Chapter - 13

Surface Areas and Volumes

Surface Area of Combination of Solids

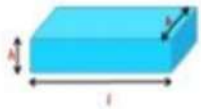
Surface Area and Volumes

Introduction

In our daily life, we come across many objects which are a combination of two or more of the basic solids.

Solids have three dimensions - length, breadth, height or depth.

Example: book, football, ice cream cone.



• A conical circus tent with a cylindrical base.

• An ice cream cone is a combination of a cone and a hemisphere.

• A test tube used in Science Laboratory is a combination of cylinder and hemisphere.

The surface area of a combination of solids

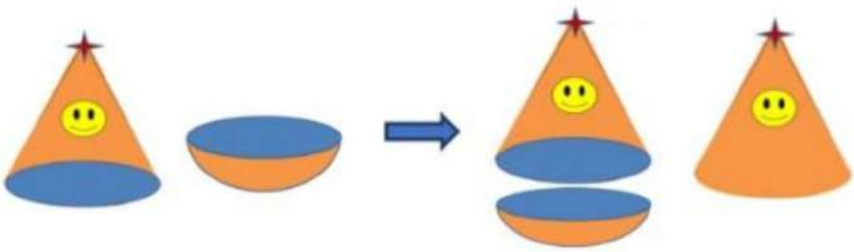
Let us consider an example of a circus tent, the tent consists of a cylindrical base surmounted by a conical roof.

The total surface area of the tent

= Curved surface area of the cone + Curved Surface area of the cylinder

$= \pi rl + 2\pi rh$

Suppose we have to make a toy by putting together a hemisphere and a cone.



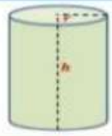
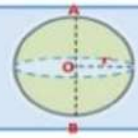
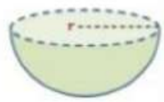
The surface area of the toy = Curved Surface Area of hemisphere + Curved Surface Area of cone

$= 2\pi r^2 + \pi rl$

When we are calculating the surface area of combined figures, we should only calculate the areas which are visible to our eyes. For example, an ice cream cone is a combination of a cone and a hemisphere. If we have to find the total surface area of this solid, then we add the curved surface areas of the cone and the hemisphere, leaving their base areas as they are attached together and not visible to our eyes.

Important Formulae

TSA = Total surface Area and CSA = Curved Surface Area

Cuboid	TSA = $2(lb + bh + lh)$ Volume = $l \times b \times h$	
Right Circular Cylinder	CSA= $2\pi rh$ TSA = CSA + Area of two circular ends TSA = $2\pi rh + 2\pi r^2$ Volume $=\pi r^2 h$	
Sphere	Surface Area of Sphere = $4\pi r^2$ Volume of sphere = $\frac{4}{3}\pi r^3$	
Hemisphere	Surface Area of Hemisphere = $2\pi r^2$ TSA = CSA + Area of circular end $= 2\pi r^2 + \pi r^2 = 3\pi r^2$ Volume = $\frac{2}{3}\pi r^3$	
Right Circular Cone	Slant height, $l = \sqrt{r^2 + h^2}$ CSA = πrl TSA = CSA + Area of circular end $= \pi rl + \pi r^2 = \pi r(l + r)$ Volume = $\frac{1}{3}\pi r^2 h$	

Example: A circus tent is cylindrical upto a height of 3 m and conical above it. If the diameter of the base is 140 m and the slant height of the conical part is 53 m, find the total canvas used in making the tent.

The total canvas used = Curved Surface Area of cylinder + Curved Surface area of the cone.

The slant height of the conical part = 53 m

Curved Surface Area of cylinder = $2\pi rh$

Curved Surface Area of cone = πrl

Total canvas used $= 2\pi rh + \pi rl$

$$= \pi r(2h + l) = \frac{22}{7} \times \frac{140}{2} \times (2 \times 3 + 53)$$

$$= \frac{22}{7} \times \frac{140}{2} \times (2 \times 3 + 53) = \frac{22}{7} \times 70 \times 59 = 22 \times 10 \times 59$$

$= 12980$ cm^2

Example: From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2

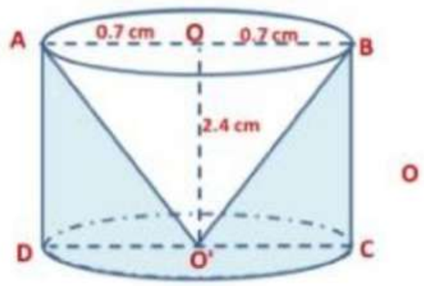
(REFERENCE: NCERT)

Total surface area of the remaining solid = Curved Surface Area of the cylinder + Area of the base of the cylinder + Curved Surface Area of the cone

Now, Curved Surface Area of cylinder = $2\pi rh$

Curved Surface Area of base of the cylinder = πr^2

Curved Surface Area of cone = πrl



Slant height of the , l = $\sqrt{r^2 + h^2}$

l = $\sqrt{(0.7)^2 + (2.4)^2} = 2.5$ cm

Total surface area of the remaining solid $= 2\pi rh + \pi r^2 + \pi rl$

$= \frac{22}{7} \times \frac{1.4}{2} \times 2 \times 2.4 + \frac{22}{7} \times (\frac{1.4}{2})2 + \frac{22}{7} \times \frac{1.4}{2} \times 2.5$

$= 11 \times 0.4 \times 2.4 + 22 \times 0.1 \times 0.7 + 22 \times 0.1 \times 2.5$

$= 10.56 + 1.54 + 5.5 = 17.6\ cm^2$

Example: A wooden article was made by scooping out a hemisphere from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm,

find the total surface area of the article.

(REFERENCE: NCERT)

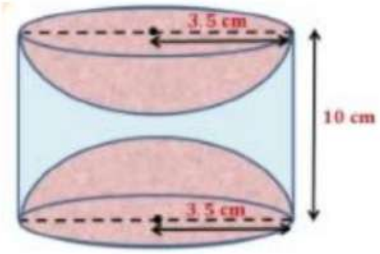
Total Surface Area of the article

= Curved Surface Area of the cylinder + 2 × Curved Surface Area of the hemisphere

Now, Curved Surface Area of cylinder = $2\pi rh$

Curved Surface Area of hemisphere = $2\pi r^2$

Total Surface Area of the article



$=2\pi rh + 2 \times 2\pi r^2 = 2\pi r(h + 2r)$

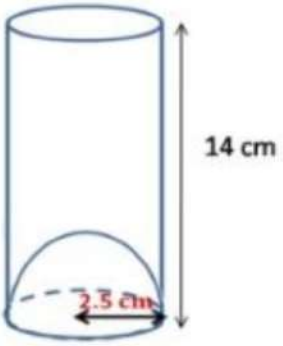
$= 2 \times \frac{22}{7} \times 3.5 \times (10 + 2 \times 3.5) = 44 \times 0.5 \times 17 = 374\ cm^2$

Volume of a Combination of Solids

Volume of a combination of Solids

In this section, we will learn how to calculate the volume of a combined solid. The volume of the solid formed by joining two basic solids is the sum of the volumes of the constituents.

Example: A juice seller was serving his customers using glasses. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of the glass was 14 cm, find what the apparent capacity of the glass was and what the actual capacity was (Use $\pi = 3.14$)



Now, height of the glass = 14 cm

Diameter of the glass = 5 cm

Radius $=\frac{5}{2}$ cm

The apparent capacity of the glass $=\pi r^2 h$

$= 3.14 \times (\frac{5}{2})^2 \times 14 = 274.75\ cm^3$

Volume of the hemispherical part at the base of the glass $=\frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times (\frac{5}{2})^3 = 32.73\ cm^3$

The actual capacity of the glass is less by the volume of the hemisphere at the base of the glass.'

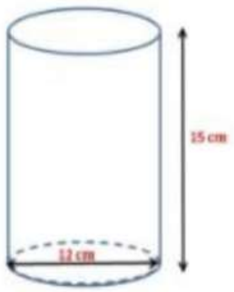
The actual capacity of the glass

= Apparent capacity of the glass - Volume of the hemispherical part

$= 274.75\ cm^3 - 32.73\ cm^3 = 242.02\ cm^3$

Example: A right circular cylinder having a diameter of 12 cm and a height 15 cm is full of ice - cream. The ice – cream is to be filled in cones of height 12 cm and diameter 6 cm having a hemispherical shape on the top. Find the number of such cones which can be filled with ice – cream.

(REFERENCE: NCERT)



Diameter of the cylinder = 12 cm

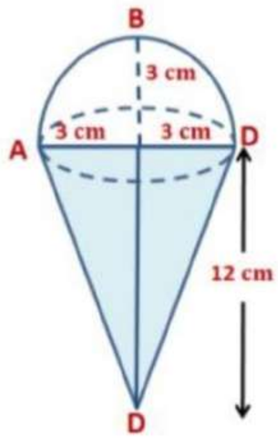
Radius of the cylinder = 6 cm

Height of the cylinder = 15 cm

Volume of the cylinder = $\pi r^2 h = \pi \times (6)^2 \times 15 = 540\pi\ cm^3$

Radius of the ice cream cone = $\frac{6}{2} = 3$ cm

Height of the ice cream cone = 12 cm



Volume of the conical part of the ice cream cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times (3)^2 \times 12 = 36\ \pi\ cm^3$

Volume of the hemispherical top of the ice cream cone = $2/3\ \pi r^3$

$= \frac{2}{3} \times \pi \times (3)^3 = 18\ \pi\ cm^3$

Total volume of the ice cream cone = $36\ \pi\ cm^3 + 18\ \pi\ cm^3$

$= 54\ \pi\ cm^3$

Number of ice cream cones = $\frac{Volume\ of\ the\ cylinder}{Total\ volume\ of\ the\ ice\ cream\ cone}$

$= \frac{540\pi}{54\pi} = 10$

Conversion of Solid from One Shape to Another

Conversion of Solid from one shape to another

Let's say there is a pumpkin and we cut it into slices. So, here we are converting one solid shape to another.

Now, even if the shape and size of the pumpkin slices are different but the volume of all the slices together will be equal to the volume of the original pumpkin.

When we convert one solid object to another, the surface area of the new solid changes but the volume remains the same.

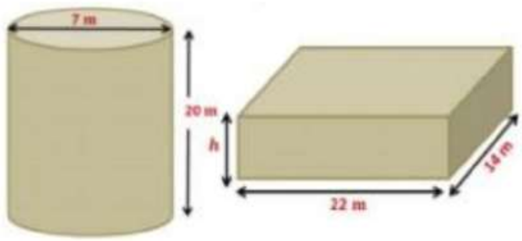
Example: A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

(REFERENCE: NCERT)

The earth obtained from digging a well of cylindrical shape is evenly spread to make a platform of cuboidal shape. Therefore, the volume of the earth will be equal to the volume of the cylindrical well and will also be equal to the volume of the cuboidal platform. The volume of the earth taken out of the well = Volume of the cylinder

Height of the well = 20 m

The diameter of the well = 7 m



Radius = $\frac{7}{2}$ m

The volume of the earth taken out from the well = $\pi r^2 h$

$= \frac{22}{7} \times (\frac{7}{2})^2 \times 20 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 770\ m^3$

Let the height of the platform be h m.

Volume of the earth in platform = $l \times b \times h = (22 \times 14 \times h) m^3$

The volume of the earth in platform = Volume of the earth taken out of the well

$22 \times 14 \times h = 770$

$h = \frac{770}{22X14} = \frac{35}{14} = \frac{5}{2}$ m = 2. 5 m

Example: A cylindrical pipe has an inner diameter of 7 cm and water flows through it at 385 litres per minute. Find the rate of flow in kilometres per hour.

The volume of water flowing per minute = 385 litres

Volume of water flowing per hour = (385× 60) litres

(∵ 1 hr = 60 min)

= (385× 60 × 1000) cm^3 (1 litre = $1000cm^3$)

The inner diameter of the pipe = 7 cm

Inner radius of the pipe = $\frac{7}{2}$ cm = 3.5 cm

Let h cm be the length of the water column of water that flows in one hour. The water column is cylindrical in shape with a radius of 3.5 cm and length h cm.

∴ Volume of water that flows in one hour

= Volume of the cylinder of radius 3.5 cm and length h cm

$= \pi r^2 h = \frac{22}{7} \times (\frac{7}{2})^2 \times h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h = \frac{77Xh}{2}$ cm^3

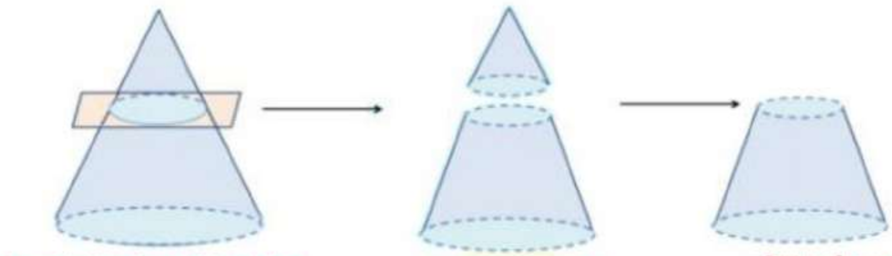
Now, $\frac{77Xh}{2} = 385 \times 60 \times 1000$

$h = \frac{385X60X1000X2}{77} = 600000$ cm = 6 km

Frustum of a Cone

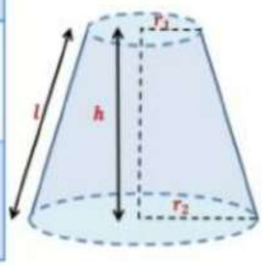
Frustum of a cone

A frustum is the Latin word meaning 'piece cut off'. A solid obtained by cutting off a right circular cone by a plane parallel to the base of the cone, then the portion of the cone between the cutting plane and the base of the cone is called a frustum of the cone.

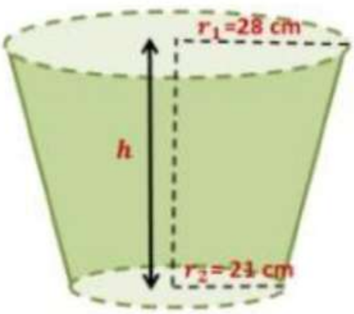


A cone sliced by a plane parallel to the base — The two parts seperated — Frustum of a cone

Volume of the frustum of the cone	$\frac{1}{3}\pi h({r_1}^2 + {r_2}^2 + r_1 r_2)$
Curved Surface area of the frustum of the cone	$\pi(r_1 + r_2)l$ Where, $l = \sqrt{h^2 + (r_1 - r_2)^2}$
Total Surface Area of the frustum of the cone	$\pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$ Where, $l = \sqrt{h^2 + (r_1 - r_2)^2}$



Example: A bucket is in the form of the frustum of a cone and holds 56.980 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.



Let the height of the bucket be h cm.

Now, $r_1 = 28$ cm, $r_2 = 21$ cm

Volume of the bucket = 56.980 litres

$= (56.980 \times 1000)\ cm^3 = 56980\ cm^3$

Volume of the bucket $= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

$56980 = \frac{1}{3} \times \frac{22}{7} \times h \times (28^2 + 21^2 + 28 \times 21)$

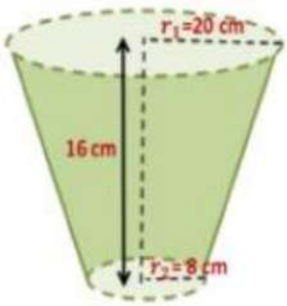
$\frac{1}{3} \times \frac{22}{7} \times h \times (784 + 441 + 588) = 56980$

$h = \frac{56980 X 3 X 7}{22 X 1813} = 30$ cm

$h = 30$ cm

Example: A container, open from the top, made up of a metal sheet is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of Rs 15 per liter and the cost of metal sheet used, if it costs Rs 5 per 100 cm^2. (Use= 3. 14)

(REFERENCE: NCERT)



Here, $r_1 = 20$ cm, $r_2 = 8$cm and $h = 16$ cm

Let l be the slant height of the frustum.

$l = \sqrt{h^2 + (r_1 - r_2)_2}$

$l = \sqrt{16^2 + (20-8)^2} = \sqrt{256 + 12^2} = \sqrt{256 + 144} = \sqrt{400} = 20$

$l = 20$ m

Volume of the container $= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{1}{3} \times 3.14 \times 16 \times (20^2 + 8^2 + 20 \times 8)$

$= \frac{1}{3} \times 3.14 \times 16 \times (400 + 64 + 160) = \frac{1}{3} \times 3.14 \times 16 \times 624$

$= 10449.92 cm^3$

$= \frac{10449.92}{1000}$ liters

$= 10.44992$ litres $\cong 10.45$ litres

The volume of the container $= 10.45$ litres

Cost of 1 litre of milk = Rs. 15

Cost of 10.45 litres of milk = Rs. (15×10.45) = Rs 156.75

To calculate the cost of metal sheet used, we need to find the Total Surface Area of the frustum.

Total Surface Area $= \pi l(r_1 + r_2) + \pi r_2^2$ ($\because$ top is open)

$= 3.14 \times 20 \times (20 + 8) + 3.14 \times 8^2$

$= 3.14 \times 20 \times 28 + 3.14 \times 64$

$= 3.14 \times (560 + 64) = 3.14 \times 624 = 1959.36 \text{ cm}^2$

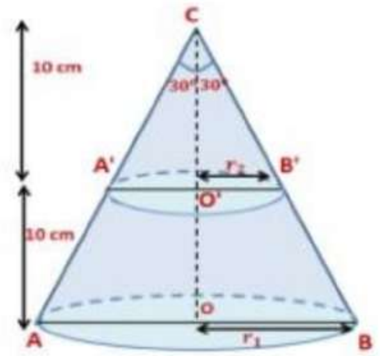
Cost of 100 cm2 of metal used = Rs. 5

Cost of 1959.36 cm2 of metal used = Rs. $\frac{5}{100} \times 1959.36$

= Rs 97.96 (Approx)

Example: A solid metallic right circular cone 20 cm high with vertical angle 60° is cut into two parts at the middle point of its height by a plane parallel to the base. If the frustum, so obtained, be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.

(REFERENCE: NCERT)



Let ABC be the solid right circular cone of height 20 cm and it is cut by a plane parallel to its base at the point O' such that CO' = O'O.

So, O' is the midpoint of CO. Let r_1 and r_2 be the radii of the circular ends of the frustum A'ABB'.

In triangles CAO and CA'O'

$\tan 30^\circ = \frac{OA}{OC}$ and $\tan 30^\circ = \frac{O'A'}{O'A}$

$\frac{1}{\sqrt{3}} = \frac{r1}{20}$ and $\frac{1}{\sqrt{3}} = \frac{r2}{10}$

r_1 =c $\frac{20}{\sqrt{3}}$ and $r_2 = \frac{10}{\sqrt{3}}$

Volume of the frustum =13h($r_1^2 + r_2^2 + r_1 r_2$)

$$\frac{1}{3} \times \pi \times 10 \times ((\frac{20}{\sqrt{3}})^2 + (\frac{10}{\sqrt{3}})^2 + \frac{20}{\sqrt{3}} + \frac{10}{\sqrt{3}})$$

$$\frac{1}{3} \times \pi \times 10 \times (\frac{400}{3} + \frac{100}{3} + \frac{200}{3})$$

$$= \frac{10\pi}{3} \times \frac{700}{3} = \frac{7000\pi}{9} cm^2$$

Let the length of the wire of diameter $\frac{1}{16}$ be l cm

Radius of the wire = $\frac{1/16}{2} = \frac{1}{32}$ cm

Volume of the metal used in wire = $\pi r^2 h = \pi \times (\frac{1}{32})^2 \times l = \frac{\pi l}{1024} cm^2$

The volume of the metal used in wire = Volume of the frustum

$$\frac{\pi l}{1024} = \frac{7000\pi}{9}$$

$$l = \frac{7000X1024}{9} = 796444.4cm = 7964.44m$$