Determinants

Multiple Choice Questions

Choose and write the correct option in the following questions.

1. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ then the possible value(s) of 'x' is/are [*CBSE Sample Paper 2023*]

   (*a*) 3 (*b*) $\sqrt{3}$ (*c*) $-\sqrt{3}$ (*d*) $\sqrt{3}, -\sqrt{3}$

2. The area of a triangle with vertices (–3, 0),(3, 0) and (0, k) is 9 sq. units. The value of k will be [*NCERT Exemplar*]

   (*a*) 9 (*b*) 3 (*c*) –9 (*d*) 6

3. If $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$, then [*NCERT Exemplar*]

   (*a*) $f(a) = 0$ (*b*) $f(b) = 0$ (*c*) $f(0) = 0$ (*d*) $f(1) = 0$

4. If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$, then the value of x is [*CBSE 2020 (65/4/1)*]

   (*a*) 3 (*b*) 0 (*c*) –1 (*d*) 1

5. If $x = -4$ is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then the sum of the other two roots is [*CBSE (Term-1) 2021-22 (65/2/4)*]

   (*a*) 4 (*b*) –3 (*c*) 2 (*d*) 5

6. If A is a square matrix of order 3, $|A'| = -3$, then $|AA'| =$ [*CBSE Sample Paper 2023*]

   (*a*) 9 (*b*) –9 (*c*) 3 (*d*) –3

7. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 27$, then the value of α is: [*CBSE Examination Paper 2022 (Term-1)*]

   (*a*) ± 1 (*b*) ± 2 (*c*) $\pm\sqrt{5}$ (*d*) $\pm\sqrt{7}$

8. Value of k, for which $A = \begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix is [*CBSE Sample Paper 2022 (Term-1)*]

   (*a*) 4 (*b*) -4 (*c*) ± 4 (*d*) 0

9. The value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ is [*CBSE 2023 (65/4/1)*]

   (*a*) 0 (*b*) 1 (*c*) $x + y + z$ (*d*) $2(x + y + z)$

10. If $|A| = |kA|$, where A is a square matrix of order 2, then sum of all possible values of k is [CBSE 2023 (65/2/1)]

(a) 1 (b) −1 (c) 2 (d) 0

11. If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and $a \in A$, then the set A is [CBSE 2023 (65/2/1)]

(a) $\mathbb{R}$ (b) $\{0\}$ (c) $\{4\}$ (d) $\mathbb{R} - \{4\}$

12. If C_{ij} denotes the cofactor of element P_{ij} of the matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$, then the value of $C_{31} \cdot C_{23}$ is [CBSE (Term-1) 2021-22 (65/2/1)]

(a) 5 (b) 24 (c) −24 (d) −5

13. For matrix $A = \begin{bmatrix} 2 & 5 \\ -11 & 7 \end{bmatrix}$, $(adj\ A)'$ is equal to [CBSE Sample Paper 2022 (Term-1)]

(a) $\begin{bmatrix} -2 & -5 \\ 11 & -7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 5 \\ 11 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$

14. If $A \cdot (\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $|A| + |\text{adj } A|$ is equal to [CBSE 2023 (65/3/2)]

(a) 12 (b) 9 (c) 3 (d) 27

15. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then the value of $|\text{adj } A|$ is [CBSE 2020 (65/3/1)]

(a) 64 (b) 16 (c) 0 (d) − 8

16. Let A be a skew-symmetric matrix of order 3. If $|A| = x$, then $(2023)^x$ is equal to : [CBSE 2023 (65/3/2)]

(a) 2023 (b) $\frac{1}{2023}$ (c) $(2023)^2$ (d) 1

17. The inverse of $\begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}$ is: [CBSE Examination Paper 2022 (Term-1)]

(a) $\begin{bmatrix} -5 & -3 \\ 7 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} -5 & 7 \\ 3 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -5 & -3 \\ -7 & -4 \end{bmatrix}$

18. If $|A| = 2$, where A is a 2×2 matrix, then $|4A^{-1}|$ equals : [CBSE 2023 (65/1/1)]

(a) 4 (b) 2 (c) 8 (d) $\frac{1}{32}$

19. If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to [CBSE 2023 (65/2/1)]

(a) $2\Delta^2$ (b) $4\Delta^2$ (c) 2Δ (d) 4Δ

20. If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then : [CBSE 2022 (Term-1)]

(a) $a = 1 = b$

(b) $a = \cos 2\theta, b = \sin 2\theta$

(c) $a = \sin 2\theta, b = \cos 2\theta$

(d) $a = \cos\theta, b = \sin\theta$

Answers

1. (d)	2. (b)	3. (c)	4. (c)	5. (a)	6. (a)	7. (d)
8. (c)	9. (a)	10. (d)	11. (d)	12. (a)	13. (c)	14. (a)
15. (a)	16. (d)	17. (b)	18. (c)	19. (b)	20. (b)	

Solutions of Selected Multiple Choice Questions

1. $2 - 20 = 2x^2 - 24 \quad \Rightarrow \quad 2x^2 = 6$

$\Rightarrow \quad x^2 = 3 \quad \Rightarrow \quad x = \pm\sqrt{3}$

$\therefore$ Option (d) is correct.

2. We know that, area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right| = \frac{1}{2}\left|\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix}\right|$$

Expanding along R_1, we get

$$9 = \frac{1}{2}\left|[-3(-k) - 0 + 1(3k)]\right|$$

$$\Rightarrow \quad 18 = |3k + 3k| = |6k|$$

$$\therefore \quad k = \pm\frac{18}{6} = \pm 3 = 3, -3$$

$\therefore$ Option (b) is correct.

3. We have, $f(x) = \begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix}$

$$\Rightarrow \quad f(a) = \begin{vmatrix} 0 & 0 & a-b \\ 2a & 0 & a-c \\ a+b & a+c & 0 \end{vmatrix} = [(a-b)\{2a.(a+c)\}] \neq 0$$

and $$f(b) = \begin{vmatrix} 0 & b-a & 0 \\ b+a & 0 & b-c \\ 2b & b+c & 0 \end{vmatrix} = -(b-a)[-2b(b-c)] = 2b(b-a)(b-c) \neq 0$$

and $$f(0) = \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix} = a(bc) - b(ac) = abc - abc = 0$$

$\therefore$ Option (c) is correct.

4. We have,

$$\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix} + 3 = 0$$

Expanding along R_2, we get

$\Rightarrow \quad -x(3-18)+x(2-8)-x(18-12)+3=0$

$\Rightarrow \quad -x(-15)+x(-6)-x(6)+3=0$

$\Rightarrow \quad x(15-6-6)+3=0$

$\Rightarrow \quad 3x+3=0 \quad \Rightarrow \quad 3(x+1)=0$

$\Rightarrow \quad x+1=0 \quad \Rightarrow \quad x=-1$

$\therefore$ Option (*c*) is correct.

5. $\because \quad \Delta = \begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$ (Given)

$\Rightarrow \quad x(x^2-2)-1(2x-6)+3(2-3x)=0$

$\Rightarrow \quad x^3-2x-2x+6+6-9x=0$

$\Rightarrow \quad x^3-13x+12=0$

Given $x=-4$ is a root of $\Delta=0$.

$\Rightarrow \quad (x+4)$ is a factor of $x^3-13x+12$.

$\Rightarrow \quad x^3+4x^2-4x^2-16x+3x+12=0$

$\Rightarrow \quad x^2(x+4)-4x(x+4)+3(x+4)=0$

$\Rightarrow \quad (x+4)(x^2-4x+3)=0$

Other two roots of $\Delta=0$ are the roots of $x^2-4x+3=0$.

If α and β are roots of above equation, then

$$\alpha+\beta=-\frac{(-4)}{1}=4$$

$\therefore$ Sum of other two roots of $\Delta=0$ is 4.

$\therefore$ Option (*a*) is correct.

6. $AA'| = |A||A'| = (-3)(-3) = 9$ $[\because |A'|=|A|]$

$\therefore$ Option (*a*) is the correct.

8. As A is singular matrix.

$\Rightarrow \quad |A|=0$

$\Rightarrow \quad 2k^2-32=0 \Rightarrow k=\pm 4$

$\therefore$ Option (*c*) is correct.

9. We have,

$$\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

Expanding along R_1 we get

$$\Delta = (x+y)(x-y)-(y+z)(z-y)+(z+x)(z-x)$$
$$= x^2-y^2+y^2-z^2+z^2-x^2=0$$

$\therefore$ Option (*a*) is correct.

10. Given A is square matrix of order 2 *i.e.*, $n=2$

Also, $|A|=|kA| \Rightarrow |A|=k^n|A| \Rightarrow 1=k^2 \quad \Rightarrow k=\pm 1$

$\therefore \quad k=1,-1$

Sum of values of $k=1+(-1)=0$

$\therefore$ Option (*d*) is correct.

11. Given that $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non singular matrix

$\therefore \quad \begin{vmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{vmatrix} \neq 0 \Rightarrow 1(3-a) - 2(2-3) + 1(2a-9) \neq 0$

$\Rightarrow \quad 3 - a + 2 + 2a - 9 \neq 0 \quad \Rightarrow a - 4 \neq 0 \quad \Rightarrow a \neq 4$

$\therefore$ Set $A = R - \{4\}$

$\therefore$ Option (*d*) is correct.

12. We are given matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$ (Given)

C_{ij} = cofactor of $p_{ij} = (-1)^{i+j} P_{ij}$

Where P_{ij} denote the minor of p_{ij}

$\because \quad P_{31} = \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = 3 - 4 = -1$

$P_{23} = \begin{vmatrix} 1 & -1 \\ 3 & 2 \end{vmatrix} = 2 + 3 = 5$

$\therefore \quad C_{31} = (-1)^{3+1} P_{31} = (-1)^4 (-1) = (-1)^5 = -1$

$C_{23} = (-1)^{2+3} P_{23} = (-1)^5 \times 5 = -5$

$\therefore \quad C_{31} . C_{23} = -1 \times (-5) = 5$

$\therefore$ Option (*a*) is correct.

13. Given matrix $A = \begin{bmatrix} 2 & 5 \\ -11 & 7 \end{bmatrix}$

Its co-factors are

$C = \begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix} \quad \Rightarrow \quad \text{adj}(A) = \begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$

$\therefore \quad (\text{adj } A)' = \begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}' = \begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$

$\therefore$ Option (*c*) is correct.

14. Given, $A . (\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$\Rightarrow \quad |A| I_3 = 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3I_3$

$\Rightarrow \quad |A| = 3$

Also, $|\text{adj. } A| = |A|^{n-1} = |A|^{3-1} = |A|^2 = (3)^2 = 9$

$\therefore \quad |A| + |\text{adj. } A| = 3 + 9 = 12$

$\therefore$ Option (*a*) is correct.

15. We have a matrix A of order 3×3 given by

$A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \quad \Rightarrow \quad |A| = -2\,((-2) \times (-2) - 0) \quad \Rightarrow \quad |A| = -8$

$\therefore \quad |\text{adj } A| = |A|^{n-1} = |A|^{3-1} = (-8)^2 = 64$

$\therefore$ Option (*a*) is correct.

16. Since A is a Skew-Symmetric Matrix.

$\therefore \quad A = -A^T \quad \Rightarrow \quad |A| = (-1)^3\,|A^T| \quad \Rightarrow \quad |A| = -|A^T|$

$\Rightarrow \quad |A| = -|A| \Rightarrow \quad 2|A| = 0 \quad \Rightarrow \quad |A| = 0 \quad \Rightarrow \quad x = 0$

$\therefore \quad (2023)^x = (2023)^0 = 1$

$\therefore$ Option (*d*) is correct.

17. Let $A = \begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} -4 & 3 \\ 7 & -5 \end{vmatrix} = 20 - 21 = -1 \neq 0$

$\therefore \quad A^{-1}$ exists

Co-factors of A are

$C_{11} = -5, \quad C_{21} = -3$

$C_{12} = -7, \quad C_{22} = -4$

$\therefore$ adj $A = \begin{bmatrix} -5 & -3 \\ -7 & -4 \end{bmatrix} \quad \Rightarrow A^{-1} = \dfrac{adj\ A}{|A|} = \dfrac{\begin{bmatrix} -5 & -3 \\ -7 & -4 \end{bmatrix}}{-1}$

$A^{-1} = \begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$

$\therefore$ Option (*b*) is correct.

18. Given, $|A| = 2 \quad \Rightarrow \quad |A| = \dfrac{1}{|A|} = \dfrac{1}{2}$

Since A is of order 2×2 *i.e.*, $n = 2$.

$\Rightarrow |4A^{-1}| = 4^2|A^{-1}| = 16 \times \dfrac{1}{|A|} = 16 \times \dfrac{1}{2} = 8$

$\therefore$ Option (*c*) is correct.

19. Since (a, b), (c, d) and (e, f) are the vertices of ΔABC

$\therefore \quad \Delta = \dfrac{1}{2}\begin{bmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{bmatrix} \quad \Rightarrow \begin{bmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{bmatrix} = 2\Delta$

$C \longleftrightarrow R$

$\Rightarrow \quad \begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2 = 4\Delta^2$

$\therefore$ Option (*b*) is correct.

20. Let $A = \begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix} \quad \Rightarrow |A| = \begin{vmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{vmatrix} = 1 + \tan^2\theta = \sec^2\theta$

$\therefore$ Co-factors of A are

$C_{11} = 1 \qquad C_{21} = -\tan\theta$

$C_{12} = \tan\theta \quad C_{22} = 1$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{\sec^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$$

Now, $$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\frac{1}{\sec^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \quad \frac{1}{\sec^2\theta}\begin{bmatrix} 1-\tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1-\tan^2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \dfrac{1-\tan^2\theta}{\sec^2\theta} & \dfrac{-2\tan\theta}{\sec^2\theta} \\ \dfrac{2\tan\theta}{\sec^2\theta} & \dfrac{1-\tan^2\theta}{\sec^2\theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \quad a = \frac{1-\tan^2\theta}{\sec^2\theta} = \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta} \times \cos^2\theta = \cos 2\theta$$

and, $$b = \frac{\dfrac{2\sin\theta}{\cos\theta}}{\sec^2\theta} = \frac{\dfrac{2\sin\theta}{\cos\theta}}{\dfrac{1}{\cos^2\theta}} = 2\sin\theta.\cos\theta = \sin 2\theta$$

$\therefore$ $a = \cos 2\theta$, $b = \sin 2\theta$

$\therefore$ Option (*b*) is correct.

Assertion-Reason Questions

The following questions consist of two statements—Assertion(A) and Reason(R). Answer these questions selecting the appropriate option given below:

(a) Both A and R are true and R is the correct explanation for A.

(b) Both A and R are true but R is not the correct explanation for A.

(c) A is true but R is false.

(d) A is false but R is true.

1. **Assertion (A) :** Determinant is a number associated with a square matrix.

   Reason (R) : Determinant is a square matrix.

2. **Assertion (A) :** If $A = \begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$, then the matrix A is singular if $x = 3$.

   Reason (R) : A square matrix is a singular matrix if its determinant is zero.

3. **Assertion (A) :** If A is a 3×3 matrix, $|A| \neq 0$ and $|5A| = K|A|$, then the value of $K = 125$.

   Reason (R) : If A be any square matrix of order $n \times n$ and k be any scalar then $|KA| = K^n|A|$.

4. **Assertion (A) :** If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ then $x = \pm 6$.

Reason (R) : If A is a skew-symmetric matrix of odd order, then $|A| = 0$.

5. **Assertion (A) :** If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, then $C_{22} = 1$, where C_{ij} denotes the co-factor of i^{th} row and j^{th} column.

Reason (R) : The co-factor C_{ij} of a_{ij} in the matrix $A = [a_{ij}]_{n \times n}$ equal to $(-1)^{i+j} M_{ij}$.

6. **Assertion (A) :** If A is an invertible square matrix, then A^T is invertible.

Reason (R) : Inverse of invertible symmetric matrix is a symmetric matrix.

7. **Assertion (A) :** If A is an invertible matrix of order 3 and $|A| = 5$ then, $|\text{adj } A| = 25$.

Reason (R) : If B is a non-singular matrix of order n. Then, $|\text{adj } A| = |A|^{n-1}$.

8. **Assertion (A) :** If $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ then $|\text{adj}(\text{adj } A)| = 16$.

Reason (R) : $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

9. **Assertion (A) :** Given a system of linear equations:

$2x - y = 17$ and $3x + 5y = 6$ are consistent.

Reason (R) : For a system of linear equations $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ is said to be consistent if determinant of the coefficient is non-zero.

10. **Assertion (A) :** Solution of system of equations

$2x - y = 17$ and $3x + 5y = 6$ is $x = 7, y = -3$.

Reason (R) : For a system of equations $AX = B$. If $|A| \neq 0$ then solution of the above system given by $X = A^{-1}B$.

Answers

1. (c)	2. (a)	3. (a)	4. (b)	5. (a)	6. (b)	7. (a)
8. (a)	9. (a)	10. (a)				

Solutions of Assertion-Reason Questions

1. Clearly, Assertion (A) is true and Reason (R) is false.
Hence, option (c) is correct.

2. For singular matrix, we have $\begin{vmatrix} 5-x & x+1 \\ 2 & 4 \end{vmatrix} = 0 \Rightarrow 20 - 4x - 2x - 2 = 0$

$\Rightarrow \quad 18 - 6x = 0 \Rightarrow 6x = 18 \quad \Rightarrow x = \frac{18}{6} = 3 \quad \Rightarrow x = 3$

So, Assertion (A) and Reason (R) both are true and Reason (R) is the correct explanation of Assertion (A).
Hence, option (a) is correct.

3. We have, A is a square matrix of order 3×3

$\therefore \quad |5A| = 5^3|A| = 125|A|$

$\Rightarrow 125|A| = K|A| \Rightarrow K = 125$

So, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Hence, option (a) is correct.

4. We have, $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$\Rightarrow x^2 - 36 = 36 - 36 \quad \Rightarrow x^2 - 36 = 0$

$\Rightarrow x^2 = 36 \qquad \Rightarrow x = \pm 6$

Clearly, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

Hence, option (b) is correct.

5. We have, $A = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

$\therefore \; M_{22} = -9 - 5(-2) = -9 + 10 = 1$

$\therefore \; C_{22} = (-1)^{2+2}.\, M_{22} = 1 \times 1 = 1$

Clearly, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Hence, option (a) is correct.

6. $\because$ A is invertible.

$\Rightarrow \quad |A| \neq 0 \quad \Rightarrow \quad |A^T| \neq 0 \qquad\qquad [\because \; |A^T| = |A|]$

$\Rightarrow$ A^T is invertible.

So, A is true.

For R : Let B be invertible symmetric matrix.

Then, $|B| \neq 0$ and $B^T = B$

Now $(B^{-1})^T = (B^T)^{-1} = (B)^{-1} \qquad\qquad [\because \; (B^{-1})^T = (B^T)^{-1} \;\&\; B^T = B]$

$\Rightarrow$ B^{-1} is also symmetric matrix.

So, R is true but R does not explain A.

Hence, option (b) is correct.

7. $\because$ A is invertible matrix of order 3.

$\therefore \; |\text{adj } A| = |A|^{3-1} = |A|^2 = 5^2 = 25$

So, A is true and R is also correct and R gives correct explanation of A.

Hence, option (a) is correct.

8. $|A| = 1\begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix} = 2 \qquad\qquad$ [Along 2nd column]

$|\text{adj}\,(\text{adj } A)| = |A|^{(n-1)^2} = 2^{(3-1)^2} = 2^4 = 16$

So, A and R are true and R gives the correct explanation of A.

Hence, option (a) is correct.

9. We have, $\quad 2x - y = 17$ and $3x + 5y = 6$

i.e., $AX = B$ where $A = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}, \; X = \begin{bmatrix} x \\ y \end{bmatrix}, \; B = \begin{bmatrix} 17 \\ 6 \end{bmatrix}$

$\because \quad |A| = 10 + 3 = 13 \neq 0$, so $|A| \neq 0$

$\Rightarrow$ A^{-1} exists and $X = A^{-1}B$ be the solution of the given system of equation.

So, the given system of equations is consistent.

So, A and R are true and R gives the correct explanation for A.

Hence, option (a) is correct.

10. Given $A = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 17 \\ 6 \end{bmatrix}$

$$|A| = 10 + 3 = 13 \neq 0, \text{ so } A^{-1} \text{ exists.}$$

Now, $A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{13}\begin{bmatrix} 5 & 1 \\ -3 & 2 \end{bmatrix}$

$$X = A^{-1}B = \frac{1}{13}\begin{bmatrix} 5 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 17 \\ 6 \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 85+6 \\ -51+12 \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 91 \\ -39 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$$

$\therefore \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \end{bmatrix} \quad \Rightarrow \quad x = 7, y = -3$

So, A and R are true and R gives the correct explanation for A.

Hence, option (*a*) is correct.

Case-based/Data-based Questions

Each of the following questions are of 4 marks.

1. **Read the following passage and answer the following questions.**

Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop. Vikram buys 2 pens. 1 beg and 3 instrument boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen 2 bages and 4 instrument boxes and pays a sum of ₹ 250.

(*i*) Convert the given above situation into a matrix equation of the form $AX = B$.

(*ii*) Find $|A|$.

(*iii*) (*a*) Find A^{-1}. [*CBSE 2023 (65/5/1)*]

OR

(*iii*) (*b*) Determine $P = A^2 - 5A$.

Sol. Let cost of 1 pen, 1 bag and 1 instrument box are ₹ x, ₹ y and ₹ z respectively.

From question.

$$5x + 3y + z = 160$$
$$2x + y + 3z = 190$$
$$x + 2y + 4z = 250$$

(*i*) From above

$$\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix} \Rightarrow AX = B$$

Where $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$

(*ii*) We have $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$

$$|A| = \begin{vmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} = 5(4-6) - 2(12-2) + 1(9-1)$$

$$= -10 - 20 + 8 = -30 + 8 = -22$$

(*iii*) (*a*) Cofactors of the matrix A is obtained

$$C_{11} = (-1)^{1+1}(4-6) = -2,\ C_{12} = (-1)^{1+2}(8-3) = -5,$$
$$C_{13} = (-1)^{1+3}(4-1) = 3,\ C_{21} = (-1)^{2+1}(12-2) = -10$$
$$C_{22} = (-1)^{2+2}(20-1) = 19,\ C_{23} = (-1)^{2+3}(10-3) = -7,$$
$$C_{31} = (-1)^{3+1}(9-1) = 8,\ C_{32} = (-1)^{3+2}(15-2) = -13,$$
$$C_{33} = (-1)^{3+3}(5-6) = -1$$

$$\therefore \quad \text{adj}(A) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}' = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}' = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}\ \text{adj}\,(A) = -\frac{1}{22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix} = \frac{1}{22}\begin{bmatrix} 2 & 10 & -8 \\ 5 & -19 & 13 \\ -3 & 7 & 1 \end{bmatrix}$$

OR

(*iii*) (*b*)
$$A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix},\ A^2 = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix}$$

$$\therefore \quad P = A^2 - 5A = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} + \begin{bmatrix} -25 & -15 & -5 \\ -10 & -5 & -15 \\ -5 & -10 & -20 \end{bmatrix} = \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$$

2. **Read the following passage and answer the following questions.**

Three friends Rahul, Ravi and Rakesh went to a vegetable market to purchase vegetable. From a vegetable shop Rahul purchased 1 kg of each Potato, Onion and Brinjal for a total of ₹21. Ravi purchased 4 kg of potato, 3 kg of onion and 2 kg of Brinjal for ₹60 while Rakesh purchased 6 kg potato, 2 kg onion and 3 kg brinjal for ₹70.

(*i*) **If the cost of potato, onion and brinjal, are ₹x, ₹y and ₹z per kg respectively, then convert above situation into system of linear equations**

(*ii*) **Convert the above system of linear equations in (*i*) in the form of $AX = B$.**

(*iii*) (*a*) **Find A^{-1}.**

OR

(*iii*) (*b*) **Find the cost of potato, onion and brinjal.**

Sol. (*i*) From question

For Rahul $x + y + z = 21$

For Ravi $4x + 3y + 2z = 60$

For Rakesh $6x + 2y + 3z = 70$

Therefore, algebraical representation is

$$x + y + z = 21$$
$$4x + 3y + 2z = 60$$
$$6x + 2y + 3z = 70$$

(*ii*) The given algebraical system of linear equation in (*i*) can be written in matrix system as

$$AX = B \qquad ...(i)$$

Where, A is co-efficient matrix

$$\therefore \qquad A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

X is variable matrix

$$\therefore \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and B is constant matrix

$$\therefore \qquad B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

(*iii*) (*a*) We have $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}$

$$\therefore \qquad |A| = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix} = 1(9-4) - 1(12-12) + (8-18)$$

$$= 5 - 0 - 10 = -5 \neq 0$$

Now, $A_{11} = 9 - 4 = 5$; $A_{21} = -(3-2) = -1$; $A_{31} = 2 - 3 = -1$

$A_{12} = -(12-12) = 0$; $A_{22} = (3-6) = -3$; $A_{32} = -(2-4) = 2$

$A_{13} = (8-18) = -10$; $A_{23} = -(2-6) = 4$; $A_{33} = (3-4) = -1$

$$\therefore \text{ Adj } A = \begin{bmatrix} 5 & 0 & -10 \\ -1 & -3 & 4 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

$$\because \qquad A^{-1} = \frac{1}{|A|} \text{ Adj } A$$

$$= -\frac{1}{5} \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

OR

(*iii*) (*b*) We have $X = A^{-1}B$

$$\Rightarrow \qquad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5} \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix} \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -25 \\ -40 \\ -40 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$\Rightarrow \quad x = 5, y = 8, z = 8$

$\Rightarrow$ Cost of potato,onion and brinjal are ₹5, ₹8 and ₹8.

3. Read the following passage and answer the following questions.

The monthly incomes of two sister Reshma and Ritam are in the ratio 3 : 4 and their monthly expenditures are in the ratio 5 : 7. Each sister saves ₹15,000 per month.

(*i*) If monthly income of Reshma and Ritam are ₹$3x$ and ₹$4x$ and their monthly expenditure are ₹$5y$ and ₹$7y$ respectively, then express information provided in problem in system of linear equations.

(*ii*) Express the system of linear equations in (*i*) in matrix form $AX = B$.

(*iii*) (*a*) Find A^{-1}.

OR

(*iii*) (*b*) Find $C = A^2 - 2I$.

Sol. (*i*) Monthly savings of Reshma

= Monthly income of Reshma – Monthly Expenditure of Reshma

$$15000 = 3x - 5y$$

Similarly for Ritham

$$15000 = 4x - 7y$$

Here required system of linear equation

$$3x - 5y = 15000$$
$$4x - 7y = 15000$$

(*ii*) We have system of linear equations

$$3x - 5y = 15000$$
$$4x - 7y = 15000$$

This may be written in matrix system as

$$AX = B, \text{ where } A = \begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

Because,

$$\begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 3x - 5y \\ 4x - 7y \end{bmatrix} = \begin{bmatrix} 15000 \\ 15000 \end{bmatrix}$$

Equating, we get

$$3x - 5y = 15000$$

and $\quad 4x - 7y = 15000$

(*iii*) (*a*) We have $A = \begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}$

$\therefore \quad |A| = -21 + 20 = -1$

$A_{11} = -7 \quad A_{12} = -4$

$A_{21} = 5 \quad A_{22} = 3$

$$\text{Adj } A = \begin{bmatrix} -7 & -4 \\ 5 & 3 \end{bmatrix}^T = \begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \text{ adj A} = \frac{1}{-1} \begin{bmatrix} -7 & 5 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 7 & -5 \\ 4 & -3 \end{bmatrix}$$

OR

(*iii*) (*b*) $\because \quad A = \begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix}$

$$\therefore \quad A^2 = \begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ 4 & -7 \end{bmatrix} = \begin{bmatrix} -11 & 20 \\ -16 & 29 \end{bmatrix}$$

$$\Rightarrow \quad 2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$\therefore \quad C = A^2 - 2I$

$$= \begin{bmatrix} -11 & 20 \\ -16 & 29 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -13 & 20 \\ -16 & 27 \end{bmatrix}$$

4. Read the following passage and answer the following questions.

On the occasion of children's day, class teacher of class XII Shri Singh, decided to donate some money to students of class XII.

If there were 8 students less, every one would have got ₹10 more. However, if there were 16 students more, every one would have got ₹10 less.

(*i*) (*a*) If number of students in class be x and Shri Singh has decided to donate ₹y to each student, express the information provided in problem in system of linear equation.

(*b*) Express system of linear equation obtained in (*a*) in matrix equation.

(*ii*) Find the number of students in class XII and the amount distributed by Shri Singh.

Sol. (*i*) (*a*) From question

$$(x-8)(y+10) = xy \Rightarrow xy + 10x - 8y - 80 = xy \Rightarrow 10x - 8y = 80$$

$$\Rightarrow \quad 5x - 4y = 40 \qquad ...(i)$$

Also, $(x+16)(y-10) = xy \Rightarrow xy - 10x + 16y - 160 = xy \Rightarrow 10x - 16y = -160$

$$\Rightarrow 5x - 8y = -80 \qquad ...(ii)$$

(*b*) We have system of linear equations

$$5x - 4y = 40$$
$$5x - 8y = -80$$

This may be written in matrix system as

$$AX = B, \text{ where } A = \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$$

(*ii*) We have $A = \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}$

$\therefore \quad |A| = -40 + 20 = -20 \neq 0$

Here, $A_{11} = -8 \qquad A_{12} = -5$

$A_{21} = 4 \qquad A_{22} = 5$

$$\text{Adj } A = \begin{bmatrix} -8 & -5 \\ 4 & 5 \end{bmatrix}^T = \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$$

$\therefore \quad A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{1}{-20} \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$

$\because \quad X = A^{-1} B$

$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{20} \begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix} \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{20} \begin{bmatrix} -320-320 \\ -200-400 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{20} \begin{bmatrix} -640 \\ -600 \end{bmatrix}$

$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$

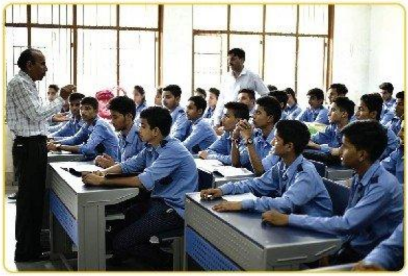
$\Rightarrow \quad x = 32, \text{ and } y = 30$

Number of students = 32 and the amount donated by Sri Singh to each student in ₹ 30.

5. Read the following passage and answer the following questions.

In coaching institutes, the students not only get academic guidance but also they get to know about career options and right goals as per their interest and academic record.

A coaching institute conduct classes in two sections A and B and fees for rich and poor children are different. In section A, there are 20 poor and 5 rich children and total monthly collection is ₹9,000, where as in section B, there are 5 poor and 25 rich children and total monthly collection is ₹26,000.



(*i*) (*a*) If ₹x and ₹y be the fees for rich and poor children respectively then the information provided in problem in system of linear equation express.

(*b*) Express the system of linear equations obtained in (*a*) as matrix equation.

(*ii*) Find the fees for rich and poor children.

Sol. (*i*) (*a*) From question

$$5x + 20y = 9000$$

and $25x + 5y = 26000$

(*b*) We have system of linear equation

$$5x + 20y = 9000$$
$$25x + 5y = 26000$$

They may be written in matrix system as

$$AX = B, \text{ where } A = \begin{bmatrix} 5 & 20 \\ 25 & 5 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$$

Because,

$$\begin{bmatrix} 5 & 20 \\ 25 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} 5x + 20y \\ 25x + 5y \end{bmatrix} = \begin{bmatrix} 9000 \\ 26000 \end{bmatrix}$$

Equating, we get

$$5x + 20 = 9000$$
$$25x + 5y = 26000$$

(*ii*) $A = \begin{bmatrix} 5 & 20 \\ 25 & 5 \end{bmatrix}$

$\therefore\ |A| = 25 - 500 = -475 \neq 0$

Also, $A_{11} = 5 \qquad A_{12} = -25$

$A_{21} = -20 \qquad A_{22} = 5$

$$\text{Adj } A = \begin{bmatrix} 5 & -25 \\ -20 & 5 \end{bmatrix}^T = \begin{bmatrix} 5 & -20 \\ -25 & 5 \end{bmatrix}$$

$$\therefore\ A^{-1} = \frac{1}{|A|} \text{ adj } A = \frac{1}{-475}\begin{bmatrix} 5 & -20 \\ -25 & 5 \end{bmatrix}$$

We have $X = A^{-1} B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-475}\begin{bmatrix} 5 & -20 \\ -25 & 5 \end{bmatrix}\begin{bmatrix} 9000 \\ 26000 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-475}\begin{bmatrix} 45000 - 520000 \\ -225000 + 130000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-475}\begin{bmatrix} -475000 \\ -95000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1000 \\ 200 \end{bmatrix} \quad \Rightarrow \quad x = 1000, y = 200$$

$\therefore$ Fees of rich and poor students are respectively

₹1000 and ₹200.

CONCEPTUAL QUESTIONS

1. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, then write the value of x. *[CBSE Delhi 2013]*

Sol. Given $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$

$$\Rightarrow (x + 1)(x + 2) - (x - 1)(x - 3) = 12 + 1$$
$$\Rightarrow x^2 + 2x + x + 2 - x^2 + 3x + x - 3 = 13$$
$$\Rightarrow 7x - 1 = 13$$
$$\Rightarrow 7x = 14 \quad \Rightarrow \quad x = 2$$

2. **Evaluate:** $\begin{vmatrix} \cos 15° & \sin 15° \\ \sin 75° & \cos 75° \end{vmatrix}$ *[CBSE (AI) 2011]*

Sol. Expanding the determinant, we get

$\cos 15°. \cos 75° - \sin 15°. \sin 75° = \cos (15° + 75°) = \cos 90° = 0$

[**Note :** $\cos (A + B) = \cos A. \cos B - \sin A. \sin B$]

3. **If A is a square matrix of order 3 and $|A| = 2$, then find the value of $|-AA'|$.** *[CBSE 2020 (65/1/3)]*

Sol. $|-AA'| = -|A|^2$ ½

$= -4$ ½

[CBSE Marking Scheme 2020 (65/1/3)]

Detailed solution:

Given matrix A is a square matrix of order 3 and $|A| = 2$

$\therefore \quad |-AA'| = (-1)^3 |AA'| = -|A|.|A'|$

$= -|A|.|A| = -|A|^2 = -(2)^2$

$= -4$

4. **If A is a square matrix of order 2 and $|A| = 4$, then find the value of $|2AA^T|$, where A^T is transpose of the matrix A.** *[CBSE 2019 (65/5/1)]*

Sol. $|2AA^T| = 4|A||A^T|$ ½

$= 4 \times 4 \times 4 = 64$ ½

[CBSE Marking Scheme 2019 (65/5/1)]

Detailed solution:

We have $|A| = 4$ and A is a matrix of order 2.

$\therefore \quad |2AA^T| = 2^2|AA^T|$ [$\because$ A is a matrix of order 2]

$= 4|AA^T| = 4|A||A^T|$

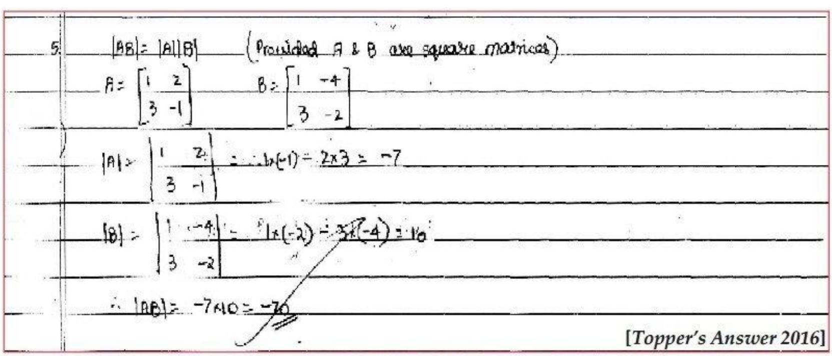
For any square matrix A,

$|A^T| = |A|$

$\therefore \quad |2AA^T| = 4 \times 4 \times 4 = 64$

5. **If $A = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -4 \\ 3 & -2 \end{pmatrix}$, find $|AB|$.** *[CBSE 2016 (South Region)]*

Sol.



5. $|AB| = |A||B|$ (Provided A & B are square matrices)

$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$

$|A| = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = 1\times(-1) - 2\times 3 = -7$

$|B| = \begin{vmatrix} 1 & -4 \\ 3 & -2 \end{vmatrix} = 1\times(-2) - 3\times(-4) = 10$

$\therefore |AB| = -7\times 10 = -70$

[Topper's Answer 2016]

6. **If for any 2 × 2 square matrix A, $A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of $|A|$.**

[CBSE (AI) 2017]

Sol.

$A(\text{adj } A) = |A| I_n$

$A(\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$

$|A| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 8 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$|A| = 8$

[Topper's Answer 2017]

7. **If A and B are square matrices of the same order 3, such that $|A| = 2$ and $AB = 2I$, write the value of $|B|$.**

[CBSE Delhi 2019]

Sol. $|A| = 2$ and $AB = 2I$

$\Rightarrow \quad |AB| = |2I| = 8$

$\Rightarrow \quad |A|\,|B| = 8 \quad \Rightarrow \quad 2\,|B| = 8 \quad \Rightarrow \quad |B| = 4$

8. **If A is a square matrix of order 3, with $|A| = 9$, then write the value of $|2.\ \text{adj } A|$.**

[CBSE 2019 (65/4/1)]

Sol. $|2 \text{ adj } A| = 2^3 |A|^{3-1} = 8 \times 81 = 648$ ½ + ½

[CBSE Marking Scheme 2019 (65/4/1)]

Detailed solution:

$\because \; |2 . \text{adj } A| = 2^3 |\text{adj } A|$ [$\because$ For any matrix A of order n, $|kA| = k^n |A|$, where k is some constant]

$= 8\,|\text{adj } A| = 8\,|A|^{3-1}$ [$\because$ $|\text{adj } A| = |A|^{n-1}$, for any square matrix of order n]

$= 8 \times (9)^2 = 8 \times 81 = 648$

9. **If $A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 2 & 3 \\ 3 & 3 & 5 \end{bmatrix}$, then find $A(\text{adj } A)$.**

[CBSE 2020 (65/1/1)]

Sol. $A . \text{adj } (A) = |A| \text{I}$ ½

$\therefore \; A . \text{adj } (A) = 2\text{I}$ or $\begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}$ ½

[CBSE Marking Scheme 2020 (65/1/1)]

Very Short Answer Questions

1. **If A is a square matrix of order 3 such that $A^2 = 2A$, then find the value of $|A|$.**

[CBSE 2020(65/1/2)]

Sol.

$A^2 = 2A$

Premultiplying with A^{-1}

$A^{-1}A.A = 2A^{-1}A$

$A = 2I$

$|A| = 8|I|$

$|A| = 8$.

[Topper's Answer 2020]

2. **Evaluate the determinant: $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$** [*NCERT Exemplar*]

Sol. Let $\Delta = \begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

$$= (x^2 - x + 1)(x + 1) - (x^2 - 1)$$

$$= (x + 1)\{x^2 - x + 1 - x + 1\} = (x + 1)(x^2 - 2x + 2)$$

$$= x^3 - 2x^2 + 2x + x^2 - 2x + 2 = x^3 - x^2 + 2$$

3. **If A is a skew-symmetric matrix of order 3, then prove that det $A = 0$.** [*CBSE (AI) 2017*]

Sol.

A is skew symmetric

$A = -A^T$

$|A| = (-1)^3|A^T|$

$|A| = -|A|$ $\quad [\because |A| = |A^T|]$

$2|A| = 0$

$|A| = 0$

det $A = 0$

[*Topper's Answer 2017*]

4. **If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then find the value of λ for which A^{-1} exists.** [*NCERT Exemplar*]

Sol. For existence of A^{-1}

$$|A| \neq 0 \quad \Rightarrow \quad \begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix} \neq 0$$

$\Rightarrow \quad 2(6 - 5) - \lambda(0 - 5) + (-3)(0 - 2) \neq 0$ [Expanding along R_1]

$\Rightarrow \quad 2 + 5\lambda + 6 \neq 0 \Rightarrow 5\lambda \neq -8 \quad \Rightarrow \quad \lambda \neq -\frac{8}{5}$

Hence, λ can have any value other than $-\frac{8}{5}$.

5. **If A is invertible matrix of 3×3 and $|A| = 7$, then find $|A^{-1}|$.** [*NCERT Exemplar*]

Sol. $\because \quad A^{-1} = \frac{1}{|A|}.\text{adj } A$

$$|A^{-1}| = \left|\frac{1}{|A|}\text{adj } A\right| = \left(\frac{1}{|A|}\right)^3 . |\text{adj } A| \qquad [\because |KA| = K^n . |A|, \text{ where } n \text{ is order of } A]$$

$$= \frac{1}{|A|^3} . |A|^{3-1} = \frac{1}{|A|} = \frac{1}{7}.$$

6. **If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, write A^{-1} in terms of A.** [*CBSE (AI) 2011*]

Sol. $|A| = \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -4 - 15 = -19 \neq 0$

Now, $C_{11} = -2$, $C_{12} = -5$, $C_{21} = -3$ and $C_{22} = 2$

$$\text{adj } A = \begin{bmatrix} -2 & -5 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{-19}\begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19}A$$

7. Find $(AB)^{-1}$ if $A = \begin{bmatrix} 1 & 0 \\ -4 & 2 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$. [*NCERT Exemplar*]

Sol. We have,

$$A = \begin{bmatrix} 1 & 0 \\ -4 & 2 \end{bmatrix} \Rightarrow |A| = 2 \neq 0$$

Co-factors of matrix A are

$C_{11} = 2 \qquad C_{21} = 0$

$C_{12} = 4 \qquad C_{22} = 1$

$$\therefore \text{ adj } A = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} \Rightarrow A^{-1} = \frac{\text{adj } A}{|A|}$$

$$\Rightarrow A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$\therefore (AB)^{-1} = B^{-1}.A^{-1} = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} \times \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6+4 & 1 \\ 10+8 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 10 & 1 \\ 18 & 2 \end{bmatrix}$$

$$\Rightarrow (AB)^{-1} = \begin{bmatrix} 5 & 1/2 \\ 9 & 1 \end{bmatrix}$$

8. Given $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, compute A^{-1} and show that $2A^{-1} = 9I - A$. [*CBSE Examination 2018*]

Sol.

$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$|A| = 14 - 12 = 2$ $\quad \frac{1}{2}$

$|A| \neq 0$ hence inverse exist

Now, $C_{11} = 7$, $C_{12} = +4$

$C_{21} = +3$, $C_{22} = 2$

$\text{Adj}(A) = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ Ans. $\quad \frac{1}{2}$

$2A^{-1} = 9I - A$

$2 \times \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$

$\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ Hence Proved.

[*Topper's Answer* 2019]

Short Answer Questions

1. If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$, find $(A')^{-1}$. *[CBSE Delhi 2015]*

Sol. Given $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$ and $A' = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$$|A'| = 1(-1-8) - 0 - 2(-8+3) = -9 + 10 = 1 \neq 0$$

Hence, $(A')^{-1}$ will exist.

$$A_{11} = \begin{vmatrix} -1 & 2 \\ 4 & 1 \end{vmatrix} = -1 - 8 = -9; \qquad A_{12} = -\begin{vmatrix} -2 & 2 \\ 3 & 1 \end{vmatrix} = -(-2-6) = 8$$

$$A_{13} = \begin{vmatrix} -2 & -1 \\ 3 & 4 \end{vmatrix} = -8 + 3 = -5; \qquad A_{21} = -\begin{vmatrix} 0 & -2 \\ 4 & 1 \end{vmatrix} = -(0+8) = -8$$

$$A_{22} = \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} = 1 + 6 = 7; \qquad A_{23} = -\begin{vmatrix} 1 & 0 \\ 3 & 4 \end{vmatrix} = -(4-0) = -4$$

$$A_{31} = \begin{vmatrix} 0 & -2 \\ -1 & 2 \end{vmatrix} = 0 - 2 = -2; \qquad A_{32} = -\begin{vmatrix} 1 & -2 \\ -2 & 2 \end{vmatrix} = -(2-4) = 2$$

$$A_{33} = \begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix} = -1 - 0 = -1$$

$$\text{adj}(A') = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$(A')^{-1} = \frac{1}{1}\begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

2. If A is a symmetric matrix and B is a skew symmetric matrix such that $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then find $|AB|$.

Sol. Given A is symmetric matrix, B is skew symmetric matrix.

$\Rightarrow \quad A' = A$ and $B' = -B$

$$\because \quad A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix} \qquad \text{...}(i)$$

$$\Rightarrow \quad (A+B)' = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}'$$

$$\Rightarrow \quad A' + B' = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$$

$$\Rightarrow \quad A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix} \qquad \text{...}(ii)$$

Adding (*i*) and (*ii*), we get

$$2A=\begin{bmatrix}4 & 8\\ 8 & -2\end{bmatrix} \Rightarrow A=\begin{bmatrix}2 & 4\\ 4 & -1\end{bmatrix}$$

From (*i*), $B=\begin{bmatrix}2 & 3\\ 5 & -1\end{bmatrix}-A=\begin{bmatrix}2 & 3\\ 5 & -1\end{bmatrix}-\begin{bmatrix}2 & 4\\ 4 & -1\end{bmatrix}=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$

$$\therefore \quad AB=\begin{bmatrix}2 & 4\\ 4 & -1\end{bmatrix}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}4 & -2\\ -1 & -4\end{bmatrix}$$

$$\therefore \quad |AB|=\begin{vmatrix}4 & -2\\ -1 & -4\end{vmatrix}=-16-2=-18$$

3. Find x, y and z if $A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ satisfies $A'=A^{-1}$. [*NCERT Exemplar*]

Sol. We have, $A=\begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}$ and $A'=\begin{bmatrix}0 & x & x\\ 2y & y & -y\\ z & -z & z\end{bmatrix}$

Also, $A'=A^{-1}=I$

$\Rightarrow \quad AA'=AA^{-1}$ $\quad [\because AA^{-1}=I]$

$$\Rightarrow \begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix}\begin{bmatrix}0 & x & x\\ 2y & y & -y\\ z & -z & z\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}4y^2+z^2 & 2y^2-z^2 & -2y^2+z^2\\ 2y^2-z^2 & x^2+y^2+z^2 & x^2-y^2-z^2\\ -2y^2+z^2 & x^2-y^2-z^2 & x^2+y^2+z^2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

By comparing the corresponding elements, we get

$2y^2-z^2=0 \Rightarrow 2y^2=z^2$ ...(*i*)

and $4y^2+z^2=1$ ...(*ii*)

$\Rightarrow 2.z^2+z^2=1 \Rightarrow z=\pm\frac{1}{\sqrt{3}}$ [From equation (*i*)]

$\therefore \quad y^2=\frac{z^2}{2} \Rightarrow y=\pm\frac{1}{\sqrt{6}}$

Also, $x^2+y^2+z^2=1$

$\Rightarrow x^2=1-y^2-z^2=1-\frac{1}{6}-\frac{1}{3}=1-\frac{3}{6}=\frac{1}{2} \Rightarrow x=\pm\frac{1}{\sqrt{2}}$

$\therefore \quad x=\pm\frac{1}{\sqrt{2}},\ y=\pm\frac{1}{\sqrt{6}}$ and $z=\pm\frac{1}{\sqrt{3}}$

4. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹ 2,800 as interest. However, if trust had interchanged money in bonds, they would have got ₹ 100 less as interest. Using matrix method, find the amount invested by the trust. [*CBSE (Central) 2016*]

Sol. Let ₹ x and ₹ y be the amount of money invested in 1st and 2nd bond respectively.

According to information given in question

$$x\times\frac{10}{100}+y\times\frac{12}{100}=2800$$

$$x\times\frac{12}{100}+y\times\frac{10}{100}=2700$$

$\Rightarrow \qquad 10x + 12y = 280000 \qquad \ldots(i)$

and $\qquad 12x + 10y = 270000 \qquad \ldots(ii)$

Above system of equation can be written in matrix form as

$AX = B$, where

$$A = \begin{bmatrix} 10 & 12 \\ 12 & 10 \end{bmatrix}, \; X = \begin{bmatrix} x \\ y \end{bmatrix}, \; B = \begin{bmatrix} 280000 \\ 270000 \end{bmatrix}$$

$$AX = B \quad \Rightarrow \quad X = A^{-1}B \qquad \ldots(iii)$$

Now, $\quad |A| = \begin{vmatrix} 10 & 12 \\ 12 & 10 \end{vmatrix} = 100 - 144 = -44$

$$A^{-1} = \frac{1}{|A|}.\text{adj } A = \frac{1}{|A|}.\begin{bmatrix} 10 & -12 \\ -12 & 10 \end{bmatrix}^T = \frac{1}{-44}\begin{bmatrix} 10 & -12 \\ -12 & 10 \end{bmatrix}$$

Putting the value of X, A^{-1} and B in (iii), we get

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-44}\begin{bmatrix} 10 & -12 \\ -12 & 10 \end{bmatrix}\begin{bmatrix} 280000 \\ 270000 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{44}\begin{bmatrix} 2800000 - 3240000 \\ -3360000 + 2700000 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{44}\begin{bmatrix} -440000 \\ -660000 \end{bmatrix} \qquad \Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10000 \\ 15000 \end{bmatrix}$$

$\Rightarrow \qquad x = 10000, y = 15000.$

Hence, invested amount in 1st and 2nd bonds are ₹10000 and ₹15000.

5. If the matrices $A = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, $B = \text{adj } A$ and $C = 3A$, then find the value of $\frac{|\text{adj } B|}{|C|}$.

Sol. We have $B = \text{adj } A$ and $C = 3A$

$\therefore \quad \text{adj } B = \text{adj (adj } A)$

$\Rightarrow \quad |\text{adj } B| = |\text{adj (adj } A)| = |A|^{(3-1)^2} = |A|^4$

$$\left[\begin{array}{l} \because |\text{adj . adj . adj ....}(A)| = |A|^{(n-1)^k} \\ \qquad k \text{ times} \\ \text{where } n \text{ is the order of matrix.} \end{array}\right]$$

$C = 3A \quad \Rightarrow \quad |C| = |3A| = 3^3|A| = 27|A|$

$$\therefore \quad \frac{|\text{adj } B|}{|C|} = \frac{|A|^4}{27|A|} = \frac{|A|^3}{27}$$

$$\because \qquad A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{bmatrix}$$

$\Rightarrow \quad |A| = 1(9 + 4) - 1\,(3 + 2) + 1\,(4 - 6)$

$= 13 - 5 - 2 = 13 - 7 = 6$

$$\therefore \quad \frac{|\text{adj } B|}{|C|} = \frac{|A|^3}{27} = \frac{6^3}{27} = \left(\frac{6}{3}\right)^3 = 2^3 = 8$$

Long Answer Questions

1. If $A = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 2 & -2 \\ 2 & -1 & 1 \end{bmatrix}$ then find A^{-1} and use it to solve the following system of the equations:

$$x + 2y - z = 6$$
$$3x + 2y - 2z = 3$$
$$2x - y + z = 2$$

[CBSE 2020 (65/1/1)]

Sol. $|A| = 7$; adj $(A) = \begin{vmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{vmatrix}$; $A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{7}\begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}$ **1 + 1½ + ½**

The system of equations in Matrix form can be written as:

$A \,.\, X = B$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix}$ **1**

$X = A^{-1}B \quad \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{7} = \begin{bmatrix} 0 & 1 & 2 \\ -7 & 7 & -7 \\ -7 & 5 & -4 \end{bmatrix}\begin{bmatrix} 6 \\ 3 \\ 2 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ -35 \\ -35 \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$ **1**

$\therefore\ x = 1, y = -5, z = -5$ **1**

[CBSE Marking Scheme 2020 (65/1/1)]

2. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Hence solve the following system of equations :

$2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$ [CBSE 2019 (65/1/2)]

Sol.

$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$adj\ A = \begin{bmatrix} 0 & +2 & 1 \\ -1 & -9 & -5 \\ 2 & +23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$|A| = 2(0) + 3(-2) + 5(1)$

$= 0 - 6 + 5 = -1$

$A^{-1} = \frac{1}{|A|} adj\ A = -1\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

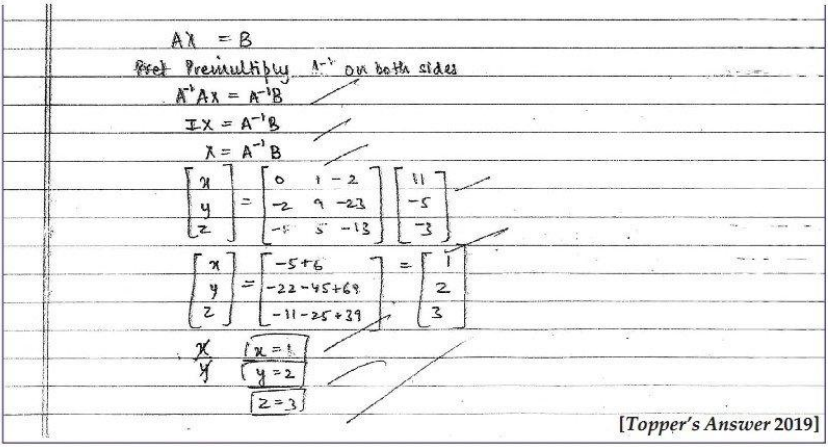
$A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

Given Eq^n:

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$



$AX = B$

~~Frst~~ Premultiply A^{-1} on both sides

$A^{-1}AX = A^{-1}B$

$IX = A^{-1}B$

$X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -5 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$x = 1$

$y = 2$

$z = 3$

[*Topper's Answer* 2019]

3. $A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{bmatrix}$, **find A^{-1} and use it to solve the following system of equations:**

$$5x - y + 4z = 5$$
$$2x + 3y - 5z = 2$$
$$5x - 2y + 6z = -1$$

[*CBSE 2020 (65/2/1)*]

Sol. $|A| = 51$ **1**

$$\text{Cofactors:} = \begin{matrix} A_{11} = 28 & A_{12} = 13 & A_{13} = -19 \\ A_{21} = -2 & A_{22} = 10 & A_{23} = 5 \\ A_{31} = -17 & A_{32} = -17 & A_{33} = 17 \end{matrix}$$ **2**

$$A^{-1} = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}$$ **1**

Given system is $AX = B$ where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$ **½**

$$\Rightarrow X = A^{-1}B = \frac{1}{51}\begin{bmatrix} 28 & -2 & -17 \\ 13 & 10 & -17 \\ -19 & 5 & 17 \end{bmatrix}\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix}$$ **1**

$x = 3, y = 2, z = -2$ **½**

[*CBSE Marking Scheme 2020 (65/2/1)*]

4. **Solve the following system of equations by matrix method**

$$x - y + 2z = 7$$
$$2x - y + 3z = 12$$
$$3x + 2y - z = 5$$

[*CBSE 2020 (65/3/1)*]

Sol. Writing given equations in matrix form

$$\begin{bmatrix}1 & -1 & 2\\2 & -1 & 3\\3 & 2 & -1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}7\\12\\5\end{bmatrix}$$

Which is of the form $AX = B$ **1**

Here $|A| = -2 \neq 0$ **1**

$$A^{-1}=\frac{1}{-2}\begin{bmatrix}-5 & 3 & -1\\11 & -7 & 1\\7 & -5 & 1\end{bmatrix}$$ **2**

$$\therefore \quad X = A^{-1}B=\frac{1}{-2}\begin{bmatrix}-5 & 3 & -1\\11 & -7 & 1\\7 & -5 & 1\end{bmatrix}\begin{bmatrix}7\\12\\5\end{bmatrix}=\begin{bmatrix}2\\1\\3\end{bmatrix}$$ **1**

$\Rightarrow \quad x = 2, y = 1, z = 3$ **1**

[CBSE Marking Scheme 2020 (65/3/1)]

5. If $A=\begin{bmatrix}3 & 2\\5 & -7\end{bmatrix}$, then find A^{-1} and use it to solve the following system of equations :

$3x + 5y = 11, 2x - 7y = -3.$ *[CBSE 2023 (65/3/2)]*

Sol. Given, $A=\begin{bmatrix}3 & 2\\5 & -7\end{bmatrix}$

$\Rightarrow \quad |A| = 3 \times -7 - 2 \times 5$

$= -21 - 10 = -31 \neq 0$

$\therefore \quad A^{-1}$ exists

$\therefore \quad$ its co-factors

$C_{11} = -7, C_{21} = -2$

$C_{12} = -5, C_{22} = 3$

$$\therefore \quad C=\begin{bmatrix}-7 & -5\\-2 & 3\end{bmatrix} \Rightarrow \text{adj}.A = C^T=\begin{bmatrix}-7 & -2\\-5 & 3\end{bmatrix}$$

$$\therefore \quad A^{-1}=\frac{adj.A}{|A|}=-\frac{1}{31}\begin{bmatrix}-7 & -2\\-5 & 3\end{bmatrix}$$

Now, given system of equations

$3x + 5y = 11$

$2x - 7y = -3$

We can write this as

$$\underset{P}{\begin{bmatrix}3 & 5\\2 & -7\end{bmatrix}}\underset{X}{\begin{bmatrix}x\\y\end{bmatrix}}=\underset{B}{\begin{bmatrix}11\\-3\end{bmatrix}}$$

$\Rightarrow PX = B \quad \Rightarrow X = P^{-1}.B = (A^T)^{-1}.B = (A^{-1})^T.B$

$\Rightarrow X = (A^{-1})^T.B$ ...(*i*)

$$\Rightarrow X = -\frac{1}{31}\begin{bmatrix}-7 & -2\\-5 & 3\end{bmatrix}^T \times \begin{bmatrix}11\\-3\end{bmatrix}$$

$$\Rightarrow \quad X = -\frac{1}{31}\begin{bmatrix}-7 & -5\\ -2 & 3\end{bmatrix}\begin{bmatrix}11\\ -3\end{bmatrix} = -\frac{1}{31}\begin{bmatrix}-77+15\\ -22-9\end{bmatrix}$$

$$\Rightarrow \quad X = -\frac{1}{31}\begin{bmatrix}-62\\ -31\end{bmatrix} = \begin{bmatrix}2\\ 1\end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}2\\ 1\end{bmatrix} \Rightarrow x = 2, y = 1$$

6. If $A = \begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$, $B = \begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$, then find AB and use it to solve the following system of equations :

$$x - 2y = 3$$
$$2x - y - z = 2$$
$$-2y + z = 3$$

[CBSE 2023 (65/2/1)]

Sol. Given matrices

$$A = \begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}, B = \begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$$

$$\therefore \; AB = \begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix} = \begin{bmatrix}-3+4+0 & -6+2+4 & 0+4-4\\ 2-2+0 & 4-1-2 & 0-2+2\\ 2-2+0 & 4-1-3 & 0-2+3\end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = I \quad \Rightarrow B^{-1} = A$$

$$\Rightarrow (B^{-1})^T = A^T \qquad \Rightarrow (B^T)^{-1} = A^T \qquad ...(i)$$

Given system of equations

$$x - 2y = 3$$
$$2x - y - z = 2$$
$$-2y + z = 3$$

We can write this as $\underset{P}{\begin{bmatrix}1 & -2 & 0\\ 2 & -1 & -1\\ 0 & -2 & 1\end{bmatrix}} \underset{X}{\begin{bmatrix}x\\ y\\ z\end{bmatrix}} = \underset{Q}{\begin{bmatrix}3\\ 2\\ 3\end{bmatrix}}$

$$\Rightarrow PX = Q \Rightarrow X = P^{-1}.Q = (B^T)^{-1}.Q = A^TQ \qquad \text{(from (i))}$$

$$\Rightarrow X = \begin{bmatrix}-3 & 2 & 2\\ -2 & 1 & 1\\ -4 & 2 & 3\end{bmatrix}\begin{bmatrix}3\\ 2\\ 3\end{bmatrix} = \begin{bmatrix}-9+4+6\\ -6+2+3\\ -12+4+9\end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ -1\\ 1\end{bmatrix}$$

$\Rightarrow x = 1, y = -1$ and $z = 1$

7. If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$. [*CBSE 2023 (65/1/1)*]

Sol. Given matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ $\Rightarrow |A| = 1 \times 3 - 2 \times (-1) + (-2) \times 2$

$= 3 + 2 - 4$

$= 3 - 2 = 1 \neq 0$

$\therefore$ Its co-factors are $\therefore A^{-1}$ exists

$C_{11} = 3$, $C_{21} = 2$, $C_{31} = 6$

$C_{12} = 1$, $C_{22} = 1$, $C_{32} = 2$

$C_{13} = 2$, $C_{23} = 2$, $C_{33} = 5$

$\therefore C = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}$ $\Rightarrow \text{adj } A = C^T = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Also, given $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

$$\therefore (AB)^{-1} = B^{-1}.A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1+2 & 6-1+2 & 18-2+5 \\ -45+6-10 & -30+6-10 & -90+12-25 \\ 15-2+4 & 10-2+4 & 30-4+10 \end{bmatrix}$$

$$\Rightarrow (AB)^{-1} = \begin{bmatrix} 10 & 7 & 21 \\ -49 & -34 & 103 \\ 17 & 12 & 36 \end{bmatrix}$$

8. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, find adj A and verify that $A(\text{adj } A) = (\text{adj } A)A = |A| I_3$. [*CBSE (F)* 2016]

Sol. Given, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because$ $A_{11} = \begin{vmatrix} \cos\alpha & 0 \\ 0 & 1 \end{vmatrix} = \cos\alpha;$ $A_{12} = -\begin{vmatrix} \sin\alpha & 0 \\ 0 & 1 \end{vmatrix} = -\sin\alpha;$ $A_{13} = 0$

$A_{21} = -\begin{vmatrix} -\sin\alpha & 0 \\ 0 & 1 \end{vmatrix} = \sin\alpha;$ $A_{22} = \begin{vmatrix} \cos\alpha & 0 \\ 0 & 1 \end{vmatrix} = \cos\alpha;$ $A_{23} = -\begin{vmatrix} \cos\alpha & \sin\alpha \\ 0 & 0 \end{vmatrix} = 0$

$A_{31} = \begin{vmatrix} -\sin\alpha & 0 \\ \cos\alpha & 0 \end{vmatrix} = 0;$ $A_{32} = -\begin{vmatrix} \cos\alpha & 0 \\ \sin\alpha & 0 \end{vmatrix} = 0;$ $A_{33} = \begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix} = 1$

$$\therefore \text{adj}A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now $A.\text{adj } A = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \cos\alpha.\sin\alpha - \sin\alpha.\cos\alpha & 0 \\ \sin\alpha.\cos\alpha - \sin\alpha.\cos\alpha & \sin^2\alpha + \cos^2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A.\text{adj } A = |A| I_3$ ...(*i*) $\left[\because |A| = \begin{vmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{vmatrix} = \cos^2\alpha + \sin^2\alpha = 1\right]$

Again, $\text{adj } A.A = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & -\sin\alpha.\cos\alpha + \sin\alpha.\cos\alpha & 0 \\ -\sin\alpha.\cos\alpha + \sin\alpha.\cos\alpha & \sin^2\alpha + \cos^2\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $\text{adj } A.A = |A|\ I_3$ ...(*ii*)

From (*i*) and (*ii*), we get

$A\ .\ \text{adj } A = \text{adj } A\ .\ A = |A| I_3.$

9. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:

$x - y + 2z = 1,\ 2y - 3z = 1,\ 3x - 2y + 4z = 2$ ***[CBSE (F) 2011]***

OR

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations:

$x + 3z = 9,\ -x + 2y - 2z = 4,\ 2x - 3y + 4z = -3.$ ***[CBSE Delhi 2017]***

Sol. Given system of equations are

$x - y + 2z = 1,\ \ 2y - 3z = 1,\ \ 3x - 2y + 4z = 2$

Above system of equations can be written in matrix form

as $AX = B \Rightarrow X = A^{-1} B$

where, $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix},\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

Let $C = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

Now, $AC = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} = \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow$ $AC = I \Rightarrow A^{-1}(AC) = A^{-1} I$ [Pre-multiply by A^{-1}]

$\Rightarrow$ $(A^{-1} A)C = A^{-1}$ [By Associativity]

$\Rightarrow \quad IC = A^{-1} \quad \Rightarrow \quad A^{-1} = C$

$$\Rightarrow \quad A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

Putting X, A^{-1} and B in $X = A^{-1}B$, we get

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \quad \Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2+0+2 \\ 9+2-6 \\ 6+1-4 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix} \quad \Rightarrow \quad x = 0, y = 5 \text{ and } z = 3$$

OR

Solution is similar as above.

Ans.: $x = 0$, $y = 5$ and $z = 3$

10. Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse, A^{-1}, solve the system of linear equations $x - y + 2z = 1$; $2y - 3z = 1$; $3x - 2y + 4z = 3$. **[*CBSE 2023 (65/4/1)*]**

Sol. Given matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

$\Rightarrow \quad |A| = 1(8-6) + 1(9) + 2(-6) = 2 + 9 - 12 = -1 \neq 0$

$\therefore \quad A^{-1}$ exists.

Co-factors of A are

$$C_{11} = 2, \quad C_{21} = 0, \quad C_{31} = -1$$
$$C_{12} = -9, \quad C_{22} = -2, \quad C_{32} = 3$$
$$C_{13} = -6, \quad C_{23} = -1, \quad C_{33} = 2$$

$$\therefore \quad C = \begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix} \quad \Rightarrow \quad \text{adj. } A = C^T = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{adj.A}{|A|} = \frac{1}{-1}\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

System of linear equations be

$$x - y + 2z = 1$$
$$2y - 3z = 1$$
$$3x - 2y + 4z = 3$$

We can write this as

$$\underset{A}{\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}}\underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}}$$

$\Rightarrow \qquad AX = B \quad \Rightarrow X = A^{-1}.B$

$$\therefore \qquad X = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -2+0+3 \\ 9+2-9 \\ 6+1-6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \Rightarrow x = 1, y = 2 \text{ and } z = 1$$

11. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ **and use it to solve the system of equations**

$x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1.$ **[*CBSE (AI) 2017*]**

Sol.

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} (-4)1 + 4(1) + 4(2) & -4(-1) + 4(-2) + 4(1) & -4(1) + 4(-2) + 4(3) \\ -7(1) + 1(1) + 3(2) & -7(-1) + 1(-2) + 3(1) & -7(1) + 1(-2) + 3(3) \\ 5(1) - 3(1) - 1(2) & 5(-1) - 3(-2) - 1(1) & 5(1) - 3(-2) - 1(3) \end{bmatrix}$$

$$\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I$$

$x - y + z = 4$

$x - 2y - 2z = 9$

$2x + y + 3z = 1$

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$AX = B$

$X = A^{-1}B$

$$x = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} A = 8I$$

$$A^{-1} = \frac{1}{8} \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$$

$$= \frac{1}{8} \begin{bmatrix} -4(4) + 4(9) + 4(1) \\ -7(4) + 1(9) + 3(1) \\ 5(4) - 3(9) - 1(1) \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix}$$

$$= \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$ Hence $x = 3$, $y = -2$, $z = -1$

[*Topper's Answer* 2017]

12. A shopkeeper has 3 varieties of pens '*A*', '*B*' and '*C*'. Meenu purchased 1 pen of each variety for a total of ₹ 21. Jeevan purchased 4 pens of '*A*' variety, 3 pens of '*B*' variety and 2 pens of '*C*' variety for ₹ 60. While Shikha purchased 6 pens of '*A*' variety, 2 pens of '*B*' variety and 3 pens of '*C*' variety for ₹ 70. Using matrix method, find cost of each variety of pen. [*CBSE (Central) 2016*]

Sol. Let the cost of varieties of pens *A*, *B* and *C* be ₹ x, ₹ y, and ₹ z respectively.

From question

$$x + y + z = 21,\quad 4x + 3y + 2z = 60,\quad 6x + 2y + 3z = 70$$

The given system of linear equation in matrix equation is as follows

$$AX = B, \text{ where } A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix},\quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$\because \quad AX = B \quad \Rightarrow X = A^{-1}B \qquad \ldots(i)$

Now $$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{vmatrix} = 1(9-4) - 1(12-12) + 1(8-18)) = 5 - 0 - 10 = -5 \neq 0$$

$$A_{11} = (9-4) = 5 \qquad A_{21} = -(3-2) = -1 \qquad A_{31} = (2-3) = -1$$
$$A_{12} = -(12-12) = 0 \qquad A_{22} = (3-6) = -3 \qquad A_{32} = -(2-4) = 2$$
$$A_{13} = (8-18) = -10 \qquad A_{23} = -(2-6) = 4 \qquad A_{33} = (3-4) = -1$$

$$\therefore \text{ Adj } A = \begin{bmatrix} 5 & 0 & -10 \\ -1 & -3 & 4 \\ -1 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}.\text{adj} A = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$

Now from (i) $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -25 \\ -40 \\ -40 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \Rightarrow x = 5,\ y = 8,\ z = 8$$

$\Rightarrow$ Cost of pen A = ₹ 5; cost of pen B = ₹ 8 and cost of pen C = ₹ 8

13. **If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}. Use it to solve the system of equations** **[*CBSE 2018*]**

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

Sol. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

$$|A| = 2(-4 + 4) + 3(-6 + 4) + 5(3 - 2) = -6 + 5 = -1 \neq 0$$

$\therefore$ A^{-1} exist.

Calculation of cofactor of A

$A_{11} = 0$ $\quad A_{12} = 2$ $\quad A_{13} = 1$

$A_{21} = -1$ $\quad A_{22} = -9$ $\quad A_{23} = -5$

$A_{31} = 2$ $\quad A_{32} = 23$ $\quad A_{33} = 13$

Cofactors of $A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}$

$$adj\,(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

Equations are:

$$2x - 3y + 5z = 11;\ 3x + 2y - 4z = -5;\ x + y - 2z = -3$$

$$\Rightarrow \quad \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \quad AX = B$$

Using pre-multiplication with A^{-1}, we get

$$A^{-1}AX = A^{-1}B$$

$$\Rightarrow \quad IX = A^{-1}B$$

$$\Rightarrow \quad X = A^{-1}B$$

$$\Rightarrow \quad X = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore$ $\quad x = 1, y = 2, z = 3$

14. Solve the following system of equations by matrix method :

$$x + 2y + 3z = 6$$
$$2x - y + z = 2$$
$$3x + 2y - 2z = 3$$

[CBSE 2023 (65/1/1)]

Sol. Given system of equations

$$x + 2y + 3z = 6$$
$$2x - y + z = 2$$
$$3x + 2y - 2z = 3$$

We can write this in matrix form as

$$\underset{A}{\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 2 & -2 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}}$$

$\Rightarrow \quad AX = B \quad \Rightarrow X = A^{-1}. B \qquad ...(i)$

Now, $\quad A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 2 & -2 \end{bmatrix}$

$\Rightarrow \quad |A| = (1 \times 0) - (2 \times -7) + (3 \times 7) = 14 + 21$

$\qquad = 35 \neq 0$

$\therefore \quad A^{-1}$ exists

Its co-factors are

$$C_{11} = 0, \quad C_{21} = 10, \quad C_{31} = 5$$
$$C_{12} = 7, \quad C_{22} = -11, \quad C_{32} = 5$$
$$C_{13} = 7, \quad C_{23} = 4, \quad C_{33} = -5$$

$\therefore \quad C = \begin{bmatrix} 0 & 7 & 7 \\ 10 & -11 & 4 \\ 5 & 5 & -5 \end{bmatrix} \qquad \Rightarrow \qquad \text{adj } A = C^T = \begin{bmatrix} 0 & 10 & 5 \\ 7 & -11 & 5 \\ 7 & 4 & -5 \end{bmatrix}$

$\therefore \quad A^{-1} = \dfrac{\text{adj}A}{|A|} = \dfrac{1}{35}\begin{bmatrix} 0 & 10 & 5 \\ 7 & -11 & 5 \\ 7 & 4 & -5 \end{bmatrix}$

Putting in (*i*), we have

$$X = A^{-1}.B = \frac{1}{35}\begin{bmatrix} 0 & 10 & 5 \\ 7 & -11 & 5 \\ 7 & 4 & -5 \end{bmatrix}\begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 0 + 20 + 15 \\ 42 - 22 + 15 \\ 42 + 8 - 15 \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 35 \\ 35 \\ 35 \end{bmatrix}$$

$\Rightarrow \quad x = 1, y = 1, z = 1$

15. **If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Use A^{-1} to solve the following system of equations:**

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

Sol. Given equation is of the form $AX = B$ where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = 2(0) + 3(-2) + 5(1) = -1$$

$$A^{-1} = \frac{adjA}{|A|}$$

$$adjA = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

Now, $$A^{-1} = \frac{1}{(-1)}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-1)}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \frac{1}{(-1)}\begin{bmatrix} 0+5-6 \\ 22+45-69 \\ 11+25-39 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{(-1)}\begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \Rightarrow x = 1, y = 2, z = 3$$

Questions for Practice

■ Multiple Choice Questions

1. **Choose and write the correct option in each of the following questions.**

(*i*) If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is [*CBSE 2023 (65/5/1)*]

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

(*ii*) If A, B are non-singular square matrices of the same order, then $(AB^{-1})^{-1} =$

(*a*) $A^{-1}B$ (*b*) $A^{-1}B^{-1}$ (*c*) BA^{-1} (*d*) AB

[*CBSE Sample Paper 2023*]

(*iii*) If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1} = kA$, then k equals

(*a*) 19 (*b*) $\frac{1}{19}$ (*c*) -19 (*d*) $\frac{-1}{19}$

(*iv*) If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then

(*a*) $a = 1, b = 1$ (*b*) $a = \cos 2\theta, b = \sin 2\theta$

(*c*) $a = \sin 2\theta, b = \cos 2\theta$ (*d*) none of these

(*v*) If A is a square matrix of order 3, such that $A(\text{adj } A) = 10\, I$, then $|\text{adj } A|$ is equal to **[CBSE 2020 (65/5/1)]**

(*a*) 1 (*b*) 10 (*c*) 100 (*d*) 101

(*vi*) If A satisfies the equation $x^3 - 5x^2 + 4x + \lambda = 0$, then A^{-1} exists if

(*a*) $\lambda \neq 1$ (*b*) $\lambda \neq 2$ (*c*) $\lambda \neq -1$ (*d*) all of them

(*vii*) If A is a square matrix of order 3 and $|A| = 5$, then $|adj\ A| =$

(*a*) 20 (*b*) 10 (*c*) 0 (*d*) 100

■ Conceptual Questions

2. If A_{ij} is the cofactor of the element a_{ij} of the determinant

$\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then write the value of $a_{32} \cdot A_{32}$. **[CBSE (AI) 2013]**

3. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then for any natural number n, find the value of det (A^n). **[CBSE Ajmer 2015]**

4. Find the cofactors of all the elements of $\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$. **[CBSE 2020 (65/5/3)]**

5. Write the adjoint of the matrix $\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$. **[CBSE (AI) 2010]**

6. If A is a square matrix of order 3 such that $|\text{adj } A| = 64$, find $|A|$.

7. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then find adj (AB).

8. Find $|AB|$, if $A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$. **[CBSE 2019 (65/2/2)]**

9. Let A be a square matrix of order 3×3. Write the value of $|2A|$, where $|A| = 4$. **[CBSE Delhi 2012]**

10. A is a square matrix with $|A| = 4$. Then find the value of $|A.(\text{adj } A)|$. **[CBSE 2019 (65/4/3)]**

11. For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, write A^{-1}. **[CBSE 2020 (65/5/1)]**

■ Very Short Answer Questions

12. Find the equation of line Joining (3, 1) and (9, 3) using determinant.

13. Using co-factors of elements of third column,

evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$.

14. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, show that $A^{-1} = \frac{1}{19}A$.

15. Solve the following system of equations by matrix method:

$$3x + y = 19$$
$$3x - y = 23$$

Short Answer Questions

16. Show that the points $A(a, b + c)$, $B(b, c + a)$, $C(c, a + b)$ are collinear.

17. A typist charges ₹ 145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are ₹ 180. Using matrices, find the charges of typing one English and one Hindi page separately. **[CBSE (North) 2016]**

18. If A, B are square matrices of the same order, then prove that adj (AB) = (adj B) (adj A).

19. Let $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 4 & -6 \\ -2 & 4 \end{bmatrix}$. Then compute AB. Hence, solve the following system of equations:

$2x + y = 4,\ 3x + 2y = 1.$ **[*CBSE Sample Paper* 2016]**

20. Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

21. Without using properties of determinant prove that $\begin{vmatrix} 0 & 2 & -3 \\ -2 & 0 & 4 \\ 3 & -4 & 0 \end{vmatrix} = 0$.

22. Without using properties of determinant prove that $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$.

Long Answer Questions

23. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}. Use it to solve the system of equations:

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

[CBSE 2018]

24. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$. Find A^{-1}. Hence, solve the system of equations

$x + y + z = 6, x + 2z = 7, 3x + y + z = 12$. **[CBSE 2019 (65/1/1)]**

25. Using matrix, solve the following system of equations:

$2x - 3y + 5z = 11,\ \ 3x + 2y - 4z = -5,\ \ x + y - 2z = -3$ **[CBSE Delhi 2009]**

26. Find the adjoint of the matrix $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ and hence, show that $A.(\text{adj } A) = |A| I_3$.

[CBSE Allahabad 2015]

27. Find the inverse of matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and hence show that $A^{-1}. A = I$.

[CBSE Chennai 2015]

28. Using matrices, solve the following system of linear equations: [CBSE (F) 2009]

$$3x - 2y + 3z = 8$$
$$2x + y - z = 1$$
$$4x - 3y + 2z = 4$$

29. Using matrices, solve the following system of equations: [CBSE (AI) 2011]

$$x + 2y + z = 7$$
$$x + 3z = 11$$
and $$2x - 3y = 1$$

30. Using matrices, solve the following system of equations: [CBSE (F) 2012]

$$x - y + z = 4$$
$$2x + y - 3z = 0$$
$$x + y + z = 2$$

31. Using matrices, solve the following system of equations: [CBSE (AI) 2012]

$$2x + 3y + 3z = 5$$
$$x - 2y + z = -4$$
$$3x - y - 2z = 3$$

32. Using matrices, solve the following system of equations: [CBSE (AI) 2012]

$$x + y - z = 3$$
$$2x + 3y + z = 10$$
$$3x - y - 7z = 1$$

33. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. [CBSE Sample Paper 2021]

Hence solve the system of equations:

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

34. Evaluate the product AB, where [CBSE Sample Paper 2021]

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Hence solve the system of linear equations

$$x - y = 3$$
$$2x + 3y + 4z = 17$$
$$y + 2z = 7$$

Answers

1. (*i*) (*d*) (*ii*) (*c*) (*iii*) (*b*) (*iv*) (*b*) (*v*) (*c*) (*vi*) (*d*) (*vii*) (*b*)

2. 110

3. 1

4. $C_{11} = 3,\ C_{21} = 2,\ C_{12} = -4,\ C_{22} = 1$

5. $\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$

6. ± 8

7. $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

8. 0

9. 32

10. 4^n

11. $\begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$

12. $x - 3y = 0$ **13.** $(x-y)(y-z)(z-x)$

15. $x = 7, y = -2$ **17.** ₹ 10 and ₹ 15 respectively

19. $AB = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, x = 7, y = -10$

23. $A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}; \ x = 1, y = 2, z = 3$

24. $A^{-1} = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}; \ x = 3, y = 1, z = 2$

25. $x = 1, \ y = 2, \ z = 3$

26. $\text{adj } A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$

27. $A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

28. $x = 1, \ y = 2, \ z = 3$

29. $x = 2, \ y = 1, \ z = 3$

30. $x = 2, \ y = -1, \ z = 1$

31. $x = 1, \ y = 2, \ z = -1$

32. $x = 3, \ y = 1, \ z = 1$

33. $A^{-1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$x = 0, y = -5, z = -3$

34. $\begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$x = 2, y = -1, z = 4$