CBSE Class 10th Mathematics
Basic Sample Paper - 01

Maximum Marks:
Time Allowed: 3 hours

General Instructions:

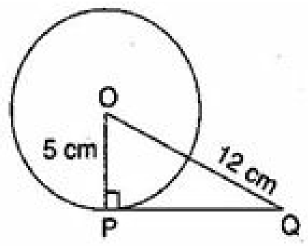
a. All questions are compulsory
b. The question paper consists of 40 questions divided into four sections A, B, C & D.
c. Section A comprises of 20 questions of 1 mark each. Section B comprises of 6 questions of 2 marks each. Section C comprises of 8 questions of 3 marks each. Section D comprises 6 questions of 4 marks each.
d. There is no overall choice. However internal choices have been provided in two questions of 1 mark each, two questions of 2 marks each, three questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
e. Use of calculators is not permitted.

Section A

1. The decimal form of $\frac{5}{8}$ is:
   a. 0.625
   b. 0.600
   c. 0.750
   d. 0.375
2. For any positive integer ‘a’ and 3, there exist unique integers ‘q’ and ‘r’ such that a = 3q + r where ‘r’ must satisfy
   a. 1 < r < 3
   b. $0 < r \leqslant 3$
   c. $0 \leqslant r < 3$
   d. 0 < r < 3
3. Every positive odd integer is of the form 2q + 1, where ‘q’ is some
   a. None of these

b. whole number
c. natural number
d. integer

4. A tangent PQ at a point P o a circle of radius 5 cm meets a line through the centre O at point Q, so that OQ = 12 cm. find the length PQ.



a. $\sqrt{113}$ cm.
b. 13 cm
c. 26 cm
d. $\sqrt{119}\ cm.$

5. The percentage of marks obtained by 100 students in an examination are as follows:

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Frequency	14	16	18	23	18	8	3

The median class is
a. 40 – 45
b. 50 – 55
c. 45 – 50
d. none of these

6. A bag contains 6 red, 8 white, 4 green and 7 black balls. One ball is drawn at random. The probability that the ball drawn is neither green nor white is
a. $\frac{8}{25}$
b. $\frac{12}{25}$
c. $\frac{13}{25}$
d. $\frac{4}{25}$

7. The polynomial to be added to the polynomial $x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$ is
a. $x^2 + 1$
b. 2 – x
c. x – 2

d. x + 2

8. If $x^3 + x^2 - 2x - 3 = (x-2)(x^2 + ax + b) + 5$, then
   a. a = – 3 and b = – 4
   b. a = 5 and b = 6
   c. a = 4 and b = 5
   d. a = 3 and b = 4
9. The distance of a point from the y – axis is called
   a. origin
   b. None of these
   c. abscissa
   d. ordinate
10. The co – ordinates of the point which divides the join of (– 6, 10) and (3, – 8) in the ratio 2 : 7 is
    a. (4, – 6)
    b. (– 4, 6)
    c. (1, – 3)
    d. (– 1, 3)
11. Fill in the blanks:

    The distance of point P(3, 4) from the origin is ________.
12. Fill in the blanks:

    A system of two linear equations in two variables has no solution, if their graphs _____________ at any point.

    OR

    Fill in the blanks:

    The equations x + 2y = 4 and 2x + y = 5 will have ________ solution.
13. Fill in the blanks:

    In right angled triangle, the square of the ________ is equal to the sum of the squares of the other two sides.
14. Fill in the blanks:

    If x tan 45^{o}cos60^{o} = sin60^{o} cot 60^{o}, then the value of x is _______.
15. Fill in the blanks:
16. All ________ triangles are similar.

17. Solve: $2\cos 3\theta = 1$

OR

Without using trigonometric tables, evaluate: $\frac{\sin 16^\circ}{\cos 74^\circ}$

18. What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40 cm and 9 cm?

19. The probability of getting a bad egg in a lot of 400 eggs is 0.035. Find the number of bad eggs in the lot.

20. If $\triangle$ABC ~ $\triangle$DEF such that 2AB = DE and BC = 6 cm, find EF.

21. Show that a - b, a and a + b form consecutive terms of an AP.

Section B

22. All kings and queens are removed from a pack of 52 cards. The remaining cards are well-shuffled and then a card is randomly drawn from it. Find the probability that this card is

    i. a red face card
    ii. a black card.

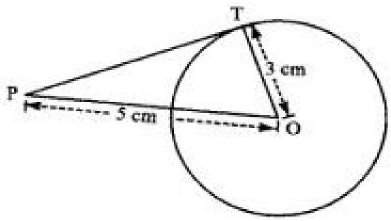
23. Cards marked with numbers 13, 14, 15, ..., 60 are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that number on the card drawn is

    i. divisible by 5
    ii. a number is a perfect square

24. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80^o, then find $\angle$POA.

OR

Find the length of the tangent from a point which is at a distance of 5 cm from the centre of the circle of radius 3 cm.

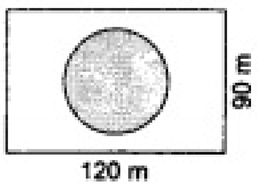


25. If θ is an acute angle and $\sin\theta = \cos\theta$, find the value of $2\tan^2\theta + \sin^2\theta - 1$.

OR

Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$.

26. A park is in the form of a rectangle 120 m by 90 m. At the centre of the park, there is a circular lawn as shown in the figure. The area of the park excluding the lawn is 2950 m^2. Find the radius of the circular lawn. [Given, π = 3.14.]



27. In the class test of mathematics, a teacher asked his students to write different kinds of polynomials. 6 students wrote the following polynomials. Identify the type of polynomials written by these students:

i. $f(p) = 3 - p^2 + \sqrt{7}p$
ii. $p(v) = \sqrt{3}v^4 - \frac{2}{3}v + 7$
iii. $q(x) = \frac{\sqrt{2}}{5}x^3 + 1$
iv. $p(z) = \sqrt{5}z + 2\sqrt{2}$
v. $r(t) = \frac{-t+3t^2-4t^3}{t}$

Section C

28. Divide $p(x) = 6x^5 + 4x^4 - 27x^3 - 7x^2 - 27x - 6$ by $q(x) = 2x^2 - 3$.

29. Construct a $\triangle ABC$ in which AB = 6.5cm, $\angle B = 60^\circ$ and BC = 5.5 cm. Also construct a triangle ABC similar to $\triangle ABC$ whose each side is $\frac{3}{2}$ times the corresponding side of the $\triangle ABC$.

OR

Draw a pair of tangents to a circle of radius 4.5 cm, which are inclined to each other at an angle of 45°.

30. A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of rice. How much canvas cloth is required to just cover the heap?

31. If $\cos\theta = \frac{12}{13}$, show that $\sin\theta(1 - \tan\theta) = \frac{35}{156}$

OR

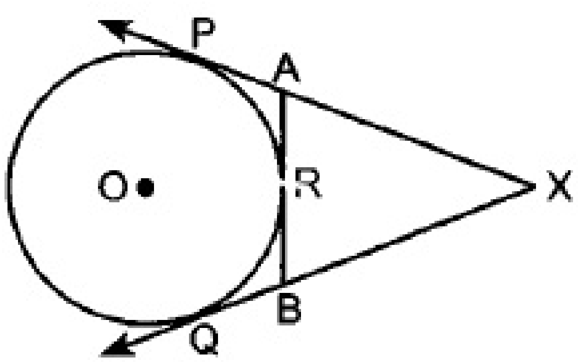
If 3sinθ + 5cosθ = 5, prove that 5sinθ - 3cosθ = ± 3.

32. Find the largest number which divides 546 and 764, leaving remainders 6 and 8 respectively.

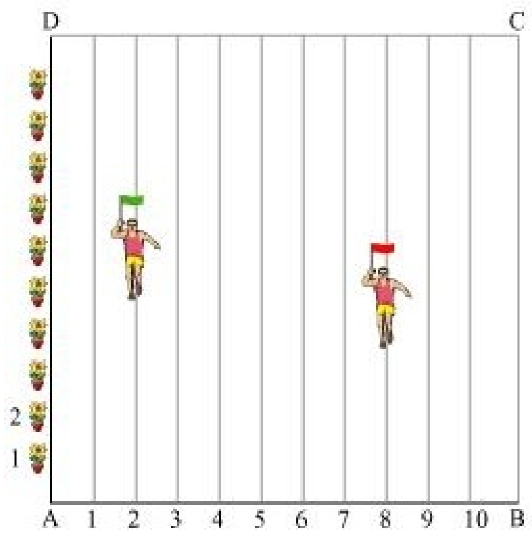
OR

Show that $3\sqrt{2}$ is an irrational number.

33. In figure, XP and XQ are two tangents to a circle with centre O from a point X outside the circle. ARB is tangent to circle at R. Prove that XA + AR = XB + BR.



34. To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in Fig. Niharika runs the distance AD on the 2nd line and posts a green flag. Preet runs the distance AD on the eighth line and posts a red flag.



i. Calculate the distance Niharika and Preet posted the green flag and reg flag respectively.
ii. What is the distance between both the flags?
iii. If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

35. Solve: $\frac{x}{a} + \frac{y}{b} = 2$ ax - by =a^2 - b^2

Section D

36. The difference of two numbers is 5 and the 1 difference of their reciprocals is $\frac{1}{10}$. Find the numbers

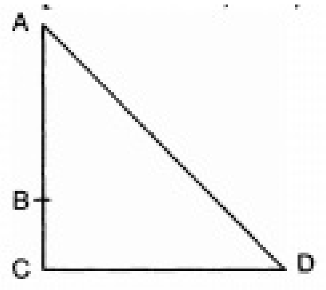
37. The sum of the first three numbers in an Arithmetic Progression is 18. If the product of the first and the third term is 5 times the common difference, find the three numbers.

OR

If the sum of the 3^{rd} and 7^{th} terms of an A.P. is 6 and their product is 8. Find the sum of the first 20 terms of the A.P.

38. A round balloon of radius r subtends an angle α at the eye of the observer while the angle of elevation of its centre is β. Prove that the height of the centre of the balloon is r sin βcosec $\frac{\alpha}{2}$.

39. In the right triangle, B is a point on AC such that AB + AD = BC + CD. If AB = x, BC = h and CD = d, then find x (in term of h and d).



OR

A ladder reaches a window which is 12 m above the ground on one side of the street. Keeping its foot at the same point, the ladder is turned to the other side of the street to reach a window 9 m high. Find the width of the street if the length of the ladder is 15 m.

40. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of the base of the cylinder or the cone is 24 m. The height of the cylinder is 11 m. If the vertex of the cone is 16 m above the ground, find the area of the canvas required for making the tent. (Use π = 22/7)

OR

Due to some floods, some welfare associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but of height 2.8 m, and the canvas to be used costs Rs.100 per square metre, find the amount the associations will have to pay?

41. The following table shows the marks scored by 140 students in an examination of a certain paper:

Marks	0-10	10-20	20-30	30-40	40-50
Number of students	20	24	40	36	20

Calculate the average marks by using all the three methods: direct method, assumed mean deviation and shortcut method.

Solution

Section A

1. (a) 0.625

   Explanation:

   Use long division:

$$\begin{array}{r|l} & 0.625 \\ 8 & \overline{5.0000} \\ & 48 \\ & 20 \\ & 40 \\ & 0 \end{array}$$

   Thus '$\frac{5}{8}$ =0.625'.

2. (c) $0 \leqslant r \ < \ 3$

   Explanation:

   Since a is a positive integer, therefore, r = 0, 1, 2 only.

   So, that a = 3q, 3q + 1, 3q + 2.

3. (d) integer

   Explanation:

   Euclid's Division Lemma states that

   for given positive integer a and b, there exist unique integers q and r

   satisfying $a = bq + r$; $0 \leqslant r < b$.

   let b=6 then possible values of r will be 0,1,2,3,4,5

   when b =6 , r = 0 then a = 6q +0

r = 1	a = 6q +1

r = 2	a = 6q +2
r = 3	a = 6q + 3
r = 4	a = 6q +4
r = 5	a = 6q + 5

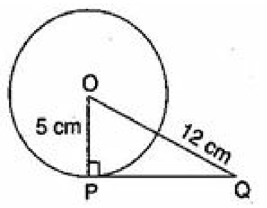
but 6q , 6q + 2, 6q + 4 cannot be because they are all positive even integers while a is odd integer

thus we can say that a can be 6q + 1 or 6q + 3 or 6q + 5

Hence, general form is bq + 1.

4. (d) $\sqrt{119}\,cm$

   Explanation:

   $\angle$OPQ = 90° [Angle between tangent and radius through the point of contact]

   $\therefore$ $OQ^2 = OP^2 + PQ^2 \Rightarrow (12)^2 = (5)^2 + PQ^2$

   $\Rightarrow PQ^2 = 144 - 25 \Rightarrow PQ^2 = 119$

   $\Rightarrow \mathrm{PQ} = \sqrt{119}$

5. (c) 45 – 50

   Explanation:

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Frequency	14	16	18	23	18	8	3
Cumulative Frequency	14	30	48	71	89	97	100

Here N = 100

$\Rightarrow \frac{\mathrm{N}}{2} = 50$

Therefore, Median class is 45 – 50.

6. (c) $\frac{13}{25}$

   Explanation:

   Total number of balls = 25

   Number of Green and White balls = 4 +8 = 12

   Number of balls neither green nor white = 25 - 12 =13

   Number of possible outcomes = 13
   Number of total outcomes = 25
   $\therefore$ Required Probability = $\frac{13}{25}$

7. (c) x – 2
   Explanation:

$$\begin{array}{r|l} & \qquad\qquad\qquad x^2 \\ \hline x^2+2x-3 & x^4+2x^3-2x^2+x-1 \\ & x^4+2x^3-3x^2 \\ & - \quad - \qquad + \\ \hline & \qquad\qquad x^2+x-1 \end{array}$$

   Term to be added is obtained by subtracting the remainder from the divisor Now,
   $\left(x^2+2x-3\right)-\left(x^2+x-1\right)=x-2$
   Therefore, $(x-2)$ is the polynomial which to be added to the given polynomial(+1 is mistakenly placed).

8. (d) a = 3 and b = 4
   Explanation:

   Given: $x^3+x^2-2x-3=(x-2)(x^2+ax+b)+5$ Dividing L.H.S. by $(x-2)$

$$
\begin{array}{r|l}
 & x^2+3x+4 \\
\hline
x-2 & x^3+x^2-2x-3 \\
 & x^3-2x^2 \\
 & -\quad + \\
\hline
 & \quad 3x^2-2x-3 \\
 & \quad 3x^2-6x \\
 & \quad -\quad + \\
\hline
 & \qquad 4x-3 \\
 & \qquad 4x-8 \\
 & \qquad -\quad + \\
\hline
 & \qquad\quad 5
\end{array}
$$

$\therefore (x-2)\left(x^2+3x+4\right)+5=(x-2)(x^2+ax+b)+5$

Comparing both side, we have $a=3, b=4$

9. (c) abscissa

   Explanation:

   The distance of a point from the y – axis is the x (horizontal) coordinate of the point and is called abscissa.

10. (b) (– 4, 6)

    Explanation:

    Given: $(x_1, y_1)=(-6,10),(x_2, y_2)=(3,-8)$
    and $m_1 : m_2=2 : 7$
    $\therefore x=\frac{m_1 x_2+m_2 x_1}{m_1+m_2}$
    = $\frac{2 \times 3+7 \times(-6)}{2+7}=\frac{6-42}{9}=\frac{-36}{9}=-4$
    And $y=\frac{m_1 y_2+m_2 y_1}{m_1+m_2}$ = $\frac{2 \times(-8)+7 \times 10}{2+7}=\frac{-16+70}{9}=\frac{54}{9}=6$
    Therefore, the required coordinates are $(-4,6)$.

11. 5 units

12. do not intersect

    OR

Unique

13. hypotenuse

14. x = 1

15. equilateral

16. Now we have,
$2\cos 3\theta = 1$
$\Rightarrow \cos 3\theta = \frac{1}{2} \Rightarrow \cos 3\theta = \cos 60^\circ$ [Since, cos60°=(1/2)]
$\Rightarrow 3\theta = 60^\circ$
$\therefore \theta = 20^\circ$

OR

$\frac{\sin 16^\circ}{\cos 74^\circ}$
$= \frac{\sin(90^\circ - 74^\circ)}{\cos 74^\circ}$
$= \frac{\cos 74^\circ}{\cos 74^\circ} = 1\ [\because \sin(90^\circ - \theta) = \cos\theta]$

17. Area of the circle = sum of areas of two circles
$\pi r^2 = \pi \times (40)^2 + \pi(9)^2$
or, r^2 = 1600 + 81
or, $r = \sqrt{1681}$
$= 41\text{cm}.$
Diameter of circle is double of radius.
$\therefore$ Diameter of given circle =$2 \times r$
= 41×2
$= 82$ cm.

18. Here, P(bad eggs) = 0.035

Total no. of eggs = 400
$Probablity\ of\ event\ happen\ P(E) = \frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$
P(bad eggs) = $\frac{No.\text{ of bad eggs}}{\text{Total no. of eggs}}$
0.035 = $\frac{\text{No. of bad eggs}}{400}$

$\therefore$ No of bad eggs $= 400 \times 0.035$
= 14

19. Given that $\triangle$ABC ~ $\triangle$DEF

We know that when two triangles are similar, then the ratios of the lengths of their corresponding sides are equal.
$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF}$
$\Rightarrow \frac{1}{2} = \frac{6}{EF}$
$\Rightarrow$ EF = 12 cm

20. $a_1 = a - b,$

$a_2 = a,$

$a_3 = a + b$
Now, $a_2 - a_1 = a - (a - b) = b$

and $a_3 - a_2 = (a + b) - a = b$

$\because a_2 - a_1 = a_3 - a_2$

$\therefore$ Given terms are the consecutive terms in AP.

Section B

21. Face cards queen and king, that is 4 queens and 4 kings are removed.

Thus, in total 4 + 4 = 8 cards are removed.
The remaining number of cards = 52 - 8 = 44
n(total remaining cards) = 44

i. Total number of face cards = 6 + 6 = 12
Number of red face cards = 2 + 2 + 2 = 6
When we remove four red cards queen and king, two jack red face cards are remaining.
$\Rightarrow$n(a red face card) = 2
$\Rightarrow$P(a red face card) = $\frac{2}{44} = \frac{1}{22}$
ii. When we remove four black face cards queen and king, the number of black cards is 26 - 4 = 22

$\Rightarrow$n(a black card) = 22

$\Rightarrow$P(a black card) = $\frac{22}{44} = \frac{1}{2}$.

22. According to the question,we are given that,

Total cards = 48

{13,14,15...60}

i. Favourable outcomes of a card divisible by 5 = 10

{15, 20, 25, 30, 35, 40, 45, 50, 55, 60}

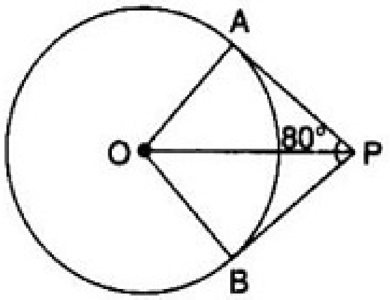
Therefore,Probability of card divisible by 5 = $\frac{10}{48} = \frac{5}{24}$

ii. Favourable outcomes of a card which is a perfect sqaure = 4

{16,25,36,49}

Therefore, Probability of a card which is a perfect square = $\frac{4}{48} = \frac{1}{12}$.

23. $\angle$OAP = 90^o [Angle between tangent and radius through the point of contact]



$\angle$OAP = $\frac{1}{2}\angle$BPQ [The centre lies on the bisector of the angle between the two tangents]

= $\frac{1}{2}(80^o) = 40^o$

In $\triangle$OPQ,

$\angle$OAP + $\angle$OPA + $\angle$POA = 180^o [$\because$ The sum of the three angle of a triangle is 180^o]

$\Rightarrow 90^o + 40^o + \angle POA = 180^o$

$\Rightarrow 130^o + \angle POA = 180^o$

$\Rightarrow \angle POA = 180^o - 130^o$

$\Rightarrow \angle POA = 50^o$

Hence, the $\angle$POA is 50^o.

OR

From Figure,

OT = 3 cm, OP = 5 cm [given]

Since, the radius of the circle is perpendicular to the tangent at the point of contact.

$\therefore \angle OTP = 90^o$

In right triangle OTP, OP is hypotenuse,

$\therefore OP^2 = OT^2 + TP^2$ [By Pythagoras theorem]

$\Rightarrow TP^2 = OP^2 - OT^2$

$\Rightarrow TP^2 = (5)^2 - (3)^2 = 25 - 9 = 16$

$\Rightarrow TP = \sqrt{16} = 4$

Hence, the length of the tangent is 4 cm.

24. We have,

$\sin\theta = \cos\theta$

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$ [Dividing both sides by $\cos\theta$]

$\Rightarrow \quad \tan\theta = 1$

$\Rightarrow \tan\theta = \tan 45^\circ$

$\Rightarrow \theta = 45^\circ$

$\therefore \quad 2\tan^2\theta + \sin^2\theta - 1$

$= 2\tan^2 45^\circ + \sin^2 45^\circ - 1$

$= 2(1)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 - 1$

$= 2 + \frac{1}{2} - 1$

$= \frac{5}{2} - 1 = \frac{3}{2}$

OR

We have to show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$.

Here, $\tan^2\theta$ or $\tan^4\theta$ can be converted into $\sec^2\theta$.

So, LHS = $\tan^4\theta + \tan^2\theta$

$= \tan^2\theta\,(\tan^2\theta + 1)$

$= (\sec^2\theta - 1) \cdot \sec^2\theta$ [because, $\tan^2\theta = \sec^2\theta - 1$ and $\tan^2\theta + 1 = \sec^2\theta$]

$= \sec^4 \theta - \sec^2 \theta$

= RHS

Hence, proved.

25. Area of rectangle $= (120 \times 90)\mathrm{m}^2$

$= 10800\ \mathrm{m}^2$

Area of circular lawn

= [Area of rectangle - Area of park excluding circular lawn]

$= [10800 - 2950]\ \mathrm{m}^2$

$= 7850\ \mathrm{m}^2$

Area of circular lawn = 7850

$\Rightarrow \pi r^2 = 7850\ \mathrm{m}^2$

$3.14 \times r^2 = 7850\ \mathrm{m}^2$

$r^2 = \left(\frac{7850}{3.14}\right) m^2$

$= 2500\ \mathrm{m}^2$

$r = \sqrt{2500}\,\mathrm{m}$

or r = 50 m

Hence, radius of the circular lawn = 50 m

26. i. Quadratic polynomial
    ii. Biquadratic polynomial
    iii. Cubic polynomial
    iv. Linear polynomial
    v. Quadratic polynomial

Section C

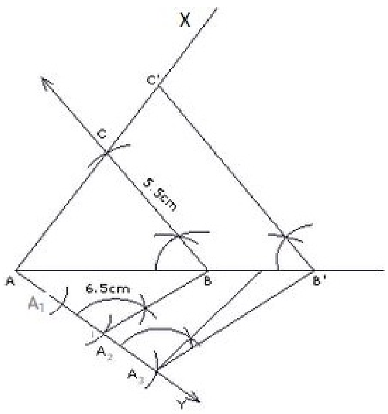
27. The given polynomial is $p(x) = 6x^5 + 4x^4 - 27x^3 - 7x^2 - 27x - 6$

and $q(x) = 2x^2 - 3$

$$\begin{array}{r|l} & 3x^3 + 2x^2 - 9x - \frac{1}{2} \\ \hline 2x^2 - 3 & 6x^5 + 4x^4 - 27x^3 - 7x^2 - 27x - 6 \\ & 6x^5 \qquad - 9x^3 \\ & - \qquad\quad + \\ \hline & \quad 4x^4 - 18x^3 - 7x^2 - 27x - 6 \\ & \quad 4x^4 \qquad\quad - 6x^2 \\ & \quad - \qquad\qquad + \\ \hline & \qquad - 18x^3 - x^2 - 27x - 6 \\ & \qquad - 18x^3 \qquad + 27x \\ & \qquad + \qquad\qquad - \\ \hline & \quad - x^2 - 54x - 6 \\ & \quad - x^2 \qquad + \frac{3}{2} \\ & \quad + \qquad\quad - \\ \hline & \qquad - 54x - \frac{15}{2} \\ \hline \end{array}$$

So, g(x) = $3x^3 + 2x^2 - 9x - \frac{1}{2}$

and r(x) = $-54x - \frac{15}{2}$

28. 

Steps of construction:

i. Draw a line segment AB = 6.5cm.
ii. At B construct $\angle ABX = 60^\circ$.
iii. With B as centre and radius BC = 5.5cm draw an arc intersecting BX at C.
iv. Join AC. Triangle so obtained is the required triangle.
v. Construct an acute angle ∠BAY at A on opposite side of vertex C of ΔABC.
vi. Locate 3 points A_1, A_2, A_3 on AY such that $AA_1 = A_1A_2 = A_2A_3$.

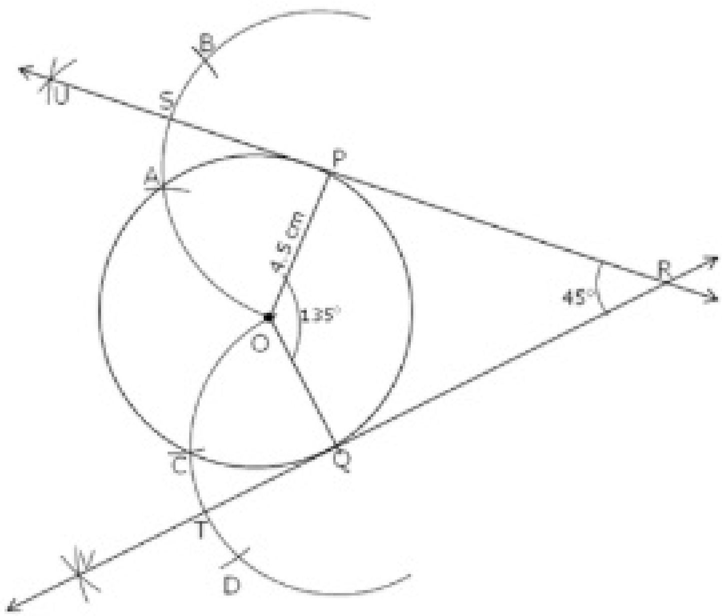
vii. Join A_2 to B and draw the line through A_3 parallel to A_2B intersecting the extended line segment AB at B'.

viii. Draw a line through B' parallel to BC intersecting the extended line segment AC at C'.

ix. $\triangle$AB'C' so obtained is the required triangle

OR

Steps of construction:-



i. Draw a circle having a centre O and a radius of 4.5 cm.

ii. Take point P on the circle and join OP.

iii. Angle between the tangents = 45°

Hence, the angle at the centre

$= 180^{\circ} - 45^{\circ} = 135^{\circ}$ (supplement of the angle between the tangents)

$\therefore$ Construct $\angle POQ = 135^{\circ}$

iv. Keeping a radius of 4.5 cm, draw arcs of circle taking the points P, and Q as the centres.

v. Name the points of intersection of arcs and circle as A and C respectively.

vi. Taking A as the centre and with the same radius mark B such that OA = AB.

vii. Similarly, taking C as the centre and with the same radius mark D such that OC = CD.

viii. Taking A and B as the centres and the same radius draw two arcs intersecting each other at U.

ix. Join P, S and U and extend it on both the sides to draw a tangent at point P.

x. Taking C and D as the centres and the same radius draw two arcs intersecting each other at V.

xi. Join Q, T and V and extend it on both the sides to draw a tangent at point Q.

xii. Extended tangents at P and Q intersect at R.

xiii. Hence, the required tangents are UR and VR such that the angle between them is 45° .

29. Heap of rice is in shape of cone, so

r = $\frac{9}{2}$ m = 4.5 m

h = 3.5 m

∴ V = $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2} \times \frac{9}{2} \times 3.5$

⇒ V = $\frac{22\times9\times9\times35}{3\times7\times2\times2\times10} = \frac{33\times9}{4} = \frac{297}{4}$

⇒ V = 74.25 m^3

Hence, volume of rice = 74.25 m^3.

For canvas:

Area of canvas = Curved surface area of cone

= πrl

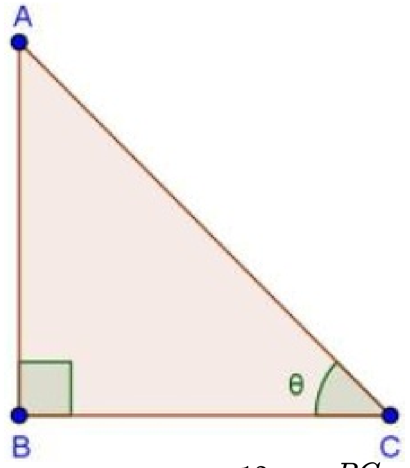
Here, $l^2 = r^2 + h^2 = (4.5)^2 + (3.5)^2$ = 20.25 + 12.25

⇒ l^2 = 32.50

⇒ l = $\sqrt{32.5}$ = 5.7 m

∴ Area of canvas = $\frac{22}{7}$ × 4.5 × 5.7 = 80.614

⇒ Area of canvas = 80.61 m^2

30. 

Given $\cos\theta = \frac{12}{13} = \frac{BC}{AC}$

Let BC = 12K

and, AC = 13K

In ΔABC, By Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$AB^2 + (12K)^2 = (13K)^2$

$AB^2 + 144K^2 = 169K^2$

$AB^2 = 169K - 144K^2 = 25K^2$

$AB = \sqrt{25K^2} = 5K$

$\therefore \sin\theta = \frac{AB}{AC} = \frac{5K}{13K} = \frac{5}{13}$

$\tan\theta = \frac{AB}{BC} = \frac{5K}{12K} = \frac{5}{12}$

$\therefore\ LHS = \sin\theta(1-\tan\theta)$

$= \frac{5}{13}\left(1-\frac{5}{12}\right)$

$= \frac{5}{13}\left(\frac{12-5}{12}\right)$

$= \frac{5}{13} \times \frac{7}{12}$

$= \frac{35}{156} = RHS$

OR

GIven $3\sin\theta + 5\cos\theta = 5$

$\Rightarrow 3\sin\theta = 5 - 5\cos\theta$

$\Rightarrow 3\sin\theta = 5(1-\cos\theta)$

$\Rightarrow 3\sin\theta = \frac{5(1-\cos\theta)(1+\cos\theta)}{(1+\cos\theta)}$

$\Rightarrow 3\sin\theta = \frac{5(1-\cos^2\theta)}{1+\cos\theta}$

$\Rightarrow 3\sin\theta = \frac{5\sin^2\theta}{1+\cos\theta}\ [\because 1-\cos^2\theta = \sin^2\theta]$

$\Rightarrow 3 = \frac{5\sin\theta}{1+\cos\theta}$

$\Rightarrow 3 + 3\cos\theta = 5\sin\theta$

$\Rightarrow 3 = 5\sin\theta - 3\cos\theta$

Hence proved.

31. 546 and 764 are divided by the largest number leaving remainders 6 and 8 respectively.

So,

$546 - 6 = 540$

$764 - 8 = 756$

So, 540 and 756 are exactly divisible by the required number.

Thus, the required number is the HCF of 540 and 756.

$540 = 2^2 \times 3^3 \times 5$

$756 = 2^2 \times 3^3 \times 7$

HCF (540, 756) $= 2^2 \times 3^3$ = 108

Hence the largest number which divides 546 and 764, leaving remainders 6 and 8 respectively is 108.

OR

Let us preassume that $3\sqrt{2}$ is a rational number.

In that case, $3\sqrt{2}$ can be writtin as $\frac{p}{q}$, where p and q are co-prime integers and q is not zero.

So, $\frac{p}{q} = \frac{3\sqrt{2}}{1}$

$\Rightarrow \frac{p}{3q} = \frac{\sqrt{2}}{1}$

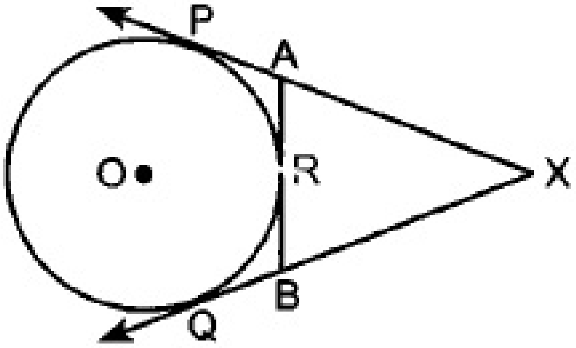
Since, p is an integer and 3q is also an integer where 3q is not zero.

So, $\frac{p}{3q}$ is a rational number but the equalient number $\sqrt{2}$ should also be a rational number.

But this contradicts the fact that $\sqrt{2}$ is an irrational number.

so, this assumption is wrong and $3\sqrt{2}$ is an irrational number.

32. Given,



$$AP = AR$$

and BQ = BR ..(i)

Also XQ = XP ..(ii)[Tangents drawn from an external point]

$\therefore$ XA + AP = XB + BQ

$\therefore XA + AR = XB + BR$ [From (i) and (ii)]

33. i. It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e., $\frac{1}{4} \times 100$ = 25m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

    Similarly, Preet posted a red flag at the distance AD i.e., $\frac{1}{5} \times 100$ = 20m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

    ii. According to distance formula,

    Distance between these flags by using the distance formula, D

    = $[(8 - 2)^2 + (25 - 20)^2]^{1/2}$ = $(36 + 25)^{1/2}$ = $\sqrt{61}m$

    iii. The point at which Rashmi should post her blue flat is the mid-point of the line joining these points. Let this point be A(x, y)

    Now by midpoint formula,

    $(x, y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

    $x = \frac{2+8}{2} = 5$

    $y = \frac{25+20}{2} = 22.5$

    Hence, A(x, y) = (5, 22.5)

    Therefore, Rashmi should post her blue flag at 22.5 m on 5th line.

34. The given system of equations may be written as

$\frac{x}{a}+\frac{y}{b}=2$

So, bx + ay - 2ab =0 .... (i)

And ax - by =a^2 - b^2

So, ax - by - (a^2 - b^2)=0 .... (ii)

By cross-multiplication, we have

$\frac{x}{-a\left(a^2-b^2\right)-(-b)(-2ab)}=\frac{-y}{-b\left(a^2-b^2\right)-a(-2ab)}=\frac{1}{b\times-b-a\times a}$

$\Rightarrow \quad \frac{x}{-a\left(a^2-b^2\right)-2ab^2}=\frac{-y}{-b\left(a^2-b^2\right)+2a^2b}=\frac{1}{-b^2-a^2}$

$\Rightarrow \quad \frac{x}{-a\left(a^2-b^2+2b^2\right)}=\frac{-y}{-b\left(a^2-b^2-2a^2\right)}=\frac{1}{-\left(a^2+b^2\right)}$

$\Rightarrow \quad \frac{x}{-a\left(a^2+b^2\right)}=\frac{-y}{-b\left(-a^2-b^2\right)}=\frac{1}{-\left(a^2+b^2\right)}$

$\Rightarrow \quad x=\frac{-a\left(a^2+b^2\right)}{-\left(a^2+b^2\right)}=a \text{ and } y=\frac{-b\left(a^2+b^2\right)}{-\left(a^2+b^2\right)}=b$

Hence, the solution of the given system of equations is x = a,y = b.

Section D

35. Let the first number be x

$\therefore$ Second number = x + 5

Now according to the question

$\frac{1}{x}-\frac{1}{x+5}=\frac{1}{10}$

$\Rightarrow \quad \frac{x+5-x}{x(x+5)}=\frac{1}{10}$

$\Rightarrow \quad 50=x^2+5x$

$\Rightarrow \quad x^2+5x-50=0$

$\Rightarrow \quad x^2+10x-5x-50=0$

$\Rightarrow x(x+10)-5(x+10)=0$

$\Rightarrow \quad (x+10)(x-5)=0$

x = 5 ,- 10 rejected

The numbers = 5 and 10.

36. Let first three terms be a - d, a and a + d

$a - d + a + a + d = 18$

So $a = 6$

$(a - d)(a + d) = 5d$

$\Rightarrow 6^2 - d^2 = 5d$

or $d^2 + 5d - 36 = 0$

$(d + 9)(d - 4) = 0$

So $d = -9$ or 4

For $d = -9$ three numbers are 15, 6 and -3

For $d = 4$ three numbers are 2, 6 and 10.

OR

Here according to question the sum of the 3rd and 7th terms of an A.P. is 6 and their product is 8. We have to find the sum of the first 20 terms of the A.P.

According to question, $a_3 + a_7 = 6$ (where a_3, a_7 represents 3rd and 7th terms)

And ,$a_3 \times a_7 = 8$.

Now ,$a_3 = a+2d$ and $a_7 = a+6d$, are 3rd and 7th terms respectively.

So,$(a+2d)+(a+6d)=6$

$\Longrightarrow 2a + 8d = 6$

$\Longrightarrow 2(a + 4d) = 6$

$\Longrightarrow a + 4d = \frac{6}{2}$

$\Longrightarrow a = 3 - 4d$.

And $(a_3 \times a_7) = (a + 2d)(a + 6d) = 8$.......(i).

Substituting the value of $a = (3 - 4d)$ in (i) we get

$(3 - 4d + 2d)(3 - 4d + 6d) = 8$

$\Rightarrow (3 + 2d)(3 - 2d) = 8$

$\Rightarrow$ 9 - 4d^2 = 8

$\therefore$.4d^2 = 1,

d^2 = $\frac{1}{4}$

d = $\pm\frac{1}{2}$

Case (i):When d = $\frac{1}{2}$

S$_{20}$ = $\frac{n}{2}$[2a+ (n - 1)d]

$\Longrightarrow$ S$_{20}$= $\frac{20}{2}\left[2+\frac{19}{2}\right]$

S$_{20}$ = 115

Now, Case(ii):When d = -$\frac{1}{2}$

S$_{20}$ = $\frac{20}{2}\left[2\times5+19\times\left(-\frac{1}{2}\right)\right]$

$=10\left[10-\frac{19}{2}\right]$ =5

37. Let O be the centre of the balloon of radius r and P the eye of the observer. Let PA, PB be tangents from P to the balloon. Then, $\angle APB=\alpha$.

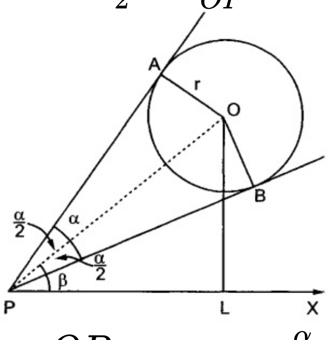
$\therefore\angle APO=\angle BPO=\frac{\alpha}{2}$

Let OL be perpendicular from O on the horizontal PX. We are given that the angle of the elevation of the centre of the balloon is β i.e, $\angle OPL=\beta$.

In $\triangle OAP$, we have

$\sin\frac{\alpha}{2}=\frac{OA}{OP}$

$\Rightarrow\sin\frac{\alpha}{2}=\frac{r}{OP}$



$\Rightarrow OP=r\ cosec\frac{\alpha}{2}$

In $\triangle OPL$, we have

$\sin\beta=\frac{OL}{OP}$

$\Rightarrow OL=OP\sin\beta$ = r cosec $\frac{\alpha}{2}$ sinβ [Using equation (i)]

Hence, the height of the centre of the balloon is r sin β cosec$\frac{\alpha}{2}$

38. Here we are given that

$\therefore \quad AB + AD = BC + CD$

or, $AD = BC + CD - AB$

or, $AD = h + d - x$

In the right angled ΔACD,

$AD^2 = AC^2 + DC^2$

or, $(h + d - x)^2 = (x + h)^2 + d^2$

or, $(h + d - x)^2 - (x + h)^2 = d^2$

$(h + d - x - x - h)(h + d - x + x + h) = d^2$

$Because\ a^2 - b^2 = (a - b)(a + b)$

or, $(d - 2x)(2h + d) = d^2$

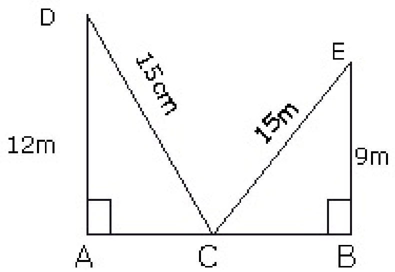
or, $2hd + d^2 - 4hx - 2xd = d^2$

or, $2hd = 4hx + 2xd$

$2hd = 2x(2h + d)$

Hence $x = \frac{hd}{2h+d}$

OR



Let AB be the width of the street and C be the foot of the ladder.

Let D and E be the windows at heights 12 m and 9 m respectively from the ground.

In CAD,$\triangle$right angled at A, we have

$CD^2 = AC^2 + AD^2$

$\Rightarrow 15^2 = AC^2 + 12^2$

$\Rightarrow AC^2 = 225 - 144 = 81$

$\Rightarrow AC = 9$ m

In $\triangle$CBE right angled at B, we have

$CE^2 = BC^2 + BE^2$

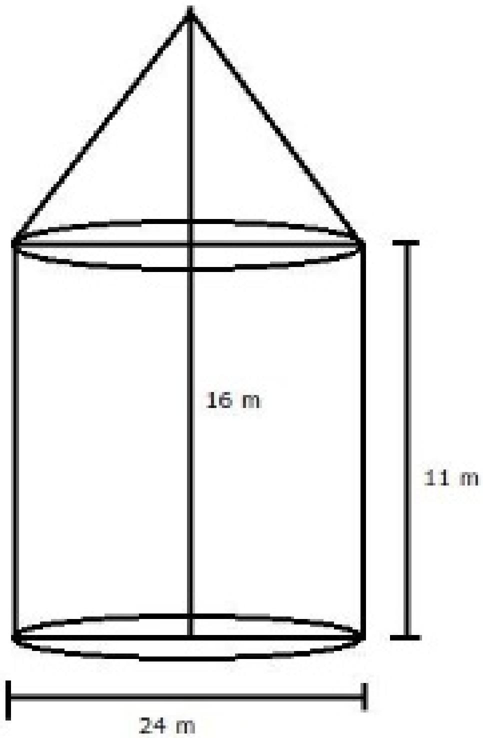
$\Rightarrow 15^2 = BC^2 + 9^2$

$\Rightarrow BC^2 = 225 - 81$

$\Rightarrow BC^2 = 144$

$\Rightarrow BC = 12m$

Hence, width of the street AB = AC + BC = 9 + 12 = 21m

39. 

Diameter of cyliner = 24m

$\therefore$ radius of cylinder = radius of cone = 12m

Height of cylinder = 11m

Total height of tent = 16m

$\therefore$ Height of cone = 16-11 = 5m

Now, $l^2 = r^2 + h^2$

$\Rightarrow l^2 = 12^2 + 5^2$

$\Rightarrow l^2 = 144 + 25 = 169$

$\Rightarrow l = \sqrt{169} = 13m$

$\therefore$ Canvas required for tent = curved surface area of cone + curved surface area of cylinder

$= \pi rl + 2\pi rh$

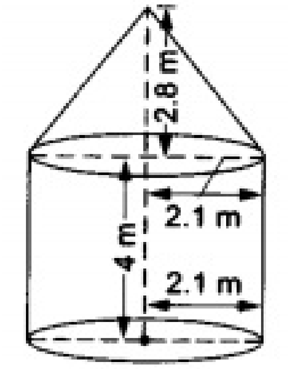
$= \frac{22}{7} \times 12 \times 13 + 2 \times \frac{22}{7} \times 12 \times 11$

$= \frac{22}{7} \times 12[13 + 2 \times 11]$

$= \frac{22}{7} \times 12 \times 35$

$= 22 \times 12 \times 5 = 1320 m^2$

OR



Radius of the cylinder, r = 2.1 m

Height of the cylinder, h = 4 m

Radius of the cone = radius of the cylinder = 2.1 m

Height of the cone, H = 2.8 m.

Slant height of the cone, $l = \sqrt{r^2 + H^2}$

$= \sqrt{(2.1)^2 + (2.8)^2}$

$= \sqrt{4.41 + 7.84}$

$= \sqrt{12.25} = 3.5\text{m}$

Area of the canvas required for each tent = curved surface area of the cylinder + curved surface area of the cone

$= 2\pi rh + \pi rl = \left[\left(2 \times \frac{22}{7} \times 2.1 \times 4\right) + \left(\frac{22}{7} \times 2.1 \times 3.5\right)\right]$

= (52.8 + 23.1) = 75.9 m^2

Total area of the canvas required for 100 tents

= (75.9 $\times$ 100) m^2 = 7590 m^2

Total cost of 100 tents = Rs. (7590 $\times$100) = Rs. 759000.

Amount to be paid by the associations

= 50% of Rs. 759000 = $Rs\left(\frac{50}{100} \times 759000\right) = Rs. 379500$

Hence, the associations will have to pay Rs. 379500.

40. i. Direct method:

Class interval	Mid value x_i	Frequency(f_i)	$f_i x_i$
0 – 10	5	20	100
10 – 20	15	24	360
20 – 30	25	40	1000
30 – 40	35	36	1260
40 – 50	45	20	900
		N = 140	$\sum f_i u_i$ **= 3620**

Mean = $\frac{\sum f_i u_i}{N}$

= $\frac{3620}{140}$

$= 25.857$

ii. Assumed mean method:

Class interval	Mid value x_i	$u_i = (x_i – A)$	Frequency f_i	$f_i u_i$
0 – 10	5	-20	20	-400
10 – 20	15	-10	24	-240
20 – 30	25	0	40	0
30 – 40	35	10	36	360
40 – 50	45	20	20	400
			N = 140	$\sum f_i u_i$ **= 120**

Let the assumed mean is 25

$Mean = A + \left(\frac{\sum f_i u_i}{N}\right)$

= $25 + \left(\frac{120}{140}\right)$

$= 25 + 0.857$

$= 25.857$

iii. Step deviation method:

Class interval	Mid value x_i	$d_i = x_i - 25$	$u_i = \frac{(x_i - 25)}{10}$	Frequency f_i	$f_i u_i$
0 – 10	5	-20	-2	20	-40
10 – 20	15	-10	-1	24	-24
20 – 30	25	0	0	40	0
30 – 40	35	10	1	36	36
40 – 50	45	20	2	20	40
				N = 140	$\sum f_i u_i$ = **12**

Let the assumed mean (A) = 25

h=10

$$Mean = A + h\left(\frac{\sum f_i u_i}{N}\right)$$

= $25 + 10\left(\frac{12}{140}\right)$

$= 25 + 0.857$

$= 25.857$