CBSE Sample Paper-04
SUMMATIVE ASSESSMENT –II
MATHEMATICS
Class – X

Time allowed: 3 hours Maximum Marks: 90

General Instructions:

a) All questions are compulsory.
b) The question paper consists of 31 questions divided into four sections – A, B, C and D.
c) Section A contains 4 questions of 1 mark each which are multiple choice questions, Section B contains 6 questions of 2 marks each, Section C contains 10 questions of 3 marks each and Section D contains 11 questions of 4 marks each.
d) Use of calculator is not permitted.

Section A

1. A box contains 3 blue, 2 white and 4 red marbles. If a marble is drawn at random from the box, the probability that it will not be a white marble is:

   (a) $\frac{2}{9}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) $\frac{7}{9}$

2. A point on $y-$axis equidistant from the points A (6, 5) and B $-4, 3$ is:

   (a) (0, 3) (b) (0, 4) (c) (0, 6) (d) (0, 9)

3. The famous mathematician associated with finding the sum of first 100 natural number s is:

   (a) Pythagoras (b) Euclid (c) Newton (d) Gauss

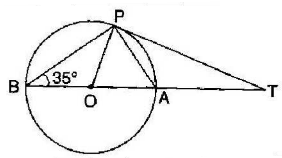
4. If the string of a kite is 75 m long and it makes an angle of 60° with the ground, then the height of kite is:

   (a) $\frac{75}{2}$ m (b) $75\sqrt{3}$ m (c) $\frac{75\sqrt{3}}{2}$ m (d) 75 m

Section B

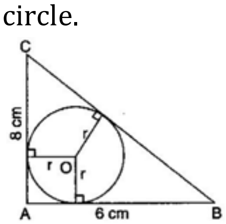
5. The radii of two circles are 3 cm and 4 cm. Find the radius of the circle whose area is equal to the sum of areas of two circles.
6. A solid metallic sphere of radius 12 cm is melted and recast into a number of small cones, each of radius 4 cm and height 3 cm. Find the number of cones so formed.
7. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tank will rise by 21 m.
8. Show that $x=-3$ is a solution of the equation $x^2+6x+9=0$.
9. Which term of the AP 21, 42, 63, 84, …… is 420?

10. In figure, BOA is a diameter of the circle and the tangent at a point P meets BA extended at T. If $\angle PBO = 35°$, then find $\angle PTA$.



Section C

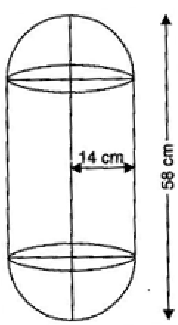
11. Find the value of p for which the points $(-1,3),(2,p)$ and $(5,-1)$ are collinear.
12. Prove that the points (3, 0), (6, 4) and $(-1,3)$ are vertices of a right angled triangle. Also, prove that the vertices of an isosceles triangle.
13. A copper wire when bent in the form of a square encloses an area of 121 cm². If the same wire is bent into the form of a circle, then find the area of the circle. $\left(\text{Use } \pi=\dfrac{22}{7}\right)$
14. The circumference of a circular plot is 220 m. A 15 m wide concrete track runs around outside the plot. Find the area of the track. $\left(\text{Use } \pi=\dfrac{22}{7}\right)$
15. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped small bottles each of diameter 3 cm and height 4 cm. How many bottles are needed to empty the bowl?
16. Solve the quadratic equation by quadratic formula: $\frac{1}{2}x^2-\sqrt{11}x+1=0$
17. Find the sum of all two digit natural numbers which when divided by 3 yield 1 as remainder.
18. In figure, ABC is a right angled triangle with AB = 6 cm and AC = 8 cm. A circle with centre O has been inscribed inside the triangle. Calculate the value of r, the radius of the inscribed circle.



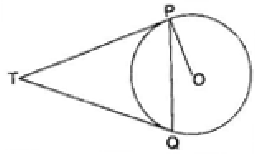
19. From the top of a hill 200 m high, the angles of depression of the top and the bottom of a pillars are 30° and 60° respectively. Find the height of the pillar and its distance from the hill
20. Red kings, queens and jacks are removed from a deck of 52 playing cards and then well-shuffled. A card is drawn from the remaining cards. Find the probability of getting:
    (i) a king.
    (ii) ared card.
    (iii) aspade.

Section D

21. Construct a Δ ABC in which BC = 6.5 cm, AB = 4.5 cm and $\angle$ACB = 60°. Construct another triangle similar to Δ ABC such that each side of new triangle is $\frac{4}{5}$ of the corresponding sides of Δ ABC.
22. An aeroplane flying at a height of 4000 m from the ground passes vertically above another aeroplane at an instant when the angle of elevation of the two planes from the same point as the ground are 60° and 45° respectively. Find the vertical distance between the aeroplanes at that instant.
23. A bag contains 4 white balls, 6 red balls, 7 black balls and 3 blue balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is:
(i) white (ii) not black
(iii) neither white nor black (iv) red or white
24. If 'a' is the length of one of the sides of an equilateral triangle ABC, base BC lies on $x-$axis and vertex B is at the origin, then find the coordinates of the vertices of the triangle ABC.
25. A solid is in the form of a right circular cylinder with hemispherical ends. The total height of the solid is 28 cm. Find the total surface area of the solid. $\left(\text{Use } \pi = \frac{22}{7}\right)$



26. A bucket is in the form of a frustum of a cone with capacity 12305.8 cm^3 of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the total height of the bucket and the area of the metal sheet used in its making. (Use π = 3.14)
27. Solve for $x: \frac{1}{p+q+x} = \frac{1}{p} + \frac{1}{q} + \frac{1}{x}$; $p \neq 0, q \neq 0, p+q+x \neq 0$
28. A trader bought a number of articles for Rs.900. Five articles were found damaged. He sold each of the remaining articles at Rs.80 in the whole transaction. Find the number of articles he bought.
29. Nidhi saves Rs. 2 on first day of the month, Rs. 4 on second day, Rs. 6 on third day and so on. Read the above passage and answer the following questions:
(i) What will be her saving in the month of February?
(ii) What value is depicted by Nidhi?
30. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle$PTQ = 2$\angle$OPQ.



31. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

    Using the above result, find the length of PQ, if a tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm.

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(Solutions)

SECTION-A

1. (d)
2. (d)
3. (d)
4. (c)
5. According to the question,

   Area of new circle = Sum of areas of two circles

   $\Rightarrow \quad \pi r^2 = \pi r_1^2 + \pi r_2^2$

   $\Rightarrow \quad \pi r^2 = \pi\left(r_1^2 + r_2^2\right)$

   $\Rightarrow \quad r^2 = r_1^2 + r_2^2$

   $\Rightarrow \quad r^2 = 3^2 + 4^2 = 9 + 16 = 25 \qquad \Rightarrow \quad r$ =5 cm

6. Let n cones be formed. Then,

   $n\times$ Volume of one cone = Volume of sphere

   $\Rightarrow \quad n.\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r_1^3 \qquad \Rightarrow \quad n.r^2 h = 4r_1^3$

   $\Rightarrow \quad n\times(4)^2(3) = 4\times(12)^3 \Rightarrow \quad n = \frac{4\times12\times12\times12}{4\times4\times3}$

   $\Rightarrow \quad n = 144$

7. Volume of water that flows out through the pipe in 1 hour.

   $= \frac{22}{7}\times\left(\frac{14}{2}\right)^2\times15\times1000\times100$ cm³

   Volume of tank = 50 x 44 x 21 x 100 x 100 cm³

   $\therefore \quad$ Required time = $\frac{50\times44\times21\times100\times100}{\frac{22}{7}\times\left(\frac{14}{2}\right)^2\times15\times1000\times100}$ = 2 hours

8. $x^2 + 6x + 9 = 0$

   $\Rightarrow \quad (x)^2 + 2.3.x + (3)^2$

   $\Rightarrow \quad (x+3)^2 \qquad \Rightarrow \quad x = -3, -3$

   Hence $x = -3$ is a solution.

9. $a = 21, d = 42 - 21 = 21$

$a_n = a + (n-1)d$

$\Rightarrow \quad 420 = 21 + (n-1)21 \quad \Rightarrow \quad (n-1)21 = 399$

$\Rightarrow \quad n - 1 = 19 \quad \Rightarrow \quad n = 20$

$\therefore$ 20th term is 420.

10. $\angle$BPA = 90° and $\angle$PBA = 35°

$\therefore \quad \angle \text{PAB} = 180^\circ - (90^\circ + 35^\circ) = 55^\circ$

$\because \quad$ OA = OP

$\therefore \quad \angle$OAP = $\angle$OPA = 55°

$\angle$OPT = 90°

$\Rightarrow \quad \angle$OPA = $\angle$APT = 90°

$\Rightarrow \quad 55^\circ + \angle$APT = 90°

$\Rightarrow \quad \angle$APT = 35°

$\therefore \quad \angle \text{PTA} = 55^\circ - 35^\circ = 20^\circ$

11. For collinear,

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$

$\Rightarrow \quad (-1)[p - (-1)] + 2(-1-3) + 5(3-p) = 0$

$\Rightarrow \quad (-p - 1 - 2 - 6 + 15 - 5p) = 0$

$\Rightarrow \quad -6p + 6 = 0$

$\Rightarrow \quad p = 1$

12. Let A $\to (3,0)$, B $\to (6,4)$ and C $\to (-1,3)$.

AB = $\sqrt{(6-3)^2 + (4-0)^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$ units

BC = $\sqrt{(-1-6)^2 + (3-4)^2} = \sqrt{(-7)^2 + (-1)^2} = \sqrt{50} = 5\sqrt{2}$ units

CA = $\sqrt{(3+1)^2 + (0-3)^2} = \sqrt{(4)^2 + (-3)^2} = \sqrt{25} = 5$

$\therefore \quad \Delta$ ABC is an isosceles triangle.

$AB^2 + CA^2 = BC^2$

$\Rightarrow \quad 5^2 + 5^2 = \left(5\sqrt{2}\right)^2$

$\Rightarrow \quad 25 + 25 = 25 \times 2$

$\Rightarrow \quad 50 = 50$

$\therefore \quad \Delta$ ABC is a right angled triangle.

13. Side of square = $\sqrt{121}$ = 11 cm

$\therefore \quad$ Perimeter of square = 4 x 11 = 44 cm

$\therefore \quad$ Circumference of circle = 44 cm

$\Rightarrow \quad 2\pi r = 44$

$\Rightarrow \quad r = \frac{44 \times 7}{2 \times 22} = 7$ cm

$\therefore$ Area of the circle $= \pi r^2$

$$= \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$$

14. $\because \quad 2\pi r = 220$

$\Rightarrow \quad r = 35$ m

Width of track = 15 m

$\therefore$ External radius = 35 + 15 = 50 m

$\therefore$ Area of circular track = $\pi R^2 - \pi r^2$

$$= \pi(50)^2 - \pi(35)^2$$

$$= \pi(2500 - 1225)$$

$$= \frac{22}{7} \times 1275$$

$$= \frac{28050}{7} \text{ m}^2$$

15. Volume of hemispherical bowl = $\frac{2}{3}\pi r^3$

$$= \frac{2}{3}\pi(9)^3 \text{ cm}^3$$

Volume of one bottle $= \pi r^2 h$

$$= \pi\left(\frac{3}{2}\right)^2(4)$$

$\therefore$ Number of bottles required = $\frac{\text{Volume of hemisphere}}{\text{Volume of 1 bottle}}$

$$= \frac{\frac{2}{3}\pi(9)^3}{\pi\left(\frac{3}{2}\right)^2(4)} = 54$$

16. Given, $a = \frac{1}{2}, b = -\sqrt{11}, c = 1$

$\because \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\therefore \quad x = \frac{-(-\sqrt{11}) \pm \sqrt{(-\sqrt{11})^2 - 4.\frac{1}{2}.1}}{2\left(\frac{1}{2}\right)}$

$\Rightarrow \quad x = \frac{\sqrt{11} \pm \sqrt{11 - 2}}{1} \qquad \Rightarrow \quad x = \sqrt{11} \pm 3$

17. According to questions, the number are 10, 13, 16, 19, ……..97

Here, $a = 10, d = 3, l = 97$

$\because \quad l = a + (n-1)d$

$\therefore \quad 97 = 10 + (n-1)3 \;\; 97 \qquad \Rightarrow \qquad 3(n-1) = 87$

$\Rightarrow \quad n - 1 = 29 \qquad \Rightarrow \qquad n = 30$

$\because \quad S_n = \frac{n}{2}(a + l)$

$\therefore \quad S_n = \frac{30}{2}(10 + 97)$ = 15 x 107 = 1605

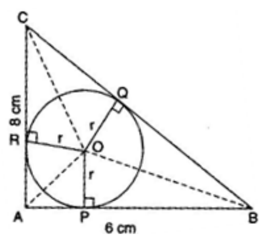
18. $\because$ Area of triangle = $\frac{1}{2}$ Base x Height

$\therefore$ Area of Δ ABC = Area of Δ AOB + Area of Δ BOC + Area of Δ COA

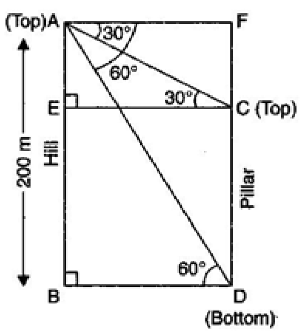
$\Rightarrow \quad \frac{6 \times 8}{2} = \frac{6 \times r}{2} + \frac{\sqrt{6^2 + 8^2} \times r}{2} + \frac{8 \times r}{2}$

$\Rightarrow \quad 24 = 3r + 5r + 4r$

$\Rightarrow \quad 24 = 12r \qquad \Rightarrow \qquad r = 2$ cm



19. In right triangle ABD,



$\tan 60^\circ = \frac{AB}{BD}$

$\Rightarrow \quad \sqrt{3} = \frac{200}{BD}$

$\Rightarrow \quad$ BD = $\frac{200}{\sqrt{3}}$ = 115.33 m

In right triangle AEC,

$$\tan 30^\circ = \frac{AE}{EC}$$

$\Rightarrow$ $\frac{1}{\sqrt{3}} = \frac{AE}{\left(\frac{200}{\sqrt{3}}\right)}$ [$\because$ EC = BD]

$\Rightarrow$ AE = $\frac{200}{3}$ = 66.67 m

$\therefore$ CD = EB = AB – AE

= 200 – 66.67 = 133.33 m

20. There are 52 cards in a pack, out of which six cards are removed.

$\therefore$ Total number of possible outcome = 52 – (2 + 2 + 2) = 46

(i) Number of kings = 2

Hence, Required probability = $\frac{2}{46} = \frac{1}{23}$

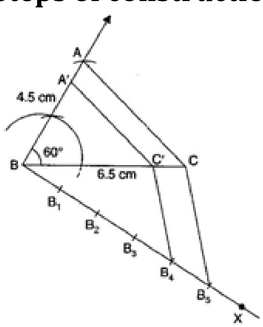
(ii) Number of red cards = 26 – 6 = 20

Hence required probability = $\frac{20}{46} = \frac{10}{23}$

(iii) Number of spades = 3

Hence required probability = $\frac{13}{46}$

21. **Steps of construction**:



(a) Draw a right angled triangle ABC with given measurements.

(b) Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.

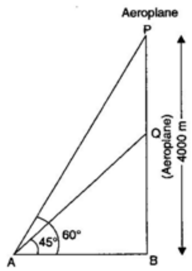
(c) Locate 5 points B_1, B_2, B_3, B_4, B_5 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.

(d) Join B_5C and draw a line through B_4 parallel to B_5C, intersecting the extended line segment BC at C'.

(e) Draw a line through C' parallel to CA intersecting the extended line segment BA at A'.

The A'BC' is the required triangle.

22. In right triangle PBA,



$\tan 60^\circ = \frac{4000}{AB}$

$\Rightarrow$ AB = $\frac{4000}{\sqrt{3}}$ ....(i)

In right triangle ABQ, $\tan 45^\circ = \frac{BQ}{AB}$

$\Rightarrow$ AB = BQ $[\because \tan 45^\circ = 1]$

$\therefore$ BQ = $\frac{4000}{\sqrt{3}}$ ....(ii)

$\therefore$ PQ = PB – BQ

$\Rightarrow$ PQ = $4000 - \frac{4000}{\sqrt{3}}$ = $\frac{4000\sqrt{3} - 4000}{\sqrt{3}}$ = 1690.4 m

23. Total number of balls in the bag = 4 + 6 + 7 + 3 = 20

$\therefore$ Number of all possible outcomes = 20

(i) Number of white balls = 4

$\therefore$ Required probability = $\frac{4}{20} = \frac{1}{5}$

(ii) Number of balls 'not black' = 4 + 6 + 3 = 13

$\therefore$ Required probability = $\frac{13}{20}$

(iii) Number of balls 'neither white nor black = 6 + 3= 9

$\therefore$ Required probability = $\frac{9}{20}$

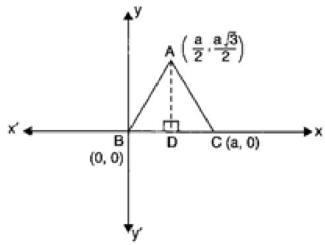
(iv) Number of balls 'red or white' = 6 + 4 = 10

$\therefore$ Required probability = $\frac{10}{20} = \frac{1}{2}$

24. $B \to (0,0)$

$\because$ BC lies on $x-$axis.

$\therefore$ $C \to (a,0)$



Draw AD = BC, then BD = DC = $\dfrac{a}{2}$

In right ΔADB, AB^2 = AD^2 + BD^2 [Pythagoras theorem]

$\Rightarrow \quad a^2 = \text{AD}^2 + \left(\dfrac{a}{2}\right)^2 \qquad \Rightarrow \quad \text{AD} = \dfrac{a\sqrt{3}}{2}$

$\therefore \quad \text{A} \rightarrow \left(\dfrac{a}{2}, \dfrac{a\sqrt{3}}{2}\right)$

25. Total surface area of the solid

= Curved surface area of cylinder + 2 x Curved surface area of the hemisphere

= $2\pi rh + 2\times 2\pi r^2$

= $2\pi r(h+2r)$

= $2\times\dfrac{22}{7}\times 14\left[58-(14+14)+2\times 14\right]$

= $88[58-28+28]$

= 5104 cm^2

26. Volume = 12308.8 cm^3

r_1 = 20 cm

r_2 = 12 cm

Volume (V) = $\dfrac{1}{3}\pi h\left(r_1^2+r_2^2+r_1r_2\right)$

$\Rightarrow \quad 12308.8 = \dfrac{1}{3}\times 3.14\times h\left[(20)^2+(12)^2+20\times 12\right]$

$\Rightarrow \quad 12308.8 = \dfrac{3.14}{3}h[400+144+240]$

$\Rightarrow \quad 12308.8 = h\times 1.05\times 784$

$\Rightarrow \quad h$ = 15 cm

And $l = \sqrt{h^2+(r_1-r_2)^2}$ = $\sqrt{(15)^2+(20-12)^2}$ = $\sqrt{225-64}$ = 17 cm

$\therefore$ Area of metal sheet used = $\pi l(r_1+r_2)+\pi r_2^2$

= $3.14\times 17(20+12)+3.14\times(12)^2$

= 3.14 x 17 x 32 + 3.14 x 144

= 2160.32 cm^2

27. $\frac{1}{p+q+x}=\frac{1}{p}+\frac{1}{q}+\frac{1}{x}$

$\Rightarrow \quad \frac{1}{p+q+x}-\frac{1}{x}=\frac{1}{p}+\frac{1}{q}$ $\quad \Rightarrow \quad \frac{x-(p+q+x)}{(p+q+x)x}=\frac{p+q}{pq}$

$\Rightarrow \quad \frac{-1}{(p+q+x)}=\frac{1}{pq}$ $\quad \Rightarrow \quad (p+q+x)x+pq=0$

$\Rightarrow \quad x^2+(p+q)x+pq=0$ $\quad \Rightarrow \quad (x+p)(x+q)=0$

$\Rightarrow \quad x=-p,-q$

28. Let he buys x articles.

C.P. of x articles = Rs. 900

$\therefore$ C.P. of 1 article = Rs. $\frac{900}{x}$

No. of articles damaged = 5

$\therefore$ No. of articles not damaged = $(x-5)$

S.P. of 1 article = Rs. $\left(\frac{900}{x}+2\right)$

$\therefore$ S.P. of $(x-5)$ articles = Rs. $\left(\frac{900}{x}+2\right)(x-5)$

According to the question,

$\left(\frac{900}{x}+2\right)(x-5)-900=80$

$\Rightarrow \quad (900+2x)(x-5)=980x$ $\quad \Rightarrow \quad 900x-4500+2x^2-10x=980x$

$\Rightarrow \quad 2x^2-90x-4500=0$ $\quad \Rightarrow \quad x^2-45x-2250=0$

$\Rightarrow \quad x^2-75x+30x-2250=0$ $\quad \Rightarrow \quad x^2(x-75)+30(x-75)=0$

$\Rightarrow \quad (x-75)(x+30)=0$ $\quad \Rightarrow \quad x=75,-30$

$x=-30$ is inadmissible as x is the number of articles.

Hence, No. of articles = 75

29. (i) Nidhi saves on first day = Rs.2, on second day = Rs.4, on third day = Rs.4 and so on.

Thus, savings form an AP, whose first term $a=2$

Common difference (d) = 4 – 2 = 2

We know that year 2012 is a leap year. So there are 29 days in month of February.

So, we have to find her savings in 29 days, i.e. $n=29$

Now, $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$\Rightarrow \quad S_{29}=\frac{29}{2}\left[2\times2+(29-1)2\right]=\frac{29}{2}\left[4+28\times2\right]$

$=\frac{29}{2}\times60=870$

(ii) Economy, Saving

30. $\because$ Tangent segments from an external point to a circle are equal in length.

$\therefore$ TP = TQ

And $\angle TQP = \angle TPQ$ ....(i) [Angles opposite to equal sides of a Δ are equal]

$\because$ Tangent is perpendicular to the radius through the point of contact.

$\therefore$ $\angle OPT = 90^\circ$ ....(ii)

$\Rightarrow$ $\angle OPQ + \angle TPQ = 90^\circ$

$\Rightarrow$ $\angle TPQ = 90^\circ - \angle OPQ$ ....(iii)

In ΔTPQ,

$\angle PTQ + \angle TPQ + \angle TQP = 180°$ [Angle sum property of a Δ]

$\Rightarrow$ $\angle PTQ + 2\angle TPQ = 180°$ ....(iv) [From eq. (ii)]

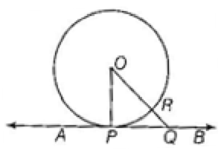
$\Rightarrow$ $\angle PTQ + 2(90^\circ - \angle OPQ) = 180°$ [From eq. (iii)]

$\Rightarrow$ $\angle PTQ = 2\angle OPQ$

31. **First part**:<u>Given</u> : A circle with centre O and radius *r* and a tangent AB at a point P.

<u>To Prove</u> : $OP \perp AB$

<u>Construction</u>: Take any point Q, other than P on the tangent AB. Join OQ. Suppose OQ meets the circle at R.



<u>Proof</u>: Clearly OP = OQ [Radii]

Now, OQ = OR + RQ

$\Rightarrow$ OQ> OR

$\Rightarrow$ OQ> OP [OP = OQ]

$\Rightarrow$ OP <OQ

Thus, OP is shorter than any segment joining O to any point of AB.

So, OP is perpendicular to AB.

Hence,

OT = OT' .... (Radii of the same circle)

and OP = OP ....(Common)

$\therefore$ $\Delta OTP \cong \Delta OT'P$ ....(RHS congruency)

Hence, $OP \perp AB$

Second part: Using the above, we get,

$\angle OPQ = 90^\circ$

$OP^2 + PQ^2 = OQ^2$ [By Pythagoras theorem]

$\Rightarrow$ $12^2 = 5^2 + PQ^2$

$\Rightarrow$ $PQ = \sqrt{119}$ cm