Probability

- Some of the terms related to probability are:
  - **Experiment**: When an operation is planned and done under controlled conditions, it is known as an experiment. For example, tossing a coin, throwing a die etc., are all experiments.
  - **Outcomes**: Different results obtained in an experiment are known as outcomes. For example, on tossing a coin, if the result is a head, then the outcome is a head; if the result is a tail, then the outcome is a tail.
  - **Random**: An experiment is random if it is done without any conscious decision. For example, drawing a card from a well-shuffled pack of playing cards is a random experiment if it is done without seeing the card.
  - **Trial**: A trial is an action or an experiment that results in one or several outcomes. For example, if a coin is tossed five times, then each toss of the coin is called a trial.
  - **Sample space**: The set of all possible outcomes of an experiment is called the sample space. It is denoted by the letter 'S'. Sample space in the experiment of tossing a coin is {H, T}.
  - **Event**: The event of an experiment is one or more outcomes of the experiment. For example, tossing a coin and getting a head or a tail is an event.

- The outcomes of an experiment having the same chances of occurrence are known as equally-likely outcomes. For example, if we toss a coin, then the possible outcomes are head or tail, and both of them have an equal chance of occurring. So, these are equally-likely outcomes.

- When the outcomes of the experiment are equally-likely, the probability of an event is given by:

$$\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

Example:

What is the probability of getting one head and one tail when two coins are tossed together?

Solution:

When two coins are tossed together, the possible outcomes are:

- Head on first coin, head on second coin(H, H)
- Head on first coin, tail on second coin (H, T)
- Tail on first coin, head on second coin (T, H)
- Tail on first coin, tail on second coin (T, T)

∴ Total number of outcomes = 4

Outcomes in favour of the event are (H, T) and (T, H).

Number of favourable outcomes = 2

Therefore, probability of getting one head and one tail

$$\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{2}{4} = \frac{1}{2}$$

- **Experimental Probability:** The probability obtained from the result of an experiment when we actually perform the experiment is called experimental (or empirical) probability.
- **Theoretical Probability:** The probability we find through the theoretical approach without actually performing the experiment is called theoretical probability.
- The theoretical probability (or classical probability) of an event E, is denoted by P(E) and is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

- Experimental probability may or may not be equal to the theoretical probability.

- **Complementary events**

For an event E such that $0 \leq P(E) \leq 1$ of an experiment, the event $\bar{E}$ represents 'not E' , which is called the complement of the event E. We say, E and $\bar{E}$ are **complementary** events.

$P(E) + P(\bar{E}) = 1$

$\Rightarrow P(\bar{E}) = 1 - P(E)$

Example:

A pair of dice is thrown once. Find the probability of getting a different number on each die.

Solution:

When a pair of dice is thrown, the possible outcomes of the experiment can be listed as:

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

The number of all possible outcomes = 6 × 6 = 36

Let E be the event of getting the same number on each die.

Then, $\bar{E}$ is the event of getting different numbers on each die.

Now, the number of outcomes favourable to E is 6.

$\therefore P(\bar{E}) = 1 - P(E) = 1 - \frac{6}{36} = \frac{5}{6}$

Thus, the required probability is $\frac{5}{6}$.