Sample Question Paper - 27
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed : 2 hours ***Maximum Marks : 40***

General Instructions :

1. *The question paper consists of 14 questions divided into 3 sections A, B, C.*
2. *Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.*
3. *Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.*
4. *Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.*

SECTION - A

1. Following table shows the weight of 15 students :

Weight (in kg)	67	70	72	73	75
Number of students	5	2	3	4	1

Find the mean weight of the students.

2. Using factorisation method, find the roots of the quadratic equation $2ax^2 - (2a - b^2)x - b^2 = 0$.

3. A solid right circular cone is cut into two parts at the middle of its height by a plane parallel to its base. Find the ratio of the volume of the smaller cone to the whole cone.

OR

A solid is hemispherical at the bottom and conical (of same radius) above it. If the surface areas of the two parts are equal, then find the ratio of the radius and the slant height of the conical part.

4. The ninth term of an A.P. is –32 and the sum of its eleventh and thirteenth terms is –94. Find the common difference of the A.P.

5. Draw a circle of radius 5 cm. From a point *P*, 8 cm away from its centre, construct a pair of tangents to the circle.

6. Using quadratic formula solve the following quadratic equation : $13x^2 + 9(x + 1) - (2x + 3)(x + 2) = 6$

OR

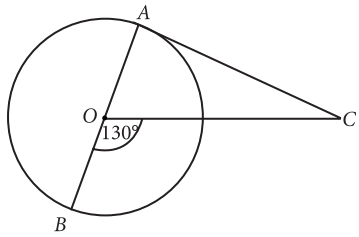
If 2 and 1 are the two roots of the quadratic equation $ax^2 + bx + 2 = 0$, then find a and b.

SECTION - B

7. A survey regarding the heights in (cm) of 51 girls of class X of a school was conducted and the following data was obtained. Find the median height.

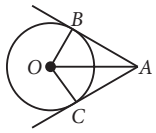
Height(in cm)	Less than 140	Less than 145	Less than 150	Less than 155	Less than 160	Less than 165
Number of Girls	4	11	29	40	46	51

8. In the given figure, *AOB* is a diameter of a circle with centre *O* and *AC* is a tangent to the circle at *A*. $\angle BOC = 130°$, then find $\angle ACO$.



OR

In the given figure, *AB* and *AC* are tangents to a circle with centre *O* and radius 8 cm. If *OA* = 17 cm, then find the length of *AC*.



9. A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the distance of the cliff from the ship. [Take $\sqrt{3} = 1.732$]

10. The table shows the daily expenditure on grocery of 25 households in a locality. Find the modal daily expenditure on grocery by a suitable method.

Daily Expenditure (in ₹)	100-150	150-200	200-250	250-300	300-350
No of households	4	5	12	2	2

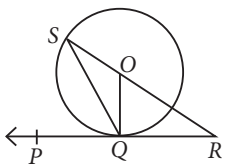
SECTION - C

11. The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left. $\left[\text{Use } \pi = \frac{22}{7}\right]$

OR

A sphere of diameter 6 cm is dropped into a cylindrical vessel, partly filled with water, whose diameter is 12 cm. If the sphere is completely submerged in water, by how much will the surface of water be raised in the cylindrical vessel? Also, find the curved surface area of cylinder if total height of cylinder is 14 cm.

12. In the given figure, *PQR* is a tangent to the circle with centre *O*. *OQ* is the radius of the circle at the point of contact. *R* and *O* are joined and produced to the point *S* on the circle. If $\angle QRO = 28°$, $\angle QOR = x$ and $\angle OQS = y$, then find the values of *x* and *y*.



Case Study - 1

13. Suppose a straight vertical tree is broken at some point due to cyclone and the broken part is inclined at a certain distant from the foot of the tree.

(i) If the top of upper part of broken tree touches ground at a distance of 6 m (from the foot of the tree) and makes an angle of inclination 60°, then find the height of remaining part of the tree.

(ii) If the top of broken part of a tree touches the ground at a point whose distance from foot of the tree is equal to height of remaining part, then find its angle of inclination.

Case Study - 2

14. Do you know, we can find A.P. in many situations in our day-to-day life. One such example is a tissue paper roll, in which the first term is the diameter of the core of the roll and twice the thickness of the paper is the common difference. If the sum of first n rolls of tissue on a roll is $S_n = 0.1\ n^2 + 7.9n$, then answer the following questions.

(i) Find the radius of the core.

(ii) Find the thickness of each tissue sheet.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

1. The frequency distribution table from the given data can be drawn as :

Weight (in kg) (x_i)	Frequency (f_i)	f_ix_i
67	5	335
70	2	140
72	3	216
73	4	292
75	1	75
	$\sum f_i = 15$	$\sum f_ix_i = 1058$

$\therefore$ Mean, $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i} = \dfrac{1058}{15} = 70.53 \text{ kg}$

2. We have, $2ax^2 - (2a - b^2)x - b^2 = 0$

$\Rightarrow \quad 2ax^2 - 2ax + b^2x - b^2 = 0$

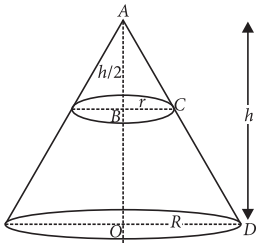
$\Rightarrow \quad 2ax(x - 1) + b^2(x - 1) = 0 \Rightarrow (x - 1)(2ax + b^2) = 0$

$\Rightarrow \quad x - 1 = 0 \text{ or } 2ax + b^2 = 0$

$\Rightarrow \quad x = 1 \text{ or } x = -\dfrac{b^2}{2a}$

Hence, 1 and $-\dfrac{b^2}{2a}$ are the two roots of the given equation.

3. Let radius of whole cone be R and height be h.
And radius of smaller cone be r and height be $h/2$.



Here, $\Delta ABC \sim \Delta AOD$

$\Rightarrow \quad \dfrac{h/2}{h} = \dfrac{r}{R} \Rightarrow r = \dfrac{R}{2}$ ...(i)

$\therefore$ Required ratio $= \dfrac{\text{Volume of smaller cone}}{\text{Volume of whole cone}}$

$= \dfrac{\frac{1}{3}\pi r^2 (h/2)}{\frac{1}{3}\pi R^2 h} = \dfrac{\frac{R^2}{4}}{2R^2} = \dfrac{1}{8}$ *i.e.*, 1 : 8 (using (i))

OR

Let r be the radius of hemisphere and conical part. Also, let l be the slant height of conical part.

Given that, surface area of hemisphere = surface area of conical part

$\Rightarrow \quad 2\pi r^2 = \pi rl \Rightarrow 2r = l$

$\Rightarrow \quad \dfrac{r}{l} = \dfrac{1}{2} \Rightarrow r : l = 1 : 2$

4. Let the first term be a and d be the common difference of the A.P.

Given, $a_9 = -32 \Rightarrow a + 8d = -32$ ...(i)

Also, $a_{11} + a_{13} = -94$

$\Rightarrow \quad a + 10d + a + 12d = -94$

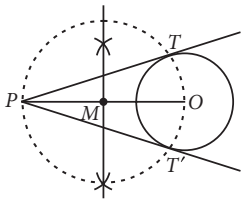
$\Rightarrow \quad 2a + 22d = -94$

$\Rightarrow \quad a + 11d = -47$ ...(ii)

Subtracting (ii) from (i), we get

$-3d = 15 \Rightarrow d = -5$

5.



6. We have, $13x^2 + 9(x + 1) - (2x + 3)(x + 2) = 6$

$\Rightarrow \quad 13x^2 + 9x + 9 - (2x^2 + 4x + 3x + 6) = 6$

$\Rightarrow \quad 11x^2 + 2x - 3 = 0$

Here, $a = 11$, $b = 2$ and $c = -3$.

$\therefore \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-2 \pm \sqrt{(2)^2 - 4(11)(-3)}}{2 \times 11}$

$= \dfrac{-2 \pm \sqrt{136}}{22} = \dfrac{-1 \pm \sqrt{34}}{11}$

Thus, roots of the given equation are

$\dfrac{-1 + \sqrt{34}}{11}$ and $\dfrac{-1 - \sqrt{34}}{11}$.

OR

Given, 2 and 1 are the two roots of the quadratic equation $ax^2 + bx + 2 = 0$

$\therefore \quad a(2)^2 + b(2) + 2 = 0$ and $a(1)^2 + b(1) + 2 = 0$

$\Rightarrow \quad 4a + 2b + 2 = 0$ and $a + b + 2 = 0$

$\Rightarrow \quad 2a + b + 1 = 0$ ...(i)

and $a + b + 2 = 0$ ...(ii)

Subtracting (ii) from (i), we get $a - 1 = 0 \Rightarrow a = 1$

Putting $a = 1$ in (i), we get $2 + b + 1 = 0 \Rightarrow b = -3$

Hence, $a = 1$ and $b = -3$.

7.

Height (in cm)	Number of girls (f)	*c.f.*
Below 140	4	4
140-145	11 – 4 = 7	11
145-150	29 – 11 = 18	29
150-155	40 – 29 = 11	40
155-160	46 – 40 = 6	46
160-165	51 – 46 = 5	51
	$N = \Sigma f_i = 51$	

Here, $N = 51 \Rightarrow N/2 = 51/2 = 25.5$

As 29 is just greater than 25.5, therefore median class is 145-150.

Now, median $= l + \dfrac{\left(\frac{N}{2} - c.f.\right)}{f} \times h$

$\Rightarrow$ Median $= 145 + \dfrac{(25.5 - 11)}{18} \times 5 = 149.03$

Thus, median height = 149.03 cm

8. Given, $\angle BOC = 130°$

Since, AC is a tangent to the circle at A.

$\therefore$ $\angle OAC = 90°$ [$\because$ Radius is perpendicular to the tangent at the point of contact]

In ΔAOC, $\angle ACO + \angle OAC = \angle BOC$ [Exterior angle property]

$\Rightarrow$ $\angle ACO = 130° - 90° = 40°$

OR

We have, AB and AC are the tangents to a circle with centre O.

$OC = 8$ cm, $OA = 17$ cm

Since, tangent is perpendicular to radius through point of contact.

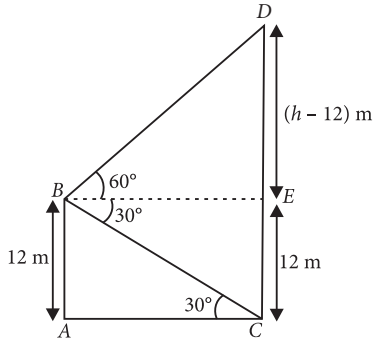
$\therefore$ $\angle OCA = 90°$

In right ΔAOC, $(OA)^2 = (AC)^2 + (OC)^2$

$\Rightarrow$ $(AC)^2 = (17)^2 - (8)^2 = 289 - 64 = 225$

$\Rightarrow$ $AC = 15$ cm

9.



Let AB represents the deck of the ship above water level, DC represents the cliff and AC represents the water level. Point B represents the position of the man.

In right ΔCAB,

$\dfrac{AB}{AC} = \tan 30° \Rightarrow \dfrac{12}{AC} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow$ $AC = 12\sqrt{3}$ m

$\therefore$ Distance of the cliff from the ship $= 12\sqrt{3}$ m

$= (12 \times 1.732)$ m $= 20.784$ m

10. As the class 200-250 has the maximum frequency, so it is modal class.

Now, we have $l = 200$, $h = 50$, $f_1 = 12$, $f_0 = 5$, $f_2 = 2$

$\therefore$ Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$= 200 + \dfrac{12 - 5}{24 - 5 - 2} \times 50 = 200 + 20.59 = 220.59$

Hence, modal daily expenditure on grocery = ₹220.59

11. Side of cube = Diameter of sphere = 7 cm

$\therefore$ Volume of cube $= a^3 = (7)^3$

Volume of sphere $= \dfrac{4}{3}\pi r^3 = \dfrac{4}{3} \times \dfrac{22}{7} \times \left(\dfrac{7}{2}\right)^3 = \dfrac{11}{21}(7)^3$

Hence, volume of the wood left

= Volume of cube – Volume of sphere

$= (7)^3\left[1 - \dfrac{11}{21}\right] = (7)^3 \times \dfrac{10}{21} = 163.33 \text{ cm}^3$

OR

Radius (r) of sphere = 6/2 cm = 3 cm

Radius (R) of cylindrical vessel = 12/2 = 6 cm

Let rise in water level of vessel be h cm.

Then, according to question, we have

Volume of sphere = Volume of water raised in cylindrical vessel

$\Rightarrow$ $\dfrac{4}{3}\pi r^3 = \pi R^2 h \Rightarrow \dfrac{4}{3} \times (3)^3 = (6)^2 \times h$

$\Rightarrow$ $h = \dfrac{4}{3} \times \dfrac{(3)^3}{(6)^2} = 1$

$\therefore$ Rise in water level is 1 cm.

Curved surface area of cylinder $= 2 \times \dfrac{22}{7} \times 6 \times 14$

$= 528 \text{ cm}^2$

12. In ΔOQR, $\angle OQR = 90°$

[$\because$ Radius is $\perp^r$ to the tangent at the point of contact]

Now, $\angle QOR + \angle OQR + \angle QRO = 180°$ (By angle sum property of a triangle)

$\Rightarrow$ $x + 90° + 28° = 180° \Rightarrow x = 62°$

Now, $\angle SOQ + \angle QOR = 180°$ (Linear pair)

$\Rightarrow$ $\angle SOQ + 62° = 180° \Rightarrow \angle SOQ = 118°$

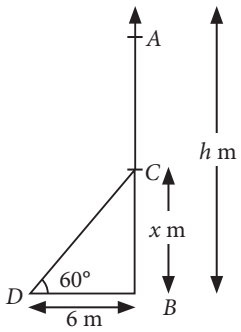
In ΔSOQ, $OS = OQ$ (Radii of same circle)

$\Rightarrow$ $\angle OSQ = \angle OQS = y$ (given)

Also, $\angle OSQ + \angle OQS + \angle SOQ = 180°$

$\Rightarrow \quad y + y + 118° = 180° \Rightarrow 2y = 62° \Rightarrow y = 31°$

13. (i) Let AB be the tree of height h m and let it broken at height of x m, as shown in figure.



Clearly $CD = AC = (h - x)$ m

Now, In ΔCBD, we have

$$\tan 60° = \frac{x}{6} \quad \Rightarrow \quad x = 6\sqrt{3}\,\text{m}$$

Thus, the height of remaining part of the tree is $6\sqrt{3}$ m.

(ii) In this case = $BD = BC = x$ m

$\therefore$ If θ be the angle of inclination, then

$$\tan\theta = \frac{BC}{BD} = 1$$

$\Rightarrow \quad \tan\theta = \tan 45° \Rightarrow \theta = 45°$

14. Here $S_n = 0.1n^2 + 7.9n$

(i) $S_1 = t_1 = a = 0.1(1)^2 + 7.9(1) = 8$ cm

= Diameter of core

So, radius of the core = 4 cm

(ii) $S_2 = (0.1)\,(2)^2 + (7.9)\,(2)$

$= 0.4 + 15.8 = 16.2$

$\therefore \quad t_2 = S_2 - S_1 = 16.2 - 8 = 8.2$

Now, $d = t_2 - t_1 = 8.2 - 8 = 0.2$ cm

So, thickness of tissue = $0.2 \div 2 = 0.1$ cm = 1 mm