

**CBSE Class 11 Mathematics**

**Important Questions**

**Chapter 2**

**Relations and Functions**

**1 Marks Questions**

**1. Find a and b if  $(a - 1, b + 5) = (2, 3)$  If  $A = \{1,3,5\}$ ,  $B = \{2,3\}$  find : (Question-2, 3)**

**Ans.**  $a = 3, b = -2$

**2.  $A \times B$**

**Ans.**  $A \times B = \{(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)\}$

**3.  $B \times A$  Let  $A = \{1,2\}$ ,  $B = \{2,3,4\}$ ,  $C = \{4,5\}$ , find (Question- 4,5)**

**Ans.**  $B \times A = \{(2,1), (2,3), (2,5), (3,1), (3,3), (3,5)\}$

**4.  $A \times (B \cap C)$**

**Ans.**  $\{(1,4), (2,4)\}$

**5.  $A \times (B \cup C)$**

**Ans.**  $\{(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)\}$

**6. If  $P = \{1,3\}$ ,  $Q = \{2,3,5\}$ , find the number of relations from A to B**

**Ans.**  $2^6 = 64$

**7. If  $A = \{1,2,3,5\}$  and  $B = \{4,6,9\}$ ,  $R = \{(x, y) : |x - y| \text{ is odd, } x \in A, y \in B\}$  Write R in roster form**

**Which of the following relations are functions. Give reason.**

**Ans.**  $R = \{ (1,4), (1,6), (2,9), (3,4), (3,6), (5,4), (5,6) \}$

**8.**  $R = \{ (1,1), (2,2), (3,3), (4,4), (4,5) \}$

**Ans.** Not a function because 4 has two images.

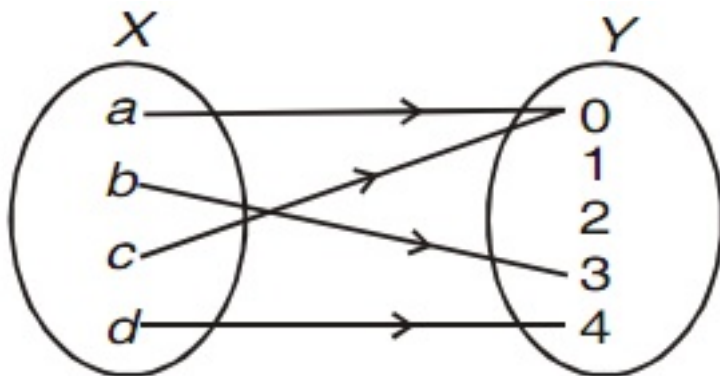
**9.**  $R = \{ (2,1), (2,2), (2,3), (2,4) \}$

**Ans.** Not a function because 2 does not have a unique image.

**10.**  $R = \{ (1,2), (2,5), (3,8), (4,10), (5,12), (6,12) \}$  Which of the following arrow diagrams represent a function? Why?

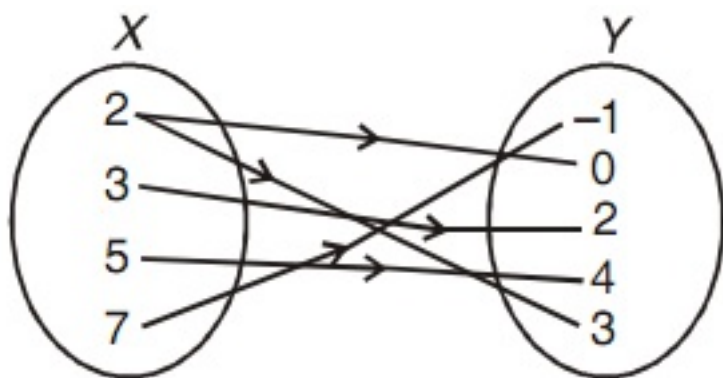
**Ans.** Function

**11.**



**Ans.** Function

**12.**



Let  $f$  and  $g$  be two real valued functions, defined by,  $f(x) = x^2$ ,  $g(x) = 3x + 2$ .

Ans. Not a function

13.  $(f + g)(-2)$

Ans. 0

14.  $(f - g)(1)$

Ans. -4

15.  $(fg)(-1)$

Ans. -1

16.  $\left(\frac{f}{g}\right)(0)$

Ans. 0

17. If  $f(x) = x^3$ , find the value of,  $\frac{f(5) - f(1)}{5 - 1}$

Ans. 31

18. Find the domain of the real function,  $f(x) = \sqrt{x^2 - 4}$

**Ans.**  $(-\infty, -2] \cup [2, \infty)$

**19. Find the domain of the function,  $f(x) = \frac{x^2 + 2x + 3}{x^2 - 5x + 6}$  Find the range of the following functions, (Question- 20,21)**

**Ans.**  $\mathbb{R} - \{2, 3\}$

**20.  $f(x) = \frac{1}{1-x^2}$**

**Ans.**  $(-\infty, -0] \cup [1, \infty)$

**21.  $f(x) = x^2 + 2$**

**Ans.**  $[2, \infty)$

**22. Find the domain of the relation,  $R = \{(x, y) : x, y \in \mathbb{Z}, xy = 4\}$  Find the range of the following relations : (Question-23, 24)**

**Ans.**  $\{-4, -2, -1, 1, 2, 4\}$

**23.  $R = \{(a, b) : a, b \in \mathbb{N} \text{ and } 2a + b = 10\}$**

**Ans.**  $\{2, 4, 6, 8\}$

**24.  $R = \left\{ \left( x, \frac{1}{x} \right) : x \in \mathbb{Z}, 0 < x < 6 \right\}$**

**Ans.**  $\left\{ 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \right\}$

**25. If the ordered Pairs  $(x-1, y+3)$  and  $(2, x+4)$  are equal, find  $x$  and  $y$**

(i) (3,3) (ii) (3,4) (iii) (1,4) (iv) (1,0)

Ans. (3,4)

26. If,  $n(A) = 3, n(B) = 2$ ,  $A$  And  $B$  are two sets Then no. of relations of  $A \times B$  have.

(i) (6) (ii) (12) (iii) (32) (iv) (64)

Ans. 64

27. Let  $f(x) = -|x|$  then Range of function

(i)  $(0, \infty)$  (ii)  $(-\infty, \infty)$  (iii)  $(-\infty, 0)$  (iv) none of there

Ans.  $(-\infty, 0)$

28. A real function  $f$  is defined by  $f(x) = 2x - 5$ . Then the Value of  $f(-3)$

(i) -11 (ii) 1 (iii) 0 (iv) none of there

Ans. -11

29. If  $P = \{a, b, c\}$  and  $Q = \{d\}$ , form the sets  $P \times Q$  and  $Q \times P$  are these two Cartesian products equal?

Ans. Given  $P = \{a, b, c\}$  and  $Q = \{d\}$ , by definition of cartesian product, we set

$$P \times Q = [(a, d), (b, d), (c, d)] \text{ and } Q \times P = [(d, a), (d, b), (d, c)]$$

By definition of equality of ordered pairs the pair  $(a, d)$  is not equal to the pair  $(d, a)$  therefore  $P \times Q \neq Q \times P$ .

**30..If  $A$  and  $B$  are finite sets such that  $n(A) = m$  and  $n(B) = k$  find the number of relations from  $A$  to  $B$**

**Ans.** Given  $n(A) = m$  and  $n(B) = k$

$$\therefore n(A \times B) = n(A) \times n(B) = mk$$

$\therefore$  the number of subsets of  $A \times B = 2^{mk}$

$\because n(A) = m$ , then the number of subsets of  $A = 2^m$

Since every subset of  $A \times B$  is a relation from A to B therefore the number of relations from A to B =  $2^{mk}$

**31.Let  $f = \{(1,1), (2,3), (0,-1), (-1,3), \dots\}$  be a function from  $\mathbb{Z}$  to  $\mathbb{Z}$  defined by  $f(x) = ax + b$ , for same integers  $a$  and  $b$  determine  $a$  and  $b$ .**

**Ans.** Given  $f(x) = ax + b$

Since  $(1,1) \in f, f(1) = 1 \Rightarrow a + b = 1 \dots (i)$

$(2,3) \in f, f(2) = 3 \Rightarrow 2a + b = 3 \dots (ii)$

Subtracting (i) from (ii) we get  $a = 2$

Substituting  $a = 2$  in (i) we get  $2 + b = 1$

$$\Rightarrow b = -1$$

Hence  $a = 2, b = -1$

**32.Express  $\{(x,y) : y + 2x = 5, x, y \in \mathbb{W}\}$  as the set of ordered pairs**

**Ans.** Since  $y + 2x = 5$  and  $x, y \in \mathbb{W}$ ,

Put  $x = 0, y + 0 = 5 \Rightarrow y = 5$

$$x=1, y+2 \times 1=5 \Rightarrow y=3$$

$$x=2, y+2 \times 2=5 \Rightarrow y=1$$

For another values of  $x \in W$ , we do not get  $y \in W$ .

Hence the required set of ordered pairs is  $\{(0, 5), (1, 3), (2, 1)\}$

**33. If  $A = \{1, 2\}$ , find  $(A \times A \times A)$**

**Ans.** We have

$$A \times A \times A = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (2, 2, 1), (2, 2, 2)\}$$

**34. A function  $f$  is defined by  $f(x) = 2x - 3$  find  $f(5)$**

**Ans.** Here  $f(x) = 2x - 3$

$$f(x) = (2 \times 5 - 3) = 7$$

**35. Let  $f = \{(0, -5), (1, -2), (2, 1), (3, 4), (4, 7)\}$  be a linear function from  $Z$  into  $Z$  find  $f$**

**Ans.**  $f(x) = 3x - 5$

**36. If the ordered pairs  $(x - 2, 2y + 1)$  and  $(y - 1, x + 2)$  are equal, find  $x$  &  $y$**

**Ans.**  $x = 3, \quad y = 2$

**37. Let  $A = \{-1, 2, 5, 8\}$ ,  $B = \{0, 1, 3, 6, 7\}$  and  $R$  be the relation, is one less than from  $A$  to  $B$  then find domain and Range of  $R$**

**Ans.** Given  $A = \{-1, 2, 5, 8\}$ ,  $B = \{0, 1, 3, 6, 7\}$ , and  $R$  is the relation 'is one less than' from

$A$  to  $B$  therefore  $R = [(-1,0), (2,3), (5,6)]$

Domain of  $R = \{-1, 2, 5\}$  and range of  $R = \{0, 3, 6\}$

38. Let  $R$  be a relation from  $N$  to  $N$  define by  $R = [(a, b) : a, b \in N \text{ and } a = b^2]$ .

Is the following true  $a, b \in R$  implies  $(b, a) \in R$

Ans. No; let  $a = 4, b = 2$ . As  $4 = 2^2$ , so  $(4, 2) \in R$  but  $2 \neq 4^2$ , so  $(2, 4) \notin R$

39. Let  $N$  be the set of natural numbers and the relation  $R$  be define in  $N$  by  $R = [(x, y) : y = 2x, x, y \in N]$ . what is the domain, co domain and range of  $R$ ? Is this relation a function?

Ans. Given  $R = [(x, y) : y = 2x, x, y \in N]$

$\therefore$  Domain of  $R = N$ , co domain of  $R = N$ , and Range of  $R$  is the set of even natural numbers.

Since every natural number  $x$  has a unique image  $2x$  therefore, the relation  $R$  is a function.

40. Let  $R = \{(x, y) : y = x + 1\}$  and  $y \in \{0, 1, 2, 3, 4, 5\}$  list the element of  $R$

Ans.  $R = \{(-1, 0), (0, 1), (1, 2), (2, 3), (3, 4), (4, 5)\}$

41. Let  $f$  be the subset of  $Q \times Z$  defined by

$f = \left\{ \left( \frac{m}{n}, m \right) : m, n \in Z, n \neq 0 \right\}$ . Is  $f$  a function from  $Q$  to  $Z$ ? Justify your answer

Ans.  $f$  Is not a function from  $Q$  to  $Z$



$$f\left(\frac{1}{2}\right)=1 \text{ and } f\left(\frac{2}{4}\right)=2$$

$$\text{But } \frac{1}{2} = \frac{2}{4}$$

∴ One element  $\frac{1}{2}$  have two images

∴  $f$  is not function

42. The function ' $f$ ' which maps temperature in Celsius into temperature in Fahrenheit is defined by  $f(c) = \frac{9}{5}c + 32$  find  $f(0)$

$$\text{Ans. } f(0) = \frac{9}{5} \times 0 + 32$$

$$f(0) = 32$$

43. If  $f\left(x = x^3 - \frac{1}{x^3}\right)$  Prove that  $f(x) + f\left(\frac{1}{x}\right) = 0$

$$\text{Ans. } f(x) = x^3 - \frac{1}{x^3}$$

$$f\left(\frac{1}{x}\right) = \frac{1}{x^3} - x^3$$

$$f(x) + f\left(\frac{1}{x}\right) = \cancel{x^3} - \cancel{\frac{1}{x^3}} + \cancel{\frac{1}{x^3}} - \cancel{x^3} = 0$$

44. If  $A$  and  $B$  are two sets containing  $m$  and  $n$  elements respectively how many different relations can be defined from  $A$  to  $B$ ?

$$\text{Ans. } 2^{m+n}$$

**CBSE Class 12 Mathematics**  
**Important Questions**  
**Chapter 2**  
**Relations and Functions**

**4 Marks Questions**

1. Let  $A = \{1,2,3,4\}$ ,  $B = \{1,4,9,16,25\}$  and  $R$  be a relation defined from  $A$  to  $B$  as,  $R = \{(x, y) : x \in A, y \in B \text{ and } y = x^2\}$

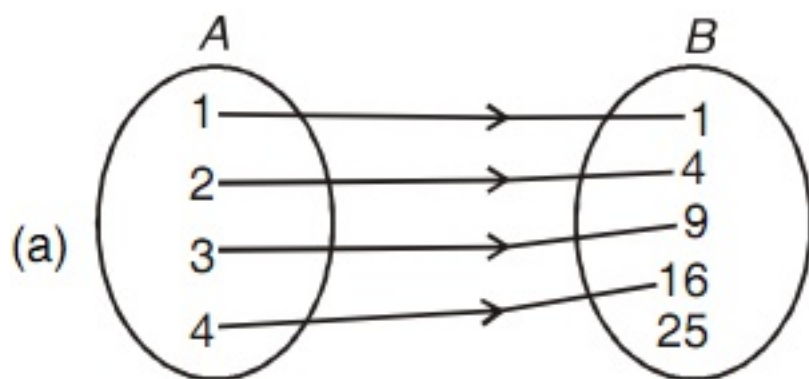
(a) Depict this relation using arrow diagram.

(b) Find domain of  $R$ .

(c) Find range of  $R$ .

(d) Write co-domain of  $R$ .

Ans.



(b)  $\{1,2,3,4\}$

(c)  $\{1,4,9,16\}$

(d)  $\{1,4,9,16,25\}$

2. Let  $R = \{(x, y) : x, y \in \mathbb{N} \text{ and } y = 2x\}$  be a relation on  $\mathbb{N}$ . Find :

**(i) Domain**

**(ii) Codomain**

**(iii) Range**

**Is this relation a function from N to N**

**Ans. (i) N**

**(ii) N**

**(iii) Set of even natural numbers**

yes, R is a function from N to N.

**3. Find the domain and range of,  $f(x) = |2x - 3| - 3$**

**Ans.** Domain is R

Range is  $[-3, \infty)$

**4. Draw the graph of the Constant function,  $f : R \rightarrow R; f(x) = 2 \quad x \in R$ . Also find its domain and range.**

**Ans.** Domain = R

Range =  $\{2\}$

**5. Let  $R = \{(x, -y) : x, y \in W, 2x + y = 8\}$  then**

**(i) Find the domain and the range of R (ii) Write R as a set of ordered pairs.**

**Ans. (i)** Given  $2x + y = 8$  and  $x, y \in W$

Put

$$x = 0, 2 \times 0 + y = 8 \Rightarrow y = 8,$$

$$x = 1, 2 \times 1 + y = 8 \Rightarrow y = 6,$$

$$x = 2, 2 \times 2 + y = 8 \Rightarrow y = 4,$$

$$x = 3, 2 \times 3 + y = 8 \Rightarrow y = 2,$$

$$x = 4, 2 \times 4 + y = 8 \Rightarrow y = 0$$

for all other values of  $x \in W$ , we do not get  $y \in W$

$\therefore$  Domain of  $R = \{0, 1, 2, 3, 4\}$  and range of  $R = \{8, 6, 4, 2, 0\}$

(ii)  $R$  as a set of ordered pairs can be written as

$$R = \{(0, 8), (1, 6), (2, 4), (3, 2), (4, 0)\}$$

**6. Let  $R$  be a relation from  $Q$  to  $Q$  defined by  $R = \{(a, b) : a, b \in Q \text{ and } a - b \in Z\}$  show that (i)  $(a, a) \in R$  for all  $a \in Q$  (ii)  $(a, b) \in R$  implies that  $(b, a) \in R$**

**(iii)  $(a, b) \in R$  and  $(b, c) \in R$  implies that  $(a, c) \in R$**

$$\text{Ans. } R = \{(a, b) : a, b \in Q \text{ and } a - b \in Z\}$$

(i) For all  $a \in Q$ ,  $a - a = 0$  and  $0 \in Z$ , it implies that  $(a, a) \in R$ .

(ii) Given  $(a, b) \in R \Rightarrow a - b \in Z \Rightarrow -(a - b) \in Z$

$$\Rightarrow b - a \in Z \Rightarrow (b, a) \in R.$$

(iii) Given  $(a, b) \in R$  and  $(b, c) \in R \Rightarrow a - b \in Z$  and  $b - c \in Z \Rightarrow (a - b) + (b - c) \in Z$

$$\Rightarrow a - c \in Z \Rightarrow (a, c) \in R.$$

$$7. \text{ If } f(x) = \frac{x^2 - 3x + 1}{x - 1}, \text{ find } f(-2) + f\left(\frac{1}{3}\right)$$

$$\text{Ans. Given } f(x) = \frac{x^2 - 3x + 1}{x - 1}, Df = R - \{1\}$$

$$\therefore f(-2) = \frac{(-2)^2 - 3(-2) + 1}{-2 - 1} - \frac{4 + 6 + 1}{-3} = 1\frac{1}{3} \text{ and}$$

$$f\left(\frac{1}{3}\right) = \frac{\left(\frac{1}{3}\right)^2 - 3 \times \frac{1}{3} + 1}{\frac{1}{3} - 1} = \frac{\frac{1}{9} - 1 + 1}{-\frac{2}{3}} = \frac{\frac{1}{9}}{-\frac{2}{3}} = \frac{1}{9} \times \left(-\frac{3}{2}\right) = -\frac{1}{6}$$

$$\therefore f(-2) + f\left(\frac{1}{3}\right) = -\frac{11}{3} - \frac{1}{6} = \frac{-22 - 1}{6} = \frac{-23}{6} = 3\frac{5}{6}.$$

**8. Find the domain and the range of the function  $f(x) = 3x^2 - 5$ . Also find  $f(-3)$  and the numbers which are associated with the number 43 in its range.**

**Ans.** Given  $f(x) = 3x^2 - 5$

For  $Df$ ,  $f(x)$  must be real number

$\Rightarrow 3x^2 - 5$  must be a real number

Which is a real number for every  $x \in \mathbb{R}$

$\Rightarrow Df = \mathbb{R} \dots \dots (i)$

for  $Rf$ , let  $y = f(x) = 3x^2 - 5$

We know that for all  $x \in \mathbb{R}$ ,  $x^2 \geq 0 \Rightarrow 3x^2 \geq 0$

$\Rightarrow 3x^2 - 5 \geq -5 \Rightarrow y \geq -5 \Rightarrow Rf = [-5, \infty]$

Further, as  $-3 \in Df$ ,  $f(-3)$  exists and  $f(-3)$

$= 3(-3)^2 - 5 = 22$ .

As  $43 \in Rf$  on putting  $y = 43$  in (i) we get

$3x^2 - 5 = 43 \Rightarrow 3x^2 = 48 \Rightarrow x^2 = 16 \Rightarrow x = -4, 4$ .

Therefore  $-4$  and  $4$  are number

(is  $Df$ ) which are associated with the number  $43$  in  $Rf$

9.If  $f(x) = x^2 - 3x + 1$ , find  $x$  such that  $f(2x) = f(x)$

Ans. Given  $f(x) = x^2 - 3x + 1, Df = R$

$$\therefore f(2x) = (2x)^2 - 3(2x) + 1 = 4x^2 - 6x + 1$$

As  $f(2x) = f(x)$  (Given)

$$\Rightarrow 4x^2 - 6x + 1 = x^2 - 3x + 1$$

$$\Rightarrow 3x^2 - 3x = 0 \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0$$

$$\Rightarrow x = 0, 1.$$

10. Find the domain and the range of the function  $f(x) = \sqrt{x-1}$

Ans. Given  $f(x) = \sqrt{x-1}$ ,

for  $Df$ ,  $f(x)$  must be a real number

$$\Rightarrow \sqrt{x-1} \text{ must be a real number}$$

$$\Rightarrow x-1 \geq 0 \Rightarrow x \geq 1$$

$$\Rightarrow Df = [1, \infty]$$

for  $Rf$ , let  $y = f(x) = \sqrt{x-1}$

$$\Rightarrow \sqrt{x-1} \geq 0 \Rightarrow y \geq 0$$

$$\Rightarrow Rf = [0, \infty]$$

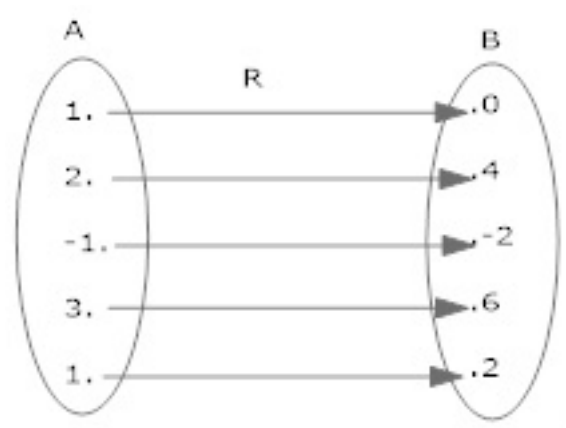
11. Let a relation  $R = \{(0, 0), (2, 4), (-1, 2), (3, 6), (1, 2)\}$  then

**(i) write domain of R**

**(ii) write range of R**

**(iii) write R the set builder form**

**(iv) represent R by an arrow diagram**



**Ans.** Given  $R = [(0, 0), (2, 4), (-1, -2), (3, 6), (1, 2)]$

**(i) Domain of  $R = [0, 2, -1, 3, 1]$**

**(ii) Rang of  $R = [0, 4, -2, 6, 2]$**

**(iii) R in the builder form can be written as**

$$R = [(x, y) : x \in I, -1 \leq x \leq 3, y = 2x]$$

**(iv) The reaction R can be represented by the arrow diagram are shown.**

**12. Let  $A = \{1, 2, 3\}$ ,  $B = \{1, 2, 3, 4\}$  and  $R = \{(x, y) : (x, y) \in A \times B, y = x + 1\}$**

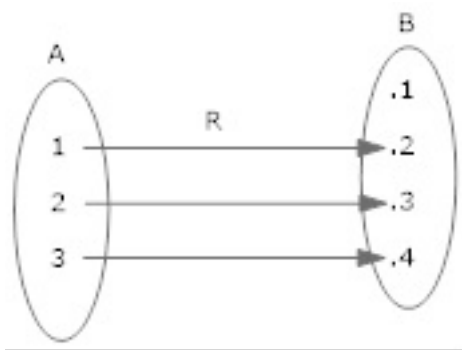
**(i) find  $A \times B$**

**(ii) write R in roster form**

**(iii) write domain & range of R**

(iv) represent  $R$  by an arrow diagram

Ans. (i)  $\{(1,1), (1,2), (1,3), (1,4)\}$



$(2,1), (2,2), (2,3), (2,4)$

$(3,1), (3,2), (3,3), (3,4)\}$

(ii)  $R = [(1,2), (2,3), (3,4)]$

(iii) Domain of  $R = \{1, 2, 3\}$  and range of  $R = \{2, 3, 4\}$

(iv) The relation  $R$  can be represented by the arrow diagram as shown.

13. The cartesian product  $A \times A$  has 9 elements among which are found  $(-1, 0)$  and  $(0, 1)$ . find the set and the remaining elements of  $A \times A$

Ans. Let  $n(A) = m$

Given  $n(A \times A) = 9 \Rightarrow n(A) \cdot n(A) = 9$

$\Rightarrow m \cdot m = 9 \Rightarrow m^2 = 9 \Rightarrow m = 3 \quad (\because m > 0)$

Given  $(-1, 0) \in A \times A \Rightarrow -1 \in A$  and  $0 \in A$

Also  $(0, 1) \in A \times A \Rightarrow 0 \in A$  and  $1 \in A$

This  $-1, 0, 1 \in A$  but  $n(A) = 3$



Therefore  $A = [-1, 0, 1]$

The remaining elements of  $A \times A$  are  $(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0), (1, 1)$

**14. Find the domain and the range of the following functions**  $f(x) = \frac{1}{\sqrt{5-x}}$

**Ans.** Given  $f(x) = \frac{1}{\sqrt{5-x}}$

For  $D_f$ ,  $f(x)$  must be a real number

$\Rightarrow \frac{1}{\sqrt{5-x}}$  Must be a real number

$\Rightarrow 5-x > 0 \Rightarrow 5 > x \Rightarrow x < 5$

$\Rightarrow D_f = (-\infty, 5)$

For  $R_f$  let  $y = \frac{1}{\sqrt{5-x}}$

As  $x < 5, 0 < 5-x$

$\Rightarrow 5-x > 0 \Rightarrow \sqrt{5-x} > 0$

$\Rightarrow \frac{1}{\sqrt{5-x}} > 0 \left( \because \frac{1}{a} > 0 \text{ if and only if } a > 0 \right)$

$\Rightarrow y > 0$

$\Rightarrow R_f = (0, \infty)$

**15. Let  $f(x) = x+1$  and  $g(x) = 2x-3$  be two real functions. Find the following**

**functions** (i)  $f+g$  (ii)  $f-g$  (iii)  $fg$  (iv)  $\frac{f}{g}$  (v)  $f^2-3g$

**Ans.** Given  $f(x) = x+1$  and  $g(x) = 2x-3$  we note that  $D_f = R$  and  $D_g = R$  so these functions have the same Domain  $R$

$$(i) (f+g)(x) = f(x) + g(x) = (x+1) + (2x-3) = 3x-2, \text{ for } x \in R$$

$$(ii) (f-g)(x) = f(x) - g(x) = (x+1) - (2x-3) = -x+4, \text{ for all } x \in R$$

$$(iii) (fg)(x) = f(x)g(x) = (x+1)(2x-3) = 2x^2 - x - 3, \text{ for all } x \in R$$

$$(iv) \left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)} = \frac{x+1}{2x-3}, x \neq \frac{3}{2}, x \in R$$

$$(v) (f^2 - 3g)(x) = (f^2)(x) - (3f)(x) = (f(x))^2 - 3g(x)$$

$$= (x+1)^2 - 3(2x-3) = x^2 + 2x + 1 - 6x + 9$$

$$= x^2 - 4x + 10, \text{ for all } x \in R$$

**16. Find the domain and the range of the following functions**

$$(i) f(x) = \frac{x-3}{2x+1} \quad (ii) f(x) = \frac{x^2}{1+x^2} \quad (iii) f(x) = \frac{1}{1-x^2}$$

**Ans. (i)** Given  $f(x) = \frac{x-3}{2x+1}$

For  $D_f$ ,  $f(x)$  must be a real number

$$\Rightarrow \frac{x-3}{2x+1} \text{ must be a real number}$$

$$\Rightarrow 2x+1 \neq 0 \Rightarrow x \neq -\frac{1}{2}$$

$$\Rightarrow D_f = \text{set of all real number except } -\frac{1}{2}$$

$$-\frac{1}{2}i.e.R - \left[-\frac{1}{2}\right]$$

For  $R_F$ , let  $y = \frac{x-3}{2x+1} \Rightarrow 2xy + y = x-3$

$$\Rightarrow (2y-1)x = -y-3 \Rightarrow x = \frac{y+3}{1-2y} \text{ but } x \in R$$

$$\Rightarrow \frac{y+3}{1-2y} \text{ Must be a real number } \Rightarrow 1-2y \neq 0 \Rightarrow y \neq \frac{1}{2}$$

$$\Rightarrow R_F = \text{Set of all real number except } \frac{1}{2} R - \left[\frac{1}{2}\right]$$

(ii) Given  $f(x) = \frac{x^2}{1+x^2}$

For  $D_F$ ,  $f(x)$  must be a real number  $\Rightarrow \frac{x^2}{1+x^2}$

Must be a real number

$$\Rightarrow D_F = R \quad (\because x^2 + 1 \neq 0 \text{ for all } x \in R)$$

For  $R_F$  let  $y = \frac{x^2}{1+x^2} \Rightarrow x^2 y + y = x^2$

$$\Rightarrow (y-1)x^2 = -y \Rightarrow x^2 = \frac{-y}{y-1}, y \neq 1$$

But  $x^2 \geq 0$  for all  $x \in R \Rightarrow \frac{-y}{y-1} \geq 0, y \neq 1$

Multiply both sides by  $(y-1)^2$ , a positive real number

$$\Rightarrow -y(y-1) \geq 0$$

$$\Rightarrow y(y-1) \leq 0 \Rightarrow (y-0)(y-1) \leq 0$$

$$\Rightarrow 0 \leq y \leq 1 \text{ but } y \neq 1$$

$$\Rightarrow 0 \leq y < 1$$

$$\Rightarrow R_F = (0,1)$$

(iii) Given  $f(x) = \frac{1}{1-x^2}$

For  $D_F, f(x)$  must be a real number

$$\Rightarrow \frac{1}{1-x^2} \text{ Must be a real number}$$

$$\Rightarrow 1-x^2 \neq 0 \Rightarrow x \neq -1, 1$$

$$\Rightarrow D_F = \text{Set of all real number except } -1, 1 \Rightarrow D_F = \mathbb{R} - [-1, 1]$$

For  $R_F$  let  $y = \frac{1}{1-x^2}, y \neq 0$

$$\Rightarrow 1-x^2 = \frac{1}{y} \Rightarrow x^2 = 1 - \frac{1}{y} \neq 0$$

But  $x^2 \geq 0$  for all  $x \in D_F \Rightarrow 1 - \frac{1}{y} \geq 0$

But  $y^2 > 0, y \neq 0$

Multiplicity both sides by  $y^2$  a positive real number

$$\Rightarrow y^2 \left( 1 - \frac{1}{y} \right) \geq 0 \Rightarrow y(y-1) \geq 0 \Rightarrow (y-0)(y-1) \geq 0$$

Either  $y \leq 0$  or  $y \geq 1$  but  $y \neq 0$

$$\Rightarrow R_F = (-\infty, 0) \cup (1, \infty).$$

17. If  $A = \{1, 2, 3\}$ ,  $B = \{3, 4\}$  and  $C = \{4, 5, 6\}$

find (i)  $A \times (B \cup C)$  (ii)  $A \times (B \cap C)$  (iii)  $(A \times B) \cap (B \times C)$

Ans. We have

$$(i) (B \cup C) = \{3, 4\} \cup \{4, 5, 6\} = \{3, 4, 5, 6\}$$

$$\therefore A \times (B \cup C)$$

$$= \{1, 2, 3\} \times \{3, 4, 5, 6\}$$

$$= \{(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4),$$

$$(2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)\}$$

$$(ii) (B \cap C) = \{3, 4\} \cap \{4, 5, 6\} = \{4\}$$

$$\therefore A \times (B \cap C) = \{1, 2, 3\} \times \{4\} = \{(1, 4), (2, 4), (3, 4)\}$$

$$(iii) (A \times B) = \{1, 2, 3\} \times \{3, 4\}$$

$$= \{(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)\}$$

$$(B \times C) = \{3, 4\} \times \{4, 5, 6\}$$

$$= \{(3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)\}$$

$$\therefore (A \times B) \cap (B \times C) = \{(3, 4)\}$$

18. For non empty sets  $A$  and  $B$  prove that  $(A \times B) = (B \times A) \Leftrightarrow A = B$

Ans. First we assume that  $A = B$

Then  $(A \times B) = (A \times A)$  and  $(B \times A) = (A \times A)$

$$\therefore (A \times B) = (B \times A)$$

This, when  $A = B$ , then  $(A \times B) = (B \times A)$

Conversely, Let  $(A \times B) = (B \times A)$ , and let be  $x \in A$ .

Then,  $x \in A \Rightarrow (x, b) \in A \times B$  for some  $b \in B$

$$\Rightarrow (x, b) \in B \times A \quad [\because A \times B = B \times A]$$

$$\Rightarrow x \in B.$$

$$\therefore A \subseteq B$$

similarly,  $B \subseteq A$

Hence,  $A = B$

**19. Let  $m$  be a given fixed positive integer. let**

**$R = \{(a, b) : a, b \in \mathbb{Z} \text{ and } (a - b) \text{ is divisible by } m\}$  show that  $R$  is an equivalence relation on  $\mathbb{Z}$ .**

**Ans.**  $R = \{(a, b) : a, b \in \mathbb{Z} \text{ and } (a - b) \text{ is divisible by } m\}$

**(i)** Let  $a \in \mathbb{Z}$ . Then,

$a - a = 0$ , which is divisible by  $m$

$$\therefore (a, a) \in R \text{ for all } a \in \mathbb{Z}$$

so  $R$  is reflexive

**(ii)** Let  $(a, b) \in R$  Then

$$(a, b) \in R \Rightarrow (a - b) \text{ is divisible by } m$$

$$\Rightarrow -(a - b) \text{ is divisible by } m$$

$$\Rightarrow (b - a) \text{ is divisible by } m$$

$$\Rightarrow (b, a) \in R$$

$$\text{Then } (a, b) \in R \Rightarrow (b, a) \in R.$$

So  $R$  is symmetric.

$$\text{(iii) Let } (a, b) \in R \text{ and } (b, c) \in R$$

$$\Rightarrow (a - b) \text{ is divisible by } m \text{ and } (b - c) \text{ is divisible by } m$$

$$\Rightarrow [(a - b) + (b - c)] \text{ is divisible by } m$$

$$\Rightarrow (a - c) \text{ is divisible by } m$$

$$\Rightarrow (a, c) \in R$$

$$\therefore (a, b) \in R \text{ and } (b, c) \in R \Rightarrow (a, c) \in R.$$

So,  $R$  is transitive this  $R$  is reflexive symmetric and transitive Hence,  $R$  is an equivalence relation and  $\mathbb{Z}$ .

**20. Let  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{1, 3, 4\}$  let  $R$  be the relation, is greater than from  $A$  to  $B$ . Write  $R$  as a set of ordered pairs. find domain ( $R$ ) and range ( $R$ )**

$$\text{Ans. } R = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 1), (5, 2), (5, 3), (5, 4)\}$$

$$\text{Domain of } R = \{2, 3, 4, 5\} \text{ Range of } R = \{1, 2, 3, 4\}$$

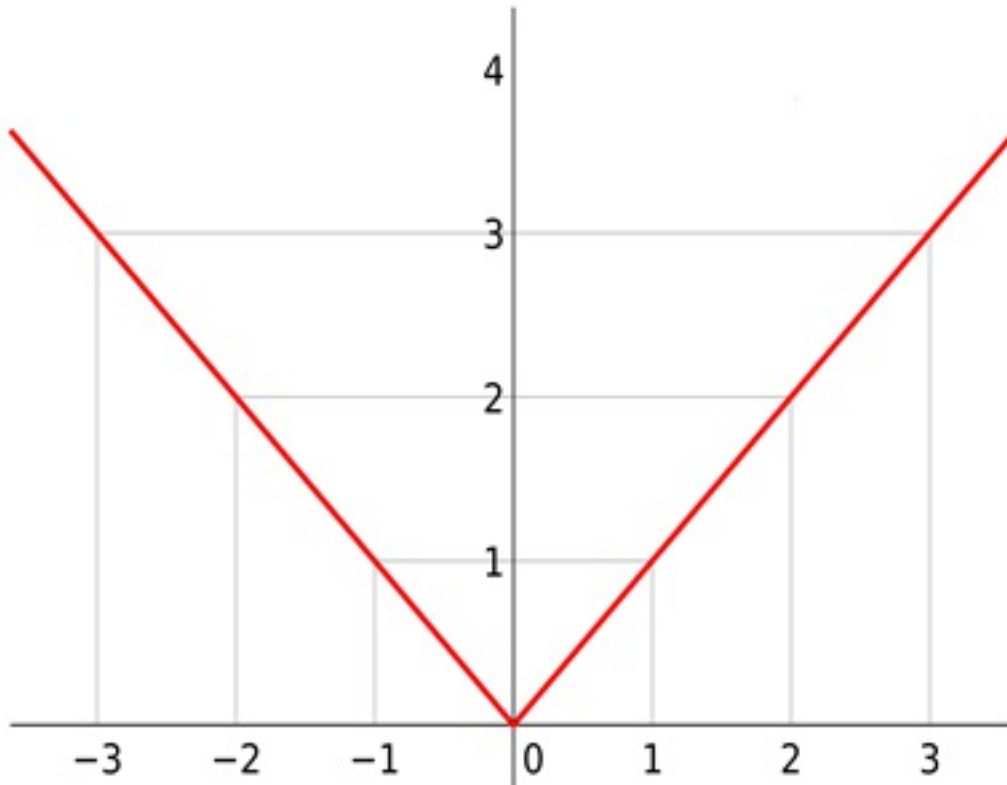
**21. Define modulus function Draw graph.**

$$\text{Ans. let } f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = |x| \text{ for each } x \in \mathbb{R}. \text{ then } f(x) = |x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$$

we know that  $|x| \geq 0$  for all  $x$

$\therefore \text{dom}(f) = \mathbb{R}$  and  $\text{range}(f) = \text{set of non negative real number}$

Drawing the graph of modulus function defined by



$$f: \mathbb{R} \rightarrow \mathbb{R}: f(x) = |x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$$

We have

$x$	3	-2	-1	0	1	2	3	4
$f(x)$	3	2	1	0	1	2	3	4

Scale: 5 small divisions = 1 unit

On a graph paper, we plot the points

$A(-3,3), B(-2,2), C(-1,1), O(0,0), D(1,1), E(2,2), F(3,3)$  and  $G(4,4)$

Join them successively to obtain the graph lines AO and OG, as show in the figure above.



22. Let  $f(x) = \begin{cases} x^2, & \text{when } 0 \leq x \leq 3 \\ 3x, & \text{when } 3 \leq x \leq 10 \end{cases}$   $g(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 2x, & 3 \leq x \leq 10 \end{cases}$  Show that  $f$  is a

function, while  $g$  is not a function.

Ans. Each element in  $\{0, 10\}$  has a unique image under  $f$ .

But,  $g(3) = 3^2 = 9$  and

$$g(3) = (2 \times 3) = 6$$

So  $g$  is not a function

23. Let  $A = \{1, 2\}$  and  $B = \{3, 4\}$  write  $A \times B$  how many subsets will  $A \times B$  have? List them.

Ans.  $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$ ; 16 Subsets of  $A \times B$  have

Subsets =  $\phi, \{(1, 3)\}, \{(1, 4)\}, \{(2, 3)\}, \{(2, 4)\},$

$\{(1, 4)\}, \{(1, 3), (2, 3)\}, \{(1, 3), (2, 4)\},$

$\{(1, 4), (2, 3)\}, \{(1, 4), (2, 4)\}, \{(2, 3)\},$

$\{(2, 4)\}, \{(1, 3), (1, 4), (2, 3)\}, \{(1, 3), (1, 4)\},$

$\{(2, 4)\}, \{(1, 3), (2, 3), (2, 4)\}, \{(1, 4), (2, 3)\},$

$\{(2, 4)\};$

24. Let  $A = \{1, 2\}$ ,  $B = \{1, 2, 3, 4\}$ ,  $C = \{5, 6\}$  and  $D = \{5, 6, 7, 8\}$  verify that

(i)  $A \times (B \cap C) = (A \times B) \cap (A \times C)$  (ii)  $A \times C$  is subset of  $B \times D$

Ans. L.H.S.  $B \cap C = \phi$

**Part-I**

$$\text{L.H.S } A \times (B \cap C) = \phi$$

$$\text{R.H.S. } A \times B = \left\{ (1,1), (1,2), (1,3), (1,4) \right\} \\ \left\{ (2,1), (2,2), (2,3), (2,4) \right\}$$

$$A \times C = \{(1,5), (1,6), (2,5), (2,6)\}$$

$$(A \times B) \cap (A \times C) = \phi \quad \text{L.H.S} = \text{R.H.S}$$

**Part-II**

$$B \times D = \left\{ (1,5), (1,6), (1,7), (1,8) \right\} \\ \left\{ (2,5), (2,6), (2,7), (2,8) \right\}$$

$$(A \times C) \subset (B \times D)$$

**25. Find the domain and the range of the relation  $R$  defined by**

$$R = \left[ (x+1, x+3) : x \in (0, 1, 2, 3, 4, 5) \right]$$

**Ans.** Given  $x \in \{0, 1, 2, 3, 4, 5\}$

$$\text{put } x=0, x+1=0+1=1 \text{ and } x+3=0+3=3$$

$$x=1, x+1=1+1=2 \text{ and } x+3=1+3=4,$$

$$x=2, x+1=2+1=3 \text{ and } x+3=2+3=5,$$

$$x=3, x+1=3+1=4 \text{ and } x+3=3+3=6,$$

$$x=4, x+1=4+1=5 \text{ and } x+3=4+3=7$$

$$x=5, x+1=6 \text{ and } x+3=5+3=8$$

$$\text{Hence } R = \left[ (1,3), (2,4), (3,5), (4,6), (5,7), (6,8) \right]$$

$$\therefore \text{Domain of } R = [1, 2, 3, 4, 5, 6] \text{ and range of } R = [3, 4, 5, 6, 7, 8]$$

**26. Find the linear relation between the components of the ordered pairs of the relation  $R$  where  $R = \{(2, 1), (4, 7), (1, -2), \dots\}$**

**Ans.** Given  $R = \{(2, 1), (4, 7), (1, -2), \dots\}$

Let  $y = ax + b$  be the linear relation between the components of  $R$

Since  $(2, 1) \in R, \therefore y = ax + b \Rightarrow 1 = 2a + b \dots\dots (i)$

Also  $(4, 7) \in R, \therefore y = ax + b \Rightarrow 7 = 4a + b \dots\dots (ii)$

Subtracting  $(i)$  from  $(ii)$ , we get  $2a = 6 \Rightarrow a = 3$

Subtracting  $a = 3$  in  $(i)$ , we get  $1 = 6 + b \Rightarrow b = -5$

Substituting the values of  $a$  and  $b$  in  $y = ax + b$ , we get

$y = 3x - 5$ , which is the required linear relation between the components of the given relation.

**27. Let  $A = \{1, 2, 3, 4, 5, 6\}$  define a relation  $R$  from  $A$  to  $A$  by**

$$R = \{(x, y) : y = x + 1, x, y \in A\}$$

**(i) write  $R$  in the roster form**

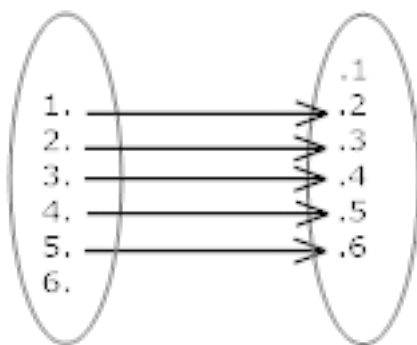
**(ii) write down the domain, co-domain and range of  $R$**

**(iii) Represent  $R$  by an arrow diagram**

**Ans. (i)**  $\{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

**(ii)** Domain =  $\{1, 2, 3, 4, 5\}$  co domain =  $A$ , range =  $\{2, 3, 4, 5, 6\}$

**(iii)**



28. A relation ' $f$ ' is defined by  $f: x \rightarrow x^2 - 2$  where  $x \in \{-1, -2, 0, 2\}$

(i) list the elements of  $f$

(ii) is  $f$  a function?

Ans. Relation  $f$  is defined by  $f: x \rightarrow x^2 - 2$

(i) is  $f(x) = x^2 - 2$  where  $x \in \{-1, -2, 0, 2\}$

$$f(-1) = (-1)^2 - 2 = 1 - 2 = -1$$

$$f(-2) = (-2)^2 - 2 = 4 - 2 = 2$$

$$f(0) = 0^2 - 2 = 0 - 2 = -2$$

$$f(2) = 2^2 - 2 = 4 - 2 = 2$$

$$\therefore f = \{(-1, -1), (-2, 2), (0, -2), (2, 2)\}$$

(ii) We note that each element of the domain of  $f$  has a unique image; therefore, the relation  $f$  is a function.

29. If  $y = \frac{6x-5}{5x-6}$ . Prove that  $f(y) = x$ ,  $x \neq \frac{6}{5}$

Ans.  $y = \frac{6x-5}{5x-6}$

$$y = f(x) = \frac{6x-5}{5x-6}$$

$$f(y) = \frac{6\left[\frac{6x-5}{5x-6}\right]-5}{5\left[\frac{6x-5}{5x-6}\right]-6}$$

$$f(y) = \frac{36x - \cancel{30} - 25x + \cancel{30}}{\cancel{30}x - 25 - \cancel{30}x + 36}$$

$$f(y) = \frac{11x}{11} = x, x \neq \frac{6}{5}$$

30. Let  $f : X \rightarrow Y$  be defined by  $f(x) = x^2$  for all  $x \in X$  where  $X = \{-2, -1, 0, 1, 2, 3\}$  and  $Y = \{0, 1, 4, 7, 9, 10\}$  write the relation  $f$  in the roster form. Is  $f$  a function?

Ans.  $f : X \rightarrow Y$  defined by

$$f(x) = x^2, x \in X$$

$$\text{and } X = \{-2, -1, 0, 1, 2, 3\}$$

$$Y = \{0, 1, 4, 7, 9, 10\}$$

$$f(-2) = (-2)^2 = 4$$

$$f(-1) = (-1)^2 = 1$$

$$f(0) = 0^2 = 0$$

$$f(1) = 1^2 = 1$$

$$f(2) = 2^2 = 4$$

$$f(3) = 3^2 = 9$$

$$\therefore f = \{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9)\}$$

$f$  is a function because different elements of  $X$  have different images in  $Y$

**31. Determine a quadratic function ' $f$ ' defined by**

$$f(x) = ax^2 + bx + c \text{ if } f(0) = 6, f(2) = 11 \text{ and } f(-3) = 6$$

**Ans.**  $f(x) = ax^2 + bx + c$

$$f(0) = 6$$

$$a \times 0^2 + b \times 0 + c = 6$$

$$c = 6$$

$$f(2) = 11$$

$$a \times 2^2 + b \times 2 + c = 11$$

$$4a + 2b + c = 11$$

$$4a + 2b + 6 = 11$$

$$4a + 2b = 11 - 6$$

$$[4a + 2b = 5] \text{ --- (i)}$$

$$f(-3) = 6$$

$$a \times (-3)^2 + b \times (-3) + c = 6$$

$$9a - 3b + 6 = 6$$

$$[9a - 3b = 0] \text{ --- (ii)}$$

Multiplying eq. (i) by 3 and eq. (ii) by 2

$$12a + 6b = 15$$

$$18a - 6b = -12$$

$$\hline 30a = 3$$

$$a = \frac{3}{30} = \frac{1}{10}$$

$$4 \times \frac{1}{5} + 2b = 5$$

$$2b = 5 - \frac{4}{5}$$

$$2b = \frac{25-4}{5} = \frac{21}{5}$$

$$b = \frac{21}{10}$$

$$\therefore f(x) = \frac{1}{10}x^2 + \frac{21}{10}x + 6$$

32. Find the domain and the range of the function  $f$  defined by  $f(x) = \frac{x+2}{|x+2|}$

Ans.  $f(x) = \frac{x+2}{|x+2|}$

For Df,  $f(x)$  must be a real no.

$$\Rightarrow |x+2| \neq 0 \Rightarrow x+2 \neq 0 \Rightarrow x \neq -2$$

$\therefore$  Domain of  $f$  = set of all real numbers

except -2 i.e.  $Df = \mathbb{R} - \{-2\}$

for Rf

case I if  $x+2 > 0$  then  $|x+2| = x+2$

$$\therefore f(x) = \frac{x+2}{|x+2|} = 1$$

case II if  $x+2 < 0$ ,  $|x+2| = -(x+2)$

$$\therefore f(x) = \left( \frac{x+2}{-x+2} \right) = -1$$

$$\therefore \text{Range of } f = \{-1, 1\}$$

33. Find the domain and the range of  $f(x) = \frac{x^2}{1+x^2}$

$$\text{Ans. } f(x) = \frac{x^2}{1+x^2}$$

Domain of  $f$  = all real no. =  $R$

for Range let  $f(x) = y$

$$y = \frac{x^2}{1+x^2}$$

$$y(1+x^2) = x^2$$

$$y + yx^2 = x^2$$

$$y = x^2 - yx^2$$

$$y = (1-y)x^2$$

$$x^2 = \frac{y}{1-y}$$

$$x = \sqrt{\frac{y}{1-y}}$$

$$\frac{y}{1-y} \geq 0 \quad 1-y \neq 0$$

$$y \neq 1$$

also  $y \geq 0$  and  $1-y > 0$



$$y < 1$$

$\therefore$  Range of  $f = [0, 1)$ .

34. If

$A = \{1, 2, 3\}$ ,  $B = \{1, 2, 3, 4\}$  and

$R = \{(x, y) : (x, y) \in A \times B, y = x + 1\}$  then

(i) find  $A \times B$  (ii) write domain and Range

Ans.

(i)

$$A \times B = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4)\}$$

$$(ii) R = \{(1, 2), (2, 3), (3, 4)\}$$

$$\text{Domain of } R = \{1, 2, 3\}$$

$$\text{Range of } R = \{2, 3, 4\}$$

## CBSE Class 12 Mathematics

### Important Questions

#### Chapter 2

#### Relations and Functions

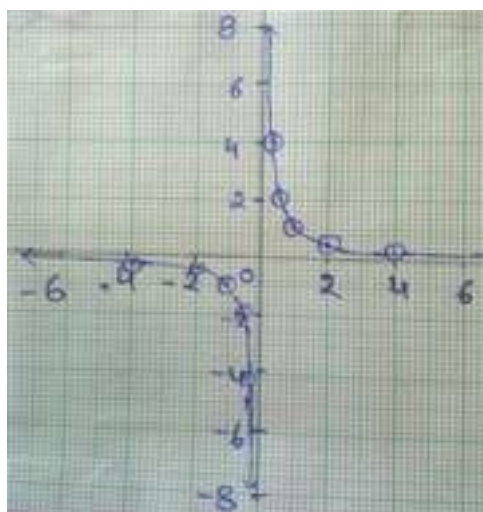
#### 6 Marks Questions

1. Draw the graphs of the following real functions and hence find their range

$$f(x) = \frac{1}{x}, x \in \mathbb{R}, x \neq 0$$

Ans. Given  $f(x) = \frac{1}{x}, x \in \mathbb{R}, x \neq 0$

Let  $y = f(x) = \frac{1}{x}$  if  $y = \frac{1}{x}, x \in \mathbb{R}, x \neq 0$



(Fig for Answer 11)

$x$	-4	-2	-1	-0.5	-0.25	0.5	1	2	4
$y = \frac{1}{x}$	-0.25	-0.5	-1	-2	-4	2	1	0.5	0.25

Plot the points shown in the above table and join these points by a free hand drawing.

Portion of the graph are shown the right margin

From the graph, it is clear that  $Rf = R - [0]$

This function is called reciprocal function.

**2.If**  $f(x) = x - \frac{1}{x}$ , **Prove that**  $[f(x)]^3 = f(x^3) + 3f\left(\frac{1}{x}\right)$

**Ans.** If  $f(x) = x - \frac{1}{x}$ , prove that  $[f(x)]^3 = f(x^3) + f\left(\frac{1}{x}\right)$

**Given**  $f(x) = x - \frac{1}{x}$ ,  $Df = R - [0]$

$$\Rightarrow f(x^3) = x^3 - \frac{1}{x^3} \text{ and } f\left(\frac{1}{x}\right) = \frac{1}{x} - \frac{1}{\frac{1}{x}} = \frac{1}{x} - x \dots\dots (i)$$

$$\therefore [f(x)]^3 = \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3x \cdot \frac{1}{x} \left(x - \frac{1}{x}\right)$$

$$= x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

$$= x^3 - \frac{1}{x^3} + 3\left(\frac{1}{x} - x\right)$$

$$= f(x^3) + 3f\left(\frac{1}{x}\right) [\text{using } (i)]$$

**3.Draw the graphs of the following real functions and hence find their range**

$$(i) f(x) = 2x - 1 \quad (ii) f(x) = \frac{x^2 - 1}{x - 1}$$

**Ans. (i)** Given  $f(x)$  i.e.  $y = x - 1$ , which is first degree equation in  $x, y$  and hence it represents a straight line. Two points are sufficient to determine straight line uniquely

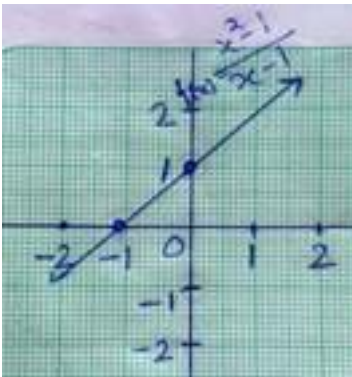
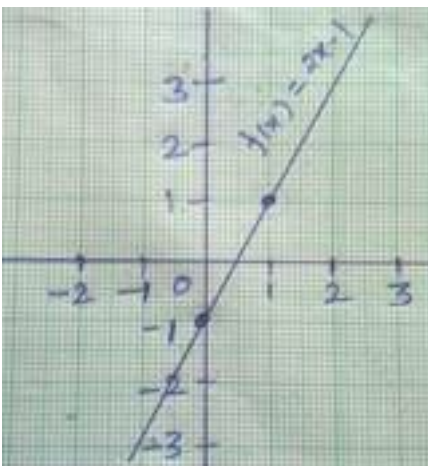


Table of values

$x$	0	1
$y$	-1	1

A portion of the graph is shown in the figure from the graph, it is clear that  $y$  takes all real values. It therefore that  $R_f = R$

**(ii)** Given  $f(x) = \frac{x^2 - 1}{x - 1} \Rightarrow D_f = R - \{1\}$



Let  $y = f(x) = \frac{x^2 - 1}{x - 1} = x + 1 (\because x \neq 1)$

i.e.  $y = x + 1$ , which is a first degree equation in  $x, y$  and hence it represents a straight line. Two points are sufficient to determine a straight line uniquely

Table of values

$x$	-1	0
$y$	0	1

A portion of the graph is shown in the figure from the graph it is clear that  $y$  takes all real values except 2. It follows that  $R_f = \mathbb{R} - \{2\}$ .

4. Let  $f$  be a function defined by  $f : x \rightarrow 5x^2 + 2, x \in \mathbb{R}$

(i) find the image of 3 under  $f$

(ii) find  $f(3) + f(2)$

(iii) find  $x$  such that  $f(x) = 22$

Ans. Given  $f(x) = 5x^2 + 2, x \in \mathbb{R}$

$$(i) f(3) = 5 \times 3^2 + 2 = 5 \times 9 + 2 = 47$$

$$(ii) f(2) = 5 \times 2^2 + 2 = 5 \times 4 + 2 = 22$$

$$\therefore f(3) \times f(2) = 47 \times 22 = 1034$$

$$(iii) f(x) = 22$$

$$\Rightarrow 5x^2 + 2 = 22$$

$$\Rightarrow 5x^2 = 20$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = 2, -2$$

5. The function  $f(x) = \frac{9x}{5} + 32$  is the formula to connect  $^{\circ}\text{C}$  to Fahrenheit

units find (i)  $f(0)$  (ii)  $f(-10)$  (iii) the value of  $x$   $f(x) = 212$  interpret the result in each case

Ans.  $f(x) = \frac{9x}{5} + 32$  (given)

$$(i) f(0) = \left( \frac{9 \times 0}{5} + 32 \right) = 32 \Rightarrow f(0) = 32 \Rightarrow 0^\circ C = 32^\circ F$$

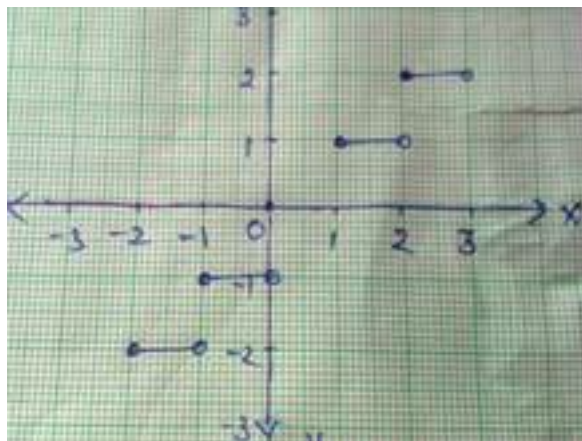
$$(ii) f(-10) = \left( \frac{9 \times (-10)}{5} + 32 \right) = 14 \Rightarrow f(-10) = 14^\circ \Rightarrow (-10)^\circ C = 14^\circ F$$

$$(iii) f(x) = 212 \Leftrightarrow \frac{9x}{5} + 32 = 212 \Leftrightarrow 9x = 5 \times (180) \\ \Leftrightarrow x = 100$$

$$\therefore 212^\circ F = 100^\circ C$$

6. Draw the graph of the greatest integer function,  $f(x) = [x]$ .

Ans. Clearly, we have



$$f(x) = \begin{cases} -2, & \text{when } x \in [-2, -1) \\ -1, & \text{when } x \in [-1, 0) \\ 0, & \text{when } x \in [0, 1) \\ 1, & \text{when } x \in [1, 2) \end{cases}$$

$x$	.....	$-2 \leq x < 1$	$-1 \leq x < 0$	$0 \leq x < 1$	$1 \leq x < 2$	$2 \leq x < 3$	.....
$y$	.....	-2	-1	0	1	2	.....

7. Find the domain and the range of the following functions:

$$(i) f(x) = \sqrt{x^2 - 4} \quad (ii) f(x) = \sqrt{16 - x^2} \quad (iii) f(x) = \frac{1}{\sqrt{9 - x^2}}$$

**Ans.** (i) Given  $f(x) = \sqrt{x^2 - 4}$

For  $D_f$ ,  $f(x)$  must be a real number

$$\Rightarrow \sqrt{x^2 - 4} \text{ Must be a real number}$$

$$\Rightarrow x^2 - 4 \geq 0 \Rightarrow (x+2)(x-2) \geq 0$$

$$\Rightarrow \text{either } x \leq -2 \text{ or } x \geq 2$$

$$\Rightarrow D_f = (-\infty, -2] \cup [2, \infty).$$

For  $R_f$ , let  $y = \sqrt{x^2 - 4}$ .....(i)

As square root of a real number is always non-negative,  $y \geq 0$

On squaring (i), we get  $y^2 = x^2 - 4$

$$\Rightarrow x^2 = y^2 + 4 \text{ but } x^2 \geq 0 \text{ for all } x \in D_f$$

$$\Rightarrow y^2 + 4 \geq 0 \Rightarrow y^2 \geq -4, \text{ which is true for all } y \in \mathbb{R}. \text{ also } y \geq 0$$

$$\Rightarrow R_f = [0, \infty)$$

**(ii)** Given  $f(x) = \sqrt{16 - x^2}$

For  $D_f$ ,  $f(x)$  must be a real number

$\Rightarrow \sqrt{16-x^2}$  must be a real number

$$\Rightarrow \sqrt{16-x^2} \geq 0 \Rightarrow -(x^2-16) \geq 0$$

$$\Rightarrow x^2-16 \leq 0$$

$$\Rightarrow (x+4)(x-4) \leq 0 \Rightarrow -4 \leq x \leq 4$$

$$\Rightarrow D_f = [-4, 4].$$

For  $R_f$ , let  $y = \sqrt{16-x^2}$  .....(i)

As square root of real number is always non-negative,  $y \geq 0$

Squaring (i) we get

$$y^2 = 16 - x^2$$

$$\Rightarrow x^2 = 16 - y^2 \text{ but } x^2 \geq 0 \text{ for all } x \in D_f$$

$$\Rightarrow 16 - y^2 \geq 0 \Rightarrow -(y^2 - 16) \geq 0 \Rightarrow y^2 - 16 \leq 0$$

$$\Rightarrow (y+4)(y-4) \leq 0 \Rightarrow -4 \leq y \leq 4 \text{ but } y \geq 0$$

$$\Rightarrow R_f = [0, 4]$$

(iii) Given  $f(x) = \frac{1}{\sqrt{9-x^2}}$

For  $D_f$ ,  $f(x)$  must be a real number

$$\Rightarrow \frac{1}{\sqrt{9-x^2}} \text{ must be a real number}$$



$$\Rightarrow 9 - x^2 > 0 \Rightarrow -(x^2 - 9) > 0 \Rightarrow x^2 - 9 < 0$$

$$\Rightarrow (x+3)(x-3) < 0 \Rightarrow -3 < x < 3 \Rightarrow D_f = (-3, 3)$$

For  $R_f$ , let  $y = \frac{1}{\sqrt{9-x^2}}, y \neq 0 \dots\dots\dots(i)$

Also as the square root of a real number is always non-negative,  $y > 0$ .

on squaring (i) we get

$$y^2 = \frac{1}{9-x^2} \Rightarrow 9-x^2 = \frac{1}{y^2} \Rightarrow x^2 = 9 - \frac{1}{y^2}$$

But  $x^2 \geq 0$  for all  $x \in D_f \Rightarrow 9 - \frac{1}{y^2} \geq 0$

$$y^2 > 0$$

(Multiply both sides by  $y^2$ , a positive real number)

$$\Rightarrow 9y^2 - 1 \geq 0 \Rightarrow y^2 - \frac{1}{9} \geq 0$$

$$\Rightarrow \left(y + \frac{1}{3}\right)\left(y - \frac{1}{3}\right) \geq 0$$

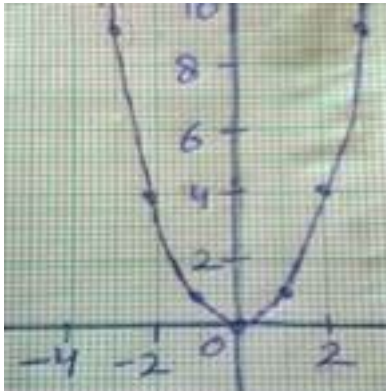
$$\Rightarrow \text{either } y \leq -\frac{1}{3} \text{ or } y \geq \frac{1}{3}$$

$$y > 0 \Rightarrow y \geq \frac{1}{3}$$

$$\Rightarrow R_f = \left[\frac{1}{3}, \infty\right).$$

**8. Draw the graphs of the following real functions and hence find range:  $f(x) = x^2$**

**Ans.**



Given  $f(x) = x^2 \Rightarrow D_f = \mathbb{R}$

Let  $y = f(x) = x^2, x \in \mathbb{R}$

$x$	-4	-3	-2	-1	0	1	2	3	4
$y = x^2$	16	9	4	1	0	1	4	9	16

Plot the points

$(-4, 16), (-3, 9), (-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4), (3, 9), (4, 16), \dots$

And join these points by a free hand drawing. A portion of the graph is shown in sigma (next)

From the graph, it is clear that  $y$  takes all non-negative real values, it follows that

$$R_f = [0, \infty)$$

**9. Define polynomial function. Draw the graph of  $f(x) = x^3$  find domain and range**

**Ans.** A function  $f: \mathbb{R} \rightarrow \mathbb{R}$  define by

$$f(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

where  $a_0, a_1, a_2, \dots, a_n \in \mathbb{R}$

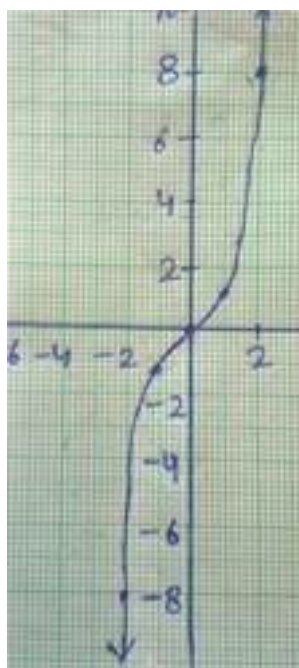
And  $n$  is non negative integer is called polynomial function

Graph of  $f(x) = x^3$

$x$	0	1	2	-1	-2
$f(x)$	0	1	8	-1	-8

Domain of  $f = \mathbb{R}$

Range of  $f = \mathbb{R}$



10.(a) If  $A, B$  are two sets such that  $n(A \times B) = 6$  and some elements of  $A \times B$  are  $(-1, 2), (2, 3), (4, 3)$ , then find  $A \times B$  and  $B \times A$

(b) Find domain of the function  $f(x) = \frac{1}{\sqrt{x + [x]}}$

**Ans. (a)** Given  $A$  and  $B$  are two sets such that

$$n(A \times B) = 6$$

Some elements of  $A \times B$  are

$$(-1, 2), (2, 3) \text{ and } (4, 3)$$

$$\text{then } A = \{-1, 2, 4\} \text{ and } B = \{2, 3\}$$

$$A \times B = \{(-1, 2), (-1, 3), (2, 2), (2, 3), (4, 2), (4, 3)\}$$

$$B \times A = \{(2, -1), (3, -1), (2, 2), (3, 2), (2, 4), (3, 4)\}$$

(b)

$$f(x) = \frac{1}{\sqrt{x + [x]}}$$

we knowe that

$$x + [x] > 0 \text{ for all } x > 0$$

$$x + [x] = 0 \text{ for all } x = 0$$

$$x + [x] < 0 \text{ for all } x < 0$$

$$\text{also } f(x) = \frac{1}{\sqrt{x + [x]}} \text{ is defined for all}$$

$$x \text{ satisfying } x + [x] > 0$$

$$\text{Hence, Domain } (f) = (0, \infty)$$