NUMBER SYSTEM

1
CHAPTER

CONTENTS

➢ NUMBER SYSTEM

❖ **Natural Numbers :**

The simplest numbers are 1, 2, 3, 4....... the numbers being used in counting. These are called natural numbers.

❖ **Whole numbers :**

The natural numbers along with the zero form the set of whole numbers i.e. numbers 0, 1, 2, 3, 4 are whole numbers. W = {0, 1, 2, 3, 4....}

❖ **Integers :**

The natural numbers, their negatives and zero make up the integers.

Z = {....–4, –3, –2, –1, 0, 1, 2, 3, 4,....}

The set of integers contains positive numbers, negative numbers and zero.

❖ **Rational Number :**

(i) A rational number is a number which can be put in the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$.

(ii) A rational number is either a terminating or non-terminating but recurring (repeating) decimal.

(iii) A rational number may be positive, negative or zero.

❖ **Complex numbers :**

Complex numbers are imaginary numbers of the form a + ib, where a and b are real numbers and $i = \sqrt{-1}$, which is an imaginary number.

❖ **Factors :**

A number is a factor of another, if the former exactly divides the latter without leaving a remainder (remainder is zero) 3 and 5 are factors of 12 and 25 respectively.

❖ **Multiples :**

A multiple is a number which is exactly divisible by another, 36 is a multiple of 2, 3, 4, 9 and 12.

❖ **Even Numbers :**

All integers which are multiples of 2 are even number (i.e.) 2,4, 6, 8............... are even numbers.

❖ **Odd numbers :**

All integers which are not multiples of 2 are odd numbers.

❖ **Prime and composite Numbers :**

All natural numbers which cannot be divided by any number other than 1 and itself is called a prime number. By convention, 1 is not a prime number.

2, 3, 5, 7, 11, 13, 17 .... are prime numbers.

Numbers which are not prime are called composite numbers.

❖ **The Absolute Value (or modulus) of a real Number :**

If a is a real number, modulus a is written as $|a|$; $|a|$ is always positive or zero.It means positive value of 'a' whether a is positive or negative

$|3| = 3$ and $|0| = 0$, Hence $|a| = a$; if $a = 0$ or $a > 0$ (i.e.) $a \geq 0$

$|-3| = 3 = -(-3)$. Hence $|a| = -a$ when $a < 0$

Hence, $|a| = a$, if $a > 0$; $|a| = -a$, if $a < 0$

❖ **Irrational number :**

(i) All real numbers are irrational if and only if there decimal representation is non-terminating and non-repeating. e.g. $\sqrt{2}$, $\sqrt{3}$, π............ etc.

(ii) Rational number and irrational number taken together form the set of real numbers.

(iii) If a and b are two real numbers, then either (i) a > b or (ii) a = b or (iii) a < b

(iv) Negative of an irrational number is an irrational number.

(v) The sum of a rational number with an irrational number is always irrational.

(vi) The product of a non-zero rational number with an irrational number is always an irrational number.

(vii) The sum of two irrational numbers is not always an irrational number.

(viii) The product of two irrational numbers is not always an irrational number.

❖ **Rational Numbers :**

3, 4, $\frac{7}{3}$, $\frac{5}{2}$, $-\frac{3}{7}$, 2.7, 3.923, $1.42\overline{7}$, 1.2343434, etc.

❖ **Irrational Numbers :**

$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \pi$, 1.327185...........

❖ **Imaginary Numbers :**

$\sqrt{-2}, \sqrt{-49}, 3i, \left(\frac{5}{7} + \frac{\sqrt{-3}}{8}\right)$, ...

Note:- π = 3.14159265358979............ while $\frac{22}{7}$ = 3.1428571428............

$\therefore\ \pi \neq \frac{22}{7}$ but for calculation we can take $\pi \approx \frac{22}{7}$.

❖ EXAMPLES ❖

Ex.1 Is zero a rational number? can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Sol. Yes, zero is a rational number. It can be written as $\frac{0}{1} = \frac{0}{2} = \frac{0}{3}$ etc. where denominator $q \neq 0$, it can be negative also.

Ex.2 Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Sol. A rational number between r and s is $\frac{r+s}{2}$.

A rational number between

$\frac{3}{5}$ and $\frac{4}{5} = \frac{1}{2}\left(\frac{3}{5} + \frac{4}{5}\right) = \frac{7}{10}$.

And a rational number between

$\frac{3}{5}$ and $\frac{7}{10} = \frac{1}{2}\left(\frac{3}{5} + \frac{7}{10}\right) = \frac{13}{20}$

Similarly; $\frac{5}{8}, \frac{27}{40}, \frac{31}{40}$ are between $\frac{3}{5}$ and $\frac{4}{5}$.

So, five rational number between

$\frac{3}{5}$ and $\frac{4}{5}$ are $\frac{5}{8}, \frac{13}{20}, \frac{7}{10}, \frac{31}{40}, \frac{27}{40}$

Ex.3 Find six rational numbers between 3 and 4.

Sol. We can solve this problem in two ways.

Method 1 :

A rational number between r and s is $\frac{r+s}{2}$.

Therefore, a rational number between 3 and 4 = $\frac{1}{2}$ (3 + 4) = $\frac{7}{2}$

A rational number between 3 and $\frac{7}{2} = \frac{1}{2}\ \frac{6+7}{2} = \frac{13}{4}$. We can accordingly proceed in this manner to find three more rational numbers between 3 and 4.

Hence, six rational numbers between 3 and 4 are $\frac{15}{8}, \frac{13}{4}, \frac{27}{8}, \frac{7}{2}, \frac{29}{8}, \frac{15}{4}$.

Method 2 :

Since, we want six numbers, we write 3 and 4 as rational numbers with denominator 6 + 1, i.e., 3 = $\frac{21}{7}$ and 4 = $\frac{28}{7}$. Then we can check that $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$, and $\frac{27}{7}$ are all between 3 and 4.

Hence, the six numbers between 3 and 4 are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}$, and $\frac{27}{7}$

Ex.4 Are the following statement true or false? Give reasons for your answer.

(i) Every natural number is a whole number.

(ii) Every integer is a whole number.

(iii) Every rational number is a whole number.

Sol. (i) True, because natural number starts from 1 to ∞ and whole number starts from 0 to ∞.

(ii) False, because negative integers are not whole number.

(iii) False, because rational number such that $\frac{1}{2}$ is not whole number.

Ex.5 Find 3 irrational numbers between 3 & 5.

Sol. Θ 3 and 5 both are rational

The irrational are 3.127190385……………

3.212325272930………

3.969129852937…………

Ex.6 Find two rational & two irrational numbers between 4 and 5.

Sol. Rational numbers $\frac{4+5}{2} = 4.5$ **Ans.**

& $\frac{4.5+4}{2} = \frac{8.5}{2} = 4.25$ **Ans.**

Irrational numbers 4.12316908……… **Ans.**

4.562381032…….. **Ans.**

➤ DECIMAL REPRESENTATION OF RATIONAL NUMBERS

Ex.7 Express $\frac{7}{8}$ in the decimal form by long division method.

Sol. We have,

```
8) 7.000 (0.875
   64
   60
   56
    40
    40
     0
```

$\therefore \frac{7}{8} = 0.875$

Ex.8 Convert $\frac{35}{16}$ into decimal form by long division method.

Sol. We have,

```
16) 35.0000 (2.1875
    32
     30
     16
     140
     128
     120
     112
       80
       80
        0
```

$\therefore \frac{35}{16} = 2.1875$

Ex.9 Express $\frac{2157}{625}$ in the decimal form.

Sol. We have,

```
625) 2154.0000 (3.4512
     1875
      2820
      2500
       3200
       3125
        750
        625
        1250
        1250
           0
```

$\therefore \frac{2157}{625} = 3.4512$

Ex.10 Express $\frac{-17}{8}$ in decimal form by long division method.

Sol. In order to convert $\frac{-17}{8}$ in the decimal form, we first express $\frac{17}{8}$ in the decimal form and the decimal form of $\frac{-17}{8}$ will be negative of the decimal form of $\frac{17}{8}$

we have,

$$\begin{array}{l} 8\overline{)17.000}(2.125 \\ \underline{16} \\ 10 \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$\therefore \ \frac{-17}{8} = -2.125$

Ex.11 Find the decimal representation of $\frac{8}{3}$.

Sol. By long division, we have

$$\begin{array}{l} 3\overline{)8.0000}(2.6666 \\ \underline{6} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

$\therefore \ \frac{8}{3} = 2.6666 \ ... = 2.\overline{6}$

Ex.12 Express $\frac{2}{11}$ as a decimal fraction.

Sol. By long division, we have

$$\begin{array}{l} 11\overline{)2.00}(0.181818 \\ \underline{1\ 1} \\ 90 \\ \underline{88} \\ 20 \\ \underline{11} \\ 90 \\ \underline{88} \\ 20 \\ \underline{11} \\ 90 \\ \underline{88} \\ 2 \end{array}$$

$\therefore \ \frac{2}{11} = 0.181818 \ ..... = 0.\overline{18}$

Ex.13 Find the decimal representation of $\frac{-16}{45}$

Sol. By long division, we have

$$\begin{array}{l} 45\overline{)160}(0.3555 \\ \underline{135} \\ 250 \\ \underline{225} \\ 250 \\ \underline{225} \\ 250 \\ \underline{225} \\ 25 \end{array}$$

$\therefore \ \frac{16}{45} = 0.3555 \ .... = 0.3\overline{5}$

Hence, $\frac{-16}{45} = -0.3\overline{5}$

Ex.14 Find the decimal representation of $\frac{22}{7}$.

Sol. By long division, we have

$$\begin{array}{l} 7\overline{)22}(3.142857142857 \\ \underline{21} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 1 \end{array}$$

$\therefore \ \frac{22}{7} = 3.142857142857 \ ..... = 3.\overline{142857}$

So division of rational number gives decimal expansion. This expansion represents two types

(A) Terminating (remainder = 0)

Ex. $\frac{6}{5}, \frac{8}{5}, \frac{7}{4}$,………are equal to 1.2, 1.6, 1.75 respectively, so these are terminating and non repeating (recurring)

(B) Non terminating recurring (repeating)

(remainder $\neq$ 0, but equal to devidend)

Ex. $\frac{10}{3}$ = 3.333 ……….. or $3.\overline{3}$

$\frac{1}{7}$ = 0.1428514285……….or $0.\overline{142857}$

$\frac{2320}{99}$ = 23.434343……..or $23.\overline{43}$

These expansion are not finished but digits are continusely repeated so we use a line on those digits, called bar ($\overline{a}$).

So we can say that rational numbers are of the form either terminating, non repeating or non terminating repeating (recurring).

CONVERSION OF DECIMAL NUMBERS INTO RATIONAL NUMBERS OF THE FORM $\frac{m}{n}$

Case I : When the decimal number is of terminating nature.

Algorithm :

Step-1 : Obtain the rational number.

Step-2 : Determine the number of digits in its decimal part

Step-3 : Remove decimal point from the numerator. Write 1 in the denominator and put as many zeros on the right side of 1 as the number of digits in the decimal part of the given rational number.

Step-4 : Find a common divisor of the numerator and denominator and express the rational number to lowest terms by dividing its numerator and denominator by the common divisor.

Ex.15 Express each of the following numbers in the form $\frac{p}{q}$.

(i) 0.15 (ii) 0.675 (iii) –25.6875

Sol. (i) $0.15 = \frac{15}{100}$

$= \frac{15 \div 5}{100 \div 5}$

$\left[\begin{array}{l}\text{Dividing numerator and denominator} \\ \text{by the common divisor 5 of numerator} \\ \text{and denominator}\end{array}\right]$

$= \frac{3}{20}$

(ii) $0.675 = \frac{675}{1000}$

$= \frac{675 \div 25}{1000 \div 25} \Rightarrow = \frac{27}{40}$

(iii) $-25.6875 = \frac{-256875}{10000}$

$= \frac{-256875 \div 625}{10000 \div 625} = \frac{-411}{16}$

Case II : When decimal representation is of non-terminating repeating nature.

In a non terminating repeating decimal, there are two types of decimal representations

(i) A decimal in which all the digit after the decimal point are repeated. These type of decimals are known as pure recurring decimals.

For example: $0.\overline{6}, 0.\overline{16}, 0.\overline{123}$ are pure recurring decimals.

(ii) A decimal in which at least one of the digits after the decimal point is not repeated and then some digit or digits are repeated. This type of decimals are known as mixed recurring decimals.

For example, $2.1\overline{6}, 0.3\overline{5}, 0.7\overline{85}$ are mixed recurring decimals.

❖ Conversion of a pure recurring decimal to the form $\frac{p}{q}$

Algorithm :

Step-1 : Obtain the repeating decimal and pur it equal to x (say)

Step-2 : Write the number in decimal form by removing bar from the top of repeating digits and listing repeating digits at least twice. For sample, write x = $0.\overline{8}$ as x = 0.888.... and x = $0.\overline{14}$ as x = 0.141414......

Step-3 : Determine the number of digits having bar on their heads.

Step-4 : If the repeating decimal has 1 place repetition, multiply by 10; a two place repetition, multiply by 100; a three place repetition, multiply by 1000 and so on.

Step-5 : Subtract the number in step 2 from the number obtained in step 4

Step-6 : Divide both sides of the equation by the coefficient of x.

Step-7 : Write the rational number in its simplest form.

❖ EXAMPLES ❖

Ex.16 Express each of the following decimals in the form $\frac{p}{q}$:

(i) $0.\overline{6}$

(ii) $0.\overline{35}$

(iii) $0.\overline{585}$

Sol. (i) Let x = $0.\overline{6}$

then, x = 0.666....... ....(i)

Here, we have only one repeating digit, So, we multiply both sides of (i) by 10 to get

10 x = 6.66.... ....(ii)

Subtracting (i) from (ii), we get

10 x – x = (6.66 .....) – (0.66.....)

$\Rightarrow$ 9x = 6 $\Rightarrow$ x = $\frac{6}{9}$

$\Rightarrow$ x = $\frac{2}{3}$ Hence $0.\overline{6} = \frac{2}{3}$

(ii) Let x = $0.\overline{35}$

$\Rightarrow$ x = 0.353535.... ....(i)

Here, we have two repeating digits after the decimal point. So, we multiply sides of (i) by 10^2 = 100 to get

100x = 35.3535...... ....(ii)

Subtracting (i) from (ii), we get

100 x – x = (35.3535....) – (0.3535....)

$\Rightarrow$ 99 x = 35

$\Rightarrow$ x = $\frac{35}{99}$

Hence, $0.\overline{35}$

(iii) Let x = $0.\overline{585}$

$\Rightarrow$ x = 0.585585585... ....(i)

Here, we have three repeating digits after the decimal point. so, we multiple both sides of (i) by 10^3 = 1000 to get

1000 x = 585.585585......... ....(ii)

Subtracting (i) from (ii), we get

1000x – x = (585.585585...) – (0.585585585...)

1000x – x = 585

$\Rightarrow$ 999x = 585

$\Rightarrow$ x = $\frac{585}{999} = \frac{195}{333} = \frac{65}{111}$

The above example suggests us the following rule to convert a pure recurring decimal into a rational number in the form $\frac{p}{q}$.

Ex.17 Convert the following decimal numbers in the form $\frac{p}{q}$:

(i) $5.\overline{2}$

(ii) $23.\overline{43}$

Sol. (i) Let x = $5.\overline{2}$

$\Rightarrow$ x = 5.2222 ....(i)

Multiplying both sides of (i) by 10, we get

10 x = 52.222 ....(ii)

Subtracting (i) from (ii), we get

10x–x = (52.222...) – (5.222....)

$\Rightarrow$ 9x = 47

$\Rightarrow$ x = $\frac{47}{9}$

(ii) Let x = $23.\overline{43}$

$\Rightarrow$ x = 23.434343.....

Multiplying both sides of (i) by 100, we get

100 x = 2343.4343....... ....(ii)

Subtracting (i) from (ii), we get

100x – x = (2343.4343...) – (23.4343....)

$\Rightarrow$ 99 x = 2320

$\Rightarrow$ x = $\frac{2320}{99}$

❖ **Conversion of a mixed recurring decimal to the Form $\frac{p}{q}$:-**

Algorithm :

Step-1 : Obtain the mixed recurring decimal and write it equal to x (say)

Step-2 : Determine the number of digits after the decimal point which do not have bar on them. Let there be n digits without bar just after the decimal point

Step-3 : Multiply both sides of x by 10^n so that only the repeating decimal is on the right side of the decimal point.

Step-4 : Use the method of converting pure recurring decimal to the form $\frac{p}{q}$ and obtain the value of x

❖ **EXAMPLES** ❖

Ex.18 Express the following decimals in the form

(i) $0.3\overline{2}$ (ii) $0.12\overline{3}$

Sol. (i) Let $x = 0.3\overline{2}$

Clearly, there is just one digit on the right side of the decimal point which is without bar. So, we multiply both sides of x by 10 so that only the repeating decimal is left on the right side of the decimal point.

$\therefore \quad 10x = 3.\overline{2}$

$\Rightarrow \quad 10x = 3 + 0.\overline{2} \qquad \left[\because 0.\overline{2} = \frac{2}{9}\right]$

$\Rightarrow \quad 10x = 3 + \frac{2}{9}$

$\Rightarrow \quad 10x = \frac{9 \times 3 + 2}{9} \Rightarrow 10x = \frac{29}{9}$

$\Rightarrow \quad x = \frac{29}{90}$

(ii) Let $x = 0.12\overline{3}$

Clearly, there are two digits on the right side of the decimal point which are without bar. So, we multiply both sides of x by $10^2 = 100$ so that only the repeating decimal is left on on the right side of the decimal point.

$\therefore \quad 100x = 12.\overline{3}$

$\Rightarrow \quad 100x = 12 + 0.\overline{3}$

$\Rightarrow \quad 100x = 12 + \frac{3}{9}$

$\Rightarrow \quad 100x = \frac{12 \times 9 + 3}{9}$

$\Rightarrow \quad 100x = \frac{108 + 3}{9}$

$\Rightarrow \quad 100x = \frac{111}{9}$

$\Rightarrow \quad x = \frac{111}{900} = \frac{37}{300}$

Ex.19 Express each of the following mixed recurring decimals in the form $\frac{p}{q}$;

(i) $4.3\overline{2}$ (ii) $15.7\overline{12}$

Sol. (i) Let $x = 4.3\overline{2}$

$\Rightarrow \quad 10x = 43.\overline{2}$ [Multiplying both sides of x by 10]

$\Rightarrow \quad 10x = 43 + 0.\overline{2}$

$\Rightarrow \quad 10x = 43 + \frac{2}{9}$

$\Rightarrow \quad 10x = \frac{43 \times 9 + 2}{9}$

$\Rightarrow \quad 10x = \frac{387 + 2}{9}$

$\Rightarrow \quad 10x = \frac{389}{9}$

$\Rightarrow \quad x = \frac{389}{90}$

(ii) Let $x = 15.7\overline{12}$. Then,

$10x = 157.\overline{12}$

$\Rightarrow \quad 10x = 157 + 0.\overline{12}$

$\Rightarrow \quad 10x = 157 + \frac{12}{99}$

$\Rightarrow \quad 10x = 157 + \frac{4}{33}$

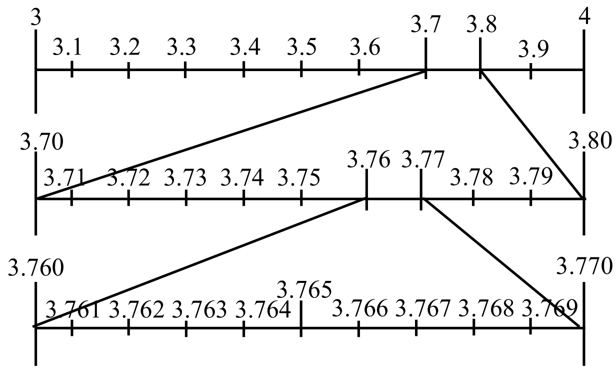
$\Rightarrow \quad 10x = \frac{157 \times 33 + 4}{33}$

$\Rightarrow \quad 10x = \frac{5181 + 4}{33}$

$\Rightarrow \quad 10x = \frac{5185}{33} \Rightarrow x = \frac{5185}{330} = \frac{1037}{66}$

Ex.20 Represent 3.765 on the number line.

Sol. This number lies between 3 and 4. The distance 3 and 4 is divided into 10 equal parts. Then the first mark to the right of 3 will represent 3.1 and second 3.2 and so on. Now, 3.765 lies between 3.7 and 3.8. We divide the distance between 3.7 and 3.8 into 10 equal parts 3.76 will be on the right of 3.7 at the sixth mark, and 3.77 will be on the right of 3.7 at the 7th mark and 3.765 will lie between 3.76 and 3.77 and soon.

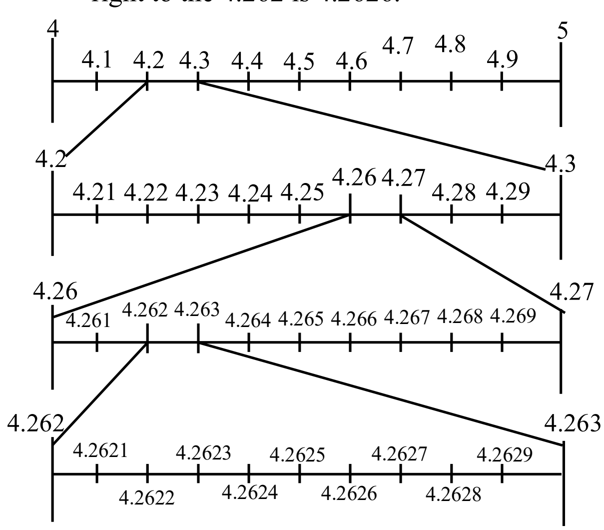


To mark 3.765 we have to use magnifying glass

Ex.21 Visualize $4.\overline{26}$ on the number line, upto 4 decimal places.

Sol. We have, $4.\overline{26} = 4.2626$

This number lies between 4 and 5. The distance between 4 and 5 is divided into 10 equal parts. Then the first mark to the right of 4 will represent 4.1 and second 4.2 and soon. Now, 4.2626 lies between 4.2 and 4.3. We divide the distance between 4.2 and 4.3 into 10 equal parts 4.2626 lies between 4.26 and 4.27. Again we divide the distance between 4.26 and 4.27 into 10 equal parts. The number 4.2626 lies between 4.262 and 4.263. The distance between 4.262 and 4.263 is again divided into 10 equal parts. Sixth mark from right to the 4.262 is 4.2626.



Ex.22 Express the decimal $0.00\overline{352}$ in the form $\frac{p}{q}$

Sol. Let x = $0.00\overline{352}$

Clearly, there is three digit on the right side of the decimal point which is without bar. So, we multiply both sides of x by $10^3 = 1000$ so that only the repeating decimal is left on the right side of the decimal point.

$\therefore$ $1000x = 3.\overline{52}$

$\Rightarrow$ $1000x = 3 + 0.52$

$\Rightarrow$ $1000x = 3 + \frac{52}{99}$

$\Rightarrow$ $1000x = \frac{3 \times 99 + 52}{99} \Rightarrow 1000x = \frac{297 + 52}{99}$

$\Rightarrow$ $1000x = \frac{349}{99} \Rightarrow x = \frac{349}{99000}$

Ex.23 Give an example of two irrational numbers, the product of which is (i) a rational number (ii) an irrational number

Sol. (i) The product of $\sqrt{27}$ and $\sqrt{3}$ is $\sqrt{81} = 9$, which is a rational number.

(ii) The product of $\sqrt{2}$ and $\sqrt{3}$ is $\sqrt{6}$, which is an irrational number.

Ex.24 Insert a rational and an irrational number between 2 and 3.

Sol. If a and b are two positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b. Also, if a,b are rational numbers, then $\frac{a+b}{2}$ is a rational number between them.

$\therefore$ A rational number between 2 and 3 is

$\frac{2+3}{2} = 2.5$

An irrational number between 2 and 3 is

$\sqrt{2 \times 3} = \sqrt{6}$

Ex.25 Find two irrational numbers between 2 and 2.5.

Sol. If a and b are two distinct positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b.

$\therefore$ Irrational number between 2 and 2.5 is

$\sqrt{2 \times 2.5} = \sqrt{5}$

Similarly, irrational number between 2 and $\sqrt{5}$ is $\sqrt{2 \times \sqrt{5}}$

So, required numbers are $\sqrt{5}$ and $\sqrt{2 \times \sqrt{5}}$.

Ex.26 Find two irrational numbers lying between $\sqrt{2}$ and $\sqrt{3}$.

Sol. We know that, if a and b are two distinct positive irrational numbers, then $\sqrt{ab}$ is an irrational number lying between a and b.

$\therefore$ Irrational number between $\sqrt{2}$ and $\sqrt{3}$ is $\sqrt{\sqrt{2}\times\sqrt{3}} = \sqrt{\sqrt{6}} = 6^{1/4}$

Irrational number between $\sqrt{2}$ and $6^{1/4}$ is $\sqrt{\sqrt{2}\times 6^{1/4}} = 2^{1/4} \times 6^{1/8}$.

Hence required irrational number are $6^{1/4}$ and $2^{1/4} \times 6^{1/8}$

Ex.27 Find two irrational numbers between 0.12 and 0.13.

Sol. Let a = 0.12 and b = 0.13. Clearly, a and b are rational numbers such that a < b.

We observe that the number a and b have a 1 in the first place of decimal. But in the second place of decimal a has a 2 and b has 3. So, we consider the numbers

c = 0.1201001000100001 ......

and, d = 0.12101001000100001.......

Clearly, c and d are irrational numbers such that a < c < d < b.

Ex.28 Find two rational numbers between 0.232332333233332.... and 0.252552555255552......

Sol. Let a = 0.232332333233332....

and b = 0.252552555255552.....

The numbers c = 0.25 and d = 0.2525

Clearly, c and d both are rational numbers such that a < c < d < b.

Ex.29 Find a rational number and also an irrational number between the numbers a and b given below:

a = 0.101001000100001....,

b = 0.1001000100001...

Sol. Since the decimal representations of a and b are non-terminating and non-repeating. So, a and b are irrational numbers.

We observed that in the first two places of decimal a and b have the same digits. But in the third place of decimal a has a 1 whereas b has zero.

$\therefore$ a > b

Construction of a rational number between a and b : As mentioned above, first two digits after the decimal point of a and b are the same. But in the third place a has a 1 and b has a zero. So, if we consider the number c given by

c = 0.101

Then, c is a rational number as it has a terminating decimal representation.

Since b has a zero in the third place of decimal and c has a 1.

$\therefore$ b < c

We also observe that c < a, because c has zeros in all the places after the third place of decimal whereas the decimal representation of a has a 1 in the sixth place.

Thus, c is a rational number such that b < c < a.

Hence , c is the required rational number between a and b.

Construction of an irrational number between a and b : Consider the number d given by

d = 0.1002000100001......

Clearly, d is an irrational number as its decimal representation is non-terminating and non-repeating.

We observe that in the first three places of their decimal representation b and d have the same digits but in the fourth place d and a 2 whereas b has only a 1.

$\therefore$ d > b

Also, comparing a and d, we obtain a > d

Thus, d is an irrational number such that b < d < a.

Ex.30 Find one irrational number between the number a and b given below :

a = 0.1111..... $= 0.\overline{1}$ and b = 0.1101

Sol. Clearly, a and b are rational numbers, since a has a repeating decimal and b has a terminating decimal. We observe that in the third place of decimal a has a 1, while b has a zero.

$\therefore$ a > b

Consider the number c given by

c = 0.111101001000100001.....

Clearly, c is an irrational number as it has non-repeating and non-terminating decimal representation.

We observe that in the first two places of their decimal representations b and c have the same digits. But in the third place b has a zero whereas c has a 1.

$\therefore$ b < c

Also, c and a have the same digits in the first four places of their decimal representations but in the fifth place c has a zero and a has a 1.

$\therefore \quad c < a$

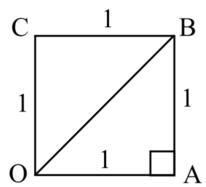
Hence, $b < c < a$

Thus, c is the required irrational number between a and b.

➤ REPRESENTING IRRATIONAL NUMBERS ON THE NUMBER LINE

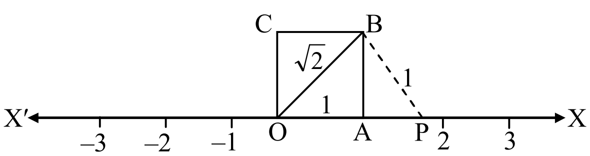
◈ Represent $\sqrt{2}$ & $\sqrt{3}$ on the number line :

Greeks discovered this method. Consider a unit square OABC, with each side 1 unit in lenght. Then by using pythagoras theorem



$OB = \sqrt{1+1} = \sqrt{2}$

Now, transfer this square onto the number line making sure that the vertex O coincides with zero



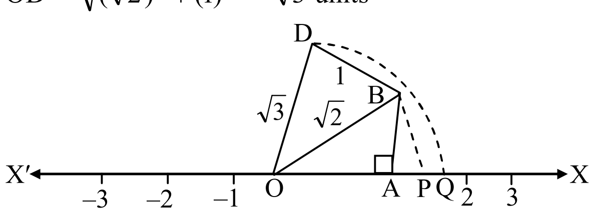
With O as centre & OB as radius, draw an arc, meeting OX at P. Then

$OB = OP = \sqrt{2}$ units

Then, the point represents $\sqrt{2}$ on the number line

Now draw, $BD \perp OB$ such that BD = 1 unit join OD. Then

$OD = \sqrt{(\sqrt{2})^2 + (1)^2} = \sqrt{3}$ units



With O as centre & OC as radius, draw an arc, meeting OX at Q. Then

$OQ = OD = \sqrt{3}$ units

Then, the point Q represents $\sqrt{3}$ on the real line

Remark : In the same way, we can locate $\sqrt{n}$ for any positive integer n, after $\sqrt{n-1}$ has been located.

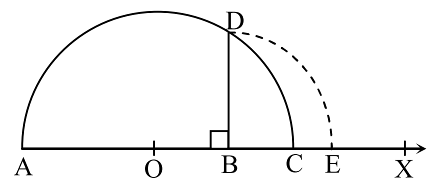
◈ Existence of $\sqrt{n}$ for a positive real number :

The value of $\sqrt{4.3}$ geometrically : -

Draw a line segment AB = 4.3 units and extend it to C such that BC = 1 unit.

Find the midpoint O of AC.

With O as centre and OA a radius, draw a semicircle.



Now, draw $BD \perp AC$, intersecting the semicircle at D. Then, $BD = \sqrt{4.3}$ units.

With B as centre and BD as radius, draw an arc, meeting AC produced at E.

Then, $BE = BD = \sqrt{4.3}$ units

➤ SURDS OR RADICALS

◈ Real Number Line :

Irrational numbers like $\sqrt{2}, \sqrt{3}, \sqrt{5}$ etc. can be represented by points on the number line. Since all rational numbers and irrational numbers can be represented on the number line, we call the number line as real number line.

◈ Surds :

$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{21}$,................ are irrational numbers, These are square roots (second roots), of some rational numbers, which can not be written as squares of any rational number.

(i) If a is rational number and n is a positive integer such that the n^{th} root of a is an irrational number, then $a^{1/n}$ is called a **surd** or **radical**.

e.g. $\sqrt{5}$, $\sqrt{2}$, $\sqrt{3}$ etc.

(ii) If is a surd then 'n' is known as order of surd and 'a' is known as **radicand**.

(iii) Every surd is an irrational number but every irrational number is not a surd.

❖ **Quadratic Surd:**

A surd of order 2 is called a quadratic surd.

Ex.31 $\sqrt{3} = 3^{1/2}$ is a quadratic surd but $\sqrt{9} = 9^{1/2}$ is not a quadratic surd, because $\sqrt{9} = 9^{1/2} = 3$ is a rational number. So, $\sqrt{9}$ is not a surd.

❖ **Cubic surd :**

A surd of order 3 is called a cubic surd.

Ex.32 The real number $\sqrt[3]{4}$ is a cubic surd but the real number $\sqrt[3]{8}$ is not a cubic surd as it not a surd.

❖ **Biquadratic surd :**

A surd of order 4 is called a biquadratic surd. A biquadratic surd is also called a quadratic surd.

Ex.33 $\sqrt[4]{5}$ is a biquadratic surd but $\sqrt[4]{81}$ is not a biquadratic surd as it is not a surd.

❖ **Laws of radicals :**

For any positive integer 'n' and a positive rational number 'a'.

(a) $\left(\sqrt[n]{a}\right)^n = a$

(b) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ [one of either a or b should be non-negative integer]

(c) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

(d) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$

(e) $\sqrt[p]{\left(a^n\right)^m} = \sqrt[p]{a^{n.m}}$

(f) $\sqrt[p]{a^n \times a^m} = \sqrt[p]{a^{n+m}}$

(iv) A surd which has unity only as rational factor is called a **pure surd**.

(v) A surd which has a rational factor other than unity is called a **mixed surd**.

(vi) Surds having same irrational factors are called similar or like surds.

(vii) Only similar surds can be added or subtracted by adding or subtracting their rational parts

(viii) Surds of same order can be multiplied or divided

(ix) If the surds to be multiplied or to be divided are not of the same order, we first reduce them to the same order and then multiply or divide.

(x) If the product of two surds is a rational number, then each one of them is called the rationalising factor of the other.

(xi) A surd consisting of one term only is called a **monomial surd**.

(xii) An expression consisting of the sum or difference of two monomial surds or the sum or difference of a monomial surds and a rational number is called **binomial surd**. e.g. $\sqrt{2}+\sqrt{5}, \sqrt{3}+2, \sqrt{2}-\sqrt{3}$ etc. are binomial surds.

(xiii) The binomial surds which differ only in sign (+ or –) between the terms connecting them, are called **conjugate surds** e.g. $\sqrt{3}+\sqrt{2}$ and $\sqrt{3}-\sqrt{2}$ or $2+\sqrt{5}$ and $2-\sqrt{5}$ are conjugate surds.

❖ EXAMPLES ❖

Ex.34. State with reasons which of the following are surds and which are not :

(i) $\sqrt{64}$ (ii) $\sqrt{45}$

(iii) $\sqrt{20} \times \sqrt{45}$ (iv) $8\sqrt{10} \div 4\sqrt{15}$

(v) $3\sqrt{12} \div 6\sqrt{27}$ (vi) $\sqrt[3]{5} \times \sqrt[3]{25}$

Sol. (i) $\sqrt{64} = 8$

8 is a rational number, hence $\sqrt{64}$ is not a surd.

(ii) $\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$

Thus $\sqrt{45}$ is an irrational number.

Because the rational number 45 is not the square of any rational number, hence $\sqrt{45}$ is a surd.

(iii) We have $\sqrt{20} \times \sqrt{45} = \sqrt{900}$

$= \sqrt{30 \times 30} = \left(\sqrt{30}\right)^2 = 30$

Which is a rational number and therefore $\sqrt{20} \times \sqrt{45}$ is not a surd.

(iv) we have

$8\sqrt{10} \div 4\sqrt{15} = \dfrac{8\sqrt{10}}{4\sqrt{15}}$

$= \dfrac{\left(\sqrt{8}\right)^2\left(\sqrt{10}\right)}{\left(\sqrt{4}\right)^2\left(\sqrt{15}\right)} \Rightarrow \dfrac{\sqrt{8} \times \sqrt{8} \times \sqrt{10}}{\sqrt{4} \times \sqrt{4} \times \sqrt{15}}$

$= \dfrac{\sqrt{8 \times 8 \times 10}}{\sqrt{4 \times 4 \times 15}} \Rightarrow \dfrac{\sqrt{640}}{\sqrt{240}}$

$\Rightarrow \frac{\sqrt{8}}{\sqrt{3}} = \sqrt{\frac{8}{3}}$

Which is an irrational number.

Because the rational number $\frac{8}{3}$ is not the square of any rational number, hence the given expression is a surd.

(v) $3\sqrt{12} \div 6\sqrt{27} = \frac{3\sqrt{12}}{6\sqrt{27}} = \frac{(\sqrt{3})^2(\sqrt{12})}{(\sqrt{6})^2\sqrt{27}}$

$= \frac{\sqrt{3}\times\sqrt{3}\times\sqrt{12}}{\sqrt{6}\times\sqrt{6}\times\sqrt{27}} \Rightarrow \frac{\sqrt{3\times3\times12}}{\sqrt{6\times6\times27}}$

$\Rightarrow \sqrt{\frac{108}{972}} \qquad \Rightarrow \sqrt{\frac{1}{9}} = \frac{1}{3}$

Since $\frac{1}{3}$ is a rational number, therefore $3\sqrt{12} \div 6\sqrt{27}$ is not a surd.

(vi) $\sqrt[3]{5} \times \sqrt[3]{25} = \sqrt[3]{5\times25}$

$= \sqrt[3]{5\times5\times5}$

$\Rightarrow \sqrt[3]{5^3} = 5$

Which is a rational number. Hence, $\sqrt[3]{5} \times \sqrt[3]{25}$ is not a surd.

Ex.35 Simplify the following :

(i) $(\sqrt[3]{5})^3$ (ii) $\sqrt[3]{64}$

Sol. (i) $(\sqrt[3]{5})^3 = 5$ (Using Ist Law)

(ii) $\sqrt[3]{64} = \sqrt[3]{4^3} = 4$ (Using Ist Law)

Ex.36 Find the value of x in each of the following:

(i) $\sqrt[3]{4x-7} - 5 = 0$ (ii) $\sqrt[4]{3x+1} = 2$

Sol. (i) $\sqrt[3]{4x-7} - 5 = 0$

$\Rightarrow \sqrt[3]{4x-7} = 5$

$\Rightarrow (\sqrt[3]{4x-7})^3 = 5^3$

$\Rightarrow 4x - 7 = 125 \qquad [(\sqrt[n]{a})^n = a]$

$\Rightarrow 4x = 132$

$\Rightarrow x = 33$

(ii) $\sqrt[4]{3x+1} = 2$

$\Rightarrow (\sqrt[4]{3x+1})^4 = 2^4$

$\Rightarrow 3x + 1 = 16 \qquad [(\sqrt[n]{a})^n = a]$

$\Rightarrow 3x = 15$

$\Rightarrow x = 5$

Ex.37 Simplify each of the following:

(i) $\sqrt[3]{3}\cdot\sqrt[3]{4}$ (ii) $\sqrt[3]{128}$

Sol. (i) $\sqrt[3]{3}\cdot\sqrt[3]{4} = \sqrt[3]{3\times4} = \sqrt[3]{12}$

[Using IInd Law]

(ii) $\sqrt[3]{128} = \sqrt[3]{64\times2} = \sqrt[3]{64}\ \sqrt[3]{2}$

[Using IInd Law]

$= \sqrt[3]{4^3}\cdot\sqrt[3]{2}$

$= 4\sqrt[3]{2}$ [Using I, $\sqrt[3]{4^3} = 4$]

Ex.38 Simplify each of the following:

(i) $\sqrt[3]{\frac{8}{27}}$ (ii) $\frac{\sqrt[4]{3888}}{\sqrt[4]{48}}$

Sol. (i) $\sqrt[3]{\frac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}}$ [Using IIIrd Law]

$= \frac{\sqrt[3]{2^3}}{\sqrt[3]{3^3}} = \frac{2}{3}$ [Using Ist Law]

(ii) $\frac{\sqrt[4]{3888}}{\sqrt[4]{48}} = \sqrt[4]{\frac{3888}{48}}$ [Using IIIrd Law]

$= \sqrt[4]{81} = \sqrt[4]{3^4} = 3$ [Using Ist Law]

Ex.39 Siplify each of the following

(i) $\sqrt[4]{\sqrt[3]{3}}$ (ii) $\sqrt[2]{\sqrt[3]{5}}$

Sol. (i) $\sqrt[4]{\sqrt[3]{3}} = \sqrt[12]{3}$ [Using IVth law]

(ii) $\sqrt[2]{\sqrt[3]{5}} = \sqrt[6]{5}$ [Using IVth law]

Ex.40 Simplify : $\sqrt[5]{\sqrt[4]{(2^3)^4}}$

Sol. Using the above property, we have

$\sqrt[5]{\sqrt[4]{(2^3)^4}} = \sqrt[5]{2^3} = \sqrt[5]{8}$.

◈ Pure And Mixed Surds :

(i) Pure Surd :

A surd which has unity only as rational factor, the other factor being irrational, is called a **pure surd**.

Ex.41 $\sqrt{3}, \sqrt[5]{2}, \sqrt[4]{3}$ are pure surds.

Ex.42 $\sqrt{6}, \sqrt[3]{12}$ are pure surds.

(ii) Mixed Surd :

A surd which has a rational factor other than unity, the other factor being irrational, is called a **mixed surd**.

Ex.43 $2\sqrt{3}, 5\sqrt[3]{12}, 2\sqrt[4]{5}$ are mixed surds.

TypeI : On expressing of mixed surds into pure surds

v EXAMPLES v

Ex.44 Express each of the following as a pure surd.

(i) $2\sqrt{3}$ (ii) $2.\sqrt[3]{4}$

(iii) $\frac{3}{4}\sqrt{32}$ (iv) $\frac{3}{4}\sqrt{8}$

Sol. (i) $2\sqrt{3} = 2 \times 3^{1/2} = (2^2)^{1/2} \times 3^{1/2} = 4^{1/2} \times 3^{1/2}$

$= (4\times3)^{1/2} = 12^{1/2} = \sqrt{12}$

(ii) $2.\sqrt[3]{4} = 2\times4^{1/3} = (2^3)^{1/3} \times 4^{1/3} = 8^{1/3} \times 4^{1/3}$

$= (8\times4)^{1/3} = (32)^{1/3} = \sqrt[3]{32}$

(iii) $\frac{3}{4}\sqrt{32} = \sqrt{\left(\frac{3}{4}\right)^2} \times \sqrt{32} = \sqrt{\left(\frac{3}{4}\right)^2 \times 32}$

$= \sqrt{\frac{9}{16}\times 32} = \sqrt{18}$

(iv) $\frac{3}{4}\sqrt{8} = \sqrt{\left(\frac{3}{4}\right)^2} \times \sqrt{8} = \sqrt{\left(\frac{3}{4}\right)^2 \times 8}$

$= \sqrt{\frac{9}{16}\times 8} = \sqrt{\frac{9}{2}}$

Ex.45 Expressed each of the following as pure surds-

(i) $\frac{2}{3}.\sqrt[3]{108}$ (ii) $\frac{3}{2}\sqrt[4]{\frac{32}{243}}$

Sol. (i) $\frac{2}{3}\sqrt[3]{108} = \frac{2}{3} \times (108)^{1/3}$

$= \left[\left(\frac{2}{3}\right)^3\right]^{1/3} \times (108)^{1/3}$

$= \left(\frac{8}{27}\right)^{1/3} \times (108)^{1/3}$

$= \left(\frac{8}{27}\times 108\right)^{1/3}$

$= (8\times4)^{1/3} = (32)^{1/3} = \sqrt[3]{32}$

(ii) $\frac{3}{2}.\sqrt[4]{\frac{32}{243}} = \frac{3}{2} \times \left(\frac{32}{243}\right)^{1/4}$

$= \left(\left(\frac{3}{2}\right)^4\right)^{1/4} \times \left(\frac{32}{243}\right)^{1/4}$

$= \left(\frac{81}{16}\right)^{1/4} \times \left(\frac{32}{243}\right)^{1/4} = \left(\frac{81}{16}\times\frac{32}{243}\right)^{1/4}$

$= \left(\frac{2}{3}\right)^{1/4} = \sqrt[4]{\frac{2}{3}}$

Ex.46 Express each of the following as pure surd :

(i) $a\sqrt{a+b}$ (ii) $a\sqrt[3]{b^2}$

(iii) $2ab\sqrt[3]{ab}$

Sol. (i) $a\sqrt{a+b} = a \times (a+b)^{1/2}$

$= (a^2)^{1/2} \times (a+b)^{1/2}$

$= [a^2\times (a+b)]^{1/2}$

$= (a^3 + a^2b)^{1/2} = \sqrt{a^3 + a^2b}$

(ii) $a\sqrt[3]{b^2} = (a^3)^{1/3} \times (b^2)^{1/3} = (a^3\times b^2)^{1/3}$

$= \sqrt[3]{a^3b^2}$

(iii) $2ab.\sqrt[3]{ab} = \left((2ab)^3\right)^{1/3} \times (ab)^{1/3}$

$= (8\,a^3b^3.ab)^{1/3}$

$= (8\,a^4b^4)^{1/3} = \sqrt[3]{8a^4b^4}$

TypeII : On expressing given surds as mixed surds in the simplest form.

v EXAMPLES v

Ex.47 Express each of the following as mixed surd in its simplest form:

(i) $\sqrt{80}$ (ii) $\sqrt[3]{72}$

(iii) $\sqrt[5]{288}$ (iv) $\sqrt{1350}$

(v) $\sqrt[5]{320}$ (vi) $5.\sqrt[3]{135}$

Sol. (i) $\sqrt{80} = \sqrt{16\times5} = \sqrt{4^2\times5} = \sqrt{4^2}\times\sqrt{5} = 4\sqrt{5}$

(ii) $\sqrt[3]{72} = \sqrt[3]{8\times9} = \sqrt[3]{2^3\times9} = \sqrt[3]{2^3} \times \sqrt[3]{9} = 2\sqrt[3]{9}$

(iii) $\sqrt[5]{288} = \sqrt[5]{32\times9} = \sqrt[5]{2^5\times9} = \sqrt[5]{2^5} \times \sqrt[5]{9} = 2\sqrt[5]{9}$

(iv) $\sqrt{1350} = \sqrt{225\times6} = \sqrt{15^2\times6} = \sqrt{15^2} \times \sqrt{6} = 15\sqrt{6}$

(v) $\sqrt[5]{320} = \sqrt[5]{32\times10} = \sqrt[5]{2^5\times10} = \sqrt[5]{2^5} \times \sqrt[5]{10}$

$= 2\times\sqrt[5]{10}$

(vi) $5.\sqrt[3]{135} = 5\sqrt[3]{27\times5} = 5\sqrt[3]{3^3\times5} = 5\sqrt[3]{3} \times \sqrt[3]{5}$

$= 5 \times 3 \times \sqrt[3]{5} = 15\sqrt[3]{5}$.

Ex.48 Express $\sqrt[4]{1280}$ as mixed surd in its simplest form :

Sol. $\sqrt[4]{1280} = \sqrt[4]{256\times5} = \sqrt[4]{256}\times\sqrt[4]{5}$

$= \sqrt[4]{4^4}\times\sqrt[4]{5} = 4\sqrt[4]{5}$

➢ SOME RULES FOR EXPONENTS

Let a, b > 0 be real numbers & p, q are rational numbes then

(i) $a^p . a^p = a^{p+q}$ i.e. if base is same with different or same powers are multiply each other then powers are add.

Ex.49 $\frac{(7)^3 \times 21}{3} = ?$

$= 7^3 \times 7 = 7^{3+1} = 7^4$ **Ans.**

(ii) $(a^p)^q = a^{pq}$

Ex.50 $\frac{(121)^3 \times 33}{3} = (11^2)^3 \times 11 = 11^6 \times 11^1$

$= 11^7$ **Ans.**

(iii) $\frac{a^p}{a^q} = \begin{cases} a^{p-q} & \text{if } p > q \\ 1/a^{q-p} & \text{if } q > p \end{cases}$

Ex.51 $\frac{(169)^2}{13^2} = \frac{[(13)^2]^2}{13^2} = \frac{13^4}{13^2}$

$= 13^{4-2} = 13^2 = 169$ **Ans.**

Ex.52 $\frac{5^3 \times 2^4 \times 49}{625 \times 32 \times 7}$

$= \frac{5^3 \times 2^4 \times 7^2}{5^4 \times 2^5 \times 7}$

$= \frac{1}{5^{4-3}} \times \frac{1}{2^{5-4}} \times 7^{2-1}$

$= \frac{7}{5 \times 2} = \frac{7}{10}$ **Ans.**

(iv) $(ab)^p = a^p b^p$

Ex.53 $\frac{(63)^4 \times 144}{132 \times 9} = ?$

$= \frac{(9 \times 7)^4 \times (12)^2}{(11 \times 12) \times 3^2}$

$= \frac{(3^2 \times 7)^4 \times (3 \times 2^2)^2}{11 \times 2^2 \times 3 \times 3^2}$

$= \frac{(3^2)^4 \times 7^4 \times 3^2 \times (2^2)^2}{11 \times 2^2 \times 3^{1+2}}$

$= \frac{3^8 \times 7^4 \times 3^2 \times 2^4}{11 \times 2^2 \times 3^3}$

$= \frac{2^4 \times 3^{8+2} \times 7^4}{2^2 \times 3^3 \times 11}$

$= \frac{2^{4-2} \times 3^{10-3} \times 7^4}{11}$

$= \frac{2^2 \times 3^7 \times 7^4}{11}$ **Ans.**

(v) $a^0 = 1$

Ex.54 $\frac{(2^0 + 3^0)5^2}{25}$

$= \frac{(1+1).5^2}{5^2}$

$= \frac{2}{5^{2-2}} = \frac{2}{5^0} = \frac{2}{1} = 2$ **Ans.**

(vi) $a^{+p} = \frac{1}{a^{-p}}$ or $a^{-p} = \frac{1}{a^p}$

Ex.55 $\frac{\sqrt{2}}{4} = ?$

$= \frac{(2)^{1/2}}{2^2}$

$= \frac{1}{(2)^{2-\frac{1}{2}}} = \frac{1}{(2)^{3/2}} = 2^{-3/2}$ **Ans.**

Ex.56 $\frac{11/3}{(11/3)^7} = ?$

$= \frac{1}{(11/3)^{7-1}}$

$= \frac{1}{(11/3)^6}$

$= \left(\frac{3}{11}\right)^6$ **Ans.**