





6. Theorems on Triangles

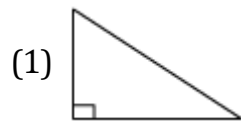
Exercise 6.1

1. Question

Match the following

- | | |
|--|----------------------------|
| (1)  | (a) Equilateral triangle |
| (2)  | (b) Acute angled triangle |
| (3)  | (c) Right-angled triangle |
| (4)  | (d) Obtuse angled triangle |

Answer



As we know, if one of the angles of the triangle is 90° then the triangle is right angle triangle.

Hence, The given triangle is a right angle triangle.

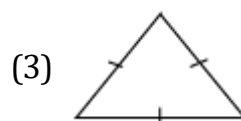
(1) - c



As we know, if one of the angles of the triangle is greater than 90° then the triangle is an obtuse angle triangle.

Hence, The given triangle is an obtuse angle triangle.

(2) - d



As we know, if all the sides of a triangle are of equal length then the triangle is an equilateral triangle.

Hence, The given triangle is a equilateral triangle.

(3) – a



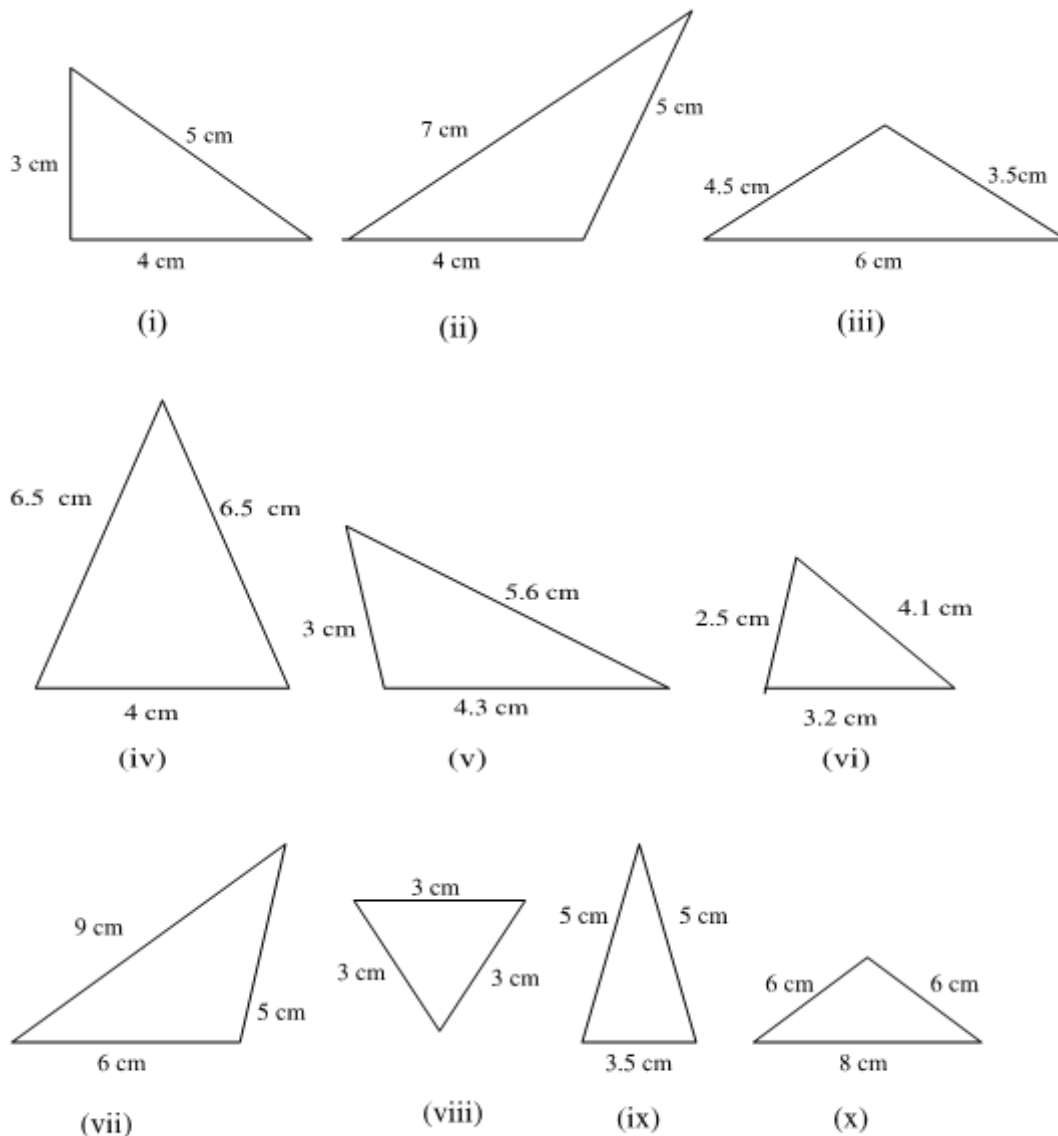
As we know, if all the angles of the triangle are less than 90° then the triangle is acute angle triangle.

Hence, The given triangle is an acute angle triangle.

(4) – b

2. Question

Based on the sides, classify the following triangles (figures not drawn to the scales).



Answer

(i) Given: three sides of the triangle: 3cm, 4cm and 5cm.

As we know, if all the sides of a triangle are of different lengths then the triangle is a scalene triangle.

So, the given triangle is scalene triangle.

(ii) Given: three sides of triangle: 4cm, 7cm and 5cm.

As we know, if all the sides of a triangle are of different lengths then the triangle is a scalene triangle.

So, the given triangle is scalene triangle.

(iii) Given: three sides of triangle: 3.5cm, 4.5cm and 6cm.

As we know, if all the sides of a triangle are of different lengths then the triangle is a scalene triangle.

So, the given triangle is scalene triangle.

(iv) Given: three sides of triangle: 6.5cm, 6.5cm and 4cm.

As we know, if two of the sides of a triangle are of equal length then the triangle is an isosceles triangle.

So, the given triangle is isosceles triangle.

(v) Given: three sides of triangle: 3cm, 5.6cm and 4.3cm.

As we know, if all the sides of a triangle are of different lengths then the triangle is a scalene triangle.

So, the given triangle is scalene triangle.

(vi) Given: three sides of triangle: 2.5cm, 4.1cm and 3.2cm.

As we know, if all the sides of a triangle are of different lengths then the triangle is a scalene triangle.

So, the given triangle is scalene triangle.

(vii) Given: three sides of triangle: 5cm, 9cm and 6cm.

As we know, if all the sides of a triangle are of different lengths then the triangle is a scalene triangle.

So, the given triangle is scalene triangle.

(viii) Given: three sides of triangle: 3cm, 3cm and 3cm.

As we know, if all the sides of a triangle are of equal length then the triangle is an equilateral triangle.

So, the given triangle is equilateral triangle.

(ix) Given: three sides of triangle: 5cm, 5cm and 3.5cm.

As we know, if two of the sides of a triangle are of equal length then the triangle is an isosceles triangle.

So, the given triangle is isosceles triangle.

(x) Given: three sides of triangle: 6cm, 6cm and 8cm.

As we know, if two of the sides of a triangle are of equal length then the triangle is an isosceles triangle.

So, the given triangle is isosceles triangle.

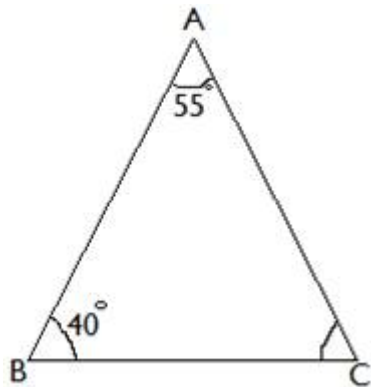
Exercise 6.2

1. Question

In a triangle ABC, if $\angle A = 55^\circ$ and $\angle B = 40^\circ$, find $\angle C$.

Answer

Given: $\angle A = 55^\circ$ and $\angle B = 40^\circ$



As we know, using theorem (1), in any triangle, sum of the three interior angles is 180° .

So,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 55^\circ + 40^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 55^\circ - 40^\circ$$

$$\Rightarrow \angle C = 85^\circ$$

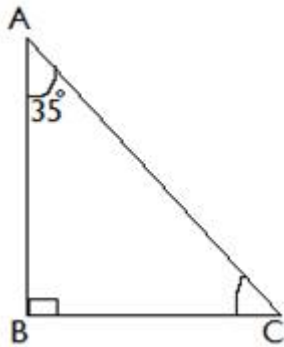
2. Question

In a right angled triangle, if one of the other two angles is 35° , find the remaining angle.

Answer

Given:

Let $\angle A = 35^\circ$ and $\angle B = 90^\circ$ (right angle)



As we know, using theorem (1), in any triangle, sum of the three interior angles is 180° .

So,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 35^\circ + 90^\circ + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - 35^\circ - 90^\circ$$

$$\Rightarrow \angle C = 55^\circ$$

3. Question

If the vertex angle of an isosceles triangle is 50° , find the other angles.

Answer

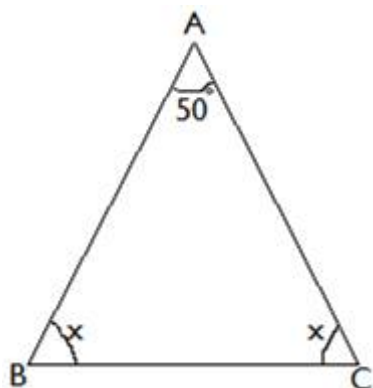
Given: vertex angle

let $\angle A = 55^\circ$

in an isosceles triangle opposite angles of opposite sides are equal.

Hence,

Let $\angle B = \angle C = x$



As we know, using theorem (1), in any triangle, sum of the three interior angles is 180° .

So,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 50^\circ + x + x = 180^\circ$$

$$\Rightarrow 2x = 180^\circ - 50^\circ$$

$$\Rightarrow 2x = 130^\circ$$

$$\Rightarrow x = 65^\circ$$

So, $\angle B = \angle C = 65^\circ$.

4. Question

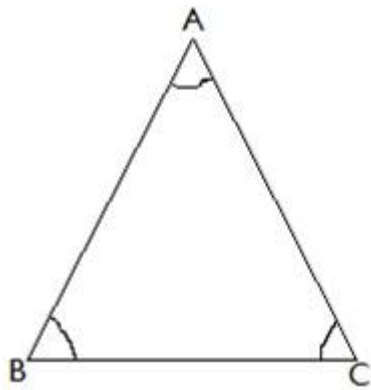
The angles of a triangle are in the ratio 1:2:3. Determine the three angles.

Answer

Given: the ratio of angles as 1:2:3

So, let the angle,

$$\angle A = x, \angle B = 2x, \angle C = 3x$$



As we know, using theorem (1), in any triangle, sum of the three interior angles is 180° .

So,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow x + 2x + 3x = 180^\circ$$

$$\Rightarrow 6x = 180^\circ$$

$$\Rightarrow x = 30^\circ$$

So, the angles are,

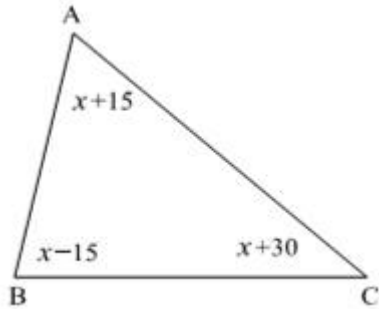
$$\angle A = x = 30^\circ$$

$$\angle B = 2x = 2 \times 30^\circ = 60^\circ$$

$$\angle C = 3x = 3 \times 30^\circ = 90^\circ$$

5. Question

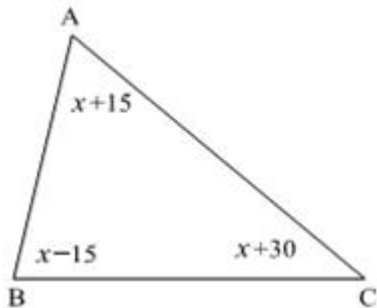
In the adjacent triangle ABC, find the value of x and calculate the measure of all the angles of the triangle.



Answer

Given:

$$\angle A = x + 15, \angle B = x - 15, \angle C = x + 30$$



As we know, using theorem (1), in any triangle, sum of the three interior angles is 180° .

So,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow x + 15 + x - 15 + x + 30 = 180^\circ$$

$$\Rightarrow 3x + 30^\circ = 180^\circ$$

$$\Rightarrow 3x = 180^\circ - 30^\circ$$

$$\Rightarrow 3x = 150^\circ$$

$$\Rightarrow x = 50^\circ$$

So, the angles are,

$$\angle A = x + 15 = 50^\circ + 15^\circ = 65^\circ$$

$$\angle B = x - 15 = 50^\circ - 15^\circ = 35^\circ$$

$$\angle C = x + 30 = 50^\circ + 30^\circ = 80^\circ$$

6. Question

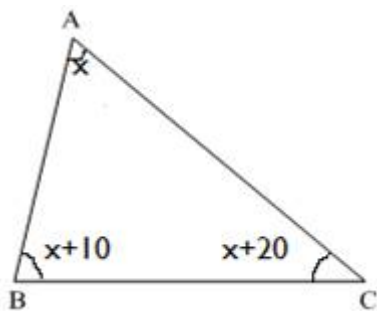
The angles of a triangle are arranged in ascending order of their magnitude. If the difference between two consecutive angles is 10° , find the three angles.

Answer

Given:

Let the angles are :

$$\angle A = x, \angle B = x + 10, \angle C = x + 20$$



As we know, using theorem (1), in any triangle, sum of the three interior angles is 180° .

So,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow x + x + 10 + x + 20 = 180^\circ$$

$$\Rightarrow 3x + 30^\circ = 180^\circ$$

$$\Rightarrow 3x = 180^\circ - 30^\circ$$

$$\Rightarrow 3x = 150^\circ$$

$$\Rightarrow x = 50^\circ$$

So, the angles are,

$$\angle A = x = 50^\circ$$

$$\angle B = x + 10 = 50^\circ + 10^\circ = 60^\circ$$

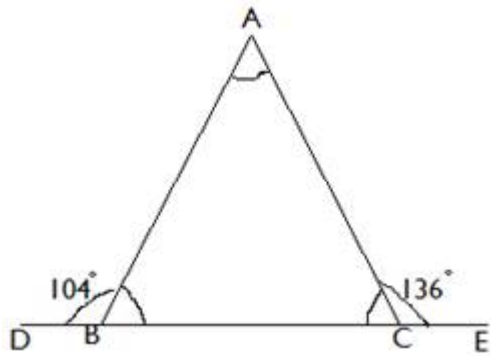
$$\angle C = x + 20 = 50^\circ + 20^\circ = 70^\circ$$

Exercise 6.3

1. Question

The exterior angles obtained on producing the base of a triangle both ways are 104° and 136° . Find the angles of the triangle.

Answer



Given: exterior angles: $\angle ABD = 104^\circ$ and $\angle ACE = 136^\circ$

As D, B and C all lie on the same line.

So,

$$\angle ABD + \angle ABC = 180^\circ$$

$$\Rightarrow 104^\circ + \angle ABC = 180^\circ$$

$$\Rightarrow \angle ABC = 180^\circ - 104^\circ$$

$$\Rightarrow \angle ABC = 76^\circ$$

Similarly, As E, B and C all lie on the same line.

So,

$$\angle ACB + \angle ACE = 180^\circ$$

$$\Rightarrow \angle ACB + 136^\circ = 180^\circ$$

$$\Rightarrow \angle ACB = 180^\circ - 136^\circ$$

$$\Rightarrow \angle ACB = 44^\circ$$

As we know, using theorem (1), in any triangle, sum of the three interior angles is 180° .

So,

$$\angle ABC + \angle ACB + \angle CAB = 180^\circ$$

$$\Rightarrow 76^\circ + 44^\circ + \angle CAB = 180^\circ$$

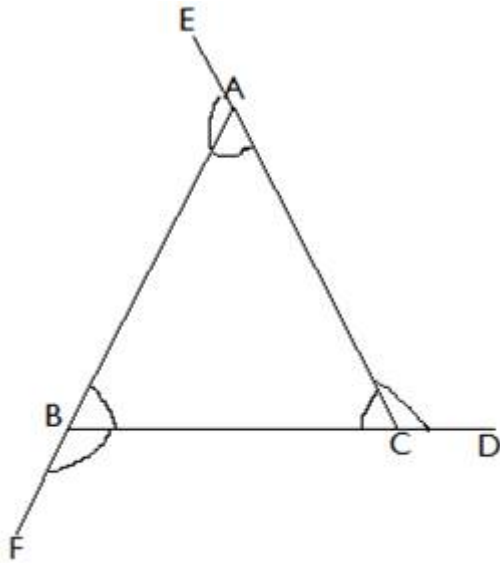
$$\Rightarrow \angle CAB = 180^\circ - 76^\circ - 44^\circ$$

$$\Rightarrow \angle CAB = 60^\circ$$

2. Question

Sides BC, CA and AB of a triangle ABC are produced in an order, forming exterior angles $\angle ACD$, $\angle BAE$, and $\angle CBF$. Show that $\angle ACD + \angle BAE + \angle CBF = 360^\circ$.

Answer



As we know, using theorem (2) i.e. if a side of the triangle is produced, the exterior angle so formed is equal to the sum of corresponding opposite interior angles.

So,

$$\angle ACD = \angle BAC + \angle ABC \dots(1)$$

$$\angle BAE = \angle ABC + \angle ACB \dots(2)$$

$$\angle CBF = \angle BAC + \angle BCA \dots(3)$$

Add (1), (2) and (3)

We get,

$$\angle ACD + \angle BAE + \angle CBF = \angle BAC + \angle ABC + \angle ABC + \angle BCA + \angle BAC + \angle BCA$$

$$\Rightarrow \angle ACD + \angle BAE + \angle CBF = 2(\angle BAC + \angle ABC + \angle BCA)$$

As we know, using theorem (1), in any triangle, sum of the three interior angles is 180° .

So,

$$\angle ABC + \angle BAC + \angle BCA = 180^\circ$$

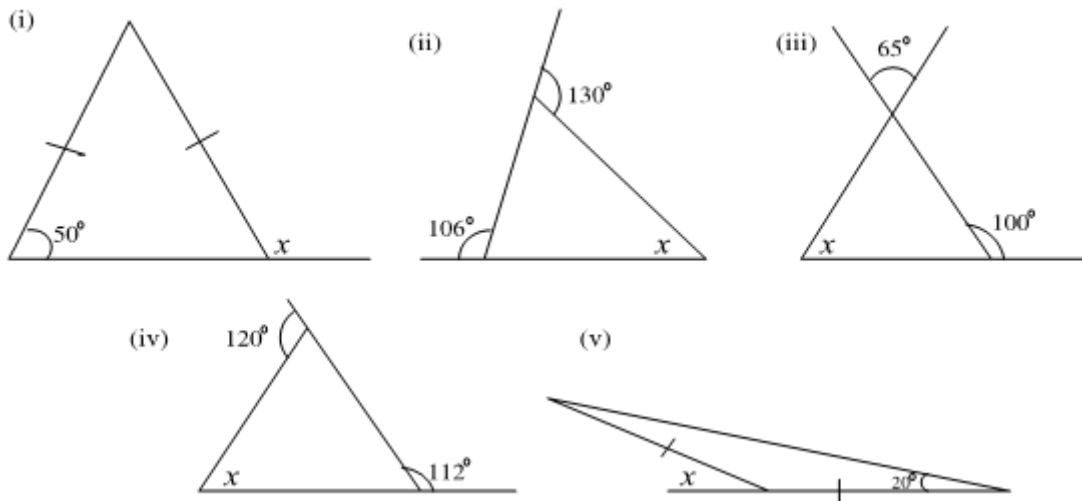
$$\Rightarrow \angle ACD + \angle BAE + \angle CBF = 2(180^\circ)$$

$$\Rightarrow \angle ACD + \angle BAE + \angle CBF = 360^\circ$$

Hence proved.

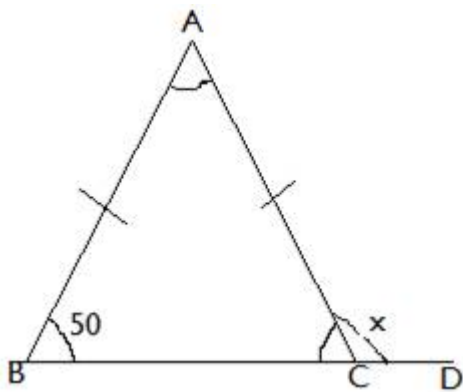
3. Question

Compute the value of x in each of the following figures:



Answer

(i) Given:



As $AB = AC$ (isosceles triangle)

So,

$$\angle ABC = \angle ACB = 50^\circ$$

As B, C and D lie on the same line.

So,

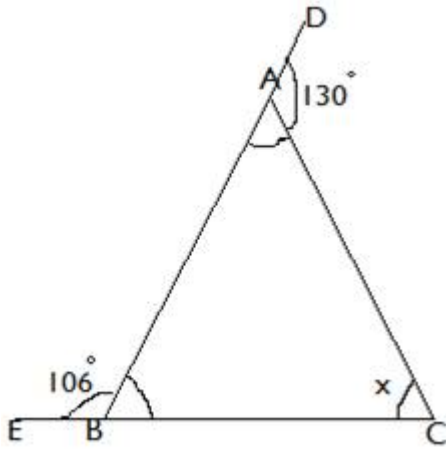
$$\angle ACB + \angle ACD = 180^\circ$$

$$\Rightarrow 50^\circ + x = 180^\circ$$

$$\Rightarrow x = 180^\circ - 50^\circ$$

$$\Rightarrow x = 130^\circ$$

(ii)



As B, A and D lie on the same line.

So,

$$\angle BAC + \angle CAD = 180^\circ$$

$$\Rightarrow \angle BAC + 130^\circ = 180^\circ$$

$$\Rightarrow \angle BAC = 180^\circ - 130^\circ$$

$$\Rightarrow \angle BAC = 50^\circ$$

As we know, using theorem (2) i.e. if a side of triangle is produced, the exterior angle so formed is equal to the sum of corresponding opposite interior angles.

Hence,

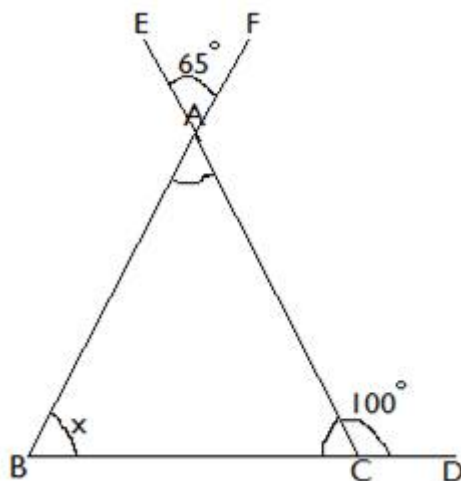
$$\angle ABE = \angle BAC + \angle ACB$$

$$\Rightarrow 106^\circ = 50^\circ + x$$

$$\Rightarrow x = 106^\circ - 50^\circ$$

$$\Rightarrow x = 56^\circ$$

(iii)



$$\angle BAC = \angle EAF = 65^\circ \text{ (vertically opposite angle)}$$

As we know, using theorem (2) i.e. if a side of triangle is produced, the exterior angle so formed is equal to the sum of corresponding opposite interior angles.

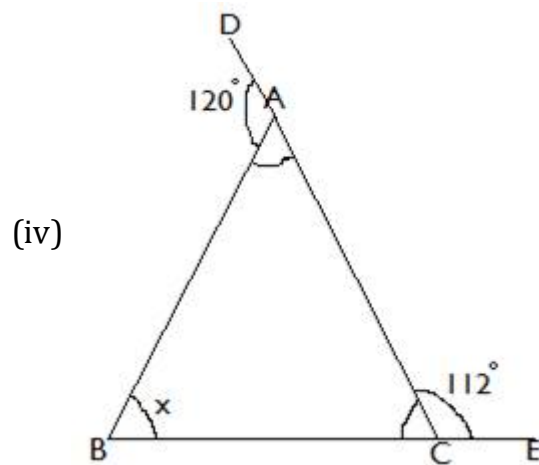
Hence,

$$\angle ACD = \angle BAC + \angle CBA$$

$$\Rightarrow 100^\circ = 65^\circ + x$$

$$\Rightarrow x = 100^\circ - 65^\circ$$

$$\Rightarrow x = 35^\circ$$



As C, A and D lie on the same line.

So,

$$\angle CAB + \angle BAD = 180^\circ$$

$$\Rightarrow \angle CAB + 120^\circ = 180^\circ$$

$$\Rightarrow \angle CAB = 180^\circ - 120^\circ$$

$$\Rightarrow \angle CAB = 60^\circ$$

As we know, using theorem (2) i.e. if a side of triangle is produced, the exterior angle so formed is equal to the sum of corresponding opposite interior angles.

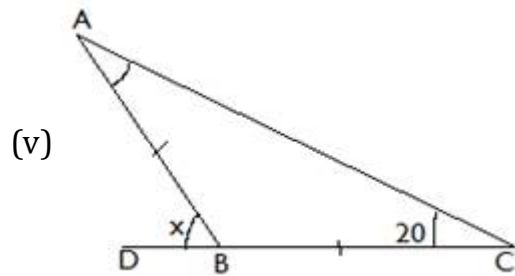
Hence,

$$\angle ACE = \angle CAB + \angle ABC$$

$$\Rightarrow 112^\circ = 60^\circ + x$$

$$\Rightarrow x = 112^\circ - 60^\circ$$

$$\Rightarrow x = 52^\circ$$



As $AB = BC$ (isosceles triangle)

So,

$$\angle BAC = \angle BCA = 20^\circ$$

As we know, using theorem (2) i.e. if a side of the triangle is produced, the exterior angle so formed is equal to the sum of corresponding opposite interior angles.

Hence,

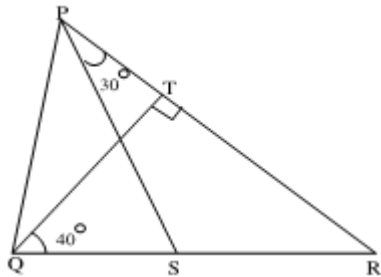
$$\angle ADB = \angle BAC + \angle BCA$$

$$\Rightarrow x = 20^\circ + 20^\circ$$

$$\Rightarrow x = 40^\circ$$

4. Question

In the figure, $QT \perp PR$, $\angle TQR = 40^\circ$ and $\angle SPR = 30^\circ$. Find $\angle TRS$ and $\angle PSQ$.



Answer

Given: $\angle TQR = 40^\circ$ and $\angle SPR = 30^\circ$.

$$\angle QTR = 90^\circ \text{ (right angle)}$$

As we know, using theorem (1), in any triangle, sum of the three interior angles is 180° .

So,

$$\angle TQR + \angle QTR + \angle TRQ = 180^\circ$$

$$\Rightarrow 40^\circ + 90^\circ + \angle TRQ = 180^\circ$$

$$\Rightarrow \angle TRQ = 180^\circ - 40^\circ - 90^\circ$$

$$\Rightarrow \angle TRQ = 50^\circ$$

$$\Rightarrow \angle TRQ = \angle TRS = 50^\circ$$

Also,

$$\angle SPR + \angle PRS + \angle RSP = 180^\circ$$

$$\angle SPR + \angle TRS + \angle RSP = 180^\circ$$

$$\Rightarrow 30^\circ + 50^\circ + \angle RSP = 180^\circ$$

$$\Rightarrow \angle RSP = 180^\circ - 30^\circ - 50^\circ$$

$$\Rightarrow \angle RSP = 100^\circ$$

As R, S and Q lie on the same line.

So,

$$\angle RSP + \angle PSQ = 180^\circ$$

$$\Rightarrow \angle PSQ + 100^\circ = 180^\circ$$

$$\Rightarrow \angle PSQ = 80^\circ$$

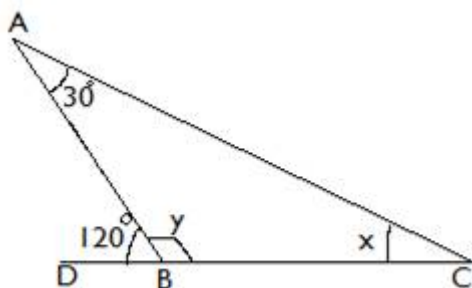
5. Question

An exterior angle of a triangle is 120° and one of the interior opposite angles is 30° . Find the other angles of the triangle.

Answer

Given: exterior angle let $\angle ABD = 120^\circ$

One interior angle let $\angle BAC = 30^\circ$



As we know, using theorem (2) i.e. if a side of the triangle is produced, the exterior angle so formed is equal to the sum of corresponding opposite interior angles.

Hence,

$$\angle ABD = \angle BAC + \angle BCA$$

$$\Rightarrow 120^\circ = 30^\circ + \angle BCA$$

$$\Rightarrow \angle BCA = 120^\circ - 30^\circ$$

$$\angle BCA = x = 90^\circ$$

As we know, using theorem (1), in any triangle, sum of the three interior angles is 180° .

So,

$$\angle BAC + \angle BCA + \angle ABC = 180^\circ$$

$$\Rightarrow 30^\circ + 90^\circ + y = 180^\circ$$

$$\Rightarrow y = 180^\circ - 30^\circ - 90^\circ$$

$$\Rightarrow y = 60^\circ$$

Additional Problems 6

1. Question

Fill up the blanks to make the following statements true:

- (a) Sum of the angles of a triangle is _____
- (b) An exterior angle of a triangle is equal to the sum of _____ opposite angles.
- (c) An exterior angle of a triangle is always _____ than either of the interior opposite angles.
- (d) A triangle cannot have more than _____ right angle.
- (e) A triangle cannot have more than _____ obtuse angle.

Answer

- (a) Sum of the angles of a triangle is 180° which is stated by the angle sum property of the triangle.
- (b) An exterior angle of a triangle is equal to the sum of the interior opposite angles which is the exterior angle property of a triangle.
- (c) An exterior angle of a triangle is always larger than either of the interior opposite angles. This is because an exterior angle of a triangle is equal to the sum of the interior opposite angles.
- (d) A triangle cannot have more than one right angle. This is because the sum of the angles of a triangle is 180° if two angles will be 90° then the third angle will be 0° which is not possible.
- (e) A triangle cannot have more than one obtuse angle. This is because the sum of the angles of a triangle is 180° if two angles will be more than 90° then the sum will exceed 180° .

2 A. Question

In a triangle ABC, $\angle A = 80^\circ$ and $AB = AC$, then $\angle B$ is _____

A. 50°

B. 60°

C. 40°

D. 70°

Answer

In the given $\triangle ABC$,

$\angle A = 80^\circ$ and $AB = AC$

$\Rightarrow \angle B = \angle C$ {angles opposite to equal sides are equal}

Also, sum of all the angles of a triangle is 180° .

$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 2\angle B = 180^\circ - 80^\circ$

$\Rightarrow 2\angle B = 100^\circ$

$\Rightarrow \angle B = \angle C = 50^\circ$

2 B. Question

In right angled triangle, $\angle A$ is right angle and $\angle B = 35^\circ$, then $\angle C$ is _____

A. 65°

B. 55°

C. 75°

D. 45°

Answer

In the given $\triangle ABC$,

$\angle A = 90^\circ$

$\Rightarrow \angle B = 35^\circ$

We know that sum of all the angles of a triangle is 180° .

$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow \angle C = 180^\circ - 90^\circ - 35^\circ$

$$\Rightarrow \angle C = 55^\circ$$

2 C. Question

In a triangle ABC, $\angle B = \angle C = 45^\circ$, then the triangle is _____

- A. right triangle
- B. acute angled triangle
- C. obtuse angle triangle
- D. equilateral triangle

Answer

Given that $\angle B = \angle C = 45^\circ$

We know that the sum of the angles of a triangle is 180° .

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A = 180^\circ - 45^\circ - 45^\circ$$

$$\Rightarrow \angle A = 90^\circ$$

Hence, $\triangle ABC$ is a right triangle.

2 D. Question

In an equilateral triangle, each exterior angle is _____

- A. 60°
- B. 90°
- C. 120°
- D. 150°

Answer

In an equilateral triangle, each angle is of 60° .

Also, an exterior angle of a triangle is equal to the sum of the interior opposite angles.

$$\Rightarrow \text{Each exterior angle} = 60^\circ + 60^\circ = 120^\circ$$

2 E. Question

Sum of the three exterior angles of a triangle is _____

- A. two right angles
- B. three right angles

C. one right angle

D. four right angles

Answer

We know that an exterior angle of a triangle is equal to the sum of the interior opposite angles.

$$\Rightarrow \angle A + \angle B = \text{external } \angle C$$

$$\angle A + \angle C = \text{external } \angle B$$

$$\angle C + \angle B = \text{external } \angle A$$

Adding above three, we get

$$2\angle A + 2\angle B + 2\angle C = \text{sum of the three external angles}$$

$$\Rightarrow 2(\angle A + \angle B + \angle C) = \text{sum of the three external angles}$$

We know that the sum of the angles of a triangle is 180° .

$$\Rightarrow 2 \times 180^\circ = \text{sum of the three external angles}$$

$$\Rightarrow \text{Sum of the three external angles} = 4 \times 90^\circ$$

3. Question

In a triangle ABC, $\angle B = 70^\circ$. Find $\angle A + \angle C$.

Answer

In the given $\triangle ABC$,

$$\angle B = 70^\circ$$

We know that the sum of all the angles of a triangle is 180° .

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle B + \angle C = 180^\circ - 70^\circ$$

$$\Rightarrow \angle B + \angle C = 110^\circ$$

4. Question

In a triangle ABC, $\angle A = 110^\circ$ and $AB = AC$. Find $\angle B$ and $\angle C$.

Answer

In the given $\triangle ABC$,

$$\angle A = 110^\circ \text{ and } AB = AC$$

$$\Rightarrow \angle B = \angle C \text{ \{angles opposite to equal sides are equal\}}$$

Also, sum of all the angles of a triangle is 180° .

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 2\angle B = 180^\circ - 110^\circ$$

$$\Rightarrow 2\angle B = 70^\circ$$

$$\Rightarrow \angle B = \angle C = 35^\circ$$

5. Question

If three angles of a triangle are in the ratio 2:3:5, determine three angles.

Answer

Let the given angles of a triangle be $2x$, $3x$ and $5x$.

We know that the sum of the angles of a triangle is 180° .

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 2x + 3x + 5x = 180^\circ$$

$$\Rightarrow 10x = 180^\circ$$

$$\Rightarrow x = 18^\circ$$

$$\text{So, } \angle A = 2 \times 18^\circ = 36^\circ$$

$$\angle B = 3 \times 18^\circ = 54^\circ$$

$$\angle C = 5 \times 18^\circ = 90^\circ$$

6. Question

The angles of a triangle are arranged in ascending order of magnitude. If the difference between two consecutive angles is 15° , find the three angles.

Answer

It is given that the angles of a triangle are arranged in ascending order of magnitude with the difference between consecutive angles = 15° .

So, the angles be $x - 15^\circ$, x and $x + 15^\circ$

We know that the sum of the angles of a triangle is 180° .

$$\Rightarrow x - 15^\circ + x + x + 15^\circ = 180^\circ$$

$$\Rightarrow 3x = 180^\circ$$

$$\Rightarrow x = 60^\circ$$

$$\text{So, } x - 15^\circ = 60^\circ - 15^\circ = 45^\circ$$

$$x + 15^\circ = 60^\circ + 15^\circ = 75^\circ$$

7. Question

The sum of two angles of a triangle is equal to its third angle. Determine the measure of the third angle.

Answer

It is given that the sum of two angles of a triangle is equal to its third angle.

In the given $\triangle ABC$,

$$\text{Let } \angle A + \angle B = \angle C \dots (1)$$

We know that the sum of the angles of a triangle is 180° .

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle C + \angle C = 180^\circ \text{ \{From (1)\}}$$

$$\Rightarrow 2\angle C = 180^\circ$$

$$\Rightarrow \angle C = 90^\circ$$

8. Question

In a triangle ABC, if $2\angle A = 3\angle B = 6\angle C$, determine $\angle A$, $\angle B$ and $\angle C$.

Answer

In a triangle ABC, it is given that

$$2\angle A = 3\angle B = 6\angle C$$

$$\Rightarrow 2\angle A = 6\angle C$$

$$\Rightarrow \angle A = 3\angle C$$

$$\text{Similarly, } 3\angle B = 6\angle C$$

$$\Rightarrow \angle B = 2\angle C$$

We know that the sum of the angles of a triangle is 180° .

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 3\angle C + 2\angle C + \angle C = 180^\circ$$

$$\Rightarrow 6\angle C = 180^\circ$$

$$\Rightarrow \angle C = 30^\circ$$

$$\text{So, } \angle A = 3 \times \angle C = 90^\circ$$

$$\angle B = 2 \times \angle C = 60^\circ$$

9. Question

The angles of a triangle are $x - 40^\circ$, $x - 20^\circ$ and $x + 15^\circ$. Find the value of x .

Answer

It is given that the angles of a triangle are $x - 40^\circ$, $x - 20^\circ$ and $x + 15^\circ$.

So, the angles be $x - 15^\circ$, x and $x + 15^\circ$

We know that the sum of the angles of a triangle is 180° .

$$\Rightarrow x - 40^\circ + x - 20^\circ + x + 15^\circ = 180^\circ$$

$$\Rightarrow 3x - 45^\circ = 180^\circ$$

$$\Rightarrow 3x = 180^\circ + 45^\circ$$

$$\Rightarrow x = 75^\circ$$

10. Question

In triangle ABC, $\angle A - \angle B = 15^\circ$ and $\angle B - \angle C = 30^\circ$, find $\angle A$, $\angle B$, and $\angle C$.

Answer

In a triangle ABC, it is given that

$$\angle A - \angle B = 15^\circ \text{ and } \angle B - \angle C = 30^\circ$$

$$\Rightarrow \angle A = \angle B + 15^\circ \text{ and } \angle C = \angle B - 30^\circ$$

We know that the sum of the angles of a triangle is 180° .

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle B + 15^\circ + \angle B + \angle B - 30^\circ = 180^\circ$$

$$\Rightarrow 3\angle B = 180^\circ + 15^\circ$$

$$\Rightarrow 3\angle B = 195^\circ$$

$$\Rightarrow \angle B = 65^\circ$$

$$\text{So, } \angle A = \angle B + 15^\circ = 65^\circ + 15^\circ = 80^\circ$$

$$\angle C = \angle B - 30^\circ = 65^\circ - 30^\circ = 35^\circ$$

11. Question

The sum of two angles of a triangle is 80° and their difference is 20° . Find the angles of the triangle.

Answer

In a triangle ABC, it is given that

$$\angle A - \angle B = 20^\circ \dots (1)$$

$$\text{and } \angle A + \angle B = 80^\circ \dots (2)$$

Add (1) and (2),

$$\Rightarrow 2\angle A = 100^\circ$$

$$\Rightarrow \angle A = 50^\circ$$

$$\text{So, } \angle B = 80^\circ - \angle A = 80^\circ - 50^\circ = 30^\circ$$

We know that the sum of the angles of a triangle is 180° .

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow 50^\circ + 30^\circ + \angle C = 180^\circ$$

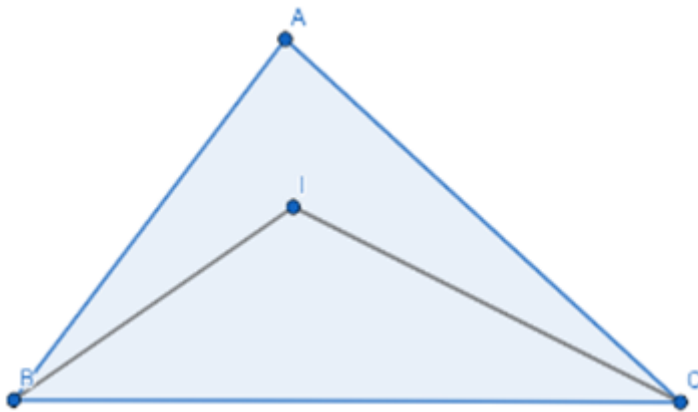
$$\Rightarrow \angle C = 180^\circ - 80^\circ$$

$$\Rightarrow \angle C = 100^\circ$$

12. Question

In a triangle ABC, $\angle B = 60^\circ$ and $\angle C = 80^\circ$. Suppose the bisector of $\angle B$ and $\angle C$ meet at I. Find $\angle BIC$.

Answer



It is given that in a triangle ABC, $\angle B = 60^\circ$, and $\angle C = 80^\circ$. IB and IC are the bisectors of $\angle B$ and $\angle C$ respectively.

$$\Rightarrow \angle IBC = 30^\circ \text{ and } \angle ICB = 40^\circ$$

We know that the sum of the angles of a triangle is 180° .

So, in $\triangle IBC$

$$\Rightarrow \angle BIC + \angle IBC + \angle ICB = 180^\circ$$

$$\Rightarrow \angle BIC = 180^\circ - 40^\circ - 30^\circ$$

$$\Rightarrow \angle BIC = 110^\circ$$

13. Question

In a triangle, each of the smaller angles is half the largest angle. Find the angles.

Answer

In a triangle ABC, let $\angle C$ be the larger angle and $\angle A$ and $\angle B$ be the smaller ones. It is given that

$$\angle A = \angle B = \frac{1}{2} \angle C$$

$$\Rightarrow 2\angle A = 2\angle B = \angle C$$

$$\Rightarrow \angle A = \angle B$$

$$\text{And } \angle C = 2\angle A$$

We know that the sum of the angles of a triangle is 180° .

$$\Rightarrow \angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + \angle A + 2\angle A = 180^\circ$$

$$\Rightarrow 4\angle A = 180^\circ$$

$$\Rightarrow \angle A = 45^\circ$$

$$\text{So, } \angle B = \angle A = 45^\circ$$

$$\angle C = 2 \times \angle A = 90^\circ$$

14. Question

In a triangle, each of the bigger angles is twice the third angle. Find the angles.

Answer

In a triangle ABC, let $\angle C$ be the third angle and $\angle A$ and $\angle B$ be the bigger ones. It is given that

$$\angle A = \angle B = 2\angle C$$

We know that the sum of the angles of a triangle is 180° .

$$\Rightarrow 2\angle C + 2\angle C + \angle C = 180^\circ$$

$$\Rightarrow 5\angle C = 180^\circ$$

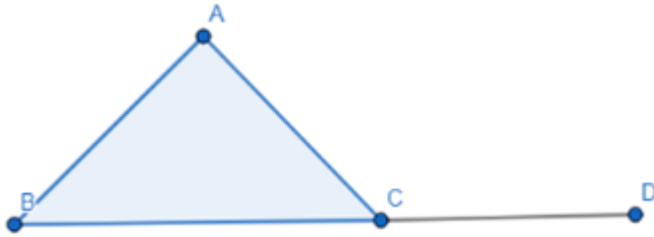
$$\Rightarrow \angle C = 36^\circ$$

$$\text{So, } \angle B = \angle A = 2 \times \angle C = 72^\circ$$

15. Question

In a triangle ABC, $\angle B = 50^\circ$ and $\angle A = 60^\circ$. Suppose BC is extended to D. Find $\angle ACD$.

Answer



It is given that in ΔABC ,

$$\angle B = 50^\circ \text{ and } \angle A = 60^\circ$$

Here, $\angle ACD$ is an external angle. We know that an exterior angle of a triangle is always larger than either of the interior opposite angles.

$$\Rightarrow \angle ACD = \angle A + \angle B = 50^\circ + 60^\circ = 110^\circ$$

16. Question

In an isosceles triangle, the vertex angle is twice the sum of the base angles. Find the angles of the triangle.

Answer

In a triangle ABC, let $\angle A$ be the vertex angle and $\angle C$ and $\angle B$ be the base angles. It is given that

$$\angle B = \angle C \text{ } \{\because \text{ it is an isosceles triangle}\}$$

$$\text{And } \angle A = 2(\angle B + \angle C)$$

$$\Rightarrow \angle A = 2 \times 2 \angle B$$

$$\angle A = 4 \angle B$$

We know that the sum of the angles of a triangle is 180° .

$$\Rightarrow 4\angle B + \angle B + \angle B = 180^\circ$$

$$\Rightarrow 6\angle B = 180^\circ$$

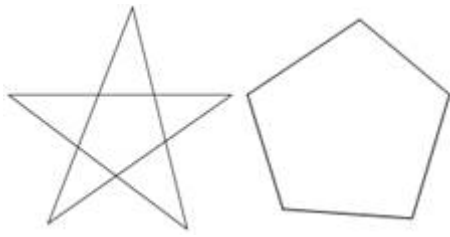
$$\Rightarrow \angle B = 30^\circ$$

$$\text{So, } \angle C = \angle B = 30^\circ$$

$$\text{And } \angle A = 4 \times \angle B = 120^\circ$$

17. Question

Find the sum of all the angles at the five vertices of the adjoining star.



Answer

The given star is of the pentagram shape. Each angle at vertices is of 36° . So, the sum of all the angles at the five vertices of the adjoining star = $5 \times 36^\circ = 180^\circ$