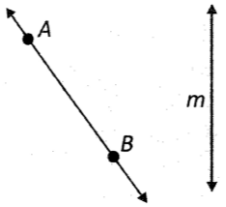
Geometry

Learning Objectives

- Line and angles
- Triangle
- Congruency of triangles
- Symmetry

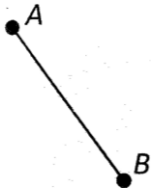
Lines and Angles

- A is an infinitely thin, infinitely long collection of points extending in two opposite directions. While drawing lines in geometry, an arrow at each end is put up to show that it can extend infinitely.



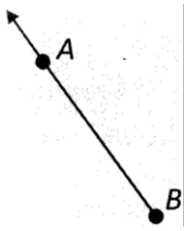
A line can be named either using two points on the line (for example $\overleftrightarrow{AB}$) or simply by a letter, usually lowercase (for example, line m).

- A line segment has two end points. It contains these endpoints and all the points of the line between them. One can measure the length of a segment, but not of a line.



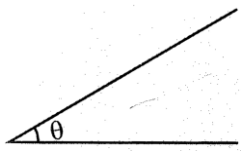
A segment is named by its two endpoints, for example AB

- A ray is a part of a line that has one endpoint and goes on infinitely in only one direction. You cannot measure the length of a ray.



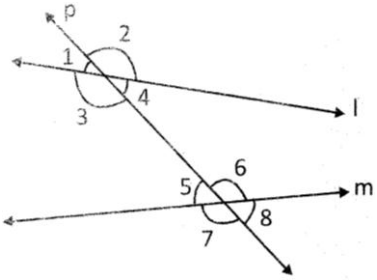
A ray is named using its endpoint first, and then any other point on the ray (for example, $\overrightarrow{BA}$)

- An angle is formed when two lines (or rays or line-segments) meet.

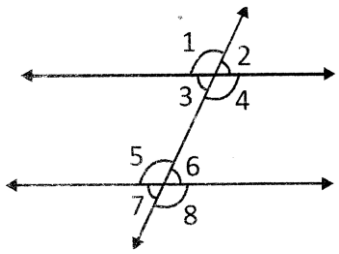


- The sum of two complementary angles is equal to $90°$.

- The sum of two supplementary angles is equal to $180°$.
- A right angle measures $90°$.
- Two adjacent angles have a common vertex and a common arm but no common interior. Linear pair of angles are adjacent and supplementary to each other.
  When two lines l and m meet, we say they intersect; the meeting point is called the point of intersection.
  (i) When two lines intersect (looking like the letter X), two pairs of opposite angles are formed. They are called vertically opposite angles. They are equal in measure.
  (ii) A transversal is a line that intersects two or more lines at distinct points, it gives rise to several types of angles as shown in the figure.



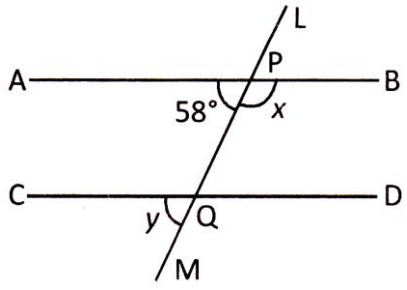
  (iii) In the figure, transversal p intersects the lines I and m.
- $\angle 3, \angle 4, \angle 5, \angle 6$ are interior angles.
- $\angle 1, \angle 2, \angle 7, \angle 8$ are exterior angles.
- $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$ are pairs of corresponding angles.
- $\angle 3$ and $\angle 6$, $\angle 4$ and $\angle 5$ are alternate interior angles.
- $\angle 1$ and $\angle 8$, $\angle 2$ and $\angle 7$ are alternate exterior angles.
- $\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$ are interior angles on the same side of transversal.
  (iv) When a transversal cuts two parallel lines as shown in the figure, following relationships can be derived:



- $\angle 1 = \angle 5, \angle 3 = \angle 7, \angle 2 = \angle 6, \angle 4 = \angle 8$ (Each pair of corresponding angles are equal).
- $\angle 3 = \angle 6, \angle 4 = \angle 5$ (Each pair of alternate interior angles are equal).
- $\angle 3 + \angle 5 = 180°, \angle 4 + \angle 6 = 180°$ (Each pair of interior angles on the same side of transversal are supplementary).

Example

In the following figure $AB \parallel CD$ and LM is transversal line for AB and CD



The values of x and y are respectively

(a) $32°, 58°$ (b) $122° 58°$

(c) $58°, 122°$ (d) $58°, 32°$

(e) None of these

Ans. (b)

Explanation: In the figure x and 58° forms a linear pair.

$\therefore \quad x° + 58° = 180°$

$\Rightarrow x° = 180° - 58° = 112°$

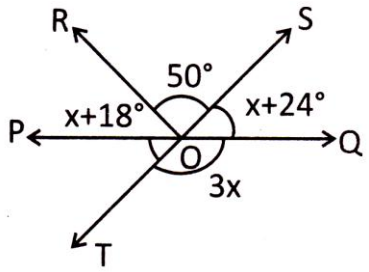
Now $\angle APQ$ and $\angle CQM$ are corresponding angles. Thus,

$\angle CQM = \angle APQ = 58°$

$\Rightarrow \quad y° = 58°$

So, option (b) is correct choice.

In the given figure, if POQ is a straight line then x is equal to:



(a) 144° (b) 145°

(c) 44° (d) 50°

(e) None of these

Ans. (c)

Explanation: $x + 18° + 50° + x + 24° = 180°$

$\Rightarrow \quad 2x + 92° = 180° = 2x = 180° - 92°$

$\Rightarrow 2x = 88° \Rightarrow x = 44°.$

Triangle

- A triangle is one of the basic shapes in geometry: a polygon with three corners or vertices and three sides or edges which are line segments.
- The line segment that joins a vertex of a triangle to the midpoint of its opposite side is called the median of the triangle. A triangle has 3 medians.
- The line segment from a vertex of a triangle that is perpendicular to its opposite side is called an altitude of the triangle. A triangle has 3 altitudes.
- An exterior angle of a triangle is formed when a side of a triangle is produced. At each Vertex, an exterior angle can be formed by producing any of the two lines joining at the vertex.
- The measure of any exterior angle of a triangle is equal to the sum of the measures of its interior opposite angles.
- The total measure of the three angles of a triangle is $180°$. This is called the angle sum property of a triangle.
- Triangles are of three types - (i) equilateral (ii) isosceles and (iii) scalene. An equilateral triangle is the triangle which has equal sides and measure of each angle is also equal to $60°$.
- An isosceles triangle has at least any two of its sides same in length. The non-equal side of an isosceles triangle is called its base.

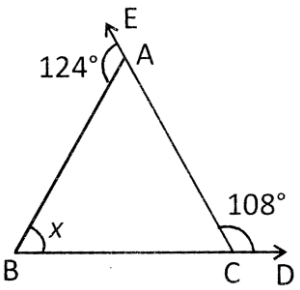
- The base angles of an isosceles triangle have equal measure.
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- The difference between the lengths of any two sides is smaller than the length of the third side.
- In a right-angled triangle, the side opposite to the right angle is called the hypotenuse and the other two sides are called its legs.

  $(\text{Hypotenuse})^2 = (\text{Side}_1)^2 + (\text{Side}_1)^2$

  This property does not hold good for triangles other than right angled triangle.

Example

In the adjoining figure, BC is produced to D and CA is produced to E, and $\angle ACD = 108°$ and $\angle BAE = 124°$, then the value of x is:



(a) 48° (b) 52°
(c) 76° (d) 128°
(e) None of these

Ans. (b)

Explanation: $\angle ACB = 180° - \angle ACD$

$\Rightarrow 180° - 108° = 72° \Rightarrow \angle BAC = 180° - \angle EAB$

$\Rightarrow 180° - 124° = 56°$

From ΔABC, we have

$\Rightarrow x + (\angle ACB + \angle BAC) = 180° \Rightarrow x + (72° - 56°) = 180°$

$\Rightarrow x = 180° - 128° = 52°$

If the angles of a triangle are in the ratio 1: 1: 2, then which one of the following statements is incorrect?

(a) The given triangle is right angled triangle.
(b) The angles of the triangles are 90°, 45° and 45°.
(c) The angles of the triangles are 90°, 45° and 45° and it is right-angled isosceles triangle.
(d) The angles of the triangles are 90°, 45° and 45° and it is scalene.
(e) None of these

Ans. (c)

Explanation: The angles of triangle are x, x and 2x therefore/ from angles sum property of triangle we get $x + x + 2x = 180°$ or, $4x = 180°$

or $x = 45°$, the other angles of the triangle are 90°, 45° and 45°

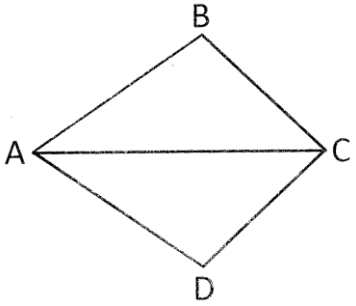
Here two angles are equal. Therefore, the given triangle is isosceles triangle.

Congruency of triangles

- Congruent objects are the exact copies of one another.
- Two plane figures say, F_1 and F_2 are congruent if the trace-copy of F_1 fits exactly on that of F_2. We write this as $F_1 \cong F_2$
- Two line segment say, $\overline{AB}$ and $\overline{CD}$, are congruent if they have equal lengths. We write this as $\overline{AB} \cong \overline{CD}$. However, it is common to write it as $\overline{AB} = \overline{CD}$.
- Two angles, say, $\angle$ABC = $\angle$PQR, are congruent if their measures are equal. It can be written as $\angle ABC \cong \angle PQR$ or as m $\angle$ABC = m$\angle$PQR. However in practice it is common to write as $\angle$ABC = $\angle$PQR
- **SSS congruence of two triangles:** Under a given correspondence, two triangles are congruent if the three sides of the one are equal to the three corresponding sides of the other.
  SAS congruence of two triangles: Under a given correspondence, two triangles are congruent if two sides and the angle included between them in one of the triangles are equal to the corresponding sides and the angle included between them of the other triangle.
  ASA congruence of two triangles: Under a given correspondence, two triangles are congruent if two angles and the side the side included between them in one of the triangles are equal to the corresponding angles and the side included between them of the other triangle.

Example

In the given figure, AC bisect $\angle$BAD and $\angle$BCD. Then which of the following methods can be used to prove ΔABC $\cong$ ΔADC ?



(a) Side-Angle-Side (SAS)
(b) Angie-side-Angle ASA)
(c) Angle-Angle-Side (AAS)
(d) Information is insufficient
(e) None of these

Ans. (B)

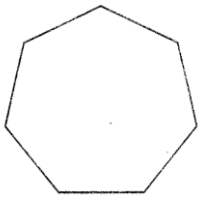
Explanation: In ΔABC and ΔADC,
AC = AC (common)
$\angle$BAC = $\angle$DAC (AC bisects ÐBAD)
$\angle$BCA = $\angle$ACD (AC bisects ÐBCD)
$\therefore \Delta ABC \cong \Delta ADC$ by ASA congruence theorem.

Symmetry

- If there is a line one a figure about which the figure may be folded so that the two parts of the figure will coincide, the figure is said to have a line of symmetry.
- Regular polygons have equal sides and equal angles. They have multiple (i.e., more than one) lines of symmetry
- Mirror reflection leads of symmetry, under which the left-right orientation have to be taken care of.
- Rotation turns an object about a fixed point. This fixed point is called the centre of rotation. The angle by which the object rotates is the angle of rotation. A half-turn means rotation by 180, a quarter-turn means rotation by 90. Rotation may be clockwise or anti-clockwise
- If, after a rotation, an object looks exactly the same, we say that it has a rotational symmetry.
- When an object completely turns (turns by 360°); the number of times it looks exactly the same is called the order of rotational symmetry. For example, the order of rotational symmetry of a square/ for example, is 4 while, for an equilateral triangle, it is 3.
- Some shapes have only one line of symmetry. For example, the letter E, while some have only rotational symmetry, like the letter S; and some have both symmetries like the letter H.

Example

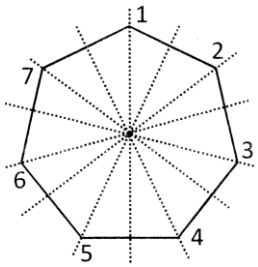
How many lines of symmetry does the following figure has?



(a) 4 (b) 5
(c) 6 (d) 7
(e) None of these

Ans. (d)

Explanation: The given figure has 7 lines of symmetry as shown below:



Which of these letter has highest order of rotational symmetry?

(a) H (b) S
(c) N (d) O
(e) None of these

Ans. (d)

Explanation: Letter O has infinite number of rotational symmetries if written circularly.