Chapter : 12. INDEFINITE INTEGRAL

Exercise : 12

Question: 1

Evaluate :

Solution:

i Given:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^7 \, dx = \frac{x^{7+1}}{7+1} + c$$

$$= \frac{x^8}{8} + c$$

ii. Given:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{-7} \, dx = \frac{x^{-7+1}}{-7+1} + c$$

$$= \frac{x^{-6}}{-6} + c$$

iii. Given:

$$\int \frac{1}{x} dx = \ln|x| + c$$

iv. Given:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\frac{5}{3}} dx = \frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1} + c$$

$$= \frac{3x^{\frac{8}{3}}}{8} + c$$

v. Given:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{-\frac{5}{4}} dx = \frac{x^{-\frac{5}{4}+1}}{-\frac{5}{4}+1} + c$$

$$= -4x^{-\frac{1}{4}} + c$$

vi. Given:

$$\int a^x \, dx = \frac{a^x}{\ln a} + c$$

$$\int 2^x\,dx = \frac{2^x}{\ln 2} + c$$

vii. Given:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\frac{2}{3}}\,dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c$$

$$= \frac{3x^{\frac{5}{3}}}{5} + c$$

viii. Given:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{-\frac{3}{4}}\,dx = \frac{x^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} + c$$

$$= 4x^{\frac{1}{4}} + c$$

ix. Given:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int 2x^{-2}\,dx = 2\frac{x^{-2+1}}{-2+1} + c$$

$$= \frac{-2}{x} + c$$

Question: 2

Evaluate :

Solution:

i Given:

$$\int \left(6x^5 - \frac{2}{x^4} - 7x + \frac{3}{x} - 5 + 4e^x + 7^x\right)dx$$
$$= 6\frac{x^{5+1}}{5+1} - 2\frac{x^{-4+1}}{-4+1} - 7\frac{x^2}{2} + 3\ln|x| - 5x + 4e^x + \frac{7^x}{\ln 7} + c$$

$$= 6\frac{x^6}{6} - 2\frac{x^{-3}}{-3} - 7\frac{x^2}{2} + 3\ln|x| - 5x + 4e^x + \frac{7^x}{\ln 7} + c$$

$$= x^6 + \frac{2}{3}x^{-3} - \frac{7}{2}x^2 + 3\ln|x| - 5x + 4e^x + \frac{7^x}{\ln 7} + c$$

ii. Given:

$$\int \left(8 - x + 2x^3 - \frac{6}{x^3} + 2x^{-5} + 5x^{-1}\right)dx$$
$$= 8x - \frac{x^2}{2} + 2\frac{x^{3+1}}{3+1} - 6\frac{x^{-3+1}}{-3+1} + 2\frac{x^{-5+1}}{-5+1} + 5\ln|x| + c$$

$$= 8x - \frac{x^2}{2} + \frac{2}{4}x^4 + \frac{6}{2}x^2 - \frac{2}{4}x^{-4} + 5\ln|x| + c$$

$$= 8x - \frac{x^2}{2} + \frac{1}{2}x^4 + 3x^2 - \frac{1}{2}x^{-4} + 5\ln|x| + c$$

iii. Given:

$$\int\left(\frac{x}{a}+\frac{a}{x}+x^{a}+a^{x}+ax\right)dx=\frac{1}{a}\frac{x^{2}}{2}+a\ln|x|+\frac{x^{a+1}}{a+1}+\frac{a^{x}}{\ln a}+a\frac{x^{2}}{2}+c$$

Question: 3

Evaluate :

Solution:

i. Given:

$\int(2-5x)(3+2x)(1-x)dx$

$=\int(6-11x-10x^2)(1-x)\,dx$

$=\int(10x^3+x^2-17x+6)dx$

$$=\frac{10x^{4}}{4}+\frac{x^{3}}{3}-\frac{17x^{2}}{2}+6x+c$$

$$=\frac{5x^{4}}{2}+\frac{x^{3}}{3}-\frac{17x^{2}}{2}+6x+c$$

ii. Given:

$$=\int\left(ax^{\frac{5}{2}}+bx^{\frac{3}{2}}+cx^{\frac{1}{2}}\right)dx$$

$$=a\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}+b\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}+c\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C$$

$$=\frac{2a}{7}x^{\frac{7}{2}}+\frac{2b}{5}x^{\frac{5}{2}}+\frac{2c}{3}x^{\frac{3}{2}}+C$$

iii. Given:

$$\int\left(x^{\frac{1}{2}}-x^{\frac{4}{3}}+7x^{\frac{-2}{3}}-6e^{\ln x^{x}}+1\right)dx$$

$$=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}-\frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1}+7\frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}-6e^{\ln x^{x}}+x+c$$

$$=\frac{2x^{\frac{3}{2}}}{3}-\frac{3x^{\frac{7}{3}}}{7}-21x^{-\frac{1}{3}}-6x^{x}+x+c$$

Question: 4

Evaluate :

Solution:

i. Given:

$=\int x^6+x^{-6}-3x^2-3x^{-2}\,dx$

$$=\frac{x^{7}}{7}+\frac{x^{-5}}{5}-x^{3}+3x^{-1}+c$$

ii. Given:

$$=\int x^{\frac{1}{2}}-x^{\frac{-1}{2}}dx$$

$$=\frac{2}{3}x^{\frac{3}{2}}-2x^{\frac{1}{2}}+c$$

iii. Given:

$$= \int \left(x + \frac{1}{x} + 2\right) dx$$

$$= \frac{x^2}{2} + \ln|x| + 2x + c$$

iv. Given:

$$= \int \frac{1 + 8x^3 + 6x + 12x^2}{x^4} dx$$

$$= \int x^{-4} + \frac{8}{x} + 6x^{-3} + 12x^{-2} dx$$

$$= -\frac{x^{-3}}{3} + 8\ln|x| - 3x^{-2} - 12x^{-1} + c$$

v. Given:

$$= \int \frac{1 + x^3 + 3x + 3x^2}{\sqrt{x}} dx$$

$$= \int x^{\frac{-1}{2}} + x^{\frac{5}{2}} + 3x^{\frac{1}{2}} + 3x^{\frac{3}{2}} dx$$

$$= 2x^{\frac{1}{2}} + \frac{2}{7}x^{\frac{7}{2}} + 2x^{\frac{3}{2}} + \frac{6}{5}x^{\frac{5}{2}} + c$$

vi. Given:

$$= \int \left(\frac{2x^2}{x-2} + \frac{x-2}{x-2}\right) dx$$

$$= 2\int \left(\frac{x^2 - 4x + 4}{x-2} + \frac{4x}{x-2} - \frac{4}{x-2}\right) dx + \int dx$$

$$= 2\left[\int \frac{(x-2)^2}{x-2} dx + 4\int \frac{x-2+2}{x-2} dx - 4\int \frac{1}{x-2} dx\right] + x + c$$

$$= 2\left[\int (x-2) dx + 4\left(\int dx + 2\int \frac{1}{x-2} dx\right) - 4\ln|x-2|\right] + x + c$$

$$= 2\left[\frac{x^2}{2} - 2x + 4x + 8\ln|x-2| - 4\ln|x-2|\right] + x + c$$

$=x^2 - 4x+8x+8\ln|x-2|+x + c$

$=x^2 +5x+8\ln|x-2|+c$

Question: 5

Evaluate :

Solution:

Given:

Since, $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$;

$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$;

$\int a^x\, dx = \frac{a^x}{\ln a} + c$ &

$$\int \frac{1}{|x|\sqrt{(x^2-1)}} dx = \sec^{-1} x + c$$

So,

$= x + \tan^{-1} x - 2\sin^{-1} x + 5\sec^{-1} x + \frac{a^x}{\ln a} + c$

Question: 6

Evaluate :

Solution:

i. Given:

$= \int \frac{x^2+1-2}{x^2+1} dx$

$= \int \frac{x^2+1}{x^2+1} - \frac{2}{x^2+1} dx$

$= x - 2\tan^{-1} x + c$

ii. Given:

$= \int \left[\frac{x^6}{x^2+1} - \frac{1}{x^2+1}\right] dx$

$= \int \left[\frac{x^6+3x^2+3x^4+1-3x^2-3x^4-1}{x^2+1} - \frac{1}{x^2+1}\right] dx$

$= \int \left[\frac{(x^2+1)^3}{x^2+1} - 3\frac{x^2}{x^2+1} - 3\frac{x^4}{x^2+1} - \frac{1}{x^2+1} - \frac{1}{x^2+1}\right] dx$

$= \int (x^2+1)^2 dx - 3\int \left[\frac{x^2+1-1}{x^2+1}\right] dx - 3\left[\int \frac{x^4+2x^2+1}{x^2+1} + \frac{-2x^2-1}{x^2+1} dx\right]$
$- 2\int \frac{1}{x^2+1} dx$

$= \int (x^4+2x^2+1) dx - 3\left[\int dx - \int \frac{1}{x^2+1} dx\right]$
$- 3\left[\int \frac{(x^2+1)^2}{x^2+1} dx - 2\int \frac{x^2}{x^2+1} dx - \int \frac{1}{x^2+1} dx\right] - 2\int \frac{1}{x^2+1} dx$

$= \int (x^4+2x^2+1) dx - 3\left[\int dx - \int \frac{1}{x^2+1} dx\right]$
$- 3\left[\int (x^2+1) dx - 2\int \frac{x^2+1-1}{x^2+1} dx - \int \frac{1}{x^2+1} dx\right]$
$- 2\int \frac{x^2+1}{x^2+1} dx$

$= \frac{x^5}{5} + \frac{2}{3}x^3 + x - 3x + 3\tan^{-1} x - x^3 - 3x + 6x - 3\tan^{-1} x - 2\tan^{-1} x + c$

$= \frac{x^5}{5} + \frac{1}{3}x^3 + x - 2\tan^{-1} x + c$

iii. Given:

$= \int \frac{x^4+2x^2+1}{x^2+1} + \frac{-2x^2-1}{x^2+1} dx$

$= \int \frac{(x^2+1)^2}{x^2+1} dx - 2\int \frac{x^2}{x^2+1} dx - \int \frac{1}{x^2+1} dx$

$= \int (x^2+1) dx - 2\int \frac{x^2+1-1}{x^2+1} dx - \int \frac{1}{x^2+1} dx$

$= \int (x^2+1) dx - 2\int dx + 2\int \frac{1}{x^2+1} dx - \int \frac{1}{x^2+1} dx$

$$= \int (x^2+1)dx - 2\int dx + \int \frac{1}{x^2+1}dx$$

$$= \frac{1}{3}x^3 - x + \tan^{-1}x + c$$

iv. Given:

$$= \int \left[\frac{x^2+1-1}{x^2+1}\right]dx$$

$$= \int dx - \int \frac{1}{x^2+1}dx$$

$=x - \tan^{-1} x + c$

Question: 7

Evaluate :

Solution:

Given:

$$= \int (9\sin x - 7\cos x - 6(\sec x)^2 + 2(\csc x)^2 + (\csc x)^2 - 1)\,dx$$

$=-9 \cos x-7 \sin x-6 \tan x-3 \cot x - x + c$

Question: 8

Evaluate :

Solution:

Given:

$$= \int (\cot x \csc x - (\sec x)^2 + 1 - \tan x \sec x + 2(\sec x)^2)$$

$=-\csc x-\tan x + x-\sec x+2 \tan x + c$

$=-\csc x + \tan x + x -\sec x + c$

Question: 9

Evaluate :

Solution:

i. Given:

$$= \int (\sec x)^2 + \sec x \tan x\,dx$$

$=\tan x+\sec x+c$

ii. Given:

$$= \int (\csc x)^2 - \cot x \csc x\,dx$$

$=-\cot x + \csc x + c$

Question: 10

Evaluate :

Solution:

i. Given:

$$= \int ((\tan x)^2 + (\cot x)^2 + 2)dx$$

$$= \int ((\sec x)^2 - 1 + (\csc x)^2 - 1 + 2)dx$$

$$= \int ((\sec x)^2 + (\csc x)^2)dx$$

=tan x-cot x+c

ii. Given:

$$= \int \left(\frac{1}{(\cos x)^2} + 2\frac{\sin x}{(\cos x)^2}\right)dx$$

$$= \int ((\sec x)^2 + 2\tan x \sec x)dx$$

=tan x+2 sec x + c

iii. Given:

$$= \int (2\cot x \csc x + 4(\csc x)^2)$$

=-2 csc x-4 cot x + c

Question: 11

Evaluate :

Solution:

i. Given:

Multiply and divide by $(1 + \cos x)$

$$= \int \frac{1 + \cos x}{1 - (\cos x)^2}dx$$

$$= \int \frac{1 + \cos x}{(\sin x)^2}dx$$

$$= \int ((\csc x)^2 + \csc x \cot x)dx$$

=-cot x-csc x + c

ii. Given:

Multiply and divide by $(1 + \sin x)$

$$= \int \frac{1 + \sin x}{1 - (\sin x)^2}dx$$

$$= \int \frac{1 + \sin x}{(\cos x)^2}dx$$

$$= \int ((\sec x)^2 + \sec x \tan x)dx$$

=tan x + sec x + c

Question: 12

Evaluate :

Solution:

i. Given:

Multiply and divide by $(\sec x - \tan x)$

$$= \int \frac{\tan x \sec x - (\tan x)^2}{(\sec x)^2 - (\tan x)^2}dx$$

$$= \int \tan x \sec x - (\tan x)^2 dx$$

$$= \int (\tan x \sec x - (\sec x)^2 + 1) dx$$

=sec x-tan x + x + c

ii. Given:

Multiply and divide by $(\csc x + \cot x)$

$$= \int \frac{(\csc x)^2 + \csc x \cot x}{(\csc x)^2 - (\cot x)^2} dx$$

$$= \int (\csc x)^2 + \csc x \cot x \, dx$$

=-cot x-csc x + c

Question: 13

Evaluate :

Solution:

i. Given:

Multiply and divide by $(1 - \cos x)$

$$= \int \frac{\cos x - (\cos x)^2}{1 - (\cos x)^2} dx$$

$$= \int \frac{\cos x - (\cos x)^2}{(\sin x)^2} dx$$

$$= \int (\cot x \csc x - (\cot x)^2) dx$$

$$= \int (\cot x \csc x - (\csc x)^2 + 1) dx$$

=-csc x + cot x + x + c

ii. Given:

Multiply and divide by $(1 + \sin x)$

$$= \int \frac{\sin x + (\sin x)^2}{1 - (\sin x)^2} dx$$

$$= \int \frac{\sin x - (\sin x)^2}{(\cos x)^2} dx$$

$$= \int (\tan x \sec x + (\tan x)^2) dx$$

$$= \int (\tan x \sec x + (\sec x)^2 - 1) dx$$

=sec x + tan x-x + c

Question: 14

Evaluate :

Solution:

i. Given:

$$= \int \sqrt{2(\cos x)^2} \, dx$$

$= \sqrt{2} \int \cos x \, dx$

$= \sqrt{2} \sin x + c$

ii. Given:

$= \int \sqrt{2(\sin x)^2} \, dx$

$= \sqrt{2} \int \sin x \, dx$

$=-\sqrt{2} \cos x + c$

Question: 15

Evaluate :

Solution:

i. Given:

$= \int \frac{1}{2(cos\ x)^2} dx$

$= \frac{1}{2} \int (\sec x)^2 \, dx$

$= \frac{1}{2} \tan x + c$

ii. Given:

$= \int \frac{1}{2(\sin x)^2} dx$

$= \frac{1}{2} \int (\csc x)^2 dx$

$= -\frac{1}{2} \cot x + c$

Question: 16

Evaluate :

Solution:

Given:

$= \int \sqrt{\left(1 + \frac{2 \tan x}{1 + (\tan x)^2}\right)} dx$

$= \int \sqrt{\left(\frac{(1 + \tan x)^2}{(\sec x)^2}\right)} dx$

$= \int \left(\frac{1 + \tan x}{\sec x}\right) dx$

$=\int(\cos x + \sin x)$ d

$=\sin x-\cos x+c$

Question: 17

Evaluate :

Solution:

Given:

$$= \int \frac{(\sin x)^3}{(\sin x)^2(\cos x)^2} + \frac{(\sin x)^3}{(\sin x)^2(\cos x)^2} dx$$

=∫(tan x sec x + csc x cot x)dx

=sec x- csc x + c

Question: 18

Evaluate :

Solution:

Given:

$$= \int \tan^{-1}\left(\frac{2 \sin x \cos x}{2(\cos x)^2}\right) dx$$

=∫$\tan^{-1}$ (tan x)dx

=∫x dx

$$= \frac{x^2}{2} + c$$

Question: 19

Evaluate :

Solution:

Given:

=∫$\cos^{-1}$(cos 2x)dx

=∫2x dx

=x^2 +c

Question: 20

Evaluate :

Solution:

Given:

$$\sin^{-1}(\sin x) + \cos^{-1}(\sin x) = \frac{\pi}{2}$$

$$= \int \left[\frac{\pi}{2} - \sin^{-1}(\sin x)\right] dx$$

$$= \int \left[\frac{\pi}{2} - x\right] dx$$

$$= \frac{\pi}{2} x - \frac{x^2}{2} + c$$

Question: 21

Evaluate :

Solution:

Given:

$$= \int \tan^{-1} \sqrt{\left(\frac{(1-\sin x)^2}{1-(\sin x)^2}\right)} dx$$

$$= \int \tan^{-1}\left(\frac{1-\sin x}{\cos x}\right) dx$$

$$= \int \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{sin\left(\frac{\pi}{2}-x\right)}\right)dx$$

$$= \int \tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right)dx$$

$$= \int \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)dx$$

$$= \int \left(\frac{\pi}{4}-\frac{x}{2}\right)dx$$

$$= \frac{\pi}{4}x - \frac{x^2}{4} + c$$

Question: 22

Evaluate :

Solution:

Given:

$$= \int (9(\cot x)^2 + 4(\tan x)^2 - 12)dx$$

$$= \int (9(\csc x)^2 + 4(\sec x)^2 - 25)\,dx$$

$=-9 \cot x+4 \tan x-25x+c$

Question: 23

Evaluate :

Solution:

Given:

$$= \int (9(\sin x)^2 + 16(\csc x)^2 + 24)dx$$

$$= \int \left(\frac{9}{2}(1-\cos 2x) + 16(\csc x)^2 + 24\right)dx$$

$$= \frac{9}{2}x - \frac{9}{4}\sin 2x - 16\cot x + 24x + c$$

$$= \frac{57}{2}x - \frac{9}{4}\sin 2x - 16\cot x + c$$

Question: 24

Evaluate :

Solution:

Given:

Multiply and divide by $\sqrt{(x+1)} - \sqrt{(x+2)}$

$$= \int \frac{\left(\sqrt{(x+1)} - \sqrt{(x+2)}\right)}{x+1-x-2}dx$$

$$= -\int \sqrt{(x+3)} + \sqrt{(x+2)}\,dx$$

$$= \frac{-2}{3}(x+1)^{\frac{3}{2}} + \frac{2}{3}(x+2)^{\frac{3}{2}} + c$$

Question: 25

Evaluate :

Solution:

Given:

Multiply and divide by $\sqrt{(x+3)}+\sqrt{(x+2)}$

$$= \int \frac{\left(\sqrt{(x+3)}+\sqrt{(x+2)}\right)}{x+3-x-2}dx$$

$$= \int \sqrt{(x+3)}+\sqrt{(x+2)}\,dx$$

$$= \frac{2}{3}(x+3)^{\frac{3}{2}}+\frac{2}{3}(x+2)^{\frac{3}{2}}+c$$

Question: 26

Evaluate :

Solution:

Given:

Multiply and divide by $(1+\cos x)$

$$= \int \left(\frac{(1+\cos x)^2}{1-(\cos x)^2}\right)dx$$

$$= \int \frac{1+(\cos x)^2+2\cos x}{(\sin x)^2}dx$$

$$= \int (\csc x)^2+(\cot x)^2+2\cot x\csc x\ dx$$

$$= \int (\csc x)^2+(\csc x)^2-1+2\cot x\csc x\ dx$$

$$= -2\cot x-2\csc x-x+c$$

$$= -2(\csc x+\cot x)-x+c$$

$$= -2\left(\frac{1+\cos x}{\sin x}\right)-x+c$$

$$= -2\left(\frac{2\cos\frac{x}{2}\cos\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)-x+c$$

$$= -2\cot\frac{x}{2}-x+c$$

Question: 27

Evaluate :

Solution:

Given:

$$= \int \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}}dx$$

$$= \int \frac{\cos x+\sin x}{\cos x-\sin x}dx$$

Let cos x-sin x=t

(-sin x-cos x) dx=dt

-(sin x + cos x)dx=dt

dx=-dt

So, I= $-\int \frac{dt}{t}$

=-ln|t|+c

=-ln|cos x-sin x |+c

Question: 28

Evaluate :

Solution:

Given:

$$= \int \frac{\cos(x+b+a-b)}{\sin(x+b)} dx$$

$$= \int \frac{\cos(x+b)\cos(a-b) - \sin(x+b)\sin(a-b)}{\sin(x+b)} dx$$

=cos(a-b)∫cot(x + b) dx-∫sin(a-b)dx

=cos (a-b)ln|sin(x + b)|-x sin(a-b)+c

Question: 29

Evaluate :

Solution:

Given:

$$= \int \frac{\sin(x-\alpha+\alpha-\alpha)}{\sin(x+\alpha)} dx$$

$$= \int \frac{\sin(x+\alpha)\cos(2\alpha) - \sin(2\alpha)\cos(x+\alpha)}{\sin(x+\alpha)} dx$$

=∫cos 2α dx-sin2α ∫cot(x+α)dx

=x cos 2α-sin 2α×ln|sin(x+α) |+c

Question: 30

Evaluate :

Solution:

Given:

$$= \int \left(x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx$$

$$= \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + c$$

$$= \frac{2}{15}x^{\frac{3}{2}}(5-3x) + c$$

Question: 31

Evaluate :

Solution:

Given:

=∫(tan x)^2 dx

$=\int((\sec x)^2-1)dx$

$=\tan x - x + c$

Question: 32

Evaluate :

Solution:

Given:

$$= \int \frac{2}{(\cos x)^2} dx - \int \frac{3 \sin x}{(\cos x)^2} dx$$

$$= 2\int (\sec x)^2 \, dx - 3\int \tan x \sec x \, dx$$

$=2 \tan x-3 \sec x+c$

Exercise : OBJECTIVE QUESTIONS

Question: 1

Mark (√) against

Solution:

Given:

$\int x^6 \, dx,$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^6 \, dx = \frac{x^{6+1}}{6+1} + c$$

$$= \frac{x^7}{7} + c$$

Question: 2

Mark (√) against

Solution:

Given:

$$\int x^{\frac{5}{3}} dx$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\frac{5}{3}} \, dx = \frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1} + c$$

$$= \frac{x^{\frac{8}{3}}}{\frac{8}{3}} + c$$

$$= \frac{3}{8} x^{\frac{8}{3}} + c$$

Question: 3

Mark (√) against

Solution:

Given:

$$\int \frac{1}{x^3} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{x^3} dx = \frac{x^{-3+1}}{-3+1} + c$$

$$= -\frac{x^{-2}}{2} + c$$

$$= -\frac{1}{2x^2} + c$$

Question: 4

Mark (√) against

Solution:

Given:

$$\int \sqrt[3]{x}\, dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sqrt[3]{x}\, dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + c$$

$$= \frac{x^{\frac{4}{3}}}{4} + c$$

$$= \frac{3}{4} x^{\frac{4}{3}} + c$$

Question: 5

Mark (√) against

Solution:

Given:

$$\int \frac{1}{\sqrt[3]{x}} dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{\sqrt[3]{x}} dx = \frac{x^{\frac{-1}{3}+1}}{\frac{-1}{3}+1} + c$$

$$= \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c$$

$$= \frac{3}{2} x^{\frac{2}{3}} + c$$

Question: 6

Mark (√) against

Solution:

Given:

$$\int \sqrt[3]{x^2}dx$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \sqrt[3]{x^2}\, dx = \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + c$$

$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + c$$

$$= \frac{3}{5}x^{\frac{5}{3}} + c$$

Question: 7

Mark (√) against

Solution:

Given:

$$\int 3^x dx$$

$$\int a^x\, dx = \frac{a^x}{\ln a} + c$$

$$\int 3^x\, dx = \frac{3^x}{\ln 3} + c$$

Question: 8

Mark (√) against

Solution:

Given:

$$\int 2^{log\, x} dx$$

As $2^{\log x} = x^{\log 2}$

$$I = \int x^{log\, 2} dx$$

$$\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$$

$$\int x^{\log 2} dx = \frac{x^{\log 2+1}}{\log 2 + 1} + c$$

Question: 9

Mark (√) against

Solution:

Given:

$$\int \mathbf{cosec}x(\mathbf{cosec}x + cot\ x)dx = \int (\csc x)^2 + \cot x \csc x\, dx$$

=-cot x-csc x+c

Question: 10

Mark (√) against

Solution:

Given:

$$\int \frac{sec\, x}{(sec\, x + tan\, x)} dx$$

Multiply and divide by (sec x-tan x)

$$= \int \frac{(\sec x)^2 - \tan x \sec x}{(\sec x)^2 - (\tan x)^2} dx$$

$=\int(\sec x)^2 - \tan x \sec x\, dx$

$=\tan x - \sec x + c$

Question: 11

Mark (√) against

Solution:

Given:

$$\int \frac{(1 - \cos 2x)}{(1 + \cos 2x)} dx = \int \frac{(2(\sin x)^2)}{(2(\cos x)^2)} dx$$

$=\int(\tan x)^2$

$=\int((\sec x)^2 - 1)\, dx$

$=\tan x - x + c$

Question: 12

Mark (√) against

Solution:

Given:

$$\int \frac{1}{sin^2\, x\, cos^2\, x} dx$$

As we know $\sin^2 x + \cos^2 x = 1$

$$\int \frac{1}{sin^2\, x\, cos^2\, x} dx = \int \frac{(\sin x)^2 + (\cos x)^2}{(\sin x)^2 (\cos x)^2} dx$$

$$= \int \frac{1}{(\cos x)^2} + \frac{1}{(\sec x)^2}\, dx$$

$=\int(\sec x)^2 + (\csc x)^2\, dx$

$=\tan x - \cot x + c$

Question: 13

Mark (√) against

Solution:

Given:

$$\int \frac{cos\, 2x}{cos^2\, x\, sin^2\, x} dx = \int \frac{(\cos x)^2 - (\sin x)^2}{(\sin x)^2 (\cos x)^2} dx$$

$$= \int \frac{1}{(\sin x)^2} - \frac{1}{(\cos x)^2}\, dx$$

$=\int(\csc x)^2-(\sec x)^2\,dx$

$=-\cot x-\tan x+c$

Question: 14

Mark (√) against

Solution:

Given:

$$\int \frac{(\cos 2x-\cos 2\alpha)}{(\cos x-\cos \alpha)}dx=\int \frac{2\sin(x+\alpha)\sin(x-\alpha)}{2\sin\left(\frac{x+\alpha}{2}\right)\sin\left(\frac{x-\alpha}{2}\right)}dx$$

$$=\int 4\cos\left(\frac{x+\alpha}{2}\right)\cos\left(\frac{x-\alpha}{2}\right)\,dx$$

$=2\int\cos(x)+\cos(\alpha)dx$

$=2\sin x+2x\cos\alpha+c$

Question: 15

Mark (√) against

Solution:

Given:

$$\int \sqrt{1+\cos 2x}\,dx=\int \sqrt{2(\cos x)^2}\,dx$$

$=\sqrt{2}\int\cos x\,dx$

$=\sqrt{2}\sin x+c$

Question: 16

Mark (√) against

Solution:

Given:

$$\int \sqrt{1+\sin 2x}\,dx=\int \sqrt{1+\frac{2\tan x}{1+(\tan x)^2}}\,dx$$

$$=\int \sqrt{\frac{(1+\tan x)^2}{(\sec x)^2}}\,dx$$

$$=\int \frac{1+\tan x}{\sec x}dx$$

$=\int\cos x+\sin x\,dx$

$=\sin x-\cos x+c$

Question: 17

Mark (√) against

Solution:

Given:

$$\int \frac{\cos 2x}{\sin^2 x\cos^2 x}dx=\int \frac{(\cos x)^2-(\sin x)^2}{(\sin x)^2(\cos x)^2}dx$$

$$=\int \frac{1}{(\sin x)^2}-\frac{1}{(\cos x)^2}\,dx$$

$=\int(\csc x)^2-(\sec x)^2\, dx$

$=-\cot x-\tan x+c$

Question: 18

Mark (√) against

Solution:

Given:

$$\int \frac{dx}{(1-\cos 2x)} = \int \frac{1}{2(\sin x)^2} dx$$

$$= \frac{1}{2}\int (\csc x)^2 dx$$

$$= -\frac{1}{2}\cot x + c$$

Question: 19

Mark (√) against

Solution:

Given:

$$\int \frac{\sin 2x}{\sin x} dx = \int \frac{2\sin x \cos x}{\sin x} dx$$

$=2\int\cos x\, dx$

$=2\sin x+c$

Question: 20

Mark (√) against

Solution:

Given:

$$\int \frac{(1-\sin x)}{\cos^2 x} dx = \int \frac{1}{(\cos x)^2} - \frac{\sin x}{(\cos x)^2} dx$$

$=\int(\sec x)^2-\tan x \sec x\, dx$

$=\tan x-\sec x+c$

Question: 21

Mark (√) against

Solution:

Given:

$\int\cot^2 x\, dx=\int((\csc x)^2-1)dx$

$=-\cot x-x+c$

Question: 22

Mark (√) against

Solution:

Given:

$\int\sec x (\sec x + \tan x)\, dx= \int(\sec x)^2+\sec x \tan x\, dx$

$=\tan x+ \sec x + c$

Question: 23

Mark (√) against

Solution:

Given:

$$\int \frac{sec^2 x}{\text{cosec}^2 x} dx = \int \frac{(\sin x)^2}{(\cos x)^2} dx$$

=∫(tan x)^2 dx

=∫(sec x)^2-1 dx

=tan x-x+c

Question: 24

Mark (√) against

Solution:

Given: $\int \frac{sin^2 x}{(1+cos\ x)} dx = \int \frac{1-(\cos x)^2}{(1+\cos x)} dx$

$$= \int \frac{(1+\cos x)(1-\cos x)}{(1+\cos x)} dx$$

=∫(1-cos x)dx

=x-sin x+c

Question: 25

Mark (√) against

Solution:

Given:

$$\int \frac{cot\ x}{(\text{cosec}x - cot\ x)} dx = \int \frac{\cot x\,(\csc x + \cot x)}{((\csc x)^2 - (\cot x)^2)} dx$$

=∫cot x csc x+(csc x)^2 dx

=-csc x-cot x+c

Question: 26

Mark (√) against

Solution:

Given:

$$\int \frac{\sin x}{(1+sin\ x)} dx$$

Multiply and divide by (1-sin x)

$$= \int \frac{\sin x - (\sin x)^2}{1-(\sin x)^2} dx$$

$$= \int \frac{\sin x - (\sin x)^2}{(\cos x)^2} dx$$

=∫(tan x sec x-(tan x)^2)dx

=∫(tan x sec x-(sec x)^2+1)dx

=sec x-tan x+x+c

Question: 27

Mark (√) against

Solution:

Given:

$$\int \frac{(1+sin\,x)}{(1-sin\,x)}dx$$

Multiply and divide with (1 + sin x) to get,

$$\int \frac{(1+sin\,x)}{(1-sin\,x)}dx$$

$$=\int \frac{1+(\sin x)^2+2\sin x}{1-(\sin x)^2}dx$$

$$=\int \frac{1+(\sin x)^2+2\sin x}{(\cos x)^2}dx$$

$=\int(\sec x)^2+(\tan x)^2+2\tan x \sec x\, dx$

$=\int 2(\sec x)^2-1+2\tan x \sec x\, dx$

$=2\tan x-x+2\sec x+c$

Question: 28

Mark (√) against

Solution:

Given:

$$\int \frac{1}{(1+cos\,x)}dx$$

Multiply and divide by (1-cos x)

$$\int \frac{1}{(1+cos\,x)}dx=\int \frac{1-\cos x}{1-(\cos x)^2}dx$$

$$=\int \frac{1-\cos x}{(\sin x)^2}dx$$

$=\int(\csc x)^2-\cot x \csc x\, dx$

$=-\cot x+\csc x+c$

Question: 29

Mark (√) against

Solution:

Given:

$$\int sin^{-1}(cos\,x)\,dx$$

$$\sin^{-1}(\cos x)+\cos^{-1}(\cos x)=\frac{\pi}{2}$$

$$=\int \frac{\pi}{2}-\cos^{-1}(\cos x)\,dx$$

$$=\int \frac{\pi}{2}-x\;dx$$

$$=\frac{\pi}{2}x-\frac{x^2}{2}+c$$

Question: 30

Mark (√) against

Solution:

Given:

$$\int tan^{-1}\left\{\sqrt{\frac{1-cos\,2x}{1+cos\,2x}}\right\}dx = \int \tan^{-1}\sqrt{\frac{2(\sin x)^2}{2(\cos x)^2}}\,dx$$

$=\int \tan^{-1}(\tan x)dx$

$=\int x\,dx$

$$=\frac{x^2}{2}+c$$

Question: 31

Mark (√) against

Solution:

Given:

$$\int cot^{-1}\left(\frac{sin\,2x}{1-cos\,2x}\right)dx = \int \cot^{-1}\left(\frac{2\sin x\cos x}{1-1+2(\sin x)^2}\right)dx$$

$=\int \cot^{-1}(\cot x)\,dx$

$=\int x\,dx$

$$=\frac{x^2}{2}+c$$

Question: 32

Mark (√) against

Solution:

Given:

$$\int sin^{-1}\left(\frac{2\,tan\,x}{1+tan^2\,x}\right)dx = \int \sin^{-1}(\sin 2x)\,dx$$

$=\int 2x\,dx$

$=x^2+c$

Question: 33

Mark (√) against

Solution:

Given:

$$\int cos^{-1}\left(\frac{1-tan^2\,x}{1+tan^2\,x}\right)dx = \int \cos^{-1}(\cos 2x)\,dx$$

$=\int 2x\,dx$

$=x^2+c$

Question: 34

Mark (√) against

Solution:

Given:

$$\int tan^{-1}(\text{cosec}x - cot\, x)dx = \int \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)dx$$

$$= \int \tan^{-1}\left(\frac{2\sin\frac{x}{2}\sin\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)dx$$

$$= \int \tan^{-1}\left(\tan\frac{x}{2}\right)dx$$

$$= \int \frac{x}{2}dx$$

$$= \frac{x^2}{4} + c$$

Question: 35

Mark (√) against

Solution:

Given:

$$\int \left(\frac{(x^4+1)}{(x^2+1)}\right)dx = \int \frac{(x^4+2x^2+1)}{(x^2+1)} - \frac{2x^2}{(x^2+1)}dx$$

$$= \int \frac{(x^2+1)^2}{(x^2+1)}dx - 2\left\{\int \frac{(x^2+1)}{(x^2+1)} - \frac{1}{(x^2+1)}dx\right\}$$

$$= \int (x^2+1)dx - 2\{x - \tan^{-1}x\} + c$$

$$= \frac{x^3}{3} - x - 2\tan^{-1}x + c$$

Question: 36

Mark (√) against

Solution:

Given:

$$\int \frac{(ax+b)}{(cx+d)}dx = \int \frac{ax}{cx+d} + \frac{b}{cx+d}dx$$

$$= a\int \frac{x}{cx+d} \times \frac{c}{c}dx + b\int \frac{1}{cx+d}dx$$

$$= \frac{a}{c}\left(\int \frac{cx+d}{cx+d}dx - \frac{d}{cx+d}\right) + b\ln|cx+d| + c$$

$$= \frac{a}{c}\left(x - \frac{d}{c}\ln|cx+d|\right) + \frac{b}{c}\ln|cx+d| + c$$

$$= \frac{a}{c}x + \frac{(bc-ad)}{c^2}\ln|cx+d| + c$$

Question: 37

Mark (√) against

Solution:

Given:

$$\int \frac{(sin^3 x + cos^3 x)}{sin^2 x\, cos^2 x}dx = \int \frac{(\sin x)^3}{(\sin x)^2(\cos x)^2} + \frac{(\sin x)^3}{(\sin x)^2(\cos x)^2}dx$$

$=\int(\tan x \sec x + \csc x \cot x)dx$

=sec x-csc x+c

Question: 38

Mark (√) against

Solution:

Given:

$$\int \frac{sin\,x}{sin(x-\alpha)}dx$$

Let x-α=t

dx=dt

$$I = \int \frac{\sin(t+\alpha)}{\sin t}dx$$

$$= \int \frac{\sin t \cos\alpha + \cos t \sin\alpha}{\sin t}dt$$

=∫cos α+sin α cot t dt

=t cos α+sin α ln|sin t |+c

=(x-α) cos α+(sinα)ln|sin(x-α) |+c

=x cos α+ (sin α)ln|sin(x-α) |+c

Question: 39

Mark (√) against

Solution:

Given:

$$\int sin\,3x\,sin\,2x\,dx = \frac{1}{2}\int 2\sin 3x \sin 2x\,dx$$

$$= \frac{1}{2}\int \cos x - \cos 5x\,dx$$

$$= \frac{1}{2}\left\{\frac{\sin x}{1} - \frac{\sin 5x}{5}\right\} + c$$

$$= \frac{\sin x}{2} - \frac{\sin 5x}{10} + c$$

Question: 40

Mark (√) against

Solution:

Given:

$$\int cos\,3x\,sin\,2x\,dx = \frac{1}{2}\int 2\cos 3x \sin 2x\,dx$$

$$= \frac{1}{2}\int \sin 5x + \cos x\,dx$$

$$= \frac{1}{2}\left\{\frac{-\cos 5x}{5} + \frac{\sin x}{1}\right\} + c$$

$$= -\frac{\cos 5x}{10} + \frac{\sin x}{2} + c$$

Question: 41

Mark (√) against

Solution:

Given:

$$\int cos\,4x\,cos\,x\,dx = \frac{1}{2}\int 2\cos 4x \cos x\,dx$$

$$= \frac{1}{2}\int \cos 5x + \cos 3x\,dx$$

$$= \frac{1}{2}\left\{\frac{\sin 5x}{5} + \frac{\sin 3x}{3}\right\} + c$$

$$= \frac{\sin 5x}{10} + \frac{\sin 3x}{6} + c$$