

CHAPTER 2

Solving Linear Equations

2-1. Writing Equations

An **equation** is a mathematical sentence with an equal sign. To translate a word sentence into an equation, choose a variable to represent one of the unspecified numbers or measures in the sentence. This is called **defining a variable**. Then use the variable to write equations for the unspecified numbers.

Example 1 □ Translate each sentence into an equation.

- Twice a number increased by fourteen is identical to fifty.
- Half the sum of seven and a number is the same as the number decreased by two.
- The quotient of m and n equals four more than one-third the sum of m and n .
- The cube of x plus the square of y is equal to fifty two.

Solution □ a. Let c be the number. Define a variable.

$$\underbrace{2c}_{\text{Twice a number } c} \quad \underbrace{+}_{\text{increased by}} \quad \underbrace{14}_{\text{fourteen}} \quad \underbrace{=}_{\text{is identical to}} \quad \underbrace{50}_{\text{fifty.}}$$

b. Let n be the number. Define a variable.

$$\underbrace{\frac{1}{2}}_{\text{Half}} \quad \underbrace{(7+n)}_{\substack{\text{the sum of 7} \\ \text{and a number}}} \quad \underbrace{=}_{\text{is the same as}} \quad \underbrace{n}_{\text{the number}} \quad \underbrace{-}_{\text{decreased by}} \quad \underbrace{2}_{\text{two.}}$$

$$\text{c. } \underbrace{\frac{m}{n}}_{\substack{\text{The quotient} \\ \text{of } m \text{ and } n}} \quad \underbrace{=}_{\text{equals}} \quad \underbrace{4}_{\text{four}} \quad \underbrace{+}_{\text{more than}} \quad \underbrace{\frac{1}{3}}_{\text{one-third}} \quad \underbrace{(m+n)}_{\text{the sum of } m \text{ and } n}.$$

$$\text{d. } \underbrace{x^3}_{\text{The cube of } x} \quad \underbrace{+}_{\text{plus}} \quad \underbrace{y^2}_{\text{the square of } y} \quad \underbrace{=}_{\text{is equal to}} \quad \underbrace{52}_{\text{fifty two.}}$$

Consecutive Numbers

Consecutive Integers $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$ $n, n+1, n+2$ are three consecutive integers if n is an integer.

Consecutive Even Integers $\dots, -6, -4, -2, 0, 2, 4, 6, \dots$ $n, n+2, n+4$ are three consecutive even integers if n is an even integer.

Consecutive Odd Integers $\dots, -5, -3, -1, 1, 3, 5, \dots$ $n, n+2, n+4$ are three consecutive odd integers if n is an odd integer.

Example 2 □ Write an equation to represent the given relationship between integers.

- The sum of four consecutive integers is -54 .
- The product of three consecutive odd integers is 693.

Solution □ a. Let n be the first integer. Define a variable.

$$\underbrace{n + (n+1) + (n+2) + (n+3)}_{\substack{\text{The sum of four consecutive integers} \\ \text{is } -54}} \quad \underbrace{=}_{\text{is}} \quad \underbrace{-54}_{-54}$$

b. Let n be the first odd integer. Define a variable.

$$\underbrace{n(n+2)(n+4)}_{\substack{\text{The product of three consecutive odd integers} \\ \text{is } 693}} \quad \underbrace{=}_{\text{is}} \quad \underbrace{693}_{693}$$

Exercise - Writing Equations

1

Eighteen more than the number n is 125.
What is the value of n ?

2

Twenty is 7 less than twice the number w .
What is the value of w ?

3

Nine less than twice x is three more than x .
What is the value of x ?

4

Eight less than four times the number c is twenty.
What is the value of c ?

5

The sum of four consecutive odd integers is 296.
What is the greatest of the four consecutive odd integers?

6

The sum of three fourths of the number a and 24 is negative 9. What is the value of a ?

- A) -44
- B) -20
- C) 20
- D) 44

7

A number g is decreased by 23 and then multiplied by $\frac{1}{2}$. The result is 8 more than twice the number g .

- A) -13
- B) $-\frac{34}{3}$
- C) $-\frac{29}{3}$
- D) -8

8

The quotient of p and q is twelve less than three times the sum of p and q .

Which of the following equations represents the statement above?

- A) $\frac{p}{q} = (3p + q) - 12$
- B) $\frac{p}{q} = 12 - (3p + q)$
- C) $\frac{p}{q} = 3(p + q) - 12$
- D) $\frac{p}{q} = 12 - 3(p + q)$

2-2. Solving Equations

To solve an equation means to find all values of the variable that make the equation a true statement. One way to do this is to isolate the variable that has a coefficient of 1 onto one side of the equation. You can do this using the rules of algebra called **properties of equality**.

Properties of Equality	Symbols	Examples
1. Addition Property	If $a = b$, then $a + c = b + c$.	If $x - 3 = 5$, then $(x - 3) + 3 = (5) + 3$.
2. Subtraction Property	If $a = b$, then $a - c = b - c$.	If $x + 2 = 6$, then $(x + 2) - 2 = (6) - 2$.
3. Multiplication Property	If $a = b$, then $ca = cb$.	If $\frac{1}{2}x = 3$, then $2 \cdot \frac{1}{2}x = 2 \cdot 3$.
4. Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.	If $3x = 15$, then $\frac{3x}{3} = \frac{15}{3}$.

Example 1 □ Solve each equation.

a. $a + (-11) = -25$

b. $-24 = 8y$

Solution □ a. $a + (-11) = -25$

$$a + (-11) + 11 = -25 + 11$$

$$a = -14$$

Add 11 to each side.

$$-11 + 11 = 0 \text{ and } -25 + 11 = -14$$

b. $-24 = 8y$

$$\frac{-24}{8} = \frac{8y}{8}$$

$$-3 = y$$

Divide each side by 8.

$$\frac{-24}{8} = 3 \text{ and } \frac{8y}{8} = y$$

Many equations require more than one operation to solve. Such equations are called **multi-step equations**. To solve multi-step equations, first simplify each side of the equation, if needed, and then use inverse operations to isolate the variable.

Example 2 □ Solve $\frac{4}{5}(x-5) - \frac{1}{5}(x-10) = 21$.

Solution □ $\frac{4}{5}(x-5) - \frac{1}{5}(x-10) = 19$

$$\frac{4}{5}x - 4 - \frac{1}{5}x + 2 = 19$$

Distributive Property

$$\frac{3}{5}x - 2 = 19$$

Simplify.

$$\frac{3}{5}x - 2 + 2 = 19 + 2$$

Add 2 to each side.

$$\frac{3}{5}x = 21$$

Simplify.

$$\frac{5}{3} \left(\frac{3}{5}x \right) = \frac{5}{3}(21)$$

Multiply each side by $\frac{5}{3}$.

$$x = 35$$

Simplify.

Exercise - Solving Equations

1

$$-11 + x = 9$$

Given the above equation, what is the value of $20 - (11 - x)$?

2

If $33 - a = a + 27 - 5a$, what is the value of $33 + 3a$?

3

If $\frac{1}{2}x + 3 = \frac{3}{4} - x$, what is the value of x ?

4

If $x - (3 - 2x) + (4 - 5x) = -7$, what is the value of x ?

5

If three quarters of a number decreased by twenty is equal to eighty two, what is that number?

6

Two and three fifths of a number equals -26 . What is the number?

- A) -15
- B) -10
- C) -5
- D) 10

7

There are one hundred forty-two students in a high school band. These students represent two ninth of the total students in the high school. How many students attend the school?

- A) 587
- B) 613
- C) 639
- D) 665

8

$$820c + 380r = 4,360$$

The above equation models the amount of calories in a snack of c cups of cashews and r cups of raisins. The amount of calories per cup of cashews is 820 and the amount of calories per cup of raisins is 380. According to the equation, how many cups of raisins are used, if 3 cups of cashews are used to make the snack?

- A) 3
- B) 4
- C) 5
- D) 6

2-3. Solving Equations with Variables on Both Sides

Some equations have variables on both sides. To solve such equations, first use the Addition or Subtraction Property of Equality to write an equivalent equation that has all of the variables on one side. Then use the Multiplication or Division Property of Equality to simplify the equation if necessary. When solving equations that contain grouping symbols, use the Distributive Property to remove the grouping symbols.

Example 1 □ Solve each equation.

a. $\frac{7}{3}x - 8 = 6 + \frac{1}{3}x$

b. $5 - 3(k + 1) = -k$

Solution □ a. $\frac{7}{3}x - 8 = 6 + \frac{1}{3}x$

$$\frac{7}{3}x - 8 - \frac{1}{3}x = 6 + \frac{1}{3}x - \frac{1}{3}x$$

$$2x - 8 = 6$$

$$2x - 8 + 8 = 6 + 8$$

$$2x = 14$$

$$\frac{2x}{2} = \frac{14}{2}$$

$$x = 7$$

Subtract $\frac{1}{3}x$ from each side.

Simplify.

Add 8 to each side.

Simplify.

Divide each side by 2.

Simplify.

b. $5 - 3(k + 1) = -k$

$$5 - 3k - 3 = -k$$

$$2 - 3k = -k$$

$$2 - 3k + k = -k + k$$

$$2 - 2k = 0$$

$$2 - 2k - 2 = 0 - 2$$

$$-2k = -2$$

$$\frac{-2k}{-2} = \frac{-2}{-2}$$

$$k = 1$$

Distributive Property

Simplify.

Add k to each side.

Simplify.

Subtract 2 from each side.

Simplify.

Divide each side by -2 .

Simplify.

Example 2 □ Four times the sum of three and a number equals nine less than the number.

a. Write an equation for the problem.

b. Then solve the equation.

Solution □ a. $4(3 + n) = n - 9$

b. $12 + 4n = n - 9$

$$12 + 4n - n = n - 9 - n$$

$$12 + 3n = -9$$

$$12 + 3n - 12 = -9 - 12$$

$$3n = -21$$

$$\frac{3n}{3} = \frac{-21}{3}$$

$$n = -7$$

Distributive Property

Subtract n from each side.

Simplify.

Subtract 12 from each side.

Simplify.

Divide each side by 3.

Simplify.

Exercise - Solving Equations with Variables on Both Sides

1

If $7n + 3 = 2n - 12$, what is the value of $-n + 3$?

2

If $7(h - 5) - 3h = \frac{3}{2}h$, what is the value of $\frac{1}{7}h$?

3

$$\frac{r}{3} + \frac{s}{11} = \frac{39}{33}$$

Given the above equation, if $s = 2$, what is the value of r ?

4

If $\frac{9 - 2k}{3} = k - 2$, what is the value of k ?

5

A \$48 shirts costs \$22 more than one half the cost of a pair of pants. How much does the pair of pants cost?

6

Twice a number n , increased by 11 is the same as six times the number decreased by 9. What is the value of n ?

7

One half of a number increased by 3 is five less than two thirds of the number.

8

Four times the greatest of three consecutive odd integers exceeds three times the least by 31. What is the greatest of the three consecutive odd integers?

2-4. Equation with No Solution and Identity

It is possible that an equation may have no solution. That is, there is no value of the variable that will result in a true equation. It is also possible that an equation may be true for all values of the variable. Such an equation is called an **identity**.

Example 1 □ Solve each equation. If the equation has no solution or it is an identity, write no solution or identity.

a. $2(1 - x) + 5x = 3(x + 1)$

b. $5w - 3(1 - w) = -2(3 - w)$

c. $\frac{1}{2}(8y - 6) = 5y - (y + 3)$

Solution □ a. $2(1 - x) + 5x = 3(x + 1)$

$$2 - 2x + 5x = 3x + 3$$

Distributive property

$$2 + 3x = 3x + 3$$

Simplify.

$$2 + 3x - 3x = 3x + 3 - 3x$$

Subtract $3x$ from each side.

$$2 = 3$$

Simplify.

The given equation is equivalent to the false statement $2 = 3$.

Therefore the equation has no solution.

b. $5w - 3(1 - w) = -2(3 - w)$

$$5w - 3 + 3w = -6 + 2w$$

Distributive property

$$8w - 3 = -6 + 2w$$

Simplify.

$$8w - 3 - 2w = -6 + 2w - 2w$$

Subtract $2w$ from each side.

$$6w - 3 = -6$$

Simplify.

$$6w - 3 + 3 = -6 + 3$$

Add 3 to each side.

$$6w = -3$$

Simplify.

$$\frac{6w}{6} = \frac{-3}{6}$$

Divide each side by 6.

$$w = -\frac{1}{2}$$

Simplify.

c. $\frac{1}{2}(8y - 6) = 5y - (y + 3)$

$$4y - 3 = 5y - y - 3$$

Distributive property

$$4y - 3 = 4y - 3$$

Simplify.

The given equation is equivalent to $4y - 3 = 4y - 3$, which is true for all values of y .

This equation is an identity.

Exercise - Equation with No Solution and Identity

1

If $\frac{1}{3}(9-6x) = 5-2x$, what is the value of x ?

- A) 3
- B) 4
- C) 5
- D) The equation has no solution.

2

If $5(x-2)-3x = 2(x-5)$, which of the following must be true?

- A) x is 3.
- B) x is 4.
- C) x is 5.
- D) The equation is true for all values of x .

3

$$\frac{1}{3}(15-6x) = 5-ax$$

If the linear equation above is an identity, what is the value of a ?

- A) 2
- B) 3
- C) 4
- D) 5

4

$$4x+13 = 7(x-2)+bx$$

If the linear equation above has no solution, which of the following could be the value of b ?

- A) -1
- B) -2
- C) -3
- D) -4

5

What is the value of n

$$\text{if } -\frac{7}{2}(2n-3)+4n = \frac{3}{2}(5+2n)?$$

6

What is the value of k

$$\text{if } \frac{13-7(k+1)}{3} = 3k-2?$$

7

What is the value of x

$$\text{if } -2[3-(x-4)]+5x = 2-x?$$

8

What is the value of m

$$\text{if } 0.4(5m-9) = -5m-4(0.3-m)?$$

2-5. Solving for a Specific Variable

A **formula** is an equation that states the relationship between two or more variables. Formulas and some equations contain more than one variable. It is often useful to solve formulas for one of the variables.

Example 1 □ Solve each equation or formula for the specified variable.

a. $3x - a = kx + b$, for x

b. $A = \frac{1}{2}(a + b)h$, for h

c. $C = \frac{5}{9}(F - 32)$, for F

Solution □ a. $3x - a = kx + b$

$$3x - a - kx = kx + b - kx$$

$$3x - a - kx = b$$

$$3x - a - kx + a = b + a$$

$$3x - kx = b + a$$

$$x(3 - k) = b + a$$

$$\frac{x(3 - k)}{3 - k} = \frac{b + a}{3 - k}$$

$$x = \frac{b + a}{3 - k}$$

Subtract kx from each side.

Simplify.

Add a to each side.

Simplify.

Distributive property

Divide each side by $3 - k$.

Simplify.

b. $A = \frac{1}{2}(a + b)h$

$$2 \cdot A = 2 \cdot \frac{1}{2}(a + b)h$$

$$2A = (a + b)h$$

$$\frac{2A}{a + b} = \frac{(a + b)h}{a + b}$$

$$h = \frac{2A}{a + b}$$

Multiply each side by 2.

Simplify.

Divide each side by $a + b$.

Simplify.

c. $C = \frac{5}{9}(F - 32)$

$$\frac{9}{5} \cdot C = \frac{9}{5} \cdot \frac{5}{9}(F - 32)$$

$$\frac{9}{5}C = F - 32$$

$$\frac{9}{5}C + 32 = F - 32 + 32$$

$$F = \frac{9}{5}C + 32$$

Multiply each side by $\frac{9}{5}$.

Simplify.

Add 32 to each side.

Simplify.

Exercise - Solving for a Specific Variable

1

If $2x + 3y = 18$, which of the following gives y in terms of x ?

A) $y = 6 + \frac{2}{3}x$

B) $y = 6 - \frac{2}{3}x$

C) $y = 6 + \frac{3}{2}x$

D) $y = 6 - \frac{3}{2}x$

2

If $P = 2l + 2w$, which of the following gives w in terms of P and l ?

A) $w = P - 2l$

B) $w = P - l$

C) $w = \frac{P}{2} - l$

D) $w = P - \frac{l}{2}$

3

If $c = \frac{a}{a+b}$, which of the following gives a in terms of b and c ?

A) $a = \frac{bc}{1-c}$

B) $a = \frac{bc}{1+c}$

C) $a = \frac{bc}{b-c}$

D) $a = \frac{bc}{b+c}$

4

If $\frac{ab-1}{3} = c$, which of the following gives b in terms of the other variables?

A) $b = \frac{3c+1}{a}$

B) $b = \frac{3c-1}{a}$

C) $b = \frac{3c}{a} + 1$

D) $b = \frac{3c}{a} - 1$

5

If $gh - f = g - h$, which of the following gives g in terms of the other variables?

A) $g = \frac{f+h}{h-1}$

B) $g = \frac{f-h}{h+1}$

C) $g = \frac{f+h}{h+1}$

D) $g = \frac{f-h}{h-1}$

6

If $n = a + (k-1)d$, which of the following gives k in terms of the other variables?

A) $k = \frac{n-a+1}{d}$

B) $k = \frac{n+a-1}{d}$

C) $k = \frac{n-a-d}{d}$

D) $k = \frac{n-a+d}{d}$

Chapter 2 Practice Test

1

If $\frac{5}{6}x = \frac{4}{5}$, what is the value of x ?

- A) $\frac{3}{2}$
- B) $\frac{2}{3}$
- C) $\frac{24}{25}$
- D) $\frac{25}{24}$

2

When one half of the number n is decreased by 4, the result is -6 . What is three times n added to 7?

- A) -7
- B) -5
- C) -3
- D) -1

3

If $4 - 7x$ is 5 less than 23, what is the value of $3x$?

- A) -12
- B) -9
- C) -6
- D) -3

4

$$P = F\left(\frac{1}{2}v^2 + 1\right)$$

The above equation gives pressure P , which is exerted by a fluid that is forced to stop moving. The pressure depends on the initial force, F , and the speed of the fluid, v . Which of the following expresses the square of the velocity in terms of the pressure and the force?

- A) $v^2 = 2(P - F) - 1$
- B) $v^2 = 2(P - F - 1)$
- C) $v^2 = 2\left(\frac{P}{F}\right) - 1$
- D) $v^2 = 2\left(\frac{P - F}{F}\right)$

5

One half of the number n increased by 10 is the same as four less than twice the number.

Which of the following equations represents the statement above?

- A) $\frac{1}{2}(n + 10) = 2(n - 4)$
- B) $\frac{1}{2}n + 10 = 2(n - 4)$
- C) $\frac{1}{2}n + 10 = 2n - 4$
- D) $\frac{1}{2}(n + 10) = 2n - 4$

6

If a is b less than one-half of c , what is b in terms of a and c ?

- A) $\frac{1}{2}c - a$
- B) $a - \frac{1}{2}c$
- C) $2a - c$
- D) $c - 2a$

7

If $x = 1 - y$ and $3x = 8 - 5y$, what is the value of x ?

- A) -2
- B) $-\frac{3}{2}$
- C) $-\frac{1}{2}$
- D) $\frac{5}{2}$

8

The quotient of a number and five equals nine less than one half of the number. What is the number?

- A) -20
- B) -10
- C) 20
- D) 30

9

If $\frac{a}{b} = 1$, what is the value of $a - b$?

10

When an object is thrown from the ground into the air with an initial upward speed of v_0 meters per second, the speed v , in meters per second, is given by the equation $v = v_0 - 9.8t$, where t is the time in seconds. The speed of an object becomes 0 when the object reaches its maximum height. If an object is thrown upward with an initial speed of 14 m/sec, how many seconds does it take to reach its maximum height? (Round your answer to the nearest hundredth of a second.)

11

When an object is dropped from a height of s feet above the ground, the height h of the object is given by the equation $h = -16t^2 + s$, where t is the time in seconds after the object has dropped. If an object is dropped from a height of 144 feet above the ground, how many seconds will it take to hit the ground?

Answer Key

Section 2-1

1. 107 2. 13.5 3. 12 4. 7 5. 77
6. A 7. A 8. C

Section 2-2

1. 29 2. 27 3. $\frac{5}{2}$ 4. 4 5. 136
6. B 7. C 8. C

Section 2-3

1. 6 2. 2 3. 3 4. 3 5. 52
6. 5 7. 48 8. 19

Section 2-4

1. D 2. D 3. A 4. C 5. $\frac{1}{2}$
6. $\frac{3}{4}$ 7. 2 8. 0.8

Section 2-5

1. B 2. C 3. A 4. A 5. D
6. D

Chapter 2 Practice Test

1. C 2. B 3. C 4. D 5. C
6. A 7. B 8. D 9. 0 10. 1.43
11. 3

Answers and Explanations

Section 2-1

1. 107

$$\underbrace{n+18}_{18 \text{ more than } n} = 125$$

$$n = 125 - 18 = 107$$

2. 13.5

$$20 = \underbrace{2w-7}_{7 \text{ less than twice } w}$$

$$20 + 7 = 2w - 7 + 7 \quad \text{Add 7 to each side.}$$

$$27 = 2w \quad \text{Simplify.}$$

$$\frac{27}{2} = \frac{2w}{2} \quad \text{Divide each side by 2.}$$

$$13.5 = w \quad \text{Simplify.}$$

3. 12

$$\underbrace{2x-9}_{9 \text{ less than twice } x} = \underbrace{x+3}_{3 \text{ more than } x}$$

$$2x - 9 - x = x + 3 - x \quad \text{Subtract } x \text{ from each side.}$$

$$x - 9 = 3 \quad \text{Simplify.}$$

$$x = 12$$

4. 7

$$\underbrace{4c-8}_{8 \text{ less than 4 times } c} = 20$$

$$4c - 8 + 8 = 20 + 8 \quad \text{Add 8 to each side.}$$

$$4c = 28 \quad \text{Simplify.}$$

$$c = 7$$

5. 77

Let n = the smallest of four consecutive odd integers. Then,

$$n + (n + 2) + (n + 4) + (n + 6) = 296$$

$$4n + 12 = 296$$

$$4n = 284$$

$$n = 71$$

The greatest of the four consecutive odd integers is $n + 6$. Therefore,

$$n + 6 = 71 + 6 = 77$$

6. A

$$\underbrace{\frac{3}{4}a + 24}_{\text{the sum of three fourths of } a \text{ and } 24} = -9$$

$$\frac{3}{4}a + 24 - 24 = -9 - 24 \quad \text{Subtract 24 from each side.}$$

$$\frac{3}{4}a = -33 \quad \text{Simplify.}$$

$$\frac{4}{3} \cdot \frac{3}{4}a = \frac{4}{3}(-33) \quad \text{Multiply each side by } \frac{4}{3}.$$

$$a = -44$$

7. A

$$\underbrace{(g-23)\frac{1}{2}}_{g \text{ is decrease by 23 and then multiplied by } \frac{1}{2}} = \underbrace{2g+8}_{8 \text{ more than twice } g}$$

$$(g-23)\frac{1}{2} \cdot 2 = (2g+8)2 \quad \text{Multiply each side by 2.}$$

$$g - 23 = 4g + 16 \quad \text{Simplify.}$$

$$g - 23 + 23 = 4g + 16 + 23 \quad \text{Add 23 to each side.}$$

$$g = 4g + 39 \quad \text{Simplify.}$$

$$g - 4g = 4g + 39 - 4g \quad \text{Subtract } 4g.$$

$$-3g = 39 \quad \text{Simplify.}$$

$$g = -13$$

8. C

$$\underbrace{\frac{p}{q}}_{\text{the quotient of } p \text{ and } q} = \underbrace{3(p+q)}_{\text{three times the sum of } p \text{ and } q} \underbrace{-12}_{\text{twelve less than}}$$

Section 2-2

1. 29

$$\text{Given } -11 + x = 9.$$

$$20 - (11 - x) = 20 - 11 + x = 20 + (-11 + x)$$

$$= 20 + 9 = 29$$

2. 27

$$33 - a = a + 27 - 5a$$

$$33 - a = 27 - 4a$$

Simplify.

$$33 - a + 4a = 27 - 4a + 4a \quad \text{Add } 4a \text{ to each side.}$$

$$33 + 3a = 27$$

Simplify.

3. $\frac{5}{2}$

$$\frac{1}{2}x - 3 = \frac{3}{4} - x$$

Multiply by 4 on both sides of the equation to simplify the given equation.

$$4\left(\frac{1}{2}x - 3\right) = 4\left(\frac{3}{4} - x\right)$$

$$2x - 12 = 3 - 4x$$

Distributive Property

$$2x - 12 + 4x = 3 - 4x + 4x$$

Add $4x$ to each side.

$$6x - 12 = 3$$

Simplify.

$$6x - 12 + 12 = 3 + 12$$

Add 12 to each side.

$$6x = 15$$

Simplify.

$$x = \frac{15}{6} = \frac{5}{2}$$

4. 4

$$x - (3 - 2x) + (4 - 5x) = -7$$

$$x - 3 + 2x + 4 - 5x = -7$$

Simplify.

$$-2x + 1 = -7$$

Simplify.

$$-2x + 1 - 1 = -7 - 1$$

Subtract 1.

$$-2x = -8$$

Simplify.

$$x = \frac{-8}{-2} = 4$$

5. 136

$$\underbrace{\frac{3}{4}x}_{\text{three quarters of a number}} \underbrace{-}_{\text{decreased by}} \underbrace{20}_{\text{twenty}} \underbrace{=}_{\text{equals}} \underbrace{82}_{\text{eighty two}}$$

$$\frac{3}{4}x - 20 + 20 = 82 + 20 \quad \text{Add 20 to each side.}$$

$$\frac{3}{4}x = 102 \quad \text{Simplify.}$$

$$x = \frac{4}{3} \cdot 102 = 136$$

6. B

$$\underbrace{2\frac{3}{5}x}_{\text{two and three fifth of a number}} \underbrace{=}_{\text{equals}} \underbrace{-26}_{\text{negative twenty six}}$$

$$\frac{13}{5}x = -26$$

$$2\frac{3}{5} = \frac{13}{5}$$

$$\frac{5}{13} \cdot \frac{13}{5}x = \frac{5}{13} \cdot -26$$

Multiply each side by $\frac{5}{13}$.

$$x = -10$$

7. C

Let x = the total students in the high school.

$$\text{Then } \frac{2}{9}x = 142.$$

$$x = \frac{9}{2} \cdot 142 = 639$$

8. C

$$820c + 380r = 4,360$$

Substitute 3 for c in the equation above since c represents the number of cups of cashews.

$$820(3) + 380r = 4,360$$

$$2,460 + 380r = 4,360 \Rightarrow 380r = 1,900$$

$$\Rightarrow r = 5$$

Section 2-3

1. 6

$$7n + 3 = 2n - 12 \Rightarrow 5n = -15 \Rightarrow n = -3$$

Therefore, $-n + 3 = -(-3) + 3 = 3 + 3 = 6$.

2. 2

$$7(h - 5) - 3h = \frac{3}{2}h \Rightarrow 7h - 35 - 3h = \frac{3}{2}h$$

$$\Rightarrow 4h - 35 = \frac{3}{2}h \Rightarrow 4h - \frac{3}{2}h = 35$$

$$\Rightarrow \frac{5}{2}h = 35 \Rightarrow h = 35 \cdot \frac{2}{5} = 14$$

$$\text{Therefore, } \frac{1}{7}h = \frac{1}{7}(14) = 2.$$

3. 3

$$\frac{r}{3} + \frac{s}{11} = \frac{39}{33} \text{ and } s = 2 \Rightarrow \frac{r}{3} + \frac{2}{11} = \frac{39}{33}$$

To simplify the equation, multiply both sides of the equation by 33, which is the LCD of 3 and 11.

$$33\left(\frac{r}{3} + \frac{2}{11}\right) = 33 \cdot \frac{39}{33} \Rightarrow 11r + 6 = 39 \\ \Rightarrow 11r = 33 \Rightarrow r = 3$$

4. 3

$$\frac{9-2k}{3} = k-2$$

To simplify the equation, multiply both sides of the equation by 3.

$$3\left(\frac{9-2k}{3}\right) = 3(k-2) \Rightarrow 9-2k = 3k-6 \\ \Rightarrow -2k-3k = -6-9 \Rightarrow -5k = -15 \\ \Rightarrow k = 3$$

5. 52

Let p = the cost of a pair of pants.

Since a \$48 shirt costs \$22 more than one half the cost of a pair of pants, you can set up the following equation.

$$48 = \frac{1}{2}p + 22 \\ \Rightarrow 26 = \frac{1}{2}p \Rightarrow 52 = p$$

6. 5

$$\underbrace{2n+11}_{\substack{\text{twice a number} \\ \text{increased by 11}}} = \underbrace{6n-9}_{\substack{\text{six times the number} \\ \text{decreased by 9}}}$$

$$2n+11 = 6n-9 \Rightarrow 20 = 4n \Rightarrow n = 5$$

7. 48

$$\underbrace{\frac{1}{2}n+3}_{\substack{\text{one half of a number} \\ \text{increased by three}}} = \underbrace{\frac{2}{3}n-5}_{\substack{\text{five less than two thirds} \\ \text{of the number}}}$$

To simplify the equation, multiply both sides of the equation by 6, which is the LCD of 2 and 3.

$$6\left(\frac{1}{2}n+3\right) = 6\left(\frac{2}{3}n-5\right) \\ 3n+18 = 4n-30 \\ \text{Solving for } n \text{ yields } n = 48.$$

8. 19

Let n be the first of the three consecutive odd integers, so n , $n+2$, and $n+4$ are the three

consecutive odd integers.

$$\underbrace{4(n+4)}_{\substack{\text{4 times the greatest of 3} \\ \text{consecutive odd integers}}} = \underbrace{3n}_{\substack{\text{3 times the least of 3} \\ \text{consecutive odd integers}}} + \underbrace{31}_{\substack{\text{exceeds} \\ \text{by 31}}}$$

$$4(n+4) = 3n+31 \\ 4n+16 = 3n+31 \Rightarrow n = 15$$

The greatest of the three consecutive odd integers is $n+4 = 15+4 = 19$.

Section 2-4

1. D

$$\frac{1}{3}(9-6x) = 5-2x \\ 3-2x = 5-2x \quad \text{Distributive Property} \\ 3-2x+2x = 5-2x+2x \quad \text{Add } 2x \text{ to each side.} \\ 3 = 5$$

The given equation is equivalent to the false statement $3 = 5$. Therefore the equation has no solution.

2. D

$$5(x-2)-3x = 2(x-10) \\ 5x-10-3x = 2x-20 \quad \text{Distributive Property} \\ 2x-10 = 2x-20 \quad \text{Simplify.}$$

The given equation is equivalent to $2x-10 = 2x-20$, which is true for all values of x .

3. A

$$\frac{1}{3}(15-6x) = 5-ax \\ 5-2x = 5-ax \quad \text{Distributive Property}$$

If the linear equation is an identity, the value of a is 2.

4. C

$$4x+13 = 7(x-2)+bx \\ 4x+13 = 7x-14+bx \\ 4x+13 = (7+b)x-14 \\ \text{If } 4 = 7+b, \text{ the linear equation has no solution.} \\ \text{Solving for } b \text{ yields } b = -3.$$

5. $\frac{1}{2}$

$$-\frac{7}{2}(2n-3)+4n = \frac{3}{2}(5+2n)$$

To simplify the equation, multiply both sides of the equation by 2.

$$\begin{aligned}
 2\left[-\frac{7}{2}(2n-3)+4n\right] &= 2\left[\frac{3}{2}(5+2n)\right] \\
 -7(2n-3)+8n &= 3(5+2n) && \text{Distributive Property} \\
 -14n+21+8n &= 15+6n && \text{Simplify.} \\
 -6n+21 &= 15+6n && \text{Simplify.} \\
 -6n+21+6n &= 15+6n+6n && \text{Add } 6n \text{ to each side.} \\
 21 &= 15+12n \\
 21-15 &= 15+12n-15 && \text{Subtract 15.} \\
 6 &= 12n \text{ or } 12n=6 && \text{Simplify.} \\
 n &= \frac{6}{12} = \frac{1}{2}
 \end{aligned}$$

6. $\frac{3}{4}$

$$\frac{13-7(k+1)}{3} = 3k-2$$

To simplify the equation, multiply both sides of the equation by 3.

$$\begin{aligned}
 3\left[\frac{13-7(k+1)}{3}\right] &= 3[3k-2] \\
 13-7(k+1) &= 9k-6 && \text{Simplify.} \\
 13-7k-7 &= 9k-6 && \text{Distributive Property} \\
 6-7k &= 9k-6 && \text{Simplify.} \\
 6-7k-6 &= 9k-6-6 && \text{Subtract 6.} \\
 -7k &= 9k-12 && \text{Simplify.} \\
 -7k-9k &= 9k-12-9k && \text{Subtract } 9k. \\
 -16k &= -12 \\
 k &= \frac{-12}{-16} = \frac{3}{4}
 \end{aligned}$$

7. 2

$$\begin{aligned}
 -2[3-(x-4)]+5x &= 2-x \\
 -2[3-x+4]+5x &= 2-x \\
 -2[7-x]+5x &= 2-x \\
 -14+2x+5x &= 2-x \\
 -14+7x &= 2-x \\
 8x &= 16 \\
 x &= 2
 \end{aligned}$$

8. 0.8

$$\begin{aligned}
 0.4(5m-9) &= -5m-4(0.3-m) \\
 2m-3.6 &= -5m-1.2+4m \\
 2m-3.6 &= -m-1.2 \\
 3m &= 2.4 \\
 m &= 0.8
 \end{aligned}$$

Section 2-5

1. B

$$\begin{aligned}
 2x+3y &= 18 \\
 2x+3y-2x &= 18-2x && \text{Subtract } 2x \text{ from each side.} \\
 3y &= 18-2x && \text{Simplify.} \\
 \frac{3y}{3} &= \frac{18}{3} - \frac{2x}{3} && \text{Divide each side by 3.} \\
 y &= 6 - \frac{2}{3}x && \text{Simplify.}
 \end{aligned}$$

2. C

$$\begin{aligned}
 P &= 2l+2w \\
 P-2l &= 2l+2w-2l && \text{Subtract } 2l \text{ from each side.} \\
 P-2l &= 2w && \text{Simplify.} \\
 \frac{P}{2} - \frac{2l}{2} &= \frac{2w}{2} && \text{Divide each side by 2.} \\
 \frac{P}{2} - l &= w && \text{Simplify.}
 \end{aligned}$$

3. A

$$\begin{aligned}
 c &= \frac{a}{a+b} \\
 (a+b)c &= (a+b)\frac{a}{a+b} && \text{Multiply each side by } a+b. \\
 ac+bc &= a && \text{Simplify.} \\
 ac+bc-ac &= a-ac && \text{Subtract } ac \text{ from each side.} \\
 bc &= a-ac && \text{Simplify.} \\
 bc &= a(1-c) && \text{Factor.} \\
 \frac{bc}{1-c} &= a && \text{Divide each side by } 1-c.
 \end{aligned}$$

4. A

$$\begin{aligned}
 \frac{ab-1}{3} &= c \\
 3\left[\frac{ab-1}{3}\right] &= 3c && \text{Multiply each side by 3.} \\
 ab-1 &= 3c && \text{Simplify.} \\
 ab-1+1 &= 3c+1 && \text{Add 1 to each side.} \\
 ab &= 3c+1 && \text{Simplify.} \\
 \frac{ab}{a} &= \frac{3c+1}{a} && \text{Divide each side by } a. \\
 b &= \frac{3c+1}{a} && \text{Simplify.}
 \end{aligned}$$

5. D

$$gh - f = g - h$$

$$gh - f + f = g - h + f \quad \text{Add } f \text{ to each side.}$$

$$gh = g - h + f \quad \text{Simplify.}$$

$$gh - g = g - h + f - g \quad \text{Subtract } g \text{ from each side.}$$

$$gh - g = f - h \quad \text{Simplify.}$$

$$g(h-1) = f - h \quad \text{Factor.}$$

$$g = \frac{f-h}{h-1} \quad \text{Divide each side by } h-1.$$

6. D

$$n = a + (k-1)d$$

$$n = a + kd - d \quad \text{Distributive Property}$$

$$n - a + d = a + kd - d - a + d \quad \text{Add } -a + d \text{ to each side.}$$

$$n - a + d = kd \quad \text{Simplify.}$$

$$\frac{n-a+d}{d} = k \quad \text{Divide each side by } d.$$

Chapter 2 Practice Test

1. C

$$\frac{5}{6}x = \frac{4}{5}$$

$$\frac{6}{5} \cdot \frac{5}{6}x = \frac{6}{5} \cdot \frac{4}{5} \quad \text{Multiply each side by } \frac{6}{5}.$$

$$x = \frac{24}{25}$$

2. B

$$\underbrace{\frac{1}{2}n}_{\frac{1}{2} \text{ of a number } n} \quad \underbrace{-4}_{\text{decreased by 4}} = \underbrace{-6}_{\text{negative 6}}$$

$\frac{1}{2}$ of a number n

$$\frac{1}{2}n - 4 + 4 = -6 + 4 \quad \text{Add 4 to each side.}$$

$$\frac{1}{2}n = -2 \quad \text{Simplify.}$$

$$2 \cdot \frac{1}{2}n = 2 \cdot -2 \quad \text{Multiply each side by 2.}$$

$$n = -4 \quad \text{Simplify.}$$

Three times n added to 7 is $3n + 7$.

$$3n + 7$$

$$= 3(-4) + 7 \quad \text{Substitute } -4 \text{ for } n.$$

$$= -5$$

3. C

$$\underbrace{4 - 7x}_{4 - 7x \text{ is 5 less than 23}} = 23 - 5$$

$$4 - 7x = 18 \Rightarrow -7x = 14 \Rightarrow x = -2$$

$$3x = 3(-2) = -6$$

4. D

$$P = F\left(\frac{1}{2}v^2 + 1\right)$$

$$\frac{P}{F} = \frac{F}{F}\left(\frac{1}{2}v^2 + 1\right) \quad \text{Divide each side by } F.$$

$$\frac{P}{F} = \frac{1}{2}v^2 + 1 \quad \text{Simplify.}$$

$$\frac{P}{F} - 1 = \frac{1}{2}v^2 + 1 - 1 \quad \text{Subtract 1 from each side.}$$

$$\frac{P}{F} - 1 = \frac{1}{2}v^2 \quad \text{Simplify.}$$

$$2\left(\frac{P}{F} - 1\right) = 2 \cdot \frac{1}{2}v^2 \quad \text{Multiply each side by 2.}$$

$$2\left(\frac{P}{F} - 1\right) = v^2 \quad \text{Simplify.}$$

$$2\left(\frac{P}{F} - \frac{F}{F}\right) = v^2 \quad \frac{F}{F} = 1$$

$$2\left(\frac{P-F}{F}\right) = v^2 \quad \text{The common denominator is } F.$$

Combine the numerators.

5. C

$$\underbrace{\frac{1}{2}n + 10}_{\substack{\frac{1}{2} \text{ of the number } n \\ \text{increased by 10}}} = \underbrace{2n - 4}_{\substack{\text{four less than twice} \\ \text{the number}}}$$

6. A

$$\underbrace{a = \frac{1}{2}c - b}_{a \text{ is } b \text{ less than } \frac{1}{2} \text{ of } c}$$

$$a - \frac{1}{2}c = \frac{1}{2}c - b - \frac{1}{2}c \quad \text{Add } -\frac{1}{2}c \text{ to each side.}$$

$$a - \frac{1}{2}c = -b \quad \text{Simplify.}$$

$$(-1)\left[a - \frac{1}{2}c\right] = (-1)(-b) \quad \text{Multiply each side by } -1.$$

$$-a + \frac{1}{2}c = b \quad \text{or} \quad \frac{1}{2}c - a = b$$

7. B

$$\begin{array}{ll} x = 1 - y & \text{First equation} \\ 3x = 8 - 5y & \text{Second equation} \end{array}$$

Solving the first equation for y yields $y = 1 - x$.
Substitute $1 - x$ for y in the second equation.

$$\begin{array}{ll} 3x = 8 - 5(1 - x) & \text{Substitution} \\ 3x = 8 - 5 + 5x & \text{Distributive property} \\ 3x = 3 + 5x & \text{Simplify.} \\ 3x - 5x = 3 + 5x - 5x & \text{Subtract } 5x \text{ from each side.} \\ -2x = 3 & \text{Simplify.} \\ \frac{-2x}{-2} = \frac{3}{-2} & \text{Divide each side by } -2. \\ x = -\frac{3}{2} & \text{Simplify.} \end{array}$$

8. D

$$\underbrace{\frac{x}{5}}_{\substack{\text{the quotient of} \\ \text{of a number and 5}}} = \underbrace{\frac{1}{2}x - 9}_{\substack{\text{nine less than one} \\ \text{half of the number}}}$$

$$\begin{array}{ll} 10\left(\frac{x}{5}\right) = 10\left(\frac{1}{2}x - 9\right) & \text{Multiply each side by 10.} \\ 2x = 5x - 90 & \text{Distributive Property} \\ 2x - 5x = 5x - 90 - 5x & \text{Subtract } 5x \text{ from each side.} \\ -3x = -90 & \text{Simplify.} \\ \frac{-3x}{-3} = \frac{-90}{-3} & \text{Divide each side by } -3. \\ x = 30 & \text{Simplify.} \end{array}$$

9. 0

$$\begin{array}{ll} \frac{a}{b} = 1 & \\ b\left(\frac{a}{b}\right) = b(1) & \text{Multiply each side by } b. \\ a = b & \text{Simplify.} \\ a - b = b - b & \text{Subtract } b \text{ from each side.} \\ a - b = 0 & \text{Simplify.} \end{array}$$

10. 1.43

As the object moves upward, its speed decreases continuously and becomes 0 as it reaches its maximum height.

$v = v_0 - 9.8t$ is the given equation. Substituting 14 for v_0 and 0 for v gives $0 = 14 - 9.8t$.

Solving the equation for t gives $t = \frac{14}{9.8} = 1.428$

seconds, which is 1.43 to the nearest hundredth of a second.

11. 3

When the object hits the ground, the height is 0. Substitute 0 for h and 144 for s in the equation

$0 = -16t^2 + 144$. Solving the equation for t^2 gives $t^2 = \frac{144}{16} = 9$.

Therefore, $t = \sqrt{9} = 3$.