Chapter 11

Linear Equations with One Variable

11.1 In previous classes, you have come across many algebraic expression and equations. Some expressions and equations are given below:

(i) $2x+5$ (ii) $x+y=0$ (iii) $3xy=5z$ (iv) $10x^2y$

In these, which are the equations?

You would remember that equality "=" sign is always used in equations.

In this chapter, we will study about the single variable linear equations i.e., equations having single variable and that variable should have maximum power.

$$x+2=7, \qquad 2x-3=0$$

In previous class, we learnt to solve such kind of equations which have one sided variables, with this, we also learnt to solve the equations having rational coefficient, such as:

$$5x-4=26$$

And

$$\frac{2x}{3} - \frac{x}{2} = 30$$

Let us practice to solve an equation:

Equation $3x+9=15$ solution $x=2$

Sr. No.	Both Sides	New Equation	Solution
1.	Adding 2	$3x+11=17$	$x = 2$
2.	Subtracting 3	$3x+6=12$	$x = $ ....
3.	Multiplying by 2	$6x+18=$......	$x = $ ....
4.	Dividing by 3	...............$=5$	$x = $ ....

In this way, by adding, subtracting, multiplying or dividing any number in both sides of the equation the formation of equation is changed but the solution remains the same. Thus, we take help of these operations accordingly to solve the equation.

Now, we would learn to solve such type of equations which may have variables on both sides.

To solve the equation $8x-13=5x-7$ means to make its formation $x=$.............

So, we have to keep a variable in LHS and a constant in RHS, for this, applying same operation or transposition on both sides.

Ex 1: Solve the equation $8x - 13 = 5x - 7$

Sol: Balancing method:

$8x - 13 + 13 = 5x - 7 + 13$ (adding 13 on both sides)

The balance is not disturbed

Or $8x = 5x + 6$

Or $8x - 5x = 5x + 6 - 5x$ (subtracting $5x$ from both sides)

Or $3x = 6$

Or $\dfrac{3x}{3} = \dfrac{6}{3}$ (by dividing both sides by 3)

Or $x = 2$

Sol 2: (Transposition method)

$8x - 13 = 5x - 7$

Or $8x - 5x = -7 + 13$ (by transposition of $5x$ and -13)

Or $3x = 6$

Or $x = 6/3$ (by transposition of coefficient 3)

Or $x = 2$

Since, transposition method is the shortest and simplest way of comparison method. Thus we will use only transposition method.

Example 2: Solve the equation $\dfrac{x}{5} = \dfrac{7 - 6x}{3}$

Sol: $\dfrac{x}{5} = \dfrac{7 - 6x}{3}$

Or $3x = 35 - 30x$

(by taking LCM 15 and multiplying both side or by cross multiplication)

or $3x + 30x = 35$

or $33x = 35$

$x = \dfrac{35}{33}$

Example 3: Solve $\dfrac{6x + 1 + 1}{3} = \dfrac{x - 3}{6}$

Sol: $\dfrac{6x + 1 + 1}{3} = \dfrac{x - 3}{6}$

Or $\dfrac{6x + 4}{3} = \dfrac{x - 3}{6}$

Or $\dfrac{6x+1+3}{3} = \dfrac{x-3}{6}$

Or $\dfrac{6x+4}{3} = \dfrac{x-3}{6}$

Or $\dfrac{6x+4}{3} \times 6 = \dfrac{x-3}{6} \times 6$ [Multiplying on both side by LCM 6]

Or

$$12x + 8 = x - 3$$

Or $\quad 12x - x = -3 - 8$

Or $\quad 11x = -11$

Or $\quad x = \dfrac{-11}{11}$

Or

$$x = -1$$

Example 4: Solve the equation $\dfrac{x+1}{2} + \dfrac{x+2}{3} = \dfrac{x+3}{4} + \dfrac{x+4}{5}$

Sol. $\quad \dfrac{x+1}{2} + \dfrac{x+2}{3} = \dfrac{x+3}{4} + \dfrac{x+4}{5}$

Multiplying both sides by 60 LCM of 2,3,4 and 5

$$60 \times \dfrac{x+1}{2} + 60 \times \dfrac{x+2}{3} = 60 \times \dfrac{x+3}{4} + 60 \times \dfrac{x+4}{5}$$

Or $\quad 30(x + 1) + 20(x + 2) \quad = \quad 15(x + 3) + 12(x + 4)$

Or $\quad 30x + 30 + 20x + 40 \quad = \quad 15x + 45 + 12x + 48$

Or $\quad 30x + 20x - 15x - 12x \quad = \quad 45 + 48 - 30 - 40$

Or $\quad 50x - 27x \quad = \quad 93 - 70$

Or $\quad 23x \quad = \quad 23$

Or $\quad x \quad = \quad \dfrac{23}{23}$

Or $\quad x \quad = \quad 1$

If coefficient or numbers are in decimal or fraction form then it is solved by changing them in simple fraction and multiply by least common multiple

Example 5: Solve $0.5m + 1.5 = 0.25m - 1.5$

Sol: $0.5m + 1.5 = 0.25m - 1.5$

$$\dfrac{5}{10}m + \dfrac{15}{10} = \dfrac{25}{100}m - \dfrac{15}{10} \quad \text{(Changing decimal fraction into simple fraction)}$$

$$50m + 150 = 25m - 150 \qquad \left[\begin{array}{l} \text{by multiplying 100 since} \\ \text{LCM of 10 and 100 is 100} \end{array} \right]$$
$$50m - 25m = -150 - 150$$

$$25m = -300$$
$$m = \frac{-300}{25}$$
$$m = -12$$

If there is a variable in numerator and denominator in fraction then apply cross multiplication and then transpose and solve.

Example 6: Solve the equation $\dfrac{(2x + 5)}{(3x + 1)} = \dfrac{3}{11}$

Sol: By cross multiplication

$$11(2x + 5) = 3(3x + 1)$$
$$22x + 55 = 9x + 3$$
$$22x - 9x = 3 - 55$$
$$13x = -52$$
$$x = \frac{-52}{13}$$
$$x = -4$$

Do and Learn: ◆

Solve the following equations-

1. $\dfrac{2x}{x + 6} = 1$ 2. $10 = x + 3$ 3. $16 = 7x - 9$ 4. $\dfrac{x + 5}{x} = 2\dfrac{2}{3}$

Exercise 11.1

1. $6x + 3 = 4x + 11$

2. $3(x + 5) = 4x + 9$

3. $3x + 2(x + 3) = 21$

4. $\dfrac{x + 1}{2} + \dfrac{x + 2}{3} = \dfrac{2x - 5}{7} + 9$

5. $\dfrac{3x - 2}{5} = 4 - \left(\dfrac{x + 2}{3}\right)$

6. $\dfrac{x + 2}{2} + \dfrac{x + 4}{3} = \dfrac{x + 6}{4} + \dfrac{x + 8}{5}$

7. $0.6x + 0.25x = 0.45x + 1.2$ 8. $2.5x - 7 = 0.5x + 13$

9. $\dfrac{7x + 4}{x + 2} = \dfrac{-4}{3}$ 10. $\dfrac{4x + 8}{5x + 8} = \dfrac{5}{6}$

11.2 Solution of word problem

Lets revise to make mathematical sentence, if there is any number x then fill in the blanks:

5 More than the number	$= x + 5$
3 Less than the number	= ....
Half of the number	= ....
7 Less half of the number	= ....
4 More than one third of the number	= ....
6 More than triple of the number	= ....
3 Less than 5 times of the number	= ....

The solution of many problems, puzzles etc of daily life can be obtained by the equations. For this purpose, following steps are used:

1. Read the given problem carefully. Find the known and unknown quantities .
2. Now write the unknown quantity in form of variable quantity x.
3. Write the all statements in mathematical form (algebraic terms and expression).
4. According to the condition of the question, equal quantities are written in form of equation.
5. Find the value of variable by solving equations.
6. Verify the answer according to the terms of the questions.

Example 7: Sum of two numbers is 60. Three times of the smaller number is equal to the double of the large number. Find the numbers.

Sol: Say, the smaller number $= x$

According to the question, the larger number $= 60 - x$

Three times of the smaller number $= 3x$

Double of the larger number $= 2(60 - x)$

According to the condition of the question,

$$3x = 2(60-x)$$
$$3x = 120 - 2x$$

$$3x + 2x = 120$$
$$5x = 120$$
$$x = \frac{120}{5}$$
$$x = 24$$

Another number $= 60 - x = 60 - 24 = 36$

Ans. 24, 36

Use the following formula to make two-digit numbers-
Number = 10 × tens digit + units digit

Example 8: In a number, units digit is 3 less than the tens digit. Number is greater than 3 of the 7 times of sum of the digits. Find the numbers.

Sol: Say, tens place digit $= x$

Then, units place digit $= x$-3

Number $= 10 ×$ tens digit $+$ units digit

$= 10 × x + x$-3

$= 11x - 3$

Sum of the digits $= (x + x - 3)$

According to the condition given in the question,

$$11x - 3 = 7(x + x - 3) + 3$$
$$11x - 3 = 7x + 7x - 21 + 3$$
$$11x - 7x - 7x = -21 + 3 + 3$$
$$-3x = -15$$
$$x = \frac{-15}{-3}$$
$$x = 5$$

Thus, the desired number,

$$= 11x - 3$$
$$= 11 × 5 - 3$$
$$= 55 - 3$$
$$= 52$$

Example 9: Age of Ramesh's father exceeds Ramesh's age by 27 years. After 5 years, ratio of Ramesh's age and his father's age will be 2:3. Find their present age.

Sol: Let the age of the Ramesh $= x$ year

Then, the age of father $= (x+27)$ year

After 5 years, age of Ramesh's = $(x + 5)$ years
After 5 years, age of Father $= x + 27 + 5 = x + 32$
According to the question,

$$\frac{\text{After 5 years, age of Ramesh}}{\text{After 5 years, age of father}} = \frac{2}{3}$$

$$\frac{x + 5}{x + 32} = \frac{2}{3} \qquad \text{(By cross multiplication)}$$

$$3(x + 5) = 2(x + 32)$$
$$3x + 15 = 2x + 64$$
$$3x - 2x = 64 - 15$$
$$x = 49$$

Age of Ramesh $x = 49$ year
Age of Father $x + 27 = 49 + 27 = 76$ years.

Example 10: Denominator of a rational number exceeds its numerator by 8. If 17 is added to numerator and 1 is subtracted from denominator then we get 3/2. Find the rational number.

Sol: Let the numerator of rational number $= x$
       According to the question, value of denominator $= x + 8$

Thus, the rational number $= \dfrac{x}{x+8}$

Now adding 17 to numerator and subtracting 1 from denominator

$$\frac{x + 17}{x + 8 - 1} = \frac{3}{2}$$

$$\frac{x + 17}{x + 7} = \frac{3}{2}$$
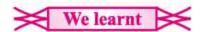
$$2(x + 17) = 3(x + 7)$$
$$2x + 34 = 3x + 21$$
$$3x - 2x = 34 - 21$$
$$x = 13$$

Thus, numerator $= 13$ and denominator $= 13 + 8 = 21$

The required rational number $= \dfrac{13}{21}$

Exercise 11.2

1. The numerator of a rational number is less than its denominator by 3. If 5 is added to both i.e., its numerator and its denominator then it becomes $\frac{3}{4}$ Find the numbers.

2. What should be added in numerator and denominator of fraction $\frac{5}{13}$ so that the fraction become $\frac{3}{5}$.

3. What should be subtracted from numerator and denominator of fraction $\frac{15}{19}$ so that the fraction becomes $\frac{5}{7}$.

4. Ramesh distributed his capital, half of the capital to his wife, one third to his son and remaining 50,000/- to his daughter. Find the total amount of his capital.

5. 5 times of any number is 48 more than its double. Find the numbers.

6. Distribute 45 in this way that one part is 7 less than three times of another part.

7. Age of Ranu is three times of Sujal's age. After 4 years, sum of their age will be 40 years. Find their present age.

8. Length of a rectangle exceeds its breath by 6 meter. If its perimeter is 64 metre then find its length and breath.

9. Sum of the digits of a two-digit number is 12. New number formed by reversing the digit is greater than the original number by 54. Find the original number.

10. In two-digit number, first digit is four times of second digit. Adding this to the new number formed by reversing the digits, 110 is obtained. Find the numbers.

We learnt

1. Equation having only linear polynomials is called a linear equation.
2. Linear equation having single variable is called single variable linear equation.
3. Substituting the value in place of variable in an equation if LHS = RHS then that value is called solution or root of an equation.
4. Like the numbers, variables in an equation can also be transposed from one side to another side.