Chapter 1

Elements of Vector Calculus and Static Fields

CHAPTER HIGHLIGHTS

INTRODUCTION

Vector analysis is a mathematical tool with which electromagnetic concepts are most conveniently expressed and best comprehended.

A quantity can either be scalar or vector.

Vector: It is a quantity that is characterized by both magnitude and direction such as electric and magnetic field. For example, force, velocity, electric field intensity, and electric flux density.

Scalar: It is a quantity that is characterized only by magnitude. For example, time, mass, temperature, entropy, electric potential, and population of a country.

Field: It is a function that specifies a particular quantity everywhere in the region. If the quantity is scalar (or vector), the field is said to be a scalar (or vector) field. For example, scalar fields and vector fields.

Scalar fields:

1. Temperature distribution in a building.
2. Electric potential in a region.

Vector fields:

Gravitational force on a body in space.

Unit vector: A vector P has both magnitude and direction. Magnitude of P is a scalar written as $|P|$. Unit vector a_p along P is defined as a vector whose magnitude is unity (i.e., 1) and its direction is along p

$$\text{i.e., } a_p = \frac{p}{|p|}$$

$$|a_p| = \frac{|p|}{|p|} = 1$$

Thus, $\bar{p}$ can be written as

$$\bar{p} = |P| a_p$$

$|P|$ = magnitude of vector $\bar{p}$

a_p – unit vector along $\bar{p}$

A vector $\bar{p}$ in Cartesian coordinated system can be represented as (p_x, p_y, p_z)

(or)

$P_x a_x + P_y a_y + P_z a_z P_x, P_y, P_z$ are components of $\bar{p}$ in the x, y, z directions, respectively.

a_x, a_y, and a_z, are unit vectors along x, y, and z directions, respectively.

$$|p| = \sqrt{p_x^2 + p_y^2 + p_z^2}$$

$$a_p = \frac{\bar{p}}{|p|} = \frac{p_x a_x + p_y a_y + p_z a_z}{\sqrt{p_x^2 + p_y^2 + p_z^2}}$$

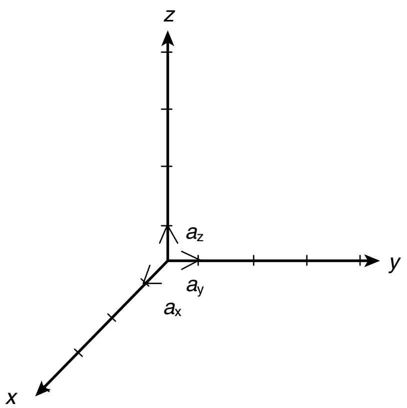


Figure 1 Unit vectors a_x, a_y and a_z are shown.

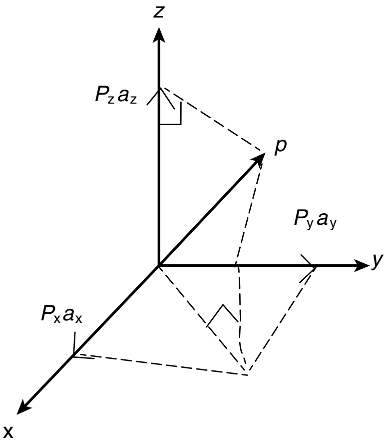


Figure 2 Components of p along the co-ordinate axis.

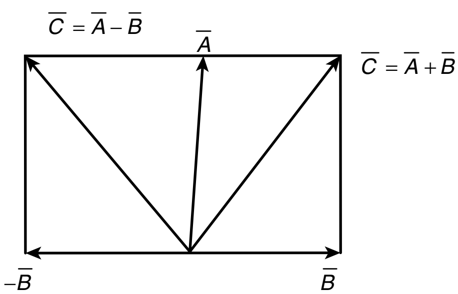
Unit vectors a_x, a_y, and a_z are shown in Figure 1 and the components of p along the coordinates axis are shown in Figure 2.

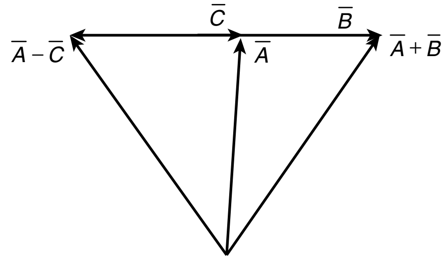
Vector Algebra

1. **Addition and subtraction:**
   Two vectors $\bar{A}$ and $\bar{B}$ can be added or subtracted together to give another vector $\bar{C}$,
   i.e., $\bar{C} = \bar{A} \pm \bar{B}$
   The abovementioned two operations on vectors are carried out component by components

   $$\bar{C} = (A_x \pm B_x)a_x + (A_y \pm B_y)a_y + (A_z \pm B_z)a_z$$

   Graphically, vector addition and subtraction are obtained by either the parallelogram rule or head-to-tail rule as follows:





Law	Addition	Multiplication
Commutative	$\bar{A}+\bar{B}=\bar{B}+\bar{A}$	$K\bar{A}=\bar{A}K$
Associative	$\bar{A}+(\bar{B}+\bar{C})=(\bar{A}+\bar{B})+\bar{C}$	$K(\ell\bar{A})=(K\ell)\bar{A}$
Distributive	$k(\bar{A}+\bar{B})=k\bar{A}+k\bar{B}$	

where k and l are scalars.

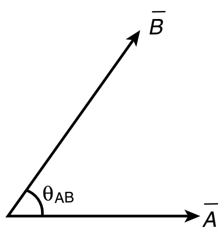
The three basic laws of algebra obeyed by any given vectors A, B, and C.

2. **Vector multiplication:**
   When two vectors are multiplied, the resultant is either a scalar or a vector depending on their multiplication.

   1. Scalar (or dot) product: $A.B$
   2. Vector (or cross) product: $A \times B$

3. **Dot product:**
   The dot product of two vectors A and B is defined as the product of magnitudes of two given vectors A and B and the cosine of the angle between them.

   $$\therefore \quad \bar{A}.\bar{B} = |\bar{A}||\bar{B}| \cos \theta_{AB}$$



θ_{AB} – angle between $\bar{A}$ and $\bar{B}$

if $\quad A = A_x a_x + A_y a_y + A_z a_z$

$\quad B = B_x a_x + B_y a_y + B_z a_z$

Two vectors are said to be orthogonal with each other, if their scalar (or dot) product is zero.

Scalar (or dot) product obeys the following:

1. **Commutative**:

   $$\bar{A}.\bar{B} = \bar{B}.\bar{A}$$

2. **Distribution**:

   $$\bar{A}.(\bar{B}+\bar{C}) = \bar{A}.\bar{B} + \bar{A}.\bar{C}$$

   $$\bar{A}.\bar{A} = |A|^2$$

3. $a_x \cdot a_x = a_y \cdot a_y = a_z . a_z = 1$
$a_x . a_y = a_y \cdot a_z = a_z . a_x = 0$

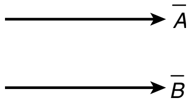
Depending on the angle between two vectors A and B

1. if $\bar{A}$ and $\bar{B}$ are parallel

$$\bar{A}.\bar{B} = |A||B|\cos\theta_{AB}$$

$$\Rightarrow \theta_{AB} = 0$$

Therefore, $\bar{A}.\bar{B} = |A||B|$

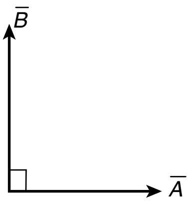


2. If $\bar{A}$ and $\bar{B}$ are perpendicular,

$$\theta_{AB} = \pi/2 \quad \therefore \cos_{AB} = 0$$

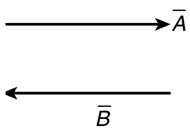
$$\bar{A}.\bar{B} = |A||B|\cos\theta_{AB} = 0$$

$$\bar{A}.\bar{B} = 0$$



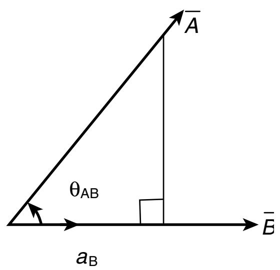
3. if $\bar{A}$ and $\bar{B}$ are opposite in direction:
$\theta = \pi$, $\cos\theta = -1$

$$\bar{A}.\bar{B} = |A||B|\cos\theta_{AB} = -|A||B|$$



Geometrical Interpretation of Dot Product

The dot product of two vectors $\bar{A}$ and $\bar{a}_B$ given the length of projection of $\bar{A}$ along the direction of $\bar{B}$.
where a_B is the unit vector along $\bar{B}$.



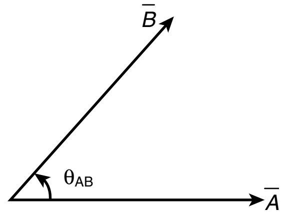
$$\bar{A}.\bar{a}_B = |\bar{A}|.\cos\theta_{AB}$$

Vector Product or Cross Product

The cross product of two vectors $\bar{A}$ and $\bar{B}$ is written as $\bar{A}\times\bar{B}$. It is a vector quantity whose magnitude is equal to the product of magnitude of two vectors and sine of the angle between them,

$$|\bar{A}\times\bar{B}| = |A||B|\sin\theta_{AB}$$

The direction of $A \times B$ is perpendicular to the plane containing $\bar{A}$ and $\bar{B}$ and is in the direction of advance of a right-handed screw as $\bar{A}$ is turned into $\bar{B}$.

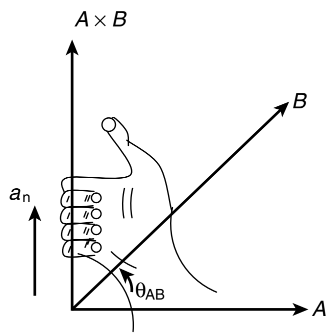


Direction of $\bar{A}\times\bar{B}$ can be determined using right-hand thumb rule.

Right-hand Thumb Rule

$$\bar{A}\times\bar{B} = |A||B|\sin\theta_{AB} . \hat{a}_n$$

The direction of a_n is taken as the direction of the right thumb when the fingers of right hand rotate from $\bar{A}$ to $\bar{B}$.



If $\bar{A} = A_x a_x + A_y a_y + A_z a_z$

$\bar{B} = B_x a_x + B_y a_y + B_z a_z$

$$\bar{A}\times\bar{B} = \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y)\, a_x + (A_z B_x - A_x B_z)\, a_y + (A_x B_y - A_y B_x)\, a_z$$

Properties

1. It is anti-commutative

$$\bar{A}\times\bar{B} \neq \bar{B}\times\bar{A}$$

$$\bar{A}\times\bar{B} = -\bar{B}\times\bar{A}$$

2. It is not associative

$$\bar{A} \times (\bar{B} \times \bar{C}) \neq (\bar{A} \times \bar{B}) \times \bar{C}$$

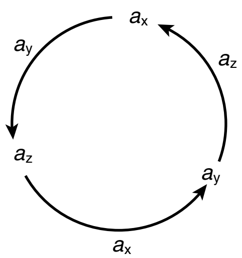
3. It is distributive:

$$\bar{A} \times (\bar{B} \times \bar{C}) = (\bar{A} \times \bar{B}) \times \bar{C}$$

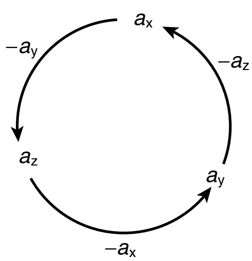
4. $\bar{A} \times \bar{A} = 0$
   and

$$a_x \times a_y = a_z$$
$$a_y \times a_z = a_x$$
$$a_z \times a_x = a_y$$

right-handed coordinate system is in which right-hand thumb rule or right-hand screw rule is satisfied.



Right-hand system: clockwise leads to positive results.

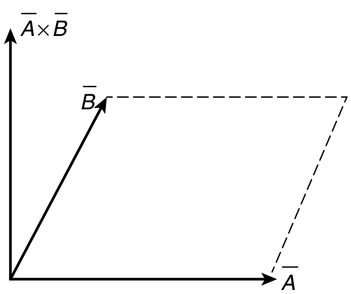


Anticlockwise leads to negative results.

$$a_y \times a_x = -a_z$$
$$a_z \times a_y = -a_x$$
$$a_x \times a_z = -a_y$$

Geometrical Interpretation of Cross Product

The magnitude of vector product of two vectors is the area of the parallelepiped formed by $\bar{A}$ and $\bar{B}$.



Coordinate System

The spatial variations of fields should be defined uniquely in space in a suitable manner. This needs the appropriate coordinate system.

Orthogonal system is one in which the coordinates are mutually perpendicular. Non-orthogonal systems are hard do work but of little practical use.

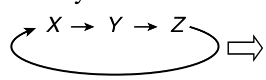
The best three orthogonal coordinate systems are

1. Cartesian coordinate system
2. Cylindrical coordinate system
3. Spherical coordinate system

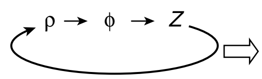
These coordinate systems are orthogonal, orthonormal, and right-handed systems.

1. Orthogonality means dot product of any two different unit vectors of same system is zero and dot product of any two same unit vectors is one.
2. Orthonormality means cross products of any two different unit vectors is the third unit vector.

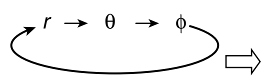
Right-handed system follows for orthonormality.

(A) 

$$a_x \times a_y = a_z$$
$$a_y \times a_z = a_x$$
$$a_z \times a_x = a_y$$

(B) 

$$a_\rho \times a_\phi = a_z$$
$$a_\phi \times a_z = a_\rho$$
$$a_z \times a_\rho = a_\phi$$

(C) 

$$a_r \times a_\theta = a_\phi$$
$$a_\theta \times a_\phi = a_r$$
$$a_\phi \times a_r = a_\theta$$

Cartesian Coordinate System

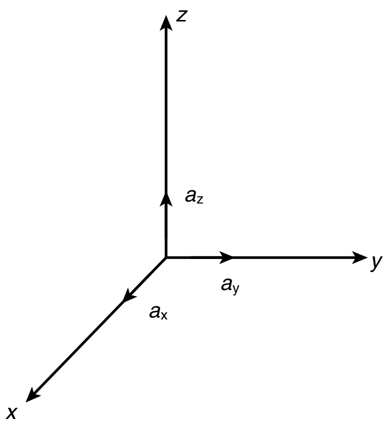
Range of coordinate variables:

$$-\infty < x < \infty$$
$$-\infty < y < \infty$$
$$-\infty < z < \infty$$

Vector $\bar{A}$ can be written as

$$\bar{A} = A_x a_x + A_y a_y + A_z a_z$$

a_x, a_y, and a_z are unit vectors along x, y, and z directions.



Cylindrical Coordinate System

1. A point P in cylindrical system is represented as (ρ, ϕ, z)
   Range of values

$$0 \leq \rho < \infty$$
$$0 \leq \phi < 2\pi$$
$$-\infty < z < \infty$$

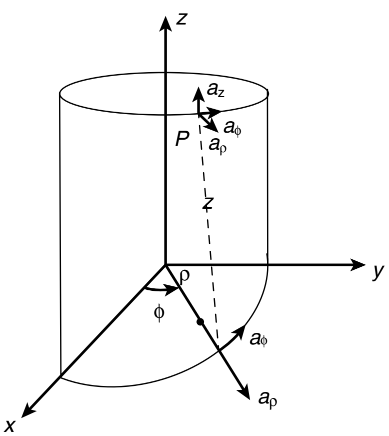
where ρ is the radius of cylinder; ϕ is the Azimuthal angle measured with respect to x-axis in xy plane; and z is same as in Cartesian coordinate system.

Vector $\overline{A}$ in cylindrical coordinate system

$$\overline{A} = A_\rho a_\rho + A_\phi a_\phi + A_z a_z$$

$$|A| = \sqrt{A_\rho^2 + A_\phi^2 + A_z^2}$$

where a_ρ is the unit vector along the direction of increasing 'ρ'; a_ϕ is the unit vector along the direction of increasing 'ϕ'; and a_z is the unit vector along positive z-direction.



Relationships between Cartesian and cylindrical systems

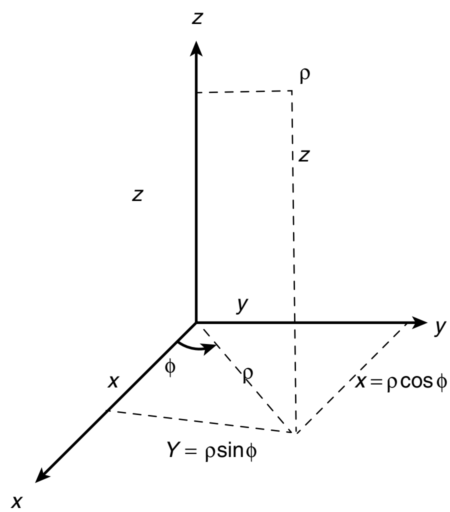
$$x = \rho \cos \phi$$
$$y = \rho \sin \phi$$
$$z = z$$

and

$$\rho = \sqrt{x^2 + y^2}, \; 0 \leq \rho < \infty$$
$$\phi = \tan^{-1} y/x, \; 0 \leq \phi < 2\pi$$
$$z = z, \; 0 \leq z < \infty$$

Dot products of a_x, a_y, and a_z with a_ρ, a_ϕ, and a_z are given by

$$a_x.a_\rho = \cos\phi$$
$$a_x.a_\phi = -\sin\phi$$
$$a_y.a_\rho = \sin\phi$$
$$a_y.a_\phi = \cos\phi$$
$$a_z.a_\rho = 0$$
$$a_z.a_\phi = 0$$



Unit Vector Transformation

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix}$$

$$\begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

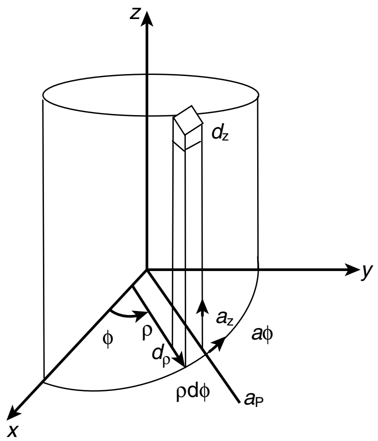
Vector Transformation

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

The cylindrical coordinate system is convenient for problem having cylindrical symmetry. For example, line charges and current wires.

Infinite small strip of cylinder is shown in figure. Differential length



$$d\ell = (d\rho)a_\rho + \rho\,(d\phi)a_\phi + (dz)a_z$$

Differential normal area:

$$dl = (dp)ap + \rho d\phi\, a_\phi + dz\, a_z$$

$$ds = (\rho d\phi)(dz)a_\rho + (d\rho)(dz)a_\phi + (d\rho)(\rho d\phi)\, a_z$$

Differential volume:

$$dv = (d\rho)\,(\rho d\phi)(dz)$$

$$dv = \rho(d\rho)\,(d\phi)(dz)$$

Spherical Coordinate System

It is the most appropriate with problems having a degree of spherical symmetry. A point is obtained by the intersection of these surfaces, namely

a spherical surface, $r = k$ (constant), meter.

a cone, $\theta = \alpha$ (constant), radian, and

a plane, $\phi = \beta$ (constant), radian

All these three surfaces are mutually perpendicular to each other.

These are said to be orthogonal

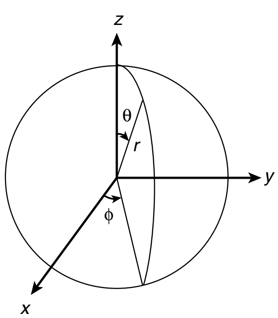


Figure 3 Spherical co-ordinates.

A point P can be represented as (r, θ, ϕ), where

r is the radius of sphere or length of line joining origin and P; θ is the angle made by the position vector $\overline{OP}$ with respect to positive z-axis;

and ϕ is same as in cylindrical coordinate system.

$$0 \le r < \infty$$
$$0 \le \theta \le \pi$$
$$-0 \le \phi \le 2\pi$$

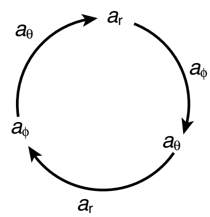
A vector $\overline{A}$ can be written as

$$A_r a_r + A_\theta a_\theta + A_\phi a_\phi$$

a_r, a_θ, and a_ϕ are unit vectors along r, θ_*, and ϕ directions, respectively, and mutually orthogonal to each other and form R.H.S

$$|\overline{A}| = \sqrt{A_r^2 + A_\theta^2 + A_\phi^2}$$

$$a_r \times a_\theta = a_\phi$$
$$a_\theta \times a_\phi = a_r$$
$$a_\phi \times a_r = a_\theta$$



$$a_r . a_r = 1 = a_\theta . a_\theta = a_\phi . a_\phi$$
$$a_r . a_\theta = a_\theta . a_\phi = a_\phi . a_r = 0$$

Relationship between space variables (x, y, z), (r, θ, ϕ) and (ρ, ϕ, z)

The variables of Cartesian and spherical coordinates are related by

$$x = r \sin\theta \cos\phi, \; -\infty < x < \infty$$
$$y = r \sin\theta \sin\phi, \; -\infty < y < \infty$$
$$z = r \cos\theta, \; -\infty < z < \infty$$

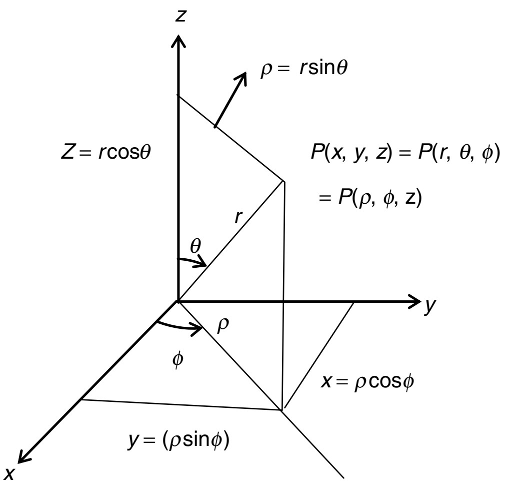
and $$r = \sqrt{x^2 + y^2 + z^2}\;, 0 \le r \le \infty$$

$$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}}\;, 0 \le \theta \le \pi$$

$$\phi = \tan^{-1} \frac{y}{x}, \; 0 \le \phi \le 2\pi$$

The relationship between the variables of cylindrical and spherical coordinates are given by

$$\rho = r \sin\theta$$
$$\phi = \phi$$
$$z = r\cos\theta$$
$$r = \sqrt{\rho^2 + z^2}$$
$$\theta = \tan^{-1} \frac{\rho}{z}$$
$$\phi = \phi$$



Relationship between unit victors and vectors of Cartesian and spherical coordinate system:

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix}$$

$$\begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

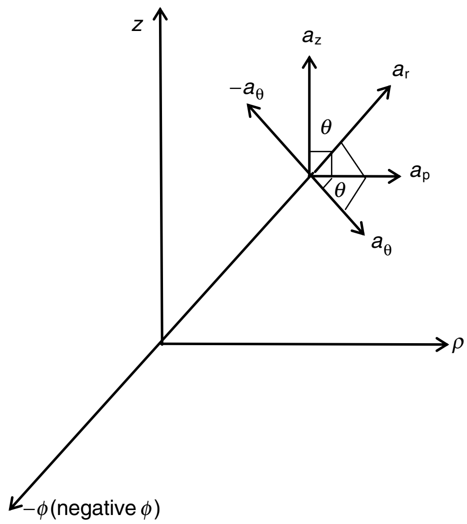
Cylindrical and spherical

$$\begin{bmatrix} a_\rho \\ a\phi \\ a_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ +\cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix}$$

$$\begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ +\cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$



Differential displacement is

$$dl = (dr)a_r + (rd\theta)a_\theta + (r\sin\theta d\phi)a_\phi$$

$$ds = (r^2 \sin\theta d\theta d\phi)a_r + (r \sin\theta dr d\phi)a_\theta + (r\, dr d\theta)a_\phi$$

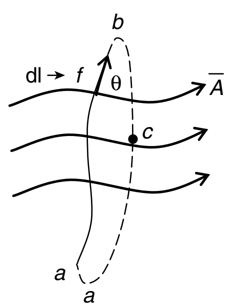
$$dv = r^2 \sin\theta dr d\theta d\phi$$

Line, Surface and Volume Integrals

Line Integral

$\int_L \bar{A}.d\ell$ is the integral of tangential component of $\bar{A}$ along curve L

$$\int_L \bar{A}d\ell = \int_a^b |A| \cos\theta\, d\ell$$

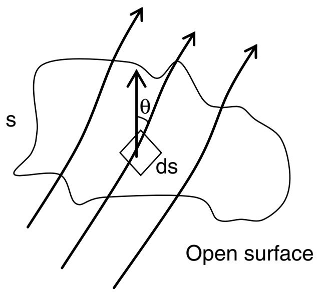


If the path of integration is closed, then it becomes a closed integral $\oint_L \bar{A}.dl$

Surface Integral or Flux

$$\psi = \oint_s \bar{A}ds$$

$$\overline{ds} = |ds| a_n$$

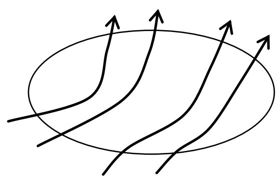


An unit vector normal to surface s.

For a closed surface, surface integral becomes $\oint_s \bar{A}.ds$. It is the net outward flux of $\bar{A}$ from s.

NOTE

1. closed path defines an open surface.
2. closed surface integral is same as volume integral of ρ_v over volume V.



Del operator:

Del operator written as ∇ and also called as vector deferential operator or gradient operator. When it operates on scalar, it gives a vector.

Cartesian:

$$\nabla = \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z$$

Cylindrical:

$$\nabla = a_\rho \frac{\partial}{\partial \rho} + a_\phi \frac{1}{\rho} \cdot \frac{\partial}{\partial \phi} + a_z \cdot \frac{\partial}{\partial z}$$

Spherical:

$$\nabla = a_r \frac{\partial}{\partial r} + a_\theta \frac{1}{r}\frac{\partial}{\partial \theta} + a_\phi \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}$$

∇ is useful in defining

1. Gradient of a scalar ∇V.
2. Divergence of a vector $\nabla . \bar{A}$.
3. Curl of a vector $\nabla \times \bar{A}$.
4. Laplacian of a scalar $\nabla^2 V$.

Gradient of a Scalar (∇V)

The gradient of a scalar field V is a vector that has its magnitude and direction as those of the maximum rate of change of 'V'.

If $A = \nabla V$, V is said to be the scalar potential of $\bar{A}$.

Cartesian:

$$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial y} . a_z$$

Cylindrical:

$$\nabla V = \frac{\partial V}{\partial \rho} . a_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi} . a_\phi + \frac{\partial V}{\partial Z} . a_z$$

Spherical:

$$\nabla V = \frac{\partial V}{\partial r} a_r + \frac{1}{r}\frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} a_\phi$$

Divergence of a Vector ($\nabla . \bar{A}$)

Divergence of a vector $\bar{A}$ is the net outward flow of flux per unit volume over a closed incremental surface.

$$\nabla . \bar{A} = \left(\frac{\partial}{\partial x} a_x + \frac{\partial}{\partial_y} a_y + \frac{\partial}{\partial z} a_z\right)(A_x\, a_x + A_y\, a_y + A_z\, a_z)$$

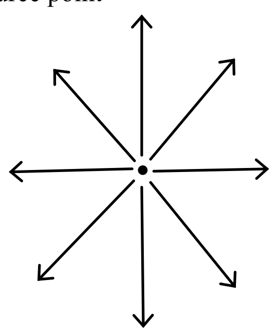
$$\nabla . \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla . \bar{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho A \rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

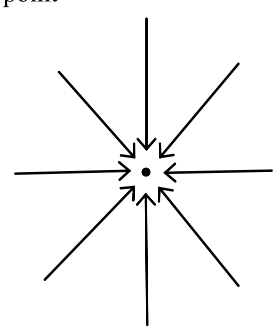
$$\nabla . \bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(A_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

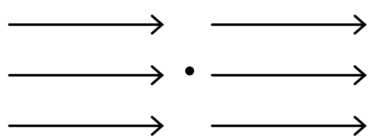
Physically, the divergence of a vector field at a given point is the measure of how much field emanates from that point.

Positive at source point



Negative at sink point





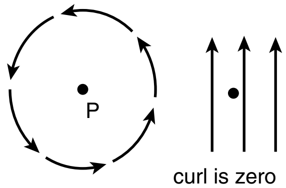
If $\nabla . \bar{A} = 0$, then $\bar{A}$ is said to be solenoidal.

Curl of a Vector

Curl of a vector field is an axial vector, which provides the maximum value of the circulation of the field per unit area and indicates the direction along which the maximum value occurs.

(or)

The curl of a vector field $\overline{A}$ at a point P is the measure of the circulation of how much field curl around 'p'.



Direction of ∇×P is out of page.

Properties:

1. Curl of a vector is another vector.
2. $\nabla \times (A + B) = \nabla \times A + \nabla \times B$
3. $\nabla \times (A \times B) = A\,(\nabla.B) - B(\nabla.A) + (B.\nabla)\,A - (A.\nabla)B$
4. $\nabla.\,(\nabla \times A) = 0$
5. $\nabla \times \nabla V = 0$

If the curl of a vector is zero, then the vector is said to be irrotational or conservative field.

$$\nabla \times A = 0.$$

Cartesian:

$$\nabla \times A = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Cylindrical:

$$\nabla \times A = \frac{1}{\rho}\begin{vmatrix} a_\rho & a_\phi & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A\rho & \rho.A_\phi & A_z \end{vmatrix}$$

Spherical:

$$\nabla \times A = \frac{1}{r^2 \sin\theta}\begin{vmatrix} a_r & r.a_\theta & r\sin\theta\, a\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r.A_\theta & r\sin\theta\, A\phi \end{vmatrix}$$

Laplacian Operator

The Laplacian of a scalar field V, written as $\nabla^2\, V$ is the divergence of gradient of V.

$$\nabla^2 V = \nabla.(\nabla V)$$

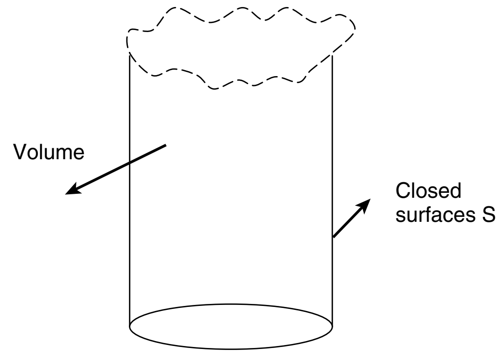
if $\nabla^2 V = 0$, V is called harmonic function.

Gauss–Ostrogradsky Theorem

It states that the total outward flux of a vector field $\overline{A}$ through the closed surface S is the same as the volume integral of the divergence of $\overline{A}$.

$$\oint \overline{A}.ds = \oint_v (\nabla.\overline{A})dv$$

This is also called divergence theorem.



Solved Examples

Example 1

Consider a closed surface 'S' surrounding volume V. If $\overline{r}$ is the position vector of a point inside 's' with $\hat{n}$ is the unit normal vector on 's', the value of integral $\oiint 3\overline{r}.\hat{n}ds =$

(A) 3 V (B) V (C) 9 V (D) $V/3$

Solution

According to the divergence theorem

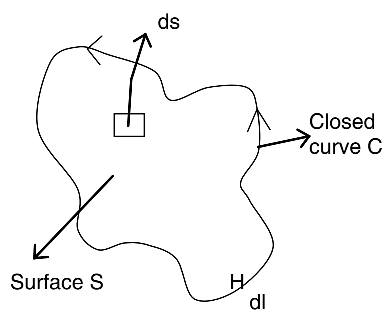
$$\oiint_s 3\overline{r}.\hat{r}ds = \iiint_v 3(\nabla.\overline{r})dv$$

$$= 3(3)\iiint_v \text{dv} = 9V\ (\nabla.\overline{r} = 3)$$

Stokes Theorem

The line integral of tangential component of vector $\overline{A}$ taken around a simple closed curve C is equal to the surface integral of the normal integral of the curl of $\overline{A}$ taken over any surface S having C as its boundary.

$$\oint_c \overline{A}.\overline{d\ell} = \int_s (\nabla \times \overline{A}).\overline{ds}$$



Electromagnetic Fields

In static electromagnetic fields, electric and magnetic fields are independent of each other. While in dynamic electromagnetic fields, both are interdependent and the latter one is of more practical use. Therefore, familiarity with the static fields provides good background for understanding dynamic EM fields.

1. A stationary charge produces electrostatic field.
2. A moving charge or steady current produces magnetic fields
3. Time-varying currents produce electromagnetic fields.

Electrostatic Fields

Coulomb's Law

This law states that the force of attraction or repulsion between two point charges (Q_1) and (Q_2) is

1. Along the line joining the charges.
2. Directly proportional to the product of the charges.
3. Inversely proportional to the square of the distance between them.

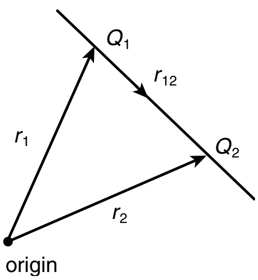
Mathematically,

$$F = \frac{KQ_1Q_2}{r^2}$$

$$K = \text{constant} = \frac{1}{4\pi \epsilon_0} \approx 9\times10^9 \text{ m/F}$$

Q_1 and Q_2 in Coulombs (C), where r is the distance in meters.

$$\epsilon_0 - 8.85 \times 10^{-12} \approx \frac{10^{-9}}{36\pi} \text{F/m}$$



1. Force due to Q_1 on Q_2: F_{12}

$$F_{12} = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q_1Q_2}{R^2} a_{r_{12}}$$

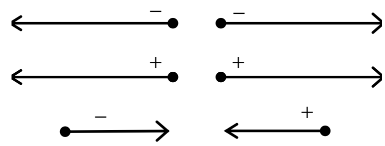
$$|r_{12}| = R$$

$a_{r_{12}}$ is the unit vector along line joining Q_1 and Q_2

$$a_{r_{12}} = \frac{r_2 - r_1}{R}$$

$$F_{12} = \frac{1}{4\pi \epsilon_0} \cdot \frac{Q_1Q_2}{R^3}(r_2 - r_1)$$

2. Like charges repel each other and unlike charge attract each other.



3. Force due to charges Q_1, Q_2 ….., Q_n on charge 'q' r_1, r_2, r_3…..r_n are position vectors of Q_1, Q_2, Q_3….., Q_n, respectively, r- position vector of q and resultant force on q is

$$Fq = \frac{q}{4\pi \epsilon_0}\left[\sum_{i=1}^{n} \frac{Qi(r - ri)}{|r - ri|^3}\right]$$

Electric Field Intensity

Force acting on a unit positive charge is called electric field intensity.

$$E_{12} = \lim_{Q_2 \to 1} \frac{F_{12}}{Q_2}$$

$$E = \frac{F}{Q} \text{vector/coulomb } (or) \text{ Volt/meter}$$

where E is in the direction of force F.

Electric field due to a charge Q at a distance is R

$$E = \frac{Q}{4\pi \epsilon_0 R^2} a_R$$

- Electric field due to n charges is

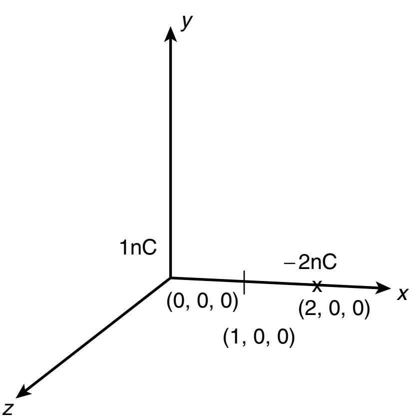
$$E = \frac{1}{4\pi \epsilon_0}\left[\sum_{i=1}^{n} Q_i \frac{(R - Ri)}{(R - R_i)^3}\right]$$

Example 2

Two point charges 1 nC and –2 nC are placed along the x-axis at (0, 0, 0) and (2, 0, 0), respectively.
The force exerted by 1 nc on –2 nC is

(A) 4.5×10^9 N (B) -4.5×10^9 N
(C) 9×10^9 N (D) 9×10^9 N

Solution



F_{12}– Force exerted by 1 on 2

$$F_{12} = \frac{Q_1 Q_2}{4\pi \epsilon_O R^2}$$

a_{12} is the unit vector $= a_x$

$$F_{12} = \frac{-2\times10^{-9}\times10^{-9}}{4\pi \in_0 \times 2^2}\times a_x$$

$$= \frac{-2\times10^{-18}\times9\times10^{-9}}{4}\times a_x$$

$$F_{12} = -4.5\times10^{9}\, a_x \text{ Newton}$$

Force is along negative X-axis direction. This means that the charge is pulling the other towards it, that is, unlike charges attract each other.

Example 3

In the previous problem, the electric field at $Q_2(-2$ nc) is

(A) $-4.5\times10^9\, a_x$ N/m (B) $-2.25\times10^{-9}\, a_x$ N/m

(C) $2.25\; a_x$ N/m (D) $4.5\; a_x$ N/m

Solution

Electric field is force acting on a unit positive charge

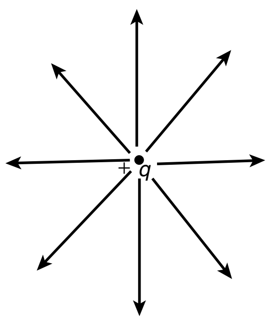
is $$E = \frac{F_{12}}{Q_2} = \frac{-4.5\times10^{-9}}{-2\times10^{-9}}ax \text{ N/m} = 2.25 \text{ N/m } ax$$

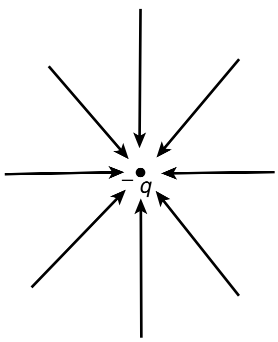
Electric field is along the positive x-direction, which is away from the 1 nc charge.

It depends on permittivity of the medium

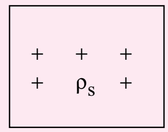
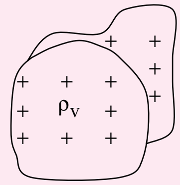
$$\in = \in_o \in_r$$

where $\in_r$ is the electric relative permittivity that originates at positive charge and terminates at negative charges.





Fields due to continuous charge distributions:

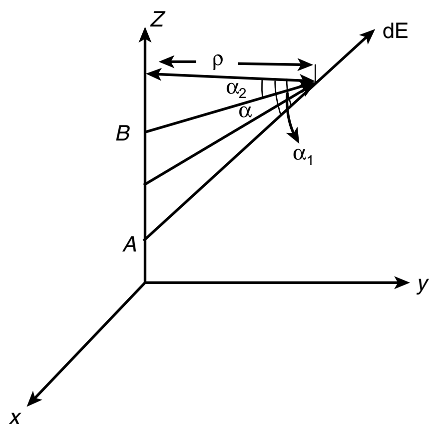
Line charge	Surface charge	Volume charge
$\rho = \oint_L d\ell$		
$d_Q = \rho_\ell d\ell$	$d_Q = \rho_s ds$	$d_Q = \rho_v dv$
$Q = \int_L \rho_\ell dl$	$Q = \int_s \rho_s d_s$	$Q = \int_v \rho_v d_v$
$E = \int \frac{\rho_\ell dl}{4\pi \in_0 R^2} a_R$	$E = \int \frac{\rho_s ds}{4\pi \in_0 R^2} a_R$	$E = \int \frac{\rho_v dv}{4\pi \in_0 R^2} a_R$

Electric fields due to a line charge of finite length placed along z-axis.

$$E = \frac{\rho_L}{4\pi \in_0 \rho}[-(\sin\alpha_2 - \sin\alpha_1)a\rho + (\cos\alpha_2 - \cos\alpha_1)a_z]$$

Electric fields due to infinite line charge placed along z-axis

$$E = \frac{\rho_L}{2\pi \in_0 \rho}a\rho$$



Example 4

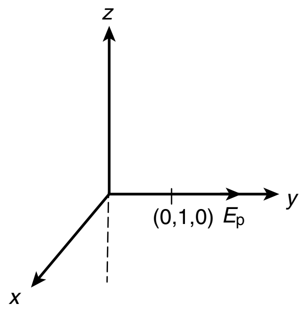
If the line $x = 0 = y$ carries a charge 2π nc/m, the electric field intensity at (0, 1, 0) is

(A) $\frac{10^{-9}}{\in_O} ay$ N/m (B) $\frac{10^{9}}{\in_O} ay$ N/m

(C) $\frac{10^{-8}}{\in_O} ay$ N/m (D) $\frac{10^{8}}{\in_O} ay$ N/m

Solution

The line $x = 0 = y$ is 'z'-axis



Electric field due to infinite line change at a distance ρ is

$$E = \frac{\rho_L}{2\pi \in_0 \rho} a_\rho$$

$$\overline{E} = \frac{2\pi\, nc/m}{2\pi \times \in_0 \times 1m} a\rho(\rho = 1m)$$

$$\overline{E} = \frac{10^{-9}}{\in_o} a_\rho \; N/m$$

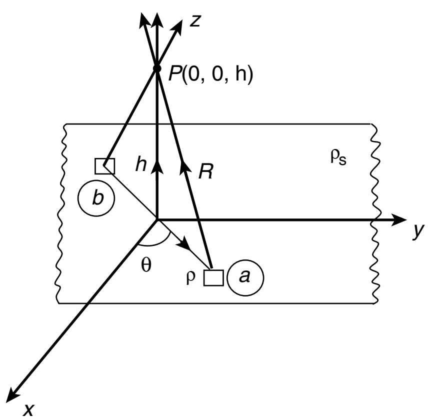
at $(0,1,0)$ $a_\rho = a_y$

$$\therefore \overline{E} = \frac{10^{-9}}{\in_o} a_y$$

Electric Fields due to Continuous Charge Distribution for Infinite Sheet

Consider an infinite sheet of charge in the x–y plane with uniform charge density ρ_s.

Electric field E at point $p(0, 0, h)$ by differential charge dQ on the element a is



$$\text{dE} = \frac{\text{dQ}}{4\pi\, \varepsilon_0 R^2} \hat{a}_R$$

⇒ differential charge $dQ = \rho_s\, ds$

ds is differential element area $= \rho d\phi\, d\rho$

$$\overline{R} = \rho(-\hat{a}_\rho) + h\hat{a}_z$$

$$\text{dE} = \frac{\rho_s \rho d\phi d\rho \vec{R}}{4\pi\, \varepsilon_0 \text{R}^{3/2}} = \frac{\rho_s \rho\, d\phi\, d\rho\left[-\rho\, \hat{a}_\rho + h\hat{a}_z\right]}{4\pi\, \varepsilon_0 (\rho^2 + h^2)^{3/2}}$$

⇒ From symmetry of charge distribution, for every element in region a, there is ⓐ corresponding element in region ⓑ. Therefore, field along a_ρ cancels.

$$E_\rho = 0$$

Electric field has only z component and the point can vary from 0 to ∞ and $\phi \in (0, 2\pi)$

$$E = \int dE_z = \frac{\rho s}{4\pi\, \varepsilon_0} \int_0^{2\pi} \int_0^{\infty} \frac{\rho\, d\phi\, d\rho h \hat{a}_z}{[\rho^2 + h^2]^{3/2}}$$

$$= \frac{2\pi\, \rho_s h}{4\pi\, \varepsilon_0} \int_0^{\infty} \frac{\rho\, d\rho}{[\rho^2 + h^2]^{3/2}} \hat{a}_z$$

$$E = \frac{\rho_s}{2\varepsilon_0} \hat{a}_z \text{ this is for } h > 0$$

$$E = \frac{-\rho_s}{2\varepsilon_0} \hat{a}_z \text{ for } h < 0$$

In general, electric field for an infinite charge sheet is $\frac{\rho_S}{2\varepsilon_0} \hat{a}_n$

Example 5

Electric field on the surface of a perfect conductor is 4 Volts/m. The conductor is immersed in water with $\in = 40\in_o$. The surface charge density on the conductor is

(A) $80\in_o$ (B) $40\in_o$
(C) $20\in_o$ (D) $160\in_o$

Solution

The electric field due to the plane sheet with surface charge density ρ_s is $\frac{\rho_s}{2\in_o}$, whereas due to conducting plane sheet is $\frac{\rho_s}{\in}$

$$\therefore E = \frac{\rho_s}{\in}$$

$$\frac{\rho_s}{\in} = 4 \text{ v/m}$$

$$\rho_s = 4 \times 40\, \in_o = 160 \times \in_o = 160 \in_o$$

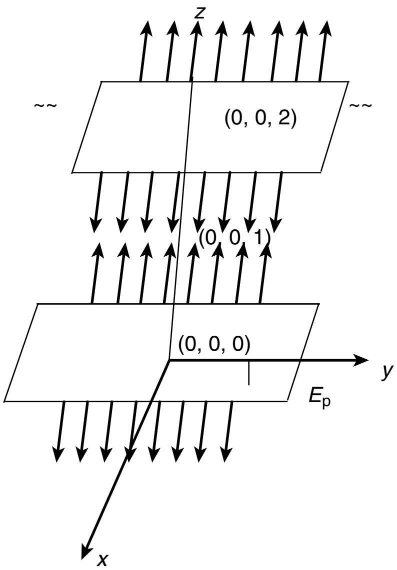
Example 6

Two infinite plane sheets of equal charge densities $1c/m^2$ are placed at (0, 0, 0) and (0, 0, 2), respectively. Then, the electric field intercity at (0, 0, 1) is

(A) 0 N/m (B) $\frac{1}{\in_o} \hat{a}_z$ N/m
(C) $\frac{-1}{\in_o}$ N/m (D) None

Solution

The direction of field lines shown in the following figure:



$$E_{(0,0,1)} = E_{q(0,0,2)} + E_{q(0,0,0)}$$

$$E_{(0,0,1)} = \frac{-\rho_s}{2\in_o} a_z + \frac{\rho_s}{2\in_o} a_z$$

ρ_s is equal for both, that is, 1 C/m^2

$$E_{(0,0,1)} = 0$$

NOTES

(a) $E_{(0,0,3)} = E_{q(0,0,2)} + E_{q(0,0,0)}$

$$= \frac{\rho_s}{2\in_o} a_z + \frac{\rho_s}{2\in_o} a_z$$

$$= \frac{\rho_s}{\in_o} a_z \text{ N/m}$$

$$E_{(0,0,3)} = \frac{1}{\in_o} a_z \text{ N/m}$$

(b) $_{(0,0,-1)} = \frac{-\rho_s}{2\in_o} a_z + \frac{-\rho_s}{2\in_o} a_z$

$$= -\frac{\rho_s}{\in_o} a_z$$

$$E_{(0,0,-1)} = \frac{-1}{\in_o} a_z \text{ N/m}$$

Electric field between two infinite plane sheets of equal surface densities is zero.

Electric Flux Density (D)

D is independent of the medium. Electric flux $\phi = \int \vec{D}.\vec{ds}$ is measured in Coulombs.

Hence, D is called electric flux density. D is measured in Coulombs/m^2, and D is also called electric displacement. The direction of $\overline{D}$ at a point is given by the direction of field lines at that point and magnitude is number of flux lines crossing a surface normal to the lines divided by the surface area.

$\overline{D} = \in_o \overline{E}$ (free space)

Gauss Law

Total electric flux through a closed surface is equal to charge enclosed by that surface.

Mathematically,

$$\psi = Q_{\text{encl}}.$$

$$\psi = \oint_s \overline{D}.ds = Q_{\text{encl}}.$$

Total charge enclosed is

$$Q_{\text{encl}}. = \int_v \rho_v (dv)$$

$$\therefore \oint_s \overline{D}.ds = \int_v \rho_v dv$$

Applying divergence theorem

$$\int_v (\nabla.\overline{D}) dv = \int_v \rho_v d_v$$

$$\nabla.\overline{D} = \rho_v$$

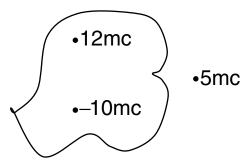
ρ_v is the volume charge density.

Gauss Law can be used to determine $\overline{E}$ or $\overline{D}$ for symmetrical charge distributions such as a point charge and infinite line charge.

Gauss Law always holds good whether the charge distribution is symmetric or not.

Example 7

The flux through the surface s shown in the following figure



(A) 7 mc (B) 2 mc (C) 27 mc (D) 3 mc

Solution

According to the Gauss law, the total dielectric flux enclosed by the surface 's' is equal to the charge enclosed by that surface

$$\Psi = 12 \text{ mc} - 10 \text{ mc} = 2 \text{ mc}.$$

Applications of Gauss Law

Procedure for applying Gauss law:

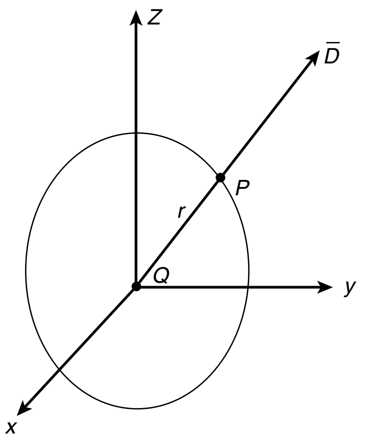
1. Check for the symmetry.
2. Construct a Gaussian surface.

Conditions for a Special Gaussian Surface

1. The surface is closed.
2. At each point of the surface, $\overline{D}$ is either normal or tangential to the surface.
3. D is sectionally constant over that part of the surface where $\overline{D}$ is normal.

Point Charge

Spherical symmetry exists for the point charge, and therefore, the Gaussian surface is a sphere that encloses the charge 'Q', as shown in the figure.



D is everywhere normal to the surface.

Therefore, $\overline{D} = D_r \; a_r$

$$\oint_s \overline{D}.\overline{ds} = \oint_s D_r \; a_r.\overline{ds} = Qencl$$

$d\overline{s}$ in spherical coordinates

$$= r^2 \sin\theta \; d\theta \; d\phi a_r + r\sin\theta \; dr d\phi a\theta + r dr \; d\theta \; a\phi$$

$$\oint \overline{D}.\overline{ds} = \oint_s D_r \; r^2 . \sin\theta d\theta d\phi$$

$$= D_r . 4\pi r^2$$

$$\oint_s \overline{D} \, ds = D_r 4\pi r^2 = Q$$

$$D_r = \frac{Q}{4\pi r^2}$$

$$\overline{D} = \frac{Q}{4\pi \; r^2} a_r$$

$$\overline{E} = \frac{Q}{4\pi \in_o r^2} a_r \quad \left[\overline{D} = \in_o \overline{E}\right]$$

Infinite Line Charge

$$\overline{D} = \frac{\rho_L}{2\pi\rho} a\rho$$

ρ_L is the linear charge density.

Uniformly Charged Sphere

$$\overline{D} = \begin{cases} \dfrac{Q.r}{4\pi a^3} \; a_r; & 0 < r \le a \\ \dfrac{Q}{4\pi r^2} \; a_r; & r \ge a \end{cases}$$

$$\overline{E} = \begin{cases} \dfrac{Q.r}{4\pi \in_o a^3}; & a_r 0 < r \le a \\ \dfrac{Q}{4\pi r \in_o r^2} a_r; & r \ge a \end{cases}$$

For example, charge sheet.

Energy Associated with Charge Distribution

If there are n point charges that are brought in infinity to specific points, then work done.

No work is required to bring initial charge

$$W = \frac{1}{4\pi\varepsilon} \sum_{i=2}^{N} \sum_{j=1}^{i-1} \frac{q_i q_j}{R_{ji}}$$

Work done for volume charge is

$$W = \frac{1}{2} \int_v \rho_v V dV$$

$$= \frac{1}{2} \int_v (\nabla . D) V dV$$

$$= \frac{1}{2} \int D.E \; dV$$

$$= \frac{1}{2} \int \varepsilon_0 E^2 dV$$

Electrostatic energy density W_E is $\dfrac{dW}{dV}$

Therefore, $W_E = \dfrac{1}{2} D.E$

$$= \frac{1}{2} \varepsilon_0 E^2 = \frac{D^2}{2\varepsilon_0}$$

Electric Potential

Electric potential at a point is defined as the work done in bringing a unit positive charge from infinity to that point in an electric field. Work done in moving a charge Q through a distance dl is

$$dw = -F d\ell$$

$$dw = -Q \overline{E}.d\ell \quad \left[\overline{F} = Q\overline{E}\right]$$

$$w = -Q \int_A^B \overline{E}.d\ell$$

$$V_{AB} = \frac{W}{Q} = -\int_A^B \overline{E}.d\ell$$

$$V_B = \frac{W}{Q} = -\int_{\infty}^{r_B} \overline{E}.d\ell$$

Here, $r_A = \infty$

Example 8

The electric scalar potential to a charge of 'q' at origin at a distance 'r' is

(A) $\frac{q}{4\pi \in_o r}$ volts (B) $\frac{q}{8\pi \in_o r^2}$ volts

(C) $\frac{q}{8\pi \in_o r}$ volts (D) $\frac{q}{8\pi \in_o r^2}$ volts

Solution

The field due to 'q' is

$$\overline{E} = \frac{q}{4\pi \in_o r^2} a_r$$

$$vr = -\int_{\infty}^{r} \frac{q}{4\pi \in_o r^2} a_r dr$$

$$v_r = -\int_{\infty}^{r} \frac{q}{4\pi \in_o r^2} dr = \frac{+q}{4\pi \in_0 r}$$

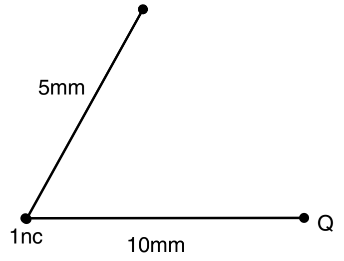
Potential Difference

$$V_{AB} = V_B - V_A$$

V_{AB} is called as potential at B with respect to A.
Potential at infinity is chosen as zero.

Example 9

A point charge of +1 nc is placed in free space, as shown in the figure.



The potential difference between two points P and Q, v_{pq} is

(A) +90 v (B) −90 v

(C) +900 v (D) −900 v

Solution

$$V_{pQ} = V_p - V_Q = \frac{q}{4\pi \in_o r_p} - \frac{q}{4\pi \in_o r_q}$$

$$= \frac{q}{4\pi \in_o}\left[\frac{1}{r_p} - \frac{1}{r_q}\right]$$

$$= \frac{q}{4\pi \in_o}\left[\frac{1}{5\text{mm}} - \frac{1}{10\text{mm}}\right]$$

$$V_{PQ} = = \frac{q}{4\pi \in_o} \times \frac{1}{10\text{mm}}$$

$$V_{PQ} = \frac{10^{-9} \times 10^3 \times 9 \times 10^9}{10}$$

$$\left(\frac{1}{4\pi \in_o} = 9 \times 10^9\right)\frac{\text{m}}{\text{F}}$$

$$V_{PQ} = 900 \text{ v}$$

V_{AB} is independent of path taken, and hence,

$$V_{AB} = -V_{BA}$$

$$V_{AB} + V_{BA} = 0$$

is $$\oint \overline{E}dl = 0$$

Applying Stokes theorem

$$\oint \left(\nabla \times \overline{E}\right) ds = 0$$

$$\Rightarrow \nabla \times E = 0$$

where $\overline{E}$ is irrotational vector or conservative field; thus, electrostatic field is conservative field.

$$E = -\nabla V$$

Example 10

The scalar potential $V = \frac{10}{r^2}$ sinθ cosϕ, and electric flux density at $(1, \pi/2, 0)$

(A) $10 \in_o$ c/m^2 (B) $-10 \in_o$ c/m^2

(C) $20 \in_o$ c/m^2 (D) $-20 \in_o$ c/m^2

Solution

$$D = \in_o \overline{E}$$

$$\overline{E} = -\nabla v$$

$$= -\left[\frac{\partial v}{\partial r} a_r + \frac{1}{r}\frac{\partial v}{\partial \theta} a\theta + \frac{1}{r \sin\theta}\frac{\partial v}{\partial \phi} a\phi\right]$$

$$= -\left[\frac{-20}{r^3} \sin\theta \cos\phi\, a_r + \frac{10}{r^3} \cos\theta \cos\phi\, a\theta + \frac{-10}{r^3} \sin\phi\, a\phi\right]$$

$$\overline{E} = \frac{20}{r^3} \sin\theta \cos\phi . a_r - \frac{10}{r^3} \cos\theta \cos\phi\, a_\theta + \frac{10}{r^3} \sin\phi\, a_\varphi$$

$$E_{\left(1, \pi/2, 0\right)} = 20 a_r$$

$$D = \in_0 \overline{E} = 20 \in_0 a_r$$

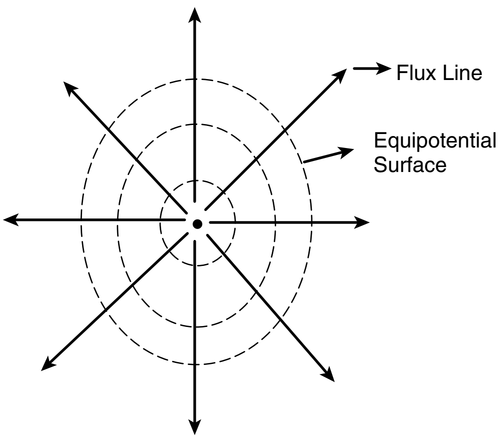
Equipotential Surface

Any surface on which the potential is same throughout the surface is known as equipotential surface. Work done in a moving a charge from one point to another along an equipotential surface is zero.

$$\int \overline{E} dl = 0$$

Properties of Equipotential Surface

1. Lines of force or flux lines are always normal to the surface.



Energy Density in Electrostatic Fields

$$W_E = \frac{1}{2} \epsilon_o E^2 = \frac{D^2}{2 \epsilon_o}$$

Energy due to continuous volume charge distribution.

$$W_E = \frac{1}{2} \int_v D.E\, dv = \frac{1}{2} \int \epsilon_o |E|^2 d\upsilon$$

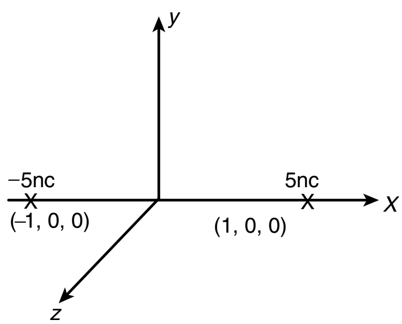
$$W_E = \frac{1}{2} \sum_{k=1}^{n} Q_k v_k$$

Example 11

Two point charges of –5 nc and 5 nc are located in free space at (–1, 0, 0) m and (1, 0, 0), respectively. The energy stored in field is

(A) 0 (B) 225 nJ
(C) 112.5 nJ (D) –112.5 nJ

Solution



$$W = \frac{q_1 q_2}{4\pi \epsilon_0 r}$$

$$W = \frac{5 \times 5 \times 10^{-18}}{4\pi \epsilon_o \times 2}$$

$$= -\frac{25 \times 10^9 \times 9 \times 10^{-18}}{2}$$

$$= -\frac{225}{2} \times 10^{-9}$$

$$= -112.5 \times 10^{-9} \text{ J}$$

Current

Current through a point is defined as the rate of charge passing through that point in unit time.

$$i = \frac{dq}{dt} \text{(amps)}$$

Current density at a given point is the current through a unit normal at that point denoted by J.

$$I = \int_s J.ds$$

Unit of J is ampere/meter ^{2}.

Convection Current

1. It does not involve conductors
2. It does not satisfy ohms law
3. It occurs when current flows through in an insulating medium such as liquid or vacuum

For example, a beam of electron in a vacuum tube.

Convection Current Density

$$J_{cv} = \rho_v . V$$

where ρ_v is the charge density and V is the velocity.

Conduction Current

1. It requires conductors
2. Large amount of free electrons in a conductor provides conduction current.

Conduction Current Density

$$J_c = \sigma E$$

The above relation is also called as Ohms law.

Conductors

1. The electric field inside a conductor placed in an electric field is zero.
2. Conductor is an equipotential surface.

$$R = \frac{\rho_c \ell}{A}$$

$$\rho_c = 1/\sigma$$

σ is the conductivity of material.

$$R = \frac{V}{I} = \frac{\int_l \bar{E} dl}{\int_s \sigma E . d\bar{s}}$$

Joule's Law

Power (P)

$$P = \int \bar{E} . J dv. = I^2 R = VI$$

Power density, $W_p = \frac{dp}{dv}$

$$W_p = \bar{E} . \bar{J} = \sigma |E|^2$$

Dielectrics

The effect of dielectric on electric field E is to increase $\bar{D}$ inside it by an amount $\bar{P}$.

$$\bar{D} = \epsilon_o E + p$$

$$\bar{D}_{dielectric} > \bar{D}_{freespace}$$

where p is $\chi_e \epsilon_o \bar{E}$ and

χ_e is electric susceptibility.

$$\bar{D} = \epsilon_0 \bar{E} + \chi_e \epsilon_o \bar{E}$$

$$\bar{D} = \epsilon_0 (1 + \chi_e) \bar{E}$$

$$\bar{D} = \epsilon_o \epsilon_r E$$

$$\bar{D} = \epsilon \bar{E}$$

$$\epsilon = \epsilon_0 (1 + \chi_e) = \epsilon_o \epsilon_r$$

$$\epsilon_r = 1 + \chi_e$$

Dielectric Strength

It is the maximum electric field that a dielectric can tolerate or withstand without breakdown.

Continuity Equation

$\nabla . J = -\frac{\partial \rho_v}{\partial t}$ for steady currents $\frac{\partial \rho_v}{\partial t} = 0$,

Therefore, $\nabla J = 0$

This means the charge entering the volume is same as the charge leaving.

$$\rho_v = \rho_{vo} . e^{-t/\tau_r}$$

where t_r is the relaxation time, that is. the time it takes a charge placed in the interior of a material to drop to e^{-1} = 36.8% of its initial value

$$\tau_r = \frac{\epsilon}{\sigma} = \frac{\epsilon_o \epsilon_r}{\sigma}$$

For a good conductor, the relaxation time is so short that most of the charge will vanish from any interior point and appear at the surface.

Boundary Conditions

Dielectric–Dielectric

1. Tangential electric fields are continuous i.e., $E_{1t} = E_{2t}$
2. Normal components of electric flux density are discontinuous by an amount of charge density is $D_{1n} - D_{2n} = \rho_s$

   For a source-free region,

$$\rho_s = 0$$

$$D_{1n} = D_{2n}$$

$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

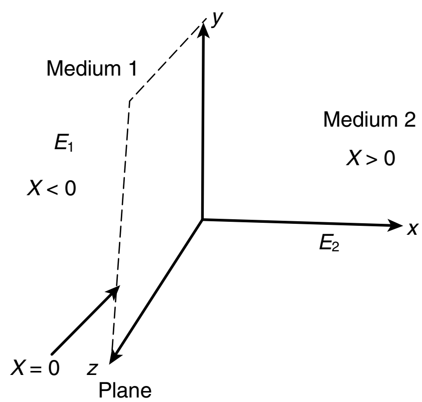
and $$\frac{\tan \theta_1}{\tan \theta_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}$$

Example 12

Medium 1 has electrical permittivity $\epsilon_1 = 3 \epsilon_0$ F/m and occupies the region in the left of $x = 0$ plane. Medium 2 has electrical permittivity $\epsilon_2 = 5\epsilon_0$ F/m and occupies the region to the right of $x = 0$ plane. If E_1 in the medium 1 is $\bar{E} = a_x - 2a_y + 3a_z$ volt/m, then $\bar{E}2$ (V/m) in medium 2 is

(A) $1.2\, a_x - 2\, a_y + 3a_z$
(B) $a_x - a_y + 3\, a_z$
(C) $0.6\, a_x - 2\, a_y + 3\, a_z$
(D) $a_x - 2\, a_y + 1.5\, a_z$

Solution



Y and Z components of $\bar{E}$ are tangential components and x-component is normal component.

$$E_{2y} = E_{1y}, E_{2z} = E_{1z}$$

(tangential electric field is continues)
Since the region is source free

$$D_{1n} = D_{2n},$$
$$D_{1x} = D_{2x},$$
$$\in_1 \bar{E}_{1x} = \in_2 \bar{E}_{2x}$$
$$E_{2x} = \frac{\in_1}{\in_2} \times E_{1x}$$
$$E_{2x} = = 0.6$$
$$\bar{E}_2 = 0.6a_x - 2a_y + 3a_z$$

Conductor Dielectric

1. $E_{1t} = 0$ [electric field in a conductor = 0]
   $E_{2t} = 0$
2. $E_{1n} = 0$ [E in a conductor = 0]
   $D_{1n} = 0$
   $D_{2n} = i_s = \in_0 \in_r E_{2n}$
   For a source-free region
   $D_{2n} = 0$

Conductor-free Space

This is a special case of conductor– dielectric condition:

1. $E_{1t} = E_{2t}$
   $E_{1t} = 0$ [electric field inside a conductor is zero]
   $E_{2t} = 0$
2. $D_{1n} - D_{2n} = \rho_s$
   $D_{1n} = 0$ (conductor)
   $D_{2n} = \rho_s = \in_o E_n$ (free space $\in_r = 1$)

Poisson's Equation for Electric Fields

$$\nabla^2 V = \frac{-\rho_v}{\in}$$

A special case of this is when $\rho_v = 0$ (charge-free region). $\nabla^2 V = 0$ called as Laplace equation.

Example 13

The potential (scalar) distribution is given as $V = 10y^3 + 2x^2$. If $\in_o$ is permittivity of free space, what is the charge density at the point (3, 0) in C/m^3 ?

(A) $4 \in_o$ (B) $-4 \in_o$
(C) $8 \in_o$ (D) $-8 \in_o$

Solution

Poisson equation for electric fields is $\nabla^2 V = -\frac{\rho_v}{\in_0}$

$$V = 10y^3 + 2x^2$$
$$\nabla^2 v = \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}$$
$$\nabla^2 v = 4 + 60y$$
$$\frac{-\rho_v}{\in_o} = 4 + 60y$$

at (2, 0)

$$\left(\frac{-\rho_v}{\in_o}\right)_{(2,0)} = 4 + 60 \times 0$$
$$-p_v = 4\in_o$$
$$p_v = -4\in_o$$

Capacitance: It is the ratio of magnitude of charge on one of the plates to the potential difference between them.

$$C = \frac{Q}{V} = \frac{\in \int \bar{E}.ds}{\int \bar{E}.dl}$$

Parallel-plate capacitor

$$C_o = \frac{\in_o A}{d}$$

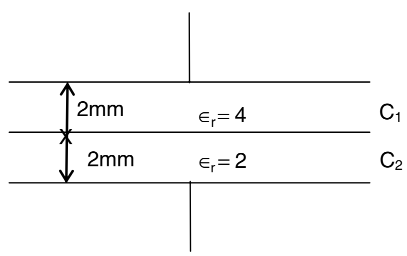
where A is the area of the plate and
d is the distance between the plates.

$$C = \frac{\in_o \in_r A}{d}$$
$$\in_r = \frac{c}{c_o}$$

Example 14

A parallel-plate capacitor is shown in the figure. It is made up of two square plates of 100 mm side. The 4 mm space between the plate is filled with two layers of dielectric $\in_r =$ 4, 2 mm thick and $\in_r = 2$, 2 mm thick. Neglecting the fringe fields at the edges, the capacitance is

(A) 2.94 pF (B) 29.4 pF
(C) 5.98 pF (D) 59.8 pF



Solution

Capacitance is in series

$$-C \in_q = \frac{C_1 C_2}{C_1 + C_2}$$

(or)

$$\frac{1}{C \in_q} = \frac{1}{C_1} + \frac{1}{C_2}$$
$$C_1 = \frac{\in_o \in_{r1} A}{d_1} = \frac{4 \in_o A}{d_1}$$

$$C_1 = \frac{\epsilon_o \epsilon_{r2} A}{d_2} = \frac{2\epsilon_o A}{d_2}$$

$$\frac{1}{C_{eq}} = \frac{d_1}{4\epsilon_o A} + \frac{d_2}{2\epsilon_o A}$$

$$d_1 = d_2 = 2 \text{ mm}$$

$$A = 100 \times 100 \times 10^{-6} \text{ m}^2$$

$$\frac{1}{C_{eq}} = \frac{2\text{mm}}{2\epsilon_o A}\left(3/2\right) = \frac{3\times 10^{-3}}{2\times \epsilon_o \times 10^{-2}}$$

$$\frac{1}{C_{eq}} = \frac{0.15}{\epsilon_o}$$

$$C_{eq} = \epsilon_0 \times \frac{20}{3}$$

$$= 8.82 \times 10^{-12} \times \frac{20}{3}$$

$$= 2.94 \times 10^{-12} \times 20$$

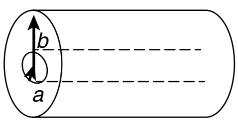
$$C_{eq} = 59.8 \text{ pF} = 59.8 \text{ pF}$$

Energy stored in a capacitor

$$W_E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

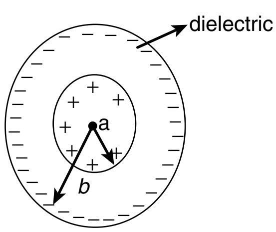
(b) Co-axial capacitor

$$C = \frac{2\pi \epsilon L}{\ell n(b/a)}$$



(c) Spherical capacitor:

$$C = \frac{4\pi \epsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$



Magnetostatic Fields

A constant current flow produces magnetostatic or static magnetic fields.

Biot–Savart Law

It states that the magnetic field intensity dH produced at a point P by the differential current element Idl is:

1. Proportional to product of Idl and sine of the angle between the element and the line joining P to the element.
2. Inversely proportional to the square of the distance between P and element.

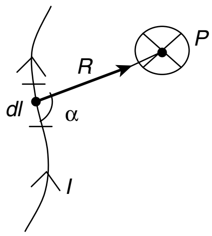
$$dH \;\alpha\; \frac{Id\ell \sin\alpha}{R^2}$$

$$dH = \frac{Id\ell \sin\alpha}{4\pi R^2}$$

$$dH = \frac{Id\ell \times a_R}{4\pi R^2}$$

$$a_R = \frac{\overline{R}}{|R|}$$

$$dH = \frac{Id\ell \times \overline{R}}{4\pi R^3}$$

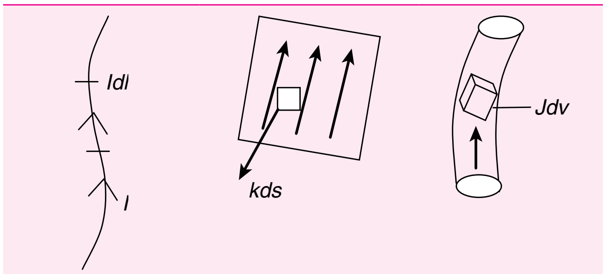


The direction of dH can be determined by the right-hand thumb rule

$$H = \int dH$$

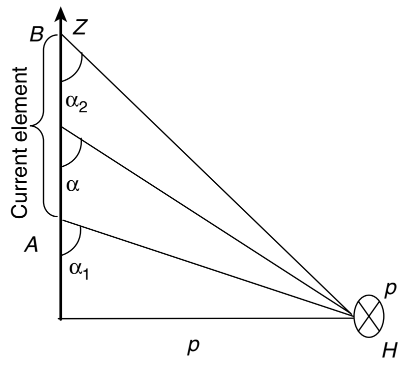
(i) 'H' is into the page

(ii) 'H' is out of the page

Line Current	Surface Current	Volume Current
		
Idl	Kds	Jdv
$\int_l \frac{Idl \times a_R}{4\pi R^2}$	$\int_s \frac{\overline{k}ds \times a_R}{4\pi R^2}$	$\int_v \frac{Jdv \times a_R}{4\pi R^2}$

H due to a Line Current

1. Finite length:



$$H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)a_\phi$$

2. Semi-infinite length:

$$\alpha_2 = 0,\ \alpha_1 = 90°$$

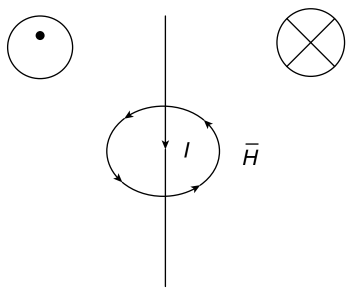
$$H = \frac{I}{4\pi\rho}a_\phi z$$

3. Infinite length:

$$\alpha_1 = 180°;\ \alpha_2 = 0$$

$$H = \frac{I}{2\pi\rho}a_\phi$$

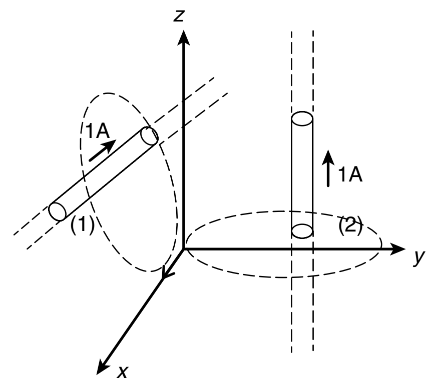
If a conducting wire of infinite length carrying current I is placed along the z-axis, the direction of $\overline{H}$ is along the tangent of circle drawn at that point with the wire as centre.



Example 15

Two infinitely long wires carrying current areas shown in the following figure one wire is in x–z plane and parallel to x-axis. The other wire is in the y–z plane and parallel to the z-axis. Which components of resulting magnetic field are non-zero at origin?

(A) x, y, z components (B) x, y components
(C) y, z components (D) x, z components



Solution

The magnetic field intensity due to infinite wire is along the direction of tangent drawn to a circle formed with the wire as centre through that point.

$\overline{H}$ due to (1) is along negative y-direction. $\overline{H}$ due to (2) is along positive x-direction

$\overline{H}$ contains only x and y components.

Ampere's Law

It states that the line integral of tangential component of magnetic field intensity around a closed path is same as the net current enclosed by that path. Mathematically,

$$\oint \overline{H}.\overline{dl} = I_{\text{enclosed}}.$$

By applying Stokes theorem

$$\int_l \overline{H}.\overline{dl} = \int_s \left(\nabla\times\overline{H}\right).\overline{ds} = I_{encl}$$

However,

$$I_{encl} = \int_s \overline{J}.\overline{ds}$$

$$\therefore \int_s \left(\nabla\times\overline{H}\right).\overline{ds} = \int \overline{J}.\overline{ds}$$

$$\therefore \nabla\times\overline{H} = \overline{J}$$

Applications of Ampere's Law

Infinite line current:

$\overline{H} = \dfrac{I}{2\pi\rho}a_\phi$ (already derived using Biot–Savarts' law).

Example 16

The Z-axis carries filamentary current of $5\pi A$ along a_2. Which of the following is incorrect?

(A) $\overline{H} = -a_x A/m$ at (0, 2.5, 0)

(B) $\overline{H} = a_\phi A/m$ at $\left(2.5, \pi/4, 0\right)$

(C) $\overline{H} = -a_y A/m$ at $(2.5, \pi, 0)$

(D) $\overline{H} = -a_\varphi A/m$ at $\left(2.5, 3\pi/2, 0\right)$

Solution

$\overline{H}$ due to infinite wire carrying current I at a distance 'ρ'

is $H = \dfrac{I}{2\pi\rho} a$

(a) $H = \dfrac{5\pi}{2\pi \times 2.5\rho} a_\phi$

$\overline{H} = a_\phi$

At (0, 2.5, 0), $\phi = 90°$

$\therefore a_\varphi = -a_x$

Therefore, (a) is correct

(b) $\overline{H}\, at \left(2,5,{}^{\pi}\!/_{4},0\right) = \dfrac{I}{2\pi\rho} a_\phi$

$\overline{H} = a_\phi$

Therefore, (b) is correct

(c) $\overline{H}$ at (2.5, π, 0)

$\overline{H} = a_\phi \, {}^{A}\!/_{m}$

At (2.5, π, o), $\phi = 180°$

$a_\phi = -a_y$

Therefore, (c) is correct

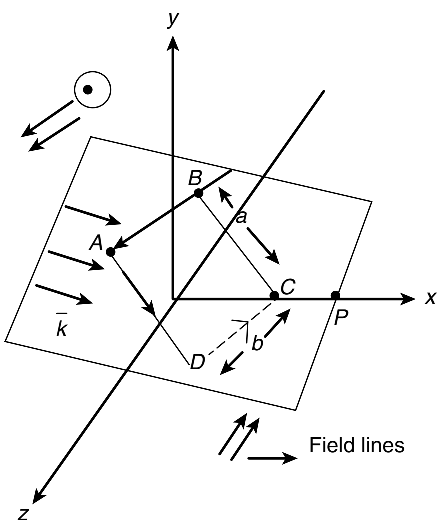
(d) $\overline{H}$ at $\left(2.5, 3{}^{\pi}\!/_{4}, 0\right)$

$\overline{H} = \dfrac{I}{2\pi\rho} a_\phi \, {}^{A}\!/_{m}$

$\overline{H} = a_\phi \, {}^{A}\!/_{m}$

Therefore, option (d) is incorrect.

(ii) Infinite sheet of current:



Current density $\left(\overline{I}_c\right) = k_x a_x$A/m applying Ampere's rule to loop ABCD

$I_{\text{enclosed}} = k_x b$

$$\therefore \oint \overline{H}.dl = k_x.b$$

$$H = \begin{cases} H_0 a_z & y > 0 \\ -H_0 a_z & y < 0 \end{cases}$$

$$\oint \overline{H}.dl = \left(\int_A^D + \int_D^C + \int_C^B + \int_B^A \right) H.dl$$

along $\int_A^D$ and $\int_c^b \overline{H}$ and dl are perpendicular.

Those two will disappear

$$\oint \overline{H}.dl = \int_D^C \overline{H}.dl + \int_B^A \overline{H}.dl$$

$$= -H_o(-b) + H_o(b)$$

$$K_x b = 2H_o b$$

$$H_o = \frac{k_x}{2}$$

$$H = \begin{cases} \dfrac{k_x}{2} a_z & y > 0 \\ \dfrac{-k_x}{2} a_z & y < 0 \end{cases}$$

$$H = \frac{1}{2}(k \times a_n)$$

Magnetic Flux Density (*B*)

The magnetic flux density is defined as the number of magnetic flux lines per unit area, and the direction of flux lines or tangent to the magnetic flux lines gives the direction of magnetic flux density.

B and *H* are related as

$$B = \mu_0 H \text{ (Wb/m}^2 \text{ or T)}$$

$$\mu_o = 4\pi \times 10^{-7}\, {}^{\text{H}}\!/_{\text{m}}$$

Flux through a surface *s* is given by

$$\psi = \int_s \overline{B}.d\overline{s}$$

Magnetic flux lines are closed lines; therefore, an isolated magnetic charge does not exist:

$$\oint_s \overline{B}.ds = 0$$

Total flux through a closed surface is zero.

Applying divergence theorem to the abovementioned integral $\oint \overline{B}.ds = \int_v (\nabla.B) dv = 0$

$$\nabla.B = 0$$

This equation is also referred to as solenoid at property of magnetic field lines.

Magnetic Scalar and Vector Potentials

Magnetic scalar potential V_m related to H as $H = -\nabla V_m$. If $J = 0$, V_m is only defined in a region where $J = 0$ (source-free region) and V_m also satisfies the Laplace equation.

$$\nabla^2 V_m = 0$$

Magnetic vector potential $\overline{A}$ is defined in a such a way that

$$\overline{B} = \nabla \times \overline{A}$$

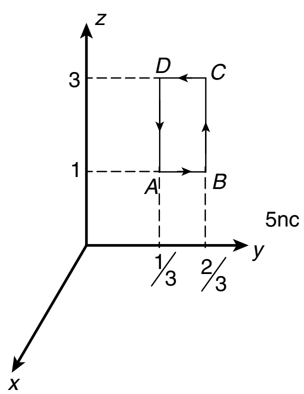
$$\psi = \int_s \overline{B}.\partial s = \int (\nabla \times \overline{A})\partial.s$$

$$= \oint_L \overline{A}.dl \text{ (Stokes theorem)}$$

$$\therefore \psi = \int \overline{A}.\partial \ell$$

Example 17

If $\overline{A} = xya_x + y^2a_y$, then $\oint \overline{A}.dl$ over the path shown in the following figure.



(A) $1/3$ (B) 0 (C) $-1/3$ (D) $2/3$

Solution

$$\oint A.dl = \int_A^B \overline{A}\,dl + \int_B^C \overline{A}dl \int_C^D \overline{A}.dl + \int_D^A \overline{A}.dl$$

$$\int_A^B \overline{A}.dl \Rightarrow dl = d_x a_x$$

$$\int_A^B \overline{A}.dl = \int_{1/3}^{2/3} xy\,dx = y\left[x^2/2\right]_{1/3}^{2/3} = \frac{1}{2}\left[1/3\right] = 1/6$$

$$\int_B^C \overline{A}.dl \Rightarrow dl = d_y.a_y$$

$$\int_B^C \overline{A}.dl = \int_1^3 y\,dy = \left[y^3/3\right]_1^3 = 26/3$$

$$\int_C^D \overline{A}.dl \Rightarrow dl = dxa_x ,$$

$$\int_C^D \overline{A}.dl \Rightarrow \int_{2/3}^{1/3} xydx$$

$$= 3\left[x^2/2\right]_{2/3}^{1/3} = \frac{3}{2}\left[-1/3\right] = -1/2$$

$$\int_D^A \overline{A}.dl \Rightarrow dl = dy.ay$$

$$\int_D^A \overline{A}.dl = \int_3^1 y^2\,dy = \frac{-26}{3}$$

$$\oint_c \overline{A}.dl = 1/6 - 1/2 + \frac{26}{3} - 26/3$$

$$\oint \overline{A}.dl = -1/3$$

Poisson's equation for magnetostatic fields.

$$\nabla^2 A = -\mu_o J$$

Lorentz Equation

Force acting on a charged particle moving in an electromagnetic field is

$$\overline{F} = Q\left(\overline{E} + \overline{V} \times \overline{B}\right).$$

where V is the velocity of charged particle.

Force on a current element:

$$\overline{F} = \int_L \overline{I}\,d\ell \times \overline{B}$$

$$= \int_s \overline{k}ds \times \overline{B}$$

$$= \int \overline{J}dv \times \overline{B}$$

Magnetic dipole moment: t is defined as the product of current and area of the loop and its direction is normal to the loop:

$$m = IS\,\hat{a}_n$$

where a_n is the normal vector

Torque $(T) = \overline{m \times B}$.

Magnetization (M)**:** It is the magnetic dipole moment per unit volume and units are Amp/meter

$$J_b = \nabla M$$

In magnetic materials,

$$B = \mu_o (H + M)$$

$$M = \chi_m H$$

χ_m is the magnetic susceptibility

$$B = \mu_o\,(1 + \chi_m)\,H$$

$$B = \mu_o\,\mu_r\,H$$

Boundary conditions

1. $B_{1n} = B_{2n}$
   $\mu_1 H_{1n} = \mu_2 H_{2n}$
   Normal components of magnetic flux density are continuous.
2. $(H_1 - H_2)$ x $a_{n12} = K$
   a_{n12} is the normal vector divided from medium 1 to 2 for a source-free region $K = 0$.
   $H_{1t} = H_{2t}$

Example 18

A current sheet of $K = 5a_y$ A/m lies on the dielectric interface $x = 0$ between two dielectric media with $L_{r1} = 5$, $\mu_{r1} = 2$ in region −1 ($x < 0$) and $L_{r2} = 5$, $\mu_{r2} = 4$ in region −2 ($x > 0$). If the magnetic field in region −1 at $x = 0$ is $\overline{H}_1 = 2a_x + 20a_y \text{A/m}$, then the magnetic field in region −2 is at $x = 0^+$.

(A) $a_x + 20\, a_y + 5\, a_z$ A/m

(B) $a_x + 20\, a_y - 5\, a_z$ A/m

(C) $a_x - 20\, a_y + 5\, a_z$ A/m

(D) $a_x - 20\, a_y\, 5 - a_z$ A/m

Solution

The tangential field due to a sheet separating two medium are discontinued by current density

$$H_{t1} - H_{t2} = \overline{k} \times a_{n12}$$
$$H_{t1} - H_{t2} = 5a_y \times a_x$$
$$H_{t2} - H_{t1} = 5a_z$$
$$H_{t2} - H_{t1} = 5a_z$$
$$H_{t2} - 20a_y = 5a_z$$

Normal components of magnetic flux densities are continuous

$$B_{n1} = B_{n2}$$

is

$$\mu_1, H_{n1} = \mu_2, H_{n2}$$
$$H_{n2} = \frac{2 \times 2}{4} = 1$$
$$\therefore \overline{H}_2 = \overline{H}_{t2} + \overline{H}_{n2}$$
$$\overline{H}_2 = a_x + 20a_y - 5a_z$$

Inductance:

$$L = \frac{N\psi}{I} = \frac{N\int \overline{B}ds}{\int \overline{J}.ds}$$

$$L = \frac{2W_m}{I^2}$$

where W_m is the magnetic energy stored in an inductor.

$$W_m = \frac{1}{2}LI^2 = \frac{1}{2}\int \overline{B}.\overline{H}dv$$
$$= \frac{\mu}{2}\int H^2 dv$$

Laplace and Poisson Equations in Electric and Magnetic Fields

Electric field (*E*)	Magnetic field (*M*)
$E = -\nabla V$	$B = \nabla \times A$
$D = L_0 E$	$B = \mu_0 H$
$\nabla D = \rho_v$	$\nabla \times H = J$
$\nabla(L_0 E) = \rho_v$	$\nabla \times \left(\frac{B}{\mu_0}\right) = J$
$L_0 \nabla(-\nabla V) = \rho_v$	$\nabla \times (\nabla \times A) = \mu_0 J$
$\nabla^2 V = -\frac{\rho_v}{\epsilon_0}$	$\nabla^2 A = -\mu_0 J$
The abovementioned expression is called Poisson's equation for electric field.	The abovementioned expression is called Poisson's equation for magnetic field.
Laplace equation for electric field is expressed as $\nabla^2 V = 0$.	Laplace equation for magnetic field is expressed as $\nabla^2 A = 0$.

Summary of Boundary Conditions

Electric field	Magnetic field
1) $D_{n1} = D_{n2}$ (charge free) $(\overline{D}_1 - \overline{D}_2).\overline{a}_{n12} = -\rho_s$ (with surface charge)	1) $B_{n1} = B_{n2}$
2) $E_{t1} = E_{t2}$	2) $H_{t1} = H_{t2}$ (current free) $(\overline{H}_1 - \overline{H}_2).\times \overline{a}_{n12} = \overline{k}$ (with current sheet)
3) $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$ (charge free)	3) $\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r2}}{\mu_{r1}}$ (current free)

Capacitors	Inductors
$C = \frac{\oint D.ds}{-\int E \cdot dl} = \frac{Q}{V}$	$L = \frac{\int B.ds}{\oint H.dl} = \frac{\psi_m}{I}$
Parallel plate $C = \frac{\epsilon_0 A}{d}$ $= \frac{\epsilon A}{d}$	Solenoid $L = \frac{N^2 \mu_0 A}{d}$ $= \frac{N^2 \mu A}{d}$
Concentric cylinder C $= \frac{2\pi \epsilon_0 l}{\ln(b/a)}$	Concentric cylinder $= \frac{\mu_0 l}{2\pi}\ln(b/a)$
$W_E = \frac{1}{2}CV^2$	$W_E = \frac{1}{2}LI^2$

Direction for questions 1 to 16: Select the correct alternative from the given choices.

Example 19

The small identical conducting spheres have charges of –1 nC and 2 nC, respectively. If they are brought in contact and separated by 4 cm, what is the force between them?

(A) 251.1×10^{-6} N (B) 1.125×10^{-5} N
(C) 1.125 N (D) 1.125×10^{-8} N

Solution

$$F = \frac{9 \times 10^9 \times (-1) \times 2 \times 10^{-18}}{16 \times 10^{-4}}$$

$$F = 1.125 \times 10^{-5}\ \text{N}$$

Example 20

If Coulomb's force $F = 2a_x + a_y + a_z$ and N is acting on a charge of 10 C, find the electric field intensity.

(A) $0.2a_x + 0.1a_y + 0.1a_z$ (B) $2a_x + a_y + a_z$
(C) $20a_x + 10a_y + 10a_z$ (D) 0.2449

Solution

$$E = \frac{F}{Q} = \frac{2a_x + a_y + a_z}{10}$$

$$E = 0.2a_x + 0.1a_y + 0.1a_z$$

Example 21

Two wires are carrying in the same direction of 500 A and 800 A are placed with their axes 5 cm apart. Calculate the force between them.

(A) 0.4 N (B) 0.15 N
(C) 0.6 N (D) 0.8 N

Solution

$$I_1 = 500\ \text{A}$$

$$I_2 = 800\ \text{A}$$

$$r = 5 \times 10^{-2}\ \text{m}$$

$$F = \frac{\mu_0 I_1 I_2}{4\pi r} = \frac{4\pi \times 10^{-7} \times 500 \times 800}{4\pi \times 5 \times 10^{-2}}$$

$$F = \frac{4}{5} = 0.8\ \text{N}$$

Example 22

A point charge Q = 10 nC is at origin in free space. Find the electric field at P (1, 0, 1). Further, find the electric flux density at 'P'.

(A) $(0.281 \times 10^{-9})\ (a_x + a_z)$
(B) (0.281×10^{-9})
(C) $281\ (a_x + a_z)$
(D) $281.62\ (a_x + a_z)$

Solution

$$D = \varepsilon_0 E = \frac{Q}{4\pi r^2} a_r$$

$$r_2 = (1, 0, 1), r_1 = (0, 0, 0)$$

$$r = r_2\ r_1 = (1, 0, 1)$$

$$|r| = \sqrt{2}, a_r = \frac{r}{|r|} = \frac{10 \times 10^{-9}}{4\pi \times 2 \times \sqrt{2}} (a_x + a_z)$$

$$D = (0.281 \times 10^{-9})(a_x + a_z)$$

Example 23

A circular coil of radius 10 cm is made up of 100 turns. It carries a current of 5 A. Compute the magnetic field intensity at the centre of the coil

(A) 25 AT/m (B) 250 AT/m
(C) 2,500 AT/m (D) 25×10^{-3} AT/m

Solution

$$a = 10 \times 10^{-2}\ \text{m}$$

$$N = 100, I = 5\ \text{A}$$

$$H = \frac{NI}{2a}$$

$$H = \frac{100 \times 5}{2 \times 0.1}$$

$$H = 2{,}500\ \text{AT/m}$$

Example 24

Find T_r of seawater whose $\varepsilon_r = 81$ and $\sigma = 5$ ℧ /m

(A) 143.37 ps (B) 14.337 ps
(C) 1.4337 ps (D) 1,433.7 ps

Solution

$$\varepsilon = \varepsilon_r \varepsilon_0$$

$$\varepsilon = 81 \times 8.854 \times 10^{-12}$$

$$\varepsilon = 715.23 \times 10^{-12}$$

$$T_r = \frac{715.23 \times 10^{-12}}{5}$$

$$T_r = 143.37 \times 10^{-12}\ \text{s}$$

$$T_r = 143.37\ \text{ps}$$

Example 25

A parallel-plate capacitor with d = 1 m and plate area 0.8 m^2 and a dielectric relative permittivity of 2.8. A DC volt of 500 V is applied between the plates. Find the energy stored.

(A) 2.479 μJ (B) 24.79 μJ
(C) 247.9 μJ (D) 24.79 J

Solution

$d = 1$ m, $A = 0.8$ m^2, $\varepsilon = 2.8$,

and $V = 500$ V

$$C = \frac{\varepsilon_0 A \varepsilon_r}{d} = \frac{8.854\times10^{-12}\times0.8\times2.8}{1}$$

$$C = 19.83 \text{ pF}$$

$$E = \frac{1}{2}CV^2 = \frac{1}{2}\times(500)^2\times19.83\times10^{-12}$$

$$E = 2.479 \ \mu\text{J}$$

Example 26

Two thin parallel wires are carrying current in the same direction. The force experienced between them is:

(A) Attractive
(B) Repulsive
(C) Perpendicular to their axis joining wires
(D) No force exists

Solution: (A)

Example 27

Laplacian of a scalar function V is:

(A) Divergence of V
(B) Gradient of V
(C) Gradient of divergence of V
(D) Divergence of gradient of V

Solution: (D)

Example 28

Units of vector magnetic potential are

(A) A/m^2 (B) A/m (C) Wb/m^2 (D) Wb/m

Solution: (A)

Example 29

In a cylindrical conductor of radius 2 mm, the current density varies with the distance from the axis according to $J = 10^3 e^{-400r}$ A/m^2. Find the total current I.

(A) 8.894 A (B) 8.964 A
(C) $I = 8.649$ A (D) $I = 8.268$ A

Solution

$$I = \oint J.ds$$

$$r = 2 \text{ mm} = 0.002 \text{ m}$$

$$I = \int_{\varphi=0}^{2\pi}\int_{r=0}^{0.002} J.dr d\phi = \int_{\phi=0}^{2\pi}\int_{r=0}^{0.002} 10^3 e^{-400r} dr d\phi$$

$$= 10^3 \int_{\phi=0}^{2\pi}\left[\frac{e^{-400r}}{-400}\right]_0^{0.002} d\phi$$

$$= \frac{-10^3}{400}\int_0^{2\pi}[0.4493-1]d\phi$$

$$= \frac{-10}{4}(-0.55067)(2\pi-0)$$

$$I = 8.649 \text{ A}$$

Example 30

Calculate the magnetic flux density due to circular coil of 100 AT and area of 70 cm^2 on the axis of the coil at distance 10 cm from the centre.

(A) 102.7 μT (B) 103.7 μT
(C) 10.27 μT (D) 10.37 μT

Solution

$$NI = 100 \text{ AT},$$

$$\pi a^2 = 70\times10^{-4}$$

$$d = 0.10 \text{ m}$$

$$a^2 = 22.28\times10^{-4}$$

Magnetic flux density

$$B = \frac{\mu_0 N I a^2}{2(b^2+d^2)^{\frac{3}{2}}}$$

$$B = \frac{4\pi\times10^{-7}\times100\times22.28\times10^{-4}}{2(22.28\times10^{-4}+0.01)^{\frac{3}{2}}}$$

$$B = 103.7\times10^{-6} \text{ T}$$

$$B = 103.7 \ \mu\text{T}$$

Example 31

Determine the force per unit length between two long parallel wires separated by 5 cm in air and carrying currents of 40 A in the same direction.

(A) 6.4 N/m (B) 6.4×10^{-6} N/m
(C) 6.4×10^{-3} N/m (D) 6.4×10^{-9} N/m

Solution

$$\text{Force/length} = \frac{\mu_0 I_1 I_2}{2\pi D}$$

$$= \frac{40\times40}{2\pi\times5\times10^{-2}}\times4\pi\times10^{-7}$$

$$= 6.4\times10^{-3} \text{ N/m}$$

Example 32

Units of magnetic dipole moment are ______

(A) A/m (B) Am (C) A/m^2 (D) Am^2

Solution: (A)

Example 33

Solutions of Laplace's equation, which are continuous through the second derivative, are called

(A) Bessel functions
(B) Odd functions
(C) Harmonic functions
(D) Fundamental functions

Solution

Harmonic functions

Example 34

Find volume charge density if the electric field, $E = x^2a_x + 2y^2a_y + z^2a_z$ V/m in a medium whose $\varepsilon_r = 2$

(A) $\rho_v = 35.416x + 70.832y + 35.416z$ C/m^3
(B) $\rho_v = 35.416x + 70.832y$ C/m^3
(C) $\rho_v = 35.416x + 35.416y + 70.832z$ C/m^3
(D) none of these

Solution

$$E = x^2a_x + 2y^2a_y + z^2a_z \text{ V/m}$$

$$D = \in E = \varepsilon_0\varepsilon_r E$$

$$= 8.854 \times 10^{-12} \times 2 \times (x^2a_x + 2y^2a_y + z^2a_z)$$

$$D = 17.708x^2a_x + 35.416y^2a_y + 17.708z^2a_z, \text{ pC/m}^2$$

From Maxwell's equation, we have

$$\nabla D = \text{Div }(D) = \frac{\partial}{\partial x}D_x + \frac{\partial}{\partial y}D_y + \frac{\partial}{\partial z}D_z = \rho_v$$

$$\rho_v = 35.416x + 70.832y + 35.416z \text{ C/m}^3$$

Exercises

Practice Problems I

Direction for questions 1 to 20: Select the correct alternative from the given choices.

1. If a vector field $\overline{V}$ is related to another vector field $\overline{A}$ through $\overline{V} = \nabla \times \overline{A}$, which of the following is true. c and s_c are any closed contour and any surface whose boundary is c

(A) $\oint \overline{v}.d\overline{l} = \iint_{s_c} \overline{A}.d\overline{s}$

(B) $\int_c \overline{A}.d\ell = \iint_{s_c} \overline{v}.ds$

(C) $\oint_C (\nabla \times \overline{V}).\overline{dl} = \iint_{Sc} (\nabla \times \overline{A})..d\overline{s}$

(D) $\oint (\nabla \times \overline{A}).\overline{dl} = \iint_{Sc} \overline{v}.d\overline{s}$

2. If n is the unit normal vector to any closed surface s, then $\iiint_v \nabla.ndv$

(A) 0 (B) s (C) $\frac{s}{3}$ (D) 3s

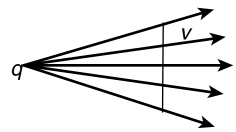
3. The electric field strength at a distance p due to a point charge $+q$ located on the origin is 10 µv/m. If the point charge now enclosed by a perfectly conducting metal sheet whose centre is at the origin, then the electric field strength at the point p, outside the sphere, becomes

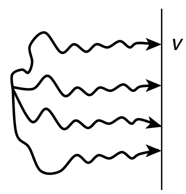
(A) 0 (B) 10 µV/m
(C) 100 µV/m (D) 50 µV/m

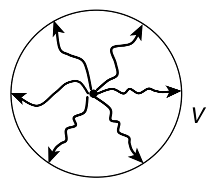
4. The infinite plane sheet at $z = 6$ m, there exists a uniform surface charge density of $\frac{1800}{\pi} n\text{C/m}^2$. Then, associated electric field strength is

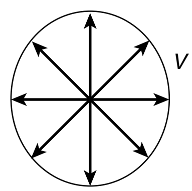
(A) 30 V/m (B) 32.4 V/m
(C) 32.4K V/m (D) 324 V/m

5. Electric field lines at the equipotential surface V are shown in the following figure. Which of the following is correct?

(A)


(B)


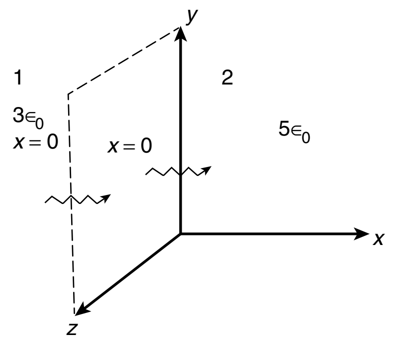
(C)


(D)


6. In an electrostatic field,

(A) $\nabla.\overline{E} = 0$ (B) $\nabla \times \overline{E} = 0$
(C) $\nabla.E = 0$ (D) none of these

7. The electric field E_1 in medium with $\varepsilon_1 = 3\epsilon_o$ is $E_1 = a_x - 5a_y + a_z$ V/m, while medium 2 has $\varepsilon_2 = 5\epsilon_o$ and $x = 0$ is boundary shown in the following figure.



Then, $\overline{E}_2$ is equal to

(A) $1.2a_x - 5a_y + a_z$ V/m

(B) $0.6a_x - 5a_y + a_z$ V/m

(C) $2\,a_x - 5a_y + a_z$ V/m

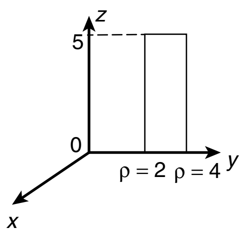
(D) $a_x - a_y + 5a_2$ V/m

8. Which of the following are true?

(A) $\overline{B} = \nabla.\overline{A}$ (B) $\overline{B} = \nabla \times \overline{A}$

(C) $\nabla.\overline{B} = 0$ (D) $\nabla \times \overline{B} = \mu_o \overline{J}$

9. Magnetic vector potential $\overline{A} = -\rho^2/4\; a_z$. Then, flux through the surface shown in the following figure is



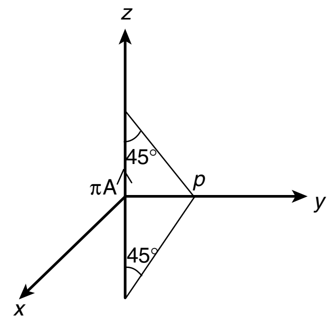
(A) 3 T (B) 5 T
(C) 15 T (D) 0 T

10. If $\overline{D} = (2y^2 + 2)a_x + 4xya_y \;\; + xa_z \; C/m^2$, then volume charge density ρ_v at (–1, 0, 3) is

(A) zero C/m^3 (B) 4 C/m^3

(C) –4 C/m^3 (D) 2 C/m^3

11. A finite length wire carrying current πA is placed along z-axis as shown in figure below.



The $\overline{H}$ at P(1, 1, 1) is

(A) $\frac{1}{\sqrt{2}}a_x$ A/m (B) $\frac{-1}{\sqrt{2}}a_x$ A/m

(C) $\frac{1}{\sqrt{2}}(a_x + a_y)$ A/m (D) $\frac{1}{\sqrt{2}}(a_x - a_y)$ A/m

12. In the field of a charge Q at the origin, the potentials at A (4, 0, 0) and B $\left(\frac{1}{2}, 0, 0\right)$ are V_A = 15 v, V_B = 60 v, respectively. Then, potential at C (2, 0, 0) is
(A) 35 V (B) 45 V (C) 30 V (D) 40 V

13. Find the work done in moving a 5 μc charge from origin to P(2, −1, 4)m via the straight line path $x = -2y$, $z = 2x$ through the field

$\overline{E} = (y\,\hat{a}_x + x\,\hat{a}_y + xy\,\hat{a}_z)$V/m.
(A) 22.2 μJ (B) 111.2 μJ
(C) 22.2 mJ (D) 111.2 mJ

14. Given $\overline{A}$ = yz $\hat{a}_x$ + xy $\hat{a}_y$ + xz $\hat{a}_z$, $|\nabla \times \overline{A}|$ at the point P(0, 1, 2) is
(A) 0 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) $\sqrt{5}$

15. $\overline{D}$ = $(4xy^2z^3\,\hat{a}_x + 3x^2z\,\hat{a}_y + 2y\,\hat{a}_z)$ nC/m^2. Find the amount of flux passing through the plane defined by $x = 3$; $0 \le y \le 2$; $0 \le z \le 1$ in a direction away from the origin.
(A) 4 nC (B) 3 nC (C) 2 nC (D) 8 nC

Direction for questions 16 and 17:
Select the value of K so that each of the fields satisfy Maxwell's equations.

16. Let $\overline{D} = (5x\,\hat{a}_x - 2y\,\hat{a}_y + Kz\,\hat{a}_z)$ μC/m^2 is defined in a region with charge-free and perfect dielectric
(A) −3 μC/m^3 (B) 3 μC/m^3
(C) −2 μC/m^3 (D) 2 μC/m^3

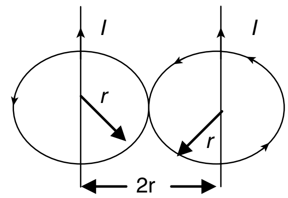
17. $\overline{E} = (Kx - 100t)\;\hat{a}_y$ V/m and $\overline{H} = (x + 20t)\,\hat{a}_z$ A/m in a region $\rho_v = 0$, σ = 0, and μ = 0.25 H/m
(A) −5 V/m^2 (B) +5 V/M^2

(C) $-\frac{1}{5}$ V/m^2 (D) $\frac{1}{5}$ V/m^2

18. The electric flux and field intensity inside a conducting sphere is
(A) zero (B) uniform
(C) maximum (D) minimum

19. A point charge of Q Coulombs is located at the origin. Find expression for the electric field at any point in the free space in spherical coordinates.
(A) $\overline{E} = \frac{Q}{4\pi \in r^2}\hat{a_r}$ (B) $\overline{E} = \frac{Q}{4\pi \in r^2}\hat{a_\varphi}$
(C) $\overline{E} = \frac{Q}{4\pi \in r^2}\hat{a_\theta}$ (D) $\overline{E} = \frac{Q}{4\pi \in r^2}(-\hat{a_r})$

20. Two infinitely parallel conductors are separated by a distance $2r$ and they carry equal and identical currents, as shown in the figure. Find the magnitude of magnetic field strength midway between these two fields.



(A) $|\overline{H}| = 0$ (B) $|\overline{H}| = \infty$
(C) $|\overline{H}| = \text{undefined}$ (D) $|\overline{H}| = 1$

Practice Problems 2

Direction for questions 1 to 15: Select the correct alternative from the given choices.

1. Which of the following is not the property of static magnetic fields?
(A) It is solenoid
(B) It is conservative
(C) It has no sinks or sources
(D) Magnetic flux lines are always closed

2. Interface of two regions of two magnetic materials is current free. Region 1 for which relative permeability $\mu_{r1} = 2$ is defined by $z < 0$ and region 2. $z > 0$ has $\mu_{r2} = 1$
If $\overline{B}_1 = 6a_x + 0.4a_y + 0.2a_z$ T, then $\overline{H}_2$(A/m) =
(A) $\frac{2}{\mu_o}[3a_x + 0.2a_y + 0.2a_z]$
(B) $\frac{1}{\mu_o}[6a_x + 0.4a_y + 0.2a_z]$
(C) $\frac{1}{\mu_o}[3a_x + 0.2a_y + 0.2a_z]$
(D) $\frac{2}{\mu_o}[6a_x + 0.4a_y + 0.1a_z]$

3. A conductor carrying a current I with a constant current density across its cross section, the magnetic field strength H at any distance $(r < R)$ from the centre of the conductor (radius R) is given by $(r < R)$
(A) $\overline{H} = \frac{I_r}{2\pi R}$ (B) $H = \frac{I_r}{2\pi R^2}$
(C) $H = \frac{I_r}{2\pi R^3}$ (D) $H = \frac{I_r}{2\pi R^4}$

4. If $V = \cosh x \cos ky.e^{2pz}$ is a solution of Laplace equation, then what is the value of K?
(A) $\sqrt{4+p^2}$ (B) $\sqrt{p^2/4 + 1}$
(C) $\sqrt{1+4p^2}$ (D) 0

5. If the magnetic flux density due to an infinite long wire at 1 m distance is $\overline{B} = 2\mu \frac{\text{wb}}{\text{m}^2} a_\phi$, then current =
(A) 1 A (B) 100 A (C) 1,000 A (D) 10 A

6. For any closed surface s, encloses a volume V. Then $\iint_s (\nabla \times F).nds =$
(A) 0 (B) S (C) V (D) 3 V

7. There are three charges that are given by $Q_1 = 1$ μc, $Q_2 = 4$ μc, and $Q_3 = 8$ μc
The field due to each charge at point p in free space is $a_x + 2a_y - 3a_z$, $a_x + 3a_y$, and $a_x - 3a_y + 3a_z$. Then, total field at P is due to all charges is
(A) $(a_x + 2a_y)$ N/C
(B) $(a_x - 2a_y)$ N/C
(C) $(a_x + 2a_y + 3a_2)$ N/C
(D) $(a_x + 2a_y + 3a_2)$ N/C

8. Two dielectric media with permittivity 2 and $\sqrt{2}$ are separated by a charge-free boundary, as shown in the figure. The $\overline{E}_1$ in medium 1 at point P_1 has magnitude E_1 and makes an angle $\alpha_1 = 30°$ with normal. The direction of $\overline{E}_2$ at point P_2 is α_2=
(A) $\sin^{-1}\left(\sqrt{\frac{1}{6}}\right)$ (B) $\sin^{-1}\left(\sqrt{\frac{1}{3}}\right)$
(C) $\tan^{-1}\left(\sqrt{\frac{1}{6}}\right)$ (D) 45°

9. $V = 4x + 2$ v, then the electric field is

(A) $4a_x$ V/m (B) $2a_x$ V/m

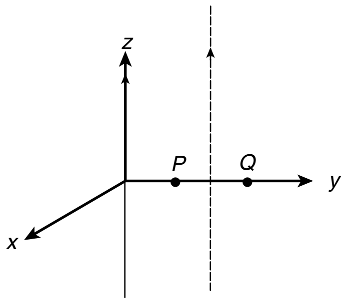
(C) $-4a_x$ V/m (D) $-2a_x$ V/m

10. Current element is represented by $4 \times 10^3\, a_y$ A/m and it is placed in a magnetic field $\overline{H} = \frac{10^{-3}}{2\mu} a_x$ A/m. Then, the force acting on the element is

(A) $2a_z$ N (B) $-2a_z$ N

(C) 0 N (D) $\frac{4}{2\mu}$ N

11. Two infinite long wires carrying current are placed along z-axis and along a line parallel to z-axis, as shown in the figure.



Find the component in the magnetic field H at Q on y-axis.

(A) x and y components
(B) Only y components
(C) Only x components
(D) x and z components

12. Two infinite plane sheets carry equal charge densities of 2×10^{-9} C/m^2 and placed at $x = 0$ and $x = 2$ planes shown in the figure. The electric displacement at the point $P(3, 0, 0)$ is shown in the following figure.

(A) 24 C/m^2 a_x

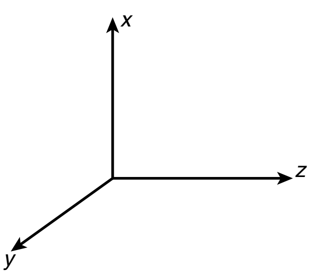
(B) -24 C/m^2 a_x

(C) 0

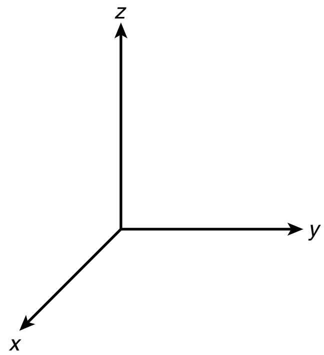
(D) $4n$C/m^2 a_x

13. Which of the following system does not form the right-handed coordinate system?

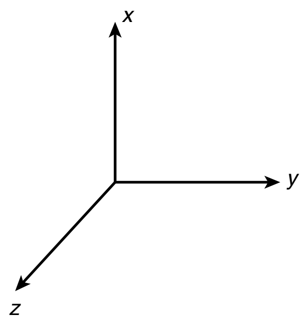
(A)



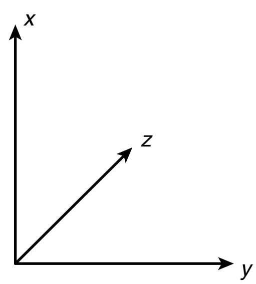
(B)



(C)



(D)



14. The line integral of the vector potential $\overline{A}$ around the boundary of a surface s represents

(A) scalar potential of the surface
(B) flux density in the surface
(C) flux through the surface
(D) current density

15. A metal sphere with 1 m radius and a surface charge density of $\frac{10}{\pi}$ coulomb/m^2 is enclosed in a cube of 10 m side. The total outward electric displacement normal to the surface of the cube is

(A) 0.4 C/m^2 (B) 4 C/m^2

(C) 40 C/m^2 (D) 400 C/m^2

Previous Years' Questions

1. For static electric and magnetic fields in an inhomogeneous source-free medium, which of the following represents the correct form of two of Maxwell's equations? **[2008]**

(A) $\nabla . E = 0$
$\nabla \times B = 0$

(B) $\nabla . E = 0$
$\nabla . B = 0$

(C) $\nabla \times E = 0$
$\nabla \times B = 0$

(D) $\nabla \times E = 0$
$\nabla . B = 0$

2. Consider a closed surface S surrounding a volume V. If $\vec{r}$ is the position vector of a point inside S, with $\hat{n}$ the unit normal on S, the value of the integral $\oiint_s 5\,\bar{r}.\,\hat{n}\, dS$ is **[2011]**

(A) 3 V (B) 5 V (C) 10 V (D) 15 V

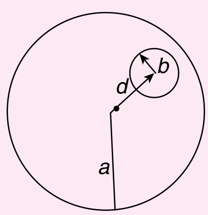
Direction for questions 3 and 4:

An infinitely long uniform solid wire of radius a carries a uniform DC current of density $\bar{j}$

3. The magnetic field at a distance r from the centre of the wire is proportional to **[2012]**

(A) r for $r < a$ and $1/r^2$ for $r > a$
(B) 0 for $r < a$ and $1/r$ for $r > a$
(C) r for $r < a$ and $1/r$ for $r > a$
(D) 0 for $r < a$ and $1/r^2$ for $r > a$

4. A hole of radius $b(b < a)$ is now drilled along the length of the wire at a distance d from the centre of the wire, as shown in the following figure.



The magnetic field inside the hole is **[2012]**

(A) uniform and depends only on d
(B) uniform and depends only on b
(C) uniform and depends on both b and d
(D) non-uniform

5. The divergence of the vector field

$\bar{A} = x\widehat{a}_x + y\widehat{a}_y + z\widehat{a}_z$ is **[2013]**

(A) 0 (B) 1/3 (C) 1 (D) 3

6. The force on a point charge $+q$ kept at a distance d from the surface of an infinite grounded metal plate in a medium of permittivity $\in$ is **[2014]**

(A) 0

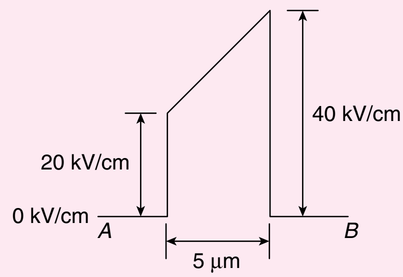
(B) $\dfrac{q^2}{16\pi \in d^2}$ away from the plate

(C) $\dfrac{q^2}{16\pi \in d^2}$ towards the plate

(D) $\dfrac{q^2}{4\pi \in d^2}$ towards the plate

7. Given the vector $A = (\cos x)(\sin y)\ \widehat{a}_x + (\sin x)(\cos x)\ \widehat{a}_y$, where $\widehat{a}_x, \widehat{a}_y$ denote unit vectors along x and y directions, respectively. The magnitude of curl of A is _____ **[2014]**

8. The electric field (assumed to be one-dimensional) between two points A and B is shown. Let Ψ_A and Ψ_B be the electrostatic potentials at A and B, respectively. The value of $\Psi_B - \Psi_A$ in volts is _____ **[2014]**

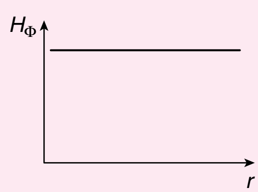


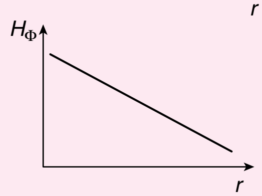
9. Given $\vec{F} = z\widehat{a}_x + x\widehat{a}_y + y\widehat{a}_z$ If S represents the portion of the sphere $x^2 + y^2 + z^2 = 1$ for $z \geq 0$, then $\int_s \nabla \times \vec{F}.\vec{ds}$ is ________ **[2014]**

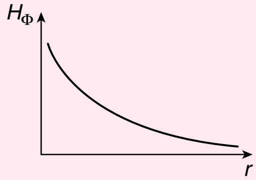
10. If $\vec{E} = -(2y^3 - 3yz^2)\hat{x} - (6xy^2 - 3xz^2)\hat{y} + (6xyz)\hat{z}$ is the electric field in a source-free region, a valid expression for the electrostatic potential is **[2014]**

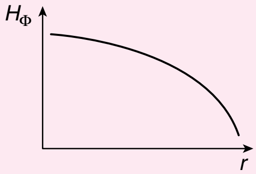
(A) $xy^3 - yz^2$ (B) $2xy^3 - xyz^2$
(C) $y^3 + xyz^2$ (D) $2xy^3 - 3xyz^2$

11. Consider a straight, infinitely long, current carrying conductor lying on the z-axis. Which one of the following plots (in linear scale) qualitatively represents the dependence of H_Φ on r, where H_Φ is the magnitude of the azimuthal component of magnetic field outside the conductor and r is the radial distance from the conductor? **[2015]**

(A)


(B)


(C) 

(D) 

12. In a source-free region in vacuum, if the electrostatic potential $\phi = 2x^2 + y^2 + cz^2$, the value of constant c must be ________. **[2015]**

13. Concentric spherical shells of radii 2m, 3m and 8m carry uniform surface charge densities of 20nC/m^2, –4nC/m^2 and ρ_s, respectively. The value of ρ (nC/m^2) required to ensure that the electric flux density $\vec{D} = \vec{0}$ at radius 10 m is ___________ . **[2016]**

14. The current density in a medium is given by $\vec{J} = \dfrac{400\sin\theta}{2\pi\left(r^2+4\right)}\ \hat{a}_r$ Am^{-2}.

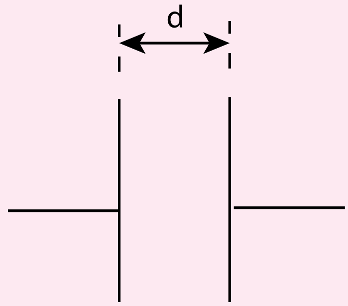
The total current and the average current density flowing through the portion of a spherical surface $r = 0.8$ m, $\dfrac{\pi}{12} \le \theta \le \dfrac{\pi}{4}$, $0 \le \phi \le 2\pi$ are given respectively, by **[2016]**

(A) 15.09A, 12.86Am^{-2}
(B) 8.73A, 13.65Am^{-2}
(C) 12.86 A, 9.23Am^{-2}
(D) 10.28A, 7.56Am^{-2}

15. A uniform and constant magnetic field $B = \hat{Z}B$ exists in the $\hat{Z}$ direction in vacuum. A particle of mass m with a small charge q is introduced in to this region with an initial velocity $V = \hat{X}V_x + \hat{Z}V_z$. Given that B, m, q, v_x and v_z are all non zero, which one of the following describes the eventual trajectory of the particle? **[2016]**

(A) Helical motion in the $\hat{Z}$ – direction
(B) Circular motion in the xy plane
(C) Linear motion in the $\hat{Z}$ – direction
(D) Linear motion in the $\hat{X}$ – direction

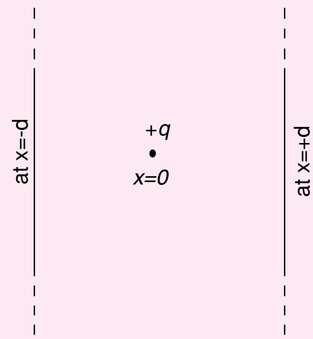
16. The parallel plate capacitor shown in the figure has movable plates. The capacitor is charged so that the energy stored in it is E when the plate separation is d. the capacitor is then isolated electrically and the plates are moved such that the plate separation becomes 2d.



At this new plate separation, what is the energy stored in the capacitor, neglecting fringing effects? **[2016]**

(A) $2E$
(B) $\sqrt{2}\ E$
(C) E
(D) $\dfrac{E}{2}$

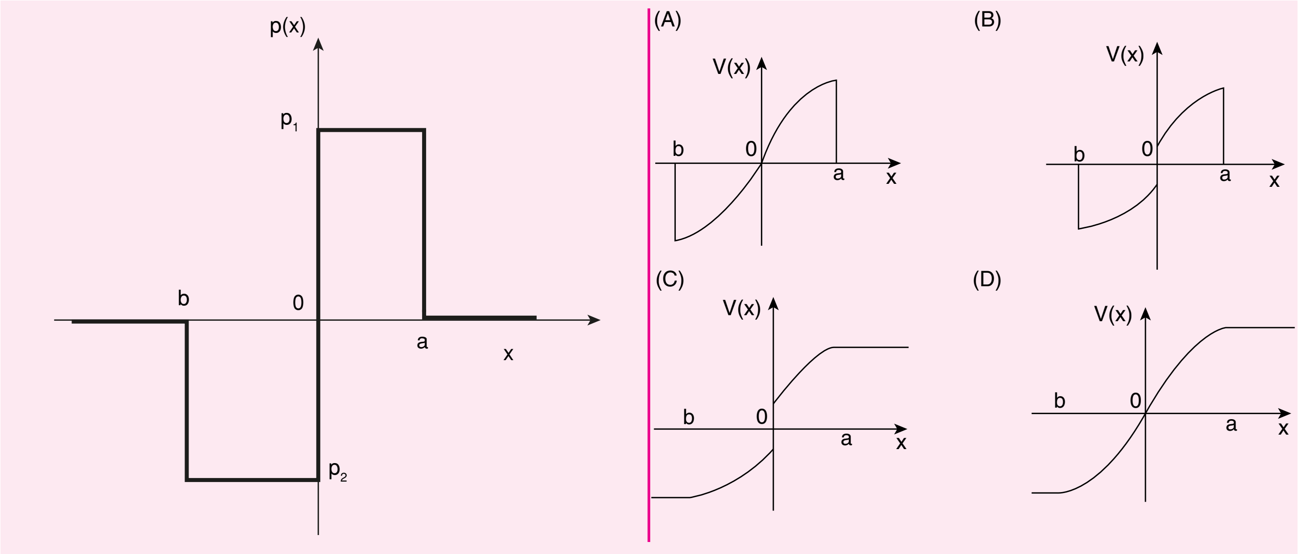
17. A positive charge q is placed at $x = 0$ between two infinite metal plates placed at $x = -d$ and at $x = +d$ respectively. The metal plates lie in the yz plane. **[2016]**



The charge is at rest at $t = 0$, when a voltage $+V$ is applied to the plate at $-d$ and voltage $-V$ is applied to the plate at $x = +d$. Assume that the quantity of the charge q is small enough that it does not perturb the field set up by the metal plates. The time that the charge q takes to reach the right plate is proportional to: **[2016]**

(A) $\dfrac{d}{V}$
(B) $\dfrac{\sqrt{d}}{V}$
(C) $\dfrac{d}{\sqrt{V}}$
(D) $\sqrt{\dfrac{d}{V}}$

18. Consider the charge profile shown in the figure. The resultant potential distribution is best described by **[2016]**



Answer Keys

Exercises

Practice Problems 1

1. B	**2.** B	**3.** B	**4.** C	**5.** D	**6.** B	**7.** B	**8.** B	**9.** C	**10.** C
11. B	**12.** C	**13.** B	**14.** B	**15.** D	**16.** A	**17.** A	**18.** A	**19.** A	**20.** A

Practice Problems 2

1. B	**2.** C	**3.** B	**4.** C	**5.** D	**6.** A	**7.** A	**8.** C	**9.** C	**10.** B
11. C	**12.** A	**13.** C	**14.** C	**15.** A					

Previous Years' Questions

1. D **2.** D **3.** C **4.** C **5.** D **6.** C **7.** 0 **8.** 14.5 to –15.5 **9.** 3.14
10. D **11.** C **12.** –3.1 to –2.9 **13.** –0.25nC/m^2 **14.** A **15.** A **16.** A **17.** C
18. D