$, we need to

Soln: (D) Change the initial condition to $-2y(0)$ and the forcing f^n to $-2x(t)$.

* Total Response

Zero state response (Z.S.R.)
(or)
forced Resp.
(or)
Steady-State Resp.
⇓
I/p is taken.
All initial conditions are zero.
⇓
Transfer function

Zero I/p response (Z.I.R.)
(or)
Natural Response.
(or)
Transient Res.
⇓
I/p is not taken.
Initial conditions are considered.
⇓
Charu. Polynomical.

⇒ T.F. of sys. is always representing the Z.S.R.

* **Differentiation in time:-**

$$\left.\begin{aligned} \frac{d}{dt} &\longrightarrow j\omega \longrightarrow s \\ \frac{d}{d\omega} &\longrightarrow -jt \\ \frac{d}{d(j\omega)} &\longrightarrow -t \\ \frac{d}{ds} &\longrightarrow -t \end{aligned}\right\} \text{ROC} = R.$$

* **Steady - State Response:-**

$\Rightarrow$ If $x(t) = A\cos(\omega_0 t + \theta)$.

then Calculate $H(s)\big|_{s=j\omega_0}$

$\Rightarrow$ $H(s)\big|_{s=j\omega_0} = K\angle\phi$.

> Steady State response,
>
> $$y_{ss}(t) = KA\cdot\cos(\omega_0 t + \theta + \phi).$$

P 5.2.3 Consider a System with T.F.

$H(s) = \dfrac{s-2}{s^2+4s+4}$. Find the Steady - State response when the input applied is $8\cos 2t$?

$\underline{\underline{\text{Sol}^n}}$: $x(t) = 8\cos 2t.$

$A = 8, \quad \omega_0 = 2, \quad \theta = 0^\circ.$

$$H(s)\Big|_{s=2j} = \frac{2j-2}{-\cancel{4}+8j+\cancel{4}} = \frac{j-1}{4j}.$$

$$= \underset{\underset{K}{\downarrow}}{0.3536}\ \underset{\underset{\phi}{\downarrow}}{\angle -45^\circ}.$$

$\therefore$ S.S.R. $= y_{ss}(t) = (0.3536 \times 8)\cos(2t + 0^\circ + (-45^\circ))$

$$\therefore \boxed{y_{ss}(t) = 2\sqrt{2}\cdot\cos(2t - 45^\circ).}$$

* Initial & Final Value Theorem:-

$\Rightarrow$ $\boxed{x(0) = \lim_{s\to\infty} sX(s) \quad \text{I.V.T}}$

$\boxed{X(\infty) = \lim_{s\to 0} sX(s). \quad \text{F.V.T.}}$

Note:-

→ Poles at Imaginary axis, F.V.T is not applicable.

→ F.V.T. is valid only if All Poles have −ve Real Parts except a simple Poles have −ve Real Parts except a simple Pole at $s=0$.

$\boxed{P\ 5.2.4.}$ Find the initial & final value for the following T.F.?

a) $$X(s) = \frac{2s+5}{s^2+5s+6}.$$

$\underline{\underline{Sol^n}}$: I.V.T.
$$x(0) = \lim_{s\to\infty} s.\frac{2s+5}{s^2+5s+6}.$$

$$= \lim_{s\to\infty} \frac{\cancel{s}(2+5/s)}{s^{\cancel{2}}\left(1+\frac{5}{s}+\frac{6}{s^2}\right)}$$

$\therefore$ ~~$x(0) = 0$~~ $\boxed{x(0) = 2}$

$\Rightarrow$

F.V.T.
$$x(\infty) = \lim_{s\to 0} \frac{s(2s+5)}{s^2+5s+6}.$$

$$\boxed{x(\infty) = 0}$$

(b) $$X(s) = \frac{4s+5}{2s+1}.$$

$\underline{\underline{Sol^n}}$: $\Downarrow$ Proper T.F.

We require strically proper f^n.

$$\Rightarrow X(s) = \frac{4s+2+3}{2s+1}$$

$$X(s) = 2 + \left.\frac{3}{2s+1}\right\}$$ Strically Proper.

$$\Rightarrow \quad x(0) = \lim_{s\to\infty} s\left[\frac{3}{2s+1}\right]$$

$$= \lim_{s\to\infty} \frac{3}{2+1/s}$$

$$\boxed{x(0) = 3/2}$$

$$\Rightarrow \quad x(\infty) = \lim_{s\to 0} s\left[\frac{3}{2s+1}\right]$$

$$\therefore \boxed{x(\infty) = 0}$$

(c) $X(s) = \dfrac{12(s+2)}{s(s^2+4)}$

Sol^n: $x(0) = \lim_{s\to\infty} \dfrac{\cancel{s}\cdot 12(s+2)}{\cancel{s}(s^2+1)} = \dfrac{12(1+\frac{2}{s})}{s(1+\frac{4}{s^2})}$

$= 0$.

here, Pole is on imaginary axis.

so, finally value can not be determine.

(d) $X(s) = e^{-s}\left[\dfrac{-2}{s(s+2)}\right]$

Sol^n: $x(0) = \lim_{s\to\infty} \cancel{s}\cdot e^{-s}\left[\dfrac{-2}{\cancel{s}(s+2)}\right]$

$$\therefore \boxed{x(0) = 0}$$

$$\Rightarrow \quad x(\infty) = \lim_{s\to 0} \cancel{s}\cdot e^{-s}\left[\frac{-2}{\cancel{s}(s+2)}\right]$$

$$= \frac{-2}{2}$$

$$\boxed{x(\infty) = -1.}$$

$\boxed{Q}$ *** $X(s) = \dfrac{s}{s^2+\omega_0^2}$.

$\underline{\underline{\text{Sol}^n}}$: $x(\infty) = \lim_{s \to 0} \dfrac{s^2}{s^2+\omega_0^2} = 0$. X

I.L.T.

but

$x(t) = \cos \omega_0 t \cdot u(t)$

$$\boxed{-1 \le x(t)\Big|_{t\to\infty} \le 1}$$

***

$\boxed{Q}$ $X(s) = \dfrac{1}{s-2}$,

$\underline{\underline{\text{Sol}^n}}$: $x(\infty) = \lim_{s \to 0} \dfrac{s}{s-2} = 0$ X

I.L.T.

$x(t) = e^{2t}$

$x(t)\big|_{t\to\infty} = \infty$

$\boxed{P\ 5.2.5}$ A LTI, Causal continuous time system has a rational T.F with simple poles at $s=-2$, and $s=-4$ and one of the simple ~~pole~~ zero at $s=-1$. A unit step $u(t)$ is applied as the input of the system. At steady state, the output has a constant value of 1. Find the impulse response?

$\underline{\underline{\text{Sol}^n}}$: $H(s) = \dfrac{K(s+1)}{(s+2)(s+4)}$.

$$\Rightarrow \quad y(\infty) = \lim_{s \to 0} s \cdot Y(s) = 1.$$

$$\therefore \quad \lim_{s \to 0} s \cdot H(s) \cdot X(s) = 1.$$

$$\Rightarrow \quad \lim_{s \to 0} \cancel{s} \cdot \frac{K(s+1)}{(s+2)(s+4)} \cdot \frac{1}{\cancel{s}} = 1$$

$$\Rightarrow \quad \frac{K(0+1)}{(0+2)(0+4)} = 1$$

$$\boxed{K = 8}$$

$$\Rightarrow \quad H(s) = \frac{8(s+1)}{(s+2)(s+4)}.$$

$$\Rightarrow \quad H(s) = \frac{12}{(s+4)} - \frac{4}{(s+2)}.$$

$$\Rightarrow \quad \boxed{h(t) = 12e^{-4t} \cdot u(t) - 4e^{-2t} \cdot u(t).}$$

$\boxed{\text{P 5.2.6.}}$ An LTI system having TF $\frac{s^2+1}{s^2+2s+1}$ & input $x(t) = \sin(t+1)$ is in steady state. The output is sampled at ω_s rad/sec to obtain final output $\{y(k)\}$ which of the following is true?

(a) $y(\cdot) = 0$ for all ω_s.

(b) $y(\cdot) \neq 0$ for all ω_s.

(C) $y(\cdot) \neq 0$ for $\omega_s > 2$ but zero for $\omega_s < 2$.

(D) $y(\cdot) = 0$ for $\omega_s > 2$ but nonzero for $\omega_s < 2$.

Soln:

$$H(s) = \frac{s^2+1}{s^2+2s+1}.$$

$$H(j\omega) = \frac{-\omega^2+1}{-\omega^2+2j\omega+1}.$$

$$H(j\omega)\Big|_{\omega=1} = \frac{-1+1}{-1+2j+1} = 0.$$

So, ans: (A) $y(\cdot) = 0$ for all ω_s.

P 5.2.7 What is the output as $t \to \infty$ for a system that has T.F., $G(s) = \frac{2}{s^2-s-2}$ when subjected to a step input?

(A) -1 (B) 1 (C) 2 (D) unbounded.

Soln:

$$G(s) = \frac{2}{s^2-s-2}$$

$$= \frac{2}{s^2-s+\frac{1}{4}-\frac{9}{4}}.$$

$$G(s) = \frac{2}{(s-\frac{1}{2})^2-(3/2)^2}.$$

Poles: $s_1, s_2 : \frac{1}{2} \pm 3/2 = 2, -1.$

Poles are Right hand side of the

s plane. So, Ans : (D) unbounded.

$\boxed{P\ 5.2.8}$ Consider a system described by the T.F. $G(s) = \frac{2s+3}{s^2+2s+5}$ when it is subjected to an input of $10u(t)$, find the initial & final values of the response.

Soln:

$$Y(s) = H(s) \cdot X(s).$$

$$Y(s) = \frac{2s+3}{s^2+2s+5} \times \frac{10}{s}.$$

$$\therefore\ y(0) = \lim_{s \to \infty} \cancel{s} \times \frac{10}{\cancel{s}} \cdot \times \frac{2s+3}{(s^2+2s+5)}$$

$$\boxed{y(0) = 0}$$

$$\Rightarrow\ y(\infty) = \lim_{s \to 0} \cancel{s} \times \frac{10}{\cancel{s}} \times \frac{2s+3}{(s^2+2s+5)}$$

$$= \frac{\overset{2}{\cancel{10}} \times 3}{(0+0+\cancel{5})}$$

$$\boxed{y(\infty) = 6}$$

$\boxed{P\ 5.2.9}$ Let a signal $a_1 \sin(\omega_1 t + \phi_1)$ be applied to a stable LTI system. Let the corresponding steady state output be represented as $F_2(\omega_2 t + \phi_2)$. Then which of the following statement is TRUE?

(A) F is not neecssarily a "sine" or "Cosine" function but must be periodic & $\omega_1 = \omega_2$

(B) F must be "sine" (or) "Cosine" with $a_1 = a_2$

(C) F must be "sine", $\omega_1 = \omega_2$, $a_1 \neq a_2$.

(D) F must be "sine" (or) "cosine" functions with $\omega_1 = \omega_2$.

Soln: Input freq. and output freq. Should remain same. so. $\omega_1 = \omega_2$.

→ Function can be sine (or) cosine.

→ Amplitude can be change.

so, Ans-(D).

* Causality & Stability :-

⇒ For a causal system $h(t) = 0$; $t < 0$ and thus is right - sided and the Roc associated with the system f^n for a causal system is a right-half plane.

⇒ An LTI system is stable if and only if the Roc of the system function $H(s)$ include $j\omega$ axis.

$\Rightarrow$ For the T.F. to be both causal and stable all poles must lie in the left half of S-plane with -ve Real parts.

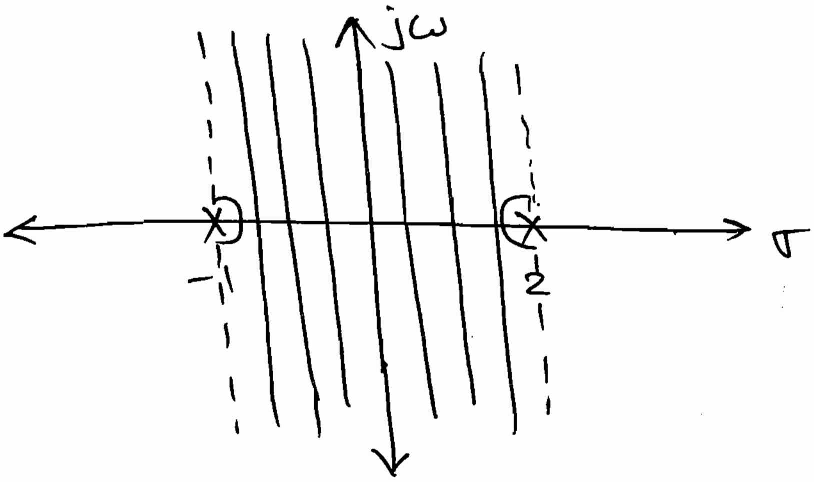
$\boxed{\text{P 5 3.1}}$ Given $H(s) = \dfrac{s-1}{(s+1)(s-2)}$. Find $h(t)$ for each of the following cases.

(i) Stable.

Soln:

$$H(s) = \frac{s-1}{(s+1)(s-2)}.$$

$$\Rightarrow H(s) = \frac{\frac{2}{3}}{(s+1)} + \frac{\frac{1}{3}}{(s-2)}.$$



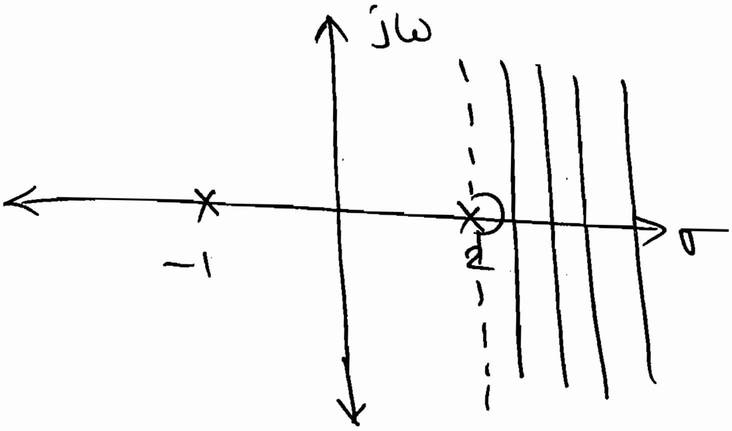
$$\Rightarrow \boxed{-1 < \sigma < 2}$$

$$\therefore \boxed{h(t) = \frac{2}{3} e^{-t} u(t) - \frac{1}{3} e^{2t} \cdot u(-t).}$$

(ii) Causal:

$$\Rightarrow H(s) = \frac{\frac{2}{3}}{(s+1)} + \frac{\frac{1}{3}}{s-2}.$$

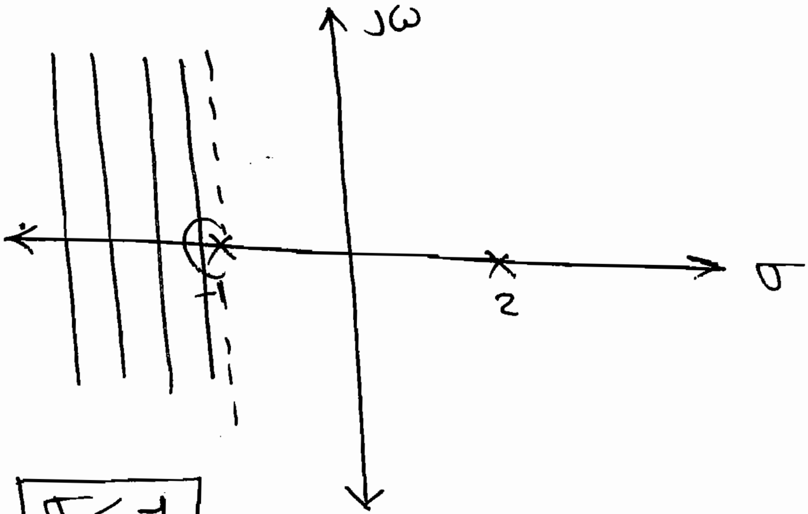
$\Rightarrow$



$\boxed{\sigma > 2}$

So, $\boxed{h(t) = \frac{2}{3} \cdot e^{-t} u(t) + \frac{1}{3} e^{2t} \cdot u(t).}$

(iii) neither causal nor stable.



$\boxed{\sigma < -1}$

So, $\boxed{h(t) = -\frac{2}{3} e^{-t} u(-t) + \left(-\frac{1}{3}\right) \cdot e^{2t} \cdot u(-t).}$

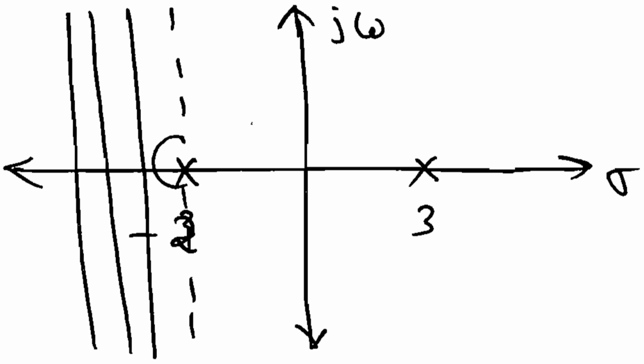
$\boxed{\text{PS. 3.2}}$ Given $X(s) = \dfrac{5-s}{s^2-s-6}$ & F.T. of the signal is defined then $x(t)$ is ______.

Soln: The given system is unstable.

$$X(s) = \frac{5-s}{s^2-s-6}.$$

$$X(s) = \frac{5-s}{(s-3)(s+2)}.$$

$$\Rightarrow X(s) = \frac{\frac{2}{5}}{(s-3)} - \frac{7/5}{(s+2)}.$$



$\boxed{\sigma < -2}$

$$\therefore \boxed{x(t) = -\frac{2}{5} e^{3t} u(-t) + \frac{7}{5} e^{-2t} \cdot u(-t).}$$

P 5.3.3 Consider an LTI system for which we are given the following information $X(s) = \frac{s+2}{s-2}$ and $x(t) = 0, t > 0$, and output is $y(t) = -\frac{2}{3} e^{2t} u(-t) + \frac{1}{3} e^{-t} u(t)$.

(a) Find T.F. & R.o.C.?

(b) Find the output if input is $x(t) = e^{3t}$ using part (a)?

$\underline{\text{Sol}^{n}}$: $X(s) = \frac{s+2}{s-2}$ and $\underline{x(t) = 0, \ t > 0}$

i.e. signal is left sided.

Hence, Ro.C $\boxed{\sigma < 2}$

Now, $y(t) = -\frac{2}{3} e^{2t} u(-t) + \frac{1}{3} e^{-t} u(t).$

$$Y(s) = \frac{\frac{2}{3}}{(s-2)} + \frac{\frac{1}{3}}{(s+1)}$$

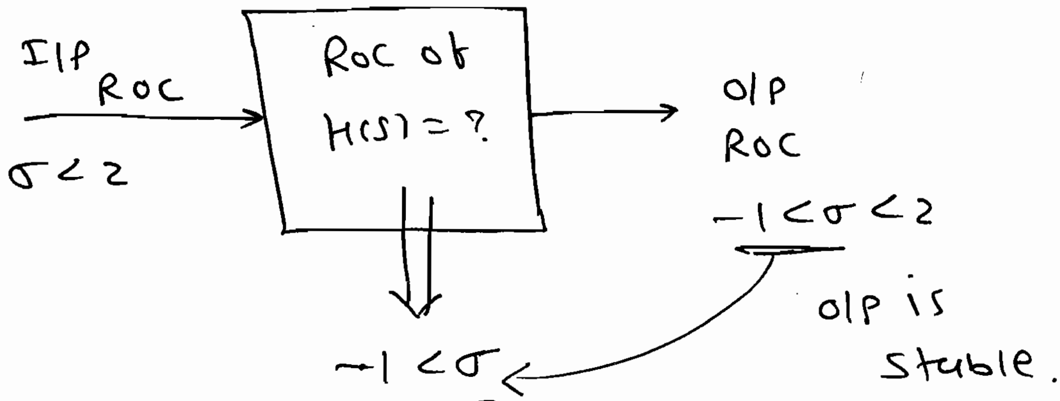
$$= \frac{1}{3}\left[\frac{2s+\cancel{2} + s-\cancel{2}}{(s-2)(s+1)}\right]$$

$$Y(s) = \frac{s}{\underset{\sigma<2}{(s-2)}\underset{\sigma>-1}{(s+1)}} \Rightarrow \boxed{-1<\sigma<2}$$

$$\text{T.F.} \Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{\dfrac{s}{(s-2)(s+1)}}{\dfrac{(s+2)}{(s-2)}}$$

$$= \frac{s}{\cancel{(s-2)}(s+1)} \times \frac{\cancel{s-2}}{(s+2)}$$

$$H(s) = \frac{s}{(s+1)(s+2)}$$



$\therefore \boxed{\sigma>-1}$

$$\therefore \boxed{H(s) = \frac{s}{(s+1)(s+2)} \; ; \; \text{ROC}: \sigma>-1}$$

(b) ~~Find~~ ~~f~~

$x(t) = e^{3t}$

if i/p $x(t) = e^{st} \Rightarrow$ o/p $y(t) = e^{st} \cdot H(s)$.

here, $\boxed{s=3}$

$$\therefore \quad y(t) = e^{3t} \cdot H(s)\Big|_{s=3}$$

$$= \frac{e^{3t} \times 3}{(4)(5)}.$$

$$\boxed{y(t) = \frac{3}{20} \cdot e^{3t}}$$

$\boxed{\text{P 5.3.4}}$ Consider an LTI system with input $x(t)$ and output $y(t)$ related as

$$\frac{dy(t)}{dt} + 3y(t) = \frac{d^2x(t)}{dt^2} + \frac{dx(t)}{dt} - 2x(t).$$

Find the T.F. of inverse system. Does a stable & causal inverse system exist?

Soln:

$\xrightarrow{\text{L.T.}}$ $SY(S) + 3Y(S) = S^2X(S) + SX(S) - 2X(S).$

$$\therefore \quad \frac{Y(S)}{X(S)} = H(S) = \frac{(S^2+S-2)}{(S+3)}.$$

$$\therefore \quad H_{Inv}(S) = \frac{(S+3)}{(S^2+S-2)}.$$

$$H_{Inv}(S) = \frac{(S+3)}{(S+2)(S-1)}.$$

for causal system poles must be lies on left hand side so can't be causal & stable simultaneously.

$\Rightarrow$ The system is stable if $\boxed{-2 < \sigma < 1}$.

P5.3.5 Which one of the following statements is NOT TRUE for a continuous time causal and stable LTI sys?

(A) All the poles of the system must lie on the left side of the $j\omega$ axis.

(B) Zeros of the system can lie anywhere in the s-plane.

(C) All the poles must lie within $|s| = 1$.

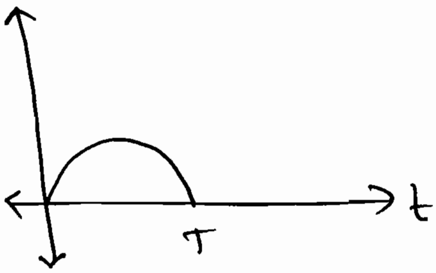
(D) All the roots of the characteristic equation must be located on the left side of the $j\omega$ axis.

<u>Ans</u>: (C)

* <u>L.T.</u> <u>of</u> <u>Switched</u> <u>Periodic</u> <u>Signal</u>:-

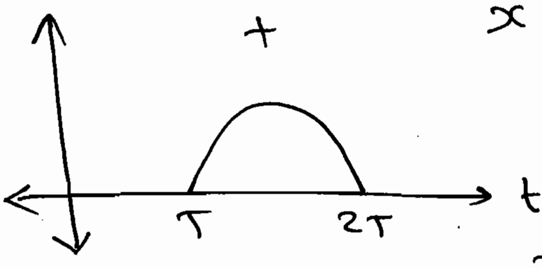
⇒

$x(t)\cdot u(t) \leftrightarrow X(s)$



+

$x(t-T)\cdot u(t-T) \leftrightarrow e^{-sT}\cdot X(s).$

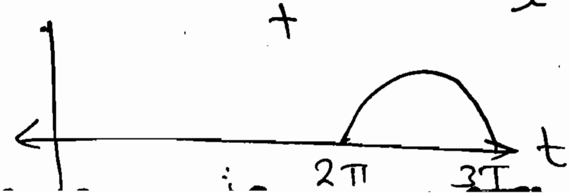


+

$x(t-2T)\cdot u(t-2T) \leftrightarrow e^{-2sT}\cdot X(s).$

+

⋮



$$\Rightarrow Y(s) = X(s)\left[1 + e^{-sT} + e^{-2sT} + e^{-3sT} + \cdots\right].$$

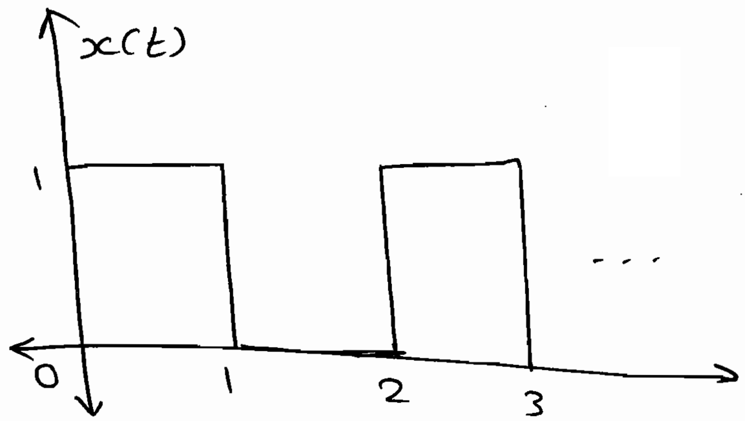
$$\therefore Y(s) = X(s) \times \frac{1}{1-e^{-sT}}.$$

$$Y(s) = \frac{X(s)}{1-e^{-sT}}.$$

$$\Rightarrow \boxed{Y_p(s) = \frac{\int_0^T x(t).e^{-sT}.dt}{1-e^{-sT}}}$$

e.g. Find the L.T. of the periodic signal shown in fig.1.

Soln: $T = 2$



$$X_p(s) = \frac{\int_0^T x(t).e^{-st}.dt}{1-e^{-sT}}.$$

$$= \frac{\int_0^2 x(t).e^{-st} dt}{1-e^{-2s}}.$$

$$= \frac{\int_0^1 1.e^{-st}.dt}{1-e^{-2s}}.$$

$$= \frac{\left[\frac{e^{-st}}{-s}\right]_0^1}{1-e^{-2s}}$$

$$= \frac{\frac{1}{s} - \frac{e^{-s}}{s}}{1-e^{-2s}}.$$

$$\therefore \boxed{X_p(s) = \frac{1-e^{-s}}{s(1-e^{-2s})}}$$