CBSE Sample Paper-01
SUMMATIVE ASSESSMENT –II
MATHEMATICS
Class – X

Time allowed: 3 hours Maximum Marks: 90

General Instructions:

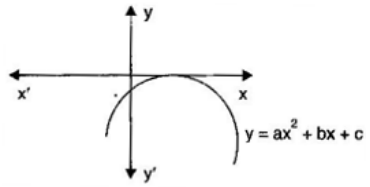
a) All questions are compulsory.
b) The question paper consists of 31 questions divided into four sections – A, B, C and D.
c) Section A contains 4 questions of 1 mark each which are multiple choice questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
d) Use of calculator is not permitted.

Section A

1. If P(E) = 0.07, then the probability of 'not E' is:
   (a) 0 (b) 0.03 (c) 0.93 (d) 0.3
2. If A is a point on $y-$axis, whose ordinate os 3 and B is a point $(-5, 2)$, then the distance AB is:
   (a) $\sqrt{26}$ units (b) $\sqrt{24}$ units (c) 5 units (d) $\sqrt{65}$
3. 12^{th} term of the AP 5, 8, 11, 14, ….. is:
   (a) 43 (b) 40 (c) 38 (d) 3
4. A 6 ft tall man finds that the angle of elevation of a 24 ft high pillar and the angle of depression of its base are complimentary angles. The distance of the man from the pillar is:
   (a) $4\sqrt{3}$ m (b) $6\sqrt{3}$ m (c) $8\sqrt{3}$ m (d) $10\sqrt{3}$

Section B

5. If the diameter of a semicircular protractor is 14 cm, then find its perimeter.
6. A heap of rice is in the form of a cone of radius 3 m and height 3 m. Find the volume of the rice. How much cloth is required to just cover the heap?
7. A solid metallic hemisphere of radius 6 cm is melted and recasted into a right circular cone of base radius 3 cm. Determine the height of the cone.
8. The graph of the polynomial $y = ax^2 + bx + c$ is shown in the figure. Write one value of $b^2 - 4ac$.

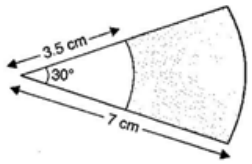


9. Find the sum of first 10 terms of the AP 10, 6, 2, ……..

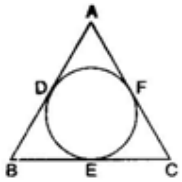
10. If $d_1, d_2\,(d_2 > d_1)$ be the diameters of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, then prove that $d_2^2 = c^2 + d_1^2$.

Section C

11. Find the ratio in which the line segment joining the points (6, 4) and $(1,-7)$ is divided by $x-$ axis.
12. Three consecutive vertices of a parallelogram are $(-2,-1),(1,0)$ and $(4,3)$. Find the coordinates of the fourth vertex.
13. In the given figure, sectors of two concentric circles of radii 7 cm and 3.5 cm are shown. Find the area of the shaded region. $\left(\text{Use } \pi = \frac{22}{7}\right)$



14. The inner circumference of a circular track is 440 m. The track is 14 m wide. Find the diameter of the outer circle of the track. $\left(\text{Use } \pi = \frac{22}{7}\right)$
15. Water flows out through a circular pipe whose internal radius is 1 cm, at the rate of 80 cm/second into an empty cylindrical tank, the radius of whose base is 40 cm. By how much will the level of water rise in the tank in half an hour?
16. Find the value of k for which the roots of the quadratic equation $kx^2 - 10x + 5 = 0$ are equal.
17. Find the 10^{th} term from the end of the AP 8, 10, 12, ……., 126.
18. In the figure, if AB = AC, then prove that BE = EC.



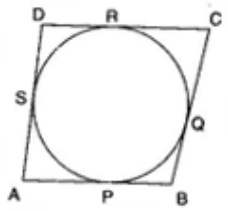
19. A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point A on the ground is 60° and the angle of the depression of point A from top of the tower is 45°. Find the height of the tower. $\left(\text{Take } \sqrt{3} = 1.732\right)$
20. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball from the bag is four times that of a red ball, then find the number of blue balls in the bag.

Section D

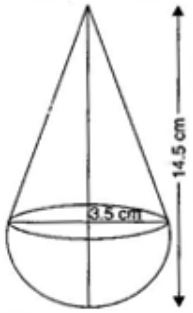
21. Prove that the length of tangents drawn from an external point to a circle are equal.
    Using the above result, prove the following:

If a circle touches all the four sides of a quadrilateral ABCD, then prove that:

AB + CD = BC + DA



22. Draw a right triangle ABC, in which $\angle B = 90^\circ$, AB = 5 cm, BC = 4 cm. Then construct another triangle A'BC' whose sides are $\frac{5}{3}$ times the corresponding sides of Δ ABC.
23. A bag contains 5 white balls, 7 red balls, 4 black balls and 2 blue balls. One ball is drawn at random from the bag. What is the probability that the ball drawn is:
    (i) white or blue? (ii) red or black?
    (iii) not white? (iv) neither white nor black?
24. The vertices of a triangle are $(-1,3),(1,-1)$ and (5, 1). Find the lengths of medians through vertices $(-1,3)$ and (5, 1).
25. A solid right circular cone of diameter 14 cm and height 8 cm is melted to form a hollow sphere. If the external diameter of the sphere is 10 cm, then find the internal diameter of the sphere.
26. A toy is in the form of a cone mounted on a hemisphere with same radius. The diameter of the base of the conical portion is 7 cm and the total height of the toy is 14.5 cm. Find the volume of the toy.



27. Solve for x: $\frac{1}{x-2}+\frac{1}{x}=\frac{8}{2x+5}, \quad x \neq 0, 2, \frac{-5}{2}$
28. There are two windows in a house. A window of the house is at a height of 1.5 m above the ground and the other window is 3 m vertically above the lower window. Ram and Shyam are sitting inside the two windows. At an instant, the angle of elevation of a balloon from these windows are observed as 45° and 30° respectively.
    Read the above passage and answer the following questions:
    (a) Find the height of the balloon from the ground.

(b) Among Ram and Shyam, who is more closer to the balloon?

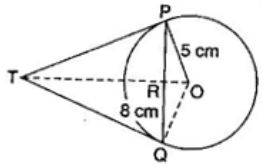
(c) Why windows are essential in any construction, commercial or residential?

(d) If the balloon is moving towards the building, then both angles of elevation will remain same or not?

29. Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, then find the sides of two squares.

30. The sum of three numbers in an AP is (-3) and their product id 8. Find the numbers.

31. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of TP.



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(Solutions)

SECTION-A

1. (c)
2. (a)
3. (c)
4. (b)
5. $r = \frac{14}{2}$ = 7 cm

   Perimeter of protractor = $\pi r + 2r = r(\pi + 2)$

   $= 7\left(\frac{22}{7} + 2\right)$ = 36 cm

6. Volume of rice = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times(3)^2(3) = \frac{198}{7}$ = 28.29 m^3 (approx.)

   Cloth required = $\pi r\sqrt{r^2+h^2} = \frac{22}{7}\times 3\sqrt{3^2+3^2} = \frac{198\sqrt{2}}{7}$ m^2

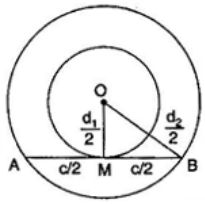
7. Let the height of the cone be H cm. Then

   $\frac{1}{3}\pi(3)^2\ \text{H} = \frac{2}{3}\pi(6)^3 \quad \Rightarrow \quad \text{H} = 48$ cm

8. Since the graph intersects the $x-$axis at one point only, so the polynomial ax^2+bx+c has only one real zero. Consequently, the roots of the quadratic equation $ax^2+bx+c=0$ are real and equal.

   $\therefore\ b^2 - 4ac = 0$

9. $a = 10, d = 6-10 = -4, n = 10$



   $S_n = \frac{n}{2}\left[2a+(n-1)d\right] = \frac{10}{2}\left[2\times 10+(10-1)(-4)\right] = 5(20-36)$

   $S_n = -80$

10. $\left(\frac{d_2}{2}\right)^2 = \left(\frac{d_1}{2}\right)^2 + \left(\frac{c}{2}\right)^2$

$\Rightarrow \quad d_2^2 = c^2 + d_1^2$

11. Let the point of division be $(x, 0)$ and the ratio be $k:1$.

Then, $\frac{(k)(-7)+(1)(4)}{k+1} = 0 \quad \Rightarrow \quad \frac{-7k+4}{k+1} = 0$

$\Rightarrow \quad -7k+4=0 \quad \Rightarrow \quad k = \frac{4}{7}$

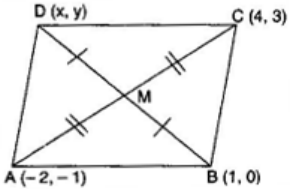
$\Rightarrow \quad k:1 = 4:7$

12. Let the fourth vertex be (x, y).

Since diagonals of a parallelogram bisect each other.

$\therefore \quad \frac{x+1}{2} = \frac{-2+4}{2} \quad \Rightarrow \quad x = 1$

And $\frac{y+0}{2} = \frac{-1+3}{2} \Rightarrow \quad y = 2$



13. Area of shaded region = $\pi(7)^2 \times \frac{30^\circ}{360^\circ} - \pi\left(\frac{7}{2}\right)^2 \times \frac{30^\circ}{360^\circ}$

$= \frac{147}{48}\pi = \frac{147}{48} \times \frac{22}{7} = 9.625 \text{ cm}^2$

14. $2\pi r = 440$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 440 \quad \Rightarrow \quad r = \frac{440 \times 7}{2 \times 22} = 70 \text{ m}$

Width of track = 14 m

$\therefore$ Radius of outer circle = 70 + 14 = 84 m

$\therefore$ Diameter of the outer circle = 2 x 84 = 168 m

15. Volume of water that flows out through the pipe in half an hour,

$= \pi(1)^2 \times 80 \times 60 \times 30$

Let the water level rise by x cm. Then,

$\pi(40)^2 \times x = \pi(1)^2 \times 80 \times 60 \times 30$

$\Rightarrow \quad x = 90 \text{ cm}$

16. Here, $a = k, b = -10, c = 5$

For equal roots, $b^2 - 4ac = 0 \quad \Rightarrow \quad (-10)^2 - 4.k.5 = 0$

$\Rightarrow \quad 100-20k=0 \quad \Rightarrow \quad k=5$

17. $a=8, \quad d=10-8=2, \quad l=126$

$l=a+(n-1)d \quad \Rightarrow \quad 126=8+(n-1)2 \quad \Rightarrow \quad 118=(n-1)2$

$\Rightarrow \quad n-1=59 \quad \Rightarrow \quad n=60$

10^{th} term from the end = $(60 - 10 + 1)^{th}$ term from beginning

$= a_{51}=a+50d = 8 + 50\ (2) = 108$

18. Since Tangent segments from an external point to a circle are equal in length.

$\therefore$ AD = AF, BD = BE and CE = CF

Now, AB = AC

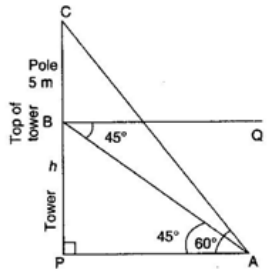
$\Rightarrow$ AB – AD = AC – AD

$\Rightarrow$ AB – AD = AC – AF

$\Rightarrow$ BD = CF

$\Rightarrow$ BE = CE

19. Let BP = h m



In right Δ CPA,

$$\tan 60^\circ = \frac{h+5}{PA}$$

$\Rightarrow \quad \sqrt{3}=\frac{h+5}{PA}$ .....(i)

In right Δ BPA,

$$\tan 45^\circ = \frac{h}{PA} \quad \Rightarrow \quad 1=\frac{h}{PA}$$

$\Rightarrow$ PA = h m .....(ii)

From eq. (i) and (ii),

$$\sqrt{3}=\frac{h+5}{h} \Rightarrow h=\frac{5}{\sqrt{3}-1} = 6.83 \text{ m}$$

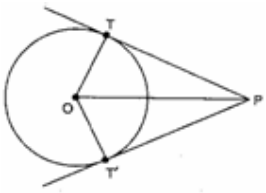
20. Let the number of blue balls in the box be x.

$\therefore$ Total number of balls in the bag = $5+x$

Probability of drawing a blue ball = 4 x Probability of drawing a red ball

$\Rightarrow \quad \frac{x}{5+x}=4\left(\frac{5}{5+x}\right) \Rightarrow \quad x=20$

21. **First part**: Given : A circle with centre O and a point P outside the circle. PT and PT' are tangents from P to the circle.



To Prove : We need to prove thatPT = PT'

Construction: Joined OP, OT and OT'

Proof : ∵ OT is a radius and PT is a tangent.

$\therefore \quad \angle OTP = 90^\circ$

Similarly, $\angle OT'P = 90^\circ$

Now in right triangles OTP and OT'P,

OT = OT' ……. (Radii of the same circle)

and OP = OP ……..(Common)

$\therefore \quad \Delta OTP \cong \Delta OT'P$ ……..(RHS congruency)

Hence, PT = PT'

Second part: Using the above, we get,

AP = AS BP = BQ

CR = CQ DR = DS

On adding, we get,

(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\Rightarrow$ AB + CD = DA + BC

22. **Steps of construction**:

(a) Draw a right angled triangle ABC with given measurements.

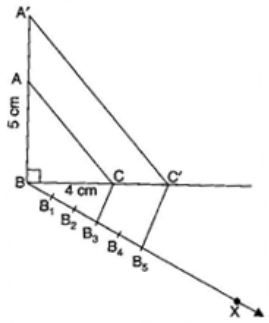
(b) Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.

(c) Locate its points B_1, B_2, B_3, B_4, B_5 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.

(d) Join B_3 to C and draw a line through B_5 parallel to B_3C, intersecting the extended line segment BC at C'.

(e) Draw a line through C' parallel to CA intersecting the extended line segment BA at A'.

The A'BC' is the required triangle.



23. Total number of balls in the bag = 5 + 7 + 4 + 2 = 18

∴ Number of all possible outcomes = 18

(i) Number of balls white or blue = 5 + 2 = 7

$\therefore$ Required probability = $\frac{7}{18}$

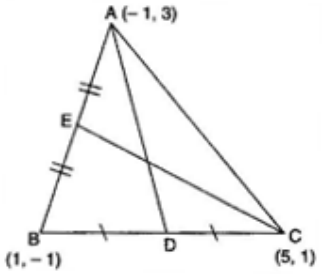
(ii) Number of balls red or black = 7 + 4 = 11

$\therefore$ Required probability = $\frac{11}{18}$

(iii) Number of balls not white = 7 + 4 + 2 = 13

$\therefore$ Required probability = $\frac{13}{18}$

(iv) Number of balls neither white nor black = 7 + 2 = 9



$\therefore$ Required probability = $\frac{9}{18} = \frac{1}{2}$

24. $D \rightarrow \left(\frac{5+1}{2}, \frac{1-1}{2}\right) \Rightarrow \quad D \rightarrow (3, 0)$

$\therefore \quad AD = \sqrt{(3+1)^2 + (0-3)^2} = 5$

$E \rightarrow \left(\frac{1-1}{2}, \frac{3-1}{2}\right) \Rightarrow \quad E \rightarrow (0, 1)$

$\therefore \quad CE = \sqrt{(5-0)^2 + (1-1)^2} = 5$

25. For cone, $r = \frac{14}{2} = 7$ cm and $h = 8$ cm

Let the internal radius of the sphere be x cm.

External radius of the sphere = $\frac{10}{2} = 5$ cm

Volume of the hollow sphere = Volume of cone

$\Rightarrow \quad \frac{4}{3}\pi\left(5^3 - x^3\right) = \frac{1}{3}\pi(7)^2 8 \quad \Rightarrow \quad x = 3$ cm

$\Rightarrow \quad 2x = 6$ cm

26. For conical portion, $r = \frac{7}{2} = 3.5$ cm and $h = 14.5 - 3.5 = 11$ cm

For hemisphere, $R = \frac{7}{2} = 3.5$ cm

Volume of the toy = $\frac{2}{3}\pi R^3 + \frac{1}{3}\pi r^2 h$

$$= \frac{2}{3}\times\frac{22}{7}\times(3.5)^3+\frac{1}{3}\times\frac{22}{7}\times(3.5)^2\times 11$$

$$= 231 \text{ cm}^3$$

27. $\frac{1}{x-2}+\frac{1}{x}=\frac{8}{2x+5} \quad \Rightarrow \quad \frac{x+(x-2)}{(x-2)x}=\frac{8}{2x+5}$

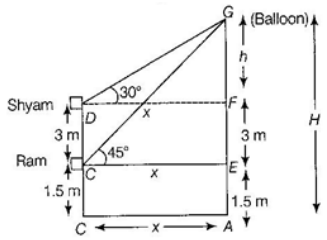
$\Rightarrow \quad 2(x-1)(2x+5)=8(x^2-2x) \Rightarrow \quad 2(2x^2+3x-5)=8(x^2-2x)$

$\Rightarrow \quad 2x^2+3x-5=4x^2-8x \quad \Rightarrow \quad 2x^2-11x+5=0$

$\Rightarrow \quad 2x^2-10x-x+5=0 \quad \Rightarrow \quad 2x(x-5)-1(x-5)=0$

$\Rightarrow \quad (x-5)(2x-1)=0 \quad \Rightarrow \quad x=5,\frac{1}{2}$

28. (a) Let H be the height of the balloon from the ground and C and D be the position of the windows.



At C and G, angle of elevation are $\angle ECG = 45^\circ$ and $\angle FDG = 30^\circ$.

Let CE = DF = x m and FG = h m

In Δ CEG , we have

$$\tan 45^\circ = \frac{EG}{EC}$$

$$\Rightarrow \quad 1 = \frac{EF + FG}{EC}$$

$$\Rightarrow \quad 1 = \frac{3+h}{x}$$

$$\Rightarrow \quad x = 3+h \qquad \text{.....(i)}$$

In Δ DFG, we have

$$\tan 30^\circ = \frac{GF}{DF}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x} \quad \Rightarrow x = \sqrt{3}h \qquad \text{.....(ii)}$$

Substituting $x = \sqrt{3}h$ in eq, (i), we get,

$$\sqrt{3}h = 3+h$$

$\Rightarrow \quad \sqrt{3}h-h=3 \quad \Rightarrow \quad h(\sqrt{3}-1)=3 \quad \Rightarrow \quad h=\frac{3}{\sqrt{3}-1}$

$\Rightarrow \quad h=\frac{3}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1} \quad \Rightarrow \quad h=\frac{3(\sqrt{3}+1)}{3-1} \quad \Rightarrow \quad h=\frac{3\times(1.732+1)}{2}$

$\Rightarrow \quad h = 4.098$ m

Hence, the height of the balloon from the ground is

H = EA + FE + h = 1.5 + 3 + 4.098 = 8.598 m

(b) The person who makes small angle of elevation is more closer to the balloon. Hence, Shyam is more closer to the balloon.

(c) Windows are most important part of any building they add value to it.

They are useful for the proper ventilation, which is very much required as natural air, keeps the building freah and suffocation free.

(d) No, when the balloon is moving towards the building then the angle of elevation will automatically increase.

29. Let the side of the larger square be x m. Then its perimeter = $4x$ m

Perimeter of the larger square – Perimeter of the smaller square = 24 m

$\Rightarrow$ $4x -$ Perimeter of the smaller square = 24

$\Rightarrow$ Perimeter of the smaller square = $(4x-24)$ m

$\Rightarrow$ Side of the smaller square = $\dfrac{4x-24}{4} = (x-6)$ m

According to the question,

Area of the larger square + Area of the smaller square = 468 m^2

$\Rightarrow \quad x^2 + (x-6)^2 = 468$ $\qquad \Rightarrow \quad x^2 + x^2 - 12x - 432 = 0$

$\Rightarrow \quad 2x^2 - 12x - 432 = 0$ $\qquad \Rightarrow \quad x^2 - 6x - 216 = 0$

$\Rightarrow \quad x^2 - 18x + 12x - 216 = 0$ $\qquad \Rightarrow \quad x(x-18) + 12(x-18) = 0$

$\Rightarrow \quad (x-18)(x+12) = 0$ $\qquad \Rightarrow \quad x = 18, -12$

$x = -12$ is inadmissible as x is the length of a side which cannot be negative.

$\therefore \quad x = 18$ and $x - 6 = 12$

Hence, the sides of the two squares are 18 m and 12 m.

30. Let the three numbers in an AP be $(a-d), a, (a+d)$. Then,

$a - d + a + a + d = -3 \quad \Rightarrow \quad a = -1$ .....(i)

$(a-d)(a)(a+d) = 8 \Rightarrow \quad d = \pm 3$ [Using eq. (i)]

Case 1: When $a = -1, d = 3$

Then the numbers are $-4, -1, 2$.

Case 2: When $a = -1, d = -3$

Then the numbers are $2, -1, -4$.

31. In right triangles OPT and OQT,

	OP = OQ	[Radii of the same circle]
	OT = OT	[Common]
$\therefore$	Δ OPT $\cong \Delta$ OQT	[RHS congruence axiom]
$\therefore$	$\angle$PTO = $\angle$QTO	[C.P.C.T.]
$\Rightarrow$	$\angle$PTR = $\angle$QTR	.....(i)

In Δ PTR and Δ QTR,

TP = TQ [Tangents segments from an external point T]

$\angle$PTR = $\angle$QTR [From eq. (i)]

TR = TR [Common]

$\therefore$ Δ PTR $\cong$ Δ QTR [SAS]

$\therefore$ PR = QR = $\frac{1}{2}$ PQ = $\frac{1}{2}\times 8$ = 4 cm .....(ii)

And $\angle$PRT = $\angle$QRT [C.P.C.T.]

But $\angle$PRT + $\angle$QRT = 180°

$\therefore$ $\angle$PRT = $\angle$QRT = 90°

In right angled triangle OPR,

$OP^2 = OR^2 + PR^2$ [By Pythagoras theorem]

$\Rightarrow$ $5^2 = OR^2 + 4^2$

$\Rightarrow$ OR = 3 cm .....(iii)

$\therefore$ In right angles triangle TRP,

$TP^2 = TR^2 + PR^2$ [By Pythagoras theorem]

$\Rightarrow$ $TP^2 = TR^2 + 16$ [From eq. (ii)] .....(iv)

In right angles triangle OPT,

$OP^2 = PT^2 + OT^2$ [By Pythagoras theorem]

$\Rightarrow$ $5^2 + PT^2 = (TR + 3)^2$

$\Rightarrow$ $25 + PT^2 = TR^2 + 6TR + 9$ .....(v)

Subtracting eq. (iv) from eq. (v),

$25 = 6TR - 7$ $\Rightarrow$ TR = $\frac{16}{3}$ cm

$\therefore$ From eq. (iv), $TP^2 = \left(\frac{16}{3}\right)^2 + 16$

$\Rightarrow$ TR = 6.67 cm (approx.)