Chapter 7

Exponential and Logarithm Series

Contents

Charles Hermite

The transcendence of e was proved by Charles Hermite *in* 1873 A.D. *In* 1926 A.D., D.H. Lehmer *computed the value of e to 709 decimal places by using a continued-fraction expansion.*

Newton (born 1642 A.D) *also expressed* log (1+x) *as an infinite series by expanding* $\frac{1}{(x+1)}$ *as* $(1 - x + x^2 - x^3 + ...)$. *However, it was* Nicolaus Mercator *who first published in 1668 A.D., the series* $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - .....$

The expansion of $\frac{1}{2}\log(1+x)(1-x)$ *was found by* John Wallis *in* 1695 A.D.

Exponential Series

7.1 Definition (The number *e*)

The limiting value of $\left(1+\frac{1}{n}\right)^n$ when n tends to infinity is denoted by e

i.e., e = $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+.........\infty$ = 2.71 (Nearly)

7.2 Properties of *e*

(1) e lies between 2.7 and 2.8. *i.e.,* 2.7 < e < 2.8 (since $\frac{1}{n!}\le\frac{1}{2^{n-1}}$ for $n \ge 2$)

(2) The value of e correct to 10 places of decimals is 2.7182818284

(3) e is an irrational (incommensurable) number

(4) e is the base of natural logarithm (Napier logarithm) *i.e.* $\ln x = \log_e x$ and $\log_{10} e$ is known as Napierian constant. $\log_{10} e = 0.43429448$, $\ln x = 2.303 \log_{10} x$

$$\left(\text{since } \ln x = \log_{10} x.\log_e 10 \text{ and } \log_e 10 = \frac{1}{\log_{10} e} = 2.30258509\right)$$

7.3 Exponential Series

For $x \in R$, $e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+......+\frac{x^r}{r!}+......\infty$ or $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$

The above series known as exponential series and e^x is called exponential function. Exponential function is also denoted by exp. *i.e.* $\exp A = e^A$; $\therefore$ $\exp x = e^x$

7.4 Exponential Function a^x, where a > 0

$\because a^x = e^{\log_e a^x} = e^{x\log_e a}$

$\therefore a^x = e^{\alpha x}$ ....(i) , where $\alpha = \log_e a$. We have, $e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}........+\frac{x^r}{r!}+........\infty$

Replacing x *by* αx in this series, $e^{\alpha x} = 1+\frac{\alpha x}{1!}+\frac{\alpha^2 x^2}{2!}+\frac{\alpha^3 x^3}{3!}+.....+\frac{\alpha^r x^r}{r!}+.....\infty$

Hence from (i), $a^x = 1 + \frac{\log_e a}{1!} x + \frac{(\log_e a)^2}{2!} x^2 + ... + \frac{(\log_e a)^r x^r}{r!} + ...\infty$

7.5 Some Important Results from Exponential Series

We have the exponential series

(1) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + ...\infty = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ .....(i)

(2) Replacing x by $-x$ in (i), we obtain $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + ...\infty = \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!}$ .....(ii)

(3) Putting $x = 1$ in (i) and (ii), we get, $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ...\infty = \sum_{n=0}^{\infty} \frac{1}{n!}$

$e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + ...\infty = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

(4) From (i) and (ii), we obtain $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + ...\infty = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

$\frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + ...\infty = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$

(5) $\frac{e + e^{-1}}{2} = 1 + \frac{1}{2!} + \frac{1}{4!} + ...\infty = \sum_{n=0}^{\infty} \frac{1}{(2n)!}$, $\frac{e - e^{-1}}{2} = 1 + \frac{1}{3!} + \frac{1}{5!} + ...\infty = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!}$

Note : ❑ $e - 1 = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ...\infty = \sum_{r=1}^{\infty} \frac{1}{r!}$ ❑ $e - 2 = \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ...\infty = \sum_{r=2}^{\infty} \frac{1}{r!}$

7.6 Some Standard results

(1) $\sum_{n=0}^{\infty} \frac{1}{n!} = \sum_{n=0}^{\infty} \frac{1}{(n-1)!} = \sum_{n=0}^{\infty} \frac{1}{(n-k)!} = e$

(2) $\sum_{n=1}^{\infty} \frac{1}{n!} = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ...\infty = e - 1$

(3) $\sum_{n=2}^{\infty} \frac{1}{n!} = \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ...\infty = e - 2$

(4) $\sum_{n=0}^{\infty} \frac{1}{(n+1)!} = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ...\infty = e - 1$

(5) $\sum_{n=0}^{\infty} \frac{1}{(n+2)!} = \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ...\infty = e - 2$

(6) $\sum_{n=1}^{\infty} \frac{1}{(n+1)!} = \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + ...\infty = e - 2$

(7) $\sum_{n=0}^{\infty} \frac{1}{(2n)!} = 1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + ...\infty = \frac{e + e^{-1}}{2} = \sum_{n=1}^{\infty} \frac{1}{(2n-2)!}$

(8) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)!} = \frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + ...\infty = \frac{e - e^{-1}}{2} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!}$

(9) $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^n}{n!} + ...\infty$

$\therefore T_{n+1} =$ General term in the expansion of $e^x = \frac{x^n}{n!}$ and coefficient of x^n in $e^x = \frac{1}{n!}$

(10) $e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + ... + (-1)^n \frac{x^n}{n!} + ...\infty$

$\therefore T_{n+1} =$ General term in the expansion of $e^{-x} = (-1)^n \frac{x^n}{n!}$ and coefficient of x^n in $e^{-x} = \frac{(-1)^n}{n!}$

(11) $e^{ax} = 1 + \frac{(ax)}{1!} + \frac{(ax)^2}{2!} + \frac{(ax)^3}{3!} + ... + \frac{(ax)^n}{n!} + ...\infty$

$\therefore T_{n+1} =$ General term in the expansion of $e^{ax} = \frac{(ax)^n}{n!}$ and coefficient of x^n in $e^{ax} = \frac{a^n}{n!}$

(12) $\sum_{n=0}^{\infty} \frac{n}{n!} = e = \sum_{n=1}^{\infty} \frac{n}{n!}$

(13) $\sum_{n=0}^{\infty} \frac{n^2}{n!} = 2e = \sum_{n=1}^{\infty} \frac{n^2}{n!}$

(14) $\sum_{n=0}^{\infty} \frac{n^3}{n!} = 5e = \sum_{n=1}^{\infty} \frac{n^3}{n!}$

(15) $\sum_{n=0}^{\infty} \frac{n^4}{n!} = 15e = \sum_{n=1}^{\infty} \frac{n^4}{n!}$

Example: 1 $\frac{2}{1!} + \frac{4}{3!} + \frac{6}{5!} + \frac{8}{7!} + ... \infty =$ **[JMI CET 2000]**

(a) $1/e$ (b) e (c) $2e$ (d) $3e$

Solution: (b) $\frac{2}{1!} + \frac{4}{3!} + \frac{6}{5!} + \frac{8}{7!} + ... \infty = \frac{(1+1)}{1!} + \frac{(1+3)}{3!} + \frac{(1+5)}{5!} + \frac{(1+7)}{7!} + ... \infty$

$= \left(\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + ... \infty\right) + \left(1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + ... \infty\right) = \frac{e - e^{-1}}{2} + \frac{e + e^{-1}}{2} = e$

Example: 2 $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + ... \infty =$ **[MNR 1979; MP PET 1995, 2002]**

(a) e (b) $2e$ (c) e^2 (d) $1/e$

Solution: (d) Here $T_n = \frac{(2n+1)-1}{(2n+1)!} = \frac{1}{(2n)!} - \frac{1}{(2n+1)!} \Rightarrow S = \sum_{n=1}^{\infty} T_n = \left(\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + ... \infty\right) - \left(\frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + ... \infty\right)$

$\Rightarrow S = \left(\frac{e + e^{-1}}{2} - 1\right) - \left(\frac{e - e^{-1}}{2} - 1\right) \Rightarrow e^{-1} = \frac{1}{e}$

Example: 3 $1 + \frac{2^3}{2!} + \frac{3^3}{3!} + \frac{4^3}{4!} + ... \infty =$ **[MNR 1976; MP PET 1997]**

(a) $2e$ (b) $3e$ (c) $4e$ (d) $5e$

Solution: (d) $S = \frac{1^3}{1!} + \frac{2^3}{2!} + \frac{3^3}{3!} + ... + \frac{n^3}{n!} + ...$

Here $T_n = \frac{n^3}{n!} \Rightarrow S_n = \sum_{n=1}^{\infty} \frac{n^3}{n!} = 5e$

Example: 4 The coefficient of x^n in the expansion of $\frac{e^{7x} + e^x}{e^{3x}}$ is

(a) $\frac{4^{n-1} + (-2)^n}{n!}$ (b) $\frac{4^{n-1} + 2^n}{n!}$ (c) $\frac{4^{n-1} + (-2)^{n-1}}{n!}$ (d) $\frac{4^n + (-2)^n}{n!}$

Solution: (d) We have $\frac{e^{7x} + e^x}{e^{3x}} = e^{4x} + e^{-2x} = \sum_{n=0}^{\infty} \frac{(4x)^n}{n!} + \sum_{n=0}^{\infty} \frac{(-2x)^n}{n!}$ $\therefore$ coefficient of x^n in $\frac{e^{7x} + e^x}{e^{3x}} = \frac{4^n + (-2)^n}{n!}$

Example: 5 $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \frac{1+2+3+4}{4!} + ... \infty =$ **[Roorkee 1999; MP PET 2003]**

(a) e (b) $3e$ (c) $e/2$ (d) $3e/2$

Solution: (d) $T_n = \frac{\sum n}{n!} = \frac{n(n+1)}{2.n!} = \frac{1}{2}\left[\frac{(n+1)}{(n-1)!}\right] = \frac{1}{2}\left[\frac{n-1}{(n-1)!} + \frac{2}{(n-1)!}\right] = \frac{1}{2}\left[\frac{1}{(n-2)!} + \frac{2}{(n-1)!}\right]$

$$Sn = \sum_{n=1}^{\infty} T_n = \frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{(n-2)!} + \sum_{n=1}^{\infty}\frac{1}{(n-1)!} = \frac{e}{2} + e = \frac{3e}{2}$$

Logarithmic Series

7.7 Logarithmic Series

An expansion for $\log_e(1+x)$ as a series of powers of x which is valid only when, $|x| < 1$,

Expansion of $\log_e(1+x)$; if $|x| < 1$, then $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\infty$

7.8 Some Important Results from the Logarithmic Series

(1) Replacing x by $-x$ in the logarithmic series, we get

$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots\ldots\infty$ or $-\log_e(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots\ldots\infty$

(2) (i) $\log_e(1+x) + \log_e(1-x) = \log_e(1-x^2) = -2\left\{\frac{x^2}{2} + \frac{x^4}{4} + \ldots\ldots\infty\right\}, (-1 < x < 1)$

(ii) $\log_e(1+x) - \log_e(1-x) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots\infty\right]$ or $\log_e\left(\frac{1+x}{1-x}\right) = 2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\ldots\infty\right]$

(3) The series expansion of $\log_e(1+x)$ may fail to be valid if $|x|$ is not less than 1. It can be proved that the logarithmic series is valid for x=1. Putting x=1 in the logarithmic series.

We get, $\log_e 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots\ldots\infty = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots\ldots\infty$

(4) When $x = -1$, the logarithmic series does not have a sum. This is in conformity with the fact that log(1-1) is not a finite quantity.

7.9 Difference between the Exponential and Logarithmic Series

(1) In the exponential series $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots\infty$ all the terms carry positive signs whereas in the logarithmic series $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots\infty$ the terms are alternatively positive and negative.

(2) In the exponential series the denominator of the terms involve factorial of natural numbers. But in the logarithmic series the terms do not contain factorials.

(3) The exponential series is valid for all the values of x. The logarithmic series is valid when $|x| < 1$.

Example: 6 $0.5-\frac{(0.5)^2}{2}+\frac{(0.5)^3}{3}-\frac{(0.5)^4}{4}+........$ **[MP PET 1995]**

(a) $\log_e\left(\frac{3}{2}\right)$ (b) $\log_{10}\left(\frac{1}{2}\right)$ (c) $\log_e(n!)$ (d) $\log_e\left(\frac{1}{2}\right)$

Solution: (a) We know that, $x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}........\infty=\log_e(1+x)$

Putting x = 0.5, we get, $0.5-\frac{(0.5)^2}{2}+\frac{(0.5)^3}{3}-\frac{(0.5)^4}{4}+........\infty=\log_e(1+0.5)=\log_e\left(\frac{3}{2}\right)$

Example: 7 $\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\frac{1}{4.5}+........\infty=$ **[Roorkee 1992; MP PET 1999; AIEEE 2003]**

(a) $\log_e\left(\frac{4}{e}\right)$ (b) $\log_e\frac{e}{4}$ (c) $\log_e 4$ (d) $\log_e 2$

Solution: (a) We know that, $\log_e 2=\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+.......\infty$ ....(i)

Also $\log_e 2=1-\left(\frac{1}{2.3}\right)-\left(\frac{1}{4.5}\right)-\left(\frac{1}{6.7}\right)-.......\infty$ ....(ii)

By adding (i) and (ii), we get, $2\log_e 2=1+\left(\frac{1}{1.2}-\frac{1}{2.3}\right)+\left(\frac{1}{3.4}-\frac{1}{4.5}\right)+........$

$\Rightarrow 2\log_e 2-1=\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\frac{1}{4.5}+........ \Rightarrow \frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}-\frac{1}{4.5}+........\infty=\log_e 4-\log_e e=\log_e\left(\frac{4}{e}\right)$

Example: 8 The coefficient of x^n in the expansion of $\log_e(1+3x+2x^2)$ is

(a) $(-1)^n\left[\frac{2^n+1}{n}\right]$ (b) $\frac{(-1)^{n+1}}{n}[2^n+1]$ (c) $\frac{2^n+1}{n}$ (d) None of these

Solution: (b) We have, $\log_e(1+3x+2x^2)=\log_e(1+x)+\log_e(1+2x)$

$$=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n}+\sum_{n=1}^{\infty}(-1)^{n-1}\frac{(2x)^n}{n}=\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{n}+\frac{2^n}{n}\right)x^n=\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1+2^n}{n}\right)x^n$$

So coefficient of $x^n=(-1)^{n-1}\left(\frac{2^n+1}{n}\right)=\frac{(-1)^{n+1}(2^n+1)}{n}$ $\left[\because (-1)^n=(-1)^{n+2}=........\right]$

Example: 9 The equation $x^{\log_x(2+x)^2}=25$ holds for **[MP PET 1992]**

(a) $x=6$ (b) $x=-3$ (c) $x=3$ (d) $x=7$

Solution: (c) Given equation $x^{\log_x(2+x)^2}=25 \Rightarrow (2+x)^2=25$ hold for $x=3$

Example: 10 If $y=-\left(x^3+\frac{x^6}{2}+\frac{x^9}{3}+........\right)$, then $x=$ **[MNR 1975]**

(a) $\frac{1+e^y}{3}$ (b) $\frac{1-e^y}{3}$ (c) $(1-e^y)^{\frac{1}{3}}$ (d) $(1-e^y)^3$

Solution: (c) $y=-\left[x^3+\frac{(x^3)^2}{2}+\frac{(x^3)^3}{3}+........\right]=\log_e(1-x^3) \Rightarrow e^y=1-x^3 \Rightarrow x=(1-e^y)^{\frac{1}{3}}$

Assignment

Exponential series

Basic Level

1. $1+\frac{4^2}{3!}+\frac{4^4}{5!}+........\infty$

(a) $\frac{e^4+e^{-4}}{4}$ (b) $\frac{e^4-e^{-4}}{4}$ (c) $\frac{e^4+e^{-4}}{8}$ (d) $\frac{e^4-e^{-4}}{8}$

2. $\frac{1}{2!}+\frac{1+2}{3!}+\frac{1+2+3}{4!}+.....\infty=$ **[EAMCET 2003]**

(a) e (b) $2e$ (c) $\frac{e}{2}$ (d) None of these

3. $1+\frac{(\log_e x)^2}{2!}+\frac{(\log_e x)^4}{4!}+........\infty$

(a) x (b) $\frac{1}{x}$ (c) $\frac{1}{2}(x+x^{-1})$ (d) $\frac{1}{2}(e^x+e^{-x})$

4. $1+\frac{2^2}{2!}+\frac{3^2}{3!}+\frac{4^2}{4!}+........\infty=$

(a) e (b) $2\,e$ (c) $3\,e$ (d) None of these

5. $\frac{1}{1!}+\frac{4}{2!}+\frac{7}{3!}+\frac{10}{4!}+........\infty=$

(a) $e+4$ (b) $2+e$ (c) $3+e$ (d) e

6. $\frac{2}{1!}+\frac{2+4}{2!}+\frac{2+4+6}{3!}+........\infty=$ **[MNR 1985]**

(a) e (b) $2\,e$ (c) $3\,e$ (d) None of these

7. $\left(1+\frac{1}{2!}+\frac{1}{4!}+........\infty\right)^2-\left(1+\frac{1}{3!}+\frac{1}{5!}+........\infty\right)^2=$

(a) 0 (b) 1 (c) –1 (d) 2

8. $1+\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+........\infty=$ **[MP PET 1991]**

(a) e^{-1} (b) e (c) $\frac{e+e^{-1}}{2}$ (d) $\frac{e-e^{-1}}{2}$

9. $1+\frac{\log_e x}{1!}+\frac{(\log_e x)^2}{2!}+\frac{(\log_e x)^3}{3!}+........\infty=$ **[Kurukshetra CEE 1998; JMI CET 2000]**

(a) $\log_e x$ (b) x (c) x^{-1} (d) $-\log_e(1+x)$

10. $\frac{x^2-y^2}{1!}+\frac{x^4-y^4}{2!}+\frac{x^6-y^6}{3!}+..........\infty=$

(a) e^x-e^y (b) $e^{x^2}-e^{y^2}$ (c) $2+e^{x^2}-e^{y^2}$ (d) $\frac{e^x-e^y}{2}$

11. $1+x\log_e a+\frac{x^2}{2!}(\log_e a)^2+\frac{x^3}{3!}(\log_e a)^3+........=$ **[EAMCET 2002]**

(a) a^x (b) x (c) $a^{\log_a x}$ (d) a

12. $3+\frac{5}{1!}+\frac{7}{2!}+\frac{9}{3!}+........\infty=$

(a) $3e$ (b) $5\,e$ (c) $5e-1$ (d) None of these

13. $1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+.....\infty=$ **[MP PET 1986]**

(a) e^x (b) e^{-x} (c) e (d) e^{x^2}

14. $\frac{2}{1!}\log_e 2+\frac{2^2}{2!}(\log_e 2)^2+\frac{2^3}{3!}(\log_e 2)^3+....\infty=$

(a) 2 (b) 3 (c) 4 (d) None of these

15. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8(2)!}+\frac{1}{16(3)!}+\frac{1}{32(4)!}+....\infty=$

(a) e (b) $\sqrt{e}$ (c) $\frac{\sqrt{e}}{2}$ (d) None of these

16. Sum to infinity of the series $1+\frac{x^2}{2!}+\frac{x^4}{4!}+.......$ is **[MP PET 1994]**

(a) $\frac{e^x-e^{-x}}{2}$ (b) $\frac{e^x+e^{-x}}{2}$ (c) $\frac{e^{-x}-e^x}{2}$ (d) $\frac{-(e^x+e^{-x})}{2}$

17. Sum of the infinite series $1+2+\frac{1}{2!}+\frac{2}{3!}+\frac{1}{4!}+\frac{2}{5!}+....$ is **[Roorkee 2000]**

(a) e^2 (b) $e+e^{-1}$ (c) $\frac{e-e^{-1}}{2}$ (d) $\frac{3e-e^{-1}}{2}$

18. The value of $1-\log 2+\frac{(\log 2)^2}{2!}-\frac{(\log 2)^3}{3!}+.....$ is **[MP PET 1998]**

(a) 2 (b) $\frac{1}{2}$ (c) $\log 3$ (d) None of these

19. The sum of the series $\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-.....$ is **[DCE 2002]**

(a) e (b) $e^{-\frac{1}{2}}$ (c) e^{-2} (d) None of these

20. If $S=\sum_{n=2}^{\infty}{}^nC_2\frac{3^{n-2}}{n!}$, then $2S$ equals

(a) $e^{3/2}$ (b) e^3 (c) $e^{-3/2}$ (d) e^{-3}

21. The coefficient of x^r in the expansion of $1+\frac{a+bx}{1!}+\frac{(a+bx)^2}{2!}+....+\frac{(a+bx)^n}{n!}+......$ is **[MP PET 1989]**

(a) $\frac{(a+b)^r}{r!}$ (b) $\frac{b^r}{r!}$ (c) $\frac{e^a b^r}{r!}$ (d) e^{a+b^r}

22. In the expansion of $\frac{a+bx}{e^x}$, the coefficient of x^r is

(a) $\frac{a-b}{r!}$ (b) $\frac{a-br}{r!}$ (c) $(-1)^r\frac{a-br}{r!}$ (d) None of these

23. In the expansion of $(e^x-1)(e^{-x}+1)$, the coefficient of x^3 is

(a) 0 (b) 1/3 (c) 2/3 (d) 1/6

24. In the expansion of $\frac{a+bx+cx^2}{e^x}$, the coefficient of x^n will be

(a) $\frac{a(-1)^n}{n!}+\frac{b(-1)^{n-1}}{(n-1)!}+\frac{c(-1)^{n-2}}{(n-2)!}$ (b) $\frac{a}{n!}+\frac{b}{(n-1)!}+\frac{c}{(n-2)!}$ (c) $\frac{(-1)^n}{n!}+\frac{(-1)^{n-1}}{(n-1)!}+\frac{(-1)^{n-2}}{(n-2)!}$ (d) None of these

25. If n is even, then in the expansion of $\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+....\right)^2$, the coefficient of x^n is

(a) $\frac{2^n}{n!}$ (b) $\frac{2^n-2}{n!}$ (c) $\frac{2^{n-1}-1}{n!}$ (d) $\frac{2^{n-1}}{n!}$

26. If $e^x = y+\sqrt{1+y^2}$, then $y =$ **[MNR 1990; UPSEAT 2000]**

(a) $\frac{e^x+e^{-x}}{2}$ (b) $\frac{e^x-e^{-x}}{2}$ (c) e^x+e^{-x} (d) e^x-e^{-x}

27. $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{2n} =$

(a) e (b) e^{-1} (c) e^2 (d) e^{-2}

Advance Level

28. $\frac{1^2}{2!}+\frac{2^2}{3!}+\frac{3^2}{4!}+....\infty =$

(a) e (b) $e-1$ (c) $e+1$ (d) e^2

29. The sum of the series $1+\frac{3}{2!}+\frac{7}{3!}+\frac{15}{4!}+........\infty$ is **[AMU 1992; Kurukshetra CEE 1999]**

(a) $e(e+1)$ (b) $e(1-e)$ (c) $e(e-1)$ (d) $3e$

30. $\left(1+\frac{1}{2!}+\frac{1}{4!}+....\right)\left(1+\frac{1}{3!}+\frac{1}{5!}+....\right) =$

(a) e^4 (b) $\frac{e^2-1}{e^2}$ (c) $\frac{e^4-1}{4e^2}$ (d) $\frac{e^4+1}{4e^2}$

31. $\frac{\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+......\infty}{1+\frac{1}{3!}+\frac{1}{5!}+\frac{1}{7!}+....\infty} =$

(a) $\frac{e+1}{e-1}$ (b) $\frac{e-1}{e+1}$ (c) $\frac{e^2+1}{e^2-1}$ (d) $\frac{e^2-1}{e^2+1}$

32. $\frac{1+\frac{2^2}{2!}+\frac{2^4}{3!}+\frac{2^6}{4!}+......\infty}{1+\frac{1}{2!}+\frac{2}{3!}+\frac{2^2}{4!}+....\infty} =$

(a) e^2 (b) e^2-1 (c) $e^{3/2}$ (d) None of these

33. $1+\frac{2^4}{2!}+\frac{3^4}{3!}+\frac{4^4}{4!}+.....\infty =$

(a) $5e$ (b) e (c) $15e$ (d) $2e$

34. $(1+3)\log_e 3+\frac{1+3^2}{2!}(\log_e 3)^2+\frac{1+3^3}{3!}(\log_e 3)^3+.....\infty =$ **[Roorkee 1989]**

(a) 28 (b) 30 (c) 25 (d) 0

35. $\frac{1}{1!}+\frac{1+2}{2!}+\frac{1+2+2^2}{3!}+.....\infty =$ **[AMU 1992; Kurukshetra CEE 1999; EAMCET 2002]**

(a) e^2 (b) e^2-1 (c) e^2-e (d) e^3-e^2

36. The sum of the series $\sum_{n=0}^{\infty}\frac{n^2-n+1}{n!}$ is **[AMU 1991]**

(a) e (b) $\frac{3}{2}e$ (c) $2e$ (d) $3e$

37. $\frac{1^2.2}{1!}+\frac{2^2.3}{2!}+\frac{3^2.4}{3!}+.....\infty=$ **[UPSEAT 1999]**

(a) $6e$ (b) $7e$ (c) $8e$ (d) $9e$

38. The value of $\sqrt{e}$ will be **[UPSEAT 1999]**

(a) 1.648 (b) 1.547 (c) 1.447 (d) 1.348

39. The sum of the infinite series

$\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\frac{4}{5!}+.....$ is **[AMU 1999]**

(a) $e-2$ (b) $\frac{2}{3}e-1$ (c) 1 (d) 3/2

40. The sum of $\frac{2}{1!}+\frac{6}{2!}+\frac{12}{3!}+\frac{20}{4!}+.....$ is **[UPSEAT 2000]**

(a) $\frac{3e}{2}$ (b) e (c) $2e$ (d) $3e$

41. $1+\frac{(\log_e n)^2}{2!}+\frac{(\log_e n)^4}{4!}+....=$ **[MP PET 1996]**

(a) n (b) $1/n$ (c) $\frac{1}{2}(n+n^{-1})$ (d) $\frac{1}{2}(e^n+e^{-n})$

42. The sum of the series

$\frac{1^2}{1.2!}+\frac{1^2+2^2}{2.3!}+\frac{1^2+2^2+3^2}{3.4!}+......+\frac{1^2+2^2+.....+n^2}{n.(n+1)!}+....\infty$ equals **[AMU 2002]**

(a) e^2 (b) $\frac{1}{2}(e+e^{-1})^2$ (c) $\frac{3e-1}{6}$ (d) $\frac{4e+1}{6}$

43. The value of $(a+b)(a-b)+\frac{1}{2!}(a+b)(a-b)(a^2+b^2)+\frac{1}{3!}(a+b)(a-b)(a^4+a^2b^2+b^4)+.....$ is

(a) $e^{a^2}-e^{b^2}$ (b) $e^{a^2}+e^{b^2}$ (c) $e^{a^2-b^2}$ (d) None of these

44. Sum of the series $C=1+\frac{\cos x}{1!}+\frac{\cos 2x}{2!}+\frac{\cos 3x}{3!}+.....$ and $S=\frac{\sin x}{1!}+\frac{\sin 2x}{2!}+\frac{\sin 3x}{3!}+.....$ is equal to **[AMU 2001]**

(a) exp (ix) (b) exp [cos (sin x)+i sin(sin x)] (c) exp [exp(ix)] (d) exp (cos x)[exp (ix)]

45. The sum of the series

$\frac{4}{1!}+\frac{11}{2!}+\frac{22}{3!}+\frac{37}{4!}+\frac{56}{5!}+.....$ is **[Kurukshetra CEE 2002]**

(a) $6e$ (b) $6e-1$ (c) $5e$ (d) $5e+1$

46. The sum of the series

$\frac{1}{1.2}+\frac{1.3}{1.2.3.4}+\frac{1.3.5}{1.2.3.4.5.6}+......\infty$ is **[Kurukshetra CEE 2002]**

(a) $15e$ (b) $e^{1/2}+e$ (c) $e^{1/2}-1$ (d) $e^{1/2}-e$

47. $\frac{9}{1!}+\frac{16}{2!}+\frac{27}{3!}+\frac{42}{4!}+.....=$ **[Roorkee 1992]**

(a) $5e$ (b) $7e$ (c) $9e$ (d) $11e-6$

48. If S_n denotes the sum of the products of the first n natural numbers taken two at a time, then $\sum_{n=0}^{\infty}\frac{S_n}{(n+1)!}=$

(a) $\frac{11e}{24}$ (b) $\frac{11e}{12}$ (c) $\frac{13e}{24}$ (d) None of these

49. The sum of the series $1+\frac{1^2+2^2}{2!}+\frac{1^2+2^2+3^2}{3!}+\frac{1^2+2^2+3^2+4^2}{4!}+.....$ is

(a) $3e$ (b) $\frac{17}{6}e$ (c) $\frac{13}{6}e$ (d) $\frac{19}{6}e$

50. The sum of the series $\frac{9}{1!}+\frac{19}{2!}+\frac{35}{3!}+\frac{57}{4!}+\frac{85}{5!}+....$ is

(a) $12e-7$ (b) $12e-5$ (c) $12e-11$ (d) None of these

51. The sum of the series $\frac{1^2.2^2}{1!}+\frac{2^2.3^2}{2!}+\frac{3^2.4^2}{3!}+......$ is

(a) $27e$ (b) $24e$ (c) $28e$ (d) None of these

52. If $a=\sum_{n=0}^{\infty}\frac{x^{3n}}{(3n)!}, b=\sum_{n=1}^{\infty}\frac{x^{3n-2}}{(3n-2)!}$ and $c=\sum_{n=1}^{\infty}\frac{x^{3n-1}}{(3n-1)!}$, then the value of $a^3+b^3+c^3-3abc$ is

(a) 1 (b) 0 (c) – 1 (d) – 2

Logarithmic series

Basic Level

53. $\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+....\infty=$

(a) $\frac{x}{1+x}-\log_e(1-x)$ (b) $\frac{x}{1+x}+\log_e(1-x)$ (c) $\frac{x}{1-x}-\log_e(1-x)$ (d) $\frac{x}{1-x}+\log_e(1-x)$

54. $1+\frac{1}{3.\,2^2}+\frac{1}{5.\,2^4}+\frac{1}{7.\,2^6}+....\infty=$

(a) $\log_e 3$ (b) $2\log_e 3$ (c) $\frac{1}{2}\log_e 3$ (d) None of these

55. $\frac{1}{2}+\frac{3}{2}\cdot\frac{1}{4}+\frac{5}{3}\cdot\frac{1}{8}+\frac{7}{4}\cdot\frac{1}{16}+.....\infty=$

(a) $2-\log_e 2$ (b) $2+\log_e 2$ (c) $\log_e 4$ (d) None of these

56. $\frac{1}{x}-\frac{1}{2x^2}+\frac{1}{3x^3}-....\infty=$

(a) $\log_e\frac{x-1}{x}$ (b) $\log_e\frac{x+1}{x}$ (c) $\log_e\frac{1}{x}$ (d) None of these

57. $\left(\frac{a-b}{a}\right)+\frac{1}{2}\left(\frac{a-b}{a}\right)^2+\frac{1}{3}\left(\frac{a-b}{a}\right)^3+....=$ **[MNR 1979; MP PET 1990; UPSEAT 2001, 02]**

(a) $\log_e(a-b)$ (b) $\log_e\left(\frac{a}{b}\right)$ (c) $\log_e\left(\frac{b}{a}\right)$ (d) $e^{\left(\frac{a-b}{a}\right)}$

58. $\frac{1}{5}+\frac{1}{2}\cdot\frac{1}{5^2}+\frac{1}{3}\cdot\frac{1}{5^3}+......\infty=$

(a) $\log_e\frac{4}{5}$ (b) $\log_e\frac{\sqrt{5}}{2}$ (c) $2\log_e\frac{\sqrt{5}}{2}$ (d) None of these

59. The sum of the series $\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+....=$ **[MP PET 1998]**

(a) $\log\frac{2}{e}$ (b) $\log\frac{e}{2}$ (c) $\frac{2}{e}$ (d) $\frac{e}{2}$

60. $\frac{1}{3}+\frac{1}{2.3^2}+\frac{1}{3.3^3}+\frac{1}{4.3^4}+....\infty=$ **[MNR 1975]**

(a) $\log_e 2-\log_e 3$ (b) $\log_e 3-\log_e 2$ (c) $\log_e 6$ (d) None of these

61. $1+\frac{2}{3}-\frac{2}{4}+\frac{2}{5}-....\infty=$

(a) $\log_e 3$ (b) $\log_e 4$ (c) $\log_e\left(\frac{e}{2}\right)$ (d) $\log_e\left(\frac{2}{3}\right)$

62. $\log_e \frac{4}{5} + \frac{1}{4} - \frac{1}{2}\left(\frac{1}{4}\right)^2 + \frac{1}{3}\left(\frac{1}{4}\right)^3 - ....$

(a) $2\log_e \frac{4}{5}$ (b) $\log_e \frac{5}{4}$ (c) 1 (d) 0

63. $\frac{1}{n^2} + \frac{1}{2n^4} + \frac{1}{3n^6} + ...\infty =$

(a) $\log_e\left(\frac{n^2}{n^2+1}\right)$ (b) $\log_e\left(\frac{n^2+1}{n^2}\right)$ (c) $\log_e\left(\frac{n^2}{n^2-1}\right)$ (d) None of these

64. The sum of $\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2^3} + \frac{1}{5}\cdot\frac{1}{2^5} + ...\infty$ is **[MP PET 1991]**

(a) $\log_e \sqrt{\frac{3}{2}}$ (b) $\log_e \sqrt{3}$ (c) $\log_e \sqrt{\frac{1}{2}}$ (d) $\log_e 3$

65. If $0 < y < 2^{1/3}$ and $x(y^3 - 1) = 1$, then $\frac{2}{x} + \frac{2}{3x^3} + \frac{2}{5x^5} + ... =$ **[EAMCET 2003]**

(a) $\log\left[\frac{y^3}{y^3-2}\right]$ (b) $\log\left[\frac{y^3}{1-y^3}\right]$ (c) $\log\left[\frac{2y^3}{1-y^3}\right]$ (d) $\log\left[\frac{y^3}{1-2y^3}\right]$

66. The sum to infinity of the given series $\frac{1}{n} - \frac{1}{2n^2} + \frac{1}{3n^3} - \frac{1}{4n^4} + ...$ is **[MP PET 1994]**

(a) $\log_e\left(\frac{n+1}{n}\right)$ (b) $\log_e\left(\frac{n}{n+1}\right)$ (c) $\log_e\left(\frac{n-1}{n}\right)$ (d) $\log_e\left(\frac{n}{n-1}\right)$

67. $e^{\left(x - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + ...\right)}$ is equal to **[DCE 2001]**

(a) $\log x$ (b) $\log(x-1)$ (c) x (d) None of these

68. If the sum of $1 + \frac{1+2}{2} + \frac{1+2+3}{3} + ...$ to n terms is S, then S is equal to **[Kerala (Engg.) 2002]**

(a) $\frac{n(n+3)}{4}$ (b) $\frac{n(n+2)}{4}$ (c) $\frac{n(n+1)(n+2)}{6}$ (d) n^2

69. Th sum of the series $2\{7^{-1} + 3^{-1}.7^{-3} + 5^{-1}.7^{-5} + ...\}$ is

(a) $\log_e\left(\frac{4}{3}\right)$ (b) $\log_e\left(\frac{3}{4}\right)$ (c) $2\log_e\left(\frac{3}{4}\right)$ (d) $2\log_e\left(\frac{4}{3}\right)$

70. $\log_e \sqrt{\frac{1+x}{1-x}} =$

(a) $x + \frac{x^3}{3} + \frac{x^5}{5} + ...\infty$ (b) $2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + ...\infty\right]$ (c) $2\left[x^2 + \frac{x^4}{4} + \frac{x^6}{6} + ...\infty\right]$ (d) None of these

71. If α, β are the roots of the equation $x^2 - px + q = 0$, then $\log_e(1 + px + qx^2) =$

(a) $(\alpha+\beta)x - \frac{\alpha^2+\beta^2}{2}x^2 + \frac{\alpha^3+\beta^3}{3}x^3 - .....\infty$ (b) $(\alpha+\beta)x - \frac{(\alpha+\beta)^2}{2}x^2 + \frac{(\alpha+\beta)^3}{3}x^3 - .....\infty$

(c) $(\alpha+\beta)x + \frac{\alpha^2+\beta^2}{2}x^2 + \frac{\alpha^3+\beta^3}{3}x^3 + ...\infty$ (d) None of these

72. $\log_e(x+1) - \log_e(x-1) =$

(a) $2\left[x + \frac{x^3}{3} + \frac{x^5}{5} + ...\infty\right]$ (b) $\left[x + \frac{x^3}{3} + \frac{x^5}{5} + ...\infty\right]$ (c) $2\left[\frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + ...\infty\right]$ (d) $\left[\frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + ...\infty\right]$

73. $\log_e\left[(1+x)^{1+x}(1-x)^{1-x}\right] =$

(a) $\frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + ...\infty$

(b) $\frac{x^2}{1.2}+\frac{x^4}{3.4}+\frac{x^6}{5.6}+.....\infty$

(c) $2\left[\frac{x^2}{1.2}+\frac{x^4}{3.4}+\frac{x^6}{5.6}+.....\infty\right]$

(d) None of these

74. If m, n are the roots of the equation $x^2-x-1=0$, then the value of $\frac{\left(1+m\log_e 3+\frac{(m\log_e 3)^2}{2!}+....\infty\right)\left(1+n\log_e 3+\frac{(n\log_e 3)^2}{2!}+....\infty\right)}{\left(1+mn\log_e 3+\frac{(mn\log_e 3)^2}{2!}+.....\infty\right)}$ is

(a) 9 (b) 3 (c) 0 (d) 1

75. The value of $\log_e\left(1+ax^2+a^2+\frac{a}{x^2}\right)$ is

(a) $a\left(x^2-\frac{1}{x^2}\right)-\frac{a^2}{2}\left(x^4-\frac{1}{x^4}\right)+\frac{a^3}{3}\left(x^6-\frac{1}{x^6}\right)-.....$

(b) $a\left(x^2+\frac{1}{x^2}\right)-\frac{a^2}{2}\left(x^4+\frac{1}{x^4}\right)+\frac{a^3}{3}\left(x^6+\frac{1}{x^6}\right)-.....$

(c) $a\left(x^2+\frac{1}{x^2}\right)+\frac{a^2}{2}\left(x^4+\frac{1}{x^4}\right)+\frac{a^3}{3}\left(x^6+\frac{1}{x^6}\right)+.....$

(d) $a\left(x^2-\frac{1}{x^2}\right)+\frac{a^2}{2}\left(x^4-\frac{1}{x^4}\right)+\frac{a^3}{3}\left(x^6-\frac{1}{x^6}\right)+.....$

76. If $\log_e\left(\frac{a+b}{2}\right)=\frac{1}{2}(\log_e a+\log_e b)$, then relation between a and b will be **[UPSEAT 1999]**

(a) $a=b$ (b) $a=2b$ (c) $2a=b$ (d) $a=\frac{b}{3}$

77. The solutions of the equation $x^{\frac{1}{2}(\log_2 x-2)}=16$ are **[AMU 1999]**

(a) $\pm 2\sqrt{2}$ (b) 4,–2 (c) $16,\frac{1}{4}$ (d) $4,\frac{1}{16}$

78. The solution of $\log_\pi(\log_2(\log_7 x))=0$ is **[AMU 2002]**

(a) 7^2 (b) π^2 (c) 2^2 (d) None of these

79. If $x=\log_b a, y=\log_c b, z=\log_a c$; then xyz is **[MP PET 2002; UPSEAT 2003]**

(a) 1 (b) 0 (c) 3 (d) None of these

Advance Level

80. $\frac{x-1}{(x+1)}+\frac{1}{2}\cdot\frac{x^2-1}{(x+1)^2}+\frac{1}{3}\cdot\frac{x^3-1}{(x+1)^3}+.....\infty=$

(a) $\log_e x$ (b) $\log_e(1+x)$ (c) $\log_e(1-x)$ (d) $\log_e\frac{x}{1+x}$

81. $\frac{5}{1.2.3}+\frac{7}{3.4.5}+\frac{9}{5.6.7}+.....$ is equal to **[Karnataka CET 1997]**

(a) $\log\frac{8}{e}$ (b) $\log\frac{e}{8}$ (c) $\log 8e$ (d) None of these

82. $\frac{1}{x+1}+\frac{1}{2(x+1)^2}+\frac{1}{3(x+1)^3}+.....\infty=$

(a) $\log_e\left(1+\frac{1}{x}\right)$ (b) $\log_e\left(1-\frac{1}{x}\right)$ (c) $\log_e\left(\frac{x}{x+1}\right)$ (d) None of these

83. $\frac{(a-1)-\frac{(a-1)^2}{2}+\frac{(a-1)^3}{3}-......\infty}{(b-1)-\frac{(b-1)^2}{2}+\frac{(b-1)^3}{3}-......\infty}=$

(a) $\log_b a$ (b) $\log_a b$ (c) $\log_e a - \log_e b$ (d) $\log_e a + \log_e b$

84. $1+\left(\frac{1}{2}+\frac{1}{3}\right)\frac{1}{4}+\left(\frac{1}{4}+\frac{1}{5}\right)\frac{1}{4^2}+\left(\frac{1}{6}+\frac{1}{7}\right)\frac{1}{4^3}+.....\infty =$

(a) $\log_e(2\sqrt{3})$ (b) $2\log_e 2$ (c) $\log_e 2$ (d) $\log_e\left(\frac{2}{\sqrt{3}}\right)$

85. $1+\frac{2}{1.2.3}+\frac{2}{3.4.5}+\frac{2}{5.6.7}+.....\infty =$ **[Roorkee 1980]**

(a) $\log_e 2$ (b) $\log_e \sqrt{2}$ (c) $\log_e 4$ (d) None of these

86. $\frac{4}{1.3}-\frac{6}{2.4}+\frac{12}{5.7}-\frac{14}{6.8}+.....\infty =$

(a) $\log_e 3$ (b) $\log_e 2$ (c) $2\log_e 2$ (d) None of these

87. $\frac{m-n}{m+n}+\frac{1}{3}\left(\frac{m-n}{m+n}\right)^3+\frac{1}{5}\left(\frac{m-n}{m+n}\right)^5+......\infty =$ **[CET 1996]**

(a) $\log_e\left(\frac{m}{n}\right)$ (b) $\log_e\left(\frac{n}{m}\right)$ (c) $\log_e\left(\frac{m-n}{m+n}\right)$ (d) $\frac{1}{2}\log_e\left(\frac{m}{n}\right)$

88. If $n=(1999)!$, then $\sum_{x=1}^{1999}\log_n x$ is equal to **[AMU 2002]**

(a) 1 (b) 0 (c) $\sqrt[1999]{1999}$ (d) –1

89. If $\log(1-x+x^2)=a_1x+a_2x^2+a_3x^3+.....$, then $a_3+a_6+a_9+.....$ is equal to **[Kurukshetra CEET 2002]**

(a) $\log 2$ (b) $\frac{2}{3}\log 2$ (c) $\frac{1}{3}\log 2$ (d) $2\log 2$

90. The sum of $1+\frac{2}{1.2.3}+\frac{2}{3.4.5}+\frac{2}{5.6.7}+.....$ is **[Roorkee 1980; MP PET 2002, 03]**

(a) $2\log_e 2$ (b) $\log_e 2$ (c) $3\log_e 3$ (d) $3\log_e 2$

91. The sum of the series $\frac{1}{2}\left(\frac{1}{5}\right)^2+\frac{2}{3}\left(\frac{1}{5}\right)^3+\frac{3}{4}\left(\frac{1}{5}\right)^4+....$ is

(a) $1/4+\log(4/5)$ (b) $1/3+\log(2/3)$ (c) $1/2+\log(3/2)$ (d) None of these

92. The sum of the series $\frac{x}{1+x^2}+\frac{1}{3}\left(\frac{x}{1+x^2}\right)^3+\frac{1}{5}\left(\frac{x}{1+x^2}\right)^5+......$ is

(a) $\frac{1}{2}\log(1+x+x^2)$ (b) $\frac{1}{2}\log\left(\frac{1+x^2+x}{1+x^2-x}\right)$ (c) $\log(1-x+x^2)$ (d) None of these

93. $\log_a x$ is defined for (a > 0) **[Roorkee 1990]**

(a) All real x
(b) All negative (-) real $x \neq 1$
(c) All positive (+) real $x \neq 0$
(d) $a \geq e$

94. If $7^{\log_7(x^2-4x+5)}=x-1$, then x can have the values **[Roorkee 1990; DCE 2001]**

(a) $(2,3)$ (b) 7 (c) $(-2,-3)$ (d) (2,-3)

95. $\log_e(1+x)=\sum_{i=1}^{\infty}\left[\frac{(-1)^{i+1}x^i}{i}\right]$ is defined for **[Roorkee 1990]**

(a) $x\in(-1,1)$
(b) Any positive (+) real x
(c) $x\in(-1,1]$
(d) Any positive (+) real $x(x\neq 1)$

96. If $2^x.3^{x+4}=7^x$, then x= **[MP PET 1991]**

(a) $\frac{4\log_e 3}{\log_e 7-\log_e 6}$ (b) $\frac{4\log_e 3}{\log_e 6-\log_e 7}$ (c) $\frac{2\log_e 4}{\log_e 7+\log_e 6}$ (d) $\frac{2\log_e 4}{\log_e 7+\log_e 6}$

97. If $x=1+\log_a(bc), y=1+\log_b(ca)$ and $z=1+\log_c(ab)$, then $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=$ **[MP PET 1991]**

(a) 0 (b) 1 (c) 3 (d) xyz

98. The value of x obtained from equation $(4)^{\log_9 3} + (9)^{\log_2 4} = (10)^{\log_x 83}$ will be **[UPSEAT 1999]**

(a) 10 (b) 100 (c) 5 (d) 2

99. In the expansion of $2\log_e x - \log_e(x+1) - \log_e(x-1)$, the coefficient of x^{-4} is

(a) $\frac{1}{2}$ (b) –1 (c) 1 (d) None of these

100. If $|x| < 1$, then the coefficient of x^5 in the expansion of $(1-x).\log_e(1-x)$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{20}$ (d) $\frac{1}{10}$

101. In the expansion of $\log_e \frac{1}{1-x-x^2+x^3}$, the coefficient of x is

(a) 0 (b) 1 (c) –1 (d) 1/2

102. If n is not a multiple of 3, then the coefficient of x^n in the expansion of $\log_e(1+x+x^2)$ is **[Roorkee 1992, Kurukshetra CEE**

(a) $\frac{1}{n}$ (b) $\frac{2}{n}$ (c) $-\frac{1}{n}$ (d) $-\frac{2}{n}$

103. If n is a multiple of 3, then the coefficient of x^n in the expansion of $\log_e(1+x+x^2)$ is **[Roorkee 1994]**

(a) $\frac{1}{n}$ (b) $\frac{2}{n}$ (c) $-\frac{1}{n}$ (d) $-\frac{2}{n}$

Miscellaneous Problems

Basic Level

104. If $y = 2x^2 - 1$, then $\left[\frac{1}{y} + \frac{1}{3y^3} + \frac{1}{5y^5} + ...\right]$ is equal to

(a) $\frac{1}{2}\left[\frac{1}{x^2} - \frac{1}{2x^4} + \frac{1}{3x^6} - .....\right]$ (b) $\frac{1}{2}\left[\frac{1}{x^2} + \frac{1}{2x^4} + \frac{1}{3x^6} + ...\right]$

(c) $\frac{1}{2}\left[\frac{1}{x^2} + \frac{1}{3x^6} + \frac{1}{5x^{10}} + ...\right]$ (d) $\frac{1}{2}\left[\frac{1}{x^2} - \frac{1}{3x^6} + \frac{1}{5x^{10}} - .....\right]$

105. If $y = x - \frac{x^2}{2} + \frac{x^3}{3} - ....\infty$, then $x =$ **[MNR 1973]**

(a) $y - \frac{y^2}{2} + \frac{y^3}{3} - ....\infty$, (b) $y + \frac{y^2}{2!} + \frac{y^3}{3!} + ...\infty$ (c) $1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + ...$ (d) None of these

Advance Level

106. If $y = x - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + ...$, then $x =$

(a) $\log_e(1-y)$ (b) $\frac{1}{\log_e(1-y)}$ (c) $\log_e \frac{1}{(1-y)}$ (d) $\log_e(1+y)$

107. If $b = a - \frac{a^2}{2} + \frac{a^3}{3} - \frac{a^4}{4} + ...$, then $b + \frac{b^2}{2!} + \frac{b^3}{3!} + \frac{b^4}{4!} + ...\infty =$

(a) $\log_e a$ (b) $\log_e b$ (c) a (d) e^a

108. If $4\left[x^2 + \frac{x^6}{3} + \frac{x^{10}}{5} + ...\right] = y^2 + \frac{y^4}{2} + \frac{y^6}{3} + ...$, then

(a) $x^2y = 2x - y$ (b) $x^2y = 2x + y$ (c) $x = 2y^2 - 1$ (d) $x^2y = 2x + y^2$

109. If $t_n = \frac{1}{4}(n+2)(n+3)$ for $n = 1, 2, 3....$, then $\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + ... + \frac{1}{t_{2003}} =$ **[EAMCET 2003]**

(a) $\frac{4006}{3006}$ (b) $\frac{4003}{3007}$ (c) $\frac{4006}{3008}$ (d) $\frac{4006}{3009}$

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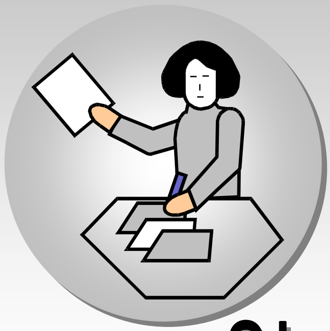

Answer Sheet

Exponential and Logarithmic ***Assignment (Basic and Advance Level)***

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
d	c	c	b	b	c	b	d	b	b	a	b	b	b	c	b	d	b	d	b
21	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**	**34**	**35**	**36**	**37**	**38**	**39**	**40**
c	c	b	a	d	b	d	b	c	c	b	b	c	a	c	c	b	a	c	d
41	**42**	**43**	**44**	**45**	**46**	**47**	**48**	**49**	**50**	**51**	**52**	**53**	**54**	**55**	**56**	**57**	**58**	**59**	**60**
c	c	a	c	b	c	d	a	b	b	a	a	d	a	a	b	b	c	b	b
61	**62**	**63**	**64**	**65**	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**	**76**	**77**	**78**	**79**	**80**
b	d	c	b	a	a	d	a	a	a	a	c	c	a	b	a	c	a	a	a
81	**82**	**83**	**84**	**85**	**86**	**87**	**88**	**89**	**90**	**91**	**92**	**93**	**94**	**95**	**96**	**97**	**98**	**99**	**100**
a	a	a	a	c	b	d	a	b	a	a	b	c	a	c	a	b	a	a	c
101	**102**	**103**	**104**	**105**	**106**	**107**	**108**	**109**											
b	a	d	b	b	c	c	a	d											