

## 2. Gravitation

### Kepler's for celestial bodies

#### Kepler's First Law (Law of Orbits)

- Every planet revolves around the sun in an elliptical orbit. The sun is situated at one foci of the ellipse.

#### Kepler's Second Law (Law of Areas)

- The line joining a planet to the sun sweeps out equal areas in equal intervals of time, i.e., the areal velocity of the planet around the sun is constant

#### Kepler's Third Law (Law of Periods)

- The square of the time period of revolution of a planet around the sun is directly proportional to the cube of the semi major axis of its elliptical orbit.

- **Universal law of gravitation**

$$|\vec{F}| = G \frac{m_1 m_2}{r^2}$$

$$\vec{F} = G \frac{m_1 m_2}{r^2} (-\hat{r}) = -G \frac{m_1 m_2}{r^2} (\hat{r})$$

- **For a point mass**

The point of attraction between a hollow spherical shell of uniform density and a point mass situated outside is just as if the entire mass of the shell is concentrated at the center of the shell.

The force of attraction due to a hollow spherical shell of uniform density, on a point mass inside it is zero.

- **Gravitation Constant,  $G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$**

- **Earth's gravitational acceleration,**

$$g = \frac{F}{m} = \frac{GM_E}{R_E^2}$$

- **Below/above Earth's surface**

$$g(h) = g \left( 1 - \frac{2h}{R_E} \right) \quad [h = \text{Height from Earth's surface}]$$

$$g(d) = g \left( 1 - \frac{d}{R_E} \right) \quad [d = \text{Depth from Earth's surface}]$$

- **Gravitational potential energy**

$$W_{12} = \int_{r_1}^{r_2} \frac{GMm}{r^2} dr$$

$$W(r) = -\frac{GM_E m}{r} + W_1 \quad (\text{Valid for } r > R_E)$$

- For any two bodies, 
$$V = -\frac{Gm_1 m_2}{r}$$

- **Escape velocity**

$$(V_i)_{\min} = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{2gR_E} \approx 11.2 \text{ km/s}$$

This is the escape velocity for earth.

- Total energy of a mass  $m$  moving with a velocity  $v$  in the vicinity of another mass  $M$ ,

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

- For circular path of radius  $a$ , 
$$E = \frac{GMm}{2a}$$

- **Earth's satellite**

- Speed and time period

$$\text{Speed, } V = \sqrt{\frac{GM_E}{(R_E + h)}}$$

$$\text{Time period, } T^2 = k(R_E + h)^3 \quad \left( k = \frac{4\pi^2}{GM_E} \right) \quad [\text{Kepler's 3rd law}]$$

- For satellite very close to earth,  $T = 2\pi \sqrt{\frac{R_E}{g}}$

- **Energy of orbiting satellite**

$$P.E. = -\frac{GmM_E}{(R_E + h)}$$

$$K.E. = \frac{GmM_E}{2(R_E + h)}$$

$$\text{Total Energy} = -\frac{GmM_E}{2(R_E + h)}$$

- **Geostationary satellite**
  - Height is about 36000 km.
  - $T = 24$  h
  - Used in television and other communication and broadcasting (INSAT group)
- **Polar satellites** are low-altitude satellites. They orbit the earth many times during a day. They are used in remote sensing, meteorology, and environment studies.
- **Weightlessness**
  - In space weightlessness is experienced because gravitational pull is used up in providing centripetal force.
  - In free fall weightlessness is experienced because there is no normal force acting opposite to the force of gravity.
- **Parking orbit** is a temporary orbit around the Earth where the satellite is temporarily parked before it is launched to its desired orbit.