Chapter : 2. FUNCTIONS

Exercise : 2A

Question: 1

Define a function

Solution:

Definition:A relation R from a set A to a set B is called a function if each element of A has a unique image in B.

It is denoted by the symbol f:A→B which reads ‘f’ is a function from A to B ‘f’ maps A to B.

Let f:A→B,then the set A is known as the domain of f & the set B is known as co - domain of f .The set of images of all the elements of A is known as the range of f.

Thus, Domain of f = {a|a ∈ A,(a,f(a)) ∈ f)

Range of f = {f(a) | a∈ A ,f(a) ∈ B }

Example: The domain of y = sin x is all values of x i.e. R , since there are no restrictions on the values for x. The range of y *is* betweeen −1 and 1. We could write this as $-1 \le y \le 1$.

Question: 2

Define each of th

Solution:

1)injective function

Definition: A function f: A → B is said to be a one - one function or injective mapping if different elements of A have different f images in B.

A function *f* is injective if and only if whenever *f(x) = f(y)*, *x = y*.

Example: f(x) = x + 9 from the set of real number R to R is an injective function. When x = 3,then :f(x) = 12,when f(y) = 8,the value of y can only be 3,so x = y.

(ii) surjective function

Definition: If the function f:A→B is such that each element in B (co - domain) is the ‘f’ image of atleast one element in A , then we say that f is a function of A ‘onto’ B .Thus f: A→B is surjective if, for all b∈B, there are some a∈A such that f(a) = b.

Example: The function f(x) = 2x from the set of natural numbers N to the set of non negative even numbers is a surjective function.

(iii) bijective function

Definition: A function f (from set A to B) is bijective if, for every y in B, there is exactly one x in A such that f(x) = y.Alternatively, f is bijective if it is a one - to - one correspondence between those sets, in other words, both injective and surjective.

Example: If f(x) = x^2,from the set of positive real numbers to positive real numbers is both injective and surjective.Thus it is a bijective function.

(iv)many - one function

Defintion : A function f: A→B is said to be a many one functions if two or more elements of A have the same f image in B.

trigonometric functions such as sinx are many - to - one since sinx = sin(2 π + x) = sin(4 π + x) and so one…

(v) into function

Definition: If f:A→B is such that there exists atleast one element in co - domain , which is not the image of any element in the domain , then f(x) is into.

Let f(x) = y = x – 1000

$\Rightarrow x = y + 1000 = g(y)$ (say)

Here g(y) is defined for each $y \in I$, but $g(y) \notin N$ for $y \le -1000$. Hence,f is into.

Question: 3

Give an example o

Solution:

(i) one - one but not onto

$f(x) = 6x$

For One - One

$f(x_1) = 6x_1$

$f(x_2) = 6x_2$

put $f(x_1) = f(x_2)$ we get

$6x_1 = 6x_2$

Hence, if $f(x_1) = f(x_2)$, $x_1 = x_2$

Function f is one - one

For Onto

$f(x) = 6x$

let $f(x) = y$,such that $y \in N$

$6x = y$

$\Rightarrow x = \frac{y}{6}$

If $y = 1$

$x = \frac{1}{6} = 0.166667$

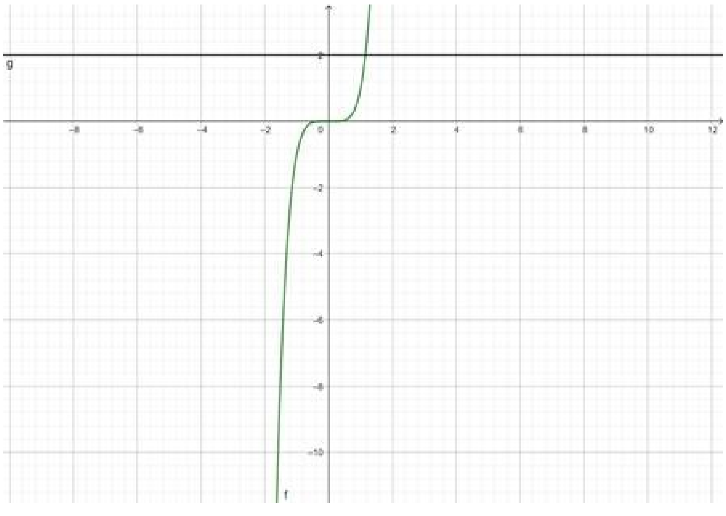
which is not possible as $x \in N$

Hence, f is not onto.

(ii) one - one and onto

$f(x) = x^5$

$\Rightarrow y = x^5$



Since the lines do not cut the curve in 2 equal valued points of y, therefore, the function f(x) is one - one.

The range of $f(x) = (-\infty, \infty) = R$(Codomain)

∴f(x) is onto

∴f(x) is one - one and onto.

(iii) neither one - one nor onto

$f(x) = x^2$

for one one:

$f(x_1) = (x_1)^2$

$f(x_2) = (x_2)^2$

$f(x_1) = f(x_2)$

$\Rightarrow (x_1)^2 = (x_2)^2$

$\Rightarrow x_1 = x_2$ or $x_1 = -x_2$

Since x_1 does not have a unique image it is not one - one

For onto

$f(x) = y$

such that $y \in R$

$x^2 = y$

$\Rightarrow x = \pm\sqrt{y}$

If y is negative under root of a negative number is not real

Hence,f(x) is not onto.

∴f(x) is neither onto nor one - one

(iv) onto but not one - one.

Consider a function $f:Z \to N$ such that $f(x) = |x|$.

Since the Z maps to every single element in N twice, this function is onto but not one - one.

Z - integers

N - natural numbers.

Question: 4

Let f : R

Solution:

i)f(2)

Since $f(x) = x^2 - 2$, when $x = 2$

$\therefore f(2) = (2)^2 - 2 = 4 - 2 = 2$

$\therefore f(2) = 2$

ii)f(4)

Since $f(x) = 3x - 1$, when $x = 4$

$\therefore f(4) = (3\times4) - 1 = 12 - 1 = 11$

$\therefore f(4) = 11$

iii)f(- 1)

Since $f(x) = x^2 - 2$, when $x = -1$

$\therefore f(-1) = (-1)^2 - 2 = 1 - 2 = -1$

$\therefore f(-1) = -1$

iv)f(- 3)

Since f(x) = 2x + 3 , when x = - 3

$\therefore f(-3) = 2\times(-3) + 3 = -6 + 3 = -3$

$\therefore f(-3) = -3$

Question: 5

Show that the fun

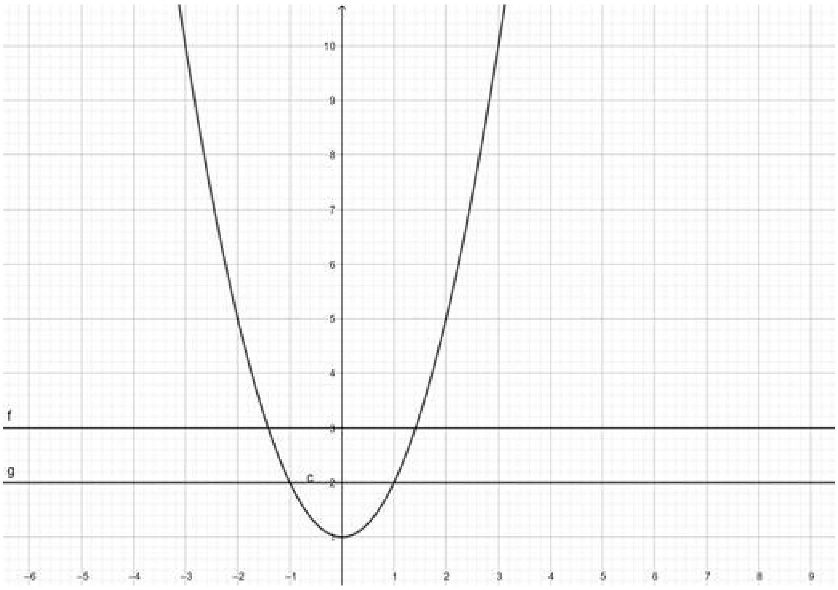
Solution:

To show: f: R → R : $f(x) = 1 + x^2$ is many - one into.

Proof:

$f(x) = 1 + x^2$

$\Rightarrow y = 1 + x^2$



Since the lines cut the curve in 2 equal valued points of y therefore the function f(x) is many one.

The range of f(x) = $[1,\infty) \neq R$(Codomain)

$\therefore$f(x) is not onto

$\Rightarrow$f(x) is into

Hence, showed that f: R → R : $f(x) = 1 + x^2$ is many - one into.

Question: 6

Show that the fun

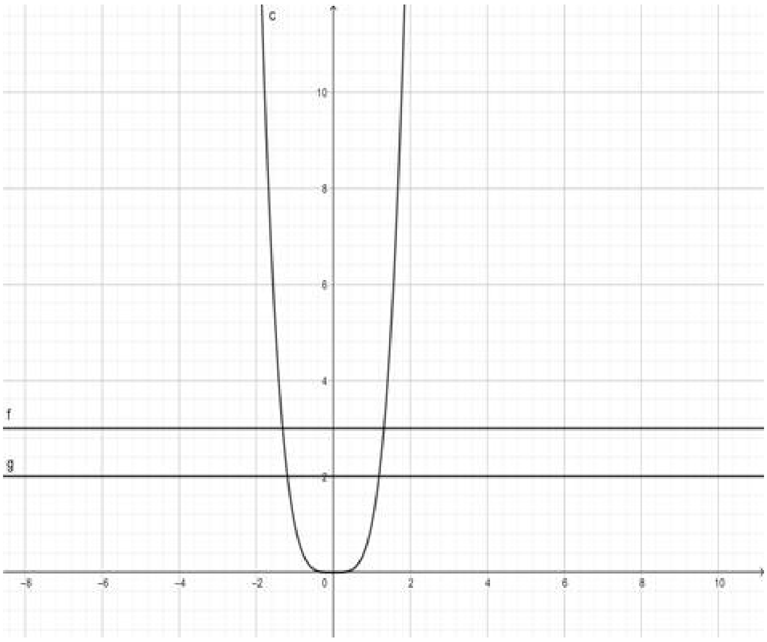
Solution:

To show: f: R → R : $f(x) = x^4$ is many - one into.

Proof:

$f(x) = x^4$

$\Rightarrow y = x^4$



Since the lines cut the curve in 2 equal valued points of y, therefore, the function f(x) is many ones.

The range of f(x) = $[0,\infty)\neq$R(Codomain)

$\therefore$f(x) is not onto

$\Rightarrow$f(x) is into

Hence, showed that f: R → R : f(x) = x^4 is many - one into.

Question: 7

Show that the fun

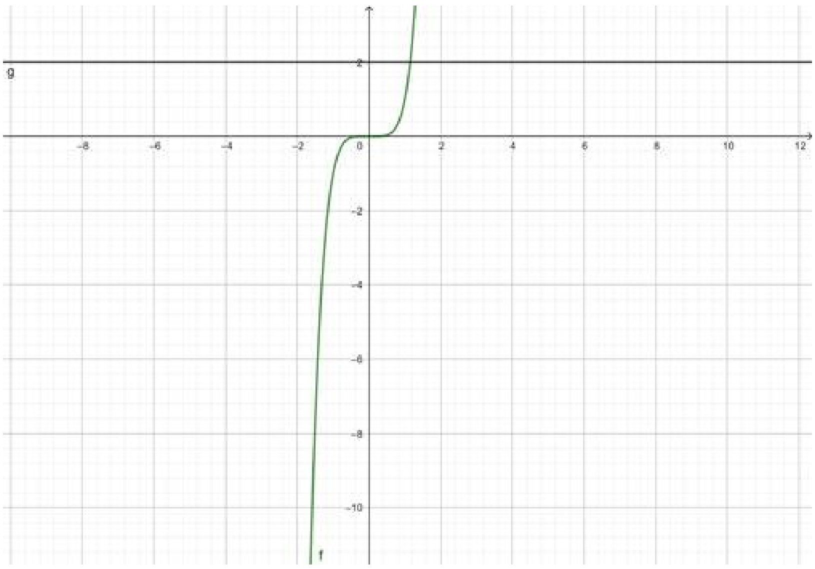
Solution:

To show: f: R → R : : f(x) = x^5 is one - one and onto.

Proof:

f(x) = x^5

$\Rightarrow$y = x^5



Since the lines do not cut the curve in 2 equal valued points of y, therefore, the function f(x) is one - one.

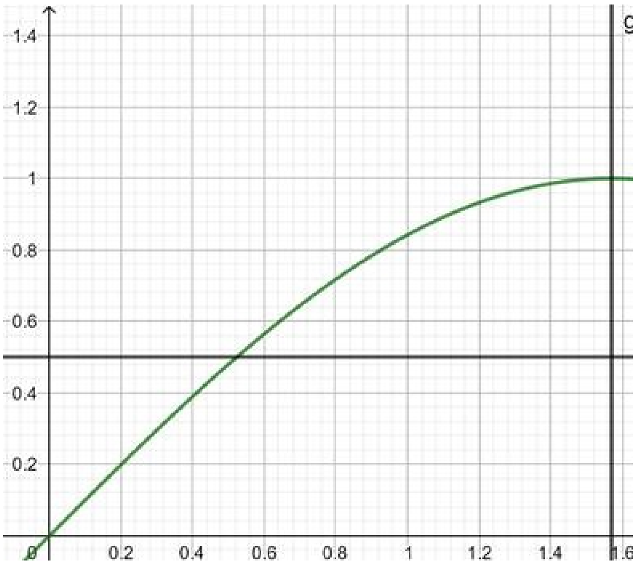
The range of f(x) = (- ∞,∞) = R(Codomain)

$\therefore$f(x) is onto

Hence, showed f: R → R : $f(x) = x^5$ is one - one and onto.

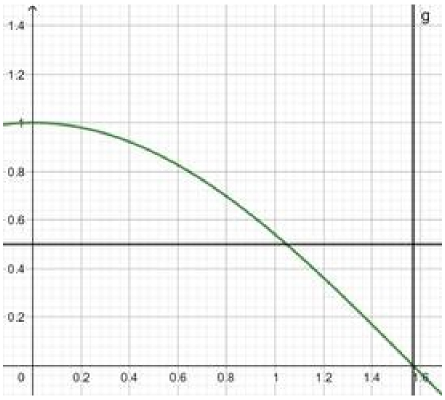
Question: 8

Let $f : \left[0, \frac{\pi}{2}\right] \rightarrow R : f(x) = \sin x$



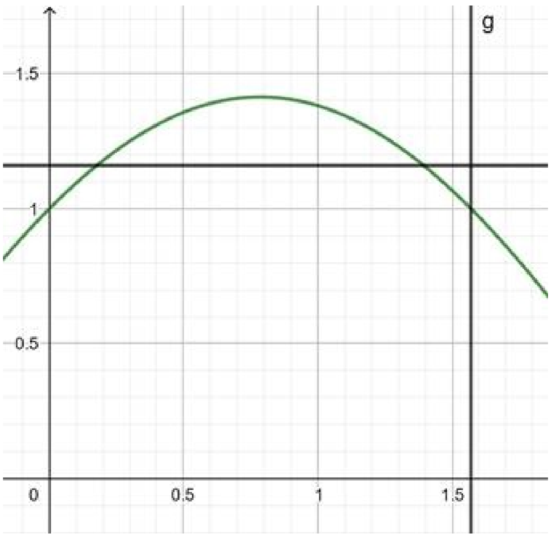
Here in this range, the lines do not cut the curve in 2 equal valued points of y, therefore, the function f(x) = sinx is one - one.

$g : \left[0, \frac{\pi}{2}\right] \rightarrow R : g(x) = \cos x.$



in this range, the lines do not cut the curve in 2 equal valued points of y, therefore, the function f(x) = cosx is also one - one.

$(f + g):[0,\frac{\pi}{2}] \rightarrow R = \sin x + \cos x$



in this range the lines cut the curve in 2 equal valued points of y, therefore, the function f(x) =

cosx + sinx is not one - one.

Hence,showed that each one of f and g is one - one but (f + g) is not one - one.

Question: 9

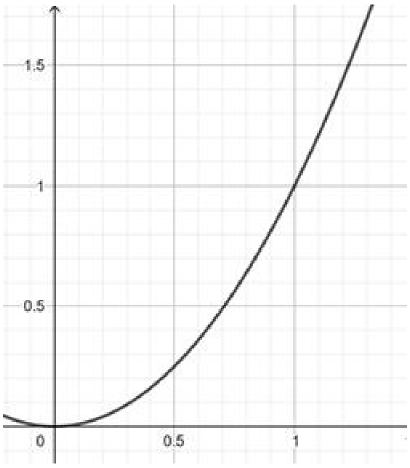
Show that the fun

Solution:

(i) f : N → N : f(x) = x^2 is one - one into.

f(x) = x^2

⇒y = x^2



Since the function f(x) is monotonically increasing from the domain N → N

∴f(x) is one –one

Range of f(x) = (0,∞)≠N(codomain)

∴f(x) is into

∴f : N → N : f(x) = x^2 is one - one into.

(ii) f : Z → Z : f(x) = x^2 is many - one into

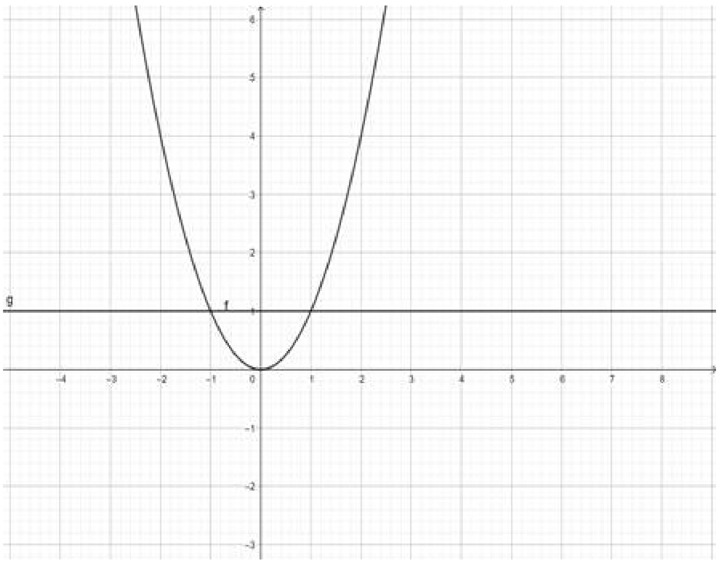
f(x) = x^2

⇒y = x^2

in this range the lines cut the curve in 2 equal valued points of y, therefore, the function f(x) = x^2 is many - one .

Range of f(x) = (0,∞)≠Z(codomain)

∴f(x) is into



∴ f : Z → Z : $f(x) = x^2$ is many - one into

Question: 10

Show that the fun

Solution:

(i) f : N → N : $f(x) = x^3$ is one - one into.

$f(x) = x^3$

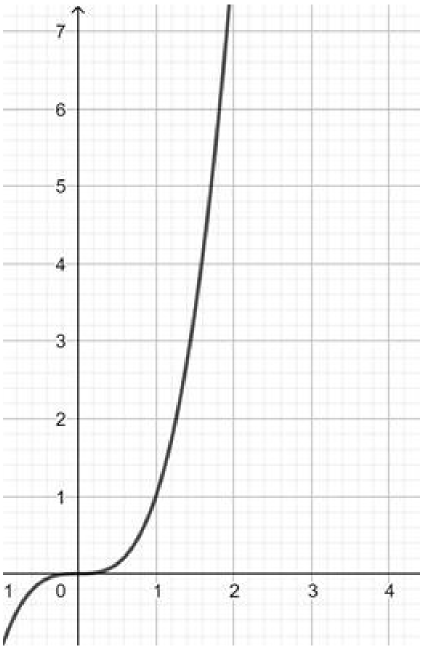
Since the function f(x) is monotonically increasing from the domain N → N

∴f(x) is one –one

Range of f(x) = (- ∞,∞)≠N(codomain)

∴f(x) is into

∴f : N → N : $f(x) = x^2$ is one - one into.



(ii) f : Z → Z : $f(x) = x^3$ is one - one into

$f(x) = x^3$

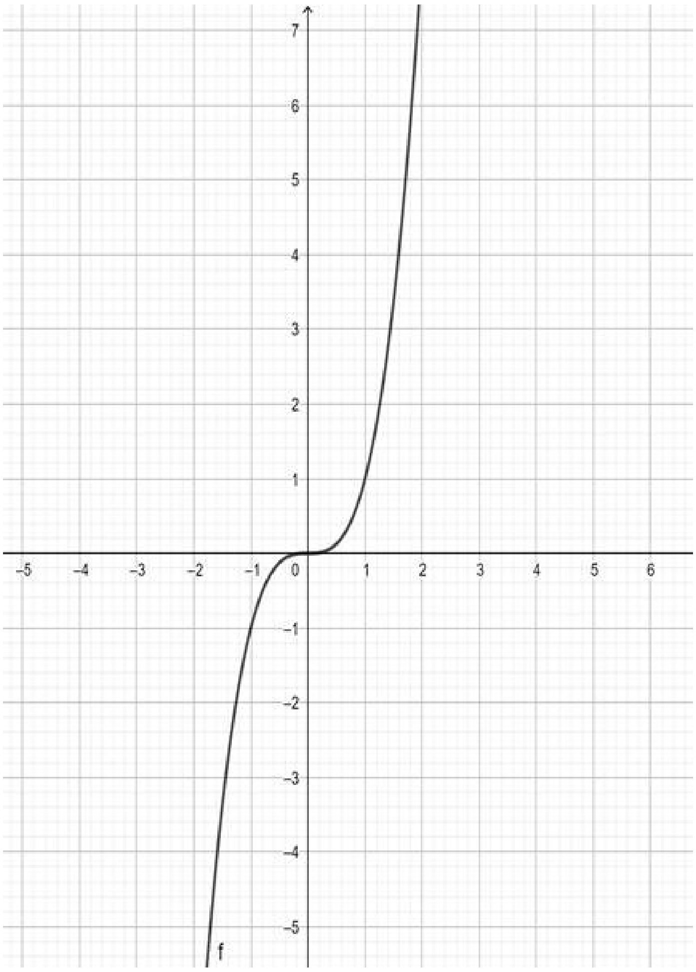
Since the function f(x) is monotonically increasing from the domain Z → Z

∴f(x) is one –one

Range of f(x) = (- ∞,∞)≠Z(codomain)

∴f(x) is into

∴ f : Z → Z : $f(x) = x^3$ is one - one into.



Question: 11

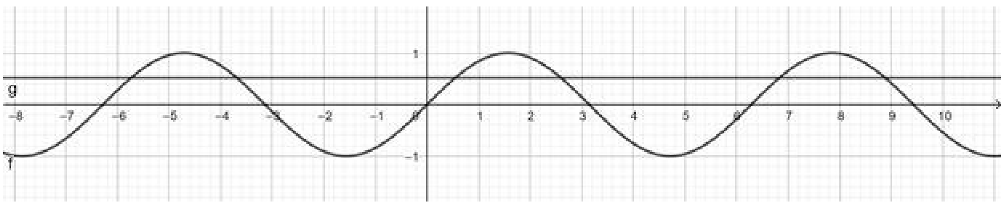
Show that the fun

Solution:

f(x) = sinx

y = sinx

Here in this range, the lines cut the curve in 2 equal valued points of y, therefore, the function f(x) = sinx is not one - one.



Range of f(x) = [- 1,1]≠R(codomain)

∴f(x) is not onto.

Hence, showed that the function f : R → R : f(x) = sin x is neither one - one nor onto.

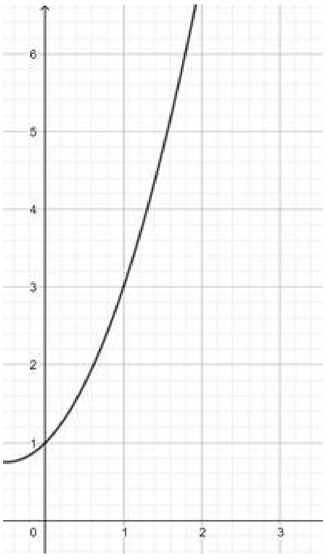
Question: 12

Prove that the fu

Solution:

In the given range of N f(x) is monotonically increasing.

∴f(n) = n^2 + n + 1 is one one.



But Range of f(n) = [0.75,∞)≠N(codomain)

Hence,f(n) is not onto.

Hence, proved that the function f : N → N : f(n) = (n^2 + n + 1) is one - one but not onto.

Question: 13

Show that the fun

Solution:

$$f(n)=\begin{cases}\frac{1}{2}(n-1), \text{ when n is odd}\\ -\frac{1}{2}n, \text{ when n is even}\end{cases}$$

f(1) = 0

f(2) = - 1

f(3) = 1

f(4) = - 2

f(5) = 2

f(6) = - 3

Since at no different values of x we get same value of y ∴f(n) is one –one

And range of f(n) = Z = Z(codomain)

∴ the function f: N → Z, defined by

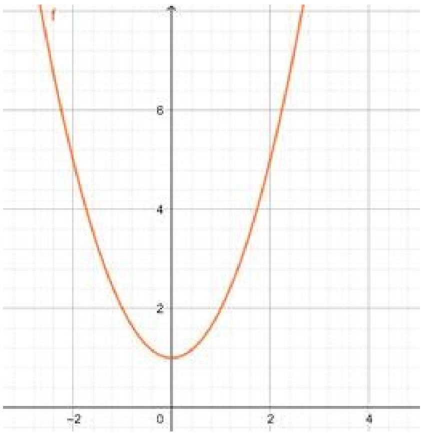
$$f(n)=\begin{cases}\frac{1}{2}(n-1), \text{ when n is odd}\\ -\frac{1}{2}n, \text{ when n is even}\end{cases}$$

is both one - one and onto.

Question: 14

Find the domain a

Solution:



Since the function f(x) can accept any values as per the given domain R, therefore, the domain of the function $f(x) = x^2 + 1$ is R.

The minimum value of f(x) = 1

⇒Range of f(x) = [- 1,∞]

i.e range (f) = {y ∈ R : y ≥ 1}

Ans: dom (f) = R and range (f) = {y ∈ R : y ≥ 1}

Question: 15

Which of the foll

Solution:

For a relation to be a function each element of 1st set should have different image in the second set(Range)

i) (i) f = {(- 1, 2), (1, 8), (2, 11), (3, 14)}

Here, each of the first set element has different image in second set.

∴f is a function whose domain = { - 1, 1, 2, 3} and range (f) = {2, 8, 11, 14}

(ii) g = {(1, 1), (1, - 1), (4, 2), (9, 3),(16, 4)}

Here, some of the first set element has same image in second set.

∴ g is not a function.

(iii) h = {(a, b), (b, c), (c, b), (d, c)}

Here, each of the first set element has different image in second set.

∴h is a function whose domain = {a, b, c, d} and range (h) = {b, c}

(range is the intersection set of the elements of the second set elements.)

Question: 16

Find the domain a

Solution:

For domain $(1 + x^2) \neq 0$

$\Rightarrow x^2 \neq -1$

⇒dom(f) = R

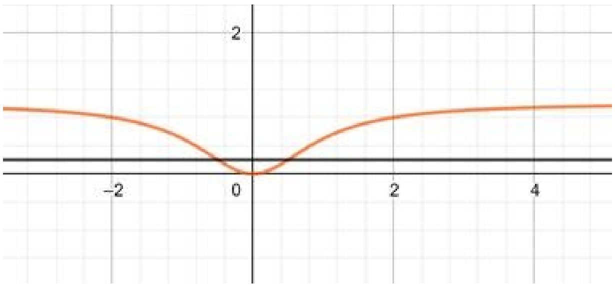
For the range of x:

$\Rightarrow y = \frac{x^2+1-1}{x^2+1} = 1 - \frac{1}{x^2+1}$

$y_{min} = 0$ (when x = 0)

$y_{max} = 1$ (when x = ∞)

∴range of f(x) = [0,1)



For many one the lines cut the curve in 2 equal valued points of y therefore the function f(x) $= \frac{x^2}{x^2+1}$ is many - one.

Ans:

dom(f) = R

range(f) = [0,1)

function f(x) $= \frac{x^2}{x^2+1}$ is many - one.

Question: 17

Show that the fun

Solution:

(i) $f\left(\frac{1}{2}\right)$

Here, x = 1/2,which is rational

∴f(1/2) = 1

(ii) $f\left(\sqrt{2}\right)$

Here, x = √2,which is irrational

∴f(√2) = - 1

(iii) $f(\pi)$

Here, x = ∏, which is irrational

$f(\pi)$ = - 1

(iv) $f\left(2+\sqrt{3}\right)$.

Here,x = 2 + √3, which is irrational

∴f(2 + √3) = - 1

Ans. (i) 1 (ii) - 1 (iii) - 1 (iv) - 1

Exercise : 2B

Question: 1

Let A = {1, 2, 3,

Solution:

(i) g o f

To find: g o f

Formula used: g o f = g(f(x))

Given: f = {(1, 4), (2, 1), (3, 3), (4, 2)} and g = {(1, 3), (2, 1), (3, 2), (4, 4)}

Solution: We have,

gof(1) = g(f(1)) = g(4) = 4

gof(2) = g(f(2)) = g(1) = 3

gof(3) = g(f(3)) = g(3) = 2

gof(4) = g(f(4)) = g(2) = 1

Ans) g o f = {(1, 4), (2, 3), (3, 2), (4, 1)}

(ii) f o g

To find: f o g

Formula used: f o g = f(g(x))

Given: f = {(1, 4), (2, 1), (3, 3), (4, 2)} and g = {(1, 3), (2, 1), (3, 2), (4, 4)}

Solution: We have,

fog(1) = f(g(1)) = f(3) = 3

fog(2) = f(g(2)) = f(1) = 4

fog(3) = f(g(3)) = f(2) = 1

fog(4) = f(g(4)) = f(4) = 2

Ans) f o g = {(1, 3), (2, 4), (3, 1), (4, 2)}

(iii) f o f

To find: f o f

Formula used: f o f = f(f(x))

Given: f = {(1, 4), (2, 1), (3, 3), (4, 2)}

Solution: We have,

fof(1) = f(f(1)) = f(4) = 2

fof(2) = f(f(2)) = f(1) = 4

fof(3) = f(f(3)) = f(3) = 3

fof(4) = f(f(4)) = f(2) = 1

Ans) f o f = {(1, 2), (2, 4), (3, 3), (4, 1)}

Question: 2

Let f : {3, 9, 12

Solution:

(i) g o f

To find: g o f

Formula used: g o f = g(f(x))

Given: f = {(3, 1), (9, 3), (12, 4)} and g = {(1, 3), (3, 3),(4, 9), (5, 9)}

Solution: We have,

gof(3) = g(f(3)) = g(1) = 3

gof(9) = g(f(9)) = g(3) = 3

$gof(12) = g(f(12)) = g(4) = 9$

Ans) $g \circ f = \{(3, 3), (9, 3), (12, 9)\}$

(ii) $f \circ g$

To find: $f \circ g$

Formula used: $f \circ g = f(g(x))$

Given: $f = \{(3, 1), (9, 3), (12, 4)\}$ and $g = \{(1, 3), (3, 3),(4, 9), (5, 9)\}$

Solution: We have,

$fog(1) = f(g(1)) = f(3) = 1$

$fog(3) = f(g(3)) = f(3) = 1$

$fog(4) = f(g(4)) = f(9) = 3$

$fog(5) = f(g(5)) = f(9) = 3$

Ans) $f \circ g = \{(1, 1), (3, 1), (4, 3), (5, 3)\}$

Question: 3

Let $f : R \rightarrow$

Solution:

To prove: $(g \circ f) \neq (f \circ g)$

Formula used: (i) $g \circ f = g(f(x))$

(ii) $f \circ g = f(g(x))$

Given: (i) $f : R \rightarrow R : f(x) = x^2$

(ii) $g : R \rightarrow R : g(x) = (x + 1)$

Proof: We have,

$g \circ f = g(f(x)) = g(x^2) = (x^2 + 1)$

$f \circ g = f(g(x)) = g(x+1) = [(x+1)^2 + 1] = x^2 + 2x + 2$

From the above two equation we can say that $(g \circ f) \neq (f \circ g)$

Hence Proved

Question: 4

Let $f : R \rightarrow$

Solution:

(i) $g \circ f$

To find: $g \circ f$

Formula used: $g \circ f = g(f(x))$

Given: (i) $f : R \rightarrow R : f(x) = (2x + 1)$

(ii) $g : R \rightarrow R : g(x) = (x^2 - 2)$

Solution: We have,

$g \circ f = g(f(x)) = g(2x + 1) = [(2x + 1)^2 – 2]$

$\Rightarrow 4x^2 + 4x + 1 – 2$

$\Rightarrow 4x^2 + 4x – 1$

Ans). $g \circ f (x) = 4x^2 + 4x – 1$

(ii) f o g

To find: f o g

Formula used: f o g = f(g(x))

Given: (i) f : R → R : f(x) = (2x + 1)

(ii) g : R → R : g(x) = (x^2 - 2)

Solution: We have,

f o g = f(g(x)) = f(x^2 - 2) = [2(x^2 - 2) + 1]

⇒ $2x^2$ - 4 + 1

⇒ $2x^2$ – 3

Ans). f o g (x) = $2x^2$ – 3

(iii) f o f

To find: f o f

Formula used: f o f = f(f(x))

Given: (i) f : R → R : f(x) = (2x + 1)

Solution: We have,

f o f = f(f(x)) = f(2x + 1) = [2(2x + 1) + 1]

⇒ 4x + 2 + 1

⇒ 4x + 3

Ans). f o f (x) = 4x+ 3

(iv) g o g

To find: g o g

Formula used: g o g = g(g(x))

Given: (i) g : R → R : g(x) = (x^2 - 2)

Solution: We have,

g o g = g(g(x)) = g(x^2 - 2) = [$(x^2 - 2)^2$ – 2]

⇒ x^4 -$4x^2$ + 4 - 2

⇒ x^4 -$4x^2$ + 2

Ans). g o g (x) = x^4 -$4x^2$ + 2

Question: 5

Let f : R →

Solution:

(i) g o f

To find: g o f

Formula used: g o f = g(f(x))

Given: (i) f : R → R : f(x) = (x^2 + 3x + 1)

(ii) g: R → R : g(x) = (2x - 3)

Solution: We have,

g o f = g(f(x)) = g(x^2 + 3x + 1) = [2(x^2 + 3x + 1) – 3]

$\Rightarrow 2x^2 + 6x + 2 – 3$

$\Rightarrow 2x^2 + 6x – 1$

Ans). g o f (x) = $2x^2 + 6x – 1$

(ii) f o g

To find: f o g

Formula used: f o g = f(g(x))

Given: (i) f : R → R : f(x) = $(x^2 + 3x + 1)$

(ii) g: R → R : g(x) = (2x - 3)

Solution: We have,

f o g = f(g(x)) = f(2x - 3) = $[(2x - 3)^2 + 3(2x – 3) + 1]$

$\Rightarrow 4x^2 - 12x + 9 + 6x – 9 + 1$

$\Rightarrow 4x^2 - 6x + 1$

Ans). f o g (x) = $4x^2 - 6x + 1$

(iii) g o g

To find: g o g

Formula used: g o g = g(g(x))

Given: (i) g: R → R : g(x) = (2x - 3)

Solution: We have,

g o g = g(g(x)) = g(2x - 3) = [2(2x – 3) - 3]

$\Rightarrow 4x – 6 - 3$

$\Rightarrow 4x - 9$

Ans). g o g (x) = 4x – 9

Question: 6

Let f : R →

Solution:

To prove: f o f = f

Formula used: f o f = f(f(x))

Given: (i) f : R → R : f(x) = |x|

Solution: We have,

f o f = f(f(x)) = f(|x|) = $||x||$ = |x| = f(x)

Clearly f o f = f.

Hence Proved.

Question: 7

Let f : R →

Solution:

|

To find: formula for h o (g o f)

To prove: Show that $[h\ o\ (g\ o\ f)]\sqrt{\frac{\pi}{4}} = 0$

Formula used: f o f = f(f(x))

Given: (i) f : R → R : f(x) = x^2

(ii) g : R → R : g(x) = tan x

(iii) h : R → R : h(x) = log x

Solution: We have,

h o (g o f) = h o g(f(x)) = h o g(x^2)

= h(g(x^2)) = h (tan x^2)

= log (tan x^2)

h o (g o f) = log (tan x^2)

For, $[h\ o\ (g\ o\ f)]\sqrt{\frac{\pi}{4}}$

$= \log\left[\tan\left(\sqrt{\frac{\pi}{4}}\right)^2\right]$

$= \log\left[\tan\frac{\pi}{4}\right]$

$= \log 1$

= 0

Hence Proved.

Question: 8

Let f : R →

Solution:

To prove: (f o g) = I_R = (g o f).

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f : R → R : f(x) = (2x - 3)

(ii) $g : R \rightarrow R : g(x) = \frac{1}{2}(x+3)$

Solution: We have,

f o g = f(g(x))

$= f\left(\frac{1}{2}(x+3)\right)$

$= \left[2\left(\frac{1}{2}(x+3)\right) - 3\right]$

= x + 3 – 3

= x

= I_R

g o f = g(f(x))

$= g(2x - 3)$

$= \frac{1}{2}(2x-3+3)$

$= \frac{1}{2}(2x)$

= x

= I_R

Clearly we can see that (f o g) = I_R = (g o f) = x

Hence Proved.

Question: 9

Let f : Z →

Solution:

To find: g : Z → Z : g o f = I_Z

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) g : Z → Z : g o f = I_Z

Solution: We have,

f(x) = 2x

Let f(x) = y

⇒ y = 2x

$\Rightarrow x = \frac{y}{2}$

$\Rightarrow x = \frac{y}{2}$

Let $g(y) = \frac{y}{2}$

Where g: Z → Z

For g o f,

⇒ g(f(x))

⇒ g(2x)

$\Rightarrow \frac{2x}{2}$

⇒ x = I_Z

Clearly we can see that (g o f) = x = I_Z

The required function is $g(x) = \frac{x}{2}$

Question: 10

Let f : N →

Solution:

To show: h o (g o f) = (h o g) o f

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f : N → N : f(x) = 2x

(ii) g : N → N : g(y) = 3y + 4

(iii) h : N → N : h(z) = sin z

Solution: We have,

LHS = h o (g o f)

⇒ h o (g(f(x))

⇒ h(g(2x))

⇒ h(3(2x) + 4)

⇒ h(6x +4)

⇒ sin(6x + 4)

RHS = (h o g) o f

⇒ (h(g(x))) o f

⇒ (h(3x + 4)) o f

⇒ sin(3x+4) o f

Now let sin(3x+4) be a function u

RHS = u o f

⇒ u(f(x))

⇒ u(2x)

⇒ sin(3(2x) + 4)

⇒ sin(6x + 4) = LHS

Hence Proved.

Question: 11

If f be a greates

Solution:

To find: $(fog)\left(\frac{-3}{2}\right)+(gof)\left(\frac{4}{3}\right)$

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f is a greatest integer function

(ii) g is an absolute value function

f(x) = [x] (greatest integer function)

g(x) = $|x|$ (absolute value function)

$f\left(\frac{4}{3}\right)= \left[\frac{4}{3}\right] = 1$ ... (i)

$g\left(\frac{-3}{2}\right)= \left|\frac{-3}{2}\right| = 1.5$ ... (ii)

Now, for $(fog)\left(\frac{-3}{2}\right)+(gof)\left(\frac{4}{3}\right)$

$\Rightarrow f\left(g\left(\frac{-3}{2}\right)\right)+g\left(f\left(\frac{4}{3}\right)\right)$

Substituting values from (i) and (ii)

$\Rightarrow f(1.5) + g(1)$

$\Rightarrow [1.5] + |1|$

$\Rightarrow 1 + 1 = 2$

Ans) 2

Question: 12

Let f : R →

Solution:

To find: f o g, g o f ,(f o g) (2) and (g o f) (-3)

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f : R → R : f(x) = $x^2 + 2$

(ii) $g : R \rightarrow R : g(x) = \frac{x}{x-1}, x \neq 1$

f o g = f(g(x))

$\Rightarrow f\left(\frac{x}{x-1}\right)$

$\Rightarrow \left(\frac{x}{x-1}\right)^2 + 2$

Ans) $\Rightarrow \frac{(x)^2}{(x-1)^2} + 2$

f o g (2) = $\frac{(2)^2}{(2-1)^2} + 2$

$= \frac{4}{1} + 2$

Ans) = 6

g o f = g(f(x))

$\Rightarrow g(x^2+2)$

$\Rightarrow \frac{x^2+2}{x^2+2-1}$

Ans) $\Rightarrow \frac{x^2+2}{x^2+1}$

(g o f) (-3) = $\frac{-3^2+2}{-3^2+1}$

$= \frac{9+2}{9+1}$

Ans) = $\frac{11}{10}$

Exercise : 2C

Question: 1

Prove that the fu

Solution:

To prove: function is one-one and onto

Given: f: R → R : f(x)= 2x

We have,

f(x) = 2x

For, $f(x_1) = f(x_2)$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

When, $f(x_1) = f(x_2)$ then $x_1 = x_2$

∴ f(x) is one-one

f(x) = 2x

Let f(x) = y such that $y \in R$

⇒ y = 2x

$\Rightarrow x = \frac{y}{2}$

Since $y \in R$,

$\Rightarrow \frac{y}{2} \in R$

⇒ x will also be a real number, which means that every value of y is associated with some x

∴ f(x) is onto

Hence Proved

Question: 2

Prove that the fu

Solution:

To prove: function is one-one and into

Given: f: N → N : f(x)= 3x

We have,

f(x) = 3x

For, $f(x_1) = f(x_2)$

$\Rightarrow 3x_1 = 3x_2$

$\Rightarrow x_1 = x_2$

When, $f(x_1) = f(x_2)$ then $x_1 = x_2$

∴ f(x) is one-one

f(x) = 3x

Let f(x) = y such that $y \in N$

⇒ y = 3x

$\Rightarrow x = \frac{y}{3}$

If y = 1,

$\Rightarrow x = \frac{1}{3}$

But as per question $x \in N$, hence x can not be $\frac{1}{3}$

Hence f(x) is into

Hence Proved

Question: 3

Show that the fun

Solution:

To prove: function is neither one-one nor onto

Given: $f : R \to R : f(x) = x^2$

Solution: We have,

$f(x) = x^2$

For, $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$ or, $x_1 = -x_2$

Since x_1 doesn't has unique image

$\therefore$ f(x) is not one-one

$f(x) = x^2$

Let f(x) = y such that $y \in R$

$\Rightarrow y = x^2$

$\Rightarrow x = \sqrt{y}$

If y = -1, as $y \in R$

Then x will be undefined as we cannot place the negative value under the square root

Hence f(x) is not onto

Hence Proved

Question: 4

Show that the fun

Solution:

To prove: function is one-one and into

Given: $f : N \to N : f(x) = x^2$

Solution: We have,

$f(x) = x^2$

For, $f(x_1) = f(x_2)$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1 = x_2$

Here we can't consider $x_1 = -x_2$ as $x \in N$, we can't have negative values

$\therefore$ f(x) is one-one

$f(x) = x^2$

Let f(x) = y such that $y \in N$

$\Rightarrow y = x^2$

$\Rightarrow x = \sqrt{y}$

If y = 2, as $y \in N$

Then we will get the irrational value of x, but $x \in N$

Hence f(x) is not into

Hence Proved

Question: 5

Show that the fun

Solution:

To prove: function is neither one-one nor onto

Given: $f : R \rightarrow R : f(x) = x^4$

We have,

$f(x) = x^4$

For, $f(x_1) = f(x_2)$

$\Rightarrow x_1^4 = x_2^4$

$\Rightarrow (x_1^4 - x_2^4) = 0$

$\Rightarrow (x_1^2 - x_2^2)(x_1^2 + x_2^2) = 0$

$\Rightarrow (x_1 - x_2)(x_1 + x_2)(x_1^2 + x_2^2) = 0$

$\Rightarrow x_1 = x_2$ or, $x_1 = -x_2$ or, $x_1^2 = -x_2^2$

We are getting more than one value of x_1 (no unique image)

$\therefore$ f(x) is not one-one

$f(x) = x^4$

Let f(x) = y such that $y \in R$

$\Rightarrow y = x^4$

$\Rightarrow x = \sqrt[4]{y}$

If y = -2, as $y \in R$

Then x will be undefined as we can't place the negative value under the square root

Hence f(x) is not onto

Hence Proved

Question: 6

Show that the fun

Solution:

To prove: function is one-one and into

Given: $f : Z \to Z : f(x) = x^3$

Solution: We have,

$f(x) = x^3$

For, $f(x_1) = f(x_2)$

$\Rightarrow x_1^3 = x_2^3$

$\Rightarrow x_1 = x_2$

When, $f(x_1) = f(x_2)$ then $x_1 = x_2$

$\therefore$ f(x) is one-one

$f(x) = x^3$

Let f(x) = y such that $y \in Z$

$\Rightarrow y = x^3$

$\Rightarrow x = \sqrt[3]{y}$

If y = 2, as $y \in Z$

Then we will get an irrational value of x, but $x \in Z$

Hence f(x) is into

Hence Proved

Question: 7

Let R_0

Solution:

To prove: function is one-one and onto

Given: $f : R_0 \to R_0 : f(x) = \frac{1}{x}$

We have,

$f(x) = \frac{1}{x}$

For, $f(x_1) = f(x_2)$

$\Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_1 = x_2$

When, $f(x_1) = f(x_2)$ then $x_1 = x_2$

$\therefore$ f(x) is one-one

$f(x) = \frac{1}{x}$

Let f(x) = y such that $y \in R_0$

$\Rightarrow y = \frac{1}{x}$

$\Rightarrow x = \frac{1}{y}$

Since $y \in R_0$,

$\Rightarrow \frac{1}{y} \in R_0$

$\Rightarrow$ x will also $\in R_0$, which means that every value of y is associated with some x

$\therefore$ f(x) is onto

Hence Proved

Question: 8

Show that the fun

Solution:

To prove: function is many-one into

Given: $f : R \to R : f(x) = 1 + x^2$

We have,

$f(x) = 1 + x^2$

For, $f(x_1) = f(x_2)$

$\Rightarrow 1 + x_1^2 = 1 + x_2^2$

$\Rightarrow x_1^2 = x_2^2$

$\Rightarrow x_1^2 - x_2^2 = 0$

$\Rightarrow (x_1 – x_2) (x_1 + x_2) = 0$

$\Rightarrow x_1 = x_2$ or, $x_1 = –x_2$

Clearly x_1 has more than one image

$\therefore$ f(x) is many-one

$f(x) = 1 + x^2$

Let f(x) = y such that $y \in R$

$\Rightarrow y = 1 + x^2$

$\Rightarrow x^2 = y – 1$

$\Rightarrow x = \sqrt{y-1}$

If y = 3, as $y \in R$

Then x will be undefined as we can’t place the negative value under the square root

Hence f(x) is into

Hence Proved

Question: 9

Let

Solution:

To find: f^{-1}

Given: $f: R \to R : f(x) = \frac{2x-7}{4}$

We have,

$f(x) = \frac{2x-7}{4}$

Let f(x) = y such that $y \in R$

$\Rightarrow y = \frac{2x-7}{4}$

$\Rightarrow 4y = 2x - 7$

$\Rightarrow 4y + 7 = 2x$

$\Rightarrow x = \frac{4y+7}{2}$

$\Rightarrow f^{-1} = \frac{4y+7}{2}$

Ans) $f^{-1}(y) = \frac{4y+7}{2}$ for all $y \in R$

Question: 10

Let f : R →

Solution:

To find: f^{-1}

Given: f : R → R : f(x) = 10x + 3

We have,

f(x) = 10x + 3

Let f(x) = y such that $y \in R$

$\Rightarrow y = 10x + 3$

$\Rightarrow y - 3 = 10x$

$\Rightarrow x = \frac{y - 3}{10}$

$\Rightarrow f^{-1} = \frac{y - 3}{10}$

Ans) $f^{-1}(y) = \frac{y - 3}{10}$ for all $y \in R$

Question: 11

Solution:

To prove: function is many-one and into

Given: $f : R \to R : f(x) = \begin{cases} 1, \text{ if } x \text{ is rational} \\ -1, \text{ if } x \text{ is irrational} \end{cases}$

We have,

f(x) = 1 when x is rational

It means that all rational numbers will have same image i.e. 1

⇒ f(2) = 1 = f (3) , As 2 and 3 are rational numbers

Therefore f(x) is many-one

The range of function is [{-1},{1}] but codomain is set of real numbers.

Therefore f(x) is into

Question: 12

Let f (x) = x + 7

Solution:

To find: (f o g) (7)

Formula used: f o g = f(g(x))

Given: (i) f (x) = x + 7

(ii) g (x) = x – 7

We have,

f o g = f(g(x)) = f(x – 7) = [(x – 7) + 7]

⇒ x

(f o g) (x) = x

(f o g) (7) = 7

Ans). (f o g) (7) = 7

Question: 13

Let f : R →

Solution:

To prove: g o f ≠ f o g

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) f : R → R : f(x) = x^2

(ii) g : R → R :g(x) = (x + 1)

We have,

f o g = f(g(x)) = f(x + 7)

f o g = $(x + 7)^2 = x^2 + 14x + 49$

g o f = g(f(x)) = $g(x^2)$

g o f = $(x^2 + 1) = x^2 + 1$

Clearly g o f ≠ f o g

Hence Proved

Question: 14

Let f : R →

Solution:

To find: f o f

Formula used: (i) f o f = f(f(x))

Given: (i) f : R → R : f(x) = $(3 - x^3)^{1/3}$

We have,

f o f = f(f(x)) = $f((3 - x^3)^{1/3})$

f o f = $[3 - \{(3 - x^3)^{1/3}\}^3]^{1/3}$

= $[3 - (3 - x^3)]^{1/3}$

$= [3 - 3 + x^3]^{1/3}$

$= [x^3]^{1/3}$

= x

Ans) f o f (x) = x

Question: 15

Let f : R →

Solution:

To find: f{f(x)}

Formula used: (i) f o f = f(f(x))

Given: (i) f : R → R : f(x) = 3x + 2

We have,

f{f(x)} = f(f(x)) = f(3x + 2)

f o f =3(3x + 2) + 2

= 9x + 6 + 2

= 9x + 8

Ans) f{f(x)} = 9x + 8

Question: 16

Let f = {(1, 2),

Solution:

To find: g o f

Formula used: g o f = g(f(x))

Given: (i) f = {(1, 2), (3, 5), (4, 1)}

(ii) g = {(1, 3), (2, 3), (5, 1)}

We have,

gof(1) = g(f(1)) = g(2) = 3

gof(3) = g(f(3)) = g(5) = 1

gof(4) = g(f(4)) = g(1) = 3

Ans) g o f = {(1, 3), (3, 1), (4, 3)}

Question: 17

Let A = {1, 2, 3,

Solution:

To find: f o f

Formula used: f o f = f(f(x))

Given: (i) f = {(1, 4), (2, 1) (3, 3), (4, 2)}

We have,

fof(1) = f(f(1)) = f(4) = 2

fof(2) = f(f(2)) = f(1) = 4

fof(3) = f(f(3)) = f(3) = 3

fof(4) = f(f(4)) = f(2) = 1

Ans) f o f = {(1, 2), (2, 4), (3, 3), (4, 1)}

Question: 18

Let f(x) = 8x

Solution:

To find: g o f and f o g

Formula used: (i) f o g = f(g(x))

(ii) g o f = g(f(x))

Given: (i) $f(x) = 8x^3$

(ii) $g(x) = x^{1/3}$

We have,

$g \text{ o } f = g(f(x)) = g(8x^3)$

$g \text{ o } f = \left(8x^3\right)^{\frac{1}{3}} = 2x$

$f \text{ o } g = f(g(x)) = f(x^{1/3})$

$f \text{ o } g = 8\left(x^{\frac{1}{3}}\right)^3 = 8x$

Ans) g o f = 2x and f o g = 8x

Question: 19

Let f : R →

Solution:

To find: the function g : R → R : g o f = f o g = I_g

Formula used: (i) g o f = g(f(x))

(ii) f o g = f(g(x))

Given: f : R → R : f(x) = 10x + 7

We have,

f(x) = 10x + 7

Let f(x) = y

⇒ y = 10x + 7

⇒ y – 7 = 10x

$\Rightarrow x = \frac{y - 7}{10}$

Let $g(y) = \frac{y - 7}{10}$ where g: R → R

$g \text{ o } f = g(f(x)) = g(10x + 7) = \frac{(10x + 7) - 7}{10}$

= x

= I_g

$f \text{ o } g = f(g(x)) = f\left(\frac{x - 7}{10}\right)$

$= 10\left(\frac{x - 7}{10}\right) + 7$

= x – 7 + 7

= x

Clearly g o f = f o g = I_gAns). $g(x) = \frac{x - 7}{10}$

Question: 20

Let A = {1, 2, 3)

Solution:

To state: Whether f is one-one

Given: f = {(1, 4), (2,5), (3, 6)}

Here the function is defined from A → B

For a function to be one-one if the images of distinct elements of A under f are distinct

i.e. 1,2 and 3 must have a distinct image.

From f = {(1, 4), (2, 5), (3, 6)} we can see that 1, 2 and 3 have distinct image.

Therefore f is one-one

Ans) f is one-one

Exercise : 2D

Question: 1

Let A = {2, 3, 4,

Solution:

To Show: that f is invertible

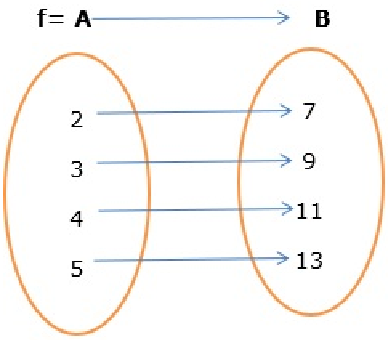
To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for x_l, $x_2 \in$ A & $f(x_1)$, $f(x_2) \in$ B, f(x1) = f(x2) ↔ $x_1 = x_2$ or $x_1 \neq x_2$ ↔ f(x1) $\neq$ f(x2)

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.



As we see that inthe above figure (2 is mapped with 7), (3 is mapped with 9), (4 is mapped with 11),

(5 is mapped with 13)

So it is one-one functions.

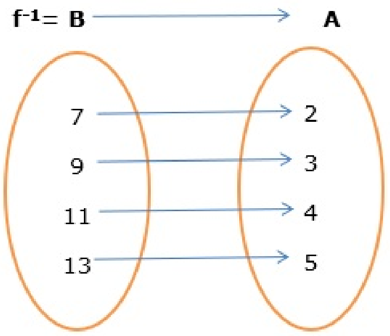
Now elements of B are known as co-domain. Also, a range of a function is also the elements of B(by definition)

So it is onto functions.

Hence Proved that f is invertible.

Now, We know that if f : A → B then f^{-1} : B → A (if it is invertible)

So,



So f^{-1} = {(7, 2), (9, 3), (11, 4), (13, 5)}

Question: 2

Show that the fun

Solution:

To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for x_1, $x_2 \in$ A & $f(x_1)$, $f(x_2) \in$ B, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let x_1, $x_2 \in$ R and f(x) = 2x+3.So $f(x_1) = f(x_2) \rightarrow 2x_1+3 = 2x_2+3 \rightarrow x_1=x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is R.

Let y = f(x) = 2x+3 , So $x = \frac{y-3}{2}$ [Range of f(x) = Domain of y]

So Domain of y is R(real no.) = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = R

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{y-3}{2}$

Question: 3

Let f : Q

Solution:

To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)] one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for x_1, $x_2 \in$ A & $f(x_1)$, $f(x_2) \in$ B, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let x_1, $x_2 \in$ Q and f(x) = 3x-4.So $f(x_1) = f(x_2) \rightarrow 3x_1 - 4 = 3x_2 - 4 \rightarrow x_1=x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is Q.

Let y = f(x) = 3x- 4 , So x = $\frac{y+4}{3}$ [Range of f(x) = Domain of y]

So Domain of y is Q = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = Q

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{y+4}{3}$

Question: 4

Let To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for x_1, $x_2 \in$ A & $f(x_1)$, $f(x_2) \in$ B, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let x_1, $x_2 \in$ Q and f(x) = $\frac{(3x+1)}{2}$.So $f(x_1) = f(x_2) \rightarrow \frac{(3x_1+1)}{2} = \frac{(3x_2+1)}{2} \rightarrow x_1=x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is R.

Let y = f(x) =$\frac{(3x+1)}{2}$, So x = $\frac{2y-1}{3}$ [Range of f(x) = Domain of y]

So Domain of y is R = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = R

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{2y-1}{3}$

Question: 5

If To Show: that f o f (x) = x

Finding (f o f) (x) = $\frac{(4\frac{(4x+3)}{(6x-4)}+3)}{(6\frac{(4x+3)}{(6x-4)}-4)} = \frac{4(4x+3)+3(6x-4)}{6(4x+3)-4(6x-4)} = \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{35x}{35} = x$.

Question: 6

Show that the fun

Solution:

To Show: that f is one-one and onto

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for x_1, $x_2 \in$ A & $f(x_1)$, $f(x_2) \in$ B, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let x_1, $x_2 \in$ Q and f(x) $=\frac{(4x+3)}{(6x-4)}$ .So $f(x_1) = f(x_2) \rightarrow \frac{(4x_1+3)}{(6x_1-4)} = \frac{(4x_2+3)}{(6x_2-4)} \rightarrow$ on solving we get $x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is R except 3x-2=0.

Let y = f(x) = $\frac{(4x+3)}{(6x-4)}$So x = $\frac{4y+3}{6y-4}$ [Range of f(x) = Domain of y]

So Domain of y is R (except 3x-2=0) = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = R except 3x-2=0

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{4y+3}{6y-4}$.

Question: 7

Show that the fun

Solution:

To Show: that f is one-one and onto

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for x_1, $x_2 \in$ A & $f(x_1)$, $f(x_2) \in$ B, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let x_1, $x_2 \in$ Q and f(x) $=\frac{4x}{(3x+4)}$ .So $f(x_1) = f(x_2) \rightarrow \frac{(4x_1)}{(3x_1+4)} = \frac{(4x_2)}{(3x_2+4)} \rightarrow$ on solving we get $x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is R except 3x+4=0.

Let y = f(x) = $\frac{(4x)}{(3x+4)}$ So x = $\frac{4y}{4-3y}$ [Range of f(x) = Domain of y]

So Domain of y is R = Range of f(x)

Hence, Range of f(x) = co-domain of f(x) = R except 3x+4=0

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{4y}{4-3y}$.

Question: 8

Let R_+

Solution:

To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for $x_1, x_2 \in A$ & $f(x_1), f(x_2) \in B$, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let $x_1, x_2 \in R$ and $f(x) = (9x^2 + 6x - 5)$.So $f(x_1) = f(x_2) \rightarrow (9x_1^2 + 6x_1 - 5) = (9x_2^2 + 6x_2 - 5)$ on solving we get→ $x_1=x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is [-5, ∞]

Let y = f(x) = $(9x^2 + 6x - 5)$, So x = $\frac{-1+\sqrt{y+6}}{3}$ [Range of f(x) = Domain of y]

So Domain of y = Range of f(x) = [-5, ∞]

Hence, Range of f(x) = co-domain of f(x) =[-5, ∞]

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{-1+\sqrt{y+6}}{3}$.

Question: 9

Let f : N

Solution:

To Show: that f is invertible

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for x_1, $x_2 \in$ A & $f(x_1)$, $f(x_2) \in$ B, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let x_1, $x_2 \in$ R and $f(x) = 4x^2 + 12x + 15$ So $f(x_1) = f(x_2) \rightarrow (4x_1{}^2 + 12x_1 + 1\,5) = (4x_2{}^2 + 12x_2 + 1\,5)$, on solving we get$\rightarrow x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is Range(f).

Let $y = f(x) = 4x^2 + 12x + 15$, So $x = \frac{-3+\sqrt{y-6}}{2}$ [Range of f(x) = Domain of y]

So Domain of y = Range of f(x) = $[6, \infty]$

Hence, Range of f(x) = co-domain of f(x) = $[6, \infty]$

So, f(x) is onto function

As it is bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{-3+\sqrt{y-6}}{2}$.

Question: 10

Let A = R – {2} a

Solution:

To Show: that f is one-one and onto

To Find: Inverse of f

[NOTE: Any functions is invertible if and only if it is bijective functions (i.e. one-one and onto)]

one-one function: A function f : A → B is said to be a one-one function or injective mapping if different elements of A have different images in B. Thus for x_1, $x_2 \in$ A & $f(x_1)$, $f(x_2) \in$ B, $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$ or $x_1 \neq x_2 \leftrightarrow f(x_1) \neq f(x_2)$

onto function: If range = co-domain then f(x) is onto functions.

So, We need to prove that the given function is one-one and onto.

Let x_1, $x_2 \in$ Q and $f(x) = \frac{x-1}{x-2}$.So $f(x_1) = f(x_2) \rightarrow \frac{x_1-1}{x_1-2} = \frac{(x_2-1)}{x_2-2}$, on solving we get $\rightarrow x_1 = x_2$

So $f(x_1) = f(x_2) \leftrightarrow x_1 = x_2$, f(x) is one-one

Given co-domain of f(x) is R – {1}

Let $y = f(x) = \frac{x-1}{x-2}$, So $x = \frac{2y-1}{y-1}$ [Range of f(x) = Domain of y]

So Domain of y = Range of f(x) = R – {1}

Hence, Range of f(x) = co-domain of f(x) = R – {1}.

So, f(x) is onto function

As it is a bijective function. So it is invertible

Invers of f(x) is $f^{-1}(y) = \frac{2y-1}{y-1}$

Question: 11

Let f and g be tw

Solution:

To Find: Inverse of f o g and g o f.

Given: $f(x) = |x| + x$ and $g(x) = |x| - x$ for all $x \in R$

$f \circ g\,(x) = f(g(x)) = |g(x)| + g(x) = ||x| - x| + |x| - x$

Case 1) when $x \geq 0$

$f(g(x)) = 0$ (i.e. $|x| - x$)

Case 2) when $x < 0$

$f(g(x)) = -4x$

$g \circ f\,(x) = g(f(x)) = |f(x)| - f(x) = ||x| + x| - |x| - x$

Case 1) when $x \geq 0$

$g(f(x)) = 0$ (i.e. $|x| - x$)

Case 2) when $x < 0$

$g(f(x)) = 0$

Exercise : OBJECTIVE QUESTIONS

Question: 1

Mark (√) against

Solution:

$f(x) = 2x$

For One - One

$f(x_1) = 2x_1$

$f(x_2) = 2x_2$

put $f(x_1) = f(x_2)$ we get

$2x_1 = 2x_2$

Hence, if $f(x_1) = f(x_2)$, $x_1 = x_2$

Function f is one - one

For Onto

$f(x) = 2x$

let $f(x) = y$, such that $y \in N$

$2x = y$

$\Rightarrow x = \frac{y}{2}$

If $y = 1$

$x = \frac{1}{2} = 0.5$

which is not possible as $x \in N$

Hence, f is not onto., f is into

Hence, option b is correct

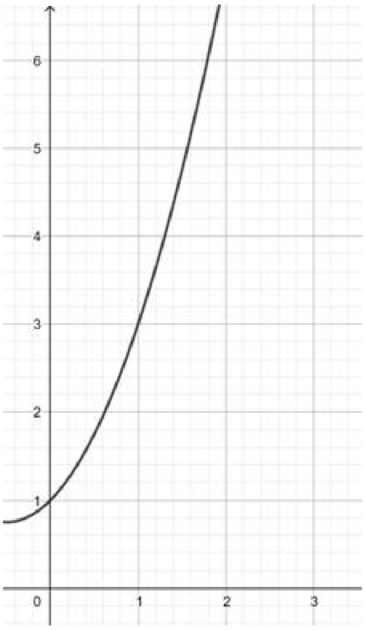
Question: 2

Mark (√) against

Solution:

In the given range of N f(x) is monotonically increasing.

∴f(x) = $x^2 + x + 1$ is one one.



But Range of f(n) = $[0.75,\infty) \neq N$(codomain)

Hence, f(x) is not onto.

Hence, the function f : N → N : f(x) = $(x^2 + x + 1)$ is one - one but not onto. i.e. into

Question: 3

Mark (√) against

Solution:

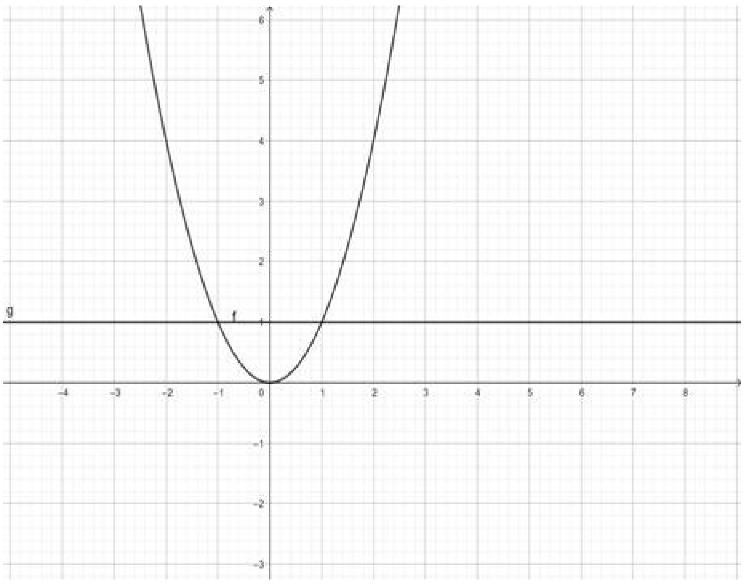
f(x) = x^2

⇒y = x^2

in this range the lines cut the curve in 2 equal valued points of y, therefore, the function f(x) = x^2 is many - one .

Range of f(x) = $(0,\infty) \neq R$(codomain)

∴f(x) is into



$\therefore$ f : R → R: f(x) = x^2 is many - one into

Question: 4

Mark (√) against

Solution:

f(x) = x^3

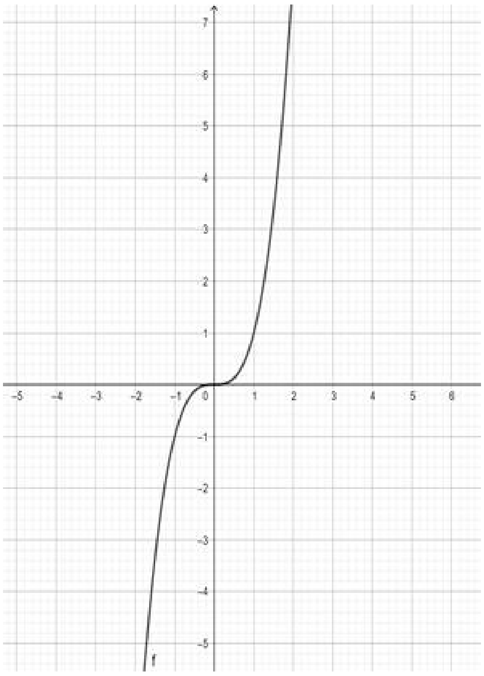
Since the function f(x) is monotonically increasing from the domain R → R

$\therefore$f(x) is one –one

Range of f(x) = (- ∞,∞)≠R(codomain)

$\therefore$f(x) is into

$\therefore$ f : R → R: f(x) = x^3 is one - one into.



Question: 5

Mark (√) against

Solution:

f(x) = e^x

Since the function f(x) is monotonically increasing from the domain $R^+ \to R^+$

$\therefore$f(x) is one –one

Range of f(x) = (1,∞) = R^+ (codomain)

$\therefore$f(x) is onto

$\therefore$ f : $R^+ \to R^+$: f(x) = e^x is one - one onto.

Question: 6

Mark (√) against

Solution:

$$f:\left[\frac{-\pi}{2},\frac{\pi}{2}\right]\to[-1,1]:f(x)=\sin x$$

Here in this range, the function is NOT repeating its value,

Therefore it is one - one.

Range = Codomain

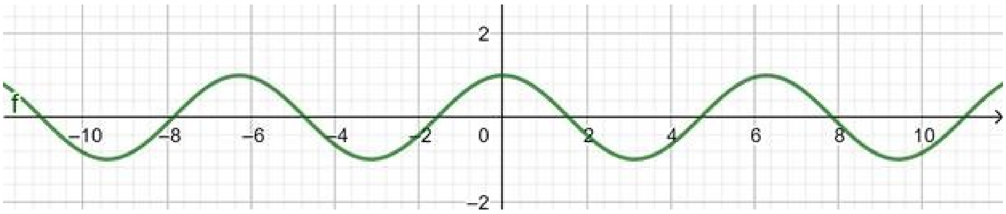
∴Function is onto

Hence, option B is the correct choice.

Question: 7

Mark (√) against

Solution:



f(x) = cosx

y = cosx

Here in this range the lines cut the curve in many equal valued points of y therefore the function f(x) = cosx is not one - one.

⇒f(x) = many one

Range of f(x) = [- 1,1]≠R(codomain)

∴f(x) is not onto.

⇒f(x) = into

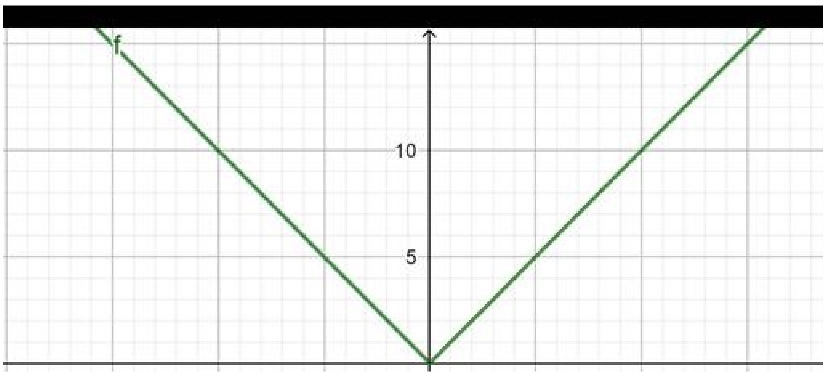
Hence,f(x) = cosx is many one and into

Ans: (c) many - one and into

Question: 8

Mark (√) against

Solution:



Here in this range the lines cut the curve in 2 equal valued points of y therefore the function f(z) = |z| is not one - one

⇒f(z) = many one.

Range of f(z) = [0,∞)≠R(codomain)

∴f(z) is not onto.

⇒f(z) = into

Hence, f(z) = |z| is many one and into

Question: 9

Mark (√) against

Solution:

$$f : A \to A : f(x) = \frac{(x-2)}{(x-3)}$$

In this function

x = 3 and y = 1 are the asymptotes of this curve and these are not included in the functions of the domain and range respectively therefore the function f(x) is one one sice there are no different values of x which has same value of y .

and the function has no value at y = 1 here range = codomain

∴ f(x) is onto

Question: 101

Mark (√) against

Solution:

f(1) = 1

f(2) = 1

f(3) = 2

f(4) = 2

f(5) = 3

f(6) = 3

Since at different values of x we get same value of y ∴f(n) is many –one

And range of f(n) = N = N(codomain)

∴ the function f: N → Z, defined by

$$f : N \to N : f(x) = \begin{cases} \frac{1}{2}(n+1), \text{when } n \text{ is odd} \\ \frac{n}{2}, \text{when } n \text{ is even.} \end{cases}$$ is both many - one and onto.

Question: 11

Mark (√) against

Solution:

SINCE, f(a, b) = (b, a).There is no same value of y at different values of x ∴function is one one

∴Range(A×B)≠Codomain(B × A)

⇒function is into

Question: 12

Mark (√) against

Solution:

f(x) = 2x + 3

⇒y = 2x + 3

x⟺y

⇒x = 2y + 3

⇒x - 3 = 2y

⇒$\frac{x-3}{2} = y$

x⟺y

$\Rightarrow \frac{y-3}{2} = x$

Question: 13

Mark (√) against

Solution:

$f(x) = \frac{4x}{3x+4}$

$\Rightarrow y = \frac{4x}{3x+4}$

$x \Longleftrightarrow y$

$\Rightarrow x = \frac{4y}{3y+4}$

$\Rightarrow 3yx + 4x = 4y$

$\Rightarrow y(3x - 4) = - 4x$

$\Rightarrow y = \frac{4x}{4-3x}$

$x \Longleftrightarrow y$

$\Rightarrow x = \frac{4y}{4-3y}$

Question: 145

Mark (√) against

Solution:

$f(x) = 4x^2 + 12x + 15$

$\Rightarrow y = 4x^2 + 12x + 15$

$\Rightarrow y = (2x + 3)^2 + 6$

$\Rightarrow \sqrt{(y - 6)} = 2x + 3$

$\Rightarrow \frac{1}{2}\left(\sqrt{y-6} - 3\right) = x$

$f^{-1}(y) = \frac{1}{2}\left(\sqrt{y-6} - 3\right)$

Question: 15

Mark (√) against

Solution:

$$f(x) = \frac{4x + 3}{6x - 4}$$

$$\Rightarrow f\big(f(x)\big) = \frac{4f(x) + 3}{6f(x) - 4} = (f \circ f)(x)$$

$$\Rightarrow f\big(f(x)\big) = \frac{4(\frac{4x+3}{6x-4}) + 3}{6(\frac{4x+3}{6x-4}) - 4}$$

$$\Rightarrow f\big(f(x)\big) = \frac{16x + 12 + 18x - 12}{24x + 18 - 24x + 16} = \frac{34x}{34} = x$$

Question: 16

Mark (√) against

Solution:

$f(x) = (x^2 – 1)$

$g(x) = (2x + 3)$

$\therefore (g \circ f)(x) = g(f(x))$

$\Rightarrow g(f(x)) = 2f(x) + 3$

$\Rightarrow g(f(x)) = 2((x^2 - 1)) + 3 = 2x^2 - 2 + 3 = 2x^2 + 1$

Question: 17

Mark (√) against

Solution:

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

$\Rightarrow f(x) = x^2 - 2$

Question: 18

Mark (√) against

Solution:

$$f(x) = \frac{1}{1 - x}$$

$\Rightarrow (f \circ f \circ f)(x) = f\Big(f(f(x))\Big)$

$$\Rightarrow f(f(x)) = \frac{1}{1 - f(x)} = \frac{1}{1 - \frac{1}{1 - x}} = \frac{1 - x}{1 - x - 1} = \frac{x - 1}{x} = 1 - \frac{1}{x}$$

$$\Rightarrow f\Big(f(f(x))\Big) = \frac{1}{1 - f(f(x))} = \frac{1}{1 - \left(1 - \frac{1}{x}\right)} = \frac{1}{\frac{1}{x}} = x$$

Question: 19

Mark (√) against

Solution:

$f(x) = \sqrt[3]{3 - x^3}$

$$\Rightarrow f(f(x)) = \sqrt[3]{3 - f(x)^3} = \sqrt[3]{3 - \left(\sqrt[3]{3 - x^3}\right)^3}$$

$\Rightarrow f(f(x)) = \sqrt[3]{3 - (3 - x^3)}$

$\Rightarrow f(f(x)) = \sqrt[3]{x^3} = x$

Question: 20

Mark (√) against

Solution:

$f(x) = x^2 - 3x + 2$

$\Rightarrow f(x) = x^2 - 2x - x + 2 = x(x - 2) - 1(x - 2)$

$\Rightarrow f(x) = (x - 2)(x - 1)$

$\Rightarrow f(x) = (x - 2)(x - 1)$

$\Rightarrow f(f(x)) = (f(x) - 2)(f(x) - 1)$

$\Rightarrow f(f(x)) = ((x - 2)(x - 1) - 2) ((x - 2)(x - 1) - 1)$

$\Rightarrow f(f(x)) = (x^2 - 3x + 2 - 2) (x^2 - 3x + 2 - 1)$

$\Rightarrow f(f(x)) = (x^2 - 3x)(x^2 - 3x + 1)$

$\Rightarrow f(f(x)) = x^4 - 3x^3 + x^2 - 3x^3 + 9x^2 - 3x$

$\Rightarrow f(f(x)) = x^4 - 6x^3 + 10x^2 - 3x$

Question: 21

Mark (√) against

Solution:

$f(x) = 8x^3$

$g(x) = x^{1/3}$

$$\Rightarrow (g \circ f)(x) = \left(f(x)\right)^{\frac{1}{3}} = (8x^3)^{\frac{1}{3}} = 2x$$

Question: 22

Mark (√) against

Solution:

$f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$

$$\Rightarrow g\left(f(x)\right) = \tan\left(f(x)\right) = \tan(x^2)$$

$$\Rightarrow h\left(g\left(f(x)\right)\right) = \log\left(g\left(f(x)\right)\right) = \log(\tan(x^2))$$

$$\Rightarrow h\left(g\left(f\left(\sqrt{\frac{\pi}{4}}\right)\right)\right) = \log\left(\tan\left(\sqrt{\frac{\pi}{4}}^{2}\right)\right) = \log\left(\tan\left(\frac{\pi}{4}\right)\right) = \log(1) = 0$$

Question: 23

Mark (√) against

Solution:

$g = \{(2, 3), (5, 1), (1, 3)\}$

$(g \circ f) = \{(\text{dom}(f), 3), (\text{dom}(f), 1), (\text{dom}(f), 3)\}$

$\Rightarrow (g \circ f) = \{(1, 3), (3, 1), (4, 3)\}$

Question: 24

Mark (√) against

Solution:

$F(x) = \sqrt{9 - x^2}$

$\sqrt{9 - x^2}$ should be ≥ 0

$\Rightarrow 9 - x^2 \geq 0$

$\Rightarrow x^2 \leq 9$

$\Rightarrow -3 \leq x \leq 3$

$\therefore \text{dom}(f) = [-3, 3]$

Question: 25

Mark (√) against

Solution:

$f(x) = \sqrt{\frac{x-1}{x-4}}$

$\sqrt{\frac{x-1}{x-4}} \geq 0$

$\Rightarrow x - 1 \geq 0$

$\Rightarrow x \geq 1$

And $x \neq 4$

$x > 4$ and $x \leq 1$

$\Rightarrow \text{dom}(f) = (-\infty, 1] \cup (4, \infty)$

Question: 26

Mark (√) against

Solution:

$f(x) = e^{\sqrt{x^2-1}} \log(x-1)$

$x - 1 > 0$

$\Rightarrow x > 1$

And

$\Rightarrow x^2 - 1 \geq 0$

$\Rightarrow x^2 \geq 1$

$\Rightarrow -1 \leq x \geq 1$

Taking the intersection we get

Dom(f) = (1, ∞)

Question: 27

Mark (√) against

Solution:

$f(x) = \frac{x}{x^2-1}$

$X^2 - 1 \neq 0$

x≠(1, - 1)

∴ Dom(f) = R - { - 1,1}

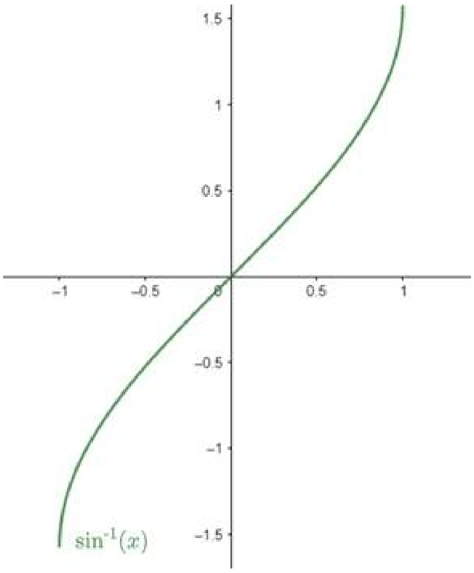
Question: 28

Given: $f(x) = \frac{\sin^{-1} x}{x}$

From f(x), x ≠ 0

Now, domain of $\sin^{-1}x$ is [-1, 1] as the values of $\sin^{-1}x$ lies between -1 and 1.

We can see that from this graph:



Domain of $f(x) = [-1, 1] – 0$

Hence, B is the correct answer.

Question: 29

Mark (√) against

Solution:

$f(x) = \cos^{-1} 2x.$

domain of $\cos^{-1}x = [-1,1]$

on multiplying by an integer the domain decreases by same number

$\Rightarrow$domain of $\cos^{-1}2x = [-1/2,1/2]$

Question: 30

Mark (√) against

Solution:

$f(x) = \cos^{-1}(3x - 1).$

domain of $\cos^{-1}x = [-1,1]$

on multiplying by an integer the domain decreases by same number

$\Rightarrow$domain of $\cos^{-1}3x = [-1/3,1/3]$

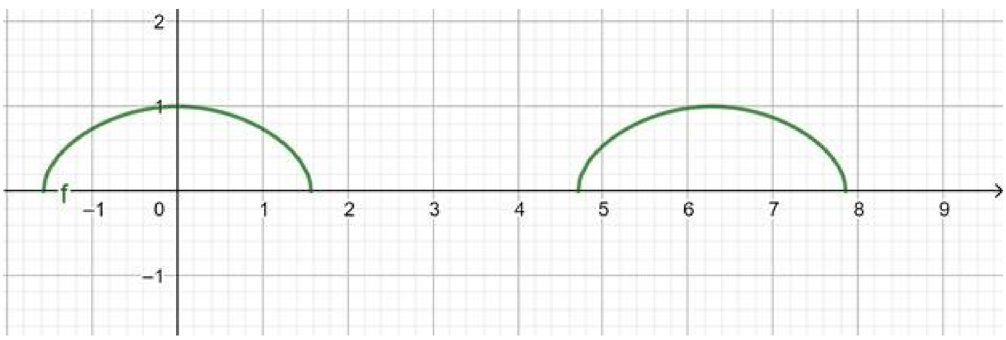
$\Rightarrow$domain of $\cos^{-1}(3x - 1) = [1/3 - 1/3,1/3 + 1/3] = [0,2/3]$

Question: 31

Mark (√) against

Solution:

$f(x) = \sqrt{\cos x}$



As per the diagram

We can imply that domain of √cosx

is $\left[0,\frac{\pi}{2}\right]\left[\frac{3\pi}{2},2\pi\right]$

Question: 32

Mark (√) against

Solution:

$f(x) = \sqrt{\log(2x - x^2)}$.

$2x - x^2 > 1$

$\Rightarrow x^2 - 2x + 1 < 0$

$\Rightarrow (x-1)^2 < 0$

$\Rightarrow x - 1 < 0$

$\Rightarrow x < 1$

$\log(2x - x^2) > 0$

$\Rightarrow 2x - x^2 > e^0 = 1$

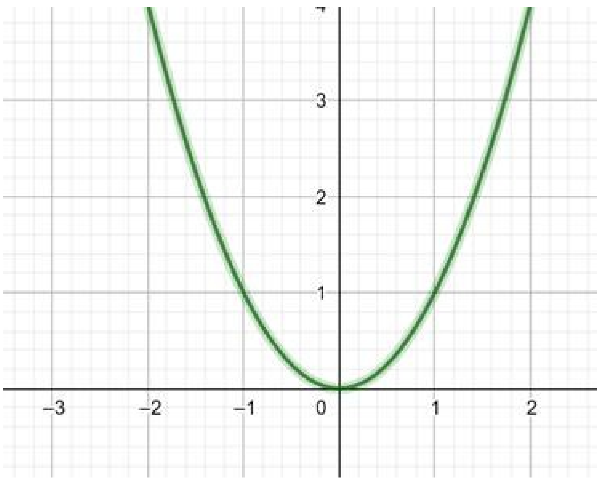
$\Rightarrow x < 1$

Dom(f) = (- ∞, 1)

Question: 33

Mark (√) against

Solution:



According to sketched graph of x^2

Domain of f(x) = R

And Range of f(x) = R^+

Question: 34

Mark (√) against

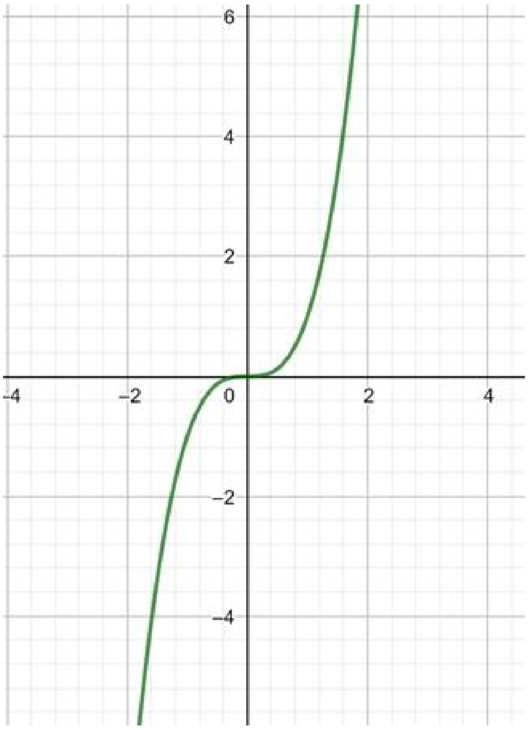
Solution:

According to sketched graph of x^3

Domain of f(x) = R

And Range of f(x) = R

Since x^3 is a, monotonically increasing function



Question: 35

Mark (√) against

Solution:

log (1 – x) + √(x^2 – 1)

1 - x>0

$x < 1$

X^2 - 1≥0

X^2 ≥1

$\Rightarrow -1 \le x \ge 1$

Taking intersection of the ranges we get

Dom (f) = (b) (- ∞, - 1]

Question: 36

Mark (√) against

Solution:

$f(x) = \frac{1}{1-x^2}$

$\Rightarrow y = \frac{1}{1-x^2}$

$\Rightarrow y - yx^2 = 1$

$\Rightarrow y - 1 = yx^2$

$\Rightarrow x = \sqrt{\frac{y-1}{y}}$

$\Rightarrow \frac{y-1}{y} \ge 0$

$\Rightarrow y \ge 1$

∴ range (f) = [1, ∞)

Question: 37

Mark (√) against

Solution:

$$f(x) = \frac{x^2}{1 + x^2}$$

$$\Rightarrow y = \frac{x^2}{1 + x^2}$$

$$\Rightarrow y + yx^2 = x^2$$

$$\Rightarrow y = x^2(1-y)$$

$$\Rightarrow x = \sqrt{\frac{y}{1-y}}$$

$$\frac{y}{1-y} \geq 0$$

$\Rightarrow y \geq 0$

And

$1 - y > 0$

$\Rightarrow y < 1$

Taking intersection we get

range (f) = [0, 1)

Question: 38

Mark (√) against

Solution:

$$f(x) = x + \frac{1}{x}$$

For this type

Range is

$$-2 \leq y \geq 2$$

Question: 39

Mark (√) against

Solution:

$f(x) = a^x$

when $x<0$

$0<a^x<1$

When $x \geq 0$

$a^x>0$

Therefore range of $f(x) = a^x = (0, \infty)$