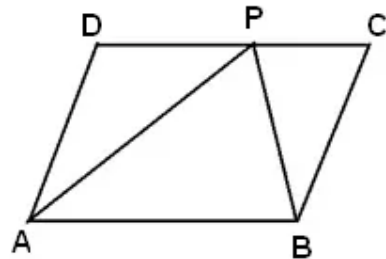


Chapter 21. Areas Theorems on Parallelograms

Ex 21.1

Answer 1.



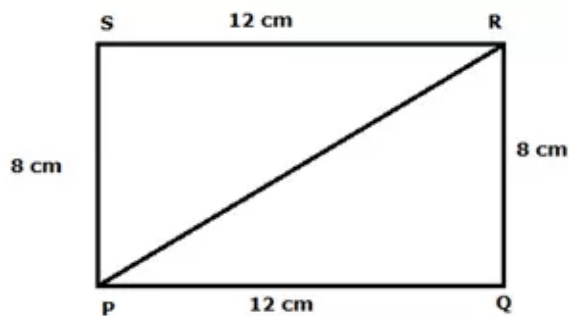
$$\text{ar}(\triangle APB) = \frac{1}{2} \times \text{ar}(\text{parallelogram } ABCD)$$

(The area of a triangle is half that of a parallelogram on the same base and between the same parallels)

$$\text{ar}(\triangle APB) = \frac{1}{2} \times 60 \text{ cm}^2$$

$$\text{ar}(\triangle APB) = 30 \text{ cm}^2$$

Answer 2.



Since PQRS is a rectangle, therefore $PQ = SR$.

$$SR = 12 \text{ cm}$$

$$PS = 8 \text{ cm}$$

$$\text{ar}(\triangle PRS) = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{ar}(\triangle PRS) = \frac{1}{2} \times SR \times PS$$

$$\text{ar}(\triangle PRS) = \frac{1}{2} \times 12 \times 8$$

$$\text{ar}(\triangle PRS) = 48 \text{ cm}^2$$

Answer 3.

$$(i) \quad \text{ar}(\triangle QTS) = \frac{1}{2} \times \text{ar}(\text{parallelogram QTSR})$$

(The area of a triangle is half that of a parallelogram on the same base and between the same parallels)

$$\Rightarrow \text{ar}(\text{parallelogram QTSR}) = 2 \times \text{ar}(\triangle QTS)$$

$$\Rightarrow \text{ar}(\text{parallelogram QTSR}) = 2 \times 60 \text{ cm}^2$$

$$\Rightarrow \text{ar}(\text{parallelogram QTSR}) = 120 \text{ cm}^2$$

$$(ii) \quad \text{ar}(\triangle QTS) = \frac{1}{2} \times \text{ar}(\text{parallelogram QTSR})$$

$$\text{ar}(\triangle QTS) = \text{ar}(\triangle RSQ) = 60 \text{ cm}^2$$

Now,

$$\text{ar}(\triangle RSQ) = \frac{1}{2} \times \text{ar}(\text{rectangle PQRS})$$

$$\Rightarrow \text{ar}(\text{rectangle PQRS}) = 2 \times \text{ar}(\triangle RSQ)$$

$$\Rightarrow \text{ar}(\text{rectangle PQRS}) = 2 \times 60 \text{ cm}^2$$

$$\Rightarrow \text{ar}(\text{rectangle PQRS}) = 120 \text{ cm}^2$$

(iii) Since PQRS is a rectangle,

Therefore $RS = PQ$ (i)

QTSR is a parallelogram,

Therefore, $RS = QT$ (ii)

From (i) and (ii)

$$PQ = QT \text{(iii)}$$

In $\triangle PSQ$ and $\triangle QST$

$$QS = QS$$

$$PQ = QT \quad (\text{from (iii)})$$

$$\angle PQS = \angle SQT = 90^\circ$$

Therefore, $\triangle PSQ \cong \triangle QST$

Area of two congruent triangles is equal.

$$\text{Hence, } \text{ar}(\triangle PSQ) = \text{ar}(\triangle QTS) = 60 \text{ cm}^2$$

Answer 7.

$$\text{ar}(\triangle APD) = \frac{\sqrt{3}s^2}{4}$$

$$\text{ar}(\triangle APD) = \frac{\sqrt{3} \times 8^2}{4}$$

$$\text{ar}(\triangle APD) = \frac{\sqrt{3} \times 64}{4}$$

$$\text{ar}(\triangle APD) = \sqrt{3} \times 16 = 16\sqrt{3}\text{cm}^2$$

$$\text{ar}(\triangle APD) = \frac{1}{2} \times \text{ar}(\text{parallelogram } ABCD)$$

(The area of a triangle is half that of a parallelogram on the same base and between the same parallels)

$$\Rightarrow \text{ar}(\text{parallelogram } ABCD) = 2 \times \text{ar}(\triangle APD)$$

$$\Rightarrow \text{ar}(\text{parallelogram } ABCD) = 2 \times 16\sqrt{3} \text{ cm}^2$$

$$\Rightarrow \text{ar}(\text{parallelogram } ABCD) = 32\sqrt{3} \text{ cm}^2$$

Answer 8.

- (i) Area of a rectangle and area of a parallelogram on the same base is equal.

Here,

For rectangle PQMN, base = PQ

For parallelogram PQRS, base = PQ

Therefore, Area of rectangle PQMN = Area of parallelogram PQRS

Area of rectangle PQMN = 84 cm^2

- (ii) $\text{ar}(\triangle PQS) = \frac{1}{2} \times \text{ar}(\text{parallelogram } PQRS)$

$$\text{ar}(\triangle PQS) = \frac{1}{2} \times 84 \text{ cm}^2$$

$$\text{ar}(\triangle PQS) = 42 \text{ cm}^2$$

- (iii) $\text{ar}(\triangle PQN) = \frac{1}{2} \times \text{ar}(\text{rectangle } PQMN)$

$$\text{ar}(\triangle PQN) = \frac{1}{2} \times 84 \text{ cm}^2$$

$$\text{ar}(\triangle PQN) = 42 \text{ cm}^2$$

Answer 9.

In quadrilateral PQST,

$$\text{ar}(\triangle PQS) = \frac{1}{2} \times \text{ar}(\text{quadrilateral PQST})$$

$$\text{ar}(\text{quadrilateral PQST}) = 2\text{ar}(\triangle PQS) \dots\dots\dots(i)$$

In $\triangle PSR$,

$$\text{ar}(\triangle PSR) = \text{ar}(\triangle PQS) + \text{ar}(\triangle QSR)$$

$$\text{but ar}(\triangle PQS) = \text{ar}(\triangle QSR) \quad (\text{since QS is median as } QS \parallel TP)$$

$$\text{ar}(\triangle PSR) = 2\text{ar}(\triangle PQS) \dots\dots\dots(ii)$$

From (i) and (ii)

$$\text{ar}(\text{quadrilateral PQST}) = \text{ar}(\triangle PSR)$$

Answer 14.

In parallelogram ABCD,

$$\text{ar}(\triangle ABC) = \frac{1}{2} \times \text{ar}(\text{parallelogram ABCD})$$

(The area of a triangle is half that of a parallelogram on the same base and between the same parallels)

$$\text{ar}(\text{parallelogram ABCD}) = 2\text{ar}(\triangle ABC) \dots\dots\dots(i)$$

In $\triangle ACE$,

$$\text{ar}(\triangle ACE) = \text{ar}(\triangle ABC) + \text{ar}(\triangle BCE)$$

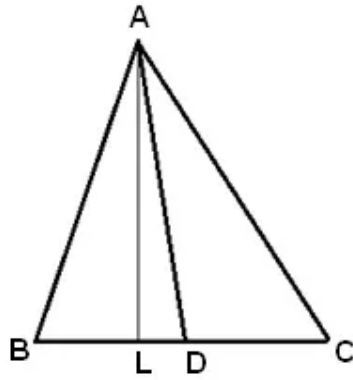
$$\text{but ar}(\triangle ABC) = \text{ar}(\triangle BCE) \quad (\text{since BC is median})$$

$$\text{ar}(\triangle ACE) = 2\text{ar}(\triangle ABC) \dots\dots\dots(ii)$$

From (i) and (ii)

$$\text{ar}(\text{parallelogram ABCD}) = \text{ar}(\triangle ACE)$$

Answer 16.



Draw AL perpendicular to BC.

Since AD is median of $\triangle ABC$. Therefore, D is the mid-point of BC.

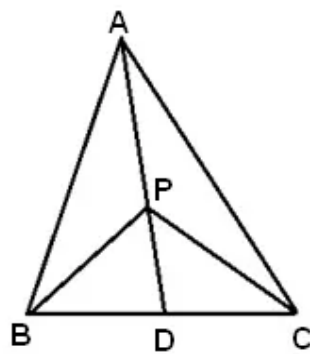
$$\Rightarrow BD = DC$$

$$\Rightarrow BD \times AL = DC \times AL \quad (\text{multiplying by AL})$$

$$\Rightarrow \frac{1}{2} (BD \times AL) = \frac{1}{2} (DC \times AL)$$

$$\Rightarrow \text{ar}(\triangle ABD) = \text{ar}(\triangle ADC)$$

Answer 17.



AD is the median of $\triangle ABC$. So, it will divide $\triangle ABC$ into two triangles of equal areas.

$$\text{Therefore, Area}(\triangle ABD) = \text{area}(\triangle ACD) \dots (1)$$

Now PD is the median of $\triangle PBC$.

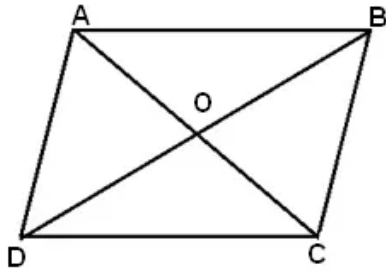
$$\text{Therefore, Area}(\triangle PBD) = \text{area}(\triangle PCD) \dots (2)$$

Subtract equation (2) from equation (1), we have

$$\text{Area}(\triangle ABD) - \text{area}(\triangle PBD) = \text{Area}(\triangle ACD) - \text{Area}(\triangle PCD)$$

$$\text{Area}(\triangle ABP) = \text{area}(\triangle ACP)$$

Answer 19.



The diagonals of a parallelogram bisect each other.

Therefore, O is the mid-point of AC and BD.

BO is the median in $\triangle ABC$. Therefore, it will divide it into two triangles of equal areas.

$$\therefore \text{ar}(\triangle AOB) = \text{ar}(\triangle BOC) \dots\dots\dots (i)$$

In $\triangle BCD$, CO is the median.

$$\therefore \text{ar}(\triangle BOC) = \text{ar}(\triangle COD) \dots\dots\dots (ii)$$

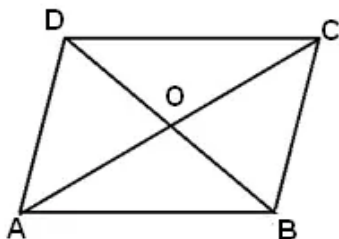
$$\text{Similarly, } \text{ar}(\triangle COD) = \text{ar}(\triangle AOD) \dots\dots\dots (iii)$$

From (i), (ii) and (iii)

$$\text{ar}(\triangle AOB) = \text{ar}(\triangle BOC) = \text{ar}(\triangle COD) = \text{ar}(\triangle AOD)$$

Hence, diagonals of a parallelogram divide it into four triangles of equal areas.

Answer 20.



In $\triangle ABD$,

$$BO = OD$$

$\Rightarrow$ O is the mid-point of BD

$\Rightarrow$ AO is a median.

$$\Rightarrow \text{ar}(\triangle AOB) = \text{ar}(\triangle AOD) \dots\dots\dots (i)$$

In $\triangle CBD$, O is the mid-point of BD

$\Rightarrow$ CO is a median.

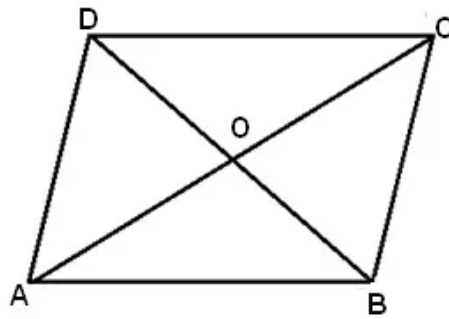
$$\Rightarrow \text{ar}(\triangle COB) = \text{ar}(\triangle COD) \dots\dots\dots (ii)$$

Adding (i) and (ii)

$$\text{ar}(\triangle AOB) + \text{ar}(\triangle COB) = \text{ar}(\triangle AOD) + \text{ar}(\triangle COD)$$

$$\text{Therefore, } \text{ar}(\triangle ABC) = \text{ar}(\triangle ADC)$$

Answer 21.



Since the diagonals of a rhombus intersect at right angles,

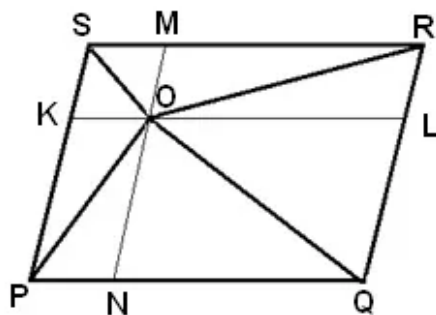
Therefore, $OB \perp AC$ and $OD \perp AC$

Now, $\text{ar}(\text{rhombus } ABCD) = \text{ar}(\triangle ABC) + \text{ar}(\triangle ADC)$

$$\begin{aligned} &= \frac{1}{2}(AC \times BO) + \frac{1}{2}(AC \times DO) \\ &= \frac{1}{2}\{AC \times (BO + DO)\} \\ &= \frac{1}{2}(AC \times BD) \end{aligned}$$

Therefore, the area of a rhombus is equal to half the rectangle contained by its diagonals.

Answer 22.



Let us draw a line segment KL, passing through point O and parallel to line segment PQ.

In parallelogram PQRS,

$PQ \parallel KL$ (By construction) ... (1)

PQRS is a parallelogram.

$\therefore PS \parallel QR$ (Opposite sides of a parallelogram)

$\Rightarrow PK \parallel QL$... (2)

From equations (1) and (2), we obtain

$PQ \parallel KL$ and $PK \parallel QL$

Therefore, quadrilateral PQLK is a parallelogram.

It can be observed that $\triangle POQ$ and parallelogram PQLK are lying on the same base PQ and between the same parallel lines PK and QL.

$$\therefore \text{Area}(\triangle POQ) = \frac{1}{2} \text{Area}(\text{parallelogram PQLK}) \dots (3)$$

Similarly, for $\triangle ROS$ and parallelogram KLRS,

$$\text{Area}(\triangle ROS) = \frac{1}{2} \text{Area}(\text{parallelogram KLRS}) \dots (4)$$

Adding equations (3) and (4), we obtain

$$\begin{aligned} \text{Area}(\triangle POQ) + \text{Area}(\triangle ROS) &= \frac{1}{2} \text{Area}(\text{parallelogram PQLK}) + \\ &\quad \frac{1}{2} \text{Area}(\text{parallelogram KLRS}) \end{aligned}$$

$$\text{Area}(\triangle POQ) + \text{Area}(\triangle ROS) = \frac{1}{2} \text{Area}(\text{PQRS}) \dots (5)$$

Let us draw a line segment MN, passing through point OP and parallel to line segment PS.

In parallelogram PQRS,

$$MN \parallel PS \text{ (By construction)} \dots (6)$$

PQRS is a parallelogram.

$$\therefore PQ \parallel RS \text{ (Opposite sides of a parallelogram)}$$

$$\Rightarrow PN \parallel SN \dots (7)$$

From equations (6) and (7), we obtain

$$MN \parallel PS \text{ and } PN \parallel SN$$

Therefore, quadrilateral PNMS is a parallelogram.

It can be observed that $\triangle POS$ and parallelogram PNMS are lying on the same base PS and between the same parallel lines PS and MN.

$$\therefore \text{Area}(\triangle SOP) = \frac{1}{2} \text{Area}(\text{PNMS}) \dots (8)$$

Similarly, for $\triangle QOR$ and parallelogram MNQR,

$$\text{Area}(\triangle QOR) = \frac{1}{2} \text{Area}(\text{MNQR}) \dots (9)$$

Adding equations (8) and (9), we obtain

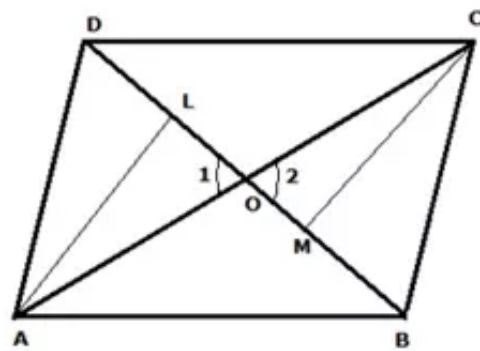
$$\text{Area}(\triangle SOP) + \text{Area}(\triangle QOR) = \frac{1}{2} \text{Area}(\text{PNMS}) + \frac{1}{2} \text{Area}(\text{MNQR})$$

$$\text{Area}(\triangle SOP) + \text{Area}(\triangle QOR) = \frac{1}{2} \text{Area}(\text{PQRS}) \quad \dots\dots\dots(10)$$

On comparing equations (5) and (10), we obtain

$$\text{Area}(\triangle POQ) + \text{Area}(\triangle ROS) = \text{Area}(\triangle SOP) + \text{Area}(\triangle QOR) = \frac{1}{2} \text{Area}(\text{|| gm PQRS})$$

Answer 23.



Join AC. Suppose AC and BD intersect at O. Draw AL and CM perpendicular to BD.

$$\text{ar}(\triangle ABD) = \text{ar}(\triangle BDC)$$

Thus $\triangle ABD$ and $\triangle ABC$ are on the same base AB and have equal area.

Therefore, their corresponding altitudes are equal i.e. $AL = CM$.

Now, in $\triangle ALO$ and $\triangle CMO$,

$$\angle 1 = \angle 2 \quad (\text{vertically opposite angles})$$

$$\angle ALO = \angle CMO \quad (\text{right angles})$$

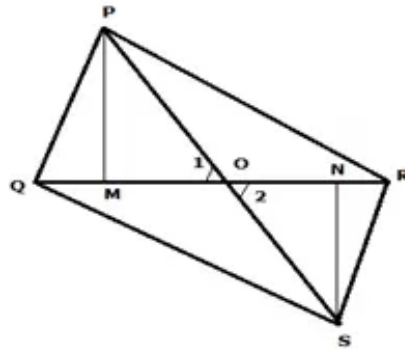
$$AL = CM$$

Therefore, $\triangle ALO \cong \triangle CMO$ (AAS axiom)

$$\Rightarrow AO = OC$$

$$\Rightarrow BD \text{ bisects } AC$$

Answer 26.



Join PS. Suppose PS and QR intersect at O. Draw PM and SN perpendicular to QR.

$$\text{ar}(\triangle PQR) = \text{ar}(\triangle SQR)$$

Thus $\triangle PQR$ and $\triangle SQR$ are on the same base QR and have equal area.

Therefore, their corresponding altitudes are equal i.e. $PM = SN$.

Now, in $\triangle PMO$ and $\triangle SNO$,

$$\angle 1 = \angle 2 \quad (\text{vertically opposite angles})$$

$$\angle PMO = \angle SNO \quad (\text{right angles})$$

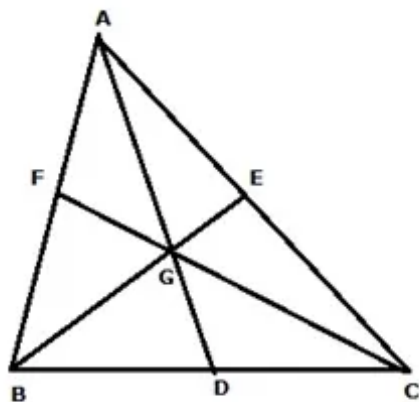
$$PM = SN$$

Therefore, $\triangle PMO \cong \triangle SNO$ (AAS axiom)

$$\Rightarrow PO = OS$$

$$\Rightarrow QR \text{ bisects } PS$$

Answer 27.



The median of a triangle divides it into two triangles of equal areas.

In $\triangle ABC$, AD is the median

$$\Rightarrow \text{ar}(\triangle ABD) = \text{ar}(\triangle ACD) \quad \text{.....(i)}$$

In $\triangle GBC$, GD is the median

$$\Rightarrow \text{ar}(\triangle GBD) = \text{ar}(\triangle GCD) \quad \text{.....(ii)}$$

Subtracting (ii) from (i),

$$\text{ar}(\triangle ABD) - \text{ar}(\triangle GBD) = \text{ar}(\triangle ACD) - \text{ar}(\triangle GCD)$$

$$\Rightarrow \text{ar}(\triangle AGB) = \text{ar}(\triangle AGC) \quad \text{.....(iii)}$$

Subtracting (ii) from (i),

$$\text{ar}(\triangle ABD) - \text{ar}(\triangle GBD) = \text{ar}(\triangle ACD) - \text{ar}(\triangle GCD)$$

$$\Rightarrow \text{ar}(\triangle AGB) = \text{ar}(\triangle AGC) \quad \text{.....(iii)}$$

$$\text{Similarly, } \text{ar}(\triangle AGB) = \text{ar}(\triangle BGC) \quad \text{.....(iv)}$$

From (iii) and (iv),

$$\text{ar}(\triangle AGB) = \text{ar}(\triangle BGC) = \text{ar}(\triangle AGC) \quad \text{.....(v)}$$

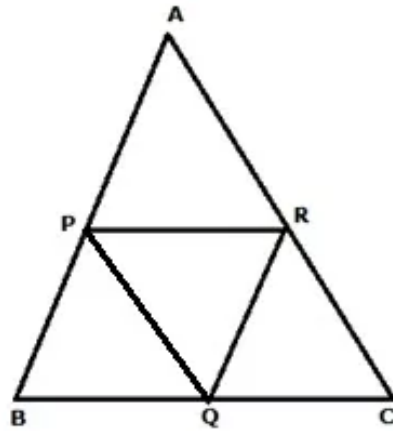
$$\text{But } \text{ar}(\triangle AGB) + \text{ar}(\triangle BGC) + \text{ar}(\triangle AGC) = \text{ar}(\triangle ABC)$$

$$\text{Therefore, } 3 \text{ ar}(\triangle AGB) = \text{ar}(\triangle ABC)$$

$$\Rightarrow \text{ar}(\triangle AGB) = \frac{1}{3} \text{ ar}(\triangle ABC)$$

$$\text{Hence, } \text{ar}(\triangle AGB) = \text{ar}(\triangle AGC) = \text{ar}(\triangle BGC) = \frac{1}{3} \text{ ar}(\triangle ABC).$$

Answer 28.



Since P and R are mid-points of AB and AC respectively.

Therefore, $PR \parallel BC$ and $PR = \frac{1}{2}BC$ (i)

Also Q is mid-point of BC,

$\Rightarrow QC = \frac{1}{2}BC$ (ii)

From (i) and (ii)

$PR \parallel BC$ and $PR = QC$

$\Rightarrow PR \parallel QC$ and $PR = QC$ (iii)

Similarly Q and R are mid-points of BC and AC respectively
 Therefore, $QR \parallel BP$ and $QR = BP$ (iv)
 Hence, BQRP is a parallelogram.

$\Rightarrow PQ$ is a diagonal of $\parallel gm$ BQRP

$ar(\triangle PQR) = ar(\triangle BQP)$ (v) (diagonal of a $\parallel gm$ divides it into two triangles of equal areas)

Similarly QCRP and QRAP are $\parallel gm$ and
 $ar(\triangle PQR) = ar(\triangle QCR) = ar(\triangle APR)$ (vi)

From (v) and (vi)
 $ar(\triangle PQR) = ar(\triangle BQP) = ar(\triangle QCR) = ar(\triangle APR)$

Now, $ar(\triangle ABC) = ar(\triangle PQR) + ar(\triangle BQP) + ar(\triangle QCR) + ar(\triangle APR)$
 $\Rightarrow ar(\triangle ABC) = ar(\triangle PQR) + ar(\triangle PQR) + ar(\triangle PQR) + ar(\triangle PQR)$
 $\Rightarrow ar(\triangle ABC) = 4ar(\triangle PQR)$

$\Rightarrow ar(\triangle PQR) = \frac{1}{4} ar(\triangle ABC)$ (vii)

$ar(\parallel gm BQRP) = ar(\triangle PQR) + ar(\triangle BQP)$

$\Rightarrow ar(\parallel gm BQRP) = ar(\triangle PQR) + ar(\triangle PQR)$ (from (v))

$\Rightarrow ar(\parallel gm BQRP) = 2ar(\triangle PQR)$

$\Rightarrow ar(\parallel gm BQRP) = 2 \times \frac{1}{4} ar(\triangle ABC)$ (from (vii))

$\Rightarrow ar(\parallel gm BQRP) = \frac{1}{2} ar(\triangle ABC)$

Answer 31.

$$\text{Area } (\triangle PQR) = \text{area } (\triangle PQS) + \text{area } (\triangle PSR) \dots(i)$$

Since PS is the median of $\triangle PQR$ and median divides a triangle into two triangles of equal area.

$$\text{Therefore, area } (\triangle PQS) = \text{area } (\triangle PSR) \dots(ii)$$

Substituting in (i)

$$\text{Area } (\triangle PQR) = \text{area } (\triangle PSR) + \text{area } (\triangle PSR)$$

$$\text{Area } (\triangle PQR) = 2\text{area } (\triangle PSR) \dots(iii)$$

$$\text{Area } (\triangle PSR) = \text{area } (\triangle PST) + \text{area } (\triangle PTR) \dots(iv)$$

Since PT is the median of $\triangle PSR$ and median divides a triangle into two triangles of equal area.

$$\text{Therefore, area } (\triangle PST) = \text{area } (\triangle PTR) \dots(v)$$

Substituting in (iv)

$$\text{Area } (\triangle PSR) = 2\text{area } (\triangle PTR) \dots(vi)$$

Substituting in (iii)

$$\text{Area } (\triangle PQR) = 2 \times 2\text{area } (\triangle PTR)$$

$$\text{Area } (\triangle PQR) = 4\text{area } (\triangle PTR) \dots(vii)$$

$$\text{Area } (\triangle PTR) = \text{area } (\triangle PMR) + \text{area } (\triangle MTR) \dots(viii)$$

Since MR is the median of $\triangle PTR$ and median divides a triangle into two triangles of equal area.

$$\text{Therefore, area } (\triangle PMR) = \text{area } (\triangle MTR) \dots(ix)$$

Substituting in (viii)

$$\text{Area } (\triangle PTR) = 2\text{area } (\triangle PMR) \dots(x)$$

Substituting in (vii)

$$\text{Area } (\triangle PQR) = 4 \times 2\text{area } (\triangle PMR)$$

$$\text{Area } (\triangle PQR) = 8 \times \text{area } (\triangle PMR)$$

$$\text{area } (\triangle PMR) = \frac{1}{8} \text{area } (\triangle PQR)$$

Answer 32.

Since the diagonals of a parallelogram divide it into four triangles of equal area

Therefore, area of $\triangle AOD = \text{area } \triangle BOC = \text{area } \triangle ABO = \text{area } \triangle CDO$.

$$\Rightarrow \text{area } \triangle BOC = \frac{1}{4} \text{area (||gm ABCD)} \quad \dots\dots\dots(i)$$

In ||gm ABCD, BD is the diagonal

Therefore, area $(\triangle ABD) = \text{area } (\triangle BCD)$

$$\Rightarrow \text{area } (\triangle BCD) = \frac{1}{2} \text{area (||gm ABCD)} \dots\dots(ii)$$

In ||gm BPCD, BC is the diagonal

Therefore, area $(\triangle BCD) = \text{area } (\triangle BPC) \quad \dots\dots(iii)$

From (iii) and (ii)

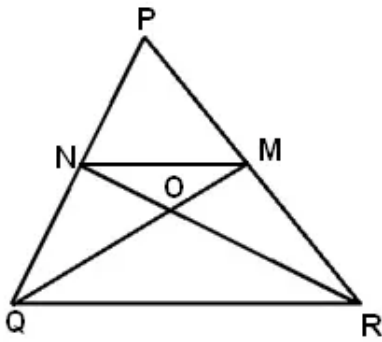
$$\text{area } (\triangle BPC) = \frac{1}{2} \text{area (||gm ABCD)} \quad \dots\dots(iv)$$

adding (i) and (iv)

$$\text{area } (\triangle BPC) + \text{area } \triangle BOC = \frac{1}{2} \text{area (||gm ABCD)} + \frac{1}{4} \text{area (||gm ABCD)}$$

$$\text{Area of OBPC} = \frac{3}{4} \text{ area of ABCD}$$

Answer 33.



Join MN. Since the line segment joining the mid-points of two sides of a triangle is parallel to the third side; so, $MN \parallel QR$

Clearly, $\triangle QMN$ and $\triangle RNM$ are on the same base MN and between the same parallel lines.

Therefore, $\text{area}(\triangle QMN) = \text{area}(\triangle RNM)$

$$\Rightarrow \text{Area}(\triangle QMN) - \text{area}(\triangle ONM) = \text{area}(\triangle RNM) - \text{area}(\triangle ONM)$$

$$\Rightarrow \text{area}(\triangle QON) = \text{area}(\triangle ROM) \quad \dots\dots(i)$$

We know that a median of a triangle divides it into two triangles of equal areas.

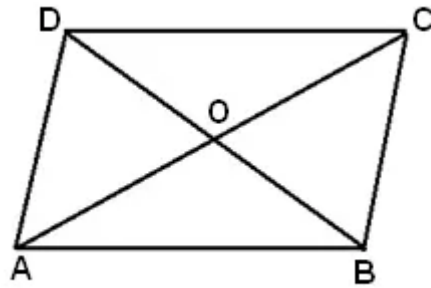
Therefore, $\text{area}(\triangle QMR) = \text{area}(\triangle PQM)$

$$\Rightarrow \text{area}(\triangle ROQ) + \text{area}(\triangle ROM) = \text{area}(\text{quad. PMON}) + \text{area}(\triangle QON)$$

$$\Rightarrow \text{area}(\triangle ROQ) + \text{area}(\triangle ROM) = \text{area}(\text{quad. PMON}) + \text{area}(\triangle ROM) \quad (\text{from (i)})$$

$$\Rightarrow \text{area}(\triangle ROQ) = \text{area}(\text{quad. PMON})$$

Answer 37.



Since the diagonals of a parallelogram bisect each other at the point of intersection.

Therefore, $OB = OD$ and $OA = OC$

In $\triangle ABC$, OB is the median and median divides triangle into two triangles of equal areas

Therefore, $\text{area}(\triangle BOC) = \text{area}(\triangle ABO)$ (i)

In $\triangle ADC$, OD is the median and median divides triangle into two triangles of equal areas

Therefore, $\text{area}(\triangle AOD) = \text{area}(\triangle CDO)$ (ii)

Adding (i) and (ii)

$\text{area}(\triangle AOD) + \text{area}(\triangle BOC) = \text{area}(\triangle ABO) + \text{area}(\triangle CDO)$