

Chapter 1 Equations and Inequalities

Ex 1.1

Answer 1e.

In order to find the reciprocal of any nonzero real number, divide one by that number.

For example, the reciprocal of the number 3 is $\frac{1}{3}$.

Therefore, the given statement can be completed as “The reciprocal of any nonzero number b is $\frac{1}{b}$.”

Answer 1gp.

Any real number whether rational or irrational can be represented as a point on a line called the real number line

Express the fraction and radical in decimal form.

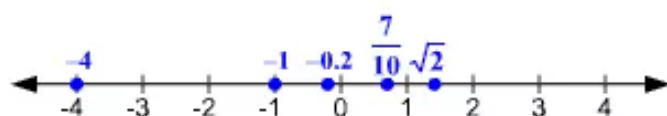
$$\frac{7}{10} = 0.7$$

$$\sqrt{2} = 1.41$$

From the decimal forms, we find that the point $\frac{7}{10}$ lies between 0 and 1, and the point

$\sqrt{2}$ lies between 1 and 2 on a number line.

Graph the given numbers on a number line.



Answer 2e.

The associative property of addition for real numbers a, b, c is given by:

$$a + (b + c) = (a + b) + c \quad \text{.....(1)}$$

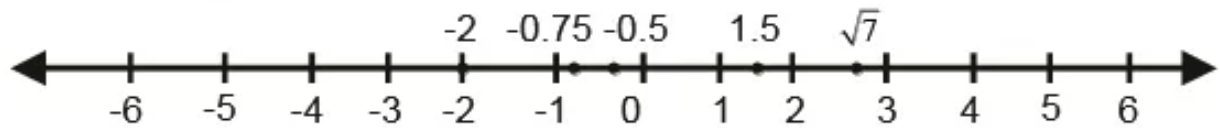
In words, associative property of addition means that the value of addition of real numbers remains same and does not change upon how the addition is done by combining any two numbers first and then adding the third number to their result. It allows us to collect any two real numbers first, add them and to their results add the third number.

In equation (1), on one side it can be seen that firstly, 2nd and 3rd number are being added and then to their result, 1st number is added. Whereas, on the other hand, firstly 1st and 2nd number are being added and then to their result 3rd number is added. Then, by equation (1), it can be seen that result of addition of these 3 real numbers remain same as done by the two above ways.

Answer 2gp.

As the number with negative sign and biggest value attached to negative sign becomes least in order and the number with positive sign and biggest value attached to positive sign becomes greatest in order. Using this rule of ordering of numbers, the list showing the numbers in increasing order is given by

$$-2, -0.75, 0.5, 1.5, \sqrt{7}$$



$$-2, -0.75, 0.5, 1.5, \sqrt{7}$$

Finally the option **C** is correct.

Answer 3e.

Any real number whether rational or irrational can be represented as a point on a line called the real number line.

Express the fractional numbers in decimal form.

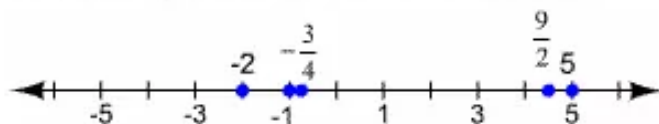
$$-\frac{3}{4} = -0.75$$

$$\frac{9}{2} = 4.5$$

From the decimal forms, we find that the point $-\frac{3}{4}$ lies between -1 and 0 , and the point

$\frac{9}{2}$ lies between 4 and 5 on a number line.

Graph the given numbers on a number line.



Answer 3gp.

Compare both sides of the given statement.

We can see that the numbers and operation symbols on both the sides are the same. The difference is that on the left side the product of 2 and 3 are grouped whereas on the right side the product of 3 and 9 are grouped. This shows that the order in which the numbers are multiplied is changed.

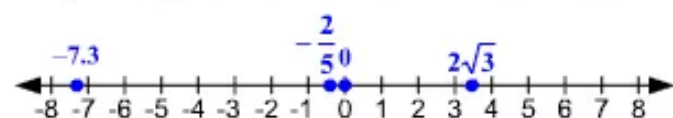
We know that according to associative property of multiplication, the order in which the numbers are multiplied does not change the result.

$$(a \cdot b) \cdot c = a \cdot (b \cdot c)$$

Therefore, the given statement illustrates the associative property.

From the decimal forms, we find that the point $-\frac{2}{5}$ lies between -1 and 0 , and the point $2\sqrt{3}$ lies between 3 and 4 on a number line.

Graph the given numbers on a number line.



Answer 4e.

Consider the list of given numbers:

$$-3, \frac{5}{2}, 2, \frac{-9}{4}, 4$$

Out of these numbers, $-3, 2, 4$ are integers. So, they can be directly plotted on the number line. However, $\frac{5}{2}$ and $\frac{-9}{4}$ are in fractions. So, for plotting them on number line, we have

to convert them into decimal form. Dividing 5 by 2 , we have $\frac{5}{2} = 2.5$

Also, dividing 9 by 4 , we have $\frac{-9}{4} = -2.25$. Thus, the list of given numbers can be written as:

$$-3, 2.5, 2, -2.25, 4.$$

Before plotting them on the number line, first arrange all of them in ascending as below

$$-3, -2.25, 2, 2.5, 4.$$

That is,

$$-3, \frac{-9}{4}, 2, \frac{5}{2}, 4$$

Now, plotting all of them on number line in ascending order from left to right as below.



Answer 4gp.

For 3 real numbers 6, 4 and 9

$$6.(4+9) = 6.4 + 6.9 \quad \text{.....(1)}$$

It can be shown by solving the left hand side and right hand side separately

$$\text{Left hand side} = 6.(4+9)$$

Simplify the brackets, it becomes

$$= 6.(13)$$

$$= 78$$

$$\text{Right hand side} = 6.4 + 6.9$$

Simplifying the multiplication, it becomes

$$= 24 + 54$$

$$= 78$$

Clearly, left hand side = right hand side

Hence, the property given by (1) represents distributive property of real numbers

Answer 4q.

Given expression is

$$6.(4+9) = 6.4 + 6.9 \quad \text{.....(1)}$$

Here, in left hand side of (1), the number 6 is being multiplied by the sum of 4 and 9. However, on right hand side, it can be seen that, 6 is being multiplied by 4 and 9 separately and then their sum is taken. This shows the multiplication of 6 is being distributive over the numbers 4 and 9 over their sum.

Hence, this shows the distributive property of real numbers. Also, as 6 is being multiplied on left side of the number 4 and 9 on both sides of given equation (1), so, this property is also called Left distributive property of real numbers

Answer 5e.

Real numbers can be graphed on a real number line, on which the numbers increase from left to right.

Convert the radical and fractions to decimal forms, approximated to the nearest tenth position.

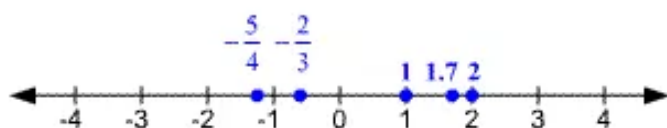
$$\sqrt{3} = 1.7$$

$$-\frac{2}{3} = -0.67$$

$$-\frac{5}{4} = -1.25$$

We find that $\sqrt{3}$ lies between 1 and 2, $-\frac{5}{4}$ lies between -1 and -2, and that $-\frac{2}{3}$ lies between 0 and -1 on a number line.

Now, plot the points on a number line.



Answer 5gp.

Observe both sides of the equation. We can see that on the left side of the equation, the sum of 5 and 25 is multiplied with 4 and on the right side of the equation, 4 is multiplied with each term inside the parentheses and calculate the sum afterwards.

According to the distributive property, for any 3 real numbers a , b , and c , we have $a(b + c) = ab + ac$.

Therefore, the statement illustrates the distributive property.

Answer 5q.

Observe the given equation.

In the expression on the left-hand side of the given equation, -5 is multiplied with 8 whereas on the right-hand side, 8 is multiplied with -5. We can see that difference is in the order of the multiplication.

The commutative property of multiplication states that changing the order of multiplication does not affect the result.

Thus, the statement illustrates the commutative property of multiplication.

Answer 6e.

Consider the list of given number as:

$$6, -\sqrt{5}, 2.7, -2, \frac{7}{3}$$

Out of these numbers, $6, -2, 2.7$ are already in decimal form and can be easily plotted on the number line. However, $\sqrt{5}, \frac{7}{3}$ can not be directly plotted. For plotting them, they have to be converted into decimal form.

Using Calculator, $-\sqrt{5} = -2.236$

Also, dividing 7 by 3, we have $\frac{7}{3} = 2.33$ Thus, list of given numbers can be written as:

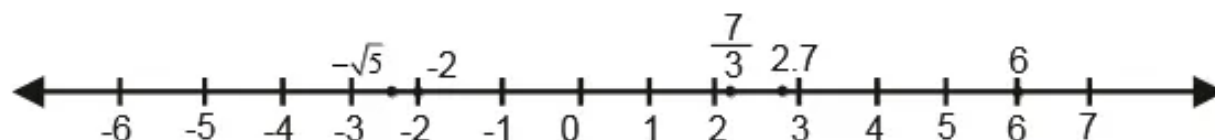
$$6, -2.236, 2.7, -2, 2.33$$

Before plotting all these on number line, first arrange all of them in increasing order from left to right as below:

$$-2.236, -2, 2.33, 2.7, 6$$

That is, $-\sqrt{5}, -2, \frac{7}{3}, 2.7, 6$

Now, plotting all these on the number line from left to right in the increasing order as below:



Answer 6gp.

Given expression is:

$$17 + (-17) = 0$$

This shows that on adding -17 to 17 , the result is 0 , which is the additive identity for real numbers.

Hence, -17 is called the additive inverse of 17 .

Thus, the given expression represents the property of additive inverse for real numbers.

Answer 6q.

Given expression is:

$$17 + (-17) = 0$$

This shows that on adding -17 to 17 , the result is 0 , which is the additive identity for real numbers.

Hence, -17 is called the additive inverse of 17 .

Thus, the given expression represents the property of additive inverse for real numbers.

Answer 7e.

Any real number can be represented as a point on a number line.

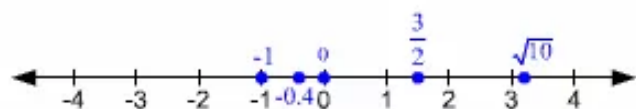
For the ease of representation, convert the rational and irrational numbers to their decimal forms.

$$\frac{3}{2} = 1.5$$

$$\sqrt{10} \approx 3.2$$

Now, graph the numbers on the number line.

The number $\frac{3}{2}$ is graphed between 1 and 2, and $\sqrt{10}$ is graphed between 3 and 4.

**Answer 7gp.**

Convert the division into multiplication. For this, multiply 4 by the reciprocal of b .

$$b \cdot (4 \div b) = b \cdot \left(4 \cdot \frac{1}{b} \right)$$

Apply the commutative property of multiplication within the parentheses.

$$b \cdot \left(4 \cdot \frac{1}{b} \right) = b \cdot \left(\frac{1}{b} \cdot 4 \right)$$

Use the associative property of multiplication.

$$b \cdot \left(\frac{1}{b} \cdot 4 \right) = \left(b \cdot \frac{1}{b} \right) \cdot 4$$

Apply the inverse property of multiplication.

$$\left(b \cdot \frac{1}{b}\right) \cdot 4 = 1 \cdot 4$$
$$= 4$$

The left side of the equation simplifies to 4. The right side expression of the given equation is also 4.

Therefore, the given statement is true.

Answer 7q.

Given expression is:

$$10m + 32 \text{ when } m = -5$$

Put $m = -5$ in given expression, it becomes

$$= 10(-5) + 32$$

$$= -50 + 32$$

$$= -18$$

So, the value of expression $10m + 32$ when $m = -5$ is -18

Answer 8e.

Consider the list of given numbers

$$-1.7, 5, \frac{9}{2}, -\sqrt{8}, -3$$

Out of these numbers, $-1.7, 5, -3$ are already in decimal form and can be easily plotted on the number line. But, $\frac{9}{2}$ and $-\sqrt{8}$ have to be converted into their decimal form

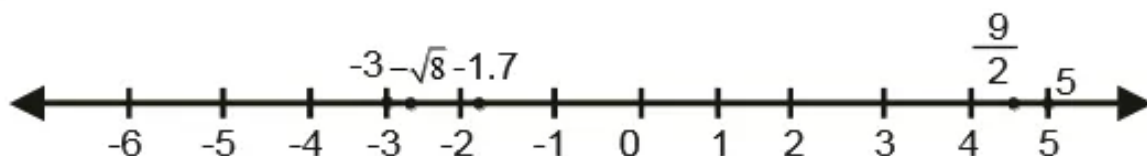
Dividing 9 by 2, we have $\frac{9}{2} = 4.5$ and using Calculator $-\sqrt{8} = -2.8$

Arranging the list of given numbers in increasing order from left to right

$$-3, -2.8, -1.7, 4.5, 5$$

That is, $-3, -\sqrt{8}, -1.7, \frac{9}{2}, 5$

Now, plotting all these on the number line from left to right in the increasing order as below:



Answer 8gp.

Given statement to be verified:

$$3x + (6 + 4x) = 7x + 6$$

$$\text{Left hand side} = 3x + (6 + 4x)$$

Apply commutative property of addition in brackets, it becomes

$$= 3x + (4x + 6)$$

Apply associative property of addition, it becomes

$$= (3x + 4x) + 6$$

Adding the like terms, it becomes

$$= 7x + 6$$

$$= \text{right hand side}$$

Hence, left hand side = right hand side

Thus, the given statement has been verified

Answer 8q.

Given expression is:

$$12 + (8 - n)^3 \text{ where } n = 5$$

Put $n = 5$, it becomes

$$= 12 + (8 - 5)^3$$

Simplifying the brackets, it becomes

$$= 12 + (3)^3$$

Simplifying power expression as $(3)^3 = 27$, it becomes

$$\begin{aligned} &= 12 + 27 \\ &= 39 \end{aligned}$$

Hence, solution of expression $12 + (8 - n)^3$ where $n = 5$ is 39

Answer 9e.

In order to arrange a set of values in an increasing order, the element with the least value should come first, followed by the next greater value. This pattern should be continued and the element with the greatest value should be at the last position.

We require the elevation from the least to the greatest, not from the greatest to the least. Thus, choice D can be eliminated.

In the increasing order, the highest elevations can be arranged as 535, 2407, 4145, 6643, 14,494.

Therefore, choice B represents the correct order.

Answer 9gp.

The amount in dollars earned per hour is the earning rate. It is given that earning for 6 hours is 69 dollars.

$$\begin{aligned}\text{Earning rate} &= \frac{69 \text{ dollars}}{6 \text{ hour}} \\ &= 11.5 \text{ dollars per hour}\end{aligned}$$

Thus, the earning rate is \$11.5.

CHECK:

We can find the total amount earned by finding the product of the earning rate and the working hours.

$$\begin{aligned}\text{Money earned} &= \frac{11.5 \text{ dollars}}{\cancel{\text{hour}}} \cdot 6 \cancel{\text{hour}} \\ &= 69 \text{ dollars}\end{aligned}$$

Thus, the solution checks.

Answer 9q.

Substitute -2 for p in the equation.

$$p^3 - 3 \cdot p^2 = (-2)^3 - 3(-2)^2$$

There is more than one operation to be performed. We can evaluate using the order of operations.

By the order, grouping symbols have to be evaluated first. But the sets of parentheses in this expression are not used for grouping. Since there are no grouping symbols in the given expression, the powers have to be evaluated first.

$$(-2)^3 - 3(-2)^2 = -8 - 3(4)$$

Multiplication has higher precedence than subtraction.

Perform the multiplication next.

$$-8 - 3(4) = -8 - 12$$

Now, subtract.

$$-8 - 12 = -20$$

Therefore, the result is -20 .

Answer 10e.

Consider the list of numbers in lowest elevation (in ft):

$$0, -282, 257, -8, 178$$

As the number with negative sign and biggest value attached to negative sign becomes least in order and the number with positive sign and biggest value attached to positive sign becomes greatest in order. Using this rule of ordering of numbers, the list which shows the lowest elevation in order from greatest to least, that is, in decreasing order is as follows:

$$257, 178, 0, -8, -282$$

Clearly, this order matches with the order of list as given in option D

Hence, the list showing lowest elevations in order from greatest to least is option **D**

Answer 10gp.

Distance traveled = 180 miles

speed = 40 miles per hour

Time taken = ?

By definition,

$$\text{Time taken} = \frac{\text{distance travelled}}{\text{speed}}$$

$$= \frac{180 \text{ miles}}{\left(\frac{40 \text{ miles}}{1 \text{ hour}} \right)}$$

$$= \frac{180 \text{ miles}}{40 \text{ miles}} \times 1 \text{ hour}$$

$$= \frac{9}{2} \text{ hour}$$

$$\boxed{\text{Time taken} = 4.5 \text{ hour}}$$

Answer 10q.

Given expression is:

$$8x + 6x^2 - 9x^2 - 4x$$

Grouping like terms, it becomes :

$$= (6x^2 - 9x^2) + (8x - 4x)$$

$$= -3x^2 + 4x$$

$$\text{Hence, } \boxed{8x + 6x^2 - 9x^2 - 4x = -3x^2 + 4x}$$

Answer 11e.

Observe the given statement.

We can see that the numbers and the operation symbols on both the sides are the same. The difference is that, on the left-side the sum of 4 and 9 are grouped whereas on the right-side the sum of 9 and 3 are grouped. This shows that the order in which the numbers are added is changed.

We know that according to the associative property of addition, the order in which the numbers are added does not change the result.

$$(a + b) + c = a + (b + c)$$

Therefore, the given statement illustrates the associative property of addition.

Answer 11gp.

One mile is equal to 1.61 km. Thus, to convert km to mile, multiply by the conversion rate 1 mile/1.61 km.

$$\frac{60 \text{ kilometer}}{1 \text{ hour}} = \frac{60 \text{ kilometer}}{1 \text{ hour}} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometer}}$$

Simplify.

$$\frac{60 \text{ kilometer}}{1 \text{ hour}} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometer}} \approx \frac{37.27 \text{ miles}}{1 \text{ hour}}$$

Therefore, the required result is 37.27 km/hour.

Answer 11q.

Apply the distributive property to clear the parentheses in the expression.

$$5(x + 9) - 2(4 - x) = 5x + 45 - 8 + 2x$$

Now, we can simplify the expression by combining the like terms. Variable terms with the same variable raised to the same exponent are called like terms. Constants are also called like terms.

The expression has $5x$ and $2x$ as variable terms, and 45 and -8 as constants. Group and combine the like terms.

$$\begin{aligned} 5x + 45 - 8 + 2x &= (5x + 2x) + (45 - 8) \\ &= 7x + 37 \end{aligned}$$

Therefore, the simplified expression is $7x + 37$.

Answer 12e.

Given expression is

$$15 \cdot 1 = 15$$

This shows that on multiplying 1 with the real number 15, the result is again the same real number, that is, 15. Thus, it shows that on multiplying 15 by 1, 1 does not change the value of 15 hence, it kept the identity of 15 same under the operation of multiplication. Due to this, 1 is called the multiplicative identity of real numbers. Thus, the property in the above given expression is the property of multiplicative identity of real numbers.

Answer 12gp.

Given conversion is :

150 yards to feet

As 1 yard = 3 feet

Multiply both sides by 150, it becomes

$$150 \times 1 \text{ yard} = 150 \times 3 \text{ feet}$$

$$150 \text{ yards} = 450 \text{ feet}$$

Answer 12q.

Given expression is :

$$24x - 6y + 15y - 18x$$

Grouping like terms, it becomes :

$$\begin{aligned} &= (24x - 18x) + (15y - 6y) \\ &= 6x + 9y \end{aligned}$$

Hence, $24x - 6y + 15y - 18x = 6x + 9y$

Answer 13e.

Observe the expression on each side of the equation.

On the left side of the equation, 6 is multiplied by 4 and on the right side, 4 is multiplied by 6. The only difference we can observe is the change in the order of multiplication.

The commutative property of multiplication states that the numbers can be multiplied in any order. The result will always remain the same.

Thus, the given statement illustrates the commutative property of multiplication.

Answer 13gp.

To convert 1 gallon to pints multiply it with 8, because 1 gallon is equal to 8 pints.

$$1 \text{ gallon} = 8 \text{ pints}$$

Thus, to convert 4 gallons to pints, multiply it by 4.

$$\begin{aligned} 4 \text{ gallon} &= 8 \cdot 4 \\ &= 32 \end{aligned}$$

Therefore, the result is 32 pints.

Answer 13q.

The amount of money left on the gift card after purchasing n CD's can be determined by finding the difference between the original amount in the gift card and the cost of n CD's.

Let us write a verbal model.

Amount in gift card	–	Number of CD's	·	Price of one CD
<i>(dollars)</i>				<i>(dollars)</i>
↓		↓		↓
100	–	n	·	8

Thus, we obtain an expression for the amount of money left in the card.

Replace n with 6.

$$100 - 8n = 100 - 8(6)$$

Perform multiplication first.

$$100 - 8(6) = 100 - 48$$

Subtract.

$$100 - 48 = 52$$

Therefore, the money left after purchasing 6 CD's is \$52.

Answer 14e.

Given expression is:

$$5 + (-5) = 0$$

This shows that on adding -5 to 5 , the result is 0 , which is the additive identity for real numbers. Hence, -5 is called the additive inverse of 5 .

Thus, the given expression represents the property of additive inverse for real numbers.

Answer 14gp.

Given Conversion is:

16 yards to seconds

As, 1 year contain 365 days which further contain 24 hours. Each hour has 60 minute and each minute contain 60 seconds.

$$\begin{aligned}\text{Thus, 1 year} &= 365 \times 24 \times 60 \times 60 \text{ seconds} \\ &= 31536000 \text{ seconds}\end{aligned}$$

$$\begin{aligned}\text{Hence, 16 years} &= 16 \times 31536000 \\ &= 504576000 \text{ seconds}\end{aligned}$$

Writing this in the standard notation,

$$\text{Hence, } \boxed{16 \text{ years} = 5.04576 \times 10^8 \text{ seconds}}$$

Answer 15e.

Observe the given statement.

The left-hand side of the given statement is the product of a number and a sum whereas the right-hand side is the sum of two products.

This is the same as the distributive property according to which $a(b + c) = a(b) + a(c)$.

Therefore, the given statement illustrates the distributive property for real numbers.

Answer 16e.

Given expression is:

$$(6.5).7 = 6.(5.7)$$

The associative property of multiplication for real numbers 6, 5, 7 is given by:

$$(6.5).7 = 6.(5.7) \quad \text{.....(1)}$$

This statement represents the associative property of real numbers and as it is known that multiplication of real numbers is always associative, that is we can multiply any two numbers first and then multiply the result with third number.

In words, associative property of multiplication means that the value of multiplication of real numbers remains same and does not change upon how the multiplication is done by combining any two numbers first and then multiplication the third number to their result. It allows us to collect any two real numbers first, multiply them and to their results multiply the third number.

In equation (1), on one side it can be seen that firstly 1st and 2nd number are being multiply and then to their result 3rd number is multiply.

Whereas, on the other hand, firstly 2nd and 3rd number are being multiply and then to their result 1st number is multiply. Then, by equation (1), it can be seen that result of multiplication of these 3 real numbers remain same as done by the two above ways.

The property given by (1) represents associative property of multiplication.

Answer 17e.

Simplify the expression on the left side and compare this with the simplified form given on the right side.

Convert the division into multiplication. For this, multiply a by the reciprocal of 3.

$$6 \cdot (a \div 3) = 6 \cdot \left(a \cdot \frac{1}{3} \right)$$

Use the commutative property of multiplication within the parentheses.

$$6 \cdot \left(a \cdot \frac{1}{3} \right) = 6 \cdot \left(\frac{1}{3} \cdot a \right)$$

Apply the associative property of multiplication.

$$6 \cdot \left(\frac{1}{3} \cdot a \right) = \left(6 \cdot \frac{1}{3} \right) \cdot a$$

Apply the associative property of multiplication.

$$6 \cdot \left(\frac{1}{3} \cdot a \right) = \left(6 \cdot \frac{1}{3} \right) \cdot a$$

Simplify the expression.

$$\begin{aligned} \left(6 \cdot \frac{1}{3} \right) \cdot a &= 2 \cdot a \\ &= 2a \end{aligned}$$

The expression simplifies to $2a$. We can see that the left side of the equation is also $2a$. Therefore, the given statement is true.

Answer 18e.

Given statement to be verified :

$$15.(3 \div b) = 45 \div b$$

$$\text{Left hand side} = 15.(3 \div b)$$

Using definition of division of real numbers

$$= 15 \cdot \left(3 \cdot \frac{1}{b} \right)$$

Apply associative property of multiplication, it becomes

$$= (15 \cdot 3) \cdot \frac{1}{b}$$

Simplify the multiplication in brackets, it becomes

$$= 45 \cdot \frac{1}{b}$$

Apply definition of division again, it becomes

$$= 45 \div b$$

$$= \text{Right hand side}$$

Hence, above statement is true.

Answer 19e.

Simplify the expression on the left side of the equation and compare this with the simplified form given on the right side.

By the definition of subtraction, convert the subtraction to addition. For this, add the opposite of the number being subtracted.

$$(c - 3) + 3 = [c + (-3)] + 3$$

Apply the associative property of addition.

$$[c + (-3)] + 3 = c + [(-3) + 3]$$

Use the inverse property of addition within the parentheses.

$$c + [(-3) + 3] = c + 0$$

Apply the identity property of addition.

$$c + 0 = c$$

The expression simplifies to c , which is the same as the right side expression of the given equation.

Thus, the statement is true.

Answer 20e.

Given statement to be verified:

$$(a + b) - c = a + (b - c)$$

$$\text{Right hand side} = a + (b - c)$$

Apply definition of subtraction in brackets, it becomes

$$= a + [b + (-c)]$$

Apply associative property of addition, it becomes

$$= (a + b) + (-c)$$

Apply definition of subtraction again, it becomes

$$= (a + b) - c$$

$$= \text{left hand side}$$

Hence, above statement is true.

Answer 21e.

Observe the equation. Simplify the expression on the left side of the equation and compare this with the expression given on the right side of the equation.

Apply the commutative property of addition within the parentheses.

$$7a + (4 + 5a) = 7a + (5a + 4)$$

Group the like terms using the associative property of addition.

$$7a + (4 + 5a) = (7a + 5a) + 4$$

Combine the like terms within the parentheses.

$$(7a + 5a) + 4 = 12a + 4$$

The expression simplifies to $12a + 4$, which is the same as the right side expression of the given equation.

Thus, the given statement is true.

Answer 22e.

Given expression to be verified:

$$(12b + 15) - 3b = 15 + 9b$$

$$\text{left hand side} = (12b + 15) - 3b$$

Apply commutative property of addition in brackets, it becomes

$$= (15 + 12b) - 3b$$

Apply definition of subtraction, it becomes

$$= (15 + 12b) + (-3b)$$

Apply associative property of addition, it becomes

$$= 15 + [12b + (-3b)]$$

Apply definition of subtraction, it becomes

$$= 15 + (12b - 3b)$$

Subtracting like terms in brackets, it becomes

$$= 15 + 9b$$

$$= \text{right hand side}$$

Hence, equation $(12b + 15) - 3b = 15 + 9b$ is verified.

Answer 23e.

We can find several values for a and b that satisfies the given condition.

Let us take a as -2 , and b as $\frac{1}{4}$.

Assign these values in $a \div b$.

$$a \div b = -2 \div \frac{1}{4}$$

Convert the division into multiplication. For this, multiply -2 by the reciprocal of $\frac{1}{4}$.

$$\begin{aligned}-2 \div \frac{1}{4} &= -2 \cdot 4 \\ &= -8\end{aligned}$$

One possible answer is $a = -2$, and $b = \frac{1}{4}$.

Similarly, other answers are also possible.

Answer 24e.

Distributive property of real numbers

Example 1 : For 3 real numbers 2, 3 and 5

$$2.(3+5) = 2.3 + 2.5$$

It can be shown by solving the left hand side and right hand side separately

$$\text{Left hand side} = 2.(3+5)$$

Simplify the brackets, it becomes

$$\begin{aligned}&= 2.(8) \\ &= 16\end{aligned}$$

$$\text{Right hand side} = 2.3 + 2.5$$

Simplifying the multiplication, it becomes

$$\begin{aligned}&= 6 + 10 \\ &= 16\end{aligned}$$

Clearly,

left hand side = right hand side

Example 2 : For 3 real numbers 3, 4 and 6

$$(3+4).6 = 3.6 + 4.6$$

It can be shown by solving the left hand side and right hand side separately

$$\text{Left hand side} = (3+4).6$$

Simplify the brackets, it becomes

$$= 7.6$$

$$= 42$$

$$\text{Right hand side} = 3.6 + 4.6$$

Simplifying the multiplication, it becomes

$$= 18 + 24$$

$$= 42$$

Clearly, left hand side = right hand side

Example 3: For 3 real numbers 2, 3 and 4

$$(2+3).4 = 2.4 + 3.4$$

It can be shown by solving the left hand side and right hand side separately

$$\text{Left hand side} = (2+3).4$$

Simplify the brackets, it becomes

$$= 5.4$$

$$= 20$$

$$\text{Right hand side} = 2.4 + 3.4$$

Simplifying the multiplication, it becomes

$$= 8 + 12$$

$$= 20$$

Clearly, left hand side = right hand side

Answer 25e.

You can find your earning rate by dividing the total amount earned by the total time worked.

The division of two numbers can be performed by multiplying one number by the reciprocal of the other number.

$$a \div b = a \cdot \frac{1}{b}, b \neq 0$$

Since the total amount earned is given as \$85 and the total time worked as 10 hours, you get a as 85, and b as 10.

$$\begin{aligned} 85 \div 10 &= 85 \cdot \frac{1}{10} \\ &= 8.5 \end{aligned}$$

Use unit analysis to check the answer.

$$85 \text{ dollars} \cdot \frac{1}{10 \text{ hours}} = 8.5 \text{ dollars per hour}$$

Therefore, the total earning rate is \$8.5 per hour.

Answer 26e.

Given is:

$$\text{Distance traveled} = 60 \text{ km}$$

$$\text{Time taken} = 1.5 \text{ hours}$$

By definition,

$$\begin{aligned} \text{Average speed} &= \frac{\text{Distance travelled}}{\text{Time taken}} \\ &= \frac{60 \text{ km}}{1.5 \text{ hr}} \\ &= \frac{60}{\left(\frac{15}{10}\right)} \text{ km/hr} \\ &= \frac{60 \times 10}{15} \text{ km/hr} \\ &= 40 \text{ km/hr} \end{aligned}$$

So, Average Speed = 40 km/hr

Answer 27e.

You can find the amount earned by multiplying the earning rate by the total time worked. The earning rate is given as \$7.25 per hour, and the total time worked as 5 hours.

$$\begin{aligned}\text{Amount earned} &= 7.25 \cdot 5 \\ &= 36.25\end{aligned}$$

Use unit analysis to check the result.

$$\frac{7.25 \text{ dollars}}{1 \text{ hour}} \cdot 5 \text{ hours} = 36.25 \text{ dollars}$$

Therefore, the total amount earned is \$36.25.

Answer 28e.

$$\begin{aligned}\text{Amount of juice bought} &= 6 \text{ gallons} \\ \text{Cost of juice per gallons} &= \$1.25 \\ \text{So, total cost of 6 gallons juice} &= \$1.25 \times 6 \\ &= \$7.5\end{aligned}$$

Hence, total cost of 6 gallons juice = \$7.5

Answer 29e.

The total distance traveled can be calculated by multiplying the speed by the total time taken for the journey. The speed is given as 65 miles per hour, and time taken for the journey as 3 hours.

$$\text{Distance} = 65 \cdot 3$$

Multiply the numbers on the right-side.

$$65 \cdot 3 = 195$$

Use unit analysis to check the result.

$$\frac{65 \text{ miles}}{1 \text{ hour}} \cdot 3 \text{ hours} = 195 \text{ miles.}$$

Therefore, total distance traveled is 195 miles.

Answer 30e.

Given is:

$$\text{Distance traveled} = 175 \text{ miles}$$

$$\text{Average speed} = 50 \text{ miles per hour}$$

By Definition,

$$\begin{aligned}
 \text{Time taken} &= \frac{\text{Distance travelled}}{\text{Average Speed}} \\
 &= \frac{175 \text{ miles}}{50 \text{ miles/hour}} \\
 &= \frac{175 \text{ miles}}{\left(\frac{50 \text{ miles}}{1 \text{ hour}} \right)} \\
 &= \frac{175}{50} \text{ hour}
 \end{aligned}$$

Hence, $\text{Time taken} = 3.5 \text{ hour}$

Answer 31e.

We know that one foot is equal to 0.33 yards. Use this information to convert 350 feet to yards.

$$\begin{aligned}
 351 \cdot 1 \text{ foot} \left(\frac{0.33 \text{ yards}}{1 \text{ foot}} \right) &= 351 \cdot 0.33 \text{ yards} \\
 &\approx 116.666 \text{ yards}
 \end{aligned}$$

Approximate the value obtained to the nearest tenth position.

$$116.666 \text{ yards} \approx 116.7 \text{ yards}$$

Therefore, 350 feet is approximately equal to 116.7 yards.

Answer 32e.

Given conversion is:

15 meters to millimeters.

$$\text{As, } 1 \text{ millimeter} = \frac{1}{1000} \text{ meter}$$

$$\text{So, } 1 \text{ meter} = 1000 \text{ millimeters}$$

$$\text{Thus, } 15 \text{ meters} = 15 \times 1000 \text{ millimeters}$$

Hence, $15 \text{ meters} = 15000 \text{ millimeters}$

Answer 33e.

We know that one kilogram is equal to 1000 grams. This means that to convert the value given in kilograms to grams, we have to multiply it by 1000.

$$2.2 \text{ kilograms} \cdot \frac{1000 \text{ grams}}{1 \text{ kilogram}} = 2200 \text{ grams}$$

Therefore, 2.2 kilograms equal 2200 grams.

Answer 34e.

Given conversion is:

5 hours to minutes

There are 60 minutes in an hour.

So, 1 hour = 60 minutes

Thus, 5 hours = 5×60 minutes
= 300 minutes

Hence, 5 hours = 300 minutes

Answer 35e.

We know that one quart equals 0.25 gallons. This means that to convert quarts to gallons, we have to multiply by 0.25.

$$7 \text{ quarts} \cdot \frac{0.25 \text{ gallons}}{1 \text{ quart}} = 1.75 \text{ gallons}$$

Therefore, 7 quarts equal 1.75 gallons.

Answer 36e.

3.5 tons to pounds

1 ton contain 2000 pounds

So, 1 ton = 2000 pounds

Thus, 3.5 tons = 3.5×2000 pounds
= 7000 pounds

Hence, 3.5 tons = 7000 pounds

Answer 37e.

Let us first convert the value given in ounces to kilograms, and then convert it from kilograms to tons. We know that one ounce is equal to 0.028 kilograms. Use this information to convert 56 ounces to kilograms.

$$56 \text{ ounces} \cdot \frac{0.028 \text{ kilograms}}{1 \text{ ounce}} = 1.58 \text{ kilograms}$$

One kilogram is equal to 0.0011 tons. In order to convert 1.58 kilograms to tons, multiply by 0.0011.

$$1.58 \text{ kilograms} \cdot \frac{0.0011 \text{ tons}}{1 \text{ kilograms}} = 0.00175 \text{ tons}$$

Therefore, 56 ounces equal 0.00175 tons.

Answer 38e.

6800 seconds to hours.

As 1 hour contain 60 minutes and each minute further contain 60 seconds.

So, 1 hour = 60 × 60 seconds

$$1 \text{ hour} = 3600 \text{ seconds}$$

Thus, 3600 seconds = 1 hour

$$\text{So, } 1 \text{ second} = \frac{1}{3600} \text{ hours}$$

$$\begin{aligned} \text{Thus, } 6800 \text{ seconds} &= \frac{1}{3600} \times 6800 \text{ hours} \\ &= \frac{17}{9} \text{ hours} \end{aligned}$$

Hence, $6800 \text{ seconds} = 1.88 \text{ hours (approx.)}$.

Answer 39e.

One dollar is equivalent to 0.82 euros. The conversion rate for the given case should be 0.82 euros / dollar.

Write the conversion using unit analysis.

$$25 \text{ dollars} \cdot \frac{0.82 \text{ euros}}{1 \text{ dollar}} \approx 20.5 \text{ euros}$$

Compare this conversion with the given conversion.

You can see that the error was in the conversion rate. Instead of taking the conversion rate as 0.82 euros/ dollar, it was incorrectly taken as 1 dollar/0.82 euros.

Thus, the error was in the conversion rate.

Answer 40e.

In the question given

$$5 \text{ pints} \times \frac{1 \text{ cup}}{2 \text{ pints}} = 2.5 \text{ cups}$$

It tells us that to convert pints into cups, we have to divide number of pints by 2 and the resulting number will be the number of cups

But, however, in actual, 1 pint = 2 Cups

This means, we have to multiply number of pints by 2

So, 5 pints = cups

Hence, 5 pints = 10 cups

Answer 41e.

There are two conversions to be performed, of which one is from miles to feet, and the other is from hours to seconds.

One mile is equal to 5280 feet. Thus, to convert 20 miles to feet, multiply by 5280.

$$\begin{aligned} 20 \text{ miles} &= 20 \text{ miles} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \\ &= 20 \cdot 5280 \text{ feet} \end{aligned}$$

You know that one hour has 60 minutes, and that 1 minute has 60 seconds. This means that, one hour has 60(60) seconds.

$$1 \text{ hour} = 60(60) \text{ seconds}$$

Use the information to convert 20 mi/h to feet per second.

$$\begin{aligned} \frac{20 \text{ miles}}{1 \text{ hour}} &= \frac{20 \cdot 5280 \text{ feet}}{60(60) \text{ seconds}} \\ &= 29.3 \text{ feet / second} \end{aligned}$$

Therefore, the required solution is 29.3 ft/sec.

Answer 42e.

Given conversion is

6ft/sec to miles per hour

1 mile contain 1.61 km, 1km contain 1000 m and 1m contain 3.279 feet

Then, 1 mile = 1.61km

$$= 1.61 \times 1000m$$

$$= 1.61 \times 1000 \times 3.279 \text{ feet}$$

$$1 \text{ mile} = 5280 \text{ feet (approx)}$$

$$\text{Thus, } 1 \text{ ft} = \frac{1}{5280} \text{ mile (approx)}$$

Also, 1 hour = 60 min

$$= 60 \times 60 \text{ sec.}$$

$$= 3600 \text{ sec.}$$

$$\text{Thus, } 1 \text{ sec.} = \frac{1}{3600} \text{ hour}$$

$$\begin{aligned} \text{Now, Given Conversion } 6\text{ft/sec} &= \frac{6\text{ft}}{1\text{second}} \\ &= \frac{6 \times \left(\frac{1}{5280} \right) \text{ mile}}{\frac{1}{3600} \text{ hour}} \\ &= 6 \times \left(\frac{1}{5280} \right) \times 3600 \text{ mile / hour} \end{aligned}$$

$$\text{Hence, } \boxed{6\text{ft / sec} = 4.09 \text{ mile / hour}}$$

Answer 43e.

One mile is equal to 1.61 km. Thus, to convert km to miles, multiply by the conversion rate 1 mile/1.61 km.

$$\frac{50 \text{ kilometers}}{1 \text{ hour}} = \frac{50 \text{ kilometers}}{1 \text{ hour}} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometers}}$$

Simplify.

$$\frac{50 \text{ kilometers}}{1 \text{ hour}} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometers}} \approx \frac{31.1 \text{ miles}}{1 \text{ hour}}$$

Therefore, the required result is 31.1 km/hour.

Answer 44e.

Given Conversion is:

40 miles/h to kilometers/hour.

$$\text{As } 1 \text{ mile / h} = 1.61 \text{ km / hour}$$

$$\begin{aligned} \text{So, } 40 \text{ miles / h} &= 40 \times 1.61 \text{ km / hour} \\ &= 64.4 \text{ km / hour} \end{aligned}$$

$$\text{Hence, } \boxed{40 \text{ mile / h} = 64.4 \text{ Kilometers / hour}}$$

Answer 45e.

There are two conversions to be performed, of which one is from gallons to ounces, and the other is from hours to seconds.

One gallon is equal to 128 ounces.

$$\begin{aligned} 1 \text{ gallon} &= 1 \text{ gallon} \cdot \frac{128 \text{ ounces}}{1 \text{ gallon}} \\ &= 128 \text{ ounces} \end{aligned}$$

You know that one hour has 60 minutes, and that 1 minute has 60 seconds. This means that, one hour has 60(60) seconds.

$$1 \text{ hour} = 60(60) \text{ seconds}$$

Use the information to convert 1 gal/h to ounces per second.

$$\begin{aligned} \frac{1 \text{ gallon}}{1 \text{ hour}} &= \frac{128 \text{ ounces}}{60(60) \text{ seconds}} \\ &= 0.04 \text{ ounce/second} \end{aligned}$$

Therefore, the required solution is 0.04 ounce/second.

Answer 46e.

Given Conversion is:

6 oz / sec to gallons per hour

$$1 \text{ gallon} = 128 \text{ oz}$$

$$\text{So, } 1 \text{ oz} = \frac{1}{128} \text{ gallons} \quad \text{.....(1)}$$

$$1 \text{ hour} = 60 \text{ min}$$

$$= 60 \times 60 \text{ sec.}$$

$$1 \text{ hour} = 3600 \text{ sec.}$$

$$\text{So, } 1 \text{ sec.} = \frac{1}{3600} \text{ hour} \quad \text{.....(2)}$$

Now, given is:

$$6 \text{ oz / sec.} = \frac{6 \text{ oz}}{1 \text{ sec.}}$$

Using (1) and (2), it becomes

$$\begin{aligned} 6 \text{ oz / sec} &= \frac{\left(6 \times \frac{1}{128}\right) \text{ gallons}}{\left(\frac{1}{3600}\right) \text{ hour}} \\ &= 6 \times \frac{1}{128} \times 3600 \text{ gallons per hour} \\ &= 168.75 \text{ gallons per hour} \end{aligned}$$

$$\text{Thus, } \boxed{6 \text{ oz / sec} = 168.75 \text{ gallons per hour}}$$

Answer 47e.

It is given that the rocket sled travels 3 miles in 6 seconds. From this, we can find out the speed of the rocket sled.

We know that the speed is the ratio of the distance traveled and the time taken to travel.

$$\begin{aligned}\text{speed} &= \frac{3 \text{ miles}}{6 \text{ seconds}} \\ &= \frac{1 \text{ miles}}{2 \text{ seconds}}\end{aligned}$$

The rocket sled will cover 1 mile in 2 seconds. We want to find out the average speed in miles per hour.

We know that 1 hour is equivalent to 3600s.

$$\begin{aligned}\text{Average speed in miles per hour} &= \frac{1 \text{ mile}}{2 \text{ seconds}} \cdot \frac{3600 \text{ seconds}}{1 \text{ hour}} \\ &= 1800 \text{ miles per hour}\end{aligned}$$

Therefore, the average speed of the rocket sled is 1800 miles per hour.

Answer 48e.

Given height raised = 500 ft

Time taken = 60 second

$$\begin{aligned}\text{Then, average speed} &= \frac{\text{height raised}}{\text{time taken}} \\ &= \frac{500 \text{ ft}}{60 \text{ sec}} \\ &= \frac{25}{3} \text{ ft / sec}\end{aligned}$$

1 mile contain 1.61 km, 1 km contain 1000 m and 1 m contain 3.279 feet

$$\begin{aligned}\text{Then, } 1 \text{ mile} &= 1.61 \text{ km} \\ &= 1.61 \times 1000 \text{ m} \\ &= 1.61 \times 1000 \times 3.279 \text{ feet} \\ 1 \text{ mile} &= 5280 \text{ feet (approx)}\end{aligned}$$

$$\text{Thus, } 1 \text{ ft} = \frac{1}{5280} \text{ mile (approx)}$$

$$\begin{aligned}\text{Also, } 1 \text{ hour} &= 60 \text{ min} \\ &= 60 \times 60 \text{ sec.} \\ &= 3600 \text{ sec.}\end{aligned}$$

$$\text{Thus, } 1 \text{ sec.} = \frac{1}{3600} \text{ hour}$$

$$\begin{aligned}\text{Now, Given Conversion } \frac{25}{3} \text{ ft/sec} &= \frac{\frac{25}{3} \text{ ft}}{1 \text{ second}} \\ &= \frac{\frac{25}{3} \times \left(\frac{1}{5280}\right) \text{ mile}}{\frac{1}{3600} \text{ hour}} \\ &= \frac{25}{3} \times \left(\frac{1}{5280}\right) \times 3600 \text{ mile / hour}\end{aligned}$$

$$\text{Hence, } \boxed{\frac{25}{3} \text{ ft / sec} = 5.681 \text{ mile / hour}}$$

Answer 49e.

Observe both sides of the equation. We can see that on the left side of the equation, the sum of a and b is added with c and on the right side of the equation, a is added with the sum of b and c .

Only the order of addition is changed. The associative property of addition states that changing the order of the addition does not affect the result.

Therefore, the statement is always true for the real numbers a , b , and c .

Answer 50e.

Given statement is :

$$a.(b.c)=(a.b).c \quad \text{for real numbers } a, b \text{ and } c.$$

This statement represents the associative property of multiplication for real numbers and as it is known that multiplication of real numbers is always associative, that is we can multiply any two numbers first and then multiply the result with third number.

In words, associative property of multiplication means that the value of multiplication of real numbers remains same and does not change upon how the multiplication is done by combining any two numbers first and then multiplication the third number to their result.

It allows us to collect any two real numbers first, multiply them and to their results multiply the third number.

In equation (1), on one side it can be seen that firstly, 2^{nd} and 3^{rd} number are being multiply and then to their result, 1^{st} number is multiply.

Whereas, on the other hand, firstly 1^{st} and 2^{nd} number are being multiply and then to their result 3^{rd} number is multiply. Then, by equation (1), it can be seen that result of multiplication of these 3 real numbers remain same as done by the two above ways.

The property given by (1) represents associative property of multiplication.

Hence the statement is always true, for all real numbers a , b and c .

Answer 51e.

Observe the expression on each side of the equation. On the left side of the equation, c is subtracted from the difference of a and b and on the right side of the equation, the difference of b and c is subtracted from a .

Apply the associative property on the left side of the equation.

$$(a - b) - c = a - (b + c)$$

The right side of the equation is given as $a - (b - c)$.

Let us assign some values for a , b , and c and check whether $a - (b + c)$ and $a - (b - c)$ evaluates to the same value.

Take $a = 8$, $b = 4$, and $c = 2$.

$$\begin{aligned} a - (b - c) &= 8 - (4 - 2) \\ &= 6 \end{aligned}$$

$$\begin{aligned} a - (b + c) &= 8 - (4 + 2) \\ &= 2 \end{aligned}$$

We can see that for the same values of a , b , and c , the two expressions evaluates to different results.

Now, take $c = 0$.

$$\begin{aligned} a - (b - c) &= a - (b - 0) \\ &= a - b \end{aligned}$$

$$\begin{aligned} a - (b + c) &= a - (b + 0) \\ &= a - b \end{aligned}$$

The two expressions give the same result when $c = 0$.

Thus, the given statement will be false except when $c = 0$.

Answer 52e.

Given statement is:

$$(a \div b) \div c = a \div (b \div c)$$

To check its validity, we simplify both the left hand side and right hand side separately.

$$\text{Left hand side} = (a \div b) \div c$$

Apply definition of division in brackets, it becomes

$$= \left(a \cdot \frac{1}{b} \right) \div c$$

Multiplying the terms in brackets, it becomes

$$= \left(\frac{a}{b} \right) \div c$$

Again, apply definition of division, it becomes

$$= \left(\frac{a}{b} \right) \cdot \left(\frac{1}{c} \right)$$

Multiplying the terms in numerator and denominator separately, it becomes

$$\begin{aligned} &= \frac{a \cdot 1}{b \cdot c} \\ &= \frac{a}{bc} \end{aligned} \quad \text{.....(1)}$$

$$\text{Right hand side} = a \div (b \div c)$$

Apply definition of division in brackets, it becomes

$$= a \div \left(b \cdot \frac{1}{c} \right)$$

Multiplying the terms in brackets, it becomes

$$= a \div \left(\frac{b}{c} \right)$$

Again, apply definition of division, that is

$$\begin{aligned}\text{Multiply } a \text{ with multiplication inverse of } \frac{b}{c}, \text{ that is, } \frac{c}{b} \\&= a \cdot \frac{c}{b} \\&= \frac{ac}{b} \quad \dots\dots(2)\end{aligned}$$

Using (1) and (2), it is clear that

left hand side \neq right hand side

Hence, statement is never true

However, if $b = 1, c = 1$, then

$$\begin{aligned}\text{Left hand side} &= \frac{a}{bc} \\&= \frac{a}{1.1} \\&= a\end{aligned}$$

$$\text{Right hand side} = \frac{ac}{b}$$

Put $b = 1, c = 1$, then

$$\begin{aligned}&= \frac{a.1}{1} \\&= a\end{aligned}$$

Hence, statement is never true, except $b = 1, c = 1$

Answer 53e.

Observe the expression on each side of the equation. On the left side of the equation, a is multiplied with the difference of b and c and on the right side of the equation, the difference of the products ab and ac is given.

Apply the distributive property on the left side of the equation.

$$\begin{aligned}a(b - c) &= a \cdot b - a \cdot c \\&= ab - ac\end{aligned}$$

The resultant expression is the same as the expression given on the right side of the equation.

Thus, the statement will be always true for the real numbers a , b , and c .

Answer 54e.

Given statement is:

$$a(b \div c) = ab \div ac$$

To check its validity, we simplify both the left hand side and right hand side separately.

$$\text{Left hand side} = a(b \div c)$$

Apply definition of division in brackets, it becomes

$$= a\left(b \cdot \frac{1}{c}\right)$$

Apply associative property of multiplication, it becomes

$$\begin{aligned}&= (ab) \cdot \frac{1}{c} \\&= (ab) \cdot \frac{1}{c}\end{aligned}$$

Again, apply definition of division, that is

$$= ab \div c$$

$$\neq \text{Right hand side}$$

Hence, statement is never true

However, if $a=1$

$$\begin{aligned}\text{Then, left hand side} &= a(b \div c) \\ &= 1.(b \div c)\end{aligned}$$

Apply property of multiplication identity, it becomes

$$= b \div c$$

Also, right hand side $= ab \div ac$

However, if $a=1$

$$= (1.b) \div (1.c)$$

Apply property of multiplication identity, it becomes

$$= b \div c$$

Clearly, left hand side = right hand side

Thus, we conclude that :

The statement is never true for all real numbers a, b and c except $a=1$

Answer 55e.

We can reason that the given statement is true, if we can show that the expression on the left-hand side is equivalent to the expression on the right-hand side.

In order to divide two numbers, multiply the first number by the reciprocal of the second number. The reciprocal of any non zero real number a , is $\frac{1}{a}$.

Use this definition to rewrite $\frac{a}{b} \div \frac{c}{d}$.

$$\begin{aligned}\frac{a}{b} \div \frac{c}{d} &= \frac{a}{b} \cdot \frac{d}{c} \\ &= \frac{a \cdot d}{b \cdot c}\end{aligned}$$

For any two real numbers x and y , the commutative law of multiplication states that $x \cdot y = y \cdot x$

Apply the commutative law to the denominator of the fraction.

$$\begin{aligned}\frac{a \cdot d}{b \cdot c} &= \frac{a \cdot d}{c \cdot b} \\ &= \frac{a}{c} \cdot \frac{d}{b}\end{aligned}$$

Now, use the reverse of the definition of division to rewrite the multiplication as division.

$$\frac{a}{c} \cdot \frac{d}{b} = \frac{a}{c} \div \frac{b}{d}$$

Thus, it can be shown that $\frac{a}{b} \div \frac{c}{d} = \frac{a}{c} \div \frac{b}{d}$.

Answer 56e.

Let the required rational number = $\frac{p}{q}$

Now, $\frac{p}{q}$ is a rational number that lies exactly halfway between $\frac{a}{b}$ and $\frac{c}{d}$ on a number line.

This means, $\frac{p}{q}$ lies at same distance away from $\frac{c}{d}$ as from $\frac{a}{b}$

If we take, $\frac{a}{b} < \frac{c}{d}$ and $\frac{p}{q}$ lies in their middle.

Then, $\frac{a}{b} < \frac{p}{q} < \frac{c}{d}$

And

$$\frac{p}{q} - \frac{a}{b} = \frac{c}{d} - \frac{p}{q}$$

Adding $\frac{p}{q}$ on both sides, it becomes

$$\frac{p}{q} - \frac{a}{b} + \frac{p}{q} = \frac{c}{d}$$

Grouping like terms and adding them,

$$\left(\frac{p}{q} + \frac{p}{q} \right) - \frac{a}{b} = \frac{c}{d}$$

$$2\left(\frac{p}{q} \right) - \frac{a}{b} = \frac{c}{d}$$

Adding $\frac{a}{b}$ on both sides, it becomes

$$2\left(\frac{p}{q} \right) = \frac{c}{d} + \frac{a}{b}$$

Multiply both sides by LCD of b and d , that is, bd , it becomes

$$2bd\left(\frac{p}{q}\right) = bc + ad$$

Dividing both sides by $2bd$, it becomes

$$\frac{p}{q} = \frac{bc + ad}{2bd}$$

Hence, the required rational number is:

$$\boxed{\frac{p}{q} = \frac{bc + ad}{2bd}}$$

Answer 57e.

- (a) Add the scores of the first player.
 $2 + 1 + 0 + 0 + (-1) + 1 + 3 + 0 + 0 = 3 - 1 + 1 + 3$
 $= 6$

Similarly, find the scores of the other players.

Second player: $(-1) + 3 + 0 + (-1) + 1 + 0 + 0 + 1 + (-1) = 2$

Third player : $1 + 0 + 1 + 0 + 0 + (-1) + 1 + 0 + 1 = 3$

Fourth player: $(-1) + (-1) + 0 + 0 + 1 + (-1) + 0 + 0 + 0 = -2$

- (b) The player with the lowest total score is the best, and the one with the highest total score is the worst player. This means that to list the players from best to worst, the total scores has to be arranged in increasing order.

The lowest total score is -2 , and the highest is 6 . Arrange the scores such that the first element is -2 and the last element is 6 .

$-2, 2, 3, 6$

Therefore, the list of players in an increasing order of their total score is fourth player, second player, third player, first player.

Answer 58e.

Consider the list of elevation (in feet) of volcano summits above or below sea level:

641, 3976, 610, -59, 1718, 1733, -137

As the number with negative sign and biggest value attached to negative sign becomes least in order and the number with positive sign and biggest value attached to positive sign becomes greatest in order. Using this rule of ordering of numbers, the list which shows the elevations (in feet) of Volcano summit above (or) below sea level from lowest to greatest, that is, in increasing order is as follows:

-137, -59, 610, 641, 1718, 1733, 3976

Answer 59e.

- (a) In order to sort in the required manner, start from the planet with the least temperature and move to the planet with the highest temperature.
Thus, the sorted list is Pluto, Neptune, Uranus, Saturn, Jupiter, Mars, Earth, Venus, and Mercury.
- (b) List the planets in the order from least to greatest distance from the Sun

Thus, the sorted list is Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto.
- (c) The surface temperature and the distance from the Sun are inversely related to each other. Temperature decreases with the increase in distance.
- (d) Mercury and Venus do not follow the general pattern. Although Mercury is closer to the Sun than Venus, its surface temperature is lower than that of Venus.

Answer 60e.

Given is:

- (a) Average weight of blue whale = 120 tons
As 1 ton = 2000 pounds
So, Average weight of blue whale = 120×2000 pounds
Thus, Average weight of blue whale = 240000 pounds(1)
Next, Average weight of bumble-bee bat = 0.07 ounce
As, 1 pound = 16 ounce
So, 1 Ounce = $\frac{1}{16}$ pound
Then, average weight of bumble bee bat = $0.07 \times \frac{1}{16}$ pound
Average weight of bumble bee bat = 0.004375 pound(2)

(b) Divide (1) by (2), it becomes:-

$$\frac{\text{Average Weight of blue Whale}}{\text{Average weight of bumble bee bat}} = \frac{240000 \text{ pounds}}{0.004375 \text{ pounds}}$$
$$= 54857142.9$$
$$\frac{\text{Average weight of blue whale}}{\text{Average weight of bumble bee bat}} = 5487143 (\text{approx})$$

Cross multiplying, it becomes

$$\text{Average weight of blue whale} = 54857143 (\text{Average weight of bumble bee bat})$$

Thus, a blue whale is 54857143 times as heavy as bumble bee bat

- c) Average weight of blue whale = 120 tons
1 ton = ounces
So, average weight of blue whale = 120×32000 ounces
= 3840000 ounces(3)
Also, average weight of bumble bee bat = 0.07 ounces(4)
Divide (3) by (4)

$$\frac{\text{Average weight of blue whale}}{\text{Average weight of bumble bee bat}} = \frac{3840000 \text{ ounces}}{0.07 \text{ ounces}}$$
$$= 54857143 (\text{approx})$$

Cross multiplying, it becomes

$$\text{Average weight of blue whale} = 54857143 (\text{Average weight of bumble bee bat})$$

Thus, a blue whale is 54857143 times as heavy as bumble bee bat

Answer 61e.

In order to convert miles per hour to feet per second, multiply the value given in miles per hour by 1.466. Similarly, to convert feet per second to mile per hour, multiply the given value by 0.681.

Find the missing values in the table using this information.

The speed of the first animal in feet per second is,

$$70 \cdot 1.466 = 102.67.$$

The speed of the second animal in miles per hour is,

$$0.22 \cdot 0.681 = 0.15.$$

The speed of the third animal in feet per second is,

$$12 \cdot 1.466 = 17.6.$$

The speed of the fourth animal in miles per hour is,

$$44 \cdot 0.681 = 30$$

- (a) Draw the completed table.

Animal	Speed (mi/h)	Speed (ft/s)
First	70	102.67
Second	0.15	0.22
Third	12	17.6
Fourth	30	44

- (b) Consider the speeds of the animals in either miles per hour or in feet per second. The first animal is the fastest, with a speed of 70 mi/hr, and the second animal is the slowest, with a speed of only 0.15mi/hr.

Now, to compare the speeds, divide the speed of the first animal by the speed of the second animal.

$$\frac{70}{0.15} = 466.67$$

Therefore, the first animal is about 467 times faster than the second animal.

Answer 62e.

Given is:

$$(a) \text{ Singapore dollar} \approx 0.605 \text{ U.S. dollars} \quad \dots\dots(1)$$

$$\text{And } \text{Canadian dollar} \approx 0.834 \text{ U.S. dollar} \quad \dots\dots(2)$$

On dividing (1) by (2); it becomes

$$\frac{\text{Singapore dollar}}{\text{Canadian dollar}} \approx \frac{0.605 \text{ U.S. dollar}}{0.835 \text{ U.S. dollars}} \approx 0.7245$$

On Cross multiplying

$$\text{Hence, Singapore dollar} \approx 0.7245 \times 1 \text{ Canadian dollar}$$

$$\text{Thus, } \boxed{1 \text{ Singapore dollar} \approx 0.7245 \text{ Canadian dollars}}$$

b) Given is:

$$\text{Hong Kong dollar} \approx 0.129 \text{ U.S. dollars} \quad \dots\dots(3)$$

$$\text{New Zealand dollar} \approx 0.695 \text{ U.S dollars} \quad \dots\dots(4)$$

On dividing (3) by (4); it becomes

$$\frac{\text{Hong Kong dollar}}{\text{New Zealand dollar}} \approx \frac{0.129 \text{ U.S. dollars}}{0.695 \text{ U.S. dollars}}$$

$$\approx 0.1856$$

On Cross multiplying, it becomes

$$\begin{aligned} \text{Hong Kong dollar} &\approx 0.1856 \times 1 \text{ New Zealand dollar} \\ &\approx 0.1856 \text{ New Zealand dollars} \end{aligned}$$

$$\text{Thus, } \boxed{\text{Hong Kong dollar} \approx 0.1856 \text{ New Zealand dollars}}$$

Answer 63e.

The operation to be performed is subtraction. In order to perform subtraction, add the additive inverse of the number being subtracted to the other number.

For example, to subtract b from a , add the additive inverse of b , which is $-b$, to a .
 $a - b = a + (-b)$

Substitute 3 for a , and 11 for b in the relation.
 $3 - 11 = 3 + (-11)$

Add.
 $3 + (-11) = -8$.

Therefore, the result is -8 .

Answer 64e.

Given operation is:

$$-4(-8)$$

For multiplying the above numbers, first multiply the numbers directly without taking their sign.

$$\text{That is, } 4(8) = 32$$

Now, for putting sign in front of it, the rule is: $(-)(-) = +$

That is, on multiplying negative number with another negative number, we get a positive number.

Now, on multiplying above numbers with sign using sign rule, it becomes

$$\boxed{-4(-8) = 32}$$

Answer 65e.

The operation to be performed is division. The division of two numbers can be performed by multiplying the dividend by the reciprocal of the divisor.

$$a \div b = a \cdot \frac{1}{b}, b \neq 0$$

Substitute 45 for a , and -9 for b .

$$45 \div (-9) = 45 \cdot \frac{-1}{9}$$

Multiply. The product of two numbers with different signs is negative.

$$45 \cdot \frac{-1}{9} = -5$$

Therefore, the required result is -5 .

Answer 66e.

Given operation is:

$$-6+13$$

As the numbers with different sign gets subtracted and the resulting number will have sign same as that of bigger number out of the two given numbers under subtraction. Here, in above operation, 6 is of negative sign and 13 is of positive sign. Thus, they will get subtracted under the above operation and the resulting number will have sign of bigger number, that is, 13 which is having positive sign.

So, $\boxed{-6+13=7}$

Answer 67e.

The operation to be performed is multiplication. The product of two numbers with different signs is negative.

$$-3(7) = -21$$

Therefore, the required solution is -21 .

Answer 68e.

Given operation is:

$$5-(-2)$$

First, apply sign rule, according to which negative of negative becomes positive

So, $5-(-2)=5+2$

On adding 5 and 2, it becomes 7

Hence, $\boxed{5-(-2)=7}$

Answer 69e.

An expression involving a variable is called an algebraic expression. Nine more than a number means that nine is added to a number.

Let x be the number to which nine is added. Thus, the required expression can be written as $x+9$.

Answer 70e.

Given phrase is:

5 less than a number

Suppose, the number = x

Now, 5 less than x means subtracting 5 from x

So, subtracting 5 from x , it becomes $x - 5$

Hence, the algebraic expression of

+ 5 less than a number x is $\boxed{x - 5}$

Answer 71e.

Use the definition “ $x\%$ of a number is $\frac{x}{100} \cdot \text{number}$ ” to find the required expression.

Let y be the number. Then, 75% of y can be written as $\frac{75}{100} \cdot y$.

Therefore, the required algebraic expression is $\frac{75}{100} \cdot y$.

Answer 72e.

Given phrase is:

7 times a numbers

Suppose, the number = x

Now, 7 times x means to multiply 7 with x .

On multiplying 7 with x , it becomes

$$7 \times x = 7x$$

Hence, the algebraic expression of

7 times a number x is $\boxed{7x}$

Answer 73e.

Assume x to be the number. Half of a number means that the number is divided by two.

Therefore, the required expression is $\frac{x}{2}$.

Answer 74e.

Given phrase is

The square of a number

Suppose, the number = x

Now, the square of x means power of x is 2.

On writing the power expression for square of x , we have x^2

Hence, the algebraic expression of

The square of a number x is x^2

Answer 75e.

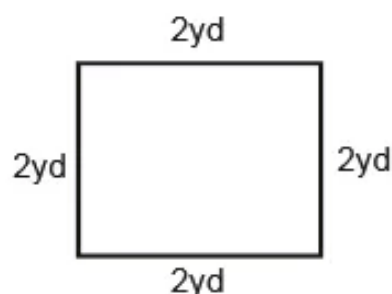
The perimeter of a triangle is the sum of its three sides.

From the given figure, we get the three sides as 14 cm, 9 cm, and 9 cm.

The perimeter of the given triangle will be 14 cm + 9 cm + 9 cm, or 32 cm.

Answer 76e.

Consider the figure:



Clearly, the above figure is a square with all sides of equal length.

Now, perimeter of given square = Sum of length of all its sides

As all sides are of same length

So, Perimeter of given square = 4 (length of each side)

$$= 4(2)$$

$$= 8$$

Hence, perimeter of given square = 8yd.

Answer 77e.

The perimeter of a parallelogram is the sum of its four sides.

The opposite sides of the parallelogram are equal. From the given figure, we get the 4 sides as 12 m, 7 m, 12 m, and 7m.

The perimeter of the given parallelogram will be 12 m + 7 m + 12 m + 7 m, or 38 m.

Answer 78e.

Consider the given figure:



Clearly, the above given figure is a rectangle

Whose length (ℓ) = 14ft

And breadth (b) = 6ft

$$\begin{aligned}\text{Now, area of rectangle } (A) &= \ell \times b \\ &= 14ft \times 6ft \\ &= 84Sq.ft\end{aligned}$$

Hence, area of a rectangle (A) = 84sq. ft

Answer 79e.

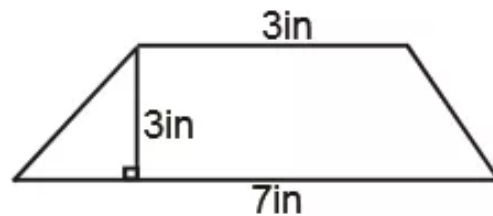
We know that the area of a triangle is half the product of the base and the altitude. The base of the triangle in the given figure is 8 m and the altitude is 3 m.

$$\begin{aligned}\text{Area} &= \frac{1}{2} \cdot 8 \text{ m} \cdot 3 \text{ m} \\ &= 12 \text{ m}^2\end{aligned}$$

Therefore, the area of the triangle is 12 m^2 .

Answer 80e.

Consider the given figure.



Clearly, the above given figure is a trapezium; which has two sides parallel and two non-parallel

Given is: length of 1st parallel side (d_1) = 3in.

Length of 2nd parallel side (d_2) = 7in.

Distance between two parallel side (h) = 3in.

Now, area of trapezium (A) = (Sum of parallel sides) (distance between them)

$$= (d_1 + d_2)h$$

$$= (3 + 7) \times 3$$

Solving brackets, it becomes = $(10) \times 3$

On multiplying, it becomes = 30

Thus, area of trapezium (A) = 30 Sq. in \square