03

Polynomials

3.01. Introduction

In previous class, we have studied polynomials in one variable and their degres. We know that in polynomial $f(x)$, the highest powerof x is called the degree of polynomial and on the bases of power, polynomial is recognized as linear, quadratic or cubic polynomial. Generally for variable x, $f(x) = ax + b$ as linear, $f(x) = ax^2 + bx + c$ as quadratic and $f(x) = ax^3 + bx^2 + cx + d$ is taken as cubic polynomials where a, b, c, d are real numbers and $a \neq 0$. In this way for variable x, polynomial of n degree can be defined as follows $f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$, where '$n$' is a natural number and $a_n, a_{n-1}, ..., a_1, a_0$ are called coefficients of these terms.

In this chapter we will study the zeroes, coefficients and division algorithm of polynomials and also about the nature of roots and solutions of quadratic equations. In previous classes we have studied HCF and LCM of real numbers. Here we will find HCF and LCM of algebraic expression.

3.02. Zeroes of a Polynomial

Consider the polynomials $f_1(x) = 4x + 2, f_2(x) = 2x^2 + 3x - \frac{2}{5}, f_3(x) = 2 - x^3$

These are examples of linear, quadratic and cubic polynomial respectively. By putting $x = 2$ in polynomials $f_1(x), f_2(x)$ and $f_3(x)$, we get the following values.

$$f_1(2) = 4 \times 2 + 2 = 10$$

$$f_2(2) = 2 \times 2^2 + 3 \times 2 - \frac{2}{5} = 8 + 6 - \frac{2}{5} = \frac{68}{5}$$

$$f_3(2) = 2 - 2^3 = -6$$

So, by putting different values of x we get different value of polynomials, so we can say that :

If $f(x)$ is a polynomial in variable x by 'a' in $f(x)$ is called the value of $f(x)$ at $x = a$ and is denoted by $f(a)$

Now consider the polynomial $f(x) = 2x^2 - 8x + 6$ then putting $x = 1$ and $x = 3$ in the polynomial, we get

$$f(1) = 2 \times 1 - 8 \times 1 + 6 = 0$$

$$f(3) = 2 \times 3^2 - 8 \times 3 + 6 = 0$$

As $f(1)$ and $f(3) = 0$ so 1 and 3 are called zeroes of the quadratic polynomial $f(x) = 2x^2 - 8x + 6$. More generally, a real number 'a' is said to be a zero of a polynomial $f(x)$, If $f(a) = 0$

Let α is zero of linear polynomial $f(x) = ax + b$, then $f(\alpha) = a\alpha + b = 0$

$$\Rightarrow \quad \alpha = \frac{-b}{a} = \frac{-(\text{constant term})}{(\text{cofficient of } x)}$$

Thus, the zero of linear polynomial is related to its coefficients.

3.03 Relationship between Zeroes and Coefficients of a Quadratic Polynomial

In previous class, we have studied about factorisation of polynomials. Quadratic polynomial is factorize by splitting the middle term. in such a way that product of both terms is equal to the product of first and third term. Here we should know that quadratic polynomial has two zeroes (real/imaginary). We can understand the relation between zeros and coefficients of polynomial.

General form : Let α and β are two zeroes of quadratic polynomial $f(x) = ax^2 + bx + c$, then $(x - \alpha)$ and $(x - \beta)$ will be factors of f(x). So for constant k we can write as :

$$f(x) = k(x - \alpha)(x - \beta)$$

i.e., $\quad ax^2 + bx + c = k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$

or $\quad ax^2 + bx + c = kx^2 - k(\alpha + \beta)x + \alpha\beta$

equating like powers on both sides, we get

$$\alpha + \beta = \frac{-b}{a} \quad \text{and} \quad \alpha\beta = \frac{c}{a}$$

$\therefore$ For polynomial $f(x) = ax^2 + bx + c$, it is clear that

$$\text{Sum of zeroes} = \frac{-b}{a} = \frac{-(\text{coefficient of } x)}{(\text{coefficient of } x^2)}$$

and $\quad \text{Product of zeroes} = \dfrac{c}{a} = \dfrac{\text{constant term}}{\text{coefficient of } x^2}$

Example 1 : Find the zeroes of quadratic polynomial $x^2 - 2x - 8$ and verify the relationship between the zeroes and the coefficients.

Solution : Let $f(x) = x^2 - 2x - 8 = x^2 - 4x + 2x - 8$

$$= x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = (x+2)(x-4)$$

Now, $\quad$ by taking $f(x) = 0$, $= (x+2)(x-4) = 0$

or $\quad x + 2 = 0 \;$ or $\; x - 4 = 0$

or $\quad x = -2 \;$ or $\; x = 4$

Thus, zeroes of polynomial $f(x) = x^2 - 2x - 8$ will be -2 and 4.

Here, $\quad$ Sum of zeroes $= -2 + 4 = 2$

also $\quad \text{Sum of zeroes} = \dfrac{-(\text{coefficient of } x)}{\text{coefficient } x^2} = \dfrac{2}{1} = 2$

Product of zeroes $= -2 \times 4 = -8$

i.e., Product of zeroes $= \dfrac{\text{constant term}}{\text{coefficient of } x^2} = \dfrac{-8}{1} = -8$

Therefore, relation between zeroes and coefficient for given polynomial is true.

Example 2. Find the zeroes of quadratic polynomial $3x^2+5x-2$ and verify the relationship between the zeroes and the coefficients.

Solution : Let $f(x) = 3x^2+5x-2$

or $f(x) = 3x^2+6x-x-2 = 3x(x+2)-1(x+2) = (3x-1)(x+2)$

Now, by taking $f(x)=0$ then $(3x-1)(x+2)=0$

or $3x-1=0$ or $x+2=0$

or $x = \dfrac{1}{3}$ or $x=-2$

Thus $1/3$ and -2 will be zeroes of given polynomial

Here, Sum of zeroes $= \dfrac{1}{3} + (-2) = \dfrac{-5}{3} = \dfrac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

Product of zeroes $= \dfrac{1}{3} \times (-2) = \dfrac{-2}{3} = \dfrac{\text{constant term}}{\text{coefficient of } x^2}$

Therefore, relation between zeroes and coefficient for given polynomial is true.

Example 3. Find a quadratic polynomial, the sum and product of whose zeroes are $1/4$ and -1 respectively.

Solution : Let α and β are zeroes of quadratic equation

$$ax^2+bx+c$$

$\therefore$ Sum of zeroes $\alpha+\beta = \dfrac{-b}{a} = \dfrac{1}{4}$ (Given) . . . (1)

and, Product of zeroes $\alpha\beta = \dfrac{c}{a} = -1$ (Given) . . . (2)

If $a=k$, where k is real number, then form equation (1) and (2), we have

$$b = -\frac{k}{4} \text{ and } c = -k$$

Thus, quadratic polynomial ax^2+bx+c is obtained in the following form :

$$kx^2 - \frac{k}{4}x - k \text{ or } \frac{k}{4}(4x^2-x-4)$$

Hence, required quadratic polynomial will be $4x^2-x-4$.

Exercise 3.1

1. Find the zeroes of the following quadratic polynomial and verify the relationship between the zeroes and the coefficients.

(i) $4x^2+8x$ (ii) $4x^2-4x+1$ (iii) $6x^2-x-2$

(iv) x^2-15 (v) $x^2-(\sqrt{3}+1)x+\sqrt{3}$ (vi) $3x^2-x-4$

2. Find a quadratic polynomial, whose sum and product of the zeroes are the following numbers respectively.

(i) –3, 2 (ii) $\sqrt{2},\frac{1}{3}$ (iii) $-\frac{1}{4},\frac{1}{4}$ (iv) $0,\sqrt{5}$

(v) 4, 1 (vi) 1,1

3. If sum of squares of zeroes of quadratic polynomial $f(x)=x^2-8x+k$ is 40, then find the value of *k*.

3.04 Division Algorithm for Polynomials with Real Coefficients

In previous chapter, we have studied that division of two integers gives quotient and remainder and the relation between them is

Dividend = Divisor × Quotient + Remainder

Here, we discuss the method of dividing one polynomial by another in same method. We stop the division process when either the remainder is zero or its degree is less than the quotient this process is called division. Algorithm we understand this process by taking a example.

Divide polynomial $f(x)=3x^2-x^3-3x+5$ by polynomial $f(x)=x-1-x^2$ by division algorithm.

Step 1. First arrange dividend and divisor in descending order of x *i.e.,* in standard form. Here, on writing $f(x), g(x)$ in standard form. $f(x)=-x^3+3x^2-3x+5$ and $g(x)=-x^2+x-1$

Step 2. Now divide highest power term of dividend $(-x^3)$ by highest power term of divisor $(-x^2)$ and obtained x as quotient *i.e.,*

$$\begin{array}{r|l} & x \\ \hline -x^2+x-1 & -x^3+3x^2-3x+5 \\ & \mp x^3 \pm x^2 \mp x \\ \hline & 2x^2-2x+5 \end{array}$$

Here remainder is $2x^2-2x+5$

Step 3. Now highest degree term of dividend $(2x^2)$ is divided by highest degree term of divisor $(-x^2)$ and obtained the quotient (–2) *i.e.,*

$$\begin{array}{r|l} & x-2 \\ \hline -x^2+x-1 & -x^3+3x^2-3x+5 \\ & \underset{+}{-x^3}\ \underset{-}{+x^2}\ \underset{+}{-x} \\ \hline & 2x^2-2x+5 \\ & \underset{-}{2x^2}\ \underset{+}{-2x}\ \underset{-}{+2} \\ \hline & 3 \end{array}$$

Here remainder is 3 and its degree is less then the degree of divisor $-x^2+x-1$, So stop the division process. Here, we get quotient (x – 2) and remainder (3). In divison algorithm verify dividend = divisor × quotient + remainder.

Here dividend $-x^3+3x^2-3x+5$, divisor $=-x^2+x-1$

quotient = $(x-2)$, remainder = 3

$\therefore$ $$(-x^2+x-1)\times(x-2)+3$$

$$=-x^3+x^2-x+2x^2-2x+2+3$$

$$=-x^3+3x^2-3x+5 \;\; = \text{dividend}$$

Thus, division algorithm can be expressed by following statement.

Divison algorithm : If $f(x)$ and $g(x)$ are any two polynomials where $g(x)\neq 0$ then from polynomials $q(x)$ and $r(x)$, we get

$$f(x)=q(x)g(x)+r(x)$$

Where $r(x)=0$ or degree $r(x) <$ degree of $g(x)$

Example 1. Using divison algorithm, divide $p(x)=x^4-3x^2+4x+5$ by $g(x)=x^2+1-x$ and find quotient and remainder.

Solution : Arrange polynomial is standard form then applying division algorithm.

$$\begin{array}{r|l}
 & x^2+x-3 \\
\hline
x^2-x+1 & x^4+0x^3-3x^2+4x+5 \\
 & x^4-x^3+x^2 \\
 & \underline{-\quad +\quad -} \\
 & x^3-4x^2+4x+5 \\
 & x^3-x^2+x \\
 & \underline{-\quad +\quad -} \\
 & -3x^2+3x+5 \\
 & -3x^2+3x-3 \\
 & \underline{+\quad -\quad +} \\
 & 8
\end{array}$$

Since degree of remainder is less than the degree of divisor so, stop the process.

Here, quotient $=x^2+x-3$ and remainder = 8

Here, divisor × quotiuent + remainder

$$=(x^2-x+1)\times(x^2+x-3)+8$$

$$=x^4-x^3+x^2+x^3-x^2+x-3x^2+3x-3+8$$

$$x^4-3x^2+4x+5 \;\; = \text{Dividend}$$

Hence, division algorithm is verified.

Example 2. Find all the zeroes of polynomial $f(x)=3x^4+6x^3-6x^3-2x^2-10x-5$, if its two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Solution : Here $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are two zeroes of given polynomial

$\therefore$ $\left(x-\sqrt{\frac{5}{3}}\right)\left(x+\frac{\sqrt{5/3}}{3}\right)=x^2-\frac{5}{3}=\frac{1}{3}(3x^2-5)$ is a factor of polynomial

i.e., $(3x^2-5)$ is also a factor of polynomial. Now divide $f(x)$ by $(3x^2-5)$

$$
\begin{array}{r|l}
 & x^2+2x+1 \\
3x^2-5 & 3x^4+6x^3-2x^2-10x-5 \\
 & \underline{\overset{-}{3x^4}\overset{+}{-5x^2}} \\
 & 6x^3+3x^2-10x-5 \\
 & \underline{\overset{-}{6x^3}\quad\overset{+}{-10x}} \\
 & 3x^2-5 \\
 & \underline{\overset{-}{-3x^2}\overset{+}{-5}} \\
 & 0
\end{array}
$$

Form division algorithm, it is clear that quotient (x^2+2x+1) is a factor of polynomial $f(x)$ because remainder is zero (0), so we can write as

$$x^2+2x+1=(x+1)^2$$

$\therefore$ Dividend = Divisor × Quotient + Remainder

$$3x^4+6x^3-2x^2-10x-5=(x+1)^2\times(3x^2-5)+0$$

$$=(x+1)^2(\sqrt{3}x-\sqrt{5})(\sqrt{3}x+\sqrt{5})$$

Since to find zeroes of polynomial $f(x)$, $f(x)=0$ should satisfy so

$$(x+1)^2(\sqrt{3}x-\sqrt{5})(\sqrt{3}x+\sqrt{5})=0$$

or $x+1=0, x+1=0, \sqrt{3}x-\sqrt{5}=0\ \sqrt{3}x+\sqrt{5}=0$

It means $-1, -1, \sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$ will be zeroes of the polynomial.

Exercise 3.2

1. Using division algorithm, divide $f(x)$ by $g(x)$ and find quotient and remainder.

   (i) $f(x)=3x^3+x^2+2x+5,$ $\quad g(x)=1+2x+x^2$

   (ii) $f(x)=x^3-3x^2+5x-3,$ $\quad g(x)=x^2-2$

   (iii) $f(x)=x^3-6x^2+11x-6,$ $\quad g(x)=x+2$

   (iv) $f(x)=9x^4-4x^2+4,$ $\quad g(x)=3x^2+x-1$

2. Divide second polynomial by first and verify that first polynomial is a factor of second polynomial.

   (i) $g(x)=x^2+3x+1,$ $\quad f(x)=3x^4+5x^3-7x^2+2x+2$

   (ii) $g(t)=t^2-3,$ $\quad f(t)=2t^4+3t^3-2t^2-9t-12$

   (iii) $g(x)=x^3-3x+1,$ $\quad f(x)=x^5-4x^3+x^2+3x+1$

3. In the following polynomials, their zerores are given, find all other zeroes.

   (i) $f(x)=2x^4-3x^3-3x^2+6x-2;$ $\sqrt{2}$ and $-\sqrt{2}$

   (ii) $f(x)=x^4-6x^3-26x^2+138x-35;$ $2\pm\sqrt{3}$

   (iii) $f(x)=x^3+13x^2+32x+20;$ -2

4. On dividing polynomial $f(x)=x^3-3x^2+x+2$ by polynomial $g(x)$, quotient $q(x)$ and remainder $r(x)$ are obtained as $x-2$ and $-2x+4$ respectively then find polynomial $g(x)$.

3.05 Standard From of Quadratic Equation

In the beginning of the chapter, we have studied about quadratic polynomial. In general $f(x)=ax^2+bx+c,\ a\neq 0$ is standard form of polynomial. We have discussed about the zeroes of polynomial $f(x)=ax^2+bx+c$. We know that value of polynomial is zero at their zeroes. This fact can be expressed as :

"If $f(x)$ is a quadratic polynomial then $f(x)=0$ is called a quadratic equation *i.e.,* $ax^2+bx+c=0,$ is a quadratic equation where *a, b, c* are real numbers and $a\neq 0$." If terms of $f(x)$ is arranged in descending order then $f(x)=0$ *i.e.,* $ax^2+bx+c=0,\ a\neq 0$ is called standard form of quadratic equation.

Let us consider some examples and test whether these are quadratic equations or not.

$$(x-2)(x+1)=(x-1)(x+3)$$

LHS.
$$=(x-2)(x+1)$$
$$=x^2-2x+x-2$$
$$=(x^2-x-2) \qquad \text{. . . (i)}$$

RHS.
$$=(x-1)(x+3)$$
$$=x^2-x+3x-3$$
$$=x^2+2x-3 \qquad \text{. . . (ii)}$$

By equating both sides according to the given equation

$$x^2 - x - 2 = x^2 + 2x - 3$$

or $\quad x^2 - x^2 - x - 2x - 2 + 3 = 0$

$\Rightarrow \quad -x + 1 = 0$ or $(x-1) = 0$

Here, equation $x - 1 = 0$ has no term of x^2 so given equation is not a quadratic equation.

In another equation, $3x^2 - 5x + 9 = x^2 - 7x + 3$, we have

$$3x^2 - x^2 - 5x + 7x + 9 - 3 = 0$$

$\Rightarrow \quad 2x^2 + 2x + 7 = 0$

Here x^2 is present so given equation is a quadratic equation.

3.06 Solution of Quadratic Equation by Factorisation

Zeroes of quadratic polynomial $f(x)$, from equation $f(x) = 0$. two values of x are obtained. Let $x = \alpha$ is a zero of polynomial $f(x) = ax^2 + bx + c$, then $f(\alpha) = 0$. It means x will satisfy the equation $ax^2 + bx + c = 0$. So we can say that a zero $x = \alpha$ of polynomial $ax^2 + bx + c$ will be root of quadratic equation $ax^2 + bx + c = 0$

So, if $f(x) = 0$ is a quadratic equation is 2. So it has polynomial $f(x)$ are called root of equation $f(x) = 0$

Highest degree of quadratic equation is 2. So it has maximum two roots.

The process to find the roots of any quadratic equation is called solution of an equation. To solve quadratic equation write it as $f(x) = 0$ in standard form, then factorise $f(x)$ and equate each factor equal to zero and then find values of x. These values of x are soluitions of quadratic equtions. Thus values of x so obtained are roots of this equation. This method can be understand by the following examples.

Example 1. Find the roots of quadratic equation $x^2 - 3x - 10 = 0$ by factorisation method.

Solution : Given equation is

$$x^2 - 3x - 10 = 0$$

On factorising,

$$x^2 - 5x + 2x - 10 = 0$$

or $\quad x(x-5) + 2(x-5) = 0$

or $\quad (x+2)(x-5) = 0$

or $\quad x + 2 = 0 \quad$ or $\quad x - 5 = 0$

or $\quad x = -2 \quad$ or $\quad x = 5$

Thus $x = -2$ and $x = 5$ are required two roots of given equation.

Example 2. Solve quadratic equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ by factorisation method.

Solution : Given equation is

$$\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$$

On factorisation

$$\sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$$

or $$x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$$

or $$(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$$

or $$\sqrt{2}x + 5 = 0 \quad \text{or} \quad x + \sqrt{2} = 0$$

or $$x = \frac{-5}{\sqrt{2}} \quad \text{or} \quad x = -\sqrt{2}$$

Thus, $x = \frac{-5}{\sqrt{2}}$ and $x = -\sqrt{2}$ are required roots of given equation.

Example 3. Find the roots of the following quadratic equation by factorisation method.

$$\frac{4}{x} - 3 = \frac{5}{2x+3} \quad \text{where} \quad x \neq 0, \frac{-3}{2}$$

Solution : Given equation is, $\frac{4}{x} - 3 = \frac{5}{2x+3}$

By taking LCM $$\frac{4-3x}{x} = \frac{5}{2x+3}$$

By cross multiplication, we get,

$$(4-3x)(2x+3) = 5x$$

or $$8x - 6x^2 + 12 - 9x = 5x$$

or $$6x^2 + 6x - 12 = 0$$

On factorising

$$6x^2 + 12x - 6x - 12 = 0$$

or $$6x(x+2) - 6(x+2) = 0$$

or $$(x+2)(6x-6) = 0$$

or $$x + 2 = 0 \quad \text{or} \quad 6x - 6 = 0$$

or $$x = -2 \quad \text{or} \quad x = 1$$

Thus, $x = -2$ and $x = 1$ are required solutions of quadratic equation.

Exercise 3.3

1. Check whether the following equations are quadratic?

   (i) $x(x+1) + 8 = (x+2)(x-2)$

   (ii) $(x+2)^3 = x^3 - 4$

   (iii) $x^2 + 3x + 1 = (x-2)^2$

   (iv) $x + \frac{1}{x} + x^2, x \neq 0$

2. Solve the following equtions by factorisation method.

   (i) $2x^2 - 5x + 3 = 0$

   (ii) $9x^2 - 3x - 2 = 0$

(iii) $\sqrt{3}x^2+10x+7\sqrt{3}=0$

(iv) $x^2-8x+16=0$

(v) $\frac{1}{x-2}+\frac{2}{x-1}=\frac{6}{x}$ where $x\neq 1,2$

(vi) $100x^2-20x+1=0$

(vii) $3x^2-2\sqrt{6}\,x+2=0$

(viii) x^2+8x+7

(ix) $\frac{x+3}{x+2}=\frac{3x-7}{2x-3}$

(x) $4x^2-4a^2x+(a^4-b^4)=0$

(xi) $abx^2+(b^2-ac)x-bc=0$

3.07 Solution by Perfect Square Method of a Quadratic Equations

Here given quadratic equations are converted into perfect square form $(x\pm A)^2=k^2$ for variable 'x' and then by taking square root of both sides, required roots of quadratic equation $x=k\pm A$ are obtained. This method will be clear by the following example.

Given equation is $2x^2-5x+3=0$ which have to solve by perfect square method.

So $\quad 2x^2-5x+3=0 \qquad$ . . . (1)

or $\quad x^2-\frac{5}{2}x+\frac{3}{2}=0 \qquad$ (Taking 1 as coefficient of x^2)

or $\quad x^2-\frac{5}{2}x=-\frac{3}{2} \qquad$ (Taking constant term in other side) . . . (2)

For making perfect square to LHS of equation (2), add half of coefficient of x on both sides, we get.

$$x^2-\frac{5}{2}x+\left(\frac{5}{4}\right)^2=-\frac{3}{2}+\left(\frac{5}{4}\right)^2$$

By writing LHS in perfect square form, simplify RHS. and obtained $(x\pm A)^2=k^2$ form.

i.e., $\quad \left(x-\frac{5}{4}\right)^2=\frac{-24+25}{16}=\frac{1}{16}$

or $\quad \left(x-\frac{5}{4}\right)^2=\left(\frac{1}{4}\right)^2$

Taking square root of both sides, we have

$$x-\frac{5}{4}=\pm\frac{1}{4} \qquad \Rightarrow \qquad \frac{5}{4}\pm\frac{1}{4}$$

or $\quad x=\frac{5}{4}+\frac{1}{4} \qquad$ or $\quad x=\frac{5}{4}-\frac{1}{4}$

or $\quad x=\frac{6}{4}=\frac{3}{2} \qquad$ or $\quad x=\frac{4}{4}=1$

So $x=\frac{3}{2}$ and $x=1$ are required roots of equation $2x^2-5x+3=0$.

Here we should note that if we get $(x\pm A)^2=-k^2$ form then value of x will not be real, it mean equation has no real , roots such quadratic equations can be solved by method, proposed by Indian mathematician Shridhar Acharya, quadratic formula.

Let quadratic equation is $ax^2+bx+c=0, a\neq 0$

Here $ax^2+bx+c=0$

$\Rightarrow \quad x^2+\frac{b}{a}x+\frac{c}{a}=0 \qquad \Rightarrow \quad x^2+\frac{b}{a}x=\frac{-c}{a}$

For making perfect square. Adding square of half of coefficient of x on both sides.

$$x^2+\frac{b}{a}x+\left(\frac{b}{2a}\right)^2=\frac{-c}{a}+\left(\frac{b}{2a}\right)^2$$

or
$$\left(x+\frac{b}{2a}\right)^2=\frac{-4ac+b^2}{4a^2}$$

Taking square root on both sides

$$x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

or
$$x=-\frac{b}{2a}\pm\sqrt{\frac{b^2-4ac}{2a}}$$

It means following are the roots of given equation

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

If $(b^2-4ac)\geq 0$, then value of x will be real, so we can use the quadratic formula for $ax^2+bx+c=0, a\neq 0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}, \text{ when } (b^2-4ac)\geq 0$$

Example 1. Solve quadratic equation $2x^2-7x+3=0$ by perfect square method and verify the roots by quadratic formula of Shrihar Acharya.

Solution : Given equation is

$$2x^2-7x+3=0$$

or
$$x^2-\frac{7}{2}x+\frac{3}{2}=0$$

or $$x^2 - \frac{7}{2}x = \frac{-3}{2}$$

For making perfect square, adding square of half of coefficient of x in both sides, we get

$$x^2 - \frac{7}{2}x + \left(\frac{7}{4}\right)^2 = \frac{-3}{2} + \left(\frac{7}{4}\right)^2$$

or $$\left(x - \frac{7}{4}\right)^2 = \frac{-24+49}{16} = \frac{25}{16}$$

or $$\left(x - \frac{7}{4}\right)^2 = \left(\frac{5}{4}\right)^2$$

Taking square roots on both sides, we get

$$x - \frac{7}{4} = \pm\frac{5}{4}$$

or $x - \frac{7}{4} = \frac{5}{4}$ or $x - \frac{7}{4} = \frac{-5}{4}$

or $x = \frac{7}{4} + \frac{5}{4} = 3$ or $x = \frac{7}{4} - \frac{5}{4} = \frac{1}{2}$

Thus $x = 3$ and 1/2 are solution of given equation.

Verification by Shridhar Acharya quadratic formula.

By comparing equation $ax^2 + bx + c = 0$ with equation $2x^2 - 7x + 3 = 0$, we get $a = 2, b = -7, c = 3$

Here $b^2 - 4ac = (-7)^2 - 4\times2\times3 = 25 \geq 0$ So roots are real. So substituting values of *a, b, c* in quadratic formula, we get.

$$x = \frac{+7 \pm \sqrt{(-7)^2 - 4\times2\times3}}{2\times2} = \frac{7\pm5}{4}$$

Thus, $x = \frac{7+5}{4}$ or $x = \frac{7-5}{4}$

i.e., $x = 3$ and $x = 1/2$ are required roots **Hence Proved**

Hence, solution of the given equation is verified by Shridhar Acharya quadratic formula.

Exercise 3.4

1. Solve the following quadratic equation by the method of perfect the square.

   (i) $3x^2 - 5x + 2 = 0$ (ii) $5x^2 - 6x - 2 = 0$ (iii) $4x^2 + 3x + 5 = 0$ (iv) $4x^2 + 4\sqrt{3}x + 3 = 0$

   (v) $2x^2 + x - 4 = 0$ (vi) $2x^2 + x + 4 = 0$ (vii) $4x^2 + 4bx - (a^2 - b^2) = 0$

2. Find the roots of the following quadratic equations, if they exist, using the quadratic formula of Shridhar Acharya.

(i) $2x^2 - 2\sqrt{2} + 1 = 0$ (ii) $9x^2 + 7x - 2 = 0$ (iii) $x + \frac{1}{x} = 3, x \neq 0$

(iv) $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$ (v) $x^2 + 4x + 5 = 0$

(vi) $\frac{1}{x} - \frac{1}{x-2} = 3,\ x \neq 0, 2$

3. Find two consecutive positive odd integers, sum of whose squares is 290.
4. The difference of square of two numbers is 45 and square of smaller number is four times the larger number. Find the two numbers.
5. Divide 16 into two parts such that 2 times the square of larger part is 164 more than the square of smaller part.

3.08 Discriminant and Nature of Roots

In previous appendix we have studied about solving of quadratic equation $ax^2 + bx + c = 0, a \neq 0$ by factorisation, perfect square method and Shridhar Acharya method. In appendix 3.07 we have used the following formula to solve quadratic equation $ax^2 + bx + c = 0$ by Shridhar Acharya method

Formula $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \text{. . . (i)}$$

Where, $(b^2 - 4ac) \geq 0$ for real roots, from this we obtained two real roots

$(b^2 - 4ac) < 0$ for unreal roots, since $(b^2 - 4ac)$ will be negative and so its square root will be imaginary. So nature of roots depends on $(b^2 - 4ac)$. Thus $(b^2 - 4ac)$ is called discriminant of quadratic equation $ax^2 + bx + c = 0$. So nature of roots can be determined by different values of discriminant as follows :

(i) If $(b^2 - 4ac) > 0$, then roots will be distinct and real.

If α, β are roots then

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

(ii) If $(b^2 - 4ac) = 0$, then roots will be equal and real *i.e.,* $\alpha = \frac{-b}{2a}, \beta = \frac{-b}{2a}$

(iii) If $(b^2 - 4ac) < 0$, then roots will be imaginary. (not real)

Now, we can clearly understand all the three types of nature of roots of quadratic equations by the following examples :

Example 1. Find the nature of the root of the following quadratic equations and if roots exits then find them.

(i) $2x^2 - 6x + 3 = 0$ **(ii)** $3x^2 - 4\sqrt{3}\,x + 4 = 0$ **(iii)** $x^2 + x + 1 = 0$

Solution : (i) Given equation is

$$2x^2 - 6x + 3 = 0$$

On comparing this equation by $ax^2 + bx + c = 0$, we have

$a = 2, b = -6, c = 3$

Now, $b^2 - 4ac = 12 > 0$ is positive, so roots of given equation will be distinct and real.

So by Shridhar Acharya quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Two required roots will be $x = \dfrac{+6 \pm \sqrt{12}}{4}$

i.e., $x = \dfrac{3 + \sqrt{3}}{2}$ or $x = \dfrac{3 - \sqrt{3}}{2}$

(ii) Here equation is $3x^2 - 4\sqrt{3}x + 4 = 0$

On comparing given equation by $ax^2 + bx + c = 0$, we have

$$a = 3,\ b = -4\sqrt{3},\ c = 4$$

and Discriminant $(b^2 - 4ac) = (-4\sqrt{3})^2 - 4 \times 3 \times 4$

$= 48 - 48 = 0$

So roots of given equation will be equation and real. By Shridhar Acharya Quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

i.e., $x = \dfrac{4\sqrt{3} \pm 0}{2 \times 3} = \dfrac{2}{\sqrt{3}}$

(iii) Given equation is $x^2 + x + 1 = 0$

On comparing given equation by $ax^2 + bx + c = 0$, we have

$$a = 1,\ b = 1,\ c = 1$$

$\therefore$ Discriminant $(b^2 - 4ac) = 1 - 4 = -3 < 0$,so roots will be imaginary.

Exercise 3.5

1. Find the nature of roots of the following quadratic equations.

(i) $2x^2 - 3x + 5 = 0$ (ii) $2x^2 - 4x + 3 = 0$ (iii) $2x^2 + x - 1 = 0$ (iv) $x^2 - 4x + 4 = 0$

(v) $2x^2 + 5x + 5 = 0$ (vi) $3x^2 - 2x + \dfrac{1}{3} = 0$

2. Find the value of k in the following quadratic equations for which roots are real and equal.

(i) $kx(x-2) + 6 = 0$ (ii) $x^2 - 2(k+1)x + k^2 = 0$ (iii) $2x^2 + kx + 3 = 0$

(iv) $(k+1)x^2 - 2(k-1)x + 1 = 0$ (v) $(k+4)x^2 + (k+1)x + 1 = 0$

(vi) $kx^2 - 5x + k = 0$

3. Find the value of k for which following quadratic equations have real and distinct roots :

(i) $kx^2+2x+1=0$ (ii) kx^2+6x+1 (iii) $x^2-kx+9=0$

4. Find the value of k so that equation $x^2+5kx+16=0$ has no real roots.

5. If roots of quadratic equation $(b-c)x^2+(c-a)x+(a-b)=0$ are real and equal then prove that $2b=a+c$.

3.09 Least common Multiple (L.C.M.) and Highest Common Multiple (H.C.M.) of Algebraic Expressions

In previous chapter we have studied LCM and HCF of positive real integers by using fundamental therorem of airthmetic. LCM is product of the greatest power of each prime factor, involved in the number and HCF is product of the smallest power of each common prime factor in the numbers. Here we will study about HCF and LCM of algebraic expressions.

Least Common multiple (LCM) : LCM of given expression $u(x)$ and $v(x)$ is a polynomial which is product of least power polynomial and least power numeric coefficient. Here sign of coefficient of highest power term is same the term of highest power of $u(x)$, $v(x)$.

Highest common factor (HCF) : In two expression $u(x)$ and $v(x)$, product of highest power factors is called HCF and its coefficient is taken as positive. So HCF of two given polynomial is obtained by the product of highest power common factor and maximum divisor of numeric coefficient. Relation between LCM and HCF of any polynomials is written as.

$$\text{LCM} \times \text{HCF} = u(x) \times v(x)$$

In this appendix, common factor means an expression which when divide each expression gives remainder as 0 and common multiple means that if f (x) is a common multiple then it should be completely divided by given polynomials.

Method to find LCM and HCF of expressions and polynomials can be easily understand by the following examples.

Example 1. Find the LCM of following expressions.

(i) $4a^2b^2c$ and $6ab^2d$

(ii) x^2-4x+3 and x^2-5x+6

(iii) $-2(x-1)(x-2)(x+3)$ and $3(x-1)(x-2)(x+3)(x+5)$

Solution : (i) Let given expression $u(x)=4a^2b^2c$ and $v(x)=6ab^2d$

Writing in factorisation form

$u=2^2\times a^2\times b^2\times c$ and $v=2\times 3\times a\times b^2\times d$

So common multiple

$$=2^2\times 3^1\times a^2\times b^2\times c\times d$$

= product of common factor of highest power

This common multiple is required LCM

i.e., LCM = 12 a^2b^2cd

(ii) Let in given polynomial $u(x) = x^2 - 4x + 3$ and $v(x) = x^2 - 5x + 6$

Writing these in factorisation form

$$u(x) = x^2 - 4x + 3 = x^2 - 3x - x + 3$$

$$= x(x-3) - 1(x-3) = (x-3)(x-1) \quad \ldots (1)$$

and $v(x) = x^2 - 5x + 6 = x^2 - 3x - 2x + 6$

$$= x(x-3) - 2(x-3) = (x-3)(x-2) \quad \ldots (2)$$

From equation (1) and (2), It is clear that product of highest power of prime factors

$$= (x-1)\times(x-2)\times(x-3)$$

$\therefore$ Required LCM $= (x-1)(x-2)(x-3)$

(iii) Let given polynomial

$u(x) = -2(x-1)(x-2)(x+3)$ and

$v(x) = 3(x-1)(x-2)(x+3)(x+5)$

By observation we can write product of common factors

$$= -2\times 3\times(x-1)\times(x-2)\times(x+3)\times(x+5)$$

In this product, highest power factorization has same sign as of highest power term $-6x^2$ of $u(x)\times v(x)$

Thus, required LCM $= -6(x-1)(x-2)(x+3)(x+5)$

Example 2. Find the highest common factor (HCF) of the folowing.

(i) $8a^2b^2c$ and $18ab^3c^2$ (ii) $20x^2 - 9x + 1$ and $5x^2 - 6x + 1$

(iii) $(x+1)^2(x+2)^2(x+3)^2$ and $(x+1)^3(x-2)^2(x+3)^3$

Solution : (i) Let given expression $u = 8a^2b^2c$ and $v = 18ab^3c^2$

writing in factorisation form $u = 2^3\times a^2\times b^2\times c$ and $v = 2\times 3^2\times a\times b^3\times c^2$

Common divisor of highest power $= 2\times a\times b^2\times c$

Product of common factors of least power

Thus required (HCF) = $2ab^2c$

(ii) Let given polynomial $u(x) = 20x^2 - 9x + 1$ and $v(x) = 5x^2 - 6x + 1$

Writing these in factorisation form

$$u(x) = 20x^2 - 9x + 1 = 20x^2 - 5x - 4x + 1$$

$$= 5x(4x-1) - 1(4x-1) = (4x-1)(5x-1) \quad \ldots (1)$$

and $v(x) = 5x^2 - 6x + 1 = 5x^2 - 5x - x + 1$

$$= 5x(x-1) - 1(x-1) = (x-1)(5x-1) \quad \ldots (2)$$

Form equation (1) and (2) it is clear that common divisor highest power is $(5x-1)$

Thus required HCF $= (5x-1)$

(iii) Let $u(x) = (x+1)^2(x+2)^2(x+3)^2$ and $v(x) = (x+1)^3(x-2)^2(x+3)^3$

$\therefore$ Common divisor of highest power $= (x+1)^2(x+3)^2$

= Product of common factors of least power

Thus required $\text{HCF} = (x+1)^2(x+3)^2$

Exercise 3.6

1. Find the LCM of following expressions :

(i) $24x^2yz$ and $27x^4y^2z^2$ (ii) x^2-3x+2 and $x^4+x^3-6x^2$

(iii) $2x^2-8$ and x^2-5x+6 (iv) x^2-1; $(x^2+1)(x+1)$ and x^2+x-1

(v) $18(6x^4+x^3-x^2)$ and $45(2x^6+3x^5+x^4)$

2. Find the HCF of the following expressions

(i) a^3b^4, ab^5, a^2b^8 (ii) $16x^2y^2, 48x^4z$

(iii) $x^2-7x+12$; $x^2-10x+21$ and $x^2+2x-15$ (iv) $(x+3)^2(x-2)$ and $(x+3)(x-2)^2$

(v) $24(6x^4-x^3-2x^2)$ and $20(6x^6 ? 5x^5 ? x^4)$

3. If $u(x)=(x-1)^2$ and $v(x)=(x^2-1)$ then verify $\text{LCM}\times\text{HCF}=u(x)\times v(x)$.

4. The product of two expressions is $(x-7)(x^2+8x+12)$. If their highest common factor (HCF) is $(x+6)$ then find their least common multiple (LCM).

5. If HCF and LCM of two quadratic expression are $(x-5)$ and $x^3-19x-30$ then find two expressions.

Miscellaneous Exercise 3

1. If one zero of polynomial $f(x)=5x^2+13x+k$ is reciprocal to other zero, then value of k will be :

(a) 0 (b) 1/5 (c) 5 (d) 6

2. Zeroes of polynomial x^2-x-6 are :

(a) 1, 6 (b) 2, −3 (c) 3, −2 (d) 1, −6

3. If 3 is a zero of polynomial $2x^2+x+k$, then value of k will be :

(a) 12 (b) 21 (c) 24 (d) − 21

4. If α, β are zeroes of polynomial $x^2-p(x+1)-c$ such that $(\alpha+1)(\beta+1)=0$, then value of c will be:

(a) 0 (b) −1 (c) 1 (d) 2

5. If quadratic equation $x^2-kx+4=0$ has equal roots, then value of k will be :

(a) 2 (b) 1 (c) 4 (d) 3

6. If $x ? 1$ is common root of equations $ax^2+ax+3=0$ and $x^2+x+b=0$, then value of ab will be :

(a) 1 (b) 3.5 (c) 6 (d) 3

7. Discriminant of quadratic equation $3\sqrt{3}\,x^2+10x+\sqrt{3}=0$ will be :

(a) 10 (b) 64 (c) 46 (d) 30

8. Nature of roots of quadratic equation $4x^2 - 12x - 9 = 0$ is :

(a) real and equal (b) real and distinct

(c) imaginary and equal (d) imaginary and distinct

9. HCF of expression $8a^2b^2c$ and $20ab^3c^2$ is :

(a) $4ab^2c$ (b) $4abc$ (c) $40a^2b^3c^2$ (d) $40abc$

10. LCM of expressions $x^2 - 1$ and $x^2 + 2x + 1$ is :

(a) $x+1$ (b) $(x^2-1)(x+1)$ (c) $(x-1)(x+1)^2$ (d) $(x^2-1)(x+1)^2$

11. If $30x^2y^4$ is LCM of expressions $6x^2y^4$ and $10xy^2$, then their HCF will be :

(a) $6x^2y^2$ (b) $2xy^2$ (c) $10x^2y^4$ (d) $60x^3y^6$

12. Write Shridhar Acharya's formula to find roots of quadratic equation $ax^2 + bx + c = 0$

13. Find the nature of roots by writing general form of discriminant of equation $ax^2 + by + c = 0$

14. Find the zeroes of qudratic polynomial $2x^2 - 8x + 6$ and verify the relation between zeroes and coefficients.

15. If α and β are zeroes of quadratic polynomial $f(x) = x^2 - px + q$, then find the value of the following.

(i) $\alpha^2 + \beta^2$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta}$

16. If polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$ and leaves remainder comes $(x + a)$, then find the value of k and a.

17. The are of a rectangular plot is 528m². Length (in m.) of plot is 1 more than twice the breadth. Determine required quadratic equation and find the lenght and breadth of the plot.

18. Solve the qudratic equation $x^2 + 4x - 5 = 0$ by perfect the square method.

19. Solve the following equation by factorisation method.

(i) $\frac{1}{x} - \frac{1}{x-2} = 3, \quad x \neq 0, 2$ (ii) $\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}, \quad x \neq 1, -5$

(iii) $x - \frac{1}{x} = 3, \quad x \neq 0$ (iv) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, \quad x \neq 4, 7$

20. If -5 is one root of quadratic equation $2x^2 + px - 15 = 0$ and roots of quadratic equation $p(x^2 + x) + k = 0$ is equal, then find the value of k.

21. Solve the following quadratic equation by using Shridhar Acharya quadratic formula.

(i) $p^2x^2 + (p^2 - q^2)x - q^2 = 0$ (ii) $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$

22. $x^3 - 7x + 6$ and $(x-1)$ are LCM and HCF of two quadratic expressions. Find the expressions.

23. LCM and HCF of two polynomials are $x^3 - 6x^2 + 3x + 10$ and $(x+1)$ respectively. If one polynomial is $x^2 - 4x + 5$, then find other polynomial.

Important Points

1. Generally $ax+b$ is linear, ax^2+bx+c is quadratic and ax^3+bx^2+cx+d is called cubical polynomial.
2. Polynomial $f(x)=0$ for which value of x, that value is called zero of $f(x)$.
3. Number of zeroes of a polynomial is equal to its highest power. A quadratic polynomial has maximum two zeroes.
4. If α, β are zeroes of ax^2+bx+c, then $(\alpha+\beta)=\frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$
5. If α, β are zeroes of any quadratic polynomial , then it can be written as $k[x^2-(\alpha+\beta)x+\alpha\beta]$
6. **Division Algorithm :** If $f(x)$ and $g(x)$ are any polynomials, then we get $g(x)$ and $r(x)$ such polynomials, so that $f(x)=q(x)\,g(x)+r(x)$, where $r(x)=0$ of $r(x)$ that power of $g(x)$
7. If $f(x)=ax^2+bx+c$ is a quadratic polynomial, then $f(x)=0, a\neq 0$ is a quadratic equation. Zeroes of polynomial $f(x)$ and roots of quadratic equation $f(x)=0$ are same.
8. **Solving of quadratic equation :** (i) By writing in standard form $f(x)=0$, factorise into a product of two linear factors then the roots of the quadratic equation can be found by equating each factor to zero.

   (ii) By Shridhar Acharya, roots of quadratic equation $ax^2+bx+c=0,\ a\neq 0$ are given by following quadratic formula.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}, \text{ where } (b^2-4ac)>0$$

9. Nature of roots of a quadratic equation $ax^2+bx+c=0,\ a\neq 0$ depends on discrimenant (b^2-4ac)

   (i) If $(b^2-4ac)>0$, then roots will be real and distinct.

   (ii) If $(b^2-4ac)=0$, then roots will be real and equal.

   (iii) If $(b^2-4ac)<0$, then roots will be imaginary.
10. Highest common factor (HCF) of given expression is common factor of highest power *i.e.,* product of common factors of least power of expressions.
11. LCM of given expression is product of common factors of smallest power i.e., common multiple. Its sign is same as the product of highest power terms have.
12. If $f(x)$ and $g(x)$ are two expressions,then relation between their LCM and HCF is

    LCM × HCF = $f(x) \times g(x)$

Answer Sheet

Exercise 3.1

1. (i) –2, 0 (ii) $\frac{1}{2}, \frac{1}{2}$ (iii) $\frac{2}{3}, -\frac{1}{2}$ (iv) $-\sqrt{15}, \sqrt{15}$ (v) $1, \sqrt{3}$ (vi) $-1, \frac{4}{3}$

2. (i) $x^2 + 3x + 2$ (ii) $3x^2 - 3\sqrt{2}x + 1$ (iii) $4x^2 + x + 1$ (iv) $x^2 + \sqrt{5}$
(v) $x^2 - 4x + 1$ (vi) $x^2 - x + 1$

3. 12

Exercise 3.2

1. (i) $3x - 5;\ 9x + 10$ (ii) $x - 3;\ 7x - 9$ (iii) $x^2 - 8x + 27;\ -60$ (iv) $3x^2 - x;\ -x + 4$

3. (i) $\frac{1}{2}, 1$ (ii) –5, 7 (iii) –10, –1

4. $x^2 - x + 1$

Exercise 3.3

1. (i) No, (ii) Yes, (iii) No, (iv) No.

2. (i) $1, \frac{3}{2}$ (ii) $-\frac{1}{3}, \frac{2}{3}$ (iii) $-\sqrt{3}, -\frac{7}{\sqrt{3}}$ (iv) 4, 4 (v) $3, \frac{4}{3}$
(vi) $\frac{1}{10}, \frac{1}{10}$ (vii) $\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$ (viii) –1, –7 (ix) –1, 5 (x) $\frac{a^2 + b^2}{2}, \frac{a^2 - b^2}{2}$
(xi) $-\frac{b}{a}, \frac{c}{b}$

Exercise 3.4

1. (i) $1, \frac{2}{3}$ (ii) $\frac{3 \pm \sqrt{19}}{5}$ (iii) not real roots, (iv) $\frac{-\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$ (v) $\frac{-1 \pm \sqrt{33}}{4}$
(vi) not real roots (vii) $\frac{-(a+b)}{2}, \frac{(a-b)}{2}$

2. (i) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ (ii) $\frac{2}{9}, -1$ (iii) $\frac{3 \pm \sqrt{5}}{2}$ (iv) $-\sqrt{2}, \frac{-5}{\sqrt{2}}$ (v) not real roots, (vi) $\frac{3 \pm \sqrt{3}}{3}$

3. 11, 13

4. 9, 6 and 9, –6

5. 10, 6

Exercise 3.5

1. (i) not real roots (ii) no any real roots (iii) roots are real and distinct
(iv) roots are real and equal (v) roots are not real (vi) roots are real and equal

2. (i) $K = 0,6$ (ii) $k = -\frac{1}{2}$ (iii) $k \le -2\sqrt{6},\ \ k \ge 2\sqrt{6}$
(iv) k = 0, 3 (v) k = 5, –3 (vi) $k = \pm\frac{5}{2}$

3. (i) $k < 1$ (ii) $k < 9$ (iii) $k < -6,\ \ k > 6$ 4. $\frac{-8}{5} < k < \frac{8}{5}$

Exercise 3.6

1. (i) $216x^4y^2z^2$ (ii) $x^2(x-1)(x-2)(x+3)$ (iii) $2(x^2-4)(x-3)$

(iv) $(x^4-1)(x^2+x-1)$ (v) $90x^4(x+1)(2x+1)(3x-1)$

2. (i) ab^4 (ii) $16x^2$ (iii) $(x+3)$ (iv) $(x+3)(x-2)$ (v) $4x^2(2x+1)$

3. $\text{LCM} = (x-1)^2(x+1)$; $\text{HCF} = (x-1)$ 4. $\text{LCM} = x^2-5x-14$

5. $x^2-3x-10$ and $x^2-2x-15$

Miscellaneous Exercise 3

1. (c) 2. (c) 3. (d) 4. (c) 5. (c) 6. (d) 7. (b)
8. (b) 9. (a) 10. (c) 11. (b)

12. $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

13. Discriminant (b^2-4ac),

(i) $b^2-4ac > 0$, real and disinct roots, (ii) $b^2-4ac = 0$, real and equal roots

(iii) $b^2-4ac < 0$, imaginary roots

14. 1, 3 15. (i) p^2-2q (ii) $\frac{p}{q}$ 16. $k = 5$ and $a = -5$

17. $2x^2+x-528=0$, breadth $= 16$ m and length $= 33$ m 18. 1, –5

19. (i) 2, –6; (ii) $\frac{3\pm\sqrt{3}}{3}$; (iii) $\frac{3\pm\sqrt{13}}{2}$; (iv) 1, 2 20. $k = \frac{7}{4}$

21. (i) $-1, \frac{q^2}{p^2}$; (ii) $\frac{2a+b}{3}, \frac{a+2b}{3}$ 22. x^2+2x-3 and x^2-3x+2 23. x^2-x-2