CHAPTER 6

AVERAGE

AVERAGE

The average or mean or arithmetic mean of a number of quantities of the same kind is equal to their sum divided by the number of those quantities.

$$\text{Average} = \frac{\text{sum of observations}}{\text{number of observations}}$$

or $\quad \bar{x} = \dfrac{x_1 + x_2 + x_3 + ... + x_n}{n}$

To calculate the sum of observations, they should be in the same unit.

Arithmetic average is used for all averages like:

averge income, average profit, average age, average marks etc.

Example ***1:*** **A man purchased 5 toys at the rate of ₹200 each, 6 toys at the rate of ₹250 each and 9 toys at the rate of ₹300 each. Calculate the average cost of one toy.**

Solution :

Price of 5 toys $= 200 \times 5 = 1000$

Price of 6 toys $= 250 \times 6 = 1500$

Price of 9 toys $= 300 \times 9 = 2700$

$$\text{Aveage cost of one toy} = \frac{1000 + 1500 + 2700}{20} = \frac{5200}{20}$$

$= ₹260$

Example ***2:*** **In three numbers, the first is twice the second and thrice the third. If the average of these three numbers is 44, then find the first number**

Solution :

Let the three numbers be x, y and z

Therefore, $x = 2y = 3z$,

$$y = \frac{x}{2} \text{ and } z = \frac{x}{3}$$

$$\text{Now, } \frac{x + \frac{x}{2} + \frac{x}{3}}{3} = 44$$

$$\Rightarrow \quad \frac{11x}{18} = 44 \text{ or } x = 72$$

Example ***3:*** **The average of five consecutive odd numbers is 61. What is the difference between the highest and lowest numbers?**

Solution :

Let the numbers be x, x + 2, x + 4, x + 6 and x + 8.

$$\text{Then, } \frac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5} = 61$$

$\Rightarrow 5x + 20 = 305$ or $x = 57$.

So, required difference $= (57 + 8) - 57 = 8$.

Remember

✧ Average of a group consisting two different groups when their averages are known :

(a) Let Group A contains *m* quantities and their average is *a* and Group B contains *n* quantities and their average is *b*, then average of group C containing a + b quantities

$$= \frac{ma + nb}{m + n}$$

(b) If average of *m* quantities is *a* and the average of n quantities out of them is b then the average of the rest of the quantities is

$$\frac{ma - nb}{m - n}$$

Example ***4:*** **There are 30 students in a class. The average age of the first 10 students is 12.5 years. The average age of the next 20 students is 13.1 years. Calculate the average age of the whole class**

Solution :

Total age of 10 students $= 12.5 \times 10 = 125$ years

Total age of 20 students $= 13.1 \times 20 = 262$ years

$$\text{Average age of 30 students } = \frac{125 + 262}{30} = 12.9 \text{ years}$$

Example ***5:*** **The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girls is 15.4 years. Find the ratio of the number of boys to the number of girls in the class is**

Solution :

Let the number of boys in the class be x.

Let the number of girls in the class be y.

$\therefore$ Sum of the ages of the boys $= 16.4\,x$

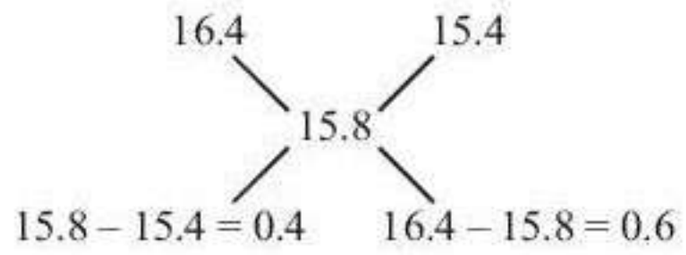
Sum of the ages of the girls $= 15.4\,y$

$\therefore \quad 15.8\,(x + y) = 16.4\,x + 15.4\,y$

$$\Rightarrow \quad 0.6x = 0.4y \Rightarrow \frac{x}{y} = \frac{2}{3}$$

$\therefore$ Required ratio = 2 : 3

Shortcut method :



$\therefore$ Required ratio $= \frac{0.4}{0.6} = \frac{2}{3}$

Example ***6:*** **Average salary of all the 50 employees including 5 officers of a company is ₹850. If the average salary of the officers is ₹2500. Find the average salary of the remaining staff of the company.**

Solution :

Here, m = 50, n = 5, a = 850, b = 2500

$\therefore$ Average salary of remaining staff $= \frac{ma - nb}{m - n}$

$= \frac{50 \times 850 - 5 \times 2500}{50 - 5}$

$= \frac{42500 - 12500}{45}$

= 667 (approx.)

WEIGHTED AVERAGE

If we have two or more groups of members whose individual averages are known, then combined average of all the members of all the groups is known as weighted average. Thus if there are k groups having member of number $n_1, n_2, n_3 \ldots\ldots n_k$ with averages $A_1, A_2, A_3 \ldots\ldots A_k$ respectively then.

(Weighted Average) $A_w = \frac{n_1A_1 + n_2A_2 + n_3A_3 + \ldots + n_kA_k}{n_1 + n_2 + n_3 + \ldots + n_k}$

Example ***7:*** **The average monthly expenditure of a family was ₹2200 during the first 3 months; ₹2250 during the next 4 months and ₹3120 during the last 5 months of a year. If the total savings during the year were ₹1260, then calculate the average monthly income.**

Solution :

Total annual income

$= 3 \times 2200 + 4 \times 2250 + 5 \times 3120 + 1260$

$= 6600 + 9000 + 15600 + 1260 = 32460$

$\therefore$ Average monthly income $= \frac{32460}{12} = ₹\,2705$

Example ***8:*** **The mean of the marks secured by 25 students of section A of class X is 47, that of 35 students of section B is 51 and that of 30 students of section C is 53. Find the combined mean of the marks of students of three sections of class X.**

Solution :

Mean of the marks of 25 students of XA = 47

$\therefore$ Sum of the marks of 25 students = 25 × 47 = 1175 .....(i)

Mean of the marks of 35 students of XB = 51

$\therefore$ Sum of the marks of 35 students = 35 × 51 = 1785 .....(ii)

Mean of the marks of 30 students of XC = 53

$\therefore$ Sum of the marks of 30 students = 30 × 53 = 1590 .....(iii)

Adding (i), (ii) and (iii)

Sum of the marks of (25 + 35 + 30) i.e., 90 students

= 1175 + 1785 + 1590 = 4550

Thus the combined mean of the marks of students of three sections $= \frac{4550}{90} = 50.56$

Example ***9:*** **Typist A can type a sheet in 5 minutes, typist B in 6 minutes and typist C in 8 minutes. Find the average number of sheets typed per hour per typist.**

Solution :

A types 12 sheets in 1 hour

B types 10 sheets in 1 hour

C types 7.5 sheets in 1 hour

Average number of sheets types per hour per typist

$= \frac{12 + 10 + 7.5}{3} = \frac{29.5}{3} = 9.83$

Remember

- If X is the average of $x_1, x_2, x_3 \ldots x_n$ then
  - (a) The average of $x_1 + a, x_2 + a, x_3 + a, \ldots x_n + a$ is $X + a$.
  - (b) The average of $x_1 - a, x_2 - a, x_3 - a \ldots x_n - a$ is $X - a$
  - (c) The average of $ax_1, ax_2, \ldots ax_n$ is aX, provided $a \neq 0$
  - (d) The average of $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \ldots \frac{x_n}{a}$ is $\frac{X}{a}$, provided $a \neq 0$
- If, in a group, one or more new quantities are added or excluded, then the new quantity or sum of added or excluded quantities = [Change in no. of quantities × original average] ± [change in average × final no. of quantities]
  Take +ve sign if quantities added and take –ve sign if quantities removed.

Example ***10:*** **The average weight of 29 students in a class is 48 kg. If the weight of the teacher is included, the average weight rises by 500 g. Find the weight of the teacher.**

Solution :

Here, weight of the teacher is added and final average of the group increases.

$\therefore$ Change in average is (+)ve. Using the formula

Sum of the quantities added

$= \begin{pmatrix} \text{Change in no. of quantities} \\ \times \\ \text{Original average} \end{pmatrix} + \begin{pmatrix} \text{Change in average} \\ \times \\ \text{Final no. of quantities} \end{pmatrix}$

$\Rightarrow$ weight of teacher = (1 × 48) + (0.5 × 30) = 63 kg.

$\therefore$ weight of teacher is 63 kg.

Example ***11:*** **The average age of 40 students in a class is 15 years. When 10 new students are admitted, the average is increased by 0.2 year. Find the average age of the new students.**

Solution :

Here, 10 new students are admitted.

∴ change in average is +ve.

Using the formula

Sum of the quantities added

$$= \begin{pmatrix} \text{Change in no. of quantities} \\ \times \\ \text{Original average} \end{pmatrix} + \begin{pmatrix} \text{Change in average} \\ \times \\ \text{Final no. of quantities} \end{pmatrix}$$

$\Rightarrow$ Sum of the weight of 10 new students admitted

$= (10 \times 15) + (0.2 \times 50) = 160\,\text{kg}$

$\therefore$ Average age of 10 new students $= \frac{s_a}{n_a} = \frac{160}{10} = 16$

$\therefore$ Average age of 10 new students is 16 years.

Example ***12:*** **Find the arithmetic mean of the sequence 1, 2, 3, ..., 100.**

Solution :

We have sum of first n natural numbers $= \frac{n}{2}(n+1)$,

here $n = 100$

$\Rightarrow \text{Sum} = \frac{100}{2} \times 101 = 101 \times 50$

$\Rightarrow \text{AM} = \frac{\text{Sum}}{100} = \frac{101 \times 50}{100} = 50.5$

Example ***13:*** **A sequence of seven consecutive integers is given. The average of the first five given integers is n. Find the average of all the seven integers.**

Solution :

Let the seven consecutive integers be

$x, x+1, x+2, ..., x+6$

The sum of the first five is

$x + x + 1 + x + 2 + x + 3 + x + 4 = 5x + 10$

The average of these five is $\frac{5x+10}{5} = x + 2 = n$

The average of the seven will be

$$\frac{5x+10+x+5+x+6}{7} = \frac{7x+21}{7} = x+3$$

As $x + 2 = n$, so $x + 3 = x + 2 + 1 = n + 1$

Example ***14:*** **The average of 11 results is 50. If the average of first six result is 49 and that of last six results is 52, find the sixth result.**

Solution :

Average of 11 results = 50

1 2 3 4 5 6 7 8 9 10 11

Average of last 6 results = 52

Average of first 6 results = 49

It is quite obvious that the sixth result is included twice. Once in the first six results and once again in the last six results.

∴ Value of the sixth result = (Sum of first six results) + (Sum of last six results) – Sum of 11 results

$= 6 \times 49 + 6 \times 52 - 11 \times 50 = 56$

Remember

- The average of first n natural numbers is $\frac{n+1}{2}$.
- The average of square of natural numbers till n is $\frac{(n+1)(2n+1)}{6}$.
- The average of cubes of natural numbers till n is $\frac{n(n+1)^2}{4}$.
- If n is odd: The average of n consecutive numbers is always the middle number.
- If n is even: The average of n consecutive numbers is always the average of the middle two numbers.
- If the average of n consecutive numbers is m, then the difference between the smallest and the largest number is $2(n-1)$.

AVERAGE SPEED FORMULA

If a person or a motor car covers three equal distances at the speed of x km/h, y km/h and z km/h, respectively, then for the entire journey average speed of the person or motor car is $\left(\frac{3xyz}{xy+yz+zx}\right)$ km/h.

If a person covers A km at a speed of x km/h, B km at a speed of y km/h and C km at a speed of z km/h, the average speed during the entire journey is $\left(\frac{A+B+C}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right)$ km / h

If a person covers Ath part of the distance at x km/h. Bth part of the distance at y km/h and the remaining Cth part at z km/h, then the average speed during the entire journey is

$$\left(\frac{1}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right) \text{km / h}$$

If a certain distance is covered at a kmph and an equal distance at b kmph, then the average speed during whole journey $= \frac{2ab}{a+b}$ km / h.

Example ***15:*** **A train covers the first 160 km at a speed of 120 km/h, another 160 km at 140 km/h and the last 160 km at 80 km/h. Find the average speed of the train for the entire journey.**

Solution :

$$\text{Average speed} = \frac{3xyz}{xy+yz+zx}$$

$$= \frac{3\times120\times140\times80}{120\times140+140\times80+80\times120}$$

$$= \frac{360\times140\times80}{16800+11200+9600} = \frac{4032000}{37600}$$

$$= 107\frac{11}{47} \text{ km/h}$$

Example ***16:*** **A person cover 9 km at a speed of 3 km/h, 25 km at a speed of 5 km/h and 30 km at a speed of 10 km/h. Find the average speed for the entire journey.**

Solution :

$$\text{The average speed} = \left(\frac{A+B+C}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right)$$

$$= \left(\frac{9+25+30}{\frac{9}{3}+\frac{25}{5}+\frac{30}{10}}\right)$$

$$= \frac{64}{11} = 5\frac{9}{11} \text{ km/h}$$

Example ***17:*** **A train covers 50% of the jouney at 30 km/h. 25% of the journey at 25 km/h and the remaining at 20 km/h. Find the average speed of the train during the entire journey.**

Solution :

The average speed

$$= \left(\frac{100}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right) = \left(\frac{100}{\frac{50}{30}+\frac{25}{25}+\frac{25}{20}}\right)$$

[Here, A = 50, B = 25 and C = 25]

$$= \frac{100}{47/12} = \frac{1200}{47}$$

$$= 25\frac{25}{47} \text{km/h}$$

Example ***18:*** **A motorist travels to a place 150 km away at an average speed of 50 km/hr and returns at 30 km/hr. calculate his average speed for the whole journey in km/hr.**

Solution :

$$\text{Average speed} = \frac{2ab}{a+b}\text{km/ hr} = \left(\frac{2\times50\times30}{50+30}\right)\text{km/ hr}$$

$$= 37.5 \text{ km/hr}$$

EXERCISE

1. Out of 40 boys in a class, average weight of 30 is 60 kg and the average weight of the remaining is 56 kg. The average weight (in kilogram) of the whole class is
   (a) 58.5 (b) 58
   (c) 57 (d) 59
2. The average of 20 numbers is zero. Of them, at the most, how many may be greater than zero?
   (a) 0 (b) 1
   (c) 10 (d) 19
3. The average age of A and B is 20 years. If C were to replace A, the average would be 19 and if C were to replace B, the averge would be 21. What are the age of A, B and C?
   (a) 22, 18, 20 (b) 20, 20, 18
   (c) 18, 22, 20 (d) None of these
4. 3 years ago the average age of a family of 5 members was 17 years. With the birth of a new baby, the average age of six members remains the same even today. Find the age of the new baby.
   (a) 1 year (b) 2 years
   (c) $1\frac{1}{2}$ years (d) cannot be determined
5. The average age of 36 students in a group is 14 years. When teacher's age is included to it, the average increases by one. What is the teacher's age in years?
   (a) 31 (b) 36
   (c) 51 (d) cannot be determined
6. A batsman in his 12th innings makes a score of 65 and thereby increases his average by 2 runs. What is his average after the 12th innings if he had never been 'not out'?
   (a) 42 (b) 43
   (c) 44 (d) 45
7. In the first 10 overs of a cricket game, the run rate was only 3.2. What should be the run rate in the remaining 40 overs to reach a target of 282 runs?
   (a) 6.25 (b) 6.50
   (c) 6.75 (d) 7.00
8. The average attendance in a school for the first 4 days of the week is 30 and for the first 5 days of the week is 32. The attendance on the fifth day is
   (a) 32 (b) 40
   (c) 38 (d) 36
9. Last year, a Home Appliance Store sold an average (arithmetic mean) of 42 microwave ovens per month. In the first 10 months of this year, the store has sold an average (arithmetic mean) of only 20 microwave ovens per month. What was the average number of microwave ovens sold per month during the entire 22 months period?
   (a) 21 (b) 30
   (c) 31 (d) 32
10. The captain of a cricket team of 11 players is 25 years old and the wicket-keeper is 3 years older. If the age of these two players are replaced by that of another two players, the average of the cricket team drops by 2 years. Find the average age of these two players.
    (a) 15 years (b) 15.5 years
    (c) 17 years (d) 16.5 years
11. A car owner buys petrol at ₹ 7.50, ₹ 8.00 and ₹ 8.50 per litre for three successive years. What is his average cost per litre of petrol if he spends ₹ 4000 each year ?
    (a) ₹ 8 (b) ₹ 9
    (c) ₹ 7.98 (d) ₹ 8.50
12. In Arun's opinion, his weight is greater than 65 kg but less than 72 kg. His brother does not agree with Arun and he thinks that Arun's weight is greater than 60 kg but less than 70 kg. His mother's view is that his weight cannot be greater than 68 kg. If all of them are correct in their estimation, what is the average of different probable weights of Arun?
    (a) 67 kg (b) 68 kg
    (c) 69 kg (d) None of these
13. The average weight of 45 students in a class is 52 kg. 5 of them whose average weight is 48 kg leave the class and other 5 students whose average weight is 54 kg join the class. What is the new average weight (in kg) of the class?
    (a) 52.6 (b) $52\frac{2}{3}$
    (c) $52\frac{1}{3}$ (d) None of these
14. The average monthly sales for the first eleven months of the year of a certain salesman were ₹ 12000, but due to his illness during the last month, the average monthly sales for the whole year came down to ₹ 11375. What was the value of sales during the last month?
    (a) 2100 (b) 875
    (c) 700 (d) 4500
15. The average mark of a class of n students is 64. When eight new students with an average mark of 73 join the class, the new average of the entire class is a whole number. Find the number of students now in the class, given that n lies between 25 and 60.
    (a) 36 (b) 28
    (c) 54 (d) 72
16. The average temperature for Monday, Tuesday and Wednesday was 55°, the average for Tuesday, Wednesday and Thursday was 60°, that for Thursday being 56°, what was the temperature on Monday?
    (a) 39° (b) 41°
    (c) 45° (d) None of these

17. In preparing a meal for 4 people, a housewife uses 600 grams of sprouts, 1 kg of potatoes, 1 cauliflower weighing 1/2 kg each and 700 grams of meat. If one quarter of the weight is lost in preparation and cooking, then what is the average weight in grams, of each person's meal ?
(a) 2100 (b) 875
(c) 700 (d) 525

18. A man travels a journey with average speed of 40 km/h and return back with average speed of 30 km/h. Find his over all average speed.
(a) 35 km/h (b) 40 km/h
(c) 30 km/h (d) 34.28 km/h

19. The respective ratio between the speed of a car, a train and a bus is 5 : 9 : 4. The average speed of the car the bus and the train is 72 km/h together. What is the average speed of the car and the train together?
(a) 82 km/h (b) 78 km/h
(c) 84 km/h (d) cannot be determined

20. The average of 5 consecutive numbers is n. If the next two numbers are also included, the average of the 7 numbers will
(a) increase by 2 (b) increase by 1
(c) remain the same (d) increase by 1.4

21. On a journey across Kolkata, a taxi averages 40 kmph for 60% of distance, 30 kmph for 20% of the distance, and 10 kmph for the remainder. The average speed of the whole journey is
(a) 25 kmph (b) 26 kmph
(c) 24 kmph (d) 30 kmph

22. A family consists of grandparents, parents and three grandchildren. The average age of the grandparents is 67 years, that of the parents is 35 years and that of the grandchildren is 6 years. What is the average age of the family?
(a) $28\frac{4}{7}$ years (b) $31\frac{5}{7}$ years
(c) $32\frac{1}{7}$ years (d) None of these

23. The average marks scored by 60 pupils in a class were 32.5. The marks of the first pupil were altered from 88 to 78, the marks of the last pupil were altered from 13 to 43. Then the new average will be
(a) 32.83 (b) 33.97
(c) 34.12 (d) 36.23

24. The average weight of 3 men A, B and C is 84 kg. Another man D joints the group and the average of the group becomes 80 kg. If another men E whose weight is 3 kg more than that of D replace A, then the average weight of B, C, D and E becomes 79 kg. Find the weight of A.
(a) 75 kg (c) 65 kg
(b) 55 kg. (d) 50 kg.

25. The average age of Mr. and Mrs Sinha at the time of their marriage in 1972 was 23 years. On the occasion of their anniversary in 1976, they observed that the average age of their family had come down by 4 years compared to their average age at the time of their marriage. This was due to the fact that their son Vicky was born during that period. What was the age of Vicky in 1980?
(a) 6 years (b) 7 years
(c) 8 years (d) 5 years

26. P is going to Delhi from Gurgaon by his car at a speed of 40 km/h. While coming back, he returns with a speed of x km/h. What should be the value of x so that his average speed during the entire journey is 80 km/h?
(a) 160 km/h (b) 40 km/h
(c) 120 km/h (d) It is not possible

27. In an exam, the average was found to be 50 marks. After deducting computational errors the marks of the 100 candidates had to be changed from 90 to 60 each and the average came down to 45 marks. The total number of candidates who took the exam were
(a) 3000 (b) 600
(c) 200 (d) 150

28. A company produces on an average 4000 items per month for the first 3 months. How many items it must produce on an average per month over the next 9 months, to average 4375 items per month over the whole ?
(a) 4500 (b) 4600
(c) 4680 (d) 4710

29. In an examination, a pupil's average marks were 63 per paper. If he had obtained 20 more marks for his Geography paper and 2 more marks for his History paper, his average per paper would have been 65. How many papers were there in the examination?
(a) 8 (b) 9
(c) 10 (d) 11

30. The average age of students of a class is 15.8 years. The average age of boys in the class is 16.4 years and that of the girls is 15.4 years. The ratio of the number of boys to the number of girls in the class is
(a) 1 : 2 (b) 2 : 3
(c) 3 : 4 (d) 3 : 5

31. Without any stoppage a person travels a certain distance at an average speed of 42 km/hr, and with stoppages he covers the same distance at an average speed of 28 km/hr. How many minutes per hour does he stop ?
(a) 14 minutes (b) 15 minutes
(c) 28 minutes (d) 20 minutes

32. The average score of 25 students in a class test in maths is 75. If the top three rankers are removed, the average drops by 2. If the paper is out of 100 marks, and the second highest score is less than 90, then what should be the least highest score, if all scores are integers ?
(a) 97 (b) 91
(c) 99 (d) 98

33. The average age of a team of 15 employees is 36. The youngest of them is 20 years old and the eldest is 56 years old. Two of them with average age 28 leave the team. If one of the two comes back on the condition that he will be made the team leader then which of the following can be the possible average age of the new team so formed?
(a) 35 (b) 36
(c) 38 (d) 39

34. The average of 10 numbers is 40.2. Later it is found that two numbers have been wrongly copied. The first is 18 greater than the actual number and the second number added is 13 instead of 31. Find the correct average.
(a) 40.2 (b) 40.4
(c) 40.6 (d) 40.8

35. I was born 30 years after my father was born. My sister was born 25 years after my mother was born. The average age of my family is 26.25 years right now. My sister will get married 4 years from now and will leave the family. Then the average age of the family will be $\frac{107}{3}$ years. What is the age of my father?
(a) 30 years (b) 35 years
(c) 40 years (d) 45 years

36. An aeroplane flies along the four sides of a square at a speed of 100, 200, 300 and 400 km/h, respectively. What is the average speed of the plane in its flight around the square?
(a) 196 km/h (b) 200 km/h
(c) 250 km/h (d) None of these

37. How much tea at ₹ 9 per kg must be mixed with 100 kg of superior tea at ₹ 13.50 per kg to give an average price of ₹ 11 per kg?
(a) 85 kg (b) 120 kg
(c) 125 kg (d) 130 kg

38. The population of a state increased from 100 million to 169 million in two decades. What is the average increase in population per decade?
(a) 20% (b) 34.5%
(c) 69% (d) 30%

39. A person invested part of ₹ 45000 at 4% and the rest at 6%. If his annual income from both are equal, then what is the average rate of interest?
(a) 4.6% (b) 4.8%
(c) 5.0% (d) 5.2%

40. Nine numbers are written in ascending order. The middle number is the average of the nine numbers. The average of the first five larger numbers is 68 and that of five smaller numbers is 44. What is the sum of all nine numbers?
(a) 450 (b) 501
(c) 504 (d) 540

41. In an examination, 40% of the candidates wrote their answers in Hindi and the others in English. The average marks of the candidates written in Hindi is 74 and the average marks of the candidates written in English is 77. What is the average marks of all the candidates?
(a) 75.5 (b) 75.8
(c) 76.0 (d) 76.8

42. The mean weight of 150 students in a certain class is 60 kg. The mean weight of the boys from the class is 70 kg, while that of girls is 55 kg. What is the number of girls in the class?
(a) 105 (b) 100
(c) 95 (d) 60

43. Out of 250 observations, the first 100 observations have mean 5 and the average of the remaining 150 observations is $\frac{25}{3}$. What is the average of the whole group of observations?
(a) 6 (b) 7
(c) 8 (d) 9

44. If the average of A and B is 30, the average of C and D is 20, then which of the following is/are correct?
I. The average of B and C must be greater than 25.
II. The average of A and D must be less than 25.
Select the correct answer using the codes given below.
(a) Only I (b) Only II
(c) Either I or II (d) Neither I or II

45. The price of a commodity increased by 5% from 2010 to 2011, 8% from 2011 to 2012 and 77% from 2012 to 2013. What is the average price increase (approximate) from 2010 to 2013?
(a) 26% (b) 32%
(c) 24% (d) 30%

46. The average weight of a class of 15 boys and 10 girls is 38.4 kg. If the average weight of the boys is 40 kg, then what is the average weight of the girls?
(a) 36.5 kg (b) 35 kg
(c) 36 kg (d) 34.6 kg

47. The average of m numbers is n^4 and the average of n numbers is m^4. The average of $(m + n)$ numbers is **(CDS)**
(a) mn (b) $m^2 + n^2$
(c) $mn(m^2 + n^2)$ (d) $mn(m^2 + n^2 - mn)$

48. The average weight of students in a class is 43 kg. Four new students are admitted to the class whose weights are 42 kg, 36.5 kg, 39 kg and 42.5 kg respectively. Now the average weight–of the students of the class is 42.5 kg, The number of students in the beginning was **(CDS)**
(a) 10 (b) 15
(c) 20 (b) 25

49. Four years ago, the average age of A and B was 18 years. Now the average age of A, B and C is 24 years. After 8 years, the age of C will be **(CDS)**
(a) 32 years (b) 28 years
(c) 36 years (d) 40 years

50. The weighted arithmetic mean of first 10 natural numbers whose weights are equal to the corresponding numbers is equal to **(CDS)**
(a) 7 (b) 14
(c) 35 (d) 38.5

HINTS & SOLUTIONS

1. (d) Average weight of 30 = 60 kg
⇒ Sum of weight of 30 boys = 1800
Average weight of 10 =56 kg
⇒ Sum of weight of 10 boys = 560
Average weight of the whole class

$$= \frac{\text{Sum of weight of all boys}}{40}$$

$$= \frac{\text{sum of weight of 30 boys + sum of weight of 10 boys}}{40}$$

$$= \frac{60\times30+56\times10}{40} = 59\text{kg}$$

2. (d) Average of 20 numbers = 0.
∴ Sum of 20 numbers = (0 × 20) = 0.
It is quite possible that 19 of these numbers may be positive and if their sum is a, then 20th number is (– a).

3. (a) Given A + B = 40 ...(i)
C + B = 38 ... (ii)
A + C = 42 ... (iii)
(i) + (ii) + (iii) ⇒ A + B + C = 60 ...(iv)
from (i) and (iv), we get
C = 20 years
∴ B = 18 years and A = 22 years

4. (b) Sum of present ages of the six members
= (17 × 6) years = 102 years.
Sum of present ages of the 5 members (excluding baby)
= 5 × (17 + 3) years = 100 years.
∴ Age of the baby = 102 – 100 = 2 years

5. (c) Age of the teacher = (37 × 15 – 36 × 14) years
= 51 years.

6. (b) Let 'x' be the average score after 12 th innings
⇒ 12 x = 11 × (x – 2) + 65
∴ x = 43

7. (a) Total runs in the first 10 overs
= 10 × 3.2 = 32
Run rate required in the remaining 40 overs

$$= \frac{282-32}{40} = \frac{250}{40} = 6.25 \text{ runs per over}$$

8. (b) Attendance on the fifth day = 32 × 5 – 30 × 4
= 160 – 120 = 40

9. (d) $\frac{42\times12+20\times10}{12+10} = \frac{504+200}{22} = \frac{704}{22} = 32$

10. (b) Let average of team = x years
Then, 25 + 28 + S_9 = 11x ... (i)
where S_9 is the sum of ages of remaining players
Also, Np + S_9 = 11(x – 2), ... (ii)
where Np is the sum of ages of new players
(i) – (ii) ⇒ 53 – Np = 22
⇒ Np = 31
∴ Average age of new two players $= \frac{31}{2} = 15.5$ years

11. (c) Let average cost of petrol per litre be ₹ x
Then $x = \frac{12000}{\frac{4000}{7.5}+\frac{4000}{8}+\frac{4000}{8.5}}$

$$= \frac{3}{\frac{2}{15}+\frac{1}{8}+\frac{2}{17}} = \frac{6120}{767} = ₹\ 7.98 \text{ per litre}$$

12. (a) Let Arun's weight be X kg.
According to Arun, 65 < X < 72.
According to Arun's brother, 60 < X < 70.
According to Arun's mother, X ≤ 68.
The values satisfying all the above conditions are 66 and 67.
∴ Required average $= \frac{66+67+68}{3} = 67\text{kg}.$

13. (b) Total weight of 45 students
= 45 × 52 = 2340 kg
Total weight of 5 students who leave
= 5 × 48 = 240 kg
Total weight of 5 students who join
= 5 × 54 = 270 kg
Therefore, new total weight of 45 students
= 2340 – 240 + 270 = 2370

⇒ New average weight $= \frac{2370}{45} = 52\frac{2}{3}\text{kg}$

14. (d) Total sales for the first eleven months
= 12,000 × 11 = ₹ 132000
Total sales for the whole year = ₹ 11375 × 12
= ₹ 136500
∴ Value of sales during the last month
= 136500 – 132000 = ₹4500

15. (b) Let 'x' be the increase in the average

$$\frac{64n+8\times73}{n+8} = 64 + x$$

$$\Rightarrow x = \frac{(73-64)8}{n+8} \Rightarrow x = \frac{9\times8}{n+8}$$

For 'x' to be a whole number 72 (= 9 × 8) should be divisible by (n + 8)
From the choices it can be said that 36 and 72 are two such factors. But 72 does not lie within the range.
∴ n + 8 = 36
n = 28
∴ number of students in class are 28.

16. (b) Sum of temp. for Monday, Tuesday and Wednesday
$= 55 \times 3 = 165°$
Sum of temp. for Tuesday, Wednesday, an Thursday
$= 60 \times 3 = 180°$
Since temp. on Thursday $= 56°$
$\Rightarrow$ Temp for Tue and Wed $= 180 - 56 = 124°$
$\Rightarrow$ Temp of Monday $= 165 - 124 = 41°$

17. (d) Sprouts 600 gms + Potatoes 1000 gms + Cauliflower 500 gms + Meat 700 gms = 2800 gms.
Hence total cooked food = 2100 gms and is served among 4 people.
Thus average weight is 525 grams.

18. (d) Average speed $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

$$= \frac{D+D}{\frac{D}{40}+\frac{D}{30}} = 34.28$$

19. (c) Total speed of car, bus and train $= 72 \times 3 = 216$ km

Speed of car & train $= \dfrac{5+9}{5+9+4} \times 216 = 168$ km/h

Average $= \dfrac{168}{2} = 84$ km/h

20. (b) Let the numbers be $n-2, n-1, n, n+1$ and $n+2$.
Their average $= n$.
Next two consecutive numbers are $n+3$ and $n+4$.
Therefore the average of 7 consecutive numbers

$$= \frac{(n-2)+(n-1)+n+(n+1)+(n+2)+(n+3)+(n+4)}{7}$$

$$= \frac{5n+2n+7}{7} = n+1$$

Hence, the average of 7 numbers will increase by 1.

21. (c) Let total distance be d.

time taken for 60% distance $= \dfrac{0.6d}{40} = \dfrac{3d}{200}$ h

time taken for 20% distance $= \dfrac{0.2d}{30} = \dfrac{d}{150}$ h

time taken for remaining 20% distance

$= \dfrac{0.2d}{10} = \dfrac{d}{50}$ h

Average speed $= \dfrac{d}{\frac{3d}{200}+\frac{d}{150}+\frac{d}{50}}$

$= \dfrac{200 \times 150 \times 50}{22500+10000+30000} = \dfrac{200 \times 150 \times 50}{62500}$

$= 24$ kmph

22. (b) Required average $= \left(\dfrac{67 \times 2 + 35 \times 2 + 6 \times 3}{2+2+3}\right)$

$= \left(\dfrac{134+70+18}{7}\right) = \dfrac{222}{7} = 31\dfrac{5}{7}$ years

23. (a) Total marks = 1950,
New total $= 1950 + (78-88) + (43-13) = 1970$

Hence average $= \dfrac{1970}{60} = 32.83$

24. (a) Therefore sum of weights of 3 men
$= 3 \times 84 = 252$ kg (i.e. A, B, C).
Now, D joins the group.
Sum of weights of 4 men (A, B, C, D) $= 252 + D = 80 \times 4$.
Therefore $D = 80 \times 4 - 252$.
Therefore wt of man D = 68 kg, weight of man E $= 68 + 3 = 71$ kg.
Sum of weight of B, C, D, E $= 79 \times 4 = 316$ kg
i.e. $252 + 68 + 71 - A = 316$.
Therefore, $A = 391 - 316 = 75$ kg.

25. (b) Sum of ages of Mr. and Mrs. Sinha in 1972 = 46 years
Sum of age of their family in 1976 $= 19 \times 3 = 57$ years
Sum of ages of Mr. and Mrs. Sinha in 1976 = (46 + 8) years
= 54 years
$\therefore$ Age of Vicky in 1980 = 57 – 54 + 4 = 7 years.

26. (d) $80 = \dfrac{2.x.40}{40+x}$
$40 + x = x$
Hence, not possible

27. (b) Let the total number of candidates $= x$

$\therefore \quad \dfrac{50x - 100(90-60)}{x} = 45$

$\therefore \quad x = 600$

28. (a) Required average $= \dfrac{(4375 \times 12) - (4000 \times 3)}{9}$

$= \dfrac{52500 - 12000}{9} = \dfrac{40500}{9} = 4500$

29. (d) Let the number of paper be x. Then, $63x + 20 + 2 = 65x$ or $2x = 22$ or $x = 11$.

30. (b) Let the ratio be k : 1. Then,
$k \times 16.4 + 1 \times 15.4 = (k+1) \times 15.8$
$\Rightarrow$ $(16.4 - 15.8)k = (15.8 - 15.4)$
$\Rightarrow$ $k = \dfrac{0.4}{0.6} = \dfrac{2}{3}$.

$\therefore$ Required ratio $= \dfrac{2}{3} : 1 = 2 : 3$.

31. (d) 42 km takes 60 minutes.

Then, 28 km will take $\dfrac{60}{42} \times 28 = 40$ min.

$\therefore$ Average stoppage time $= 60 - 40 = 20$ min.

32. (b) Total score $= 25 \times 75 = 1875$
Total score without the top three rankers
$= (25-3) \times (75-2) = 22 \times 73 = 1606$
$\therefore$ Total of top three scores $= 1875 - 1606 = 269$
$\therefore$ 2nd and 3rd highest scores could be at most 89 each
$\therefore$ The top score should be atleast $= 269 - 2 \times 89 = 91$

33. (b) Total age = 15×36
After the two left, total age = $15 \times 36 - 2 \times 28$
$= 540 - 56 = 484$ years.
Let the age of the person who returns be x years.
Then new age of the person who returns be x years.

Then new average = $\frac{484 + x}{14}$

Now x lies between 20 and 36 (both inclusive)

$\therefore$ New average min $\frac{484 + 20}{14} = \frac{520}{14} = 36$ years.

And New average max = $\frac{484 + 36}{14} = \frac{520}{14}$

$= 37.14$ years.

34. (a) Sum of 10 numbers = 402
Corrected sum of 10 numbers
$= 402 - 13 + 31 - 18 = 402$

Hence, correct average $= \frac{402}{10} = 40.2$

35. (d) Let present age of father = x year
and present age of mother = y year
therefore present age of son = $(x - 30)$
and present age of daughter = $(y - 25)$
Sum of their ages = $4 \times 26.25 = 105$ years
i.e., $x + y + x - 30 + y - 25 = 105$
$2x + 2y = 160$
$x + y = 80$ ...(1)
After 4 years, their total ages will be (excluding the daughter)
$x + 4 + y + 4 + x - 30 + 4 = 107$
$2x + y - 18 = 107$
$2x + y = 125$ ...(2)
Solving (1) and (2)
$x = 45$

36. (d) Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

$= \frac{x + x + x + x}{\frac{x}{100} + \frac{x}{200} + \frac{x}{300} + \frac{x}{400}}$

$= \frac{4x}{\frac{12x + 6x + 4x + 3x}{1200}}$

$= \frac{4x \times 1200}{25x} = 192$ km/h

37. (c) Let x kg of tea of ₹ 9 per kg.

$\frac{9 \times x + 13.5 \times 100}{x + 100} = 11$

$\Rightarrow 9x + 1350 = 11x + 1100$

$\Rightarrow 2x = 250$

$\therefore x = 125$ kg

38. (b) The difference of population in two decades
$= 169 - 100 = 69$ million

$\therefore$ Increas in population in first decade =

$\frac{69\%}{2} = 34.5\%$

39. (b) Let a person invest 4% of x.
According to question

$\therefore \frac{x \times 4}{100} = \frac{(45000 - x)}{100} \times 6$

$\Rightarrow 2x = 45000 \times 3 - 3x$

$\Rightarrow x = \frac{45000 \times 3}{5} = ₹\ 27000$

Another part is ₹ 18000.
Let r = Average rate of interest

Interest for 1st part in one year = $\frac{27000 \times 4}{100} = ₹\ 1080$

Similarly, interest for rest part in one year = 1080
$\because$ Total interest = ₹ 2160

$\therefore \frac{45000 \times r}{100} = 2160$

$\Rightarrow r = \frac{216}{45} = 4.8\%$

40. (c) Since, 5th term = average of 9 numbers = x
Sum of first five larger numbers = $68 \times 5 = 340$
Sum of first five smaller numbers = $44 \times 5 = 220$

Average of five numbers = $\frac{340 + 220 - x}{9}$

(since, x is subtracted because 5th term repeated twice)

$\therefore x = \frac{560 - x}{9}$

$\Rightarrow 9x + x = 560 \Rightarrow x = 56$

$\therefore$ Sum of 9 numbers = $56 \times 9 = 504$

41. (b) Let total number of candidates = 100
$\therefore$ Total marks of 40 candidates = 40×74
Total marks of 60 candidates = 60×77

$\therefore$ Required average marks = $\frac{40 \times 74 + 60 \times 77}{100}$

$= \frac{2960 + 4620}{100} = \frac{7580}{100} = 75.80$

42. (b) Let the no. of boys = x
No. of girls = $150 - x$

Average weight = $\frac{70x + (150 - x) \times 55}{150}$

$\Rightarrow 60 = \frac{70x + (150 - x) \times 55}{150}$

$\Rightarrow$ n = 50
No. of boys $(x) = 50$
No. of girls = $150 - x$
$= 150 - 50 = 100$

43. (b) Given, $n_1 = 100$, $\bar{x}_1 = 5$ and $n_2 = 150$, $\bar{x}_2 = \frac{25}{3}$

Average of whole group of observations

$$\bar{x}_1 = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

$$= \frac{100\times 5 + 150\times\frac{25}{3}}{100+150} = \frac{1750}{250} = 7$$

44. (d) Average of A and $B = 30$

$$\Rightarrow \frac{A+B}{2} = 30 \Rightarrow A + B = 60$$

Average of C and $D = 20$

$$\Rightarrow \frac{C+D}{2} = 20 \Rightarrow C + D = 40$$

Here, we can't find the avg of B and C, A and D so neither I nor II are is correct.

45. (d) Average price increase $= \left(\frac{5+8+77}{3}\right)\%$

$$= \frac{90}{3}\% = 30\%$$

46. (c) Let average weight of girls $= x$

Total weight of the boys = 40 kg × 15 = 600 kg.

Average weight

$$= \frac{\text{Total weight of girls + Total weight of boys}}{\text{No. of boys + No. of girls}}$$

$$\Rightarrow 38.4 = \frac{600 + 10\times x}{15+10}$$

$$\Rightarrow 38.4 = \frac{600+10x}{25}$$

$$\Rightarrow 38.4 \times 25 = 600 + 10x$$

$$\therefore x = 36 \text{ kg}$$

47. (d) Sum of 'm' numbers $= m.n^4$

Sum of 'n' numbers $= n.m^4$

Sum of 'm + n' numbers $= mn^4 + nm^4$

Average of 'm + n' numbers $= \frac{mn^4 + nm^4}{(m+n)}$

$$= \frac{mn\left(n^3 + m^3\right)}{(m+n)}$$

$$= \frac{mn(m+n)\left(m^2 + n^2 - mn\right)}{(m+n)}$$

$$= mn(m^2 + n^2 - mn)$$

48. (c) Let the no. of students in the beginning be x.

According to question-

$$\Rightarrow \frac{43x + 42 + 36.5 + 38 + 42.5}{(x+4)} = 42.5$$

$$\Rightarrow \frac{43x + 160}{(x+4)} = 42.5$$

$$\Rightarrow 43x + 160 = 42.5x + 170$$

$$\Rightarrow 0.5x = 10$$

$$\Rightarrow x = 20$$

49. (c) Let their age be A, B and C respectively.

$$\Rightarrow \frac{A+B+C}{3} = 24$$

$$\Rightarrow A + B + C = 72 \quad \text{--------(i)}$$

$$\Rightarrow \frac{A-4+B-4}{2} = 18$$

$$\Rightarrow \frac{A+B-8}{2} = 18$$

$$A + B = 44 \quad \text{--------(ii)}$$

eq(i) – eq (ii) -

$$(A+B+C) - (A+B) = 72 - 44$$

$$C = 28 \text{ years}$$

After 8 years age of C

$$= 28 + 8 = 36 \text{ years}$$

50. (a) Weighted arithmetic mean

$$= \frac{1\times1 + 2\times2 + 3\times3 + 4\times4 + ----- + 10\times10}{1+2+3+----10}$$

$$= \frac{(1)^2 + (2)^2 + (3)^2 + ----- + (10)^2}{1+2+3+----10}$$

$$= \frac{(10)(10+1)(2\times10+1)/6}{(10\times11)/2}$$

$$= \frac{10\times11\times21\times2}{6\times10\times11}$$

$$= 7$$