

Ex 7.1

Q1

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\therefore \cos A = \sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B}$$

$$\Rightarrow \cos A = \sqrt{1 - \left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \cos A = \sqrt{1 - \frac{16}{25}} \text{ and } \sin B = \sqrt{1 - \frac{25}{169}}$$

$$\Rightarrow \cos A = \sqrt{\frac{25 - 16}{25}} \text{ and } \sin B = \sqrt{\frac{169 - 25}{169}}$$

$$\Rightarrow \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Now,

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{4}{5} \times \frac{5}{13} + \frac{3}{5} \times \frac{12}{13}$$

$$= \frac{20}{65} + \frac{36}{65}$$

$$= \frac{20 + 36}{65}$$

$$= \frac{56}{65}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\therefore \cos A = \sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B}$$

$$\Rightarrow \cos A = \sqrt{1 - \left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \cos A = \sqrt{1 - \frac{16}{25}} \text{ and } \sin B = \sqrt{1 - \frac{25}{169}}$$

$$\Rightarrow \cos A = \sqrt{\frac{25 - 16}{25}} \text{ and } \sin B = \sqrt{\frac{169 - 25}{169}}$$

$$\Rightarrow \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Now,

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{3}{5} \times \frac{5}{13} - \frac{4}{5} \times \frac{12}{13}$$

$$= \frac{15}{65} - \frac{48}{65}$$

$$= \frac{15 - 48}{65}$$

$$= \frac{-33}{65}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\therefore \cos A = \sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B}$$

$$\Rightarrow \cos A = \sqrt{1 - \left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \cos A = \sqrt{1 - \frac{16}{25}} \text{ and } \sin B = \sqrt{1 - \frac{25}{169}}$$

$$\Rightarrow \cos A = \sqrt{\frac{25 - 16}{25}} \text{ and } \sin B = \sqrt{\frac{169 - 25}{169}}$$

$$\Rightarrow \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Now,

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$= \frac{4}{5} \times \frac{5}{13} - \frac{3}{5} \times \frac{12}{13}$$

$$= \frac{20}{65} - \frac{36}{65}$$

$$= \frac{20 - 36}{65}$$

$$= -\frac{16}{65}$$

We have,

$$\sin A = \frac{4}{5} \text{ and } \cos B = \frac{5}{13}$$

$$\therefore \cos A = \sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B}$$

$$\Rightarrow \cos A = \sqrt{1 - \left(\frac{4}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \cos A = \sqrt{1 - \frac{16}{25}} \text{ and } \sin B = \sqrt{1 - \frac{25}{169}}$$

$$\Rightarrow \cos A = \sqrt{\frac{25 - 16}{25}} \text{ and } \sin B = \sqrt{\frac{169 - 25}{169}}$$

$$\Rightarrow \cos A = \sqrt{\frac{9}{25}} \text{ and } \sin B = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \cos A = \frac{3}{5} \text{ and } \sin B = \frac{12}{13}$$

Now,

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{3}{5} \times \frac{5}{13} + \frac{4}{5} \times \frac{12}{13}$$

$$= \frac{15}{65} + \frac{48}{65}$$

$$= \frac{15 + 48}{65}$$

$$= \frac{63}{65}$$

Q2

We have,

$$\sin A = \frac{12}{13} \text{ and } \sin B = \frac{4}{5}$$

$$\therefore \cos A = -\sqrt{1 - \sin^2 A} \text{ and } \cos B = \sqrt{1 - \sin^2 B}$$

[$\because$ In the second quadrant $\cos \theta$ is negative]

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{12}{13}\right)^2} \text{ and } \cos B = \sqrt{1 - \left(\frac{4}{5}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{144}{169}} \text{ and } \cos B = \sqrt{1 - \frac{16}{25}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{25}{169}} \text{ and } \cos B = \sqrt{\frac{9}{25}}$$

$$\Rightarrow \cos A = \frac{-5}{13} \text{ and } \cos B = \frac{3}{5}$$

Now,

(i)

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= \frac{12}{13} \times \frac{3}{5} - \frac{5}{13} \times \frac{4}{5}$$

$$= \frac{36}{65} - \frac{20}{65}$$

$$= \frac{16}{65}$$

(ii)

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= \frac{-5}{13} \times \frac{3}{5} - \frac{12}{13} \times \frac{4}{5}$$

$$= \frac{-15}{65} - \frac{48}{65}$$

$$= \frac{-63}{65}$$

We have,

$$\sin A = \frac{3}{5} \text{ and } \cos B = \frac{-12}{13}$$

$$\therefore \cos A = -\sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B}$$

[$\because$ In the second quadrant $\cos \theta$ is negative]

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{3}{5}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{-12}{13}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{9}{25}} \text{ and } \sin B = \sqrt{1 - \frac{144}{169}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{16}{25}} \text{ and } \sin B = \sqrt{\frac{25}{169}}$$

$$\Rightarrow \cos A = -\frac{4}{5} \text{ and } \sin B = \frac{5}{13}$$

Now,

$$\begin{aligned}\sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{3}{5} \times \left(\frac{-12}{13}\right) - \frac{4}{5} \times \frac{5}{13} \\ &= -\frac{36}{65} - \frac{20}{65} \\ &= -\frac{56}{65}\end{aligned}$$

$$\therefore \sin(A+B) = -\frac{56}{65}$$

Q3

We have,

$$\cos A = -\frac{24}{25} \text{ and } \cos B = \frac{3}{5}$$

$$\therefore \sin A = -\sqrt{1 - \cos^2 A} \text{ and } \sin B = -\sqrt{1 - \cos^2 B} \quad [\because \text{In the 3rd and 4th quadrant } \sin \theta \text{ is negative}]$$

$$\Rightarrow \sin A = -\sqrt{1 - \left(-\frac{24}{25}\right)^2} \text{ and } \sin B = -\sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$\Rightarrow \sin A = -\sqrt{1 - \frac{576}{625}} \text{ and } \sin B = -\sqrt{1 - \frac{9}{25}}$$

$$\Rightarrow \sin A = -\sqrt{\frac{49}{625}} \text{ and } \sin B = -\sqrt{\frac{16}{25}}$$

$$\Rightarrow \sin A = -\frac{7}{25} \text{ and } \sin B = -\frac{4}{5}$$

Now,

$$(i) \quad \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$= -\frac{7}{25} \times \frac{3}{5} - \frac{24}{25} \times \left(-\frac{4}{5}\right)$$

$$= -\frac{21}{125} + \frac{96}{125}$$

$$= \frac{75}{125}$$

$$= \frac{3}{5}$$

$$(ii) \quad \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$= -\frac{24}{25} \times \frac{3}{5} - \left(-\frac{7}{25}\right) \times \left(-\frac{4}{5}\right)$$

$$= -\frac{72}{125} - \frac{28}{125}$$

$$= \frac{-72 - 28}{125}$$

$$= \frac{-100}{125} = -\frac{4}{5}$$

Q4

We have,

$$\tan A = \frac{3}{4}, \text{ and } \cos B = \frac{9}{41}$$

$$\begin{aligned}\therefore \sin B &= \sqrt{1 - \cos^2 B} \\&= \sqrt{1 - \left(\frac{9}{41}\right)^2} \\&= \sqrt{1 - \frac{81}{1681}} \\&= \sqrt{\frac{1600}{1681}} \\&= \frac{40}{41}\end{aligned}$$

$$\therefore \tan B = \frac{\sin B}{\cos B} = \frac{\frac{40}{41}}{\frac{9}{41}} = \frac{40}{9}$$

Now,

$$\begin{aligned}\tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\&= \frac{\frac{3}{4} + \frac{40}{9}}{1 - \frac{3}{4} \times \frac{40}{9}} \\&= \frac{\frac{27+160}{36}}{\frac{36-120}{36}} \\&= \frac{187}{-84} \\&= -\frac{187}{84}\end{aligned}$$

Q5

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{12}{13}$$

$$\therefore \cos A = -\sqrt{1 - \sin^2 A} \text{ and } \sin B = -\sqrt{1 - \cos^2 B}$$

[$\because$ cosine is negative in second quadrant and
sine is negative in fourth quadrant]

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2} \text{ and } \sin B = -\sqrt{1 - \left(\frac{12}{13}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{1}{4}} \text{ and } \sin B = -\sqrt{1 - \frac{144}{169}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = -\sqrt{\frac{25}{169}}$$

$$\Rightarrow \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = -\frac{5}{13}$$

$$\therefore \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$\text{and, } \tan B = \frac{\sin B}{\cos B} = \frac{-\frac{5}{13}}{\frac{12}{13}} = \frac{-5}{12}$$

$$\text{Now, } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\begin{aligned} &= \frac{\frac{-1}{\sqrt{3}} - \left(\frac{-5}{12}\right)}{1 + \left(\frac{-1}{\sqrt{3}}\right) \times \left(\frac{-5}{12}\right)} \\ &= \frac{\frac{-1}{\sqrt{3}} + \frac{5}{12}}{1 + \frac{5}{12\sqrt{3}}} \\ &= \frac{\frac{-12 + 5\sqrt{3}}{12\sqrt{3}}}{\frac{12\sqrt{3} + 5}{12\sqrt{3}}} \\ &= \frac{5\sqrt{3} - 12}{5 + 12\sqrt{3}} \end{aligned}$$

$$\Rightarrow \tan(A - B) = \frac{5\sqrt{3} - 12}{5 + 12\sqrt{3}}$$

Q6

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

$$\therefore \cos A = -\sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B}$$

[$\because$ cosine is negative in second quadrant]

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{1}{4}} \text{ and } \sin B = \sqrt{1 - \frac{3}{4}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{1}{2}$$

$$\therefore \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$\text{and, } \tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

Now,

$$\begin{aligned} \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ &= \frac{\frac{-1}{\sqrt{3}} + \frac{1}{\sqrt{3}}}{1 - \left(\frac{-1}{\sqrt{3}}\right) \times \left(\frac{1}{\sqrt{3}}\right)} \\ &= 0 \end{aligned}$$

$$\therefore \tan(A+B) = 0$$

We have,

$$\sin A = \frac{1}{2} \text{ and } \cos B = \frac{\sqrt{3}}{2}$$

$$\therefore \cos A = -\sqrt{1 - \sin^2 A} \text{ and } \sin B = \sqrt{1 - \cos^2 B} \text{ [}\because \text{ cosine is negative in second quadrant]}$$

$$\Rightarrow \cos A = -\sqrt{1 - \left(\frac{1}{2}\right)^2} \text{ and } \sin B = \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \cos A = -\sqrt{1 - \frac{1}{4}} \text{ and } \sin B = \sqrt{1 - \frac{3}{4}}$$

$$\Rightarrow \cos A = -\sqrt{\frac{3}{4}} \text{ and } \sin B = \sqrt{\frac{1}{4}}$$

$$\Rightarrow \cos A = -\frac{\sqrt{3}}{2} \text{ and } \sin B = \frac{1}{2}$$

$$\therefore \tan A = \frac{\sin A}{\cos A} = \frac{\frac{1}{2}}{\frac{-\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}}$$

$$\text{and, } \tan B = \frac{\sin B}{\cos B} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

$$\text{Now, } \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\frac{-1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{-1}{\sqrt{3}}\right) \times \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \frac{\frac{-2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \frac{-3}{\sqrt{3}}$$

$$= \frac{-\sqrt{3} \times \sqrt{3}}{\sqrt{3}} = -\sqrt{3}$$

$$\therefore \tan(A - B) = -\sqrt{3}$$

Q7

(i)

$$\begin{aligned}\sin 78^\circ \cos 18^\circ - \cos 78^\circ \sin 18^\circ & \quad [\sin(A - B) = \sin A \cos B - \cos A \sin B] \\&= \sin(78^\circ - 18^\circ) \\&= \sin 60^\circ \\&= \frac{\sqrt{3}}{2}\end{aligned}$$

(ii)

$$\begin{aligned}\cos 47^\circ \cos 13^\circ - \sin 47^\circ \sin 13^\circ & \quad [\cos(A + B) = \cos A \cos B - \sin A \sin B] \\&= \cos(47^\circ + 13^\circ) \\&= \cos 60^\circ \\&= \frac{1}{2}\end{aligned}$$

(iii)

$$\begin{aligned}\sin 36^\circ \cos 9^\circ + \cos 36^\circ \sin 9^\circ & \quad [\sin(A + B) = \sin A \cos B + \cos A \sin B] \\&= \sin(36^\circ + 9^\circ) \\&= \sin 45^\circ \\&= \frac{1}{\sqrt{2}}\end{aligned}$$

(iv)

$$\begin{aligned}\cos 80^\circ \cos 20^\circ + \sin 80^\circ \sin 20^\circ & \quad [\cos(A - B) = \cos A \cos B + \sin A \sin B] \\&= \cos(80^\circ - 20^\circ) \\&= \cos 60^\circ \\&= \frac{1}{2}\end{aligned}$$

Q8

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\Rightarrow \sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(\frac{-12}{13}\right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\begin{aligned} \operatorname{cosec} B &= -\sqrt{1 + \cot^2 B} & [\because \operatorname{cosec} \text{ is negative in third quadrant}] \\ &= -\sqrt{1 + \left(\frac{24}{7}\right)^2} = -\sqrt{1 + \frac{576}{49}} = -\sqrt{\frac{49 + 576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7} \end{aligned}$$

$$\Rightarrow \sin B = \frac{-7}{25} \quad \left[\because \operatorname{cosec} B = \frac{1}{\sin B} \right]$$

Now,

$$\begin{aligned} \cos B &= -\sqrt{1 - \sin^2 B} & [\because \cos \theta \text{ is negative in third quadrant}] \\ &= -\sqrt{1 - \left(\frac{-7}{25}\right)^2} = -\sqrt{1 - \frac{49}{625}} = -\sqrt{\frac{625 - 49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25} \end{aligned}$$

Now,

$$\begin{aligned} \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{5}{13} \times \left(\frac{-24}{25}\right) + \left(\frac{-12}{13}\right) \times \left(\frac{-7}{25}\right) \\ &= \frac{-120}{325} + \frac{84}{325} \\ &= \frac{-120 + 84}{325} \\ &= \frac{-36}{325} \end{aligned}$$

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(\frac{-12}{13}\right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\operatorname{cosec} B = -\sqrt{1 + \cot^2 B} \quad [\because \operatorname{cosec} \text{ is negative in third quadrant}]$$

$$= -\sqrt{1 + \left(\frac{24}{7}\right)^2} = -\sqrt{1 + \frac{576}{49}} = -\sqrt{\frac{49 + 576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7}$$

$$\Rightarrow \sin B = \frac{-7}{25} \quad \left[\because \operatorname{cosec} B = \frac{1}{\sin B} \right]$$

Now,

$$\cos B = -\sqrt{1 - \sin^2 B} \quad [\because \cos \theta \text{ is negative in third quadrant}]$$

$$= -\sqrt{1 - \left(\frac{-7}{25}\right)^2} = -\sqrt{1 - \frac{49}{625}} = -\sqrt{\frac{625 - 49}{625}} = -\sqrt{\frac{576}{625}} = \frac{-24}{25}$$

Now,

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$= \left(\frac{-12}{13}\right) \times \left(\frac{-24}{25}\right) - \left(\frac{5}{13}\right) \times \left(\frac{-7}{25}\right)$$

$$= \frac{288}{325} + \frac{35}{325}$$

$$= \frac{323}{325}$$

We have,

$$\cos A = \frac{-12}{13} \text{ and } \cot B = \frac{24}{7}$$

$$\therefore \sin A = \sqrt{1 - \cos^2 A} = \sqrt{1 - \left(\frac{-12}{13}\right)^2} = \sqrt{1 - \frac{144}{169}} = \sqrt{\frac{25}{169}} = \frac{5}{13}$$

and,

$$\begin{aligned} \operatorname{cosec} B &= -\sqrt{1 + \cot^2 B} \quad [\because \operatorname{cosec} \text{ is negative in third quadrant}] \\ &= -\sqrt{1 + \left(\frac{24}{7}\right)^2} = -\sqrt{1 + \frac{576}{49}} = -\sqrt{\frac{49 + 576}{49}} = -\sqrt{\frac{625}{49}} = -\frac{25}{7} \end{aligned}$$

$$\Rightarrow \sin B = \frac{-7}{25} \quad \left[\because \operatorname{cosec} B = \frac{1}{\sin B} \right]$$

Now,

$$\begin{aligned} \cos B &= -\sqrt{1 - \sin^2 B} \quad [\because \cos \theta \text{ is negative in third quadrant}] \\ &= -\sqrt{1 - \left(\frac{-7}{25}\right)^2} = -\sqrt{1 - \frac{49}{625}} = -\sqrt{\frac{625 - 49}{625}} = -\sqrt{\frac{576}{625}} = -\frac{24}{25} \end{aligned}$$

Now,

$$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{5}{13}}{\frac{-12}{13}} = \frac{-5}{12} \quad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\text{and, } \tan B = \frac{\sin B}{\cos B} = \frac{\frac{-7}{25}}{\frac{-24}{25}} = \frac{7}{24} \quad \left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\begin{aligned} \therefore \tan(A+B) &= \frac{\tan A + \tan B}{1 - \tan A \tan B} \\ &= \frac{\frac{-5}{12} + \frac{7}{24}}{1 - \left(\frac{-5}{12}\right) \times \frac{7}{24}} \\ &= \frac{\frac{-10 + 7}{24}}{1 + \frac{35}{288}} \\ &= \frac{\frac{-3}{24}}{\frac{288 + 35}{288}} \\ &= \frac{-3}{24} \times \frac{288}{288 + 35} \end{aligned}$$

Q9

$$\begin{aligned}\text{LHS: } & \cos 105^\circ + \cos 15^\circ \\ &= \cos(90^\circ + 15^\circ) + \cos(90^\circ - 75^\circ) \\ &= -\sin 15^\circ + \sin 75^\circ \\ &= \sin 75^\circ - \sin 15^\circ\end{aligned}$$

$$\left[\begin{array}{l} \because \cos(90 + \theta) = -\sin \theta \\ \text{and } \cos(90 - \theta) = \sin \theta \end{array} \right]$$

$$\therefore \cos 105^\circ + \cos 15^\circ = \sin 75^\circ - \sin 15^\circ$$

Hence proved.

Q10

$$\begin{aligned}\text{LHS: } & \frac{\tan A + \tan B}{\tan A - \tan B} \\ &= \frac{\frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A} - \frac{\sin B}{\cos B}} \\ &= \frac{\frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}}{\frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B}} \\ &= \frac{\sin A \cos B + \cos A \sin B}{\sin A \cos B - \cos A \sin B} \\ &= \frac{\sin(A + B)}{\sin(A - B)}\end{aligned}$$

$$\left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\left[\begin{array}{l} \because \sin(A + B) = \sin A \cos B + \cos A \sin B \\ \text{and, } \sin(A - B) = \sin A \cos B - \cos A \sin B \end{array} \right]$$

$$\therefore \frac{\tan A + \tan B}{\tan A - \tan B} = \frac{\sin(A + B)}{\sin(A - B)}$$

Hence proved.

Q11

$$\text{LHS: } \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$$

Dividing numerator and denominator by $\cos 11^\circ$, we get

$$\begin{aligned} & \frac{\frac{\cos 11^\circ}{\cos 11^\circ} + \frac{\sin 11^\circ}{\cos 11^\circ}}{\frac{\cos 11^\circ}{\cos 11^\circ} - \frac{\sin 11^\circ}{\cos 11^\circ}} \\ &= \frac{1 + \tan 11^\circ}{1 - \tan 11^\circ} \\ &= \frac{\tan 45^\circ + \tan 11^\circ}{1 - \tan 45^\circ \times \tan 11^\circ} \\ &= \tan(45^\circ + 11^\circ) \\ &= \tan 56^\circ \end{aligned}$$

$$\left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$[\tan 45^\circ = 1]$$

$$\left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

$$\therefore \frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$$

Hence proved.

$$\text{LHS: } \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$$

$$\frac{\frac{\cos 9^\circ}{\cos 9^\circ} + \frac{\sin 9^\circ}{\cos 9^\circ}}{\frac{\cos 9^\circ}{\cos 9^\circ} - \frac{\sin 9^\circ}{\cos 9^\circ}}$$

$$= \frac{1 + \tan 9^\circ}{1 - \tan 9^\circ}$$

$$= \frac{\tan 45^\circ + \tan 9^\circ}{1 - \tan 45^\circ \times \tan 9^\circ}$$

$$= \tan(45^\circ + 9^\circ)$$

$$= \tan 54^\circ$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

[Dividing numerator and denominator by $\cos 9^\circ$]

$$\left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$[\tan 45^\circ = 1]$$

$$\left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

$$\text{LHS: } \frac{\cos 8^\circ - \sin 8^\circ}{\cos 8^\circ + \sin 8^\circ}$$

$$\frac{\frac{\cos 8^\circ}{\cos 8^\circ} - \frac{\sin 8^\circ}{\cos 8^\circ}}{\frac{\cos 8^\circ}{\cos 8^\circ} + \frac{\sin 8^\circ}{\cos 8^\circ}}$$

$$= \frac{1 - \tan 8^\circ}{1 + \tan 8^\circ}$$

$$= \frac{\tan 45^\circ - \tan 8^\circ}{1 + \tan 45^\circ \times \tan 8^\circ}$$

$$= \tan(45^\circ - 8^\circ)$$

$$= \tan 37^\circ$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

[Dividing numerator and denominator by $\cos 8^\circ$]

$$\left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$[\tan 45^\circ = 1]$$

$$\left[\because \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \right]$$

Q12

$$\begin{aligned}\text{LHS: } & \sin(60^\circ - \theta) \cos(30^\circ + \theta) + \cos(60^\circ - \theta) \times \sin(30^\circ + \theta) \\ &= \sin[(60^\circ - \theta) + (30^\circ + \theta)] & [\sin(A + B) = \sin A \cos B + \cos A \sin B] \\ &= \sin[60^\circ - \theta + 30^\circ + \theta] \\ &= \sin(90^\circ) \\ &= 1 \\ &= \text{RHS}\end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

Q13

$$\begin{aligned}\text{LHS: } & \frac{\tan 69^\circ + \tan 66^\circ}{1 - \tan 69^\circ \tan 66^\circ} \\ &= \tan(69^\circ + 66^\circ) & \left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right] \\ &= \tan(135^\circ) \\ &= \tan(90^\circ + 45^\circ) \\ &= -\cot 45^\circ & [\because \tan \theta \text{ is negative in second quadrant}] \\ &= -1 \\ &= \text{RHS}\end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

Q14

We have,

$$\tan A = \frac{5}{6} \text{ and } \tan B = \frac{1}{11}$$

Now,

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{5}{6} + \frac{1}{11}}{1 - \frac{5}{6} \times \frac{1}{11}}$$

$$= \frac{\frac{55+6}{66}}{1 - \frac{5}{66}}$$

$$= \frac{\frac{61}{66}}{\frac{66-5}{66}}$$

$$= \frac{61}{66} \times \frac{66}{61}$$

$$= 1$$

$$= \tan \frac{\pi}{4}$$

$$\left[\because \tan \frac{\pi}{4} = 1 \right]$$

$$\Rightarrow \tan(A+B) = \tan \frac{\pi}{4}$$

$$\Rightarrow A+B = \frac{\pi}{4}$$

Hence proved.

We have,

$$\tan A = \frac{m}{m-1} \text{ and } \tan B = \frac{1}{2m-1}$$

$$\begin{aligned}\text{Now, } \tan(A-B) &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\&= \frac{\frac{m}{m-1} - \frac{1}{2m-1}}{1 + \frac{m}{m-1} \times \frac{1}{2m-1}} \\&= \frac{\frac{m(2m-1) - (m-1)}{(m-1)(2m-1)}}{1 + \frac{m}{(m-1)(2m-1)}} \\&= \frac{\frac{m(2m-1) - (m-1)}{(m-1)(2m-1)}}{\frac{m(2m-1) - (m-1)}{(m-1)(2m-1)} + (m)} \\&= \frac{m(2m-1) - (m-1)}{(m-1)(2m-1) + (m)} \\&= \frac{2m^2 - m - m + 1}{2m^2 - m - 2m + 1 + m} \\&= \frac{2m^2 - m - m + 1}{2m^2 - 2m + 1} \\&= \frac{2m^2 - 2m + 1}{2m^2 - 2m + 1} \\&= 1\end{aligned}$$

$$\therefore \tan(A-B) = 1 = \tan\left(\frac{\pi}{4}\right) \quad \left[\because \tan\frac{\pi}{4} = 1 \right]$$

$$\Rightarrow \tan(A-B) = \tan\left(\frac{\pi}{4}\right)$$

$$\Rightarrow A-B = \left(\frac{\pi}{4}\right)$$

Q15

$$\text{LHS: } \cos^2 45^\circ - \sin^2 15^\circ$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2 - \sin^2 15^\circ$$

$$= \frac{1}{2} - \left(\frac{1 - \cos 2 \times 15^\circ}{2}\right)$$

$$= \frac{1}{2} - \left(\frac{1 - \cos 30^\circ}{2}\right)$$

$$= \frac{1 - 1 + \cos 30^\circ}{2}$$

$$= \frac{\cos 30^\circ}{2}$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{4}$$

$$= \text{RHS}$$

$$\left[\because \cos 45 = \frac{1}{\sqrt{2}} \right]$$

$$\left[\because \cos 2\theta = 1 - \sin^2 \theta \right]$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

We have,

$$\text{LHS } \sin^2 (n+1) A - \sin^2 nA$$

$$= \sin [(n+1) A + nA] \sin [(n+1) A - nA]$$

$$\left[\because \sin^2 A - \sin^2 B = \sin (A+B) \sin (A-B) \right]$$

$$= \sin [nA + A + nA] \sin [nA + A - nA]$$

$$= \sin (2nA + A) \sin (A)$$

$$= \sin (2n+1) A \sin A$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

Q16

We have,

$$\begin{aligned}\text{LHS} &= \frac{\sin(A+B) + \sin(A-B)}{\cos(A+B) + \cos(A-B)} \\&= \frac{2 \sin A \cos B}{2 \cos A \cos B} \quad \left[\begin{array}{l} \because 2 \sin A \cos B = \sin(A+B) + \sin(A-B) \\ \text{and, } 2 \cos A \cos B = \cos(A+B) + \cos(A-B) \end{array} \right] \\&= \frac{\sin A}{\cos A} \\&= \tan A \\&= \text{RHS}\end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$\begin{aligned}\text{LHS} &= \frac{\sin(A-B)}{\cos A \cos B} + \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} \\&= \frac{\sin A \cos B - \cos A \sin B}{\cos A \cos B} + \frac{\sin B \cos C - \cos B \sin C}{\cos B \cos C} + \frac{\sin C \cos A - \cos C \sin A}{\cos C \cos A} \\&= \frac{\sin A \cos B}{\cos A \cos B} - \frac{\cos A \sin B}{\cos A \cos B} + \frac{\sin B \cos C}{\cos B \cos C} - \frac{\cos B \sin C}{\cos B \cos C} + \frac{\sin C \cos A}{\cos C \cos A} \\&\quad - \frac{\cos C \sin A}{\cos C \cos A} \\&= \tan A - \tan B + \tan B - \tan C + \tan C - \tan A \\&= 0 \\&= \text{RHS}\end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

We have,

$$\begin{aligned}
 \text{LHS} &= \frac{\sin(A-B)}{\sin A \sin B} + \frac{\sin(B-C)}{\sin B \sin C} + \frac{\sin(C-A)}{\sin C \sin A} \\
 &= \frac{\sin A \cos B - \cos A \sin B}{\sin A \sin B} + \frac{\sin B \cos C - \cos B \sin C}{\sin B \sin C} + \frac{\sin C \cos A - \cos C \sin A}{\sin C \sin A} \\
 &= \frac{\sin A \cos B}{\sin A \sin B} - \frac{\cos A \sin B}{\sin A \sin B} + \frac{\sin B \cos C}{\sin B \sin C} - \frac{\cos B \sin C}{\sin B \sin C} + \frac{\sin C \cos A}{\sin C \sin A} - \frac{\cos C \sin A}{\sin C \sin A} \\
 &= \frac{\cos B}{\sin B} - \frac{\cos A}{\sin A} + \frac{\cos C}{\sin C} - \frac{\cos B}{\sin B} + \frac{\cos A}{\sin A} - \frac{\cos C}{\sin C} \\
 &= \cot B - \cot A + \cot C - \cot B + \cot A - \cot C \quad \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta} \right] \\
 &= 0 \\
 &= \text{RHS}
 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

We have,

$$\begin{aligned}
 \text{RHS} &= \sin^2 A + \sin^2(A-B) - 2 \sin A \cos B \sin(A-B) \\
 &= \sin^2 A + \sin(A-B) [\sin(A-B) - 2 \sin A \cos B] \\
 &= \sin^2 A + \sin(A-B) [\sin A \cos B - \cos A \sin B - 2 \sin A \cos B] \\
 &= \sin^2 A + \sin(A-B) [-\sin A \cos B - \cos A \sin B] \\
 &= \sin^2 A - \sin(A-B) (\sin A \cos B + \cos A \sin B) \\
 &= \sin^2 A - \sin(A-B) (\sin(A+B)) \\
 &= \sin^2 A - \sin(A-B) \sin(A+B) \\
 &= \sin^2 A - (\sin^2 A - \sin^2 B) \quad \left[\because \sin(A-B) \sin(A+B) = \sin^2 A - \sin^2 B \right] \\
 &= \sin^2 A - \sin^2 A + \sin^2 B \\
 &= \sin^2 B \\
 &= \text{LHS}
 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$\begin{aligned}
\text{RHS} &= \cos^2 A + \cos^2 B - 2 \cos A \cos B \cos(A+B) \\
&= \cos^2 A + (1 - \sin^2 B) - 2 \cos A \cos B \cos(A+B) \\
&= [\cos^2 A - \sin^2 B] - 2 \cos A \cos B \cos(A+B) + 1 \\
&= [\cos(A+B) \cos(A-B)] - 2 \cos A \cos B \cos(A+B) + 1 \\
&= \cos(A+B) [\cos(A-B) - 2 \cos A \cos B] + 1 \\
&= \cos(A+B) [\cos A \cos B + \sin A \sin B - 2 \cos A \cos B] + 1 \\
&= \cos(A+B) [-\cos A \cos B + \sin A \sin B] + 1 \\
&= -\cos(A+B) [\cos A \cos B - \sin A \sin B] + 1 \\
&= -\cos(A+B) [\cos(A+B)] + 1 \\
&= -\cos^2(A+B) + 1 \\
&= 1 - \cos^2(A+B) \\
&= \sin^2(A+B) \qquad \qquad \qquad [\sin^2 \theta = 1 - \cos^2 \theta] \\
&= \text{RHS}
\end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

We have,

$$\begin{aligned}\text{LHS} &= \frac{\tan(A+B)}{\cot(A-B)} \\&= \frac{\tan(A+B)}{\frac{1}{\tan(A-B)}} && \left[\because \cot \theta = \frac{1}{\tan \theta} \right] \\&= \tan(A+B) \tan(A-B) \\&= \left[\frac{\tan A + \tan B}{1 - \tan A \tan B} \right] \left[\frac{\tan A - \tan B}{1 + \tan A \tan B} \right] \\&= \frac{(\tan A + \tan B)(\tan A - \tan B)}{(1 - \tan A \tan B)(1 + \tan A \tan B)} \\&= \frac{\tan^2 A - \tan^2 B}{1 - (\tan A \tan B)^2} && \left[\because (a-b)(a+b) = a^2 - b^2 \right] \\&= \frac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B} \\&= \text{RHS}\end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

Q17

We have,

$$8\theta = 6\theta + 2\theta$$

$$\begin{aligned}\Rightarrow \tan 8\theta &= \tan(6\theta + 2\theta) \\ \Rightarrow \tan 8\theta &= \frac{\tan 6\theta + \tan 2\theta}{1 - \tan 6\theta \tan 2\theta} && \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right] \\ \Rightarrow \tan 8\theta (1 - \tan 6\theta \tan 2\theta) &= \tan 6\theta + \tan 2\theta \\ \Rightarrow \tan 8\theta - \tan 8\theta \tan 6\theta \tan 2\theta &= \tan 6\theta + \tan 2\theta \\ \Rightarrow \tan 8\theta - \tan 6\theta - \tan 2\theta &= \tan 8\theta \tan 6\theta \tan 2\theta\end{aligned}$$

Hence proved.

We have,

$$45^\circ = 30^\circ + 15^\circ$$

$$\Rightarrow \tan 45^\circ = \tan(30^\circ + 15^\circ)$$

$$\Rightarrow 1 = \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \tan 15^\circ} \quad \left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

$$\Rightarrow 1 - \tan 30^\circ \tan 15^\circ = \tan 15^\circ + \tan 30^\circ$$

$$\Rightarrow 1 = \tan 15^\circ + \tan 30^\circ + \tan 30^\circ \tan 15^\circ$$

$$\Rightarrow \tan 15^\circ + \tan 30^\circ + \tan 15^\circ \tan 30^\circ = 1$$

Hence proved.

We have,

$$45^\circ = 9^\circ + 36^\circ$$

$$\Rightarrow \tan 45^\circ = \tan(9^\circ + 36^\circ)$$

$$\Rightarrow 1 = \frac{\tan 9^\circ + \tan 36^\circ}{1 - \tan 9^\circ \tan 36^\circ} \quad \left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

$$\Rightarrow 1 - \tan 9^\circ \tan 36^\circ = \tan 9^\circ + \tan 36^\circ$$

$$\Rightarrow 1 = \tan 9^\circ + \tan 36^\circ + \tan 9^\circ \tan 36^\circ$$

$$\Rightarrow \tan 9^\circ + \tan 36^\circ + \tan 9^\circ \tan 36^\circ = 1$$

Hence proved.

We have,

$$13\theta = 9\theta + 4\theta$$

$$\Rightarrow \tan 13\theta = \tan(9\theta + 4\theta)$$

$$\Rightarrow \tan 13\theta = \frac{\tan 9\theta + \tan 4\theta}{1 - \tan 9\theta \tan 4\theta} \quad \left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \right]$$

$$\Rightarrow \tan 13\theta (1 - \tan 9\theta \tan 4\theta) = \tan 9\theta + \tan 4\theta$$

$$\Rightarrow \tan 13\theta - \tan 13\theta \tan 9\theta \tan 4\theta = \tan 9\theta + \tan 4\theta$$

$$\Rightarrow \tan 13\theta - \tan 9\theta - \tan 4\theta = \tan 13\theta \tan 9\theta \tan 4\theta$$

Hence proved.

Q18

We have,

$$\begin{aligned}
\text{RHS} &= \tan 3\theta \tan \theta \\
&= \tan(2\theta + \theta) \times \tan(2\theta - \theta) \\
&= \left[\frac{\tan 2\theta + \tan \theta}{1 - \tan 2\theta \tan \theta} \right] \times \left[\frac{\tan 2\theta - \tan \theta}{1 + \tan 2\theta \tan \theta} \right] \\
&= \frac{(\tan 2\theta + \tan \theta)(\tan 2\theta - \tan \theta)}{(1 - \tan 2\theta \tan \theta)(1 + \tan 2\theta \tan \theta)} \\
&= \frac{\tan^2 2\theta - \tan^2 \theta}{1 - \tan^2 2\theta \tan^2 \theta} \quad \left[\because (a-b)(a+b) = a^2 - b^2 \right] \\
&= \text{LHS}
\end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved

Q19

$$\begin{aligned}
\frac{\sin x \cos y + \sin y \cos x}{\sin x \cos y - \sin y \cos x} &= \frac{a-b}{a+b} \\
\rightarrow \frac{\sin x \cos y + \sin y \cos x - \sin x \cos y - \sin y \cos x}{\sin x \cos y + \sin y \cos x - \sin x \cos y - \sin y \cos x} &= \frac{a-b+a-b}{a+b-a-b} \quad [\text{Using Componendo and Dividendo}] \\
\rightarrow \frac{2\sin x \cos y}{2\sin y \cos x} &= \frac{2a}{2b} \\
&= \frac{\tan x}{\tan y} = \frac{c}{d} \\
\text{Hence Proved}
\end{aligned}$$

Q20

We have,

$$\tan A = x \tan B$$

$$\frac{\sin A}{\cos A} = x \frac{\sin B}{\cos B}$$

$$\left[\because \tan \theta = \frac{\sin \theta}{\cos \theta} \right]$$

$$\Rightarrow \sin A \cos B = x \cos A \sin B \quad \text{--- (i)}$$

$$\begin{aligned} \text{Now, } \frac{\sin(A-B)}{\sin(A+B)} &= \frac{\sin A \cos B - \sin B \cos A}{\sin A \cos B + \cos A \sin B} \\ &= \frac{x \cos A \sin B - \cos A \sin B}{x \cos A \sin B + \cos A \sin B} && \text{[Using equation (i)]} \\ &= \frac{\cos A \sin B (x-1)}{\cos A \sin B (x+1)} \\ &= \frac{x-1}{x+1} \end{aligned}$$

$$\therefore \frac{\sin(A-B)}{\sin(A+B)} = \frac{x-1}{x+1}$$

Hence proved.

Q21

We have,

$$\tan(A+B) = x \text{ and } \tan(A-B) = y$$

$$\begin{aligned}\text{Now, } \tan 2A &= \tan[(A+B) + (A-B)] \\ &= \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B) \times \tan(A-B)} \\ &= \frac{x+y}{1-xy}\end{aligned}$$

$$\therefore \tan 2A = \frac{x+y}{1-xy}$$

$$\begin{aligned}\text{Now, } \tan 2B &= \tan[(A+B) - (A-B)] \\ &= \frac{\tan(A+B) - \tan(A-B)}{1 + \tan(A+B) \times \tan(A-B)} \\ &= \frac{x-y}{1+xy}\end{aligned}$$

$$\therefore \tan 2B = \frac{x-y}{1+xy}$$

Q22

We have,

$$\cos A + \sin B = m \text{ and } \sin A + \cos B = n$$

Now, $m^2 + n^2 - 2$

$$= (\cos A + \sin B)^2 + (\sin A + \cos B)^2 - 2$$

$$= \cos^2 A + \sin^2 B + 2 \cos A \sin B + \sin^2 A + \cos^2 B + 2 \sin A \cos B - 2$$

$$= (\sin^2 A + \cos^2 A) + (\sin^2 B + \cos^2 B) + 2 \cos A \sin B + 2 \sin A \cos B - 2$$

$$= 1 + 1 + 2 \cos A \sin B + 2 \sin A \cos B - 2$$

$$= 2 + 2 (\sin A \cos B + \cos A \sin B) - 2$$

$$= 2 (\sin A \cos B + \cos A \sin B)$$

$$= 2 \sin(A + B) \quad \left[\because \sin(A + B) = \sin A \cos B + \cos A \sin B \right]$$

$$\therefore 2 \sin(A + B) = m^2 + n^2 - 2$$

Hence proved

Q23

We have,

$$\tan A + \tan B = a \text{ and } \cot A + \cot B = b$$

$$\text{Now, } \cot A + \cot B = b$$

$$\Rightarrow \frac{1}{\tan A} + \frac{1}{\tan B} = b$$

$$\left[\because \cot \theta = \frac{1}{\tan \theta} \right]$$

$$\Rightarrow \frac{\tan B + \tan A}{\tan A \tan B} = b$$

$$\Rightarrow \frac{a}{\tan A \tan B} = b$$

$$\left[\because \tan A + \tan B = a \right]$$

$$\Rightarrow \frac{a}{b} = \tan A \tan B$$

$$\therefore \cot(A+B) = \frac{1}{\tan(A+B)}$$

$$= \frac{1}{\frac{\tan A + \tan B}{1 - \tan A \tan B}}$$

$$= \frac{1 - \tan A \tan B}{\tan A + \tan B}$$

$$= \frac{1 - \frac{a}{b}}{a}$$

$$\left[\because \tan A \tan B = \frac{a}{b} \right]$$

$$= \frac{b - a}{ab}$$

$$= \frac{b}{ab} - \frac{a}{ab}$$

$$= \frac{1}{a} - \frac{1}{b}$$

$$\therefore \cot(A+B) = \frac{1}{a} - \frac{1}{b}$$

Hence proved.

Q24

We have,

$$\cos \theta = \frac{8}{17}$$

$$\begin{aligned}\therefore \sin \theta &= \sqrt{1 - \cos^2 \theta} = \sqrt{1 - \frac{64}{289}} \\ &= \sqrt{\frac{225}{289}} \\ &= \frac{15}{17}\end{aligned}$$

$$\begin{aligned}
\text{Now, } & \cos\left(\frac{\pi}{6} + \theta\right) + \cos\left(\frac{\pi}{4} - \theta\right) + \cos\left(\frac{2\pi}{3} - \theta\right) \\
&= \left[\cos \frac{\pi}{6} \cos \theta - \sin \frac{\pi}{6} \sin \theta \right] + \left[\cos \frac{\pi}{4} \cos \theta + \sin \frac{\pi}{4} \sin \theta \right] \\
&\quad + \left[\cos \frac{2\pi}{3} \cos \theta + \sin \frac{2\pi}{3} \sin \theta \right] \\
&= \left[\cos \frac{\pi}{6} + \cos \frac{\pi}{4} + \cos \frac{2\pi}{3} \right] \cos \theta + \sin \theta \left[-\sin \frac{\pi}{6} + \sin \frac{\pi}{4} + \sin \frac{2\pi}{3} \right] \\
&= \left[\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} + \cos\left(\frac{\pi}{2} + \frac{\pi}{6}\right) \right] \times \frac{8}{17} + \frac{15}{17} \times \left[-\frac{1}{2} + \frac{1}{\sqrt{2}} + \sin\left(\frac{\pi}{2} + \frac{\pi}{6}\right) \right] \\
&= \left[\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \sin \frac{\pi}{6} \right] \times \frac{8}{17} + \frac{15}{17} \times \left[-\frac{1}{2} + \frac{1}{\sqrt{2}} + \cos \frac{\pi}{6} \right] \\
&= \left[\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2} \right] \times \frac{8}{17} + \frac{15}{17} \times \left[-\frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \right] \\
&\quad [\because \cos A \text{ is negative in second quadrant}] \\
&= \left[\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}} \right] \times \frac{8}{17} + \frac{15}{17} \times \left[\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}} \right] \\
&= \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}} \right) \left(\frac{8}{17} + \frac{15}{17} \right) \\
&= \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}} \right) \left(\frac{8+15}{17} \right) \\
&= \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}} \right) \times \frac{23}{17}
\end{aligned}$$

$$\therefore \cos\left(\frac{\pi}{6} + \theta\right) + \cos\left(\frac{\pi}{4} - \theta\right) + \cos\left(\frac{2\pi}{3} - \theta\right) = \left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}} \right) \times \frac{23}{17}$$

Hence proved.

Q25

We have,

$$\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3$$

$$\Rightarrow \tan x + \left[\frac{\tan x + \tan \frac{\pi}{3}}{1 - \tan x \tan \frac{\pi}{3}} \right] + \left[\frac{\tan x + \tan\left(\frac{2\pi}{3}\right)}{1 - \tan x \tan \frac{2\pi}{3}} \right] = 3$$

$$\Rightarrow \tan x + \left[\frac{\tan x + \sqrt{3}}{1 - \sqrt{3} \tan x} \right] + \left[\frac{\tan x + \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right)}{1 - \tan x \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right)} \right] = 3$$

$$\Rightarrow \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3} \tan x} + \frac{\tan x - \cot \frac{\pi}{3}}{1 + \tan x \cot \frac{\pi}{3}} = 3 \quad \left[\because \tan \theta \text{ is negative in second quadrant} \right]$$

$$\Rightarrow \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3} \tan x} + \frac{\tan x - \sqrt{3}}{1 + \sqrt{3} \tan x} = 3$$

$$\Rightarrow \tan x + \frac{(\tan x + \sqrt{3})(1 + \sqrt{3} \tan x) + (\tan x - \sqrt{3})(1 - \sqrt{3} \tan x)}{(1 - \sqrt{3} \tan x)(1 + \sqrt{3} \tan x)} = 3$$

$$\Rightarrow \tan x + \frac{\tan x + \sqrt{3} \tan^2 x + \sqrt{3} + 3 \tan x + \tan x - \sqrt{3} \tan^2 x - \sqrt{3} + 3 \tan x}{1 - (\sqrt{3} \tan x)^2} = 3$$

$$\Rightarrow \tan x + \frac{8 \tan x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{\tan x (1 - 3 \tan^2 x) + 8 \tan x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{\tan x - 3 \tan^3 x + 8 \tan x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{9 \tan x - 3 \tan^3 x}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{3(3 \tan x - \tan^3 x)}{1 - 3 \tan^2 x} = 3$$

$$\Rightarrow \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} = 1$$

Hence proved.

Q26

We have,

$$\sin(\alpha + \beta) = 1$$

$$\Rightarrow \sin(\alpha + \beta) = \sin \frac{\pi}{2}$$

$$\Rightarrow \alpha + \beta = \frac{\pi}{2} \quad \text{--- (i)}$$

$$\text{and, } \sin(\alpha - \beta) = \frac{1}{2}$$

$$\Rightarrow \sin(\alpha - \beta) = \sin \frac{\pi}{6}$$

$$\Rightarrow \alpha - \beta = \frac{\pi}{6} \quad \text{--- (ii)}$$

Adding equations (i) and (ii), we get

$$2\alpha = \frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

$$\Rightarrow \alpha = \frac{\pi}{3}$$

Putting $\alpha = \frac{\pi}{3}$ in equation (i), we get

$$\frac{\pi}{3} + \beta = \frac{\pi}{2}$$

$$\Rightarrow \beta = \frac{\pi}{2} - \frac{\pi}{3}$$

$$\Rightarrow \beta = \frac{3\pi - 2\pi}{6}$$

$$= \frac{\pi}{6}$$

$$\Rightarrow \beta = \frac{\pi}{6}$$

$$\begin{aligned}
 \text{Now, } \tan(\alpha + 2\beta) &= \tan\left(\frac{\pi}{3} + 2 \times \frac{\pi}{6}\right) \\
 &= \tan\left(\frac{\pi}{3} + \frac{\pi}{3}\right) \\
 &= \tan\frac{2\pi}{3} \\
 &= \tan\left(\frac{\pi}{2} + \frac{\pi}{6}\right) \\
 &= -\cot\frac{\pi}{6} \\
 &= -\sqrt{3}
 \end{aligned}$$

$\left[\because \tan\theta \text{ is negative in } \right.$
 $\left. \text{second quadrant} \right]$

$$\therefore \tan(\alpha + 2\beta) = -\sqrt{3}$$

$$\begin{aligned}
 \text{and, } \tan(2\alpha + \beta) &= \tan\left(2 \times \frac{\pi}{3} + \frac{\pi}{6}\right) \\
 &= \tan\left(\frac{2\pi}{3} + \frac{\pi}{6}\right) \\
 &= \tan\left(\frac{4\pi + \pi}{6}\right) \\
 &= \tan\left(\frac{5\pi}{6}\right) \\
 &= \tan\left(\frac{\pi}{2} + \frac{\pi}{3}\right) \\
 &= -\cot\frac{\pi}{3} \\
 &= -\frac{1}{\sqrt{3}}
 \end{aligned}$$

$\left[\because \tan\theta \text{ is negative in } \right.$
 $\left. \text{second quadrant} \right]$

$$\therefore \tan(2\alpha + \beta) = \frac{-1}{\sqrt{3}}$$

Q27

We have,

$$6 \cos \theta + 8 \sin \theta = 9 \quad \text{--- (i)}$$

$$\Rightarrow 8 \sin \theta = 9 - 6 \cos \theta$$

$$\Rightarrow (8 \sin \theta)^2 = (9 - 6 \cos \theta)^2 \quad [\because \text{Squaring both sides}]$$

$$\Rightarrow 64 \sin^2 \theta = 81 + 36 \cos^2 \theta - 108 \cos \theta$$

$$\Rightarrow 64 \sin^2 \theta = 81 + 36 \cos^2 \theta - 108 \cos \theta$$

$$\Rightarrow 64(1 - \cos^2 \theta) = 81 + 36 \cos^2 \theta - 108 \cos \theta$$

$$\Rightarrow 64 - 64 \cos^2 \theta = 81 + 36 \cos^2 \theta - 108 \cos \theta$$

$$\Rightarrow 36 \cos^2 \theta + 64 \cos^2 \theta - 108 \cos \theta + 81 - 64 = 0$$

$$\Rightarrow 100 \cos^2 \theta - 108 \cos \theta + 17 = 0 \quad \text{--- (ii)}$$

Since α, β are roots of equation (ii).

Therefore, $\cos \alpha$ and $\cos \beta$ are roots of equation (ii)

$$\therefore \cos \alpha + \cos \beta = \frac{17}{100} \quad \text{--- (iii)}$$

Again, $6 \cos \theta + 8 \sin \theta = 9$

$$\Rightarrow 6 \cos \theta = 9 - 8 \sin \theta$$

$$\Rightarrow (6 \cos \theta)^2 = (9 - 8 \sin \theta)^2 \quad [\because \text{Squaring both sides}]$$

$$\Rightarrow 36 \cos^2 \theta = 81 + 64 \sin^2 \theta - 144 \sin \theta$$

$$\Rightarrow 36(1 - \sin^2 \theta) = 81 + 64 \sin^2 \theta - 144 \sin \theta$$

$$\Rightarrow 36 - 36 \sin^2 \theta = 81 + 64 \sin^2 \theta - 144 \sin \theta$$

$$\Rightarrow 64 \sin^2 \theta + 36 \sin^2 \theta - 144 \sin \theta + 81 - 36 = 0$$

$$\Rightarrow 100 \sin^2 \theta - 144 \sin \theta + 45 = 0 \quad \text{--- (iv)}$$

$$\therefore \sin \alpha \times \sin \beta = \frac{45}{100} \quad \text{--- (v)}$$

$$\text{Now, } \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \frac{17}{100} - \frac{45}{100}$$

$$= -\frac{28}{100}$$

$$= -\frac{7}{25}$$

[Using equation (iii) and (v)]

$$\text{Now, } \sin(\alpha + \beta) = \sqrt{1 - (\cos \theta)^2}$$

$$= \sqrt{1 - \left(-\frac{7}{25}\right)^2}$$

$$= \sqrt{1 - \frac{49}{625}}$$

$$= \sqrt{\frac{625 - 49}{625}}$$

$$= \sqrt{\frac{576}{625}}$$

$$= \frac{24}{25}$$

$$\therefore \sin(\alpha + \beta) = \frac{24}{25}$$

Q28

$$\sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$$

$$\begin{aligned} b^2 + a^2 &= (\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 \\ \Rightarrow b^2 + a^2 &= (\cos^2 \alpha + \sin^2 \alpha) + (\cos^2 \beta + \sin^2 \beta) + 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \\ \Rightarrow b^2 + a^2 &= 1 + 1 + 2 \cos(\alpha - \beta) = 2 + 2 \cos(\alpha - \beta) \end{aligned}$$

$$\begin{aligned} \text{and, } b^2 - a^2 &= (\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 \\ b^2 - a^2 &= \cos^2 \alpha + \cos^2 \beta - \sin^2 \alpha - \sin^2 \beta + 2(\cos \alpha \cos \beta - \sin \alpha \sin \beta) \\ \Rightarrow b^2 - a^2 &= (\cos^2 \alpha - \sin^2 \beta) + (\cos^2 \beta - \sin^2 \alpha) + 2 \cos(\alpha + \beta) \\ \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) \cos(\alpha - \beta) + \cos(\beta + \alpha) \cos(\beta - \alpha) + 2 \cos(\alpha + \beta) \\ \Rightarrow b^2 - a^2 &= 2 \cos(\alpha + \beta) \cos(\alpha - \beta) + 2 \cos(\alpha + \beta) \\ &\quad \left[\because \cos(\beta - \alpha) = \cos\{-(\alpha - \beta)\} = \cos(\alpha - \beta) \right] \\ \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) \{2 \cos(\alpha - \beta) + 2\} \\ \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) (b^2 + a^2) \quad \quad \quad [\text{Using (i)}] \end{aligned}$$

$$\text{Thus, } b^2 - a^2 = (b^2 + a^2) \cos(\alpha + \beta)$$

$$\Rightarrow \cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

$$\Rightarrow \sin(\alpha + \beta) = \sqrt{1 - \left(\frac{b^2 - a^2}{b^2 + a^2}\right)^2} = \sqrt{\frac{4a^2b^2}{(a^2 + b^2)^2}} = \frac{2ab}{a^2 + b^2}$$

$$\begin{aligned}
 b^2 + a^2 &= (\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 \\
 \Rightarrow b^2 + a^2 &= (\cos^2 \alpha + \sin^2 \alpha) + (\cos^2 \beta + \sin^2 \beta) + 2(\cos \alpha \cos \beta + \sin \alpha \sin \beta) \\
 \Rightarrow b^2 + a^2 &= 1 + 1 + 2 \cos(\alpha - \beta) = 2 + 2 \cos(\alpha - \beta)
 \end{aligned}$$

$$\begin{aligned}
 \text{and, } b^2 - a^2 &= (\cos \alpha + \cos \beta)^2 - (\sin \alpha + \sin \beta)^2 \\
 b^2 - a^2 &= \cos^2 \alpha + \cos^2 \beta - \sin^2 \alpha - \sin^2 \beta + 2(\cos \alpha \cos \beta - \sin \alpha \sin \beta) \\
 \Rightarrow b^2 - a^2 &= (\cos^2 \alpha - \sin^2 \beta) + (\cos^2 \beta - \sin^2 \alpha) + 2 \cos(\alpha + \beta) \\
 \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) \cos(\alpha - \beta) + \cos(\beta + \alpha) \cos(\beta - \alpha) + 2 \cos(\alpha + \beta) \\
 \Rightarrow b^2 - a^2 &= 2 \cos(\alpha + \beta) \cos(\alpha - \beta) + 2 \cos(\alpha + \beta) \\
 &\quad \left[\because \cos(\beta - \alpha) = \cos\{-(\alpha - \beta)\} = \cos(\alpha - \beta) \right] \\
 \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) \{2 \cos(\alpha - \beta) + 2\} \\
 \Rightarrow b^2 - a^2 &= \cos(\alpha + \beta) (b^2 + a^2) \quad \text{[Using (i)]}
 \end{aligned}$$

$$\text{Thus, } b^2 - a^2 = (b^2 + a^2) \cos(\alpha + \beta)$$

$$\Rightarrow \cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$$

Q29

$$\begin{aligned}
\text{LHS} &= \frac{1}{\sin(x-a)\sin(x-b)} \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin(a-b)}{\sin(x-a)\sin(x-b)} \right] \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)} \right] \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)} \right] \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a)}{\sin(x-a)\sin(x-b)} - \frac{\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)} \right] \\
&= \frac{1}{\sin(a-b)} [\cot(x-a) - \cot(x-b)] \\
&= \frac{\cot(x-a) - \cot(x-b)}{\sin(a-b)} \\
&= \text{RHS}
\end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved

$$\begin{aligned}
\text{LHS} &= \frac{1}{\sin(x-a)\cos(x-b)} \\
&= \frac{1}{\cos(a-b)} \left[\frac{\cos(a-b)}{\sin(x-a)\cos(x-b)} \right] \\
&= \frac{1}{\cos(a-b)} \left[\frac{\cos\{(x-b)-(x-a)\}}{\sin(x-a)\cos(x-b)} \right] \\
&= \frac{1}{\cos(a-b)} \left[\frac{\cos(x-b)\cos(x-a) + \sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)} \right] \\
&= \frac{1}{\cos(a-b)} \left[\frac{\cos(x-b)\cos(x-a)}{\sin(x-a)\cos(x-b)} + \frac{\sin(x-b)\sin(x-a)}{\sin(x-a)\cos(x-b)} \right] \\
&= \frac{1}{\cos(a-b)} \left[\frac{\cos(x-a)}{\sin(x-a)} + \frac{\sin(x-b)}{\cos(x-b)} \right] \\
&= \frac{1}{\cos(a-b)} [\cot(x-a) + \tan(x-b)] \\
&= \frac{\cot(x-a) + \tan(x-b)}{\cos(a-b)} \\
&= \text{RHS}
\end{aligned}$$

∴ LHS=RHS

Hence proved

$$\begin{aligned}
\text{LHS} &= \frac{1}{\cos(x-a)\cos(x-b)} \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin(a-b)}{\cos(x-a)\cos(x-b)} \right] \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a)\cos(x-b)} \right] \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a) - \cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right] \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)\cos(x-a)}{\cos(x-a)\cos(x-b)} - \frac{\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)} \right] \\
&= \frac{1}{\sin(a-b)} \left[\frac{\sin(x-b)}{\cos(x-b)} - \frac{\sin(x-a)}{\cos(x-a)} \right] \\
&= \frac{1}{\sin(a-b)} [\tan(x-b) - \tan(x-a)] \\
&= \frac{\tan(x-b) - \tan(x-a)}{\sin(a-b)} \\
&= \text{RHS}
\end{aligned}$$

∴ LHS=RHS

Hence proved

Q30

We have,

$$\sin \alpha \sin \beta - \cos \alpha \cos \beta + 1 = 0$$

$$\Rightarrow -(\cos \alpha \cos \beta - \sin \alpha \sin \beta) = -1$$

$$\Rightarrow \cos(\alpha + \beta) = 1 \quad \text{--- (i)}$$

$$\therefore \sin(\alpha + \beta) = \sqrt{1 - \cos^2(\alpha + \beta)}$$

$$= \sqrt{1 - 1^2}$$

$$= 0$$

$$\Rightarrow \sin(\alpha + \beta) = 0 \quad \text{--- (ii)}$$

Now,

$$1 + \cot \alpha \tan \beta = 1 + \frac{\cos \alpha}{\sin \alpha} \times \frac{\sin \beta}{\cos \beta}$$

$$= \frac{\sin \alpha \times \cos \beta + \cos \alpha \times \sin \beta}{\sin \alpha \times \cos \beta}$$

$$= \frac{\sin(\alpha + \beta)}{\sin \alpha \times \cos \beta}$$

$$= \frac{0}{\sin \alpha \times \cos \beta}$$

$$= 0$$

[Using equation (ii)]

$$\therefore 1 + \cot \alpha \tan \beta = 0$$

Hence proved

Q31

We have,

$$\tan \alpha = x + 1 \text{ and } \tan \beta = x - 1$$

Now, $2 \cot(\alpha - \beta)$

$$= \frac{2}{\tan(\alpha - \beta)}$$

$$= \frac{2}{\frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}}$$

$$= \frac{2(1 + \tan \alpha \tan \beta)}{\tan \alpha - \tan \beta}$$

$$= \frac{2[1 + (x + 1)(x - 1)]}{x + 1 - (x - 1)}$$

$$= \frac{2[1 + x^2 - 1]}{x + 1 - x + 1}$$

$$= \frac{2 \times x^2}{2} = x^2$$

$$\therefore 2 \cot(\alpha - \beta) = x^2$$

Hence proved

Q32

Let the two parts of the angle be θ and $\theta - \emptyset$.

$$\tan(\theta - \emptyset) = \lambda \tan \emptyset \quad [\text{According to question}]$$

$$\Rightarrow \frac{\tan(\theta - \emptyset)}{\tan \emptyset} = \frac{\lambda}{1}$$

$$\Rightarrow \frac{\tan(\theta - \emptyset)}{\tan \emptyset} = \frac{\lambda}{1}$$

$$\Rightarrow \frac{\tan(\theta - \emptyset) + \tan \emptyset}{\tan(\theta - \emptyset) - \tan \emptyset} = \frac{\lambda + 1}{\lambda - 1} \quad [\text{Using Componendo and Dividendo}]$$

$$\Rightarrow \frac{\frac{\tan \theta - \tan \emptyset}{1 + \tan \theta \cdot \tan \emptyset} + \tan \emptyset}{\frac{\tan \theta - \tan \emptyset}{1 + \tan \theta \cdot \tan \emptyset} - \tan \emptyset} = \frac{\lambda + 1}{\lambda - 1}$$

$$\Rightarrow \frac{\frac{\tan \theta - \tan \emptyset + \tan \emptyset(1 + \tan \theta \cdot \tan \emptyset)}{1 + \tan \theta \cdot \tan \emptyset}}{\frac{\tan \theta - \tan \emptyset - \tan \emptyset(1 + \tan \theta \cdot \tan \emptyset)}{1 + \tan \theta \cdot \tan \emptyset}} = \frac{\lambda + 1}{\lambda - 1}$$

$$\Rightarrow \frac{\tan \theta - \tan \emptyset + \tan \emptyset + \tan \theta \cdot \tan^2 \emptyset}{\tan \theta - \tan \emptyset - \tan \emptyset - \tan \theta \cdot \tan^2 \emptyset} = \frac{\lambda + 1}{\lambda - 1}$$

$$\Rightarrow \frac{\tan \theta + \tan \theta \cdot \tan^2 \emptyset}{\tan \theta - 2 \tan \emptyset - \tan \theta \cdot \tan^2 \emptyset} = \frac{\lambda + 1}{\lambda - 1}$$

$$\Rightarrow \frac{\tan \theta}{\tan \theta - 2 \tan \emptyset - \tan \theta \cdot \tan^2 \emptyset} = \frac{\lambda + 1}{\lambda - 1}$$

Q33

$$\tan \theta = \frac{\sin \alpha - \cos \alpha}{\sin \alpha + \cos \alpha}$$

$$\Rightarrow \tan \theta = \frac{\tan \alpha - 1}{\tan \alpha + 1} \text{ [Dividing both Numerator and Denominator by } \cos \alpha \text{]}$$

$$\Rightarrow \tan \theta = \frac{\tan \alpha - \tan \frac{\pi}{4}}{1 + \tan \frac{\pi}{4} \cdot \tan \alpha}$$

$$\Rightarrow \tan \theta = \tan \left(\alpha - \frac{\pi}{4} \right)$$

$$\Rightarrow \theta = \alpha - \frac{\pi}{4} \text{ [Removing tan from both sides]}$$

$$\Rightarrow \cos \theta = \cos \left(\alpha - \frac{\pi}{4} \right) \text{ [Taking cos on both sides]}$$

$$\Rightarrow \cos \theta = \cos \alpha \cdot \cos \frac{\pi}{4} + \sin \alpha \cdot \sin \frac{\pi}{4}$$

$$\Rightarrow \cos \theta = \cos \alpha \cdot \frac{1}{\sqrt{2}} + \sin \alpha \cdot \frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos \theta = \frac{\cos \alpha + \sin \alpha}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2} \cos \theta = \sin \alpha + \cos \alpha$$

Hence Proved

Q34

RHS,

$$\begin{aligned}
 & \frac{p - q}{1 - pq} \\
 &= \frac{\tan(A + B) + \tan(A - B)}{1 - \tan(A + B)\tan(A - B)} \\
 &= \frac{\frac{\tan A + \tan B}{1 - \tan A \tan B} + \frac{\tan A - \tan B}{1 + \tan A \tan B}}{1 - \frac{\tan A + \tan B}{1 - \tan A \tan B} \cdot \frac{\tan A - \tan B}{1 + \tan A \tan B}} \\
 &= \frac{(\tan A + \tan B)(1 - \tan A \tan B) + (\tan A - \tan B)(1 + \tan A \tan B)}{(1 - \tan A \tan B)(1 + \tan A \tan B)} \\
 &= \frac{(1 - \tan A \tan B)(1 + \tan A \tan B) - (\tan A + \tan B)(\tan A - \tan B)}{(1 - \tan A \tan B)(1 + \tan A \tan B)} \\
 &= \frac{\tan A + \tan B + \tan^2 A \tan B - \tan A \tan^2 B + \tan A - \tan B - \tan^2 A \tan B + \tan A \tan^2 B}{1 - \tan^2 A \tan^2 B - \tan^2 A + \tan^2 B} \\
 &= \frac{2 \tan A + 2 \tan A \tan^2 B}{(1 - \tan^2 A)(1 + \tan^2 B)} = \frac{2 \tan A (1 + \tan^2 B)}{(1 - \tan^2 A)(1 + \tan^2 B)} = \frac{2 \tan A}{1 - \tan^2 A} = \tan 2A = LHS
 \end{aligned}$$

Hence Proved

Ex 7.2

Q1

Let $f(\theta) = 12 \sin \theta - 5 \cos \theta$

We know that

$$\begin{aligned} & -\sqrt{(12)^2 + (-5)^2} \leq f(\theta) \leq \sqrt{(12)^2 + (-5)^2} \\ \Rightarrow & -\sqrt{144 + 25} \leq f(\theta) \leq \sqrt{144 + 25} \\ \Rightarrow & -\sqrt{169} \leq f(\theta) \leq \sqrt{169} \\ \Rightarrow & -13 \leq f(\theta) \leq 13 \end{aligned}$$

Hence, minimum and maximum values of $12 \sin \theta - 5 \cos \theta$ are -13 and 13 respectively.

Let $f(\theta) = 12 \cos \theta + 5 \sin \theta + 4$

We know that

$$\begin{aligned} & -\sqrt{(12)^2 + (5)^2} \leq 12 \cos \theta + 5 \sin \theta \leq \sqrt{(12)^2 + (5)^2} \\ \Rightarrow & -\sqrt{144 + 25} \leq 12 \cos \theta + 5 \sin \theta \leq \sqrt{144 + 25} \\ \Rightarrow & -\sqrt{169} \leq 12 \cos \theta + 5 \sin \theta \leq \sqrt{169} \\ \Rightarrow & -13 \leq 12 \cos \theta + 5 \sin \theta \leq 13 \\ \Rightarrow & -13 + 4 \leq 12 \cos \theta + 5 \sin \theta + 4 \leq 13 + 4 \\ \Rightarrow & -9 \leq 12 \cos \theta + 5 \sin \theta + 4 \leq 17 \\ \Rightarrow & -9 \leq f(\theta) \leq 17 \end{aligned}$$

Hence, minimum and maximum values of $12 \cos \theta + 5 \sin \theta + 4$ are -9 and 17 respectively.

$$\text{Let } f(\theta) = 5 \cos \theta + 3 \sin \left(\frac{\pi}{6} - \theta \right) + 4$$

$$\begin{aligned} \text{Then, } f(\theta) &= 5 \cos \theta + 3 \left[\sin \frac{\pi}{6} \cos \theta - \cos \frac{\pi}{6} \sin \theta \right] + 4 \\ &= 5 \cos \theta + 3 \left[\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta \right] + 4 \\ &= 5 \cos \theta + \frac{3}{2} \cos \theta - \frac{3\sqrt{3}}{2} \sin \theta + 4 \\ &= \left(5 + \frac{3}{2} \right) \cos \theta - \frac{3\sqrt{3}}{2} \sin \theta + 4 \\ &= \frac{13}{2} \cos \theta - \frac{3\sqrt{3}}{2} \sin \theta + 4 \\ &= \frac{13}{2} \cos \theta - \left(\frac{-3\sqrt{3}}{2} \right) \sin \theta + 4 \end{aligned}$$

We know that

$$\begin{aligned} & -\sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{-3\sqrt{3}}{2}\right)^2} \leq \frac{13}{2} \cos \theta - \left(\frac{-3\sqrt{3}}{2}\right) \sin \theta \leq \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{-3\sqrt{3}}{2}\right)^2} \\ \Rightarrow & -\sqrt{\frac{169}{4} + \frac{27}{4}} \leq \frac{13}{2} \cos \theta - \left(\frac{-3\sqrt{3}}{2}\right) \sin \theta \leq \sqrt{\frac{169}{4} + \frac{27}{4}} \\ \Rightarrow & -\sqrt{\frac{196}{4}} \leq \frac{13}{2} \cos \theta - \left(\frac{-3\sqrt{3}}{2}\right) \sin \theta \leq \sqrt{\frac{196}{4}} \\ \Rightarrow & -\frac{14}{2} \leq \frac{13}{2} \cos \theta - \frac{3\sqrt{3}}{2} \sin \theta \leq \frac{14}{2} \\ \Rightarrow & -7 \leq \frac{13}{2} \cos \theta - \frac{3\sqrt{3}}{2} \sin \theta \leq 7 \\ \Rightarrow & -7 + 4 \leq \frac{13}{2} \cos \theta - \frac{3\sqrt{3}}{2} \sin \theta + 4 \leq 7 + 4 \\ \Rightarrow & -3 \leq \frac{13}{2} \cos \theta - \left(\frac{-3\sqrt{3}}{2}\right) \sin \theta + 4 \leq 11 \\ \Rightarrow & -3 \leq f(\theta) \leq 11 \end{aligned}$$

Let $f(\theta) = \sin \theta - \cos \theta + 1$. Then,

$$\begin{aligned}f(\theta) &= \sin \theta + (-1) \cos \theta + 1 \\&= (-1) \cos \theta + \sin \theta + 1\end{aligned}$$

We know that

$$\begin{aligned}-\sqrt{(-1)^2 + (1)^2} &\leq -\cos \theta + \sin \theta \leq \sqrt{(-1)^2 + (1)^2} \\ \Rightarrow -\sqrt{1+1} &\leq -\cos \theta + \sin \theta \leq \sqrt{1+1} \\ \Rightarrow -\sqrt{2} &\leq -\cos \theta + \sin \theta \leq \sqrt{2} \\ \Rightarrow -\sqrt{2} + 1 &\leq -\cos \theta + \sin \theta + 1 \leq \sqrt{2} + 1 \\ \Rightarrow 1 - \sqrt{2} &\leq f(\theta) \leq 1 + \sqrt{2}\end{aligned}$$

Hence, minimum and maximum values of $\sin \theta - \cos \theta + 1$ are $1 - \sqrt{2}$ and $1 + \sqrt{2}$ respectively.

Q2

$$\text{Let } f(\theta) = \sqrt{3} \sin \theta - \cos \theta$$

Multiplying and dividing by $\sqrt{(\sqrt{3})^2 + (-1)^2}$, we get

$$\begin{aligned} f(\theta) &= \sqrt{(\sqrt{3})^2 + (-1)^2} \left[\frac{\sqrt{3} \sin \theta}{\sqrt{(\sqrt{3})^2 + (-1)^2}} - \frac{\cos \theta}{\sqrt{(\sqrt{3})^2 + (-1)^2}} \right] \\ &= \sqrt{3+1} \left[\frac{\sqrt{3} \sin \theta}{\sqrt{3+1}} - \frac{\cos \theta}{\sqrt{3+1}} \right] \\ \Rightarrow f(\theta) &= 2 \left[\frac{\sqrt{3} \sin \theta}{2} - \frac{\cos \theta}{2} \right] \quad \dots (i) \\ \Rightarrow f(\theta) &= 2 \left[\frac{\sqrt{3}}{2} \times \sin \theta - \frac{1}{2} \times \cos \theta \right] \\ &= 2 \left[\cos \frac{\pi}{6} \times \sin \theta - \sin \frac{\pi}{6} \times \cos \theta \right] \\ &= 2 \left[\sin \theta \times \cos \frac{\pi}{6} - \cos \theta \times \sin \frac{\pi}{6} \right] \\ &= 2 \sin \left(\theta - \frac{\pi}{6} \right) \quad \left[\because \sin(A - B) = \sin A \cos B - \cos A \sin B \right] \\ \Rightarrow f(\theta) &= 2 \sin \left(\theta - \frac{\pi}{6} \right) \end{aligned}$$

Again,

$$\begin{aligned} f(\theta) &= 2 \left[\frac{\sqrt{3}}{2} \sin \theta - \frac{\cos \theta}{2} \right] \\ &= -2 \left[\frac{1}{2} \times \cos \theta - \frac{\sqrt{3}}{2} \times \sin \theta \right] \\ &= -2 \left[\cos \frac{\pi}{3} \times \cos \theta - \sin \frac{\pi}{3} \times \sin \theta \right] \\ &= -2 \cos \left(\frac{\pi}{3} + \theta \right) \end{aligned}$$

Let $f(\theta) = \cos \theta - \sin \theta$

Multiplying and dividing by $\sqrt{1^2 + 1^2}$, we get

$$\begin{aligned} f(\theta) &= \sqrt{1^2 + 1^2} \left[\frac{\cos \theta}{\sqrt{1^2 + 1^2}} - \frac{\sin \theta}{\sqrt{1^2 + 1^2}} \right] \\ \Rightarrow f(\theta) &= \sqrt{2} \left[\frac{\cos \theta}{\sqrt{2}} - \frac{\sin \theta}{\sqrt{2}} \right] \quad \dots (i) \end{aligned}$$

$$\begin{aligned} \text{Now, } f(\theta) &= \sqrt{2} \left[\frac{1}{\sqrt{2}} \times \cos \theta - \frac{1}{\sqrt{2}} \times \sin \theta \right] \\ &= \sqrt{2} \left[\sin \frac{\pi}{4} \times \cos \theta - \cos \frac{\pi}{4} \times \sin \theta \right] \\ &= \sqrt{2} \sin \left(\frac{\pi}{4} - \theta \right) \quad \left[\because \sin(A - B) = \sin A \cos B - \cos A \sin B \right] \end{aligned}$$

$$\Rightarrow f(\theta) = \sqrt{2} \sin \left(\frac{\pi}{4} - \theta \right)$$

Again,

$$\begin{aligned} f(\theta) &= \sqrt{2} \left[\frac{1}{\sqrt{2}} \times \cos \theta - \frac{1}{\sqrt{2}} \times \sin \theta \right] \\ &= \sqrt{2} \left[\cos \frac{\pi}{4} \times \cos \theta - \sin \frac{\pi}{4} \times \sin \theta \right] \\ &= \sqrt{2} \cos \left(\frac{\pi}{4} + \theta \right) \quad \left[\because \cos(A + B) = \cos A \cos B - \sin A \sin B \right] \end{aligned}$$

$$\Rightarrow f(\theta) = \sqrt{2} \cos \left(\frac{\pi}{4} + \theta \right)$$

Let $f(\theta) = 24 \cos \theta + 7 \sin \theta$

Multiplying and dividing by $\sqrt{(24)^2 + (7)^2}$, we get

$$\begin{aligned}
 f(\theta) &= \sqrt{(24)^2 + 7^2} \left[\frac{24 \cos \theta}{\sqrt{24^2 + 7^2}} + \frac{7 \sin \theta}{\sqrt{24^2 + 7^2}} \right] \\
 &= \sqrt{576 + 49} \left[\frac{24 \cos \theta}{\sqrt{576 + 49}} + \frac{7 \sin \theta}{\sqrt{576 + 49}} \right] \\
 &= \sqrt{625} \left[\frac{24 \cos \theta}{\sqrt{625}} + \frac{7 \sin \theta}{\sqrt{625}} \right] \\
 &= 25 \left[\frac{24}{25} \times \cos \theta + \frac{7}{25} \times \sin \theta \right] \\
 \Rightarrow f(\theta) &= 25 \left[\frac{24}{25} \times \cos \theta + \frac{7}{25} \times \sin \theta \right] \quad \text{--- (i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, } f(\theta) &= 25 \left[\frac{24}{25} \times \cos \theta + \frac{7}{25} \times \sin \theta \right] \\
 &= 25 [\sin \alpha \times \cos \theta + \cos \alpha \times \sin \theta] \\
 &\quad \text{where } \sin \alpha = \frac{24}{25} \text{ and } \cos \alpha = \frac{7}{25}
 \end{aligned}$$

$$\Rightarrow f(\theta) = 25 \sin(\alpha + \theta), \text{ where } \tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{24}{7}$$

Again,

$$\begin{aligned}
 f(\theta) &= 25 \left[\frac{24}{25} \times \cos \theta + \frac{7}{25} \times \sin \theta \right] \\
 &= 25 [\cos \alpha \times \cos \theta + \sin \alpha \times \sin \theta], \text{ where } \cos \alpha = \frac{24}{25} \text{ and } \sin \alpha = \frac{7}{25}
 \end{aligned}$$

$$\Rightarrow f(\theta) = 25 \cos(\alpha - \theta), \text{ where } \tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{7}{24}$$

Q3

We have,

$$\begin{aligned}
 & \sin 100^\circ - \sin 10^\circ \\
 &= \sqrt{2} \left(\frac{1}{\sqrt{2}} \times \sin 100^\circ - \frac{1}{\sqrt{2}} \times \cos 100^\circ \right) \quad \left[\begin{array}{l} \text{Multiplying and dividing} \\ \text{by } \sqrt{1^2 + 1^2} \text{ i.e., by } \sqrt{2} \end{array} \right] \\
 &= \sqrt{2} (\cos 45^\circ \times \sin 100^\circ - \sin 45^\circ \times \cos 100^\circ) \\
 &= \sqrt{2} (\sin 100^\circ \times \cos 45^\circ - \cos 100^\circ \times \sin 45^\circ) \\
 &= \sqrt{2} (\sin (100^\circ - 45^\circ)) \\
 &= \sqrt{2} \sin 55^\circ, \text{ which is positive real number.} \\
 & \quad \quad \quad [\because \sin \theta \text{ is positive in first quadrant}]
 \end{aligned}$$

Q4

$$(2\sqrt{3}+3)\sin\theta+2\sqrt{3}\cos\theta$$

$$\text{assume } a=2\sqrt{3}+3, b=2\sqrt{3}$$

$$\sqrt{a^2+b^2} = \sqrt{12+9+12\sqrt{3}+12} = \sqrt{33+12\sqrt{3}}$$

Dividing and multiplying the above equation with above value

$$\text{we get, } \sqrt{33+12\sqrt{3}} \left(\frac{2\sqrt{3}+3}{\sqrt{33+12\sqrt{3}}} \sin\theta + \frac{2\sqrt{3}}{\sqrt{33+12\sqrt{3}}} \cos\theta \right)$$

$$\text{Assume } \tan\phi = \frac{a}{b}, \text{ we have } \sin\phi = \frac{a}{\sqrt{a^2+b^2}}, \cos\phi = \frac{b}{\sqrt{a^2+b^2}}$$

$$\text{so above expressions changes to } \sqrt{33+12\sqrt{3}} (\sin\phi\sin\theta + \cos\phi\cos\theta)$$

$$\text{which is equal to } \sqrt{33+12\sqrt{3}} \cos(\theta-\phi)$$

We know that maximum and minimum value of any cosine term is +1 and -1

$$\sqrt{33+12\sqrt{3}} = \sqrt{15+12+6+12\sqrt{3}}$$

$$\text{we know that } 12\sqrt{3}+6 < 12\sqrt{5} \text{ because value of } \sqrt{5}-\sqrt{3} \text{ is more than } 0.5$$

so if we replace $12\sqrt{3}+6$ with $12\sqrt{5}$ the above inequality still holds

$$\text{So range of above expression can be } \sqrt{15+12+12\sqrt{5}} = 2\sqrt{3} + \sqrt{15}$$

$$-(2\sqrt{3} + \sqrt{15}) < \sqrt{33+12\sqrt{3}} \cos(\theta-\phi) < 2\sqrt{3} + \sqrt{15}$$